CPSC 121 - PREDICATE LOGIC SOLUTIONS

Problem 1. Let C be the set of European cities, let R be the set of European rivers, and S be the set of European countries. Also consider the following predicates:

$L(x, y)$: City x lies on the river y.
$P(x, y)$: A river x or a city x is at least partially contained in country y

Translate the following predicate logic statements into English.

(1) $\forall x \in C, L(x, Seine) \to P(x, France)$
Solution: Every city that lies on the Seine river is contained in France.
(2) $\exists x \in C, \sim L(x, Rhine) \wedge \sim L(x, Danube)$
Solution: There exists a city that neither lies on the Rhine river nor on the Danube river.
(3) $\forall x \in C, \sim (L(x, Rhine) \wedge L(x, Danube))$
Solution: No city lies on both the Rhine river and the Danube river.
(4) $\forall x \in R, \exists y \in C, L(y, x)$
Solution: Each river has least one city that lies on the river.
(5) $\exists x \in C, \forall y \in R, \sim L(x, y)$
Solution: There is a city that doesn't lie on a river
(6) $\forall x \in S, \exists y \in S, \exists z \in R, (x \neq y) \wedge P(z, x) \wedge P(z, y)$
Solution: Each country has at least one river that also flows through another country.
(7) $\forall x \in C, (\exists y \in S, \exists z \in S, (y \neq z) \wedge P(x, y) \wedge P(x, z)) \to (\exists q \in R, L(x, q))$
Solution: Every city that is divided between at least two countries lies on a river.

Problem 2. Let $L(x, y)$ be the statement "x loves y", where the domain for both x and y, denoted by P, consists of all people in the world. Use quantifiers, logical connectives and $L(x, y)$ to express the following statements.

(1) Everybody loves Karen.
(2) There is somebody whom Mike does not love.
(3) Everyone loves themselves.
(4) Everybody loves somebody.
(5) There is somebody whom everybody loves.
(6) Nobody loves everybody.
(7) There is somebody whom no one loves.

Here are three additional challenging questions.

(1) There is exactly one person whom everybody loves.
(2) There are exactly two people whom Jennifer loves.
(3) There is someone who loves no one besides themselves.

Solutions:

(1) Everybody loves Karen.
$\forall x \in P,\ L(x, \text{Karen})$
(2) There is somebody whom Mike does not love.
$\exists x \in P,\ \sim L(\text{Mike}, x)$
(3) Everyone loves themselves.
$\forall x \in P,\ L(x, x)$
(4) Everybody loves somebody.
$\forall x \in P, \exists y \in P,\ L(x, y)$
(5) There is somebody whom everybody loves.
$\exists x \in P, \forall y \in P,\ L(y, x)$
(6) Nobody loves everybody.
$\sim \exists x \in P, \forall y \in P,\ L(x, y)$

Or alternatively, $\forall x \in P, \exists y \in P, \sim L(x, y)$

(7) There is somebody whom no one loves.
$\exists x \in P, \sim \exists y \in P, L(y, x)$
Or alternatively, $\exists x \in P, \forall y \in P, \sim L(y, x)$

Solutions to the three additional challenging problems.

(1) There is exactly one person whom everybody loves.
$\exists x \in P, \forall y \in P, L(y, x) \wedge \forall w \in P, (\forall z \in P, L(z, w)) \rightarrow w = x$

(2) There are exactly two people whom Jennifer loves.
$\exists x \in P, \exists y \in P, x \neq y \wedge L(\text{Jennifer}, x) \wedge L(\text{Jennifer}, y) \wedge (\forall z \in P, L(\text{Jennifer}, z) \rightarrow z = x \vee z = y)$

(3) There is someone who loves no one besides themselves.
$\exists x \in P, \forall y \in P, L(x, y) \leftrightarrow x = y$