```
(*Below is the self-adjoint version of the Wheeler-
 DeWitt equation for the third ordering:*)
(*-Y''[Q] + ((8πρQ)/3 - 3/(16Q^2))Y[Q]==k Y[Q],Y[Q],Q*)
```

```
(* For the case of particles going at c and in the Euclidian metric,
Q < 0, the density is negative. The WdW equation then,
for k equal to zero, is solved by: *)
```

In[•]:=
```
DSolve[-Y''[Q] + (-(8 π * A) / (3 Q) - 3 / (16 Q^2)) Y[Q] == 0, Y[Q], Q]
```

Out[•]=

$$\left\{\left\{Y[Q] \to e^{4 i \sqrt{A} \sqrt{\frac{2\pi}{3}} \sqrt{Q}}\, Q^{1/4}\, c_1 + \frac{i\, e^{-4 i \sqrt{A} \sqrt{\frac{2\pi}{3}} \sqrt{Q}} \sqrt{\frac{3}{2\pi}}\, Q^{1/4}\, c_2}{4\sqrt{A}}\right\}\right\}$$

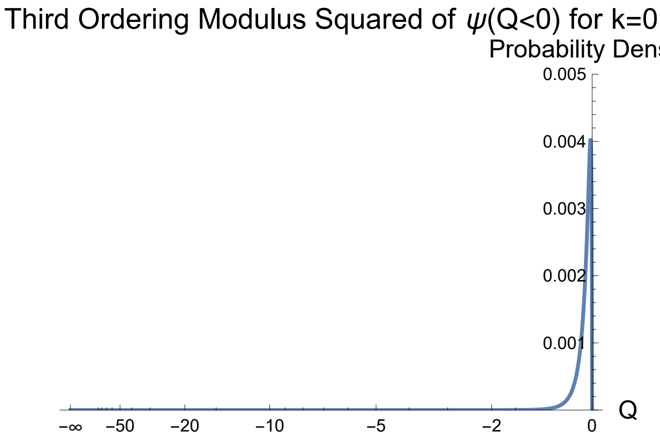
In[•]:=
```
wdwsolutionzerok3rdc[Q_, α_, β_] :=
```

$$Q^{1/4}\left(e^{4 i \sqrt{A} \sqrt{\frac{2\pi}{3}} \sqrt{Q}}\, Q^{1/4}\, \alpha + \frac{i\, e^{-4 i \sqrt{A} \sqrt{\frac{2\pi}{3}} \sqrt{Q}} \sqrt{\frac{3}{2\pi}}\, Q^{1/4}\, \beta}{4\sqrt{A}}\right)$$

In[•]:=
```
(*As the limit goes to -Infinity, positive exponentials will go to zero
 while negative exponentials will go to infinity, so need to set β=0:*)
Plot[Abs[wdwsolutionzerok3rdc[Q, 1, 0]]^2 /. A → 1,
 {Q, -Infinity, 0}, PlotRange → {0, 0.005},
 AxesLabel → {Style["Q", Black, 13], Style["Probability Density", Black, 13]},
 PlotLabel → Style["Third Ordering Modulus Squared of ψ(Q<0) for k=0", Black, 15],
 AxesStyle → Directive[Black, Thickness[0.0009]]]
```

••• General: Exp[-2863.87 + 0. i] is too small to represent as a normalized machine number; precision may be lost.

Out[•]=



In[•]:=
```
(*And for positive Q:*)
DSolve[-y''[Q] + ((8 π × A) / (3 Q) - 3 / (16 Q^2)) y[Q] == 0, y[Q], Q]
```

Out[•]=

$$\left\{\left\{y[Q] \to e^{4 \sqrt{A} \sqrt{\frac{2\pi}{3}} \sqrt{Q}}\, Q^{1/4}\, c_1 - \frac{e^{-4 \sqrt{A} \sqrt{\frac{2\pi}{3}} \sqrt{Q}} \sqrt{\frac{3}{2\pi}}\, Q^{1/4}\, c_2}{4\sqrt{A}}\right\}\right\}$$

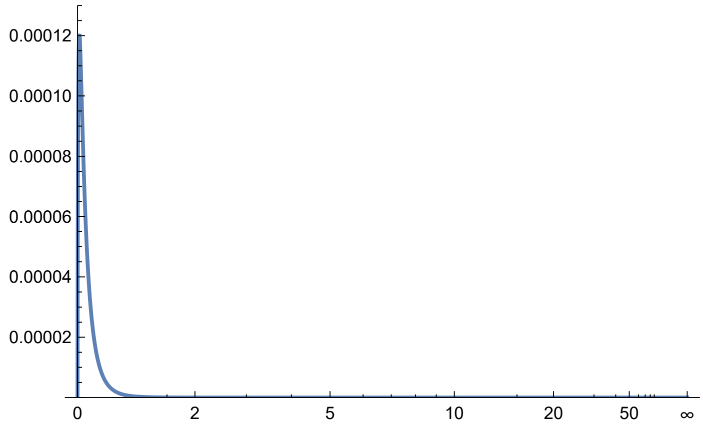
In[ ]:= (*Which then has to be multiplied by $Q^{1/4}$ to become the WdW solution:*)
wdwsolutionzerokpos3c[Q_, α_, β_] :=

$$e^{4\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}}\,Q^{1/2}\,\alpha-\frac{e^{-4\sqrt{A}\sqrt{\frac{2\pi}{3}}\sqrt{Q}}\sqrt{\frac{3}{2\pi}}\,Q^{1/2}\,\beta}{4\sqrt{A}}$$

(*Now, as Q is positive and goes to infinity, if β=0,
the limit to infinity is infinity, while if α=0, the limit to infinity is 0,
if both are non-zero the limit is indeterminent, so α=0 is chosen (β=1)*)

In[ ]:=
```
Plot[Abs[wdwsolutionzerokpos3c[Q, 0, 1]]^2 /. {A → 1},
 {Q, 0, Infinity}, PlotRange → {0, 0.00013}]
```

Out[ ]=



In[ ]:= (*Next, The behaviour of the two solutions will be investigated at Q=0:*)
```
Limit[wdwsolutionzerok3rdc[Q, 1, 0], Q → 0]
```

Out[ ]=

0

In[ ]:=
```
Limit[wdwsolutionzerokpos3c[Q, 0, 1], Q → 0]
```

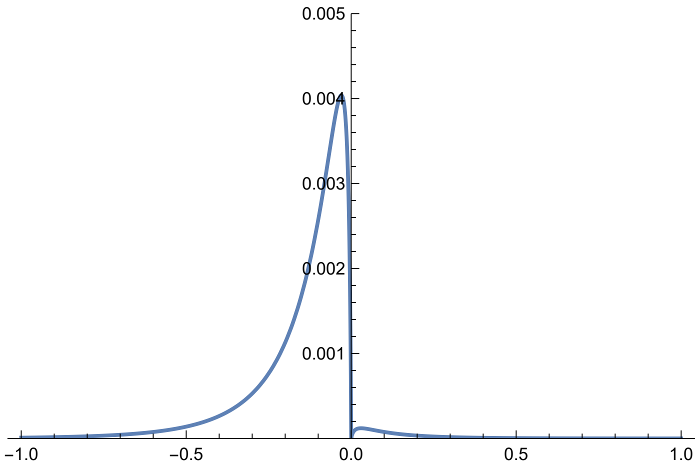
Out[ ]=

0

(*both go to zero.*)

In[⚬]:=
```
Plot[{Abs[wdwsolutionzerok3rdc[Q, 1, 0]]^2 UnitStep[-Q] +
    Abs[wdwsolutionzerokpos3c[Q, 0, 1]]^2 UnitStep[Q] /. A → 1},
 {Q, -1, 1}, PlotRange → {0, 0.005}]
```

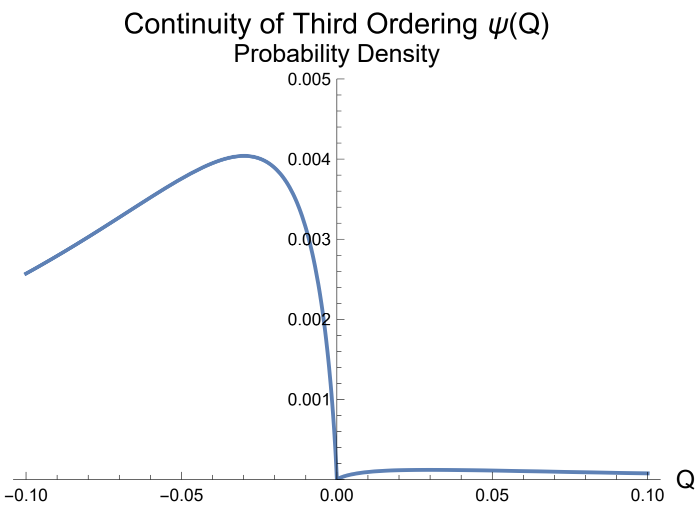
Out[⚬]=




```
(*Zooming in around Q=0:*)
```

In[⚬]:=
```
Plot[{Abs[wdwsolutionzerok3rdc[Q, 1, 0]]^2 UnitStep[-Q] +
    Abs[wdwsolutionzerokpos3c[Q, 0, 1]]^2 UnitStep[Q] /. A → 1},
 {Q, -0.1, 0.1}, PlotRange → {0, 0.005}, AxesLabel →
  {Style["Q", Black, 13], Style["Probability Density", Black, 13]},
 PlotLabel → Style["Continuity of Third Ordering ψ(Q)", Black, 15],
 AxesStyle → Directive[Black, Thickness[0.0009]]]
```

Out[⚬]=



In[⚬]:=
```
(*Next the derivatives will be checked for continuity at Q=0*)
Simplify[D[wdwsolutionzerok3rdc[Q, 1, 0], Q]]
```

Out[⚬]=

$$\frac{e^{4 i \sqrt{A} \sqrt{\frac{2\pi}{3}} \sqrt{Q}} \left(3 + 4 i \sqrt{A} \sqrt{6\pi} \sqrt{Q}\right)}{6 \sqrt{Q}}$$

In[•]:= Limit$\left[\dfrac{e^{4 i \sqrt{A} \sqrt{\frac{2\pi}{3}} \sqrt{Q}} \left(3 + 4 i \sqrt{A} \sqrt{6\pi} \sqrt{Q}\right)}{6 \sqrt{Q}}, Q \to 0, \text{Direction} \to +1\right]$

Out[•]=

$(-i)\ \infty$

In[•]:= Simplify[D[wdwsolutionzerokpos3c[Q, 0, 1], Q]]

Out[•]=

$$\frac{e^{-4 \sqrt{A} \sqrt{\frac{2\pi}{3}} \sqrt{Q}} \left(-\sqrt{\frac{6}{\pi}} + 8 \sqrt{A} \sqrt{Q}\right)}{16 \sqrt{A} \sqrt{Q}}$$

In[•]:= Limit$\left[\dfrac{e^{-4 \sqrt{A} \sqrt{\frac{2\pi}{3}} \sqrt{Q}} \left(-\sqrt{\frac{6}{\pi}} + 8 \sqrt{A} \sqrt{Q}\right)}{16 \sqrt{A} \sqrt{Q}}, Q \to 0, \text{Direction} \to -1\right]$

Out[•]=

$$\frac{-\infty}{\sqrt{A}}$$

(*Both go to -Infinity.*)

In[•]:= (* So, what needs to be done next is to 'glue' the two infinities to each other, by choosing α and β appropriately. This will be done by making the difference of the Taylor series of the derivatives vanish: *)

PowerExpand$\Bigg[$Simplify$\Bigg[$Solve$\Bigg[$

$$\frac{e^{4 i \sqrt{A} \sqrt{\frac{2\pi}{3}} \sqrt{Q}} \left(3 + 4 i \sqrt{A} \sqrt{6\pi} \sqrt{Q}\right) \alpha}{6 \sqrt{Q}} - \frac{e^{-4 \sqrt{A} \sqrt{\frac{2\pi}{3}} \sqrt{Q}} \left(-\sqrt{\frac{6}{\pi}} + 8 \sqrt{A} \sqrt{Q}\right) \beta}{16 \sqrt{A} \sqrt{Q}} == 0, \alpha\Bigg]\Bigg]\Bigg]$$

Out[•]=

$$\left\{\left\{\alpha \to \frac{e^{(-4-4 i) \sqrt{A} \sqrt{\frac{2\pi}{3}} \sqrt{Q}} \left(-3 \sqrt{\frac{6}{\pi}} + (24 + 24 i) \sqrt{A} \sqrt{Q} - 32 i A \sqrt{6\pi}\, Q\right) \beta}{8 \sqrt{A}\ (3 + 32 A \pi Q)}\right\}\right\}$$

In[•]:= (*The limit to zero is:*)

Limit$\left[\dfrac{e^{(-4-4 i) \sqrt{A} \sqrt{\frac{2\pi}{3}} \sqrt{Q}} \left(-3 \sqrt{\frac{6}{\pi}} + (24 + 24 i) \sqrt{A} \sqrt{Q} - 32 i A \sqrt{6\pi}\, Q\right) \beta}{8 \sqrt{A}\ (3 + 32 A \pi Q)}, Q \to 0\right]$

Out[•]=

$$-\frac{\sqrt{\frac{3}{2\pi}}\ \beta}{4 \sqrt{A}}$$

In[•]:= (*In numerical value*)

Simplify$\left[\text{Abs}\left[N\left[-\dfrac{\sqrt{\frac{3}{2\pi}}\ \beta}{4 \sqrt{A}}\ /.\ A \to 1\right]\right]\right]$

Out[•]=

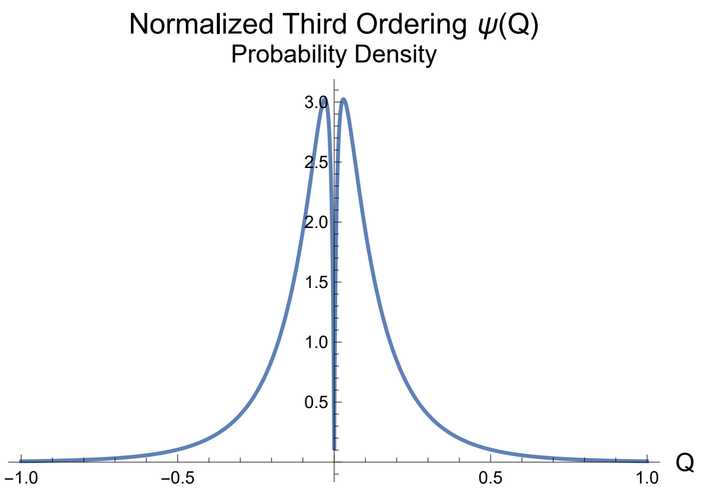
0.172747 Abs[β]

```
(* Collecting all together, leads to a WdW solution for all Q,
that goes to zero at both infinities, is normalisable on both sides,
continuous at Q = 0, and whose derivative is continous at Q = 0 as well. *)
```

In[∘]:=
```
WdWsolutionAllQ3rdc[Q_, Aa_, β_] :=
 (wdwsolutionzerok3rdc[Q, -(Sqrt[3/(2 π)] β)/(4 Sqrt[A]), 0] UnitStep[-Q] +
    wdwsolutionzerokpos3c[Q, 0, β] UnitStep[Q]) /. A → Aa

Plot[Abs[WdWsolutionAllQ3rdc[Q, 1, 158.39]]^2, {Q, -1, 1},
 AxesLabel → {Style["Q", Black, 13], Style["Probability Density", Black, 13]},
 PlotLabel → Style["Normalized Third Ordering ψ(Q)", Black, 15],
 AxesStyle → Directive[Black, Thickness[0.0009]]]
```

Out[∘]=




```
(* Now to fix the remaining overall constant β by forcing
 the absolute square of the solution to be normalised to unity. *)

(* Here is an example of what the integration will result in: *)
```

In[∘]:=
```
NIntegrate[Abs[WdWsolutionAllQ3rdc[Q, 1, 158.39]]^2, {Q, -Infinity, Infinity}]
```

Out[∘]=
```
1.00003
```

In[∘]:=
```
NIntegrate[Abs[(0.172747) * (158.39) * wdwsolutionzerok3rdc[Q, 1, 0]]^2 /. A → 1,
 {Q, -Infinity, 0}]
```

Out[∘]=
```
0.500012
```

In[∘]:=
```
NIntegrate[
 Abs[(158.39) * wdwsolutionzerokpos3c[Q, 0, 1]]^2 /. {A → 1}, {Q, 0, Infinity}]
```

Out[∘]=
```
0.500013
```

In[∘]:=
```
(*For A=100:*)
(*Gluing coefficient is:*)
Simplify[Abs[N[-(Sqrt[3/(2 π)] β)/(4 Sqrt[A]) /. A → 100]]]
```
$$\text{Simplify}\left[\text{Abs}\left[\text{N}\left[-\frac{\sqrt{\frac{3}{2\pi}}\ \beta}{4\sqrt{A}}\ /.\ A\to 100\right]\right]\right]$$

Out[∘]=

0.0172747 Abs[β]

In[∘]:=
```
(*The normalization constant is:*)
NIntegrate[Abs[WdWsolutionAllQ3rdc[Q, 100, 158390]]^2, {Q, -Infinity, Infinity}]
```

Out[∘]=

1.00003

In[∘]:=
```
(*The probability for the emergence of a Euclidean universe is:*)
NIntegrate[
 Abs[(0.017274707473566776) * (158390) * wdwsolutionzerok3rdc[Q, 1, 0]]^2 /. A → 100,
 {Q, -Infinity, 0}]
```

Out[∘]=

0.500013

```
(*The probability for the emergence of a Minkowskian universe is:*)
```

In[∘]:=
```
NIntegrate[
 Abs[(158390) * wdwsolutionzerokpos3c[Q, 0, 1]]^2 /. {A → 100}, {Q, 0, Infinity}]
```

Out[∘]=

0.500013

In[∘]:=
```
(*For A=50:*)
(*Gluing coefficient is:*)
Simplify[Abs[N[-(Sqrt[3/(2 π)] β)/(4 Sqrt[A]) /. A → 50]]]
```
$$\text{Simplify}\left[\text{Abs}\left[\text{N}\left[-\frac{\sqrt{\frac{3}{2\pi}}\ \beta}{4\sqrt{A}}\ /.\ A\to 50\right]\right]\right]$$

Out[∘]=

0.0244301 Abs[β]

In[∘]:=
```
(*The normalization constant is:*)
NIntegrate[Abs[WdWsolutionAllQ3rdc[Q, 50, 56000]]^2, {Q, -Infinity, Infinity}]
```

Out[∘]=

1.00005

In[∘]:=
```
(*The probability for the emergence of a Euclidean universe is:*)
NIntegrate[
 Abs[(0.024430125595146) * (56000) * wdwsolutionzerok3rdc[Q, 1, 0]]^2 /. A → 50,
 {Q, -Infinity, 0}]
```

Out[∘]=

0.500025

In[∘]:=
```
(*The probability for the emergence of a Minkowskian universe is:*)
NIntegrate[
 Abs[(56000) * wdwsolutionzerokpos3c[Q, 0, 1]]^2 /. {A → 50}, {Q, 0, Infinity}]
```

Out[∘]=

0.500025

In[*]:=
```
(*For A=10:*)
(*Gluing coefficient is:*)
```
$$\text{Simplify}\left[\text{Abs}\left[\text{N}\left[-\frac{\sqrt{\frac{3}{2\pi}}\,\beta}{4\sqrt{\text{A}}}\ /.\ \text{A}\to 10\right]\right]\right]$$

Out[*]=
```
0.0546274 Abs[β]
```

In[*]:=
```
(*The normalization constant is:*)
NIntegrate[Abs[WdWsolutionAllQ3rdc[Q, 10, 5008.7]]^2, {Q, -Infinity, Infinity}]
```

Out[*]=
```
1.00001
```

In[*]:=
```
(*The probability for the emergence of a Euclidean universe is:*)
NIntegrate[
 Abs[(0.05462742152960395) * (5008.7) * wdwsolutionzerok3rdc[Q, 1, 0]]^2 /. A → 10,
 {Q, -Infinity, 0}]
```

Out[*]=
```
0.500006
```

In[*]:=
```
(*The probability for the emergence of a Minkowskian universe is:*)
NIntegrate[
 Abs[(5008.7) * wdwsolutionzerokpos3c[Q, 0, 1]]^2 /. {A → 10}, {Q, 0, Infinity}]
```

Out[*]=
```
0.500006
```

In[*]:=
```
(*For A=5:*)
(*Gluing coefficient is:*)
```
$$\text{Simplify}\left[\text{Abs}\left[\text{N}\left[-\frac{\sqrt{\frac{3}{2\pi}}\,\beta}{4\sqrt{\text{A}}}\ /.\ \text{A}\to 5\right]\right]\right]$$

Out[*]=
```
0.0772548 Abs[β]
```

In[*]:=
```
(*The normalization constant is:*)
NIntegrate[Abs[WdWsolutionAllQ3rdc[Q, 5, 1770.84]]^2, {Q, -Infinity, Infinity}]
```

Out[*]=
```
1.00001
```

In[*]:=
```
(*The probability for the emergence of a Euclidean universe is:*)
NIntegrate[
 Abs[(0.07725484040463793) * (1770.84) * wdwsolutionzerok3rdc[Q, 1, 0]]^2 /. A → 5,
 {Q, -Infinity, 0}]
```

Out[*]=
```
0.500005
```

In[ ]:=
```
(*The probability for the emergence of a Minkowskian universe is:*)
NIntegrate[
 Abs[(1770.84) * wdwsolutionzerokpos3c[Q, 0, 1]]^2 /. {A → 5}, {Q, 0, Infinity}]
```
Out[ ]=
```
0.500005
```

In[ ]:=
```
(*For A=0.5*)
(*Gluing coefficient is:*)
```
$$\text{Simplify}\left[\text{Abs}\left[\text{N}\left[-\frac{\sqrt{\frac{3}{2\pi}}\,\beta}{4\sqrt{A}}\ /.\ A \to 0.5\right]\right]\right]$$

Out[ ]=
```
0.244301 Abs[β]
```

In[ ]:=
```
(*The normalization constant is:*)
NIntegrate[Abs[WdWsolutionAllQ3rdc[Q, 0.5, 55.999]]^2, {Q, -Infinity, Infinity}]
```
Out[ ]=
```
1.00001
```

In[ ]:=
```
(*The probability for the emergence of a Euclidean universe is:*)
NIntegrate[
 Abs[(0.24430125595145996) * (55.999) * wdwsolutionzerok3rdc[Q, 1, 0]]^2 /. A → 0.5,
 {Q, -Infinity, 0}]
```
Out[ ]=
```
0.500007
```

In[ ]:=
```
(*The probability for the emergence of a Minkowskian universe is:*)
NIntegrate[
 Abs[(55.999) * wdwsolutionzerokpos3c[Q, 0, 1]]^2 /. {A → 0.5}, {Q, 0, Infinity}]
```
Out[ ]=
```
0.500007
```

In[ ]:=
```
(*For A=0.01:*)
(*Gluing coefficient is:*)
```
$$\text{Simplify}\left[\text{Abs}\left[\text{N}\left[-\frac{\sqrt{\frac{3}{2\pi}}\,\beta}{4\sqrt{A}}\ /.\ A \to 0.01\right]\right]\right]$$

Out[ ]=
```
1.72747 Abs[β]
```

In[ ]:=
```
(*The normalization constant is:*)
NIntegrate[Abs[WdWsolutionAllQ3rdc[Q, 0.01, 0.15839]]^2, {Q, -Infinity, Infinity}]
```
Out[ ]=
```
1.00003
```

In[∘]:=
```
(*The probability for the emergence of a Euclidean universe is:*)
NIntegrate[
 Abs[(1.7274707473566775) * (0.15839) * wdwsolutionzerok3rdc[Q, 1, 0]]^2 /. A → 0.01,
 {Q, -Infinity, 0}]
```
Out[∘]=
```
0.500013
```

In[∘]:=
```
(*The probability for the emergence of a Minkowskian universe is:*)
NIntegrate[
 Abs[(0.15839) * wdwsolutionzerokpos3c[Q, 0, 1]]^2 /. {A → 0.01}, {Q, 0, Infinity}]
```
Out[∘]=
```
0.500013
```

In[∘]:=

```
(*For A=0.000001:*)
(*Gluing coeffifcient is:*)
```
$$\text{Simplify}\left[\text{Abs}\left[\text{N}\left[-\frac{\sqrt{\frac{3}{2\pi}}\ \beta}{4\sqrt{A}}\ /.\ A\to 0.000001\right]\right]\right]$$

Out[∘]=
```
172.747 Abs[β]
```

In[∘]:=
```
(*The normalization constant is:*)
NIntegrate[Abs[WdWsolutionAllQ3rdc[Q, 0.000001, 0.000000158389]]^2,
 {Q, -Infinity, Infinity}]
```
Out[∘]=
```
1.00001
```

In[∘]:=
```
(*The probability for the emergence of a Euclidean universe is:*)
NIntegrate[
 Abs[(172.74707473566775) * (0.000000158389) * wdwsolutionzerok3rdc[Q, 1, 0]]^2 /.
  A → 0.000001, {Q, -Infinity, 0}]
```
Out[∘]=
```
0.500006
```

In[∘]:=
```
(*The probability for the emergence of a Minkowskian universe is:*)
NIntegrate[Abs[(0.000000158389) * wdwsolutionzerokpos3c[Q, 0, 1]]^2 /. {A → 0.000001},
 {Q, 0, Infinity}]
```
Out[∘]=
```
0.500006
```