**CPSC 121 SEQUENTIAL CIRCUITS AND PROOF BY INDUCTION**

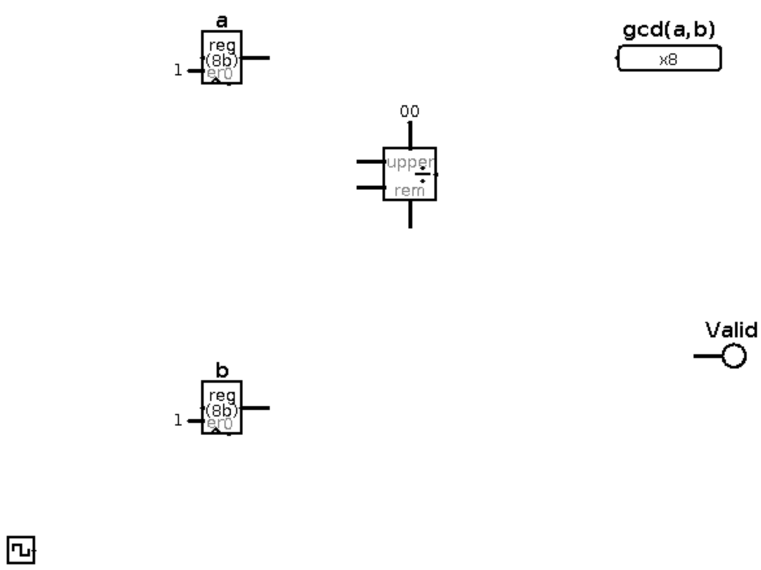
**Problem 1.** The Greatest Common Divisor (GCD) of two positive integers $a$, $b$ is the largest positive integer that is a factor of both $a$ and $b$. Being able to compute GCDs is useful because we can simplify a fraction $a/b$ by dividing both $a$ and $b$ by GCD($a$, $b$). Euclid (born around 325 BC) came up with the following algorithm to compute the GCD of two positive integers. This algorithm relies on the fact that GCD($a$, $b$) = GCD ($b$, $a$ mod $b$) as long as $b > 0$. Translated into Racket, since most of us can't read antique Greek, the algorithm becomes:

```
(define (gcd a b)
    (if (= b 0)
        a
        (gcd b (remainder a b))))
```

The following is an incomplete sequential circuit to compute the GCD of a pair of positive integers. The computation should start once the initial values of $a$ and $b$ have been entered into the two 8-bit registers `a` and `b`. A number of clock cycles later, the LED labeled `Valid` will be ON, and the output labeled `gcd(a,b)` will contain the GCD of $a$ and $b$.

Complete the implementation of this circuit. The component labeled "upper" and "rem" is a Divider subcircuit: it takes one 16-bit input $x$ (the 8 bits from the top input, and the 8 bits from the upper-left input) and one 8-bit input $y$ (the 8 bits from the bottom-left input). It produces the quotient $x/y$ (the value obtained by writing `(quotient x y)` in Dr. Racket) on the right output, and the remainder (`(remainder x y)`) on the bottom output.

Hint: consider the correspondence between the parameters in one recursive call and the parameters of the next one.




**Problem 2.** Prove the following statement using induction, $\forall n \in \mathbb{Z}^+, \sum_{i=1}^{n} \frac{2}{3^i} = 1 - \left(\frac{1}{3}\right)^n$

**Problem 3.** You have a sheet of graph paper, containing squares. The squares are all equal size, and there are an equal number of squares in each row to the number of squares in each column. You have an infinite number of "L"-shaped blocks, called trominos, that can cover three of the squares on your paper in an "L" shape.

Prove that if your graph paper has $2^n$ by $2^n$ squares on it, where $n \in \mathbb{Z}^+$, you can cover it such that *all but one square* on your graph paper is covered by a single block and with none of the blocks you used hanging off the edge of the graph paper.

*Hint:* Prove the stronger claim that you can cover any $2^n \times 2^n$ graph paper with tromino blocks such that all but one corner square is uncovered.

Below is an example of a $4 \times 4$ grid covered with trominos such that all but one square (in this example a corner square) is uncovered.