	Flujos de Potencia Newton Raphson	
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1) Se definen las bases del sistema

- Potencia base $S_b = 100 \cdot 10^6$ [VA]
- Voltaje base $V_b = 230 \cdot 10^3$ [V]

```
Sb = 100e6; %[VA]
Vb = 230e3; %[V]
```

2) Se crean las matrices con la información de la red (br_data, N_data):

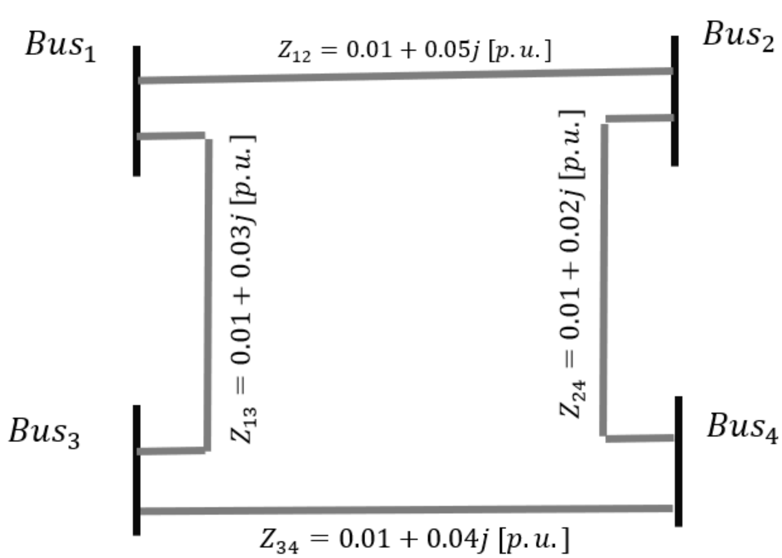
Se deben crear dos matrices para ingresar la información referente al circuito. La primera (br_data) brinda información acerca de las conexiones entre buses como la impedancia de la línea o los buses entre los cuales se realiza la conexión , mientras que la segunda (N_data) da información acerca de la naturaleza de cada uno de los buses, tipo de bus (SL, PV o PQ).

a) Para la matriz **br_data** se tiene que el formato es

$$\text{brdata} = [\text{Bus}_{\text{from}}, \text{Bus}_{\text{to}}, R, X, G, B] \text{ in } p.u.$$

- Ejemplo:

Para la red:



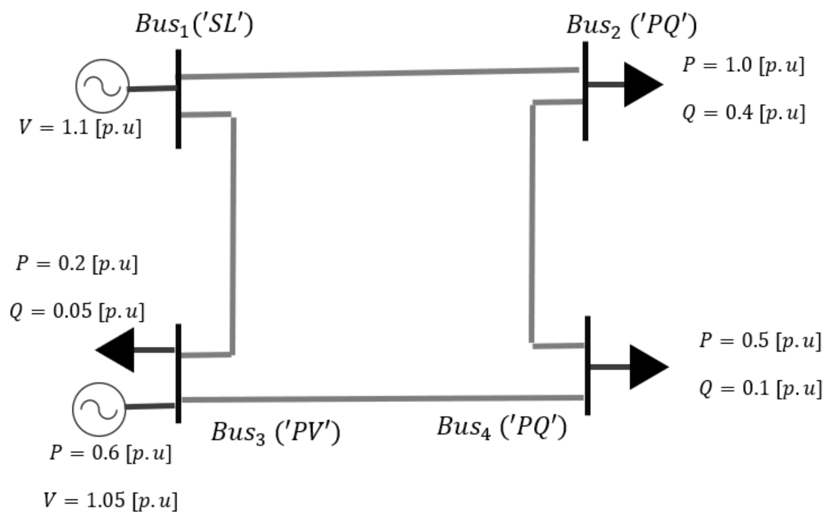
$$\text{brdata} = \begin{bmatrix} 1 & 2 & 0.01 & 0.05 & 0 & 0 \\ 2 & 4 & 0.01 & 0.02 & 0 & 0 \\ 3 & 4 & 0.01 & 0.04 & 0 & 0 \\ 1 & 3 & 0.01 & 0.03 & 0 & 0 \end{bmatrix}$$

b) Para la matriz **N_data** se tiene que el formato es:

$$N\text{data} = \{\#\text{Bus}, \text{type}('\text{SL}', '\text{PQ}', '\text{PV}'), V[p.u.], \angle \text{V}^{\text{o}}, \text{PG}[p.u.], \text{QG}[p.u.], \text{PL}[p.u.], \text{QL}[p.u.]\}$$

- Ejemplo:

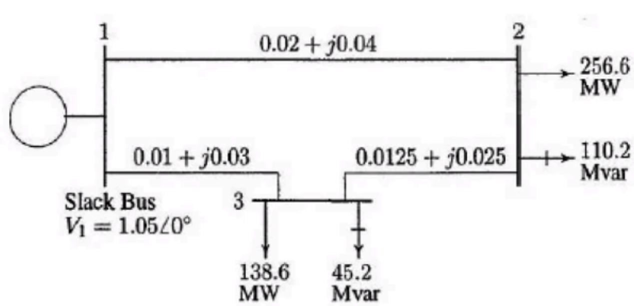
Para la red:



$$N\text{data} = \begin{Bmatrix} 1 & '\text{SL}' & 1.1 & 0 & 0 & 0 & 0 & 0 \\ 2 & '\text{PQ}' & 1.0 & 0 & 0 & 0 & 1.0 & 0.4 \\ 3 & \text{PV}' & 1.05 & 0 & 0.6 & 0 & 0.2 & 0.05 \\ 4 & '\text{PQ}' & 1.0 & 0 & 0 & 0 & 0.5 & 0.1 \end{Bmatrix}$$

Exercise:

Figure 10.2: shows the one-line diagram of a simple three-bus power system with generation at bus 1. The magnitude of voltage at bus 1 is adjusted to 1.05 per unit. The loads at bus 2 and 3 are as marked on the diagram. Line Impedance are marked as per unit on a 100 MVA base and the line charging are neglected. Using the Gauss-Seidal method, determine the phasor values of the voltage at the load buses 2 and 3 (P-Q buses) accurate to four decimal places and obtain solution using MATLAB Simulink.

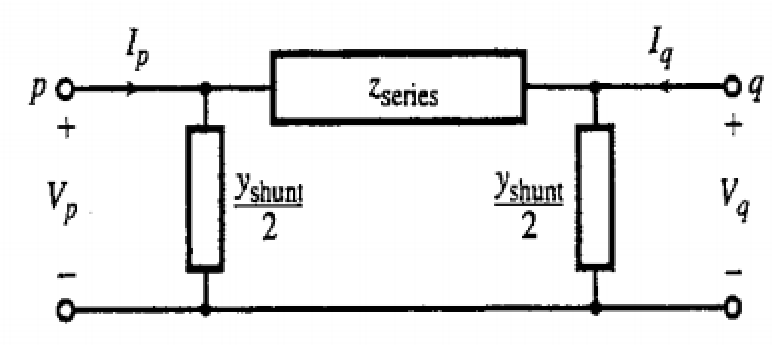


```
% Branches data R, X, G, B
% Types: General in form of impedance. (G & B are split in half at both ends).

% br_data = [ bus1, bus2, R, X, G, B] in p.u.

% br_data = [
%     1,  2,  0.03,    0.3,   0.0,    0.0;
%     2,  3,  0.06,    0.2,   0.0,    0.0];
%
% % Node definitions
% % {#, type, Vpu, <Vº, PG pu, QG pu, PL pu, QL pu}
% N_data = {
%     1,  'SL',   1.1,    0.0,    0.0,    0.0,    0.0,    0.0;
%     2,  'PQ',   1.0,    0.0,    0.0,    0.0,    1.0,    0.4;
%     3,  'PV',   1.05,   0.0,    0.6,    0.0,    0.2,    0.05};

br_data = [
    1,  2,  0.02,      0.04,    0.0,    0.0;
    2,  3,  0.0125,   0.025,    0.0,    0.0;
    1,  3,  0.01,      0.03,    0.0,    0.0];

% Node definitions
% {#, type, Vpu, <Vº, PG pu, QG pu, PL pu, QL pu}
N_data = {
    1,  'SL',   1.05,    0.0,    0.0,    0.0,       0.0   ,       0.0   ;
    2,  'PQ',   1.0 ,    0.0,    0.0,    0.0,    258.6/100,    110.2/100;
    3,  'PQ',   1.0 ,    0.0,    0.0,    0.0,    138.6/100,     45.2/100};

% % br_data = [ bus1, bus2, R, X, G, B] in p.u.
% br_data = [
%     1,  2,  0.01008,    0.05040,   0.0,    0.1025;
%     1,  3,  0.00744,    0.03720,   0.0,    0.0775;
%     2,  4,  0.00744,    0.03720,   0.0,    0.0775;
%     3,  4,  0.01272,    0.06360,   0.0,    0.1275;
%     ];

% % % Node definitions
% % % {#, type, Vpu, <Vº, PG pu, QG pu, PL pu, QL pu}
% N_data = {
%     1,  'SL',   1.0,    0.0,    0.0,    0.0,    0.5,    0.3099;
%     2,  'PQ',   1.0,    0.0,    0.0,    0.0,    1.7,    1.0535;
%     3,  'PQ',   1.0,    0.0,    0.0,    0.0,    2.0,    1.2394;
%     4,  'PV',  1.02,    0.0,   3.18,    0.0,    0.8,    0.4958  };

br_data=array2table(br_data,'VariableNames',{'FromBus','ToBus','R','X','G','B'});
N_data=cell2table(N_data,'VariableNames',{'n','type','Vm','Va','PG','QG','PL','QL'});
```

3) Se crea Ybus con las redes de 2 puertos:

Se tiene la siguiente red de dos puertos, que representa el circuito del modelo pi de la línea de transmisión

$$\begin{bmatrix} V_i \\ V_j \end{bmatrix} = \begin{bmatrix} \frac{1}{z} + \frac{y}{2} & -\frac{1}{z} \\ -\frac{1}{z} & \frac{1}{z} + \frac{y}{2} \end{bmatrix} \begin{bmatrix} I_i \\ I_j \end{bmatrix}$$



Pi-equvalent model of a transmission line *(Pag 296, Bergen)*

```
n_br = size(br_data, 1); % Number of branches
n = max(max([br_data.FromBus,br_data.ToBus]));  % Number of nodes (starting at 1)
Ybus = zeros(n, n); % Empty Ybus Matrix

for kk = 1: size(br_data, 1)    % Iter over branch Data
    ii = br_data.FromBus(kk);                % From bus
    jj = br_data.ToBus(kk);                  % To Bus
    z = br_data.R(kk) + 1j * br_data.X(kk);  % Line impedance
    y = br_data.G(kk) + 1j * br_data.B(kk);  % Shunt admitance

    Ybr=[1 / z + y / 2 ,    -1 / z;  ...     % Ybr (Two-Port Network)
            -1 / z      , 1 / z + y / 2];

    Ybus([ii,jj],[ii,jj]) = Ybus([ii,jj],[ii,jj])+ Ybr;                 % Add the Two-Por
end
```

Se extrae la información de las potencias y voltajes:

```
Vm_sp = N_data.Vm;              %   [V]  Specified Voltages
th_sp = N_data.Va.* pi ./ 180;  % [Rads] Specified Angles
PG = N_data.PG;                 % [p.u.] Active Power generators
QG = N_data.QG;                 % [p.u.] Reactive Power generators

PL = N_data.PL;                 % [p.u.] Active Power Loads
QL = N_data.QL;                 % [p.u.] Reactive Power Loads

P_sp = PG - PL;                      % [p.u.] Specified Active Power    Pnet= (PGen - PLoad)
Q_sp = QG - QL;                      % [p.u.] Specified Reactive Power  Qnet= (QGen - QLoad)
```

4) Construcción de la matriz Jacobiana

Se encuentran las dimensiones de la matriz jacobiana:

```
% Jacobian matrix size.

nL = sum(strcmp(N_data.type,'PQ'));% Number of PQ nodes  (#Loads)
nG = sum(strcmp(N_data.type,'PV'));% Number of PV nodes  (#Generators)
```

Se divide la matriz Jacobiana en la submatrices H-N-M-L

$$\begin{array}{cc} & \begin{array}{cc} nL + nG & nL \end{array} \\ \begin{array}{c} nL + nG \\ nL \end{array} & \begin{bmatrix} H & N \\ M & L \end{bmatrix}^k \end{array} \begin{bmatrix} \Delta\theta \\ \Delta V / V \end{bmatrix}^k = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}^k$$

(Eq. 3.32, pag108, Gómez - Exposito)

$$H_{ij} = \frac{\partial P_i}{\partial \theta_j}; \qquad N_{ij} = \frac{V_j \, \partial P_i}{\partial V_j}$$
$$M_{ij} = \frac{\partial Q_i}{\partial \theta_j}; \qquad L_{ij} = \frac{V_j \, \partial Q_i}{\partial V_j}$$

(Eq. 3.34, pag109, Gómez - Exposito)

```
H = zeros( nL + nG, nL + nG );
L = zeros( nL, nL );
N = zeros( nL + nG , nL );
M = zeros( nL , nL + nG );
J = [H, N; M, L];
```

Se crea el vector de estados ($X = [\theta|V]^T$) y el vector de funciones no lineales ($f(x) = [\Delta P|\Delta Q]$

```
x_th = zeros( nL + nG, 1 ); % State vector for storing angles (theta)
x_v = zeros( nL , 1);       % State vector for storing Voltages
```

$$x = [\theta|V]^{\mathrm{T}} = [\theta_1, \theta_2, \ldots, \theta_{n-1}|V_1, V_2, \ldots, V_{n_{\mathrm{L}}}]^{\mathrm{T}}$$ dim= (2nL+nG)

```
x = [x_th; x_v];
```

- Donde se tiene que:

$$\Delta P_i = P_i^{sp} - V_i \sum_{j=1}^{n} V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) \quad i = 1, 2, \ldots, n-1$$

$$\Delta Q_i = Q_i^{sp} - V_i \sum_{j=1}^{n} V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) \quad i = 1, 2, \ldots, n_L$$

(Eq. 3.30-3.31, pag108, Gómez - Exposito)

```
Dt_P = zeros( nL + nG, 1 ); % Delta P
Dt_Q = zeros( nL, 1);       % Delta Q
```

$$f(x) = [\Delta P | \Delta Q]^T = [\Delta P_1, \Delta P_2, \ldots, \Delta P_{n-1} | \Delta Q_1, \Delta Q_2, \ldots, \Delta Q_{n_L}]^T$$ dim= (2nL+nG)

```
Dt = [Dt_P;Dt_Q];  %Delta Vector (same as f(x) Vector)

%% Define Unkown Values (Voltage and angle in PQ node - Angle in PV node)
% #Buses with unknwon voltage magnitudes ('PQ' buses)  (Delta Q index)
x_v_n=N_data(strcmp(N_data.type,'PQ'),:).n;

% #Buses with unknwon voltage angles ('PQ'|'PV' buses) (Delta P index)
x_th_n=N_data(strcmp(N_data.type,'PV')|strcmp(N_data.type,'PQ'),:).n;


x_n = [x_th_n; x_v_n]; % #Buses with unknown Angles and unknown voltages

max_iter = 100; %Define Max iteration number

V = zeros(n, max_iter); % Matrix for storing node voltages per iter
Vm = zeros(n, max_iter);    % For storing node voltage magnitude per iter
th = zeros(n, max_iter);    % For storing node voltage angles per iter
V(:,1) = Vm_sp .* exp(1j .* th_sp); % Fill with initial Specified Voltages
Vm(:,1) = Vm_sp;                    % Fill with initial Specified Magnitud
th(:,1) = th_sp;                    % Fill with initial Specified Angle

err = 1.0;  % Define Error
k = 1;      % iteration counter
```

5) Inicia el método Newton Raphson

```
while (err > 1e-8 && k < max_iter)
% for cc = 1 : 3

    % **************************** Jacobian Terms Calculation.
    Vn = V( : , k ); %Phasors!!
    Vd = diag( Vn ); %Voltages in a diagonal matrix

    % Calculate active and reactive power per node
    Sn = Vd * conj( Ybus * V( : , k ) ); %  Sn=V(I)* | I=Y_bus·V | Sn=V·(Ybus·V)*
    Pn = real(Sn);  %Active Power calculated
```

```
    Qn = imag(Sn);  % Reactive Power calculated
```

Se obtienen las matrices H,L,N,M donde:

$$\text{For } i \neq j$$
$$H_{ij} = L_{ij} = V_i V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij})$$
$$N_{ij} = -M_{ij} = V_i V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij})$$

$$\text{For } i = j$$
$$H_{ii} = -Q_i - B_{ii} V_i^2 \quad L_{ii} = Q_i - B_{ii} V_i^2$$
$$N_{ii} = P_i + G_{ii} V_i^2 \quad M_{ii} = P_i - G_{ii} V_i^2$$

(Table 3.1, pag109, Gómez - Exposito)

```
    % H Matrix.   H -> Dim: (nL+nG) X (nL+nG)

    for mm = 1 : nL + nG        %Iter over rows H matrix
        for nn = 1 : nL + nG    %Iter over cols H matrix
            ii = x_th_n(mm);    % #node corresponding to row mm
            jj = x_th_n(nn);    % #node corresponding to column nn

            % From Prev Equations:
            if ii == jj
                H(mm,nn) = - Qn(ii) - imag( Ybus(ii,jj) ) .* abs( V(ii,k) ).^2;
            else
                H(mm,nn) = abs( V(ii,k) ) * abs( V(jj,k) ) .* ( ...
                    real(Ybus(ii,jj)) * sin( th(ii,k) - th(jj,k) ) - ...
                    imag(Ybus(ii,jj)) * cos( th(ii,k) - th(jj,k) ) );
            end
        end
    end

    % N Matrix. N -> Dim: (nL+nG) X (nL)
    for mm = 1 : nL + nG     %  Iter over rows N matrix
        for nn = 1 : nL      %  Iter over cols N matrix
            ii = x_th_n(mm); % #node corresponding to row mm
            jj = x_v_n(nn);  % #node corresponding to column nn

            % From Prev Equations:
            if ii == jj
                N(mm,nn) = Pn(ii) + real(Ybus(ii,jj)) .* abs( V(ii,k) ).^2;
            else
                N(mm,nn) = abs( V(ii,k) ) * abs( V(jj,k) ) .* ( ...
                    real(Ybus(ii,jj)) * cos( th(ii,k) - th(jj,k) ) + ...
                    imag(Ybus(ii,jj)) * sin( th(ii,k) - th(jj,k) ) );
            end
        end
```

```
    end

    % M Matrix. M -> Dim: (nL) X (nL+nG)
    for mm = 1 : nL                 %  Iter over rows M matrix
        for nn = 1 : nL + nG        %  Iter over cols M matrix
            ii = x_v_n(mm);         % #node corresponding to row mm
            jj = x_th_n(nn);        % #node corresponding to column nn

            % From Prev Equations:
            if ii == jj
                M(mm,nn) = Pn(ii) - real(Ybus(ii,jj)) .* abs( V(ii,k) ).^2;
            else
                M(mm,nn) = -abs( V(ii,k) ) * abs( V(jj,k) ) .* ( ...
                    real(Ybus(ii,jj)) * cos( th(ii,k) - th(jj,k) ) + ...
                    imag(Ybus(ii,jj)) * sin( th(ii,k) - th(jj,k) ) );
            end
        end
    end

    % L Matrix. L -> Dim: (nL) X (nL)
    for mm = 1 : nL             %  Iter over rows L matrix
        for nn = 1 : nL         %  Iter over cols L matrix
            ii = x_v_n(mm); % #node corresponding to row mm
            jj = x_v_n(nn); % #node corresponding to column nn

            % From Prev Equations:
            if ii == jj
                L(mm,nn) = Qn(ii) - imag(Ybus(ii,jj)) .* abs( V(ii,k) ).^2;
            else
                L(mm,nn) = abs( V(ii,k) ) * abs( V(jj,k) ) .* ( ...
                    real(Ybus(ii,jj)) * sin( th(ii,k) - th(jj,k) ) - ...
                    imag(Ybus(ii,jj)) * cos( th(ii,k) - th(jj,k) ) );
            end
        end
    end
   % The jacobian matrix results in:
    J = [H, N;...
        M, L];
```

Se calculan los términos residuales.

$$\Delta P_i = P_i^{\mathrm{sp}} - \overbrace{V_i \sum_{j=1}^{n} V_j (G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij})}^{P_n} \quad i = 1, 2, \ldots, n-1$$

$$\Delta Q_i = Q_i^{\mathrm{sp}} - \overbrace{V_i \sum^{n} V_j (G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij})}^{Q_n} \quad i = 1, 2, \ldots, n_{\mathrm{L}}$$

```
    % *************************** Residual Terms Calculations
```

```
    % Delta P
    Dt_P=P_sp(x_th_n(1:nL+nG))-Pn(x_th_n(1:nL+nG));
    % Delta Q
    Dt_Q=Q_sp(x_v_n(1:nL))-Qn(x_v_n(1:nL));


    Dt = [ Dt_P ;...
           Dt_Q ];
```

Se actualiza la solución del sistema donde:

$$\begin{bmatrix}\theta_{k+1}\\ V_{k+1}\end{bmatrix} = \begin{bmatrix}\theta_{k+1}\\ V_{k+1}\end{bmatrix} + J^{-1}\cdot\begin{bmatrix}\Delta P_k\\ \Delta Q_k\end{bmatrix}$$

```
    % *************************** System Solution
    x = J \ Dt; % Delta theta -> inv(J)*Dt
    x_th = x( 1 : nL + nG );                    % Adjust in unknown Angles
    x_v = x( nL + nG + 1 : 2 * nL + nG );    % Adjust in unknown Voltages

    Vm(:,k + 1) = Vm(:,k); % new magnitudes
    th(:,k + 1) = th(:,k); % new Angles

    th(x_th_n, k + 1) = x_th + th(x_th_n, k); % New angle    =  Adjust in unknown Angle    + prev
    Vm(x_v_n, k + 1)  = x_v.*Vm(x_v_n, k) + Vm(x_v_n, k);    % new voltage =  Adjust in unknown

    V(:,k + 1) = Vm(:,k + 1) .* exp( 1j .* th(:,k + 1) ); % Calculate phasors.

%     err = max( abs( V(:,k+1) - V(:,k) )  );
    err = max( abs( Dt )  ); % Calculate error

    k = k + 1;  % Add 1 to iterations
end
```

6) Cálculos finales:

Para el cálculo de la potencia aparente se tiene que:

$$S = \begin{bmatrix}V_1 & 0 & 0 & 0\\ 0 & V_2 & 0 & 0\\ 0 & 0 & \ddots & \vdots\\ 0 & 0 & \cdots & V_n\end{bmatrix}\cdot\left(Y_{\text{Bus}}\cdot\begin{bmatrix}V_1\\ V_2\\ \vdots\\ V_n\end{bmatrix}\right)^* = \begin{bmatrix}V_1 & 0 & 0 & 0\\ 0 & V_2 & 0 & 0\\ 0 & 0 & \ddots & \vdots\\ 0 & 0 & \cdots & V_n\end{bmatrix}\cdot\left(\begin{bmatrix}I_1\\ I_2\\ \vdots\\ I_n\end{bmatrix}\right)^* = \begin{bmatrix}V_1 & 0 & 0 & 0\\ 0 & V_2 & 0 & 0\\ 0 & 0 & \ddots & \vdots\\ 0 & 0 & \cdots & V_n\end{bmatrix}\cdot\left(\begin{bmatrix}I_1^*\\ I_2^*\\ \vdots\\ I_n^*\end{bmatrix}\right) = \begin{bmatrix}V_1 I_1^*\\ V_2 I_2^*\\ \vdots\\ V_n I_n^*\end{bmatrix} = \begin{bmatrix}S_1\\ S_2\\ \vdots\\ S_n\end{bmatrix}$$

```
% Complex power calculation for each node.
Vn = V( : , k );
Vd = diag( Vn );
Sn = Vd * conj( Ybus * V( : , k ) );
Pn = real(Sn);
```

```
Qn = imag(Sn);

SGn = Sn + (PL + 1j * QL);
SLn = SGn - Sn;

% Complex Power Flow Through Branches.

S_pf = zeros( 2*size(br_data, 1) , 5);

bc = 1;
for kk = 1: size(br_data, 1)
    ii = br_data.FromBus(kk);
    jj = br_data.ToBus(kk);
    z = br_data.R(kk) + 1j * br_data.X(kk);
    y = br_data.G(kk) + 1j * br_data.B(kk);
    Ybr=[1 / z + y / 2 ,    -1 / z;  ...     % Ybr (Two-Port Network)
            -1 / z      , 1 / z + y / 2];
    S_br=diag([Vn(ii),Vn(jj)])*conj(Ybr*[Vn(ii);Vn(jj)]);

    S_pf_br= [ii, jj, real(S_br(1)), imag(S_br(1)), abs(S_br(1));...% Apparent power from i to
              jj, ii, real(S_br(2)), imag(S_br(2)), abs(S_br(2))];  % Apparent power from j to

    S_pf(2*kk-1:2*kk,:)=S_pf_br;
end
```

7) Se presentan los resultados obtenidos

```
%% Results printing.

for i=1:1 % This "for" is used to display everything once.

    fprintf('     ******************************************** \n')
    fprintf('     **           Newton-Raphson Results        ** \n')
    fprintf('     ******************************************** \n\n')

    fprintf('-- Number of Iterarions: %d \n', k-1)
    fprintf('-- Error: %6.3e p.u.\n\n', err)

    VarNames = {'Node#', 'Type', 'V p.u.', ' ∠V(º)','PG p.u.','QG p.u.','PL p.u.','QL p.u.'};
    fprintf(1,'\t%s\t%s\t%s\t%s\t%s\t%s\t%s\t%s\n', VarNames{:})
    fprintf(1, '--------------------------------------------------------------------\n')

    for ii = 1 : n
        fprintf('   \t%d\t%s\t%6.3f\t%6.3f\t%6.3f\t%6.3f\t%6.3f\t%6.3f\t\n', ii, char(N_data{ii
            abs(Vn(ii)), angle(Vn(ii)).*180./pi, real(SGn(ii)), imag(SGn(ii)), real(SLn(ii)), i
    end
end
```

```
     ********************************************
     **           Newton-Raphson Results        **
     ********************************************
-- Number of Iterarions: 4
```

```
-- Error: 1.606e-09 p.u.
 Node# Type V p.u.  ∠V(°) PG p.u. QG p.u. PL p.u. QL p.u.
---------------------------------------------------------------------
    1 SL  1.050  0.000  4.116  1.893  0.000  0.000
    2 PQ  0.982 -3.530  0.000  0.000  2.586  1.102
    3 PQ  1.001 -2.877 -0.000 -0.000  1.386  0.452
```

```
array2table(S_pf,'VariableNames',{'FromBus','ToBus','P','Q','S'})
```

ans = 6×5 table

	FromBus	ToBus	P	Q	S
1	1	2	2.0073	0.8414	2.1765
2	2	1	-1.9214	-0.6695	2.0347
3	2	3	-0.6646	-0.4325	0.7929
4	3	2	0.6728	0.4488	0.8087
5	1	3	2.1092	1.0520	2.3569
6	3	1	-2.0588	-0.9008	2.2472

Referencias:

[1] Gomez - Exposito,A. Electrica Energy Systems, Analysis and operation. 2nd Edition. Boca Raton : Taylor & Francis, CRC Press, 2018.