CPSC 121 TUTORIAL - PROPOSITIONAL LOGIC

Problem 1. Show the following logical equivalences using the known logical equivalences. Explicitly indicate the name of every known logical equivalence that you have used.

- (1) $\sim (\sim (p \lor q \lor p) \land p) \equiv T$
- (2) $p \lor q \equiv \sim (\sim p \land ((\sim q \land \sim p) \lor (\sim q \land p)))$
- (3) $(\sim p \lor s) \land (p \to (s \to \sim p)) \equiv \sim p$
- $(4) (p \lor q) \equiv (\sim q \to p) \lor \sim (\sim p \land \sim q) \lor ((p \land \sim p) \land (q \lor \sim q))$

Problem 2. For each proposition below, indicate whether it is (1) a tautology, (2) a contradiction, or (3) neither a tautology nor a contradiction.

- $(1) \ p \to (q \to \sim p)$
- $(2) (\sim (p \oplus q)) \leftrightarrow (\sim q \oplus p)$
- $(3) ((p \leftrightarrow q) \leftrightarrow r) \to (p \lor q \lor r)$
- $(4) \sim (\sim (p \to q) \lor (p \land q)) \land p$

Problem 3. Given the truth table below, using a Karnaugh-map, design a circuit to implement the function using as few gates as possible.

p	q	r	f
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Problem 4. Given the truth table below, using a Karnaugh-map, design a circuit to implement the function using as few gates as possible.

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$					
0 0 0 0 1 0 0 0 1 1 0 0 1 0 1 0 0 1 1 0 0 1 0 0 1 0 1 0 1 1 0 1 1 0 1 1 0 0 0 0 1 0 1 0 0 1 0 1 1 0 1 1 0 0 0 1 1 0 0 0 1 1 0 1 1 1 1 0 0 0 1 1 0 1 1 1 1 0 1 1 1 1 0 1 1 1 1 0 0 0 1 1 0 0 0 1 1 0 <th>p</th> <th>q</th> <th>r</th> <th>s</th> <th>f</th>	p	q	r	s	f
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1 0 0 0 1 0 0 1 1 1 0 1 0 0 1 0 1 1 0 1 1 0 0 0 1 1 0 1 1 1 1 1 0 0	0	1	1	0	1
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1 1 1 1 1	1	1	1	0	0
	1	1	1	1	1