Ballistic Pendulum

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1 Description

This project seeks to analyze the physics of the collision between a fast moving projectile and the bob of a pendulum. In the original problem a bullet is shot at a wooden box and the two objects stick together, allowing for a calculation of the final velocity of the bullet. In this project, small tablets are shot at a spherical pendulum, and the objects separate. Conservation of mechanical energy is used to calculate the difference in the squares of the initial and final speed of the tablet. The period of the pendulum is calculated using the small angle approximation and then compared to the observed period.

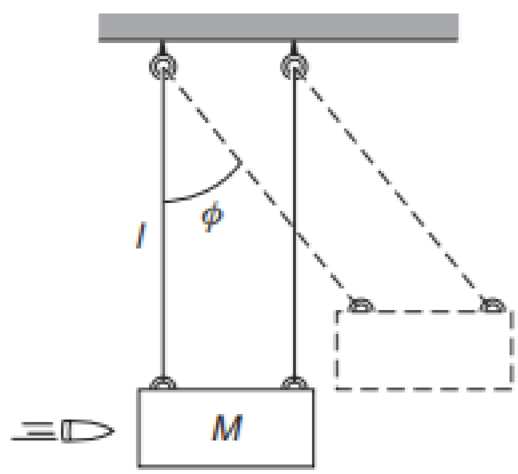

Figure 1: Initial configuration from the textbook problem

2 Construction

The apparatus consists of a wooden spherical pendulum hung from a stable wooden stand. The firing mechanism is a homemade slingshot, constructed from two wooden dowels that serve as posts and two rubber bands attached to the pouch to fire the projectile. The slingshot can be stretched to any length desired to vary the speed and angle of the projectile. For the projectile, 1 g spherical tablets were used, as they were small enough to fit in the attached slingshot. A protractor was attached to measure the maximum angle of deviation of the pendulum from the vertical. Slow motion videos of the collision were recorded, which were used to more accurately calculate the maximum angle and period of the pendulum.

Figure 2: Constructed pendulum

3 Calculations

3.1 Difference of squared velocities of projectile $v_i^2 - v_f^2$

By conservation of energy:

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}Ms^2 \tag{1}$$

where v_i and v_f are the initial and final velocity of the ball, respectively, and s is the instantaneous initial velocity of the pendulum. By conservation of energy of the pendulum:

$$\frac{1}{2}Ms^2 = Mg\Delta h \tag{2}$$

where

$$\Delta h = l(1 - \cos(\theta_f)) \tag{3}$$

Substituting Δh into (2) we have

$$\frac{1}{2}Ms^2 = Mgl(1 - \cos(\theta_f)) \tag{4}$$

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 + Mgl(1 - \cos(\theta_f)) \tag{5}$$

Rearranging,

$$v_i^2 - v_f^2 = 2gl\frac{M}{m}(1 - \cos(\theta_f)) \tag{6}$$

3.2 Trajectory and period of pendulum

To calculate the period of the pendulum once it has reached its maximum height from the collision, we begin with the pendulum's equation of motion:

$$M\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -Mg\sin(\theta) \tag{7}$$

where

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = l\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} \tag{8}$$

Substituting (8) and rearranging we have the differential equation

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g}{l}\sin(\theta) \tag{9}$$

Solving this differential equation using the first-order approximation $\sin(\theta) \approx \theta$ to calculate the pendulum's trajectory and period T we have

$$\theta(t) = \theta_0 \cos\left(\sqrt{\frac{g}{l}}t\right) \tag{10}$$

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{11}$$

4 Data

4.1 Constants

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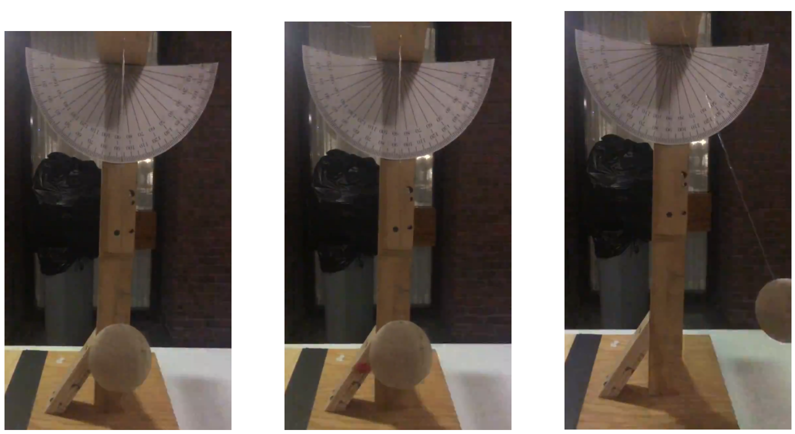
- $m = 0.001$ kg
- $M = 0.079$ kg
- $l = 0.343$ m
- $g = 9.81\ ms^{-2}$

4.2 Observations

Trial	Angle θ_f (deg)	$v_i^2 - v_f^2$ $(m^2 s^{-2})$	Period T (s)
1	34	90.8	1.24
2	35	96.1	1.24
3	25	49.8	1.25
4	30	71.2	1.26
5	25	49.8	1.26

The table displays five trials in which the pendulum was shot with the tablets. The maximum angles from the vertical vary from 25 - 35 deg. The difference in the squares of the initial and final velocities of the tablets ranges from the collision ranges from 49.8 - 96.1 ms^{-2}. The data shows that the greater the angle from the vertical, the greater the difference between the squares of the velocities. This is expected, because a greater angle deviation means more energy is given to the pendulum due to a greater amplitude. The period of the pendulum was calculated to be 1.175 s for all trials. The period is independent of the amplitude. The observed periods for the five trials were 1.24, 1.24, 1.25, 1.26, and 1.26 s for an average of 1.25 s. The deviations from the average are small, which demonstrates the consistency with theoretical expectation that period does not depend on amplitude. The error between the calculated and average period is ~ 6.00%



(a) Before collision (b) During collision (c) After collision

Figure 3: One full trial

5 Errors and Assumptions

Various assumptions were made during the calculations which may lead to observable sources of error. For example, the difference in the squares of the velocities is derived by considering only the energy of the pendulum when it reaches the maximum angle from the vertical. This does not account for energy lost as friction between the pendulum and the little tablets, the energy lost due to the sound produced by the collision, and the energy lost by the air drag as the pendulum moves towards its maximum angle from the vertical. Furthermore, the pendulum acquired a spin due to the torque caused by the collision not occurring exactly at the center of mass, as well as friction that might have resulted by the tied string and screw. The energy of the spin was also not accounted for. In other words, the calculations assume that the collisions involved were elastic, instantaneous and through the center of mass of two uniform spheres.

The period of the pendulum was calculated using the small-angle approximation. However, the actual angles reached by the pendulum ranged from 25 - 35 deg. The calculated period was 1.175 s while the average observed period captured by the videos was 1.25 s. This means that there was a 6.00% between the calculated and the observed period. While the small-angle approximation contributes to this resulted error, it should be noted that the observed period of the pendulum might have been affected by air drag, which is again a source of error.

Finally, all of the constants measured in this experiment are limited in their accuracy by the equipment used, as well as limited human observational abilities. The rulers used to measure the lengths in the calculations were readable to ± 0.5 in. The accuracy of the observed maximum angles reached by the pendulum are limited by human eye error and are readable to an estimated ± 0.5 deg. The scale used to measure the mass is readable to 0.001 kg. Overall, these assumptions, approximations, and measurements limit the accuracy of the results obtained in this experiment.

6 Results and Conclusions

In summary, this project sought to examine the motion of a ballistic pendulum resulting from shooting a sphere hanging from a string with small tablets using a slingshot. Although in the original problem, the two objects stick together, this was not the case in this experiment. This was the result of limited access to proper materials. The original problem is able to obtain a result for the final velocity of the bullet that shoots the pendulum. In this experiment, however, the two objects do not stick together after the collision. Through the described approximations and assumptions mentioned in the section above, the difference in the squares of the initial and final velocity of the little slingshot tablets are measured using the principle of conservation of energy. These differences range from 49.8 - 96.1 ms^{-2}, increasing with an increased maximum angle of the pendulum from the vertical as expected. The calculated period of the pendulum after the collision is 1.175 for all 5 trials, while the observed period was an average of 1.25 s. The accuracy of these results is limited by measurement and human errors, approximations, and assumptions.

7 Improvements

This project opens the way and leaves room for new and improved undertakings that can lead to higher accuracy and expanded physical knowledge. The original problem of the project sought to examine the motion of a ballistic pendulum in which a wooden block was shot with a bullet, with the two objects sticking together. In this project, however, small slingshot tablets are shot at a wooden sphere hanging from the string, and the two objects separate after the collision. The big deviation from the original experiment resulted from an inability to obtain the proper materials. With the right equipment, a new project more similar to the original problem can be conducted and the final velocity of the bullets measured. Moreover, with the help of a light gate, the final velocity of the tablets can estimated, even if the objects separate after the collision through the assumptions of the conservation of energy. More trials could be conducted to observe consistency or deviation of data and more accurate measuring equipment can be included for increased accuracy.