CPSC 121 - PROPOSITIONAL LOGIC PROOFS - SOLUTIONS

Problem 1. Prove b using a formal propositional logic proof given the five numbered premises below.

[1] $\sim p \vee q \to p$
[2] $\sim r \to \sim p$
[3] $\sim (r \wedge \sim a)$
[4] $\sim a \vee b$
[5] $q \vee s \to t$

Solution. Proof of b:

[6]	$\sim(\sim p \vee q) \vee p$	Def. of Implication from [1]
[7]	$(\sim\sim p \wedge \sim q) \vee p$	De Morgan's Law from [6]
[8]	$(p \wedge \sim q) \vee p$	Double Negation Law from [7]
[9]	p	Absorption Law from [8]
[10]	r	Modus Tollens from [2] and [9]
[11]	$\sim r \vee \sim\sim a$	De Morgan's Law from [3]
[12]	$\sim r \vee a$	Double Negation Law from [11]
[13]	a	Elimination from [10] and [12]
[14]	b	Elimination from [4] and [13]
		QED

Problem 2. Decide whether the following argument is valid or not. If you think it is invalid, provide a truth value assignment that proves your claim. Otherwise provide a proof for it.

[1] $p \to q$
[2] $m \vee s$
[3] $\sim s \to \sim r$
[4] $\sim q \vee s$
[5] $\sim s$
[6] $\sim p \wedge m \to u$

$\therefore$ $\sim u$

Solution. The argument is invalid. Here gives an assignment that makes the premises true but the conclusion false.

$$m = T, p = F, q = F, r = F, s = F, u = T$$

Approach I. Let's try to prove $\sim u$ from the premises [1] to [6] and see what we get.
Proof of u:

[7]	m	Elimination from [2] and [5]
[8]	$\sim r$	Modus Ponens from [3] and [5]
[9]	$\sim q$	Elimination from [4] and [5]
[10]	$\sim p$	Modus Tollens from [1] and [9]
[11]	$\sim p \wedge m$	Conjunction from [7] and [10]
[12]	u	Modus Tollens from [6] and [11]
		QED

We have proved u instead of $\sim u$ from the premises [1] to [6]. Thus we know that

- the argument of $\sim u$ is invalid, or
- the premises [1] to [6] are self-contradictory.

To make the premises [1] to [6] true, at least we need to make [5], [7], [8], [9], [10] and [12], i.e. $\sim s, m, \sim r, \sim q, \sim p$ and u, true. Then, $s = F$, $m = T$, $r = F$, $q = F$, $p = F$ and $u = T$. Using this truth assignment, we can verify that the premises [1] to [6] are true but the conclusion, $\sim u$, is false.

Approach II. Here we show how we get this assignment.

(1) To make [5] true, $s = F$.
(2) Given $s = F$, to make [2] true, $m = T$. Analogously, to make [4] true, $q = F$.
(3) Given $s = F$, to make [3] true, $r = F$.
(4) Given $q = F$, to make [1] true, $p = F$.
(5) Given $p = F$ and $m = T$, to make [6] true, $u = T$.
(6) $u = T$ makes the conclusion, $\sim u$, false. Thus the argument above is invalid, which can be verified by the assignment $s = F, m = T, q = F, r = F, p = F, u = T$.

Problem 3. Steve has bizarre powers of observation, and noticed the following facts during a staff meeting, while the TAs were nibbling on their food:

- Adam and Winnie did not both eat sandwiches.
- If Thiabaud forgot to bring his lunch, then either Adam ate a sandwich, or Yves did not eat an orange (or both).
- Winnie ate a sandwich.
- If Cindy forgot to eat her banana and Adam did not eat a sandwich, then Thiabaud forgot to bring his lunch.
- If Adam did not eat a sandwich then Yves ate an orange.

Steve thinks that Cindy did not forget to eat her banana, but he is not certain.

(1) Name each simple proposition above, e.g.: t:Thiabaud forgot to bring his lunch.
(2) Rewrite the bulleted statements using propositional logic and your propositions from the previous part.
(3) Using your statements in the previous part as premises, prove that Cindy did not forget to eat her banana. Be sure to list and number your steps and to give a justification for each step, citing the previous step(s) it depends on.

Solution

(1) (a) a: Adam ate a sandwich
  (b) w: Winnie ate a sandwich
  (c) t: Thiabaud forgot to bring his lunch
  (d) y: Yves ate an orange
  (e) q: Cindy forgot to eat her banana

(2)

$$\begin{array}{ll} [1] & \sim(a \wedge w) \\ [2] & t \rightarrow (a \vee \sim y) \\ [3] & w \\ [4] & (q \wedge \sim a) \rightarrow t \\ [5] & \sim a \rightarrow y \\ \hline \therefore & \sim q \end{array}$$

(3) Proof of $\sim q$:

[6]	$\sim a \vee \sim w$	De Morgan's Law from [1]
[7]	$\sim a$	Elimination from [3] and [6]
[8]	y	Modus Ponens from [7] and [5]
[9]	$\sim a \wedge y$	Conjunction from [7] and [8]
[10]	$\sim (\sim\sim a \vee \sim y)$	De Morgan's Law from [9]
[11]	$\sim (a \vee \sim y)$	Double Negation Law from [10]
[12]	$\sim t$	Modus Tollens from [11] and [2]
[13]	$\sim(q \wedge \sim a)$	Modus Tollens from [12] and [4]
[14]	$\sim q \vee \sim\sim a$	De Morgan's Law from [13]
[15]	$\sim q \vee a$	Double Negation Law from [14]
[16]	$\sim q$	Elimination from [7] and [15]
		QED
