```
(*Below is the self-adjoint version of the Wheeler-
 DeWitt equation for the first ordering:*)
(*-Y''[Q]-(3/(16Q^2)+((8πρQ)/(3)))Y[Q]==-k Y[Q]*)
(* For the case of particles with velocity c and in the Eucledian metric, Q < 0,
the density is negative. The WdW equation the, for k not equal zero is solved by:*)
```

In[∘]:=

```
DSolve[-Y''[Q] - (3 / (16 Q^2) + (- (8 π * A) / (3 Q))) Y[Q] == -k Y[Q], Y[Q], Q]
```

Out[∘]=

$$\left\{\left\{Y[Q] \to c_1\,\mathrm{WhittakerM}\left[-\frac{4\,A\,\pi}{3\,\sqrt{k}},\,\frac{1}{4},\,2\,\sqrt{k}\,Q\right] + c_2\,\mathrm{WhittakerW}\left[-\frac{4\,A\,\pi}{3\,\sqrt{k}},\,\frac{1}{4},\,2\,\sqrt{k}\,Q\right]\right\}\right\}$$

In[∘]:=

(*This time $Q^{1/4}$ gives the solution to the WdW equation, which is equal to:*)

SolutionNegQ1stc[Q_, k_, α_, β_] :=

$$Q^{1/4}\left(\alpha\,\mathrm{WhittakerM}\left[-\frac{4\,A\,\pi}{3\,\sqrt{k}},\,\frac{1}{4},\,2\,\sqrt{k}\,Q\right] + \beta\,\mathrm{WhittakerW}\left[-\frac{4\,A\,\pi}{3\,\sqrt{k}},\,\frac{1}{4},\,2\,\sqrt{k}\,Q\right]\right)$$

In[∘]:=

```
(*The asymptotic limit for Q →
 -Infinity can be investigated by writing a first order Taylor series:*)
Normal[Series[SolutionNegQ1stc[Q, k, α, β], {Q, -Infinity, 0}]]
```

Out[∘]=

$$2^{-\frac{4A\pi}{3\sqrt{k}}}\,e^{-\sqrt{k}\,Q}\,Q^{1/4}\left(\sqrt{k}\,Q\right)^{-\frac{4A\pi}{3\sqrt{k}}}\beta - \frac{2^{-1-\frac{4A\pi}{3\sqrt{k}}}\,e^{-\sqrt{k}\,Q}\,\sqrt{\pi}\,\left(-\sqrt{k}\,Q\right)^{\frac{1}{4}-\frac{4A\pi}{3\sqrt{k}}}\left(\sqrt{k}\,Q\right)^{3/4}\alpha}{\sqrt{k}\,Q^{3/4}\,\mathrm{Gamma}\left[\frac{3}{4}-\frac{4A\pi}{3\sqrt{k}}\right]} + \frac{2^{-1+\frac{4A\pi}{3\sqrt{k}}}\,e^{\sqrt{k}\,Q}\,\sqrt{\pi}\,Q^{1/4}\left(\sqrt{k}\,Q\right)^{\frac{4A\pi}{3\sqrt{k}}}\alpha}{\mathrm{Gamma}\left[\frac{3}{4}+\frac{4A\pi}{3\sqrt{k}}\right]}$$

In[•]:= (*For k=-1, exponential become complex, which will not converge to zero, therefore k has to equal 1 in order for the exponential to be real. Furthermore, exponentials with a negative exponent will go to Infinity for Q → -Infinity, while positive exponents will go to zero, therefore k needs to be set to 1:*)

AsymLimNegQ1stc[Q_, α_, β_] :=

$$2^{-\frac{4A\pi}{3\sqrt{k}}} e^{-\sqrt{k}Q} Q^{1/4} \left(\sqrt{k}Q\right)^{-\frac{4A\pi}{3\sqrt{k}}} \beta - \frac{2^{-1-\frac{4A\pi}{3\sqrt{k}}} e^{-\sqrt{k}Q} \sqrt{\pi} \left(-\sqrt{k}Q\right)^{\frac{1}{4}-\frac{4A\pi}{3\sqrt{k}}} \left(\sqrt{k}Q\right)^{3/4} \alpha}{\sqrt{k}\, Q^{3/4}\, \text{Gamma}\left[\frac{3}{4}-\frac{4A\pi}{3\sqrt{k}}\right]} + \frac{2^{-1+\frac{4A\pi}{3\sqrt{k}}} e^{\sqrt{k}Q} \sqrt{\pi}\, Q^{1/4} \left(\sqrt{k}Q\right)^{\frac{4A\pi}{3\sqrt{k}}} \alpha}{\text{Gamma}\left[\frac{3}{4}+\frac{4A\pi}{3\sqrt{k}}\right]} \text{ /. } k \to 1$$

SolutionNegQ1stc[Q, 1, α, β]

AsymLimNegQ1stc[Q, α, β]

Out[•]=

$$Q^{1/4} \left(\alpha\, \text{WhittakerM}\left[-\frac{4A\pi}{3}, \frac{1}{4}, 2Q\right] + \beta\, \text{WhittakerW}\left[-\frac{4A\pi}{3}, \frac{1}{4}, 2Q\right]\right)$$

Out[•]=

$$2^{-\frac{4A\pi}{3}} e^{-Q} Q^{\frac{1}{4}-\frac{4A\pi}{3}} \beta - \frac{2^{-1-\frac{4A\pi}{3}} e^{-Q} \sqrt{\pi}\, (-Q)^{\frac{1}{4}-\frac{4A\pi}{3}} \alpha}{\text{Gamma}\left[\frac{3}{4}-\frac{4A\pi}{3}\right]} + \frac{2^{-1+\frac{4A\pi}{3}} e^{Q} \sqrt{\pi}\, Q^{\frac{1}{4}+\frac{4A\pi}{3}} \alpha}{\text{Gamma}\left[\frac{3}{4}+\frac{4A\pi}{3}\right]}$$

In[•]:= (*As the limit goes to -Infinity, negative exponential will go to infinity while positive exponentials will go to zero:*)

$$\text{Solve}\left[2^{-\frac{4A\pi}{3}} e^{-Q} Q^{\frac{1}{4}-\frac{4A\pi}{3}} \beta - \frac{2^{-1-\frac{4A\pi}{3}} e^{-Q} \sqrt{\pi}\, (-Q)^{\frac{1}{4}-\frac{4A\pi}{3}} \alpha}{\text{Gamma}\left[\frac{3}{4}-\frac{4A\pi}{3}\right]} == 0, \beta\right]$$

Out[•]=

$$\left\{\left\{\beta \to \frac{\sqrt{\pi}\, (-Q)^{\frac{1}{4}-\frac{4A\pi}{3}} Q^{-\frac{1}{4}+\frac{4A\pi}{3}} \alpha}{2\, \text{Gamma}\left[\frac{3}{4}-\frac{4A\pi}{3}\right]}\right\}\right\}$$

In[•]:=

$$\text{Limit}\left[\frac{\sqrt{\pi}\, (-Q)^{\frac{1}{4}-\frac{4A\pi}{3}} Q^{-\frac{1}{4}+\frac{4A\pi}{3}} \alpha}{2\, \text{Gamma}\left[\frac{3}{4}-\frac{4A\pi}{3}\right]}, Q \to -\text{Infinity}\right]$$

Out[•]=

$$-\frac{(-1)^{\frac{3}{4}+\frac{4A\pi}{3}} \sqrt{\pi}\, \alpha}{2\, \text{Gamma}\left[\frac{3}{4}-\frac{4A\pi}{3}\right]}$$

In[∘]:=
```
Plot[Abs[SolutionNegQ1stc[Q, 1, 1, -((-1)^(3/4 + 4 A π/3) Sqrt[π])/(2 Gamma[3/4 - 4 A π/3])]]^2 /. A → 1, {Q, -Infinity, 0},
 AxesLabel → {Style["Q", Black, 13], Style["Probability Density", Black, 13]},
 PlotLabel → Style["First Ordering Modulus Squared of ψ(Q<0) for k=1", Black, 15],
 AxesStyle → Directive[Black, Thickness[0.0009]]]
```

General: Exp[−489505.] is too small to represent as a normalized machine number; precision may be lost.

General: Exp[−244753.] is too small to represent as a normalized machine number; precision may be lost.

Power: Infinite expression $\frac{1}{0.}$ encountered.

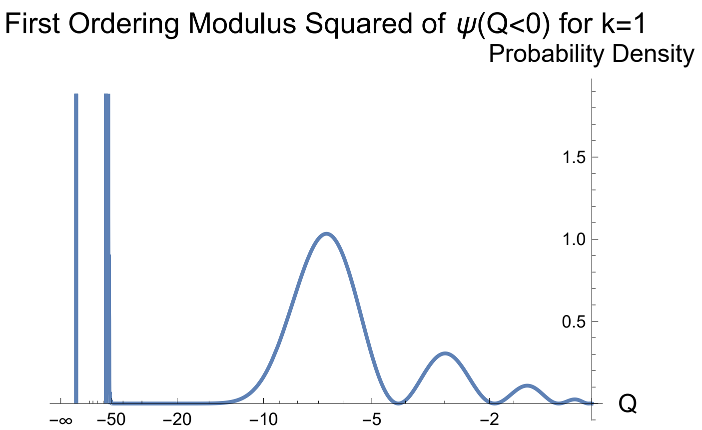
General: Exp[−244753.] is too small to represent as a normalized machine number; precision may be lost.

General: Further output of General::munfl will be suppressed during this calculation.

Power: Infinite expression $\frac{1}{0.}$ encountered.

Infinity: Indeterminate expression ComplexInfinity + ComplexInfinity encountered.

Out[∘]=



(*The solution contains a WhittakerM function, that contains Kummer's HyperGeometricM function in the form $\text{WhittakerM}_{k,\mu}(Q) = e^{-z/2} z^{1/2+\mu} M(1+\mu-k, 1+2\mu, z)$, where z=Q, the constant of integration $\beta = -\frac{(-1)^{\frac{3}{4}+\frac{4 A \pi}{3}} \sqrt{\pi}\ \alpha}{2\ \text{Gamma}\left[\frac{3}{4}-\frac{4 A \pi}{3}\right]}$ simply multiplies it by a constant. This cannot be normalized.*)