UNIVERSIDAD NACIONAL DE COLOMBIA	Flujos de Potencia Newton Raphson (Con TCSC – B Model)
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1) Se definen las bases del sistema

- Potencia base $S_b = 100 \cdot 10^6$ [VA]
- Voltaje base $V_b = 230 \cdot 10^3$ [V]

```
Sb = 100e6; %[VA]
Vb = 230e3; %[V]
```

2) Se crean las matrices con la información de la red (br_data, N_data):

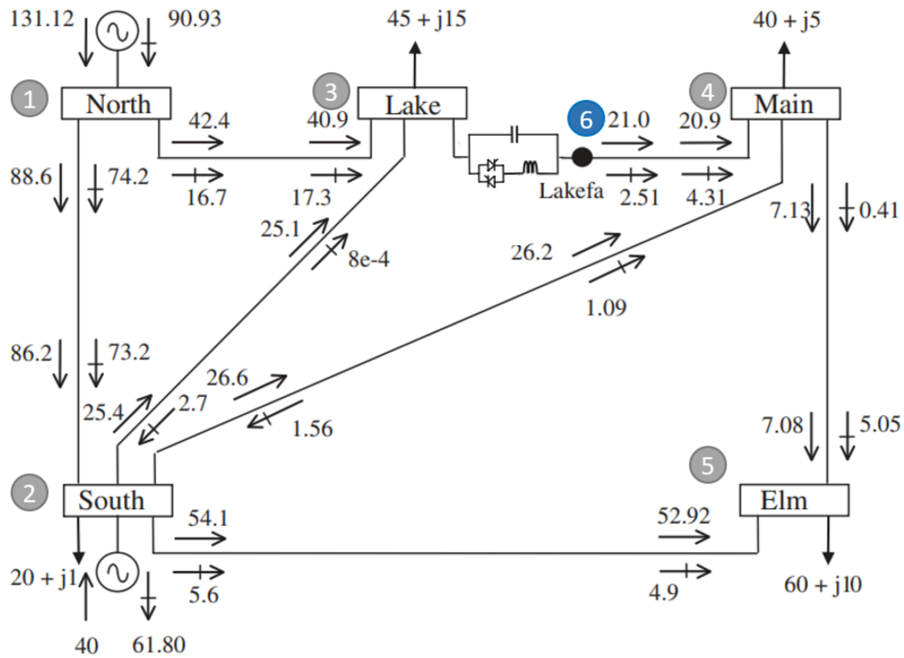
Se deben crear dos matrices para ingresar la información referente al circuito. La primera (br_data) brinda información acerca de las conexiones entre buses como la impedancia de la línea o los buses entre los cuales se realiza la conexión , mientras que la segunda (N_data) da información acerca de la naturaleza de cada uno de los buses, tipo de bus (SL, PV o PQ).

a) Para la matriz **br_data** se tiene que el formato es

$$\text{brdata} = [\text{Bus}_{\text{from}}, \text{Bus}_{\text{to}}, R, X, G, B] \text{ in } p.u.$$

b) Para la matriz **N_data** se tiene que el formato es:

$$N\text{data} = \{\#\text{Bus}, \text{type}('\text{SL}', '\text{PQ}', '\text{PV}'), V[p.u.], \angle\text{V}^{\text{o}}, \text{PG}[p.u.], \text{QG}[p.u.], \text{PL}[p.u.], \text{QL}[p.u.]\}$$



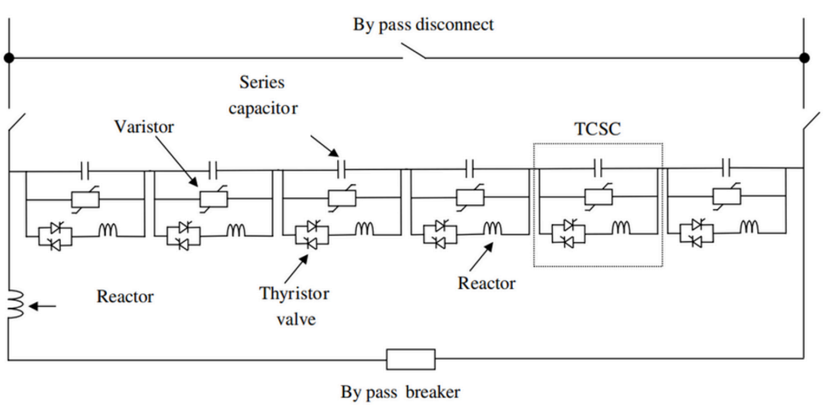

```
% Branches data R, X, G, B
% Types: General in form of impedance. (G & B are split in half at both ends).




% Node definitions
% {  #, type,   Vpu,    <Vº,    PG pu,  QG pu,  PL pu, QL pu}
N_data = {
    1,  'SL',   1.06,   0.0,    0.0,    0.0,    0.00,    0.00;
    2,  'PV',   1.0,    0.0,    0.4,    0.0,    0.20,    0.10;
    3,  'PQ',   1.0,    0.0,    0.0,    0.0,    0.45,    0.15;
    4,  'PQ',   1.0,    0.0,    0.0,    0.0,    0.40,    0.05;
    5,  'PQ',   1.0,    0.0,    0.0,    0.0,    0.60,    0.10;
    6,  'PQ',   1.0,    0.0,    0.0,    0.0,    0.00,    0.00 }; % Se crea el nodo 6 (Lakefa)


% br_data=[ bus1, bus2,   R ,      X ,     G ,     B   ] in p.u.
br_data = [ 1,       2,  0.02,    0.06,    0.0,    0.06;
            1,       3,  0.08,    0.24,    0.0,    0.05;
            2,       3,  0.06,    0.18,    0.0,    0.04;
            2,       4,  0.06,    0.18,    0.0,    0.04;
            2,       5,  0.04,    0.12,    0.0,    0.03;
            6,       4,  0.01,    0.03,    0.0,    0.02;  % || Se modifica la conexión nodo 3-4
            4,       5,  0.08,    0.24,    0.0,    0.05];


br_data=array2table(br_data,'VariableNames',{'FromBus','ToBus','R','X','G','B'});
N_data=cell2table(N_data,'VariableNames',{'n','type','Vm','Va','PG','QG','PL','QL'});
```

```
% THYRISTOR CONTROLLED SERIES COMPENSATOR reactance variable
% NTCSC : Number of TCSC’s
```

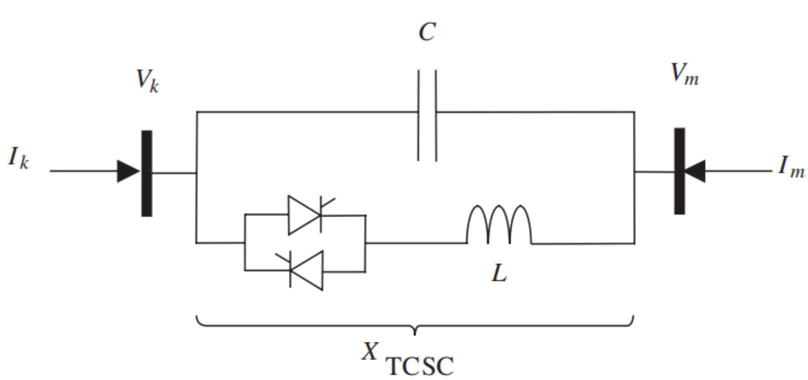
```
% TCSCsend : Sending bus
% TCSCrec : Receiving bus
% X : TCSC's reactance
% XLo : Lower reactance limit
% XHi : Higher reactance limit
% Flow : Power flow direction: 1 is for sending to receiving bus; -1
% indicates opposite direction
% Psp : Active power flow to be controlled
% status : Indicates control status for active power: 1 is on and 0 is off

% TCSC_data= [TCSCsend,TCSCrec,     X  ,  XLo  ,   XHi ,  Psp  , Flow, status];
TCSC_data  = [    3   ,    6   , -0.015 , -0.05 ,  0.05 ,  0.21 ,   1 ,   1   ];

TCSC_data=array2table(TCSC_data,'VariableNames',{'TCSCsend','TCSCrec','X','Xmin','Xmax','Psp','
```

TCSC_data = 1×8 table

	TCSCsend	TCSCrec	X	Xmin	Xmax	Psp	Flow	Status
1	3	6	-0.0150	-0.0500	0.0500	0.2100	1	1



(Fig 2.6 - Pag 19, Acha)

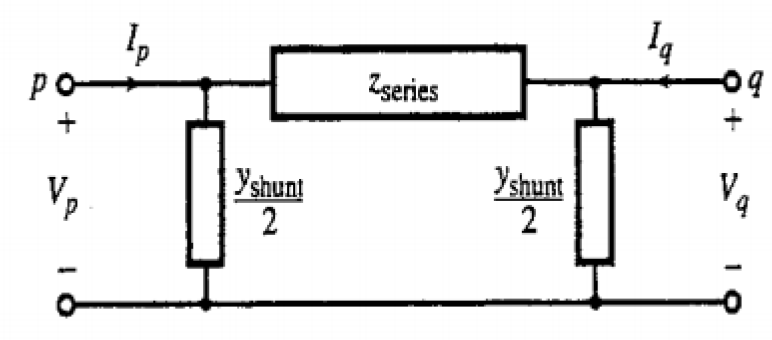


(Fig 2.12 - Pag 27, Acha)

3) Se crea Ybus con las redes de 2 puertos:

Se tiene la siguiente red de dos puertos, que representa el circuito del modelo pi de la línea de transmisión

$$\begin{bmatrix} V_i \\ V_j \end{bmatrix} = \begin{bmatrix} \frac{1}{z}+\frac{y}{2} & -\frac{1}{z} \\ -\frac{1}{z} & \frac{1}{z}+\frac{y}{2} \end{bmatrix} \begin{bmatrix} I_i \\ I_j \end{bmatrix}$$



Pi-equvalent model of a transmission line *(Pag 296, Bergen)*

```
n_br = size(br_data, 1); % Number of branches
num_nodes = max(max([br_data.FromBus,br_data.ToBus]));  % Number of nodes (starting at 1)
Ybus = zeros(num_nodes, num_nodes); % Empty Ybus Matrix

for kk = 1: size(br_data, 1)    % Iter over branch Data
    ii = br_data.FromBus(kk);                   % From bus
    jj = br_data.ToBus(kk);                     % To Bus
    z = br_data.R(kk) + 1j * br_data.X(kk);  % Line impedance
    y = br_data.G(kk) + 1j * br_data.B(kk);  % Shunt admitance

    Ybr=[1 / z + y / 2 ,    -1 / z;  ...     % Ybr (Two-Port Network)
            -1 / z     , 1 / z + y / 2];

    Ybus([ii,jj],[ii,jj]) = Ybus([ii,jj],[ii,jj])+ Ybr;                          % Add the Two-Por
end
```

Se extrae la información de las potencias y voltajes:

```
Vm_sp = N_data.Vm;                 %   [V]  Specified Voltages
th_sp = N_data.Va.* pi ./ 180; % [Rads] Specified Angles
PG = N_data.PG;                    % [p.u.] Active Power generators
QG = N_data.QG;                    % [p.u.] Reactive Power generators

PL = N_data.PL;                    % [p.u.] Active Power Loads
QL = N_data.QL;                    % [p.u.] Reactive Power Loads

P_sp = PG - PL;                         % [p.u.] Specified Active Power    Pnet= (PGen - PLoad)
Q_sp = QG - QL;                         % [p.u.] Specified Reactive Power  Qnet= (QGen - QLoad)
```

4) Construcción de la matriz Jacobiana

Se encuentran las dimensiones de la matriz jacobiana:

```
% Jacobian matrix size.

nL = sum(strcmp(N_data.type,'PQ'));% Number of PQ nodes  (#Loads)
nG = sum(strcmp(N_data.type,'PV'));% Number of PV nodes  (#Generators)
```

Se divide la matriz Jacobiana en la submatrices H-N-M-L

$$\begin{array}{cc} & \begin{array}{cc} \overbrace{}^{nL+nG} & \overbrace{}^{nL} \end{array} \\ \begin{array}{c} nL+nG \\ nL \end{array} & \begin{bmatrix} H & N \\ M & L \end{bmatrix}^{k} \end{array} \begin{bmatrix} \Delta\theta \\ \Delta V/V \end{bmatrix}^{k} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}^{k}$$

(Eq. 3.32, pag108, Gómez - Exposito)

$$H_{ij} = \frac{\partial P_i}{\partial \theta_j}; \qquad N_{ij} = \frac{V_j\,\partial P_i}{\partial V_j}$$
$$M_{ij} = \frac{\partial Q_i}{\partial \theta_j}; \qquad L_{ij} = \frac{V_j\,\partial Q_i}{\partial V_j}$$

(Eq. 3.34, pag109, Gómez - Exposito)

```
H = zeros( nL + nG, nL + nG );
L = zeros( nL, nL );
N = zeros( nL + nG , nL );
M = zeros( nL , nL + nG );
J = [H, N; M, L];
```

Se crea el vector de estados ($X = [\theta|V]^T$) y el vector de funciones no lineales ($f(x) = [\Delta P|\Delta Q]$

```
x_th = zeros( nL + nG, 1 ); % State vector for storing angles (theta)
x_v = zeros( nL , 1);       % State vector for storing Voltages
```

$$x = [\theta|V]^{\mathrm{T}} = [\theta_1, \theta_2, \ldots, \theta_{n-1}|V_1, V_2, \ldots, V_{n_L}]^{\mathrm{T}}$$ dim= (2nL+nG)

```
x = [x_th; x_v];
```

- Donde se tiene que:

$$\Delta P_i = P_i^{\mathrm{sp}} - V_i \sum_{j=1}^{n} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) \quad i = 1, 2, \ldots, n-1$$

$$\Delta Q_i = Q_i^{\mathrm{sp}} - V_i \sum_{j=1}^{n} V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) \quad i = 1, 2, \ldots, n_{\mathrm{L}}$$

(Eq. 3.30-3.31, pag108, Gómez - Exposito)

```
Dt_P = zeros( nL + nG, 1 ); % Delta P
Dt_Q = zeros( nL, 1);       % Delta Q
```

$$f(x) = [\Delta P | \Delta Q]^{\mathrm{T}} = [\Delta P_1, \Delta P_2, \ldots, \Delta P_{n-1} | \Delta Q_1, \Delta Q_2, \ldots, \Delta Q_{n_{\mathrm{L}}}]^{\mathrm{T}}$$ dim= (2nL+nG)

```
Dt = [Dt_P;Dt_Q];  %Delta Vector (same as f(x) Vector)

%% Define Unkown Values (Voltage and angle in PQ node - Angle in PV node)
% #Buses with unknwon voltage magnitudes ('PQ' buses)  (Delta Q index)
x_v_n=N_data(strcmp(N_data.type,'PQ'),:).n;

% #Buses with unknwon voltage angles ('PQ'|'PV' buses) (Delta P index)
x_th_n=N_data(strcmp(N_data.type,'PV')|strcmp(N_data.type,'PQ'),:).n;


x_n = [x_th_n; x_v_n]; % #Buses with unknown Angles and unknown voltages

max_iter = 100; %Define Max iteration number

V = zeros(num_nodes, max_iter);      % Matrix for storing node voltages per iter
Vm = zeros(num_nodes, max_iter);     % For storing node voltage magnitude per iter
th = zeros(num_nodes, max_iter);     % For storing node voltage angles per iter
V(:,1) = Vm_sp .* exp(1j .* th_sp); % Fill with initial Specified Voltages
Vm(:,1) = Vm_sp;                     % Fill with initial Specified Magnitud
th(:,1) = th_sp;                     % Fill with initial Specified Angle

err = 1.0;  % Define Error
k = 1;      % iteration counter
```

5) Inicia el método Newton Raphson

```
while (err > 1e-12 && k < max_iter)
% for cc = 1 : 3

    % *************************** Jacobian Terms Calculation.
    Vn = V( : , k ); %Phasors!!
    Vd = diag( Vn ); %Voltages in a diagonal matrix

    % Calculate active and reactive power per node
    Sn = Vd * conj( Ybus * V( : , k ) ); %  Sn=V(I)* | I=Y_bus·V | Sn=V·(Ybus·V)*
    Pn = real(Sn);  %Active Power calculated
```

```
Qn = imag(Sn);  % Reactive Power calculated
```

The transfer admittance matrix of the variable series compensator shown in Figure 5.9 is given by

$$\begin{bmatrix} I_k \\ I_m \end{bmatrix} = \begin{bmatrix} jB_{kk} & jB_{km} \\ jB_{mk} & jB_{mm} \end{bmatrix} \begin{bmatrix} V_k \\ V_m \end{bmatrix}. \tag{5.25}$$

For inductive operation, we have

$$\left.\begin{aligned} B_{kk} = B_{mm} = -\tfrac{1}{X_{\mathrm{TCSC}}}, \\ B_{km} = B_{mk} = \tfrac{1}{X_{\mathrm{TCSC}}}, \end{aligned}\right\} \tag{5.26}$$

and for capacitive operation the signs are reversed.

The active and reactive power equations at bus k are:

$$P_k = V_k V_m B_{km} \sin(\theta_k - \theta_m), \tag{5.27}$$

$$Q_k = -V_k^2 B_{kk} - V_k V_m B_{km} \cos(\theta_k - \theta_m). \tag{5.28}$$

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% TCSC Calculated Power

for n=1:size(TCSC_data,1)
    Bmm=-1/TCSC_data{n,'X'};
    Bmk= 1/TCSC_data{n,'X'};

    for kk=1:2 % for node k - node m

        % Angle difference between TCSC nodes
        A=th(TCSC_data{n,"TCSCsend"},k)-th(TCSC_data{n,"TCSCrec"},k);

        % Apparent Power at TCSC sending node
        Pcal=Vm(TCSC_data{n,"TCSCsend"},k)*Vm(TCSC_data{n,"TCSCrec"},k)*Bmk*sin(A);
        Qcal=-Vm(TCSC_data{n,"TCSCsend"},k)^2*Bmm...
             -Vm(TCSC_data{n,"TCSCsend"},k)*Vm(TCSC_data{n,"TCSCrec"},k)*Bmk*cos(A);

        % Update Apparent Power at TCSC sending node
        Pn(TCSC_data{n,"TCSCsend"})= Pn(TCSC_data{n,"TCSCsend"}) + Pcal;
        Qn(TCSC_data{n,"TCSCsend"})= Qn(TCSC_data{n,"TCSCsend"})  + Qcal;

        if kk==1
            TCSC_data{n,"TCSC_PQsend"}=Pcal + 1j*Qcal;
        else
            TCSC_data{n,"TCSC_PQrec"} = Pcal + 1j*Qcal;
        end

        % Switch send - rec definition
        send=TCSC_data{n,"TCSCsend"};
        TCSC_data{n,"TCSCsend"} = TCSC_data{n,"TCSCrec"};
        TCSC_data{n,"TCSCrec"}  = send;
```

```
        end
    end

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Se calculan los términos residuales.

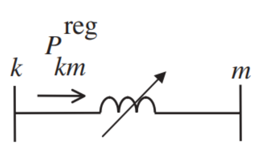
$$\Delta P_i = P_i^{\mathrm{sp}} - \overbrace{V_i \sum_{j=1}^{n} V_j (G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij})}^{P_n} \quad i = 1, 2, \ldots, n-1$$

$$\Delta Q_i = Q_i^{\mathrm{sp}} - \overbrace{V_i \sum_{j=1}^{n} V_j (G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij})}^{Q_n} \quad i = 1, 2, \ldots, n_{\mathrm{L}}$$

```
    % ************************** Residual Terms Calculations

    % Delta P
    Dt_P=P_sp(x_th_n(1:nL+nG))-Pn(x_th_n(1:nL+nG));
    % Delta Q
    Dt_Q=Q_sp(x_v_n(1:nL))-Qn(x_v_n(1:nL));


    Dt = [ Dt_P ;...
           Dt_Q ];
```

Se calcula el error en la potencia sobre el TCSC

$$\Delta P_{km}^{X_{\mathrm{TCSC}}} = P_{km}^{\mathrm{reg}} - P_{km}^{X_{\mathrm{TCSC}},\,\mathrm{cal}}$$




```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%             TCSC Power Mismatches
    if k > 1
        for n = 1:size(TCSC_data,1)

            if TCSC_data{n,'Status'} == 1
                Bmk = 1/TCSC_data{n,'X'};

                for kk = 1:2
                     % Angle difference between TCSC nodes
                     A=th(TCSC_data{n,"TCSCsend"},k)-th(TCSC_data{n,"TCSCrec"},k);

                     Pcal=Vm(TCSC_data{n,"TCSCsend"},k)*Vm(TCSC_data{n,"TCSCrec"},k)*Bmk*sin(A
```

```
                if (TCSC_data{n,'Flow'} == 1 && kk == 1) || (TCSC_data{n,'Flow'} == -1 &&
                    Dt(2*nL+nG+n) = TCSC_data{n,'Psp'}-Pcal;
                    break;
                end

                % Switch send - rec definition
                send = TCSC_data{n,"TCSCsend"};
                TCSC_data{n,"TCSCsend"} = TCSC_data{n,"TCSCrec"};
                TCSC_data{n,"TCSCrec"}  = send;
            end
        else
            Dt(2*nL+nG+n) = 0; % Add Zero Mismatch delta(Pkm)=0
        end
    end
end
```

```
%% Find Jacobian
```

Se obtienen las matrices H,L,N,M donde:

$$\text{For } i \neq j$$

$$H_{ij} = L_{ij} = V_i V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij})$$

$$N_{ij} = -M_{ij} = V_i V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij})$$

$$\text{For } i = j$$

$$H_{ii} = -Q_i - B_{ii} V_i^2 \quad L_{ii} = Q_i - B_{ii} V_i^2$$

$$N_{ii} = P_i + G_{ii} V_i^2 \quad M_{ii} = P_i - G_{ii} V_i^2$$

(Table 3.1, pag109, Gómez - Exposito)

```
  % H Matrix.   H -> Dim: (nL+nG) X (nL+nG)

    for mm = 1 : nL + nG        %Iter over rows H matrix
        for nn = 1 : nL + nG    %Iter over cols H matrix
            ii = x_th_n(mm);    % #node corresponding to row mm
            jj = x_th_n(nn);    % #node corresponding to column nn

            % From Prev Equations:
            if ii == jj
                H(mm,nn) = - Qn(ii) - imag( Ybus(ii,jj) ) .* abs( V(ii,k) ).^2;
            else
                H(mm,nn) = abs( V(ii,k) ) * abs( V(jj,k) ) .* ( ...
                    real(Ybus(ii,jj)) * sin( th(ii,k) - th(jj,k) ) - ...
                    imag(Ybus(ii,jj)) * cos( th(ii,k) - th(jj,k) ) );
            end
        end
    end
```

```
    % N Matrix. N -> Dim: (nL+nG) X (nL)
    for mm = 1 : nL + nG     %  Iter over rows N matrix
        for nn = 1 : nL      %  Iter over cols N matrix
            ii = x_th_n(mm); % #node corresponding to row mm
            jj = x_v_n(nn);  % #node corresponding to column nn

            % From Prev Equations:
            if ii == jj
                N(mm,nn) = Pn(ii) + real(Ybus(ii,jj)) .* abs(V(ii,k)).^2;
            else
                N(mm,nn) = abs( V(ii,k) ) * abs( V(jj,k) ) .* ( ...
                    real(Ybus(ii,jj)) * cos( th(ii,k) - th(jj,k) ) + ...
                    imag(Ybus(ii,jj)) * sin( th(ii,k) - th(jj,k) ) );
            end
        end
    end

    % M Matrix. M -> Dim: (nL) X (nL+nG)
    for mm = 1 : nL               %  Iter over rows M matrix
        for nn = 1 : nL + nG      %  Iter over cols M matrix
            ii = x_v_n(mm);       % #node corresponding to row mm
            jj = x_th_n(nn);      % #node corresponding to column nn

            % From Prev Equations:
            if ii == jj
                M(mm,nn) = Pn(ii) - real(Ybus(ii,jj)) .* abs(V(ii,k)).^2;
            else
                M(mm,nn) = -abs( V(ii,k) ) * abs( V(jj,k) ) .* ( ...
                    real(Ybus(ii,jj)) * cos( th(ii,k) - th(jj,k) ) + ...
                    imag(Ybus(ii,jj)) * sin( th(ii,k) - th(jj,k) ) );
            end
        end
    end

    % L Matrix. L -> Dim: (nL) X (nL)
    for mm = 1 : nL           %  Iter over rows L matrix
        for nn = 1 : nL       %  Iter over cols L matrix
            ii = x_v_n(mm); % #node corresponding to row mm
            jj = x_v_n(nn); % #node corresponding to column nn

            % From Prev Equations:
            if ii == jj
                L(mm,nn) = Qn(ii) - imag(Ybus(ii,jj)) .* abs(V(ii,k)).^2;
            else
                L(mm,nn) = abs( V(ii,k) ) * abs( V(jj,k) ) .* ( ...
                    real(Ybus(ii,jj)) * sin( th(ii,k) - th(jj,k) ) - ...
                    imag(Ybus(ii,jj)) * cos( th(ii,k) - th(jj,k) ) );
            end
        end
    end
   % The jacobian matrix results in:
    J = [H, N;...
        M, L];
```

Se agregan valores al jacobiano de la conexión del TCSC.

$$\begin{bmatrix} \Delta P_k \\ \Delta P_m \\ \Delta Q_k \\ \Delta Q_m \\ \Delta P_{km}^{X_{\mathrm{TCSC}}} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_k}{\partial \theta_k} & \frac{\partial P_k}{\partial \theta_m} & \frac{\partial P_k}{\partial V_k} V_k & \frac{\partial P_k}{\partial V_m} V_m & \frac{\partial P_k}{\partial X_{\mathrm{TCSC}}} X_{\mathrm{TCSC}} \\ \frac{\partial P_m}{\partial \theta_k} & \frac{\partial P_m}{\partial \theta_m} & \frac{\partial P_m}{\partial V_k} V_k & \frac{\partial P_m}{\partial V_m} V_m & \frac{\partial P_m}{\partial X_{\mathrm{TCSC}}} X_{\mathrm{TCSC}} \\ \frac{\partial Q_k}{\partial \theta_k} & \frac{\partial Q_k}{\partial \theta_m} & \frac{\partial Q_k}{\partial V_k} V_k & \frac{\partial Q_k}{\partial V_m} V_m & \frac{\partial Q_k}{\partial X_{\mathrm{TCSC}}} X_{\mathrm{TCSC}} \\ \frac{\partial Q_m}{\partial \theta_k} & \frac{\partial Q_m}{\partial \theta_m} & \frac{\partial Q_m}{\partial V_k} V_k & \frac{\partial Q_m}{\partial V_m} V_m & \frac{\partial Q_m}{\partial X_{\mathrm{TCSC}}} X_{\mathrm{TCSC}} \\ \frac{\partial P_{km}^{X_{\mathrm{TCSC}}}}{\partial \theta_k} & \frac{\partial P_{km}^{X_{\mathrm{TCSC}}}}{\partial \theta_m} & \frac{\partial P_{km}^{X_{\mathrm{TCSC}}}}{\partial V_k} V_k & \frac{\partial P_{km}^{X_{\mathrm{TCSC}}}}{\partial V_m} V_m & \frac{\partial P_{km}^{X_{\mathrm{TCSC}}}}{\partial X_{\mathrm{TCSC}}} X_{\mathrm{TCSC}} \end{bmatrix} \begin{bmatrix} \Delta \theta_k \\ \Delta \theta_m \\ \frac{\Delta V_k}{V_k} \\ \frac{\Delta V_m}{V_m} \\ \frac{\Delta X_{\mathrm{TCSC}}}{X_{\mathrm{TCSC}}} \end{bmatrix}$$

Es decir, se debe agregar al jacobiano las componentes correspondientes a X_{TCSC} (notar que B_{km}=-B_{mm})

Se define:

$$H_{\mathrm{km}} = -V_k V_m B_{\mathrm{mm}} \cos(\theta_k - \theta_m)$$
$$N_{\mathrm{km}} = V_k V_m B_{\mathrm{mm}} \sin(\theta_k - \theta_m)$$

$$\frac{\partial P_k}{\partial \theta_k} = -V_k^2 B_{\mathrm{kk}}$$

$$\frac{\partial P_k}{\partial \theta_m} = -V_k V_m B_{\mathrm{km}} \cos(\theta_k - \theta_m) = V_k V_m B_{\mathrm{mm}} \cos(\theta_k - \theta_m) = -H_{\mathrm{km}}$$

$$\frac{\partial P_k}{\partial V_m} = V_k V_m B_{\mathrm{km}} \sin(\theta_k - \theta_m) = -V_k V_m B_{\mathrm{mm}} \sin(\theta_k - \theta_m) = -N_{\mathrm{km}}$$

$$\frac{\partial P_k}{\partial V_k} = V_k^2 G_{\mathrm{kk}} = 0$$

$$\frac{\partial Q_k}{\partial V_k} = -V_k^2 B_{\mathrm{kk}}$$

$$\frac{\partial Q_k}{\partial \theta_m} = -V_k V_m B_{\mathrm{km}} \sin(\theta_k - \theta_m) = V_k V_m B_{\mathrm{mm}} \sin(\theta_k - \theta_m) = N_{\mathrm{km}}$$

$$\frac{\partial Q_k}{\partial V_m} = -V_k V_m B_{\mathrm{km}} \cos(\theta_k - \theta_m) = V_k V_m B_{\mathrm{mm}} \cos(\theta_k - \theta_m) = -H_{\mathrm{km}}$$

$$\frac{\partial Q_k}{\partial \theta_k} = -V_k^2 G_{kk} = 0$$ (Eq 4.30-4.37 , Acha)

Recordando que:

$$B_{km} = B_{mk} = -B_{kk} = -B_{mm} = \frac{1}{X_{TCSC}}$$

$$P_k = V_k V_m B_{mk} \sin(\theta_k - \theta_m) = \frac{V_k V_m \sin(\theta_k - \theta_m)}{X_{TCSC}}$$

$$Q_k = -V_k^2 B_{km} - V_k V_m B_{mk} \cos(\theta_k - \theta_m) = -\frac{V_k^2}{X_{TCSC}} - \frac{V_k V_m \cos(\theta_k - \theta_m)}{X_{TCSC}}$$

Se encuentra que:

$$\frac{\partial P_k}{\partial X} X = -\frac{V_k V_m \sin(\theta_k - \theta_m)}{X_{TCSC}{}^2}(X_{TCSC}) = -\frac{V_k V_m \sin(\theta_k - \theta_m)}{X_{TCSC}} = V_k V_m B_{mm} \sin(\theta_k - \theta_m) = -N_{km}$$

$$\frac{\partial Q_k}{\partial X} X = \left(+\frac{V_{kk}^2}{X_{TCSC}^2} + \frac{V_k V_m \cos(\theta_k - \theta_m)}{X_{TCSC}{}^2}\right)(X_{TCSC}) = +\frac{V_{kk}^2}{X_{TCSC}} + \frac{V_k V_m \cos(\theta_k - \theta_m)}{X_{TCSC}} = -V_{kk}^2 B_{mm} - V_k V_m B_{mm} \cos(\theta_k - \theta_m) = H_{km} - V_{kk}^2 B_{m}$$

$$\frac{\partial P_{km}}{\partial X} X = \frac{\partial P_k}{\partial X} X$$

(Appendix A Section A.2 , Acha)

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%                TCSC JACOBIAN UPDATE

 for n=1:size(TCSC_data,1)
     Bmm=-1/TCSC_data{n,'X'};
     Bmk= 1/TCSC_data{n,'X'};

     for kk=1:2 % for node k - node m

         DP_send_Index = find(x_th_n==TCSC_data{n,'TCSCsend'});% get delta(P) index of the T
         DQ_send_Index = find(x_v_n==TCSC_data{n,'TCSCsend'}); % get delta(Q) index of the T

         DP_rec_Index = find(x_th_n==TCSC_data{n,'TCSCrec'});% get delta(P) index of the TCS
         DQ_rec_Index = find(x_v_n==TCSC_data{n,'TCSCrec'}); % get delta(Q) index of the TCS



         % Angle difference between TCSC nodes
         A=th(TCSC_data{n,"TCSCsend"},k)-th(TCSC_data{n,"TCSCrec"},k);


         Hkm=-Vm(TCSC_data{n,"TCSCsend"},k)*Vm(TCSC_data{n,"TCSCrec"},k)*Bmm*cos(A);
```

```
        Nkm= Vm(TCSC_data{n,"TCSCsend"},k)*Vm(TCSC_data{n,"TCSCsend"},k)*Bmm*sin(A);

        %% Jacobians Update

        % dPk/dAk
        J(DP_send_Index,DP_send_Index)            = J(DP_send_Index,DP_send_Index)...
                                                    -Vm(TCSC_data{n,"TCSCsend"},k)^2*Bmm;
        % dPk/dAm
        J(DP_send_Index,DP_rec_Index)             = J(DP_send_Index,DP_rec_Index)...
                                                    - Hkm;
        % dPk/dVm
        J(DP_send_Index,nL+nG+DQ_rec_Index)       = J(DP_send_Index,nL+nG+DQ_rec_Index)...
                                                    - Nkm;
        % dQk/dVk
        J(nL+nG+DQ_send_Index,nL+nG+DQ_send_Index)= J(nL+nG+DQ_send_Index,nL+nG+DQ_send_Inc
                                                    -Vm(TCSC_data{n,"TCSCsend"},k)^2*Bmm;
        % dQk/dAm
        J(nL+nG+DQ_send_Index,DP_rec_Index)       = J(nL+nG+DQ_send_Index,DP_rec_Index)...
                                                    + Nkm;
        % dQk/dVm
        J(nL+nG+DQ_send_Index,nL+nG+DQ_rec_Index) = J(nL+nG+DQ_send_Index,nL+nG+DQ_rec_Inde
                                                    - Hkm;

        if k>1
            if TCSC_data{n,'Status'} == 1
                if (TCSC_data{n,'Flow'} == 1 && kk == 1) || (TCSC_data{n,'Flow'} == -1 && k

                    %dPkm/dAk
                    J((2*nL+nG)+1,DP_send_Index)       = +Hkm;

                    %dPkm/dVk * Vk
                    J((2*nL+nG)+1,nL+nG+DQ_send_Index) = -Nkm;

                    %dPkm/dAm
                    J((2*nL+nG)+1,DP_rec_Index)        = -Hkm;

                    %dPkm/dVm * Vm
                    J((2*nL+nG)+1,nL+nG+DQ_rec_Index)  = -Nkm;

                    %dPkm/dx * X
                    J((2*nL+nG)+1,(2*nL+nG)+1) = +Nkm;
                end

                %dPk/dx * X
                J(DP_send_Index,(2*nL+nG)+1)= Nkm;

                %dQk/dx * X
                J((nL+nG)+DQ_send_Index,(2*nL+nG)+1)= Hkm-Vm(TCSC_data{n,'TCSCsend'},k)^2*E
            else
                J((2*nL+nG)+1,(2*nL+nG)+1) = 1; %Delta(X)=0
            end

        end
        % Switch send - rec definition
```

```
            send=TCSC_data{n,"TCSCsend"};
            TCSC_data{n,"TCSCsend"} = TCSC_data{n,"TCSCrec"};
            TCSC_data{n,"TCSCrec"}  =send;
        end
    end
```

Se actualiza la solución del sistema donde:

$$\begin{bmatrix}\theta_{k+1}\\ V_{k+1}\end{bmatrix} = \begin{bmatrix}\theta_{k+1}\\ V_{k+1}\end{bmatrix} + J^{-1}\cdot\begin{bmatrix}\Delta P_k\\ \Delta Q_k\end{bmatrix}$$

```
    % ************************** System Solution
    x = J \ Dt; % Delta theta/V/x  -> inv(J)*Dt
    x_th = x( 1 : nL + nG );                    % Adjust in unknown Angles
    x_v = x( nL + nG + 1 : 2 * nL + nG );       % Adjust in unknown Voltages

    Vm(:,k + 1) = Vm(:,k); % new magnitudes
    th(:,k + 1) = th(:,k); % new Angles

    th(x_th_n, k + 1) = x_th + th(x_th_n, k); % New angle   =  Adjust in unknown Angle   + prev
    Vm(x_v_n, k + 1)  = x_v.*Vm(x_v_n, k) + Vm(x_v_n, k);   % new voltage =  Adjust in unknown
```

$$\Delta X_{\mathrm{TCSC}} = X_{\mathrm{TCSC}}^{(i)} - X_{\mathrm{TCSC}}^{(i-1)},$$

```
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    %%            TCSC Updating and Limits


    for n=1:size(TCSC_data,1)
        %% TCSC impedance update
        if k > 1
            if TCSC_data{n,'Status'}==1
                TCSC_data{n,'X'} = TCSC_data{n,'X'} + x((2*nL+nG)+n)*TCSC_data{n,'X'};
            end
        end


        % Check impedance Limits
        if TCSC_data{n,'X'} < TCSC_data{n,'Xmin'} || TCSC_data{n,'X'} > TCSC_data{n,'Xmax'}
                TCSC_data{n,'Status'} = 0;
            if TCSC_data{n,'X'} < TCSC_data{n,'Xmin'}
                TCSC_data{n,'X'} = TCSC_data{n,'Xmin'};
            elseif TCSC_data{n,'X'} > TCSC_data{n,'Xmax'}
                TCSC_data{n,'X'} = TCSC_data{n,'Xmax'};
            end
        end
    end

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
    V(:,k + 1) = Vm(:,k + 1) .* exp( 1j .* th(:,k + 1) ); % Calculate phasors.


    err = max( abs( Dt )  ); % Calculate error

    k = k + 1;  % Add 1 to iterations
end
```

6) Cálculos finales:

Para el cálculo de la potencia aparente se tiene que:

$$S=\begin{bmatrix}V_1&0&0&0\\0&V_2&0&0\\0&0&\ddots&\vdots\\0&0&\cdots&V_n\end{bmatrix}\cdot\left(Y_{\text{Bus}}\cdot\begin{bmatrix}V_1\\V_2\\\vdots\\V_n\end{bmatrix}\right)^*=\begin{bmatrix}V_1&0&0&0\\0&V_2&0&0\\0&0&\ddots&\vdots\\0&0&\cdots&V_n\end{bmatrix}\cdot\left(\begin{bmatrix}I_1\\I_2\\\vdots\\I_n\end{bmatrix}\right)^*=\begin{bmatrix}V_1&0&0&0\\0&V_2&0&0\\0&0&\ddots&\vdots\\0&0&\cdots&V_n\end{bmatrix}\cdot\left(\begin{bmatrix}I_1^*\\I_2^*\\\vdots\\I_n^*\end{bmatrix}\right)=\begin{bmatrix}V_1I_1^*\\V_2I_2^*\\\vdots\\V_nI_n^*\end{bmatrix}=\begin{bmatrix}S_1\\S_2\\\vdots\\S_n\end{bmatrix}$$

```
% Complex power calculation for each node.
Vn = V( : , k );
Vd = diag( Vn );
Sn = Vd * conj( Ybus * V( : , k ) );
Pn = real(Sn);
Qn = imag(Sn);

SGn = Sn + (PL + 1j * QL);
SLn = SGn - Sn;

% Complex Power Flow Through Branches.

S_pf = zeros( 2*size(br_data, 1) , 5);

bc = 1;
for kk = 1: size(br_data, 1)
    ii = br_data.FromBus(kk);
    jj = br_data.ToBus(kk);
    z = br_data.R(kk) + 1j * br_data.X(kk);
    y = br_data.G(kk) + 1j * br_data.B(kk);
    Ybr=[1 / z + y / 2 ,    -1 / z;  ...     % Ybr (Two-Port Network)
            -1 / z     , 1 / z + y / 2];
    S_br=diag([Vn(ii),Vn(jj)])*conj(Ybr*[Vn(ii);Vn(jj)]);

    S_pf_br= [ii, jj, real(S_br(1)), imag(S_br(1)), abs(S_br(1));...% Apparent power from i to
              jj, ii, real(S_br(2)), imag(S_br(2)), abs(S_br(2))];  % Apparent power from j to

    S_pf(2*kk-1:2*kk,:)=S_pf_br;
end
```

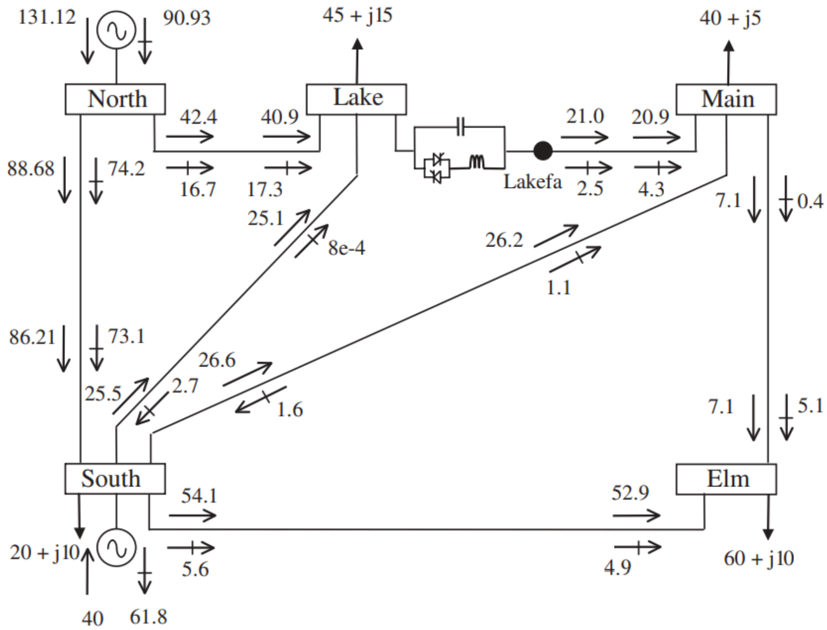
7) Se presentan los resultados obtenidos

```
%% Results printing.

for i=1:1 % This "for" is used to display everything once.

    fprintf('     ******************************************** \n')
    fprintf('     **           Newton-Raphson Results        ** \n')
    fprintf('     ******************************************** \n\n')

    fprintf('-- Number of Iterarions: %d \n', k-1)
    fprintf('-- Error: %6.3e p.u.\n\n', err)

    VarNames = {'Node#', 'Type', 'V p.u.', ' ∠V(º)','PG p.u.','QG p.u.','PL p.u.','QL p.u.'};
    fprintf(1,'\t%s\t%s\t%s\t%s\t%s\t%s\t%s\t%s\n', VarNames{:})
    fprintf(1, '--------------------------------------------------------------------\n')

    for ii = 1 : num_nodes
        fprintf('   \t%d\t%s\t%6.3f\t%6.3f\t%6.3f\t%6.3f\t%6.3f\t%6.3f\t\t\n', ii, char(N_data{ii
            abs(Vn(ii)), angle(Vn(ii)).*180./pi, real(SGn(ii)), imag(SGn(ii)), real(SLn(ii)), i
    end
end
```

```
     ********************************************
     **          Newton-Raphson Results        **
     ********************************************
-- Number of Iterarions: 7
-- Error: 8.921e-13 p.u.
 Node# Type V p.u.  ∠V(º) PG p.u. QG p.u. PL p.u. QL p.u.
--------------------------------------------------------------------
    1 SL  1.060  0.000  1.311  0.909  0.000  0.000
    2 PV  1.000 -2.038  0.400 -0.618  0.200  0.100
    3 PQ  0.987 -4.727 -0.210 -0.024  0.450  0.150
    4 PQ  0.984 -4.811 -0.000 -0.000  0.400  0.050
    5 PQ  0.972 -5.701 -0.000 -0.000  0.600  0.100
    6 PQ  0.988 -4.461  0.210  0.025  0.000  0.000
```

```
array2table(S_pf,'VariableNames',{'FromBus','ToBus','P','Q','S'});
```

```
VM =

    1.0600    1.0000    0.9870    0.9844    0.9718    0.9876

>> VA

VA =

         0   -2.0380   -4.7274   -4.8113   -5.7009   -4.4605
```



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