```
(*Self-adjoint Wheeler-deWitt equation for the second ordering:*)
(*y''[Q]+((8πρQ)/3 -k)y[Q]==0*)
(*For negative Q, ρ=0, for non-zero k this becomes:*)
```

In[∘]:=
```
DSolve[y''[Q] == k y[Q], y[Q], Q]
```

Out[∘]=

$$\left\{\left\{y[Q] \to e^{\sqrt{k}\,Q}\, c_1 + e^{-\sqrt{k}\,Q}\, c_2\right\}\right\}$$

```
(*So the solution is:*)
```

In[∘]:=
```
wdwsolutionf[Q_, k_, α_, β_] :=
 e^(√k Q) α + e^(-√k Q) β
(*The asymptotic limit for Q→ - Infinity in first order Taylor series:*)
Normal[Series[wdwsolutionf[Q, k, α, β], {Q, -Infinity, 0}]]
```

Out[∘]=

$$e^{\sqrt{k}\,Q}\,\alpha + e^{-\sqrt{k}\,Q}\,\beta$$

In[∘]:=
```
(*For negative k, function is oscillatory therefore will not converge to 0,
thus k has to be set to 1:*)
AsymptoticLimitForNegativeQf[Q_, α_, β_] :=
 e^(√k Q) α + e^(-√k Q) β /. k → 1
AsymptoticLimitForNegativeQf[Q, α, β]
```

Out[∘]=

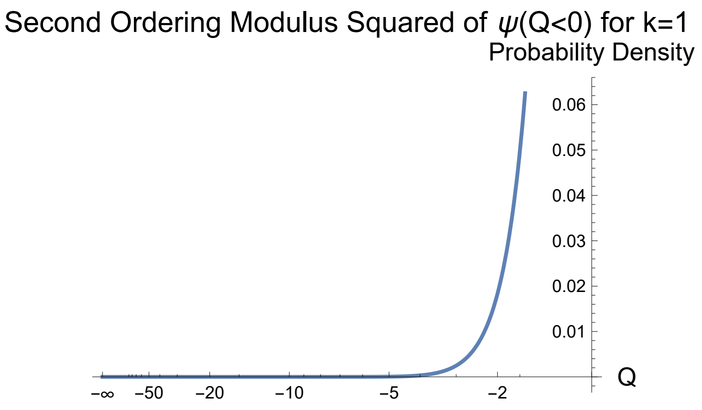
$$e^{Q}\,\alpha + e^{-Q}\,\beta$$

```
(*Now, if α=0, the limit goes to infinity, while if β=0, the limit goes to zero,
if both are non-zero limit is indeterminent, so need to set β=0*)
```

In[∘]:=
```
Plot[Abs[wdwsolutionf[Q, 1, 1, 0]]^2, {Q, -Infinity, 0},
 AxesLabel → {Style["Q", Black, 13], Style["Probability Density", Black, 13]},
 PlotLabel → Style["Second Ordering Modulus Squared of ψ(Q<0) for k=1", Black, 15],
 AxesStyle → Directive[Black, Thickness[0.0009]]]
```

General: Exp[-244753.] is too small to represent as a normalized machine number; precision may be lost.

Out[∘]=



In[∘]:=
```
(*Next, the function will be analysed for its limit towards 0:*)
Limit[wdwsolutionf[Q, 1, α, 0], Q → 0, Direction → +1]
```

Out[∘]=

α

```
(*The limit goes to α as the function goes to 0*)
(*Now the same thing will be done for Q>0:*)
```

In[•]:=
```
(*Self-adjoint Wheeler-deWitt equation for the second ordering:*)
(*y''[Q]+((8πρQ)/3 -k)y[Q]==0*)
(*For positive Q, ρ=A/Q^2 this becomes:*)
DSolve[y''[Q] + ((8 π × A) / (3 Q)) y[Q] == k y[Q], y[Q], Q]
```

Out[•]=

$$\left\{\left\{y[Q] \to \frac{1}{3} e^{\left(\sqrt{-k}-\sqrt{k}\right) Q-\sqrt{-k}\, Q}\, Q\, c_2\, \text{Hypergeometric1F1}\left[1+\frac{-108\sqrt{-k}-18\left(-6\sqrt{-k}+8 A \pi\right)}{108\sqrt{k}}, 2, 2\sqrt{k}\, Q\right] + \frac{1}{3} e^{\left(\sqrt{-k}-\sqrt{k}\right) Q-\sqrt{-k}\, Q}\, Q\, c_1\, \text{HypergeometricU}\left[1+\frac{-108\sqrt{-k}-18\left(-6\sqrt{-k}+8 A \pi\right)}{108\sqrt{k}}, 2, 2\sqrt{k}\, Q\right]\right\}\right\}$$

In[•]:=
```
(*So the solution becomes:*)
wdwsolutione[Q_, k_, α_, β_] :=
```

$$\frac{1}{3} e^{\left(\sqrt{-k}-\sqrt{k}\right) Q-\sqrt{-k}\, Q}\, Q\, \alpha\, \text{Hypergeometric1F1}\left[1+\frac{-108\sqrt{-k}-18\left(-6\sqrt{-k}+8 A \pi\right)}{108\sqrt{k}}, 2, 2\sqrt{k}\, Q\right] + \frac{1}{3} e^{\left(\sqrt{-k}-\sqrt{k}\right) Q-\sqrt{-k}\, Q}\, Q\, \beta\, \text{HypergeometricU}\left[1+\frac{-108\sqrt{-k}-18\left(-6\sqrt{-k}+8 A \pi\right)}{108\sqrt{k}}, 2, 2\sqrt{k}\, Q\right]$$

In[•]:=
```
(*The asymptotic limit for Q → Infinity in first order Taylor series:*)
Normal[Series[wdwsolutione[Q, k, α, β], {Q, Infinity, 0}]]
```

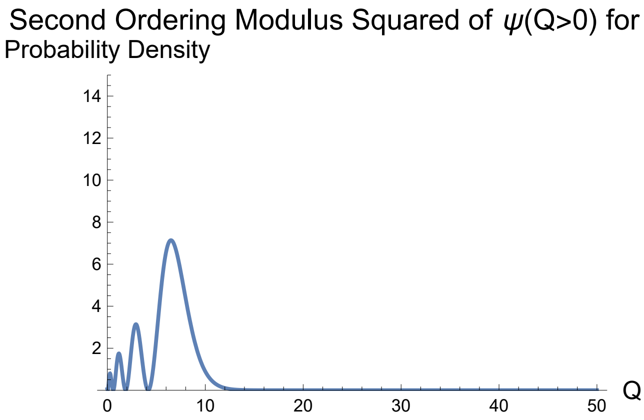
Out[•]=

$$\frac{2^{-1+\frac{4 A \pi}{3\sqrt{k}}}\, e^{-\sqrt{k}\, Q}\left(\sqrt{k}\, Q\right)^{\frac{4 A \pi}{3\sqrt{k}}} \beta}{3\sqrt{k}} - \frac{2^{-1+\frac{4 A \pi}{3\sqrt{k}}}\, e^{-\sqrt{k}\, Q}\left(-\sqrt{k}\, Q\right)^{\frac{4 A \pi}{3\sqrt{k}}} \alpha}{3\sqrt{k}\, \text{Gamma}\left[1-\frac{-108\sqrt{-k}-18\left(-6\sqrt{-k}+8 A \pi\right)}{108\sqrt{k}}\right]} + \frac{2^{-1-\frac{4 A \pi}{3\sqrt{k}}}\, e^{\sqrt{k}\, Q}\left(\sqrt{k}\, Q\right)^{-\frac{4 A \pi}{3\sqrt{k}}} \alpha}{3\sqrt{k}\, \text{Gamma}\left[1+\frac{-108\sqrt{-k}-18\left(-6\sqrt{-k}+8 A \pi\right)}{108\sqrt{k}}\right]}$$

In[∘]:=
```
(*As before, negative k values will results in oscillatory functions,
so k will be set to one:*)
AsymLimPosQ2nd[Q_, α_, β_] :=
```
$$\frac{2^{-1+\frac{4A\pi}{3\sqrt{k}}}e^{-\sqrt{k}Q}\left(\sqrt{k}Q\right)^{\frac{4A\pi}{3\sqrt{k}}}\beta}{3\sqrt{k}}-\frac{2^{-1+\frac{4A\pi}{3\sqrt{k}}}e^{-\sqrt{k}Q}\left(-\sqrt{k}Q\right)^{\frac{4A\pi}{3\sqrt{k}}}\alpha}{3\sqrt{k}\,\text{Gamma}\left[1-\frac{-108\sqrt{-k}-18\left(-6\sqrt{-k}+8A\pi\right)}{108\sqrt{k}}\right]}+\frac{2^{-1-\frac{4A\pi}{3\sqrt{k}}}e^{\sqrt{k}Q}\left(\sqrt{k}Q\right)^{-\frac{4A\pi}{3\sqrt{k}}}\alpha}{3\sqrt{k}\,\text{Gamma}\left[1+\frac{-108\sqrt{-k}-18\left(-6\sqrt{-k}+8A\pi\right)}{108\sqrt{k}}\right]}\;/.\;k\to 1$$
```
AsymLimPosQ2nd[Q, α, β]
```

Out[∘]=
$$\frac{1}{3}\times 2^{-1+\frac{4A\pi}{3}}e^{-Q}Q^{\frac{4A\pi}{3}}\beta+\frac{2^{-1-\frac{4A\pi}{3}}e^{Q}Q^{-\frac{4A\pi}{3}}\alpha}{3\,\text{Gamma}\left[1+\frac{1}{108}\left(-108\,i-18\left(-6\,i+8A\pi\right)\right)\right]}-\frac{2^{-1+\frac{4A\pi}{3}}e^{-Q}\left(-Q\right)^{\frac{4A\pi}{3}}\alpha}{3\,\text{Gamma}\left[1+\frac{1}{108}\left(108\,i+18\left(-6\,i+8A\pi\right)\right)\right]}$$

In[∘]:=
```
(*Now, as the limit goes to Infinity,
positive exponents will go to infinity while negative exponents will go to zero,
so positive exponents must vanish using one of the constants of integration. In
  this case it can vanish only if α=0, let's plot this, A=1 for example:*)
Plot[Abs[wdwsolutione[Q, 1, 0, 1]]^2 /. A → 1, {Q, 0, 50}, PlotRange → {0, 15},
 AxesLabel → {Style["Q", Black, 13], Style["Probability Density", Black, 13]},
 PlotLabel → Style["Second Ordering Modulus Squared of ψ(Q>0) for k=1", Black, 15],
 AxesStyle → Directive[Black, Thickness[0.0009]]]
```

Out[∘]=


In[∘]:=
```
(*Now let's look at the limit of the function as it goes to 0:*)
Limit[wdwsolutione[Q, 1, 0, β], Q → 0]
```

Out[∘]=
$$-\frac{\beta}{8A\pi\,\text{Gamma}\left[-\frac{4A\pi}{3}\right]}$$

In[•]:=
```
(*So both sides go to a constant value in terms of α and β for negative and
 positive Q respectively. The function has to be continuous. That can be done
 by setting the limits to each other and setting α in terms of β. That however,
will only leave one constant of integration,
with the condition of continuity of the derivatives yet to be met.*)
(*Let's check the derivatives and see if they are continuous:*)
(*For Q<0:*)
D[wdwsolutionf[Q, 1, α, 0], Q]
```

Out[•]=

$$e^{Q}\alpha$$

In[•]:=
```
(*The limit of the derivative to 0 is:*)
Limit[e^Q α, Q → 0, Direction → +1]
```

Out[•]=

$$\alpha$$

In[•]:=
```
(*this goes to α*)
D[wdwsolutione[Q, 1, 0, β], Q]
```

Out[•]=

$$\frac{1}{3} e^{-Q} \beta\, \text{HypergeometricU}\left[1+\frac{1}{108}\,(-108\,i-18\,(-6\,i+8\,A\,\pi)),\,2,\,2\,Q\right]-\frac{1}{3} e^{-Q} Q\,\beta\, \text{HypergeometricU}\left[1+\frac{1}{108}\,(-108\,i-18\,(-6\,i+8\,A\,\pi)),\,2,\,2\,Q\right]-\frac{2}{3} e^{-Q}\left(1+\frac{1}{108}\,(-108\,i-18\,(-6\,i+8\,A\,\pi))\right) Q\,\beta\, \text{HypergeometricU}\left[2+\frac{1}{108}\,(-108\,i-18\,(-6\,i+8\,A\,\pi)),\,3,\,2\,Q\right]$$

In[ ]:=
```
(*Wer'e gonna look at the Taylor series around Q = 0:*)
Normal[Series[1/3 E^-Q β HypergeometricU[1 + 1/108 (-108 I - 18 (-6 I + 8 A π)), 2, 2 Q] -
  1/3 E^-Q Q β HypergeometricU[1 + 1/108 (-108 I - 18 (-6 I + 8 A π)), 2, 2 Q] -
  2/3 E^-Q (1 + 1/108 (-108 I - 18 (-6 I + 8 A π))) Q β
   HypergeometricU[2 + 1/108 (-108 I - 18 (-6 I + 8 A π)), 3, 2 Q], {Q, 0, 1}]]
```

Out[ ]=
```
(3 β + 16 A EulerGamma π β + 8 A π β Log[2] + 8 A π β Log[Q] +
    8 A π β PolyGamma[0, 1 + 1/108 (-108 I - 18 (-6 I + 8 A π))]) /
  (24 A π Gamma[1/108 (-108 I - 18 (-6 I + 8 A π))]) +
 (Q (-9 β - 72 A π β + 128 A^2 π^2 β - 128 A^2 EulerGamma π^2 β - 64 A^2 π^2 β Log[2] -
      64 A^2 π^2 β Log[Q] - 48 A π β PolyGamma[0, 1 + 1/108 (-108 I - 18 (-6 I + 8 A π))] +
      48 A π β PolyGamma[0, 2 + 1/108 (-108 I - 18 (-6 I + 8 A π))] -
      64 A^2 π^2 β PolyGamma[0, 2 + 1/108 (-108 I - 18 (-6 I + 8 A π))])) /
  (72 A π Gamma[1/108 (-108 I - 18 (-6 I + 8 A π))])
```

In[ ]:=
```
(*Now we can look at the limit to zero:*)
Limit[(3 β + 16 A EulerGamma π β + 8 A π β Log[2] +
      8 A π β Log[Q] + 8 A π β PolyGamma[0, 1 + 1/108 (-108 I - 18 (-6 I + 8 A π))]) /
    (24 A π Gamma[1/108 (-108 I - 18 (-6 I + 8 A π))]) +
   (Q (-9 β - 72 A π β + 128 A^2 π^2 β - 128 A^2 EulerGamma π^2 β - 64 A^2 π^2 β Log[2] -
        64 A^2 π^2 β Log[Q] - 48 A π β PolyGamma[0, 1 + 1/108 (-108 I - 18 (-6 I + 8 A π))] +
        48 A π β PolyGamma[0, 2 + 1/108 (-108 I - 18 (-6 I + 8 A π))] -
        64 A^2 π^2 β PolyGamma[0, 2 + 1/108 (-108 I - 18 (-6 I + 8 A π))])) /
    (72 A π Gamma[1/108 (-108 I - 18 (-6 I + 8 A π))]) /. A → 1, Q → 0]
```

Out[ ]=
```
β ∞
```

```
(*So we have one limit going to a constant and the other to infinity,
with only one constant of integration left to "glue them together" (if possible),
this will not allow for normalisation,
and thus cannot be solved. Furthermore, the jump in the energy
  density values of the function at 0 makes the solution non-physical.*)
```