```
(*Self-adjoint Wheeler-deWitt equation for the second ordering:*)
(*y''[Q]+((8πρQ)/3 -k)y[Q]==0*)
(*For negative Q, ρ=0, for k=0 this becomes:*)
```

In[∘]:=
```
DSolve[y''[Q] == 0, y[Q], Q]
```

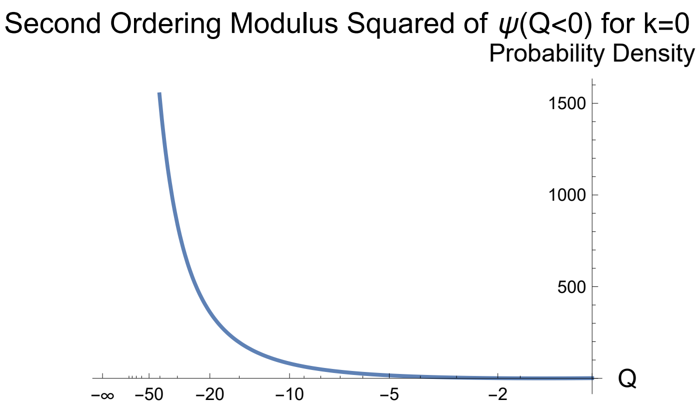
Out[∘]=

$$\{\{y[Q] \to \mathbb{c}_1 + Q\,\mathbb{c}_2\}\}$$

In[∘]:=
```
(*The limit for Q→-Infinity is -Infinity regardless of α/β (ℂ1/ℂ2) choices*)
zeroknegQ2nd[Q_, α_, β_] :=
 α + Q β

Plot[Abs[zeroknegQ2nd[Q, 1, 1]]^2, {Q, -Infinity, 0},
 AxesLabel → {Style["Q", Black, 13], Style["Probability Density", Black, 13]},
 PlotLabel → Style["Second Ordering Modulus Squared of ψ(Q<0) for k=0", Black, 15],
 AxesStyle → Directive[Black, Thickness[0.0009]]]
```

Out[∘]=




```
(*Now for positive Q:*)


(*Self-adjoint Wheeler-deWitt equation for the second ordering:*)
(*y''[Q]+((8πρQ)/3 -k)y[Q]==0*)
(*For positive Q and k=0 this becomes:*)
```

In[∘]:=
```
DSolve[y''[Q] + ((8 π × A) / (3 Q)) y[Q] == 0, y[Q], Q]
```

Out[∘]=

$$\left\{\left\{y[Q] \to 2\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}\ \text{BesselJ}\left[1, 4\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}\right]\mathbb{c}_1 + 4\,i\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}\ \text{BesselY}\left[1, 4\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}\right]\mathbb{c}_2\right\}\right\}$$

In[ ]:=
```
(*So the solution becomes:*)
wdwsolutionzerokpos2[Q_, α_, β_] :=
```
$$2\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}\,\mathrm{BesselJ}\left[1,4\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}\right]\alpha+4\,\mathrm{i}\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}\,\mathrm{BesselY}\left[1,4\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}\right]\beta$$

In[ ]:=
```
(*The asymptotic limit for Q → Infinity in first order Taylor series:*)
Normal[Series[wdwsolutionzerokpos2[Q, α, β], {Q, Infinity, 0}]]
```

Out[ ]=
$$\frac{(1+\mathrm{i})\sqrt{A}\,e^{-\mathrm{i}\pi+4\mathrm{i}\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}}\left(-\frac{1}{6\pi}\right)^{1/4}\sqrt{Q}\left(\sqrt{A}\sqrt{Q}-\mathrm{i}\sqrt{A\,Q}\right)\beta}{(A\,Q)^{3/4}}+$$
$$\frac{(1+\mathrm{i})\sqrt{A}\,e^{-\mathrm{i}\pi-4\mathrm{i}\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}}\left(-\frac{1}{6\pi}\right)^{1/4}\sqrt{Q}\left(\sqrt{A}\sqrt{Q}+\mathrm{i}\sqrt{A\,Q}\right)\beta}{(A\,Q)^{3/4}}+$$
$$\frac{\mathrm{i}\left(\sqrt{A}\sqrt{Q}+\mathrm{i}\sqrt{-A\,Q}\right)\alpha\,\mathrm{Cos}\left[\frac{\pi}{4}-4\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}\right]}{A^{1/4}\,(6\pi)^{1/4}\,Q^{1/4}}-$$
$$\frac{\left(\sqrt{A}\sqrt{Q}-\mathrm{i}\sqrt{-A\,Q}\right)\alpha\,\mathrm{Cos}\left[\frac{\pi}{4}+4\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}\right]}{A^{1/4}\,(6\pi)^{1/4}\,Q^{1/4}}+$$
$$\frac{2\times2^{3/4}\,A\,e^{-\mathrm{i}\pi-4\mathrm{i}\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}}\left(-\frac{1}{3\pi}\right)^{1/4}Q\,\beta\,\mathrm{Floor}\left[\frac{1}{2}-\frac{\mathrm{Arg}\left[\sqrt{A}\sqrt{Q}\right]}{\pi}\right]}{(A\,Q)^{3/4}}-$$
$$\frac{2\,(-2)^{3/4}\,A\,e^{-\mathrm{i}\pi+4\mathrm{i}\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}}\,Q\,\beta\,\mathrm{Floor}\left[\frac{1}{2}-\frac{\mathrm{Arg}\left[\sqrt{A}\sqrt{Q}\right]}{\pi}\right]}{(3\pi)^{1/4}\,(A\,Q)^{3/4}}$$

In[∘]:=

```
(*Written in exponential form:*)
Expand[
```

$$\text{Simplify}\Big[\text{TrigToExp}\Big[\frac{(1+i)\sqrt{A}\,e^{-i\pi+4i\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}}\left(-\frac{1}{6\pi}\right)^{1/4}\sqrt{Q}\left(\sqrt{A}\sqrt{Q}-i\sqrt{AQ}\right)\beta}{(AQ)^{3/4}}+$$

$$\frac{(1+i)\sqrt{A}\,e^{-i\pi-4i\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}}\left(-\frac{1}{6\pi}\right)^{1/4}\sqrt{Q}\left(\sqrt{A}\sqrt{Q}+i\sqrt{AQ}\right)\beta}{(AQ)^{3/4}}+$$

$$\frac{i\left(\sqrt{A}\sqrt{Q}+i\sqrt{-AQ}\right)\alpha\,\text{Cos}\left[\frac{\pi}{4}-4\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}\right]}{A^{1/4}(6\pi)^{1/4}Q^{1/4}}-$$

$$\frac{\left(\sqrt{A}\sqrt{Q}-i\sqrt{-AQ}\right)\alpha\,\text{Cos}\left[\frac{\pi}{4}+4\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}\right]}{A^{1/4}(6\pi)^{1/4}Q^{1/4}}+$$

$$\frac{2\times2^{3/4}A\,e^{-i\pi-4i\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}}\left(-\frac{1}{3\pi}\right)^{1/4}Q\beta\,\text{Floor}\left[\frac{1}{2}-\frac{\text{Arg}\left[\sqrt{A}\sqrt{Q}\right]}{\pi}\right]}{(AQ)^{3/4}}-$$

$$\frac{2(-2)^{3/4}A\,e^{-i\pi+4i\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}}Q\beta\,\text{Floor}\left[\frac{1}{2}-\frac{\text{Arg}\left[\sqrt{A}\sqrt{Q}\right]}{\pi}\right]}{(3\pi)^{1/4}(AQ)^{3/4}}\Big]\Big]\Big]$$

Out[∘]=

$$\frac{(-1)^{3/4}A^{1/4}e^{-4i\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}}Q^{1/4}\alpha}{(6\pi)^{1/4}}+\frac{(-1)^{3/4}e^{4i\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}}\sqrt{-AQ}\,\alpha}{A^{1/4}(6\pi)^{1/4}Q^{1/4}}-$$

$$\frac{(1-i)(-1)^{3/4}e^{-4i\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}}(AQ)^{1/4}\beta}{(6\pi)^{1/4}}-\frac{(1-i)(-1)^{3/4}e^{4i\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}}(AQ)^{1/4}\beta}{(6\pi)^{1/4}}-$$

$$\frac{(1+i)(-1)^{3/4}e^{-4i\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}}(AQ)^{3/4}\beta}{\sqrt{A}(6\pi)^{1/4}\sqrt{Q}}+\frac{(1+i)(-1)^{3/4}e^{4i\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}}(AQ)^{3/4}\beta}{\sqrt{A}(6\pi)^{1/4}\sqrt{Q}}-$$

$$2\times2^{3/4}e^{-4i\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}}\left(-\frac{1}{3\pi}\right)^{1/4}(AQ)^{1/4}\beta\,\text{Floor}\left[\frac{1}{2}-\frac{\text{Arg}\left[\sqrt{A}\sqrt{Q}\right]}{\pi}\right]+$$

$$\frac{2(-2)^{3/4}e^{4i\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}}(AQ)^{1/4}\beta\,\text{Floor}\left[\frac{1}{2}-\frac{\text{Arg}\left[\sqrt{A}\sqrt{Q}\right]}{\pi}\right]}{(3\pi)^{1/4}}$$

```
(*As Q is positive and the limit is to Infinity,
all terms have a complex exponential,
therefore the limit to infinity is indeterminent regardless of α/β choices*)
(*Thus the wavefunction cannot be normalised.*)
```

In[ ]:=
```
Plot[Abs[wdwsolutionzerokpos2[Q, 1, 1]]^2 /. A → 1, {Q, 0, Infinity},
  AxesLabel → {Style["Q", Black, 13], Style["Probability Density", Black, 13]},
  PlotLabel → Style["Second Ordering Modulus Squared of ψ(Q>0) for k=0", Black, 15],
  AxesStyle → Directive[Black, Thickness[0.0009]]]
```

Out[ ]=