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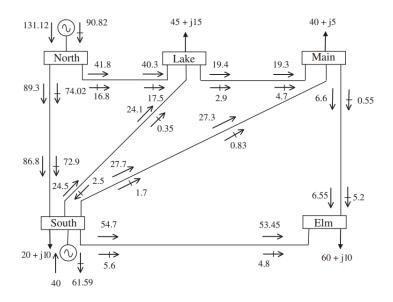
1) Se definen las bases del sistema

- Potencia base $S_b = 100 \cdot 10^6 \, \text{[VA]}$
- Voltaje base $V_b = 230 \cdot 10^3 \, [V]$

Sb = 100e6; %[VA] Vb = 230e3; %[V]

2) Se crean las matrices con la información de la red (br_data, N_data):

Se deben crear dos matrices para ingresar la información referente al circuito. La primera (br_data) brinda información acerca de las conexiones entre buses como la impedancia de la línea o los buses entre los cuales se realiza la conexión, mientras que la segunda (N_data) da información acerca de la naturaleza de cada uno de los buses, tipo de bus (SL, PV o PQ).



a) Para la matriz **br_data** se tiene que el formato es

brdata = [Bus_{from}, Bus_{to}, R, X, G, B] in p. u.

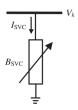
b) Para la matriz **N_data** se tiene que el formato es:

 $N \text{data} = \{ \# \text{Bus}, \text{type}(\text{'SL'}, \text{'PQ'}, \text{'PV'}), V[p.\,u.], \angle \text{V}^{\circ}, \text{PG}[p.\,u.], \text{QG}[p.\,u.], \text{PL}[p.\,u.], \text{QL}[p.\,u.] \}$

```
% Branches data R, X, G, B
% Types: General in form of impedance. (G & B are split in half at both ends).
```

```
% br data=[ bus1, bus2, R, X,
                                     G,
                                            B] in p.u.
br data = [
                  2, 0.02,
                             0.06,
                                     0.0,
                                            0.06;
          1,
                 3, 0.08,
          1,
                             0.24,
                                     0.0,
                                            0.05;
                 3, 0.06,
          2,
                             0.18,
                                     0.0,
                                            0.04;
          2,
                 4, 0.06,
                             0.18,
                                     0.0,
                                            0.04;
          2,
                 5, 0.04,
                             0.12,
                                     0.0,
                                            0.03;
                 4, 0.01,
          3,
                             0.03,
                                     0.0,
                                            0.02;
                 5, 0.08,
          4,
                           0.24,
                                     0.0,
                                            0.05];
% Node definitions
% {  #, type, Vpu,
                    <Vº, PG pu, QG pu, PL pu, QL pu}
N_data = {
   1, 'SL',
             1.06,
                     0.0,
                           0.0,
                                   0.0,
                                         0.00,
                                                   0.00;
      'PV',
            1.0,
                           0.4,
   2,
                     0.0,
                                   0.0,
                                         0.20,
                                                   0.10;
      'PQ',
            1.0,
                           0.0,
                                   0.0,
                                         0.45,
                     0.0,
                                                   0.15;
      'PQ',
   4,
            1.0,
                     0.0,
                            0.0,
                                   0.0,
                                          0.40,
                                                   0.05;
   5, 'PQ',
              1.0,
                     0.0, 0.0, 0.0, 0.60,
                                                   0.10;  };
br_data=array2table(br_data, 'VariableNames', {'FromBus', 'ToBus', 'R', 'X', 'G', 'B'});
N_data=cell2table(N_data,'VariableNames',{'n','type','Vm','Va','PG','QG','PL','QL'});
```

c) Para el SVC:



```
% VARIABLE SHUNT SUSCEPTANCE MODEL

% SVCsend : Compensated bus
% B : Initial SVC's susceptance value (p.u.)
% Bmin : Lower limit of variable susceptance (p.u.)
% Bmax : Higher limit of variable susceptance (p.u)
% TarVol : Target nodal voltage magnitude to be controlled by SVC (p.u.)
% VSta : Indicate control status for nodal voltage magnitude:1 is on and 0 is off

% SVC_data = [SVC node, SVC B, SVC Bmin, SVC Bmax, SVC TarVol, SVC Vsta]
SVC_data = [ 3 , 0.02, -0.25, 0.25 , 1 , 1];

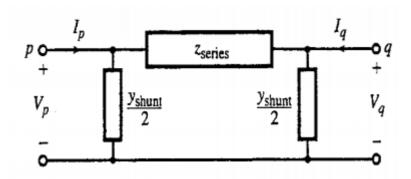
SVC_data=array2table(SVC_data,'VariableNames',{'SVCBus','B','Bmin','Bmax','Tv','Status'})
```

SVC_data = 1×6 table									
	SVCBus	В	Bmin	Bmax	Tv	Status			
1	3	0.0200	-0.2500	0.2500	1	1			

3) Se crea Ybus con las redes de 2 puertos:

Se tiene la siguiente red de dos puertos, que representa el circuito del modelo pi de la línea de transmisión

$$\begin{bmatrix} V_i \\ V_j \end{bmatrix} = \begin{bmatrix} \frac{1}{z} + \frac{y}{2} & -\frac{1}{z} \\ -\frac{1}{z} & \frac{1}{z} + \frac{y}{2} \end{bmatrix} \begin{bmatrix} I_i \\ I_j \end{bmatrix}$$



Pi-equvalent model of a transmission line (Pag 296, Bergen)

Se extrae la información de las potencias y voltajes:

```
Vm sp = N data.Vm;
                                    [V] Specified Voltages
th_sp = N_data.Va.* pi ./ 180; % [Rads] Specified Angles
PG = N data.PG;
                                % [p.u.] Active Power generators
QG = N_data.QG;
                               % [p.u.] Reactive Power generators
PL = N data.PL;
                               % [p.u.] Active Power Loads
QL = N_data.QL;
                               % [p.u.] Reactive Power Loads
%% Calculated NetPowers
                                    % [p.u.] Specified Active Power Pnet= (PGen - PLoad)
P_sp = PG - PL;
                                    % [p.u.] Specified Reactive Power Qnet= (QGen - QLoad)
Q sp = QG - QL;
```

4) Construcción de la matriz Jacobiana

Se encuentran las dimensiones de la matriz jacobiana:

```
% Jacobian matrix size.

nL = sum(strcmp(N_data.type,'PQ'));% Number of PQ nodes (#Loads)
nG = sum(strcmp(N_data.type,'PV'));% Number of PV nodes (#Generators)
```

Se divide la matriz Jacobiana en la submatrices H-N-M-L

$$\left[egin{array}{cccc} H & N \ M & L \end{array}
ight]^k \left[egin{array}{cccc} \Delta heta \ \Delta V/V \end{array}
ight]^k = \left[egin{array}{cccc} \Delta P \ \Delta Q \end{array}
ight]^k$$

(Eq. 3.32, pag108, Gómez - Exposito)

$$H_{ij} = rac{\partial P_i}{\partial \theta_j}; \qquad N_{ij} = rac{V_j}{\partial V_j} rac{\partial P_i}{\partial V_j} \ M_{ij} = rac{\partial Q_i}{\partial \theta_j}; \qquad L_{ij} = rac{V_j}{\partial V_i} rac{\partial Q_i}{\partial V_j}$$

(Eq. 3.34, pag109, Gómez - Exposito)

```
H = zeros( nL + nG, nL + nG );
L = zeros( nL, nL );
N = zeros( nL + nG , nL );
M = zeros( nL , nL + nG );
J = [H, N; M, L];
```

Se crea el vector de estados ($X = \lceil \theta \mid V \rceil^T$) y el vector de funciones no lineales ($f(x) = \lceil \Delta P \mid \Delta Q \rceil$

$$x = [\theta|V]^{T} = [\theta_{1}, \theta_{2}, \dots, \theta_{n-1}|V_{1}, V_{2}, \dots, V_{n_{L}}]^{T}$$
 dim= (2nL+nG)

```
x = [x_th; x_v];
```

• Donde se tiene que:

$$\Delta P_i = P_i^{\text{sp}} - V_i \sum_{j=1}^n V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \quad i = 1, 2, ..., n-1$$

$$\Delta Q_i = Q_i^{\text{sp}} - V_i \sum_{i=1}^n V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \quad i = 1, 2, \dots, n_{\text{L}}$$

(Eq. 3.30-3.31, pag108, Gómez - Exposito)

```
f(x) = [\Delta P | \Delta Q]^{\mathrm{T}} = [\Delta P_1, \Delta P_2, \dots, \Delta P_{n-1} | \Delta Q_1, \Delta Q_2, \dots, \Delta Q_{n_{\mathrm{L}}}]^{\mathrm{T}}  dim= (2nL+nG)
```

```
Dt = [Dt P;Dt Q]; %Delta Vector (same as f(x) Vector)
%% Define Unkown Values (Voltage and angle in PQ node - Angle in PV node)
% #Buses with unknwon voltage magnitudes ('PQ' buses) (Delta Q index)
x_v_n=N_data(strcmp(N_data.type,'PQ'),:).n;
% #Buses with unknwon voltage angles ('PQ'|'PV' buses) (Delta P index)
x th n=N data(strcmp(N data.type,'PV')|strcmp(N data.type,'PO'),:).n;
x_n = [x_{n}, x_{n}]; #Buses with unknown Angles and unknown voltages
max_iter = 100; %Define Max iteration number
V = zeros(num_nodes, max_iter); % Matrix for storing node voltages per iter
V(:,1) = Vm_sp .* exp(1j .* th_sp); % Fill with initial Specified Voltages
                               % Fill with initial Specified Magnitud
Vm(:,1) = Vm sp;
th(:,1) = th_sp;
                                % Fill with initial Specified Angle
err = 1.0; % Define Error
k = 1;
        % iteration counter
```

5) Inicia el método Newton Raphson

Una vez se tiene el cálculo de las potencias, se procede a agregar el valor de potencia reactiva suministrada por el SVC.

$$Q_{\mathrm{SVC}} = Q_k = -V_k^2 B_{\mathrm{SVC}}.$$
 (eq 5.5, Acha)

```
%% Calculate injected bus powers by the SVC
for ii = 1 : size(SVC_data,1)
        Qn(SVC_data{ii,"SVCBus"})=Qn(SVC_data{ii,"SVCBus"})-(Vm(SVC_data{ii,"SVCBus"}))^2*SVC_cend
```

A continuación, se realiza el cálculo de las componentes del Jacobiano, para ello se obtienen las matrices H.L.N.M donde:

For
$$i \neq j$$

$$H_{ij} = L_{ij} = V_i V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij})$$

$$N_{ij} = -M_{ij} = V_i V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij})$$
For $i = j$

$$H_{ii} = -Q_i - B_{ii} V_i^2 \quad L_{ii} = Q_i - B_{ii} V_i^2$$

$$N_{ii} = P_i + G_{ii} V_i^2 \quad M_{ii} = P_i - G_{ii} V_i^2$$

(Table 3.1, pag109, Gómez - Exposito)

```
% H Matrix.
         H -> Dim: (nL+nG) X (nL+nG)
 for mm = 1 : nL + nG
                      %Iter over rows H matrix
    % #node corresponding to row mm
                    % #node corresponding to column nn
       jj = x_th_n(nn);
       % From Prev Equations:
       if ii == jj
          H(mm,nn) = -Qn(ii) - imag(Ybus(ii,jj)) .* abs(V(ii,k)).^2;
       else
          H(mm,nn) = abs(V(ii,k)) * abs(V(jj,k)) .* (...
             real(Ybus(ii,jj)) * sin( th(ii,k) - th(jj,k) ) - ...
             imag(Ybus(ii,jj)) * cos( th(ii,k) - th(jj,k) ) );
       end
    end
 end
 % N Matrix. N -> Dim: (nL+nG) X (nL)
```

```
ii = x th n(mm); % #node corresponding to row mm
        jj = x v n(nn); % #node corresponding to column nn
        % From Prev Equations:
        if ii == jj
            N(mm,nn) = Pn(ii) + real(Ybus(ii,jj)) .* abs(V(ii,k)).^2;
        else
            N(mm,nn) = abs(V(ii,k)) * abs(V(jj,k)) .* (...
                real(Ybus(ii,jj)) * cos( th(ii,k) - th(jj,k) ) + \dots
                imag(Ybus(ii,jj)) * sin( th(ii,k) - th(jj,k) ) );
        end
    end
 end
% M Matrix. M -> Dim: (nL) X (nL+nG)
 for mm = 1 : nL
                           % Iter over rows M matrix
    % From Prev Equations:
        if ii == jj
            M(mm,nn) = Pn(ii) - real(Ybus(ii,jj)) .* abs(V(ii,k)).^2;
        else
            M(mm,nn) = -abs(V(ii,k)) * abs(V(jj,k)) .* (...
                real(Ybus(ii,jj)) * cos( th(ii,k) - th(jj,k) ) + \dots
                imag(Ybus(ii,jj)) * sin( th(ii,k) - th(jj,k) ) );
        end
    end
 end
% L Matrix. L -> Dim: (nL) X (nL)
 for mm = 1 : nL
                      % Iter over rows L matrix
    for nn = 1 : nL
                      % Iter over cols L matrix
        ii = x_v_n(mm); % #node corresponding to row mm
        jj = x_v_n(nn); % #node corresponding to column nn
        % From Prev Equations:
        if ii == jj
            L(mm,nn) = Qn(ii) - imag(Ybus(ii,jj)) .* abs(V(ii,k)).^2;
        else
            L(mm,nn) = abs(V(ii,k)) * abs(V(jj,k)) .* (...
                real(Ybus(ii,jj)) * sin( th(ii,k) - th(jj,k) ) - ...
                imag(Ybus(ii,jj)) * cos( th(ii,k) - th(jj,k) ) );
        end
    end
end
% The jacobian matrix results in:
 J = [H, N; \dots]
    M, L];
```

Posteriormente se hace la modificación de la matriz J del SVC (se reemplaza V_{SVC} → B_{SVC})

$$\begin{bmatrix} \Delta P_k \\ \Delta Q_k \end{bmatrix}^{(i)} = \begin{bmatrix} 0 & 0 \\ 0 & Q_k \end{bmatrix}^{(i)} \begin{bmatrix} \Delta \theta_k \\ \Delta B_{\text{SVC}}/B_{\text{SVC}} \end{bmatrix}^{(i)}.$$

```
%% SVC Jacobian Update
   for n = 1 : size(SVC_data,1)
%
         PQ_Index = find(N_data(strcmp(N_data.type, 'PQ'),:).n==SVC_data{ii, "SVCBus"});% get P(
       DQ_svc_Index =find(x_v_n==SVC_data{n,"SVCBus"});
       DP_svc_Index =find(x_th_n==SVC_data{n, "SVCBus"});
       if (SVC_data{n,'Status'} == 1)
          %Delete the voltage magnitud for the SVC bus
           JAC(2*SVCsend(ii)-1,2*SVCsend(ii)-1) = ... % <- DeltaPk/Delta(Vangle)</pre>
   %
                JAC(2*SVCsend(ii) - 1,2*SVCsend(ii)-1)-VM(SVCsend(ii))^2*B(ii); %¿Error?
           J(DP_svc_Index,DP_svc_Index)=J(DP_svc_Index,DP_svc_Index)-Vm(SVC_data{n,"SVCBus"})/
          % [deltaQk] = [Qk]*[delta(Bsvc)/Bsvc] -- k node= SVCnode
           J(nL+nG+DQ_svc_Index,nL+nG+DQ_svc_Index)= -Vm(SVC_data{n,"SVCBus"})^2*SVC_data{n,"H
           % [deltaOk]= J(nL+nG+PO Index,nL+nG+PO Index)
       end
   end
```

Se calculan los términos residuales. (DP y DQ)

$$\Delta P_i = P_i^{\text{sp}} - V_i \sum_{j=1}^n V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \quad i = 1, 2, \dots, n-1$$

$$Q_n$$

$$\Delta Q_i = Q_i^{\text{sp}} - V_i \sum_{j=1}^n V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \quad i = 1, 2, \dots, n_L$$

```
% **************************** Residual Terms Calculations

% Delta P
Dt_P=P_sp(x_th_n(1:nL+nG))-Pn(x_th_n(1:nL+nG));
% Delta Q
Dt_Q=Q_sp(x_v_n(1:nL))-Qn(x_v_n(1:nL));

Dt = [ Dt_P ;...
Dt_Q ];
```

Se actualiza la solución del sistema donde:

$$\begin{bmatrix} \theta_{k+1} \\ V_{k+1} \end{bmatrix} = \begin{bmatrix} \theta_{k+1} \\ V_{k+1} \end{bmatrix} + J^{-1} \cdot \begin{bmatrix} \Delta P_k \\ \Delta Q_k \end{bmatrix}$$

$$B_{
m SVC}^{(i)} = B_{
m SVC}^{(i-1)} + \left(rac{\Delta B_{
m SVC}}{B_{
m SVC}}
ight)^{(i)} B_{
m SVC}^{(i-1)}.$$
 (Eq. 5.7, Acha

```
SVC Update Adjust
for ii = 1 : size(SVC_data,1)
    if (SVC data{ii, 'Status'} == 1)
        DQ_Index= find(N_data(strcmp(N_data.type, 'PQ'),:).n==SVC_data{ii, "SVCBus"});% get F
        % Adjust the Voltage Magnitud target
        Vm(SVC data{ii, "SVCBus"},k + 1) =SVC data{ii, "Tv"}; %Voltage at SVCnode -> Target
        % SVC Susceptance - Delta(Bsvc)
        value = SVC_data{ii, "B"}*x(nL+nG+DQ_Index); % delta(Bsvc)
        value2 = x(nL+nG+DQ\_Index);
                                                     % delta(Bsvc)/Bsvc
        %% If high or low delta(Bsvc)
        if (value > 0.1)
            value2 = 0.1/SVC_data{ii, "B"};  % max -> delta(Bsvc)/Bsvc per iteration
        elseif (value < -0.1)</pre>
            value2 = -0.1/SVC_data{ii, "B"}; % min -> delta(Bsvc)/Bsvc per iteration
        end
        % SVC Susceptance Update
        SVC_data{ii, "B"} = SVC_data{ii, "B"} + SVC_data{ii, "B"}*value2; % Ecn 5.7 Acha
    end
   % Check susceptance limits in SVC
    if (SVC_data{ii,"B"} > SVC_data{ii,"Bmax"})
```

```
V(:,k + 1) = Vm(:,k + 1) .* exp( 1j .* th(:,k + 1) ); % Calculate phasors.

%err = max( abs( V(:,k+1) - V(:,k) ) );
err = max( abs( Dt ) ); % Calculate error

k = k + 1; % Add 1 to iterations
end
```

6) Cálculos finales:

Para el cálculo de la potencia aparente se tiene que:

$$S = \begin{bmatrix} V_1 & 0 & 0 & 0 \\ 0 & V_2 & 0 & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & V_n \end{bmatrix} \cdot \begin{pmatrix} Y_{\text{Bus}} \cdot \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \end{pmatrix}^* = \begin{bmatrix} V_1 & 0 & 0 & 0 \\ 0 & V_2 & 0 & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & V_n \end{bmatrix} \cdot \begin{pmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} \end{pmatrix}^* = \begin{bmatrix} V_1 & 0 & 0 & 0 \\ 0 & V_2 & 0 & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & V_n \end{bmatrix} \cdot \begin{pmatrix} \begin{bmatrix} I_1^* \\ I_2^* \\ \vdots \\ I_n^* \end{bmatrix} \end{pmatrix} = \begin{bmatrix} V_1 I_1^* \\ V_2 I_2^* \\ \vdots \\ V_n I_n^* \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_n \end{bmatrix}$$

```
% Complex power calculation for each node.
Vn = V(:, k);
Vd = diag( Vn );
Sn = Vd * conj( Ybus * V( : , k ) );
Pn = real(Sn);
On = imag(Sn);
SGn = Sn + (PL + 1j * QL);
SLn = SGn - Sn;
% Complex Power Flow Through Branches.
S_pf = zeros( 2*size(br_data, 1) , 5);
bc = 1;
for kk = 1: size(br_data, 1)
   ii = br_data.FromBus(kk);
    jj = br data.ToBus(kk);
   z = br_data.R(kk) + 1j * br_data.X(kk);
   y = br_data.G(kk) + 1j * br_data.B(kk);
   Ybr=[1 / z + y / 2 , -1 / z; ...]
                                            % Ybr (Two-Port Network)
           -1/z , 1/z + y/2];
```

7) Se presentan los resultados obtenidos

```
%% Results printing.
for i=1:1 % This "for" is used to display everything once.
                 *********** \n')
   fprintf('
                                                        ** \n')
   fprintf('
                            Newton-Raphson Results
                 ********* \n\n')
   fprintf('
   fprintf('-- Number of Iterarions: %d \n', k-1)
   fprintf('-- Error: %6.3e p.u.\n\n', err)
   VarNames = {'Node#', 'Type', 'V p.u.', '\angle V(^{\circ})', 'PG p.u.', 'QG p.u.', 'PL p.u.', 'QL p.u.'};
   fprintf(1,'\t%s\t%s\t%s\t%s\t%s\t%s\t1%s\n', VarNames{:})
   fprintf(1,
                                                                       ----\n')
   for ii = 1 : num_nodes
       fprintf(' \t%d\t%s\t%6.3f\t%6.3f\t%6.3f\t%6.3f\t%6.3f\t\n', ii, char(N_data{i:
           abs(Vn(ii)), angle(Vn(ii)).*180./pi, real(SGn(ii)), imag(SGn(ii)), real(SLn(ii)),
   end
end
    *************
            Newton-Raphson Results
    ************
-- Number of Iterarions: 5
-- Error: 1.242e-14 p.u.
Node# Type V p.u. \angle V(^{\circ}) PG p.u. QG p.u. PL p.u. QL p.u.
   1 SL 1.060 0.000 1.311 0.853 0.000 0.000
   2 PV 1.000 -2.053 0.400 -0.771 0.200 0.100
   3 PQ 1.000 -4.838 -0.000 0.205 0.450 0.150
   4 PQ 0.994 -5.107 -0.000 -0.000 0.400 0.050
   5 PQ 0.975 -5.797 -0.000 0.000 0.600 0.100
array2table(S_pf,'VariableNames',{'FromBus','ToBus','P','Q','S'});
```

 Table 5.1
 Nodal voltages of modified network

	Network bus						
Nodal voltage	North	South	Lake	Main	Elm		
Magnitude (p.u.)	1.06	1	1	0.994	0.975		
Phase angle (deg)	0	-2.05	-4.83	-5.11	-5.80		

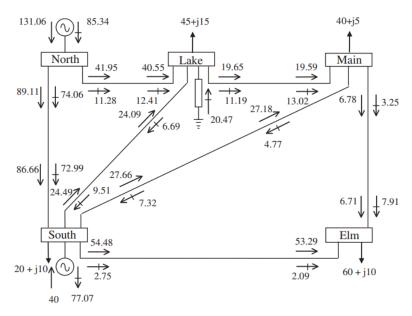


Figure 5.8 Power flow results in the five-bus network with one static VAR compensator

Referencias:

- [1] Gomez Exposito, A. Electrica Energy Systems, Analysis and operation. 2nd Edition. Boca Raton: Taylor & Francis, CRC Press, 2018.
- [2] Acha, E. FACTS Modelling and Simulation in Power Networks. 1st Edition. Boca Raton : John Wiley & Sons Inc, 2004