| UNIVERSIDAD NACIONAL DE COLOMBIA | **Flujos de Potencia Newton Raphson (Con STATCOM)** | |
|---|---|---|
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## 1) Se definen las bases del sistema

- Potencia base $S_b = 100 \cdot 10^6$ [VA]
- Voltaje base $V_b = 230 \cdot 10^3$ [V]

```
Sb = 100e6; %[VA]
Vb = 230e3; %[V]
```

## 2) Se crean las matrices con la información de la red ( br_data, N_data):

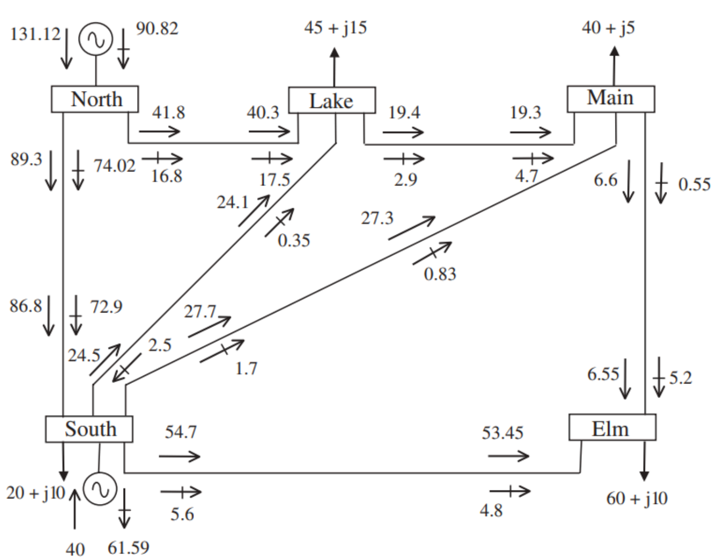
Se deben crear dos matrices para ingresar la información referente al circuito. La primera (br_data) brinda información acerca de las conexiones entre buses como la impedancia de la línea o los buses entre los cuales se realiza la conexión , mientras que la segunda (N_data) da información acerca de la naturaleza de cada uno de los buses, tipo de bus (SL, PV o PQ).

a) Para la matriz **br_data** se tiene que el formato es

$$\text{brdata} = [\text{Bus}_{\text{from}}, \text{Bus}_{\text{to}}, R, X, G, B] \text{ in } p.u.$$

b) Para la matriz **N_data** se tiene que el formato es:

$$N\text{data} = \{\#\text{Bus}, \text{type}('\text{SL}', '\text{PQ}', '\text{PV}'), V[p.u.], \angle V^{\circ}, \text{PG}[p.u.], \text{QG}[p.u.], \text{PL}[p.u.], \text{QL}[p.u.]\}$$



```matlab
% Branches data R, X, G, B
% Types: General in form of impedance. (G & B are split in half at both ends).

% br_data=[ bus1, bus2,     R,      X,      G,       B] in p.u.
br_data = [
            1,       2,  0.02,     0.06,     0.0,     0.06;
            1,       3,  0.08,     0.24,     0.0,     0.05;
            2,       3,  0.06,     0.18,     0.0,     0.04;
            2,       4,  0.06,     0.18,     0.0,     0.04;
            2,       5,  0.04,     0.12,     0.0,     0.03;
            3,       4,  0.01,     0.03,     0.0,     0.02;
            4,       5,  0.08,     0.24,     0.0,     0.05];

% Node definitions
% {  #, type,   Vpu,    <Vº,   PG pu,  QG pu,  PL pu, QL pu}
N_data = {
    1,  'SL',   1.06,   0.0,    0.0,    0.0,    0.00,    0.00;
    2,  'PV',   1.0,    0.0,    0.4,    0.0,    0.20,    0.10;
    3,  'PQ',   1.0,    0.0,    0.0,    0.0,    0.45,    0.15;
    4,  'PQ',   1.0,    0.0,    0.0,    0.0,    0.40,    0.05;
    5,  'PQ',   1.0,    0.0,    0.0,    0.0,    0.60,    0.10;  };

br_data=array2table(br_data,'VariableNames',{'FromBus','ToBus','R','X','G','B'});
N_data=cell2table(N_data,'VariableNames',{'n','type','Vm','Va','PG','QG','PL','QL'});
```

```matlab
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% STATIC SYNCHRONOUS COMPENSATOR (STATCOM)
% NSSC : Number of STATCOM's
% SSCsend: STATCOM's bus
% Xvr : Converter's reactance (p.u.)
% TarVol: Target nodal voltage magnitude (p.u.)
% VSta : Indicate the control status over nodal voltage magnitude: 1 is
% on; 0 is off
% Psp : Target active power flow (p.u.)
% PSta : Indicate the control status over active power: 1 is on; 0 is off
% Qsp : Target reactive power flow (p.u.)
% QSta : Indicate the control status over reactive power:1 is on; 0 is off
% Vvr : Initial condition for the source voltage magnitude (p.u.)
% Tvr : Initial condition for the source voltage angle (deg)
% VvrHi : Lower limit source voltage magnitude (p.u.)
% VvrLo : higher limit source voltage magnitude (p.u.)

% SSC_data= [SSCsend, Xvr , TarVol , VSta, Psp , PSta , Qsp , QSta,Vvr,Tvr,VvrHi,VvrLo];
SSC_data  = [    3  , 0.1 ,    1   ,   1 ,  0  ,   1  ,   0 ,   0 , 1 , 0 , 1.1 , 0.9 ];

SSC_data=array2table(SSC_data,'VariableNames',{'SSCsend',' Xvr ',' TarVol ',' VSta',' Psp ',' P
```

SSC_data = 1×12 table

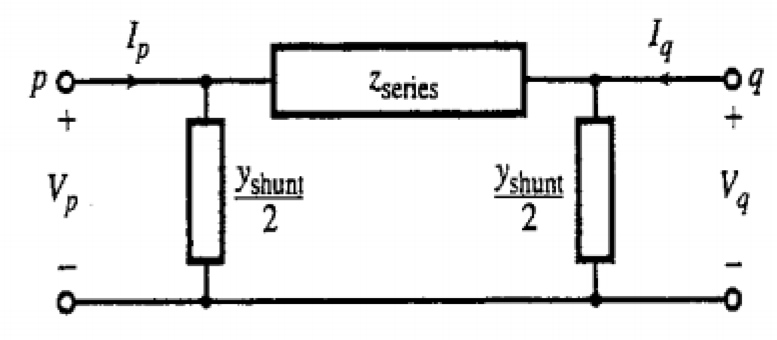
...

| | SSCsend | Xvr | TarVol | VSta | Psp | PSta | Qsp | QSta |
|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 0.1000 | 1 | 1 | 0 | 1 | 0 | 0 |

### 3) Se crea Ybus con las redes de 2 puertos:

Se tiene la siguiente red de dos puertos, que representa el circuito del modelo pi de la línea de transmisión

$$\begin{bmatrix} V_i \\ V_j \end{bmatrix} = \begin{bmatrix} \frac{1}{z}+\frac{y}{2} & -\frac{1}{z} \\ -\frac{1}{z} & \frac{1}{z}+\frac{y}{2} \end{bmatrix} \begin{bmatrix} I_i \\ I_j \end{bmatrix}$$




Pi-equvalent model of a transmission line *(Pag 296, Bergen)*

```
n_br = size(br_data, 1); % Number of branches
num_nodes = max(max([br_data.FromBus,br_data.ToBus]));  % Number of nodes (starting at 1)
Ybus = zeros(num_nodes, num_nodes); % Empty Ybus Matrix

for kk = 1: size(br_data, 1)    % Iter over branch Data
    ii = br_data.FromBus(kk);                % From bus
    jj = br_data.ToBus(kk);                  % To Bus
    z = br_data.R(kk) + 1j * br_data.X(kk);  % Line impedance
    y = br_data.G(kk) + 1j * br_data.B(kk);  % Shunt admitance

    Ybr=[1 / z + y / 2 ,    -1 / z;  ...     % Ybr (Two-Port Network)
            -1 / z     , 1 / z + y / 2];

    Ybus([ii,jj],[ii,jj]) = Ybus([ii,jj],[ii,jj])+ Ybr;                       % Add the Two-Por
end
```

Se extrae la información de las potencias y voltajes:

```
Vm_sp = N_data.Vm;                % [V]   Specified Voltages
th_sp = N_data.Va.* pi ./ 180; % [Rads] Specified Angles
PG = N_data.PG;                   % [p.u.] Active Power generators
QG = N_data.QG;                   % [p.u.] Reactive Power generators

PL = N_data.PL;                   % [p.u.] Active Power Loads
QL = N_data.QL;                   % [p.u.] Reactive Power Loads
```

```
P_sp = PG - PL;                        % [p.u.] Specified Active Power    Pnet= (PGen - PLoad)
Q_sp = QG - QL;                        % [p.u.] Specified Reactive Power  Qnet= (QGen - QLoad)
```

## 4) Construcción de la matriz Jacobiana

Se encuentran las dimensiones de la matriz jacobiana:

```
% Jacobian matrix size.

nL = sum(strcmp(N_data.type,'PQ'));% Number of PQ nodes  (#Loads)
nG = sum(strcmp(N_data.type,'PV'));% Number of PV nodes  (#Generators)
```

Se divide la matriz Jacobiana en la submatrices H-N-M-L

$$\begin{array}{cc} & \begin{array}{cc} nL+nG & nL \end{array} \\ \begin{array}{c} nL+nG \\ nL \end{array} & \begin{bmatrix} H & N \\ M & L \end{bmatrix}^k \end{array} \begin{bmatrix} \Delta\theta \\ \Delta V/V \end{bmatrix}^k = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}^k$$

*(Eq. 3.32, pag108, Gómez - Exposito)*

$$H_{ij} = \frac{\partial P_i}{\partial \theta_j}; \qquad N_{ij} = \frac{V_j\, \partial P_i}{\partial V_j}$$
$$M_{ij} = \frac{\partial Q_i}{\partial \theta_j}; \qquad L_{ij} = \frac{V_j\, \partial Q_i}{\partial V_j}$$

*(Eq. 3.34, pag109, Gómez - Exposito)*

```
H = zeros( nL + nG, nL + nG );
L = zeros( nL, nL );
N = zeros( nL + nG , nL );
M = zeros( nL , nL + nG );
J = [H, N; M, L];
```

Se crea el vector de estados ($X = [\theta|V]^T$) y el vector de funciones no lineales ($f(x) = [\Delta P|\Delta Q]$

```
x_th = zeros( nL + nG, 1 ); % State vector for storing angles (theta)
x_v = zeros( nL , 1);       % State vector for storing Voltages
```

$$x = [\theta|V]^{\mathrm{T}} = [\theta_1, \theta_2, \ldots, \theta_{n-1}|V_1, V_2, \ldots, V_{n_L}]^{\mathrm{T}}$$ dim= (2nL+nG)

```
x = [x_th; x_v];
```

- Donde se tiene que:

$$\Delta P_i = P_i^{\mathrm{sp}} - V_i \sum_{j=1}^{n} V_j (G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}) \quad i = 1, 2, \ldots, n-1$$

$$\Delta Q_i = Q_i^{\mathrm{sp}} - V_i \sum_{j=1}^{n} V_j (G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij}) \quad i = 1, 2, \ldots, n_{\mathrm{L}}$$

*(Eq. 3.30-3.31, pag108, Gómez - Exposito)*

```
Dt_P = zeros( nL + nG, 1 ); % Delta P
Dt_Q = zeros( nL, 1);       % Delta Q
```

$$f(x) = [\Delta P | \Delta Q]^{\mathrm{T}} = [\Delta P_1, \Delta P_2, \ldots, \Delta P_{n-1} | \Delta Q_1, \Delta Q_2, \ldots, \Delta Q_{n_{\mathrm{L}}}]^{\mathrm{T}}$$ dim= (2nL+nG)

```
Dt = [Dt_P;Dt_Q];  %Delta Vector (same as f(x) Vector)

%% Define Unkown Values (Voltage and angle in PQ node - Angle in PV node)
% #Buses with unknwon voltage magnitudes ('PQ' buses)  (Delta Q index)
x_v_n=N_data(strcmp(N_data.type,'PQ'),:).n;

% #Buses with unknwon voltage angles ('PQ'|'PV' buses) (Delta P index)
x_th_n=N_data(strcmp(N_data.type,'PV')|strcmp(N_data.type,'PQ'),:).n;


x_n = [x_th_n; x_v_n]; % #Buses with unknown Angles and unknown voltages

max_iter = 100; %Define Max iteration number

V = zeros(num_nodes, max_iter);     % Matrix for storing node voltages per iter
Vm = zeros(num_nodes, max_iter);    % For storing node voltage magnitude per iter
th = zeros(num_nodes, max_iter);    % For storing node voltage angles per iter
V(:,1) = Vm_sp .* exp(1j .* th_sp); % Fill with initial Specified Voltages
Vm(:,1) = Vm_sp;                    % Fill with initial Specified Magnitud
th(:,1) = th_sp;                    % Fill with initial Specified Angle

err = 1.0;  % Define Error
k = 1;      % iteration counter
```

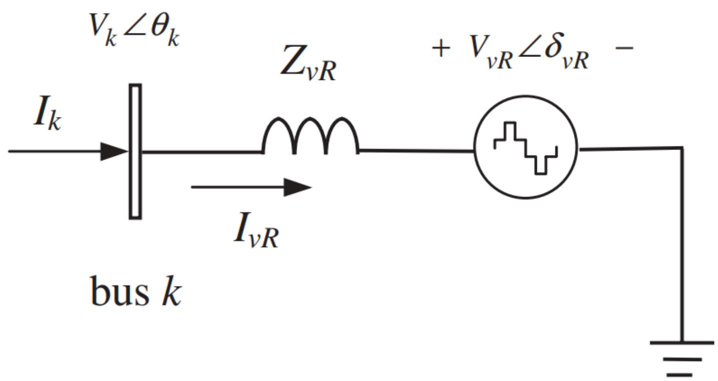
## 5) Inicia el método Newton Raphson

```
while (err > 1e-12 && k < max_iter)
% for cc = 1 : 3

    % *************************** Jacobian Terms Calculation.
    Vn = V( : , k ); %Phasors!!
    Vd = diag( Vn ); %Voltages in a diagonal matrix

    % Calculate active and reactive power per node
```

```
    Sn = Vd * conj( Ybus * V( : , k ) ); %  Sn=V(I)* | I=Y_bus·V | Sn=V·(Ybus·V)*
    Pn = real(Sn);  %Active Power calculated
    Qn = imag(Sn);  % Reactive Power calculated
```




$$P_{sh} = V_i^2 g_{sh} - V_i V_{sh}(g_{sh}\cos(\theta_i - \theta_{sh}) + b_{sh}\sin(\theta_i - \theta_{sh}))$$

$$Q_{sh} = -V_i^2 b_{sh} - V_i V_{sh}(g_{sh}\sin(\theta_i - \theta_{sh}) - b_{sh}\cos(\theta_i - \theta_{sh}))$$

(eq 2.2-2.3,pag 30, Zhang)

```
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    %% SSC Calculated Power
    for n=1:size(SSC_data,1)
        SSC_data{n,"B"} = 1/SSC_data{n,"Xvr"};
        A1 = SSC_data{n,"Tvr"} - th(SSC_data{n,"SSCsend"},k); %Thetash - Thetai

        Pn(SSC_data{n,"SSCsend"}) = Pn(SSC_data{n,"SSCsend"}) + ...
            Vm(SSC_data{n,"SSCsend"},k)*SSC_data{n,"Vvr"}*SSC_data{n,"B"}*sin(-A1);

        Qn(SSC_data{n,"SSCsend"}) = Qn(SSC_data{n,"SSCsend"}) + ...
            Vm(SSC_data{n,"SSCsend"},k)^2*SSC_data{n,"B"} - ...
            SSC_data{n,"Vvr"}*Vm(SSC_data{n,"SSCsend"},k)*SSC_data{n,"B"}*cos(-A1);

        % - -

        SSC_data{n,"PSSC"}=SSC_data{n,"Vvr"}*Vm(SSC_data{n,"SSCsend"},k)*SSC_data{n,"B"}*sin(A1

        SSC_data{n,"QSSC"}=-SSC_data{n,"Vvr"}^2*SSC_data{n,"B"} +...
            SSC_data{n,"Vvr"}*Vm(SSC_data{n,"SSCsend"},k)*SSC_data{n,"B"}*cos(A1);
    end
```

Se calculan los términos residuales.

$$\Delta P_i = P_i^{sp} - \overbrace{V_i \sum_{j=1}^{n} V_j(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij})}^{P_n} \quad i = 1, 2, \ldots, n-1$$

$$\Delta Q_i = Q_i^{sp} - \overbrace{V_i \sum_{j=1}^{n} V_j(G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij})}^{Q_n} \quad i = 1, 2, \ldots, n_L$$

```
% *************************** Residual Terms Calculations

% Delta P
Dt_P=P_sp(x_th_n(1:nL+nG))-Pn(x_th_n(1:nL+nG));
% Delta Q
Dt_Q=Q_sp(x_v_n(1:nL))-Qn(x_v_n(1:nL));


Dt = [ Dt_P ;...
       Dt_Q ];
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%     SSC Power Mismatches     -DPvr    -DQvr
for n=1:size(SSC_data,1)
    SSC_data{n,"B"} = 1/SSC_data{n,"Xvr"};

    A1 = SSC_data{n,"Tvr"} - th(SSC_data{n,"SSCsend"},k);

    Pcal = Vm(SSC_data{n,"SSCsend"},k)*SSC_data{n,"Vvr"}*SSC_data{n,"B"}*sin(-A1);

    Qcal = -Vm(SSC_data{n,"SSCsend"},k)^2*SSC_data{n,"B"} + ...
            Vm(SSC_data{n,"SSCsend"},k)*SSC_data{n,"Vvr"}*SSC_data{n,"B"}*cos(-A1);

    Dt(2*nL+nG+2*n-1) = Pcal - SSC_data{n,"Psp"};  % DPvr

    if SSC_data{n,"QSta"}==1
        Dt(2*nL+nG+2*n) = Qcal - SSC_data{n,"Qsp"}; % DQvr
    else
       Dt(2*nL+nG+2*n) = 0;
    end
end
```

```
%% Find Jacobian
```

Se obtienen las matrices H,L,N,M donde:

$$\text{For } i \neq j$$
$$H_{ij} = L_{ij} = V_i V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij})$$
$$N_{ij} = -M_{ij} = V_i V_j (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij})$$

$$\text{For } i = j$$
$$H_{ii} = -Q_i - B_{ii} V_i^2 \quad L_{ii} = Q_i - B_{ii} V_i^2$$
$$N_{ii} = P_i + G_{ii} V_i^2 \quad M_{ii} = P_i - G_{ii} V_i^2$$

*(Table 3.1, pag109, Gómez - Exposito)*

```
  % H Matrix.   H -> Dim: (nL+nG) X (nL+nG)

    for mm = 1 : nL + nG        %Iter over rows H matrix
        for nn = 1 : nL + nG    %Iter over cols H matrix
            ii = x_th_n(mm);    % #node corresponding to row mm
            jj = x_th_n(nn);    % #node corresponding to column nn

            % From Prev Equations:
            if ii == jj
                H(mm,nn) = - Qn(ii) - imag( Ybus(ii,jj) ) .* abs( V(ii,k) ).^2;
            else
                H(mm,nn) = abs( V(ii,k) ) * abs( V(jj,k) ) .* ( ...
                    real(Ybus(ii,jj)) * sin( th(ii,k) - th(jj,k) ) - ...
                    imag(Ybus(ii,jj)) * cos( th(ii,k) - th(jj,k) ) );
            end
        end
    end

    % N Matrix. N -> Dim: (nL+nG) X (nL)
    for mm = 1 : nL + nG     %  Iter over rows N matrix
        for nn = 1 : nL      %  Iter over cols N matrix
            ii = x_th_n(mm); % #node corresponding to row mm
            jj = x_v_n(nn);  % #node corresponding to column nn

            % From Prev Equations:
            if ii == jj
                N(mm,nn) = Pn(ii) + real(Ybus(ii,jj)) .* abs( V(ii,k) ).^2;
            else
                N(mm,nn) = abs( V(ii,k) ) * abs( V(jj,k) ) .* ( ...
                    real(Ybus(ii,jj)) * cos( th(ii,k) - th(jj,k) ) + ...
                    imag(Ybus(ii,jj)) * sin( th(ii,k) - th(jj,k) ) );
            end
        end
    end

    % M Matrix. M -> Dim: (nL) X (nL+nG)
    for mm = 1 : nL             %  Iter over rows M matrix
        for nn = 1 : nL + nG    %  Iter over cols M matrix
```

```
            ii = x_v_n(mm);      % #node corresponding to row mm
            jj = x_th_n(nn);     % #node corresponding to column nn

            % From Prev Equations:
            if ii == jj
                M(mm,nn) = Pn(ii) - real(Ybus(ii,jj)) .* abs( V(ii,k) ).^2;
            else
                M(mm,nn) = -abs( V(ii,k) ) * abs( V(jj,k) ) .* ( ...
                    real(Ybus(ii,jj)) * cos( th(ii,k) - th(jj,k) ) + ...
                    imag(Ybus(ii,jj)) * sin( th(ii,k) - th(jj,k) ) );
            end
        end
    end

    % L Matrix. L -> Dim: (nL) X (nL)
    for mm = 1 : nL          %  Iter over rows L matrix
        for nn = 1 : nL      %  Iter over cols L matrix
            ii = x_v_n(mm); % #node corresponding to row mm
            jj = x_v_n(nn); % #node corresponding to column nn

            % From Prev Equations:
            if ii == jj
                L(mm,nn) = Qn(ii) - imag(Ybus(ii,jj)) .* abs( V(ii,k) ).^2;
            else
                L(mm,nn) = abs( V(ii,k) ) * abs( V(jj,k) ) .* ( ...
                    real(Ybus(ii,jj)) * sin( th(ii,k) - th(jj,k) ) - ...
                    imag(Ybus(ii,jj)) * cos( th(ii,k) - th(jj,k) ) );
            end
        end
    end
   % The jacobian matrix results in:
    J = [H, N;...
        M, L];
```

Se agregan valores al jacobiano de la conexión del SSC.

$$
\begin{bmatrix} \Delta P_k \\ \\ \Delta Q_k \\ \\ \Delta P_{vR} \\ \\ \Delta Q_{vR} \end{bmatrix}
=
\begin{bmatrix}
\dfrac{\partial P_k}{\partial \theta_k} & \dfrac{\partial P_k}{\partial V_k} V_k & \dfrac{\partial P_k}{\partial \delta_{vR}} & \dfrac{\partial P_k}{\partial V_{vR}} V_{vR} \\
\dfrac{\partial Q_k}{\partial \theta_k} & \dfrac{\partial Q_k}{\partial V_k} V_k & \dfrac{\partial Q_k}{\partial \delta_{vR}} & \dfrac{\partial Q_k}{\partial V_{vR}} V_{vR} \\
\dfrac{\partial P_{vR}}{\partial \theta_k} & \dfrac{\partial P_{vR}}{\partial V_k} V_k & \dfrac{\partial P_{vR}}{\partial \delta_{vR}} & \dfrac{\partial P_{vR}}{\partial V_{vR}} V_{vR} \\
\dfrac{\partial Q_{vR}}{\partial \theta_k} & \dfrac{\partial Q_{vR}}{\partial V_k} V_k & \dfrac{\partial Q_{vR}}{\partial \delta_{vR}} & \dfrac{\partial Q_{vR}}{\partial V_{vR}} V_{vR}
\end{bmatrix}
\begin{bmatrix} \Delta \theta_k \\ \\ \dfrac{\Delta V_k}{V_k} \\ \\ \Delta \delta_{vR} \\ \\ \dfrac{\Delta V_{vR}}{V_{vR}} \end{bmatrix}
$$

$$\frac{\partial P_k}{\partial \theta_k} = -Q_k - V_k^2 G_{vR}, \qquad \frac{\partial Q_k}{\partial \theta_k} = P_k - V_k^2 G_{vR},$$

$$\frac{\partial P_k}{\partial \delta_{vR}} = V_k V_{vR}[G_{vR}\sin(\theta_k - \delta_{vR}) - B_{vR}\cos(\theta_k - \delta_{vR})], \qquad \frac{\partial Q_k}{\partial \delta_{vR}} = -V_k V_{vR}[G_{vR}\cos(\theta_k - \delta_{vR}) + B_{vR}\sin(\theta_k - \delta_{vR})],$$

$$\frac{\partial P_{vR}}{\partial \delta_{vR}} = -Q_{vR} - V_{vR}^2 B_{vR}, \qquad \frac{\partial Q_{vR}}{\partial \delta_{vR}} = P_{vR} - V_{vR}^2 G_{vR},$$

$$\frac{\partial P_{vR}}{\partial \theta_k} = V_{vR} V_k[G_{vR}\sin(\delta_{vR} - \theta_k) - B_{vR}\cos(\delta_{vR} - \theta_k)], \qquad \frac{\partial Q_{vR}}{\partial \theta_k} = -V_{vR} V_k[G_{vR}\cos(\delta_{vR} - \theta_k) + B_{vR}\sin(\delta_{vR} - \theta_k)],$$

$$\frac{\partial P_k}{\partial V_k} V_k = P_k + V_k^2 G_{vR}, \qquad \frac{\partial Q_k}{\partial V_k} V_k = Q_k - V_k^2 B_{vR},$$

$$\frac{\partial P_k}{\partial V_{vR}} V_{vR} = V_k V_{vR}[G_{vR}\cos(\theta_k - \delta_{vR}) + B_{vR}\sin(\theta_k - \delta_{vR})], \qquad \frac{\partial Q_k}{\partial V_{vR}} V_{vR} = V_k V_{vR}[G_{vR}\sin(\theta_k - \delta_{vR}) - B_{vR}\cos(\theta_k - \delta_{vR})],$$

$$\frac{\partial P_{vR}}{\partial V_{vR}} V_{vR} = P_{vR} + V_{vR}^2 G_{vR}, \qquad \frac{\partial Q_{vR}}{\partial V_{vR}} V_{vR} = Q_{vR} - V_{vR}^2 B_{vR},$$

$$\frac{\partial P_{vR}}{\partial V_k} V_k = V_{vR} V_k[G_{vR}\cos(\delta_{vR} - \theta_k) + B_{vR}\sin(\delta_{vR} - \theta_k)], \qquad \frac{\partial Q_{vR}}{\partial V_k} V_k = -V_{vR} V_k[G_{vR}\sin(\delta_{vR} - \theta_k) - B_{vR}\cos(\delta_{vR} - \theta_k)].$$

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%                 SSC JACOBIAN UPDATE

 for n=1:size(SSC_data,1)

     DP_send_Index = find(x_th_n == SSC_data{n,'SSCsend'});% get delta(P) index of the SSC s
     DQ_send_Index = find( x_v_n == SSC_data{n,'SSCsend'}); % get delta(Q) index of the SSC

     SSC_data{n,"B"} = 1/SSC_data{n,"Xvr"};

     % Change dV
     if SSC_data{n,"VSta"}==1
         J(:,(nL+nG)+DQ_send_Index) = 0;
     end

     %dPvr/dAvr    - init
         J((2*nL+nG)+2*n-1,(2*nL+nG)+2*n-1) =  1;
     %dQvr/dVvr    - init
         J((2*nL+nG)+2*n,(2*nL+nG)+2*n) = 1;     % Expand Jacobian


     % Important Values
     A1 = SSC_data{n,"Tvr"} - th(SSC_data{n,"SSCsend"},k);

     Pcal = -Vm(SSC_data{n,"SSCsend"},k)*SSC_data{n,"Vvr"}*SSC_data{n,"B"}*sin(-A1);

     DQcal = SSC_data{n,"Vvr"}*Vm(SSC_data{n,"SSCsend"},k)*SSC_data{n,"B"}*cos(-A1);

     Pssc=-SSC_data{n,"Vvr"}*Vm(SSC_data{n,"SSCsend"},k)*SSC_data{n,"B"}*sin(A1);

     DQssc=SSC_data{n,"Vvr"}*Vm(SSC_data{n,"SSCsend"},k)*SSC_data{n,"B"}*cos(A1);


     %dPk/dAk   - 1
```

```
        J(DP_send_Index,DP_send_Index)         = J(DP_send_Index,DP_send_Index)...
                                                 + Vm(SSC_data{n,"SSCsend"},k)^2*SSC_data{n,"B'

        %dQk/dAk    - 2
        J((nL+nG)+DQ_send_Index,DP_send_Index) = J((nL+nG)+DQ_send_Index,DP_send_Index)...
                                                 - Pcal;

        %dPk/dVk   - 5
        J(DP_send_Index,(nL+nG)+DQ_send_Index) = J(DP_send_Index,(nL+nG)+DQ_send_Index)...
                                                 - Pssc;

        %dQk/dVk - 6
        J((nL+nG)+DQ_send_Index,(nL+nG)+DQ_send_Index) = J((nL+nG)+DQ_send_Index,(nL+nG)+DQ_ser
                                                         - DQssc;

        if(SSC_data{n,'PSta'}==1)
            %dPvr/dAk     - 7
            J((2*nL+nG)+2*n-1,DP_send_Index) = J((2*nL+nG)+2*n-1,DP_send_Index)...
                                               + DQcal;
            %dPk/dAvr     - 8
            J(DP_send_Index,(2*nL+nG)+2*n-1) = J(DP_send_Index,(2*nL+nG)+2*n-1)...
                                               - DQssc;
            %dQk/dAvr    - 9
            J((nL+nG)+DQ_send_Index,(2*nL+nG)+2*n-1) = J((nL+nG)+DQ_send_Index,(2*nL+nG)+2*n-1)
                                                       - Pssc;
            %dPvr/dAvr   - 10
            J((2*nL+nG)+2*n-1,(2*nL+nG)+2*n-1) =  -DQssc;

            % -  -

            %dQvr/dAvr   - 13
            J((2*nL+nG)+2*n,(2*nL+nG)+2*n-1) = 0;
            %dPvr/dAvr   - 14
            J((2*nL+nG)+2*n-1,(nL+nG)+DQ_send_Index) = J((2*nL+nG)+2*n-1,(nL+nG)+DQ_send_Index)
                                                       + Pssc;

        else
            %dPvr/dAvr    - 15
            J((2*nL+nG)+2*n-1,(2*nL+nG)+2*n-1) = 1.0;
        end

        %dQk/dAvr    - 23
            J((2*nL+nG)+2*n,(2*nL+nG)+2*n) = 1;


    end
```

Se actualiza la solución del sistema donde:

$$\begin{bmatrix} \theta_{k+1} \\ V_{k+1} \end{bmatrix} = \begin{bmatrix} \theta_{k+1} \\ V_{k+1} \end{bmatrix} + J^{-1} \cdot \begin{bmatrix} \Delta P_k \\ \Delta Q_k \end{bmatrix}$$

```
% *************************** System Solution
x = J \ Dt; % Delta theta -> inv(J)*Dt
x_th = x( 1 : nL + nG );                % Adjust in unknown Angles
x_v = x( nL + nG + 1 : 2 * nL + nG );   % Adjust in unknown Voltages

Vm(:,k + 1) = Vm(:,k); % new magnitudes
th(:,k + 1) = th(:,k); % new Angles

th(x_th_n, k + 1) = x_th + th(x_th_n, k); % New angle   =  Adjust in unknown Angle   + prev
Vm(x_v_n, k + 1)  = x_v.*Vm(x_v_n, k) + Vm(x_v_n, k);   % new voltage =  Adjust in unknown
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%            SSC Updating and Limits
for n=1:size(SSC_data,1)
    if SSC_data{n,"VSta"}==1

        DP_send_Index = find(x_th_n == SSC_data{n,'SSCsend'});% get delta(P) index of the S
        DQ_send_Index = find( x_v_n == SSC_data{n,'SSCsend'}); % get delta(Q) index of the


        SSC_data{n,"Vvr"} = SSC_data{n,"Vvr"} + SSC_data{n,"Vvr"}*x(nL+nG+DQ_send_Index);
        Vm(SSC_data{n,'SSCsend'},k+1) = SSC_data{n,'TarVol'};

        if SSC_data{n,'Psp'}==0
            SSC_data{n,'Tvr'}=th(SSC_data{n,'SSCsend'},k+1);
        else
            SSC_data{n,'Tvr'}=SSC_data{n,'Tvr'}+x((2*nL+nG)+2*n-1);
        end

    else
        SSC_data{n,"Vvr"}=SSC_data{n,"Vvr"}+SSC_data{n,"Vvr"}*x((2*nL+nG)+2*n);
         SSC_data{n,'Tvr'}=th(SSC_data{n,'SSCsend'},k+1);
    end

    %% Check Limits

    if SSC_data{n,"Vvr"}>SSC_data{n,"VvrHi"}

        SSC_data{n,"Vvr"}=SSC_data{n,"VvrHi"};

    elseif SSC_data{n,"Vvr"}<SSC_data{n,"VvrLo"}

        SSC_data{n,"Vvr"}=SSC_data{n,"VvrLo"};

    end
end
```

```
V(:,k + 1) = Vm(:,k + 1) .* exp( 1j .* th(:,k + 1) ); % Calculate phasors.
```

```
%     err = max( abs( V(:,k+1) - V(:,k) )  );
    err = max( abs( Dt )  ); % Calculate error

    k = k + 1;  % Add 1 to iterations
end
```

### 6) Cálculos finales:

Para el cálculo de la potencia aparente se tiene que:

$$S = \begin{bmatrix} V_1 & 0 & 0 & 0 \\ 0 & V_2 & 0 & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & V_n \end{bmatrix} \cdot \left( Y_{\text{Bus}} \cdot \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} \right)^* = \begin{bmatrix} V_1 & 0 & 0 & 0 \\ 0 & V_2 & 0 & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & V_n \end{bmatrix} \cdot \left( \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} \right)^* = \begin{bmatrix} V_1 & 0 & 0 & 0 \\ 0 & V_2 & 0 & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & V_n \end{bmatrix} \cdot \left( \begin{bmatrix} I_1^* \\ I_2^* \\ \vdots \\ I_n^* \end{bmatrix} \right) = \begin{bmatrix} V_1 I_1^* \\ V_2 I_2^* \\ \vdots \\ V_n I_n^* \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_n \end{bmatrix}$$

```
% Complex power calculation for each node.
Vn = V( : , k );
Vd = diag( Vn );
Sn = Vd * conj( Ybus * V( : , k ) );
Pn = real(Sn);
Qn = imag(Sn);

SGn = Sn + (PL + 1j * QL);
SLn = SGn - Sn;

% Complex Power Flow Through Branches.

S_pf = zeros( 2*size(br_data, 1) , 5);

bc = 1;
for kk = 1: size(br_data, 1)
    ii = br_data.FromBus(kk);
    jj = br_data.ToBus(kk);
    z = br_data.R(kk) + 1j * br_data.X(kk);
    y = br_data.G(kk) + 1j * br_data.B(kk);
    Ybr=[1 / z + y / 2 ,    -1 / z;  ...     % Ybr (Two-Port Network)
            -1 / z     , 1 / z + y / 2];
    S_br=diag([Vn(ii),Vn(jj)])*conj(Ybr*[Vn(ii);Vn(jj)]);

    S_pf_br= [ii, jj, real(S_br(1)), imag(S_br(1)), abs(S_br(1));...% Apparent power from i to
                jj, ii, real(S_br(2)), imag(S_br(2)), abs(S_br(2))];  % Apparent power from j to

    S_pf(2*kk-1:2*kk,:)=S_pf_br;
end
```

### 7) Se presentan los resultados obtenidos

```
%% Results printing.

for i=1:1 % This "for" is used to display everything once.

    fprintf('     ******************************************** \n')
    fprintf('     **           Newton-Raphson Results       ** \n')
    fprintf('     ******************************************** \n\n')

    fprintf('-- Number of Iterarions: %d \n', k-1)
    fprintf('-- Error: %6.3e p.u.\n\n', err)

    VarNames = {'Node#', 'Type', 'V p.u.', ' ∠V(º)','PG p.u.','QG p.u.','PL p.u.','QL p.u.'};
    fprintf(1,'\t%s\t%s\t%s\t%s\t%s\t%s\t%s\t%s\n', VarNames{:})
    fprintf(1, '--------------------------------------------------------------------\n')

    for ii = 1 : num_nodes
        fprintf('   \t%d\t%s\t%6.3f\t%6.3f\t%6.3f\t%6.3f\t%6.3f\t%6.3f\t\n', ii, char(N_data{ii
            abs(Vn(ii)), angle(Vn(ii)).*180./pi, real(SGn(ii)), imag(SGn(ii)), real(SLn(ii)), i
    end
end
```

```
     ********************************************
     **          Newton-Raphson Results       **
     ********************************************
-- Number of Iterarions: 5
-- Error: 5.385e-15 p.u.
 Node# Type V p.u.  ∠V(º) PG p.u. QG p.u. PL p.u. QL p.u.
----------------------------------------------------------------------
    1 SL   1.060  0.000  1.311  0.853  0.000  0.000
    2 PV   1.000 -2.053  0.400 -0.771  0.200  0.100
    3 PQ   1.000 -4.838  0.000  0.205  0.450  0.150
    4 PQ   0.994 -5.107  0.000  0.000  0.400  0.050
    5 PQ   0.975 -5.797  0.000  0.000  0.600  0.100
```

```
array2table(S_pf,'VariableNames',{'FromBus','ToBus','P','Q','S'});
```

**Table 5.5** Bus voltages of the STATCOM-upgraded network

| | Network bus | | | | |
|---|---|---|---|---|---|
| Nodal voltage | North | South | Lake | Main | Elm |
| Magnitude (p.u.) | 1.06 | 1 | 1 | 0.994 | 0.975 |
| Phase angle (deg) | 0 | −2.05 | −4.83 | −5.11 | −5.8 |

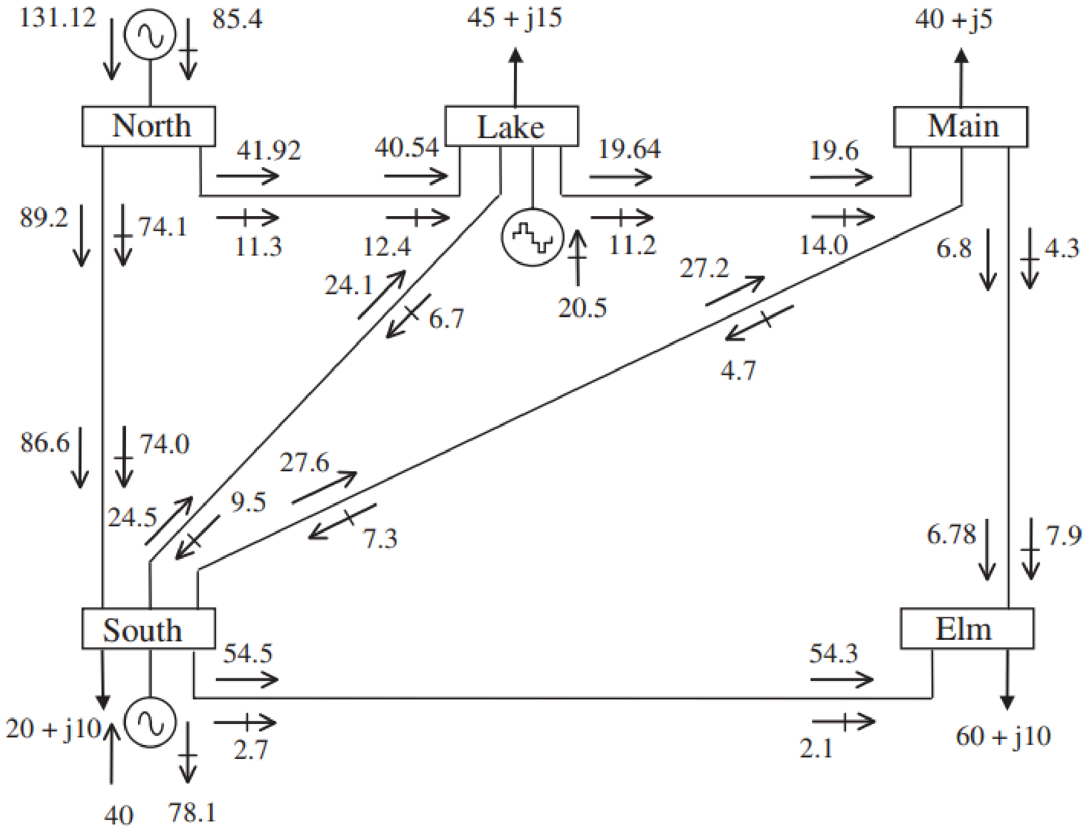



**Figure 5.16** STATCOM-upgraded test network and power flow results

## Referencias: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