

inputs		targets
x_1		t_1
1		-1
.5		-.5
0		0
-1		1

inputs		targets
x_1	x_2	t_1
2	1	1
3	1.5	1
1	3	0
2	3	0

outputs
y_1
1
1
0
0

inputs			targets	
x_1	x_2	x_3	t_1	t_2
1	0	0	.2	.4
0	1	0	.5	.7
1	0	1	.6	.7
0	1	1	.8	1

outputs	
y_1	y_2
1	3
2	-2
3	4
4	3

Suppose this is our training dataset:

x_1	x_2	x_3	t_1	t_2
1	0	0	1	-1
1	1	0	-1	1
0	1	1	1	1

Notice that $R = 3$, $N = 3$, and $M = 2$. Now suppose we have a neural network that we want to train on this data. The network has 3 inputs, and it produces $(0,0)$ every time. So these are our outputs:

y_1	y_2
0	0
0	0
0	0

To compute SSE we need to through every row, compare each targets with its corresponding output (each of these is one error), so errors together and divide by the number of rows. So we have:

$$SSE = (1 - 0)^2 + (-1 - 0)^2 + (-1 - 0)^2 + (1 - 0)^2 + (1 - 0)^2 + (1 - 0)^2 = 1 + 1 + 1 + 1 + 1 + 1 = 6$$

So for this network and this training set, $SSE = 6$. The is intuitively obvious since every output for every row is “wrong” by 1, and that’s terrible! Our network is doing a terrible job.

Now suppose we train the network a bit, and it starts doing better. So now these are the outputs:

y_1	y_2
.5	-1
-1	.5
.8	1

Better! Now we have:

$$\begin{aligned}
 SSE &= (1 - .5)^2 + (-1 - (-1))^2 + (-1 - (-1))^2 + (1 - .5)^2 + (1 - .8)^2 + (1 - 1)^2 = \\
 &= .5^2 + 0^2 + 0^2 + .5^2 + .2^2 + 0^2 = \\
 &= .25 + .25 + .04 = .54
 \end{aligned}$$

Error has gotten lower, which is the goal of supervised learning!

$$\left(\begin{array}{|c|} \hline \text{targets} \\ \hline t_1 \\ \hline 0 \\ \hline 1 \\ \hline 0 \\ \hline 1 \\ \hline \end{array} - \begin{array}{|c|} \hline \text{outputs} \\ \hline o_1 \\ \hline 0 \\ \hline 1 \\ \hline 0 \\ \hline 1 \\ \hline \end{array} \right)^2 = \left(\begin{array}{|c|} \hline \text{errors} \\ \hline 0 - 0 \\ \hline 1 - 1 \\ \hline 0 - 0 \\ \hline 1 - 1 \\ \hline \end{array} \right)^2 = \begin{array}{|c|} \hline 1 \\ \hline 1 \\ \hline 0 \\ \hline 0 \\ \hline \end{array}$$

$$\sum_R \left(\begin{array}{|c|} \hline \text{targets} \\ \hline 1 \\ \hline 2 \\ \hline 3 \\ \hline 4 \\ \hline \end{array} - \begin{array}{|c|} \hline \text{outputs} \\ \hline .9 \\ \hline 1.9 \\ \hline 2.9 \\ \hline 3.9 \\ \hline \end{array} \right)^2 = \sum_R \left(\begin{array}{|c|} \hline \text{errors} \\ \hline 1 - .9 \\ \hline 2 - 1.9 \\ \hline 3 - 2.9 \\ \hline 4 - 3.9 \\ \hline \end{array} \right)^2 = \sum_R \left(\begin{array}{|c|} \hline \text{errors} \\ \hline .1 \\ \hline .1 \\ \hline .1 \\ \hline .1 \\ \hline \end{array} \right)^2 = \sum_R \begin{array}{|c|} \hline .01 \\ \hline .01 \\ \hline .01 \\ \hline .01 \\ \hline \end{array} = .01 + .01 + .01 + .01 = .04$$

$$\sum_R \left(\begin{array}{|c|c|} \hline \text{targets} & \\ \hline 1 & 0 \\ \hline 0 & 1 \\ \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array} - \begin{array}{|c|c|} \hline \text{outputs} & \\ \hline .8 & 0 \\ \hline 0 & 1 \\ \hline 1 & 0 \\ \hline 0 & .9 \\ \hline \end{array} \right)^2 = \sum_R \left(\begin{array}{|c|c|} \hline \text{errors} & \\ \hline 1-.8 & 0-0 \\ \hline 0-0 & 1-1 \\ \hline 1-1 & 0-0 \\ \hline 0-0 & 1-.9 \\ \hline \end{array} \right)^2 = \sum_R \left(\begin{array}{|c|c|} \hline \text{errors} & \\ \hline .2 & 0 \\ \hline 0 & 0 \\ \hline 0 & 0 \\ \hline 0 & .1 \\ \hline \end{array} \right)^2 = \sum_R \left(\begin{array}{c} (.2, 0) \bullet (.2, 0) \\ (0, 0) \bullet (0, 0) \\ (0, 0) \bullet (0, 0) \\ (.1, 0) \bullet (.1, 0) \end{array} \right) = .04 + 0 + 0 + .01 = .05$$

$$\sum_R \left(\begin{array}{|c|c|} \hline \text{targets} & \\ \hline 1 & 0 \\ \hline 0 & 1 \\ \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array} - \begin{array}{|c|c|} \hline \text{outputs} & \\ \hline 1 & 1 \\ \hline 1 & 1 \\ \hline 1 & 1 \\ \hline 1 & 1 \\ \hline \end{array} \right)^2 = \sum_R \left(\begin{array}{|c|c|} \hline \text{errors} & \\ \hline 1-1 & 0-1 \\ \hline 0-1 & 1-1 \\ \hline 1-1 & 0-1 \\ \hline 0-1 & 1-1 \\ \hline \end{array} \right)^2 = \sum_R \left(\begin{array}{|c|c|} \hline \text{errors} & \\ \hline 0 & -1 \\ \hline -1 & 0 \\ \hline 0 & -1 \\ \hline -1 & 0 \\ \hline \end{array} \right)^2 = \sum_R \left(\begin{array}{c} (0, -1) \bullet (0, -1) \\ (-1, 0) \bullet (-1, 0) \\ (0, -1) \bullet (0, -1) \\ (-1, 0) \bullet (-1, 0) \end{array} \right) = 1 + 1 + 1 + 1 = 4$$

Inputs		
1	0	0
0	1	0
0	0	1
1	0	1

Outputs	
.5	0
0	.5
-1	-.5
-1	-.5

Targets	
1	0
0	1
-1	-1
-1	-1

Inputs			Targets	
1	0	0	1	0
0	1	0	0	1
0	0	1	-1	-1
1	0	1	-1	-1

Inputs			Outputs	
0	0	0	0	0
1	0	0	1	-1
1	1	1	3	-1
0	0	1	1	1
1	2	3	6	0

Inputs			Outputs	
1	0	0	.5	0
0	1	0	0	.5
0	0	1	-1	-.5
1	0	1	-1	-.5

Inputs			Targets	
1	0	0	1	0
0	1	0	0	1
0	0	1	1	1

Inputs			Targets	
1	0	0	.2	.8
0	1	0	.7	.3
0	0	1	.2	.2

Inputs		Targets
0	0	0
1	0	1
0	1	1
1	1	0

inputs			targets	
1	0	0	1	.4
0	1	0	.8	.3
0	0	1	.5	.7