CPSC 221 HW1

z0n1b, d6a2b

TOTAL POINTS

56 / 67

QUESTION 1

1 piazza posts **2 / 2**

✓ **- 0 pts** **Correct**

- 1 pts Missing one post

- 2 pts Missing both posts

QUESTION 2

math review 16 pts

2.1 a)i & ii. **4 / 4**

✓ **+ 4 pts** **Correct: i. 3(n+1)n/2 & ii. n(2n+5)**

- 1 pts i. Simplest form, but inadequate work shown

- 1 pts i. Not simplest form, but significant work shown

- 2 pts i. Incorrect or Blank

- 1 pts ii. Simplest form, but inadequate work shown

- 1 pts ii. Not simplest form, but significant work shown

- 2 pts ii. Incorrect or Blank

2.2 b)i & ii. **2 / 4**

+ 4 pts Correct: i. 9/10 & ii. (n^3-(1/n)^n)/(n-1)

- 1 pts i. Simplest form, but inadequate work shown

- 1 pts i. Not simplest form, but significant work shown

- 2 pts i. Incorrect or Blank

- 1 pts ii. Simplest form, but inadequate work shown

- 1 pts ii. Not simplest form, but significant work shown

✓ **- 2 pts** **ii. Incorrect or Blank**

2.3 c)i & ii. **2 / 4**

+ 4 pts Correct: i. 2^(n+1) - n - 2 & ii. (1-a^(2^n+1))/(1-a)

- 1 pts i. Simplest form, but inadequate work shown

- 1 pts i. Not simplest form, but significant work shown

✓ **- 2 pts** **i. Incorrect or Blank**

- 1 pts ii. Simplest form, but inadequate work shown

- 1 pts ii. Not simplest form, but significant work shown

- 2 pts ii. Incorrect or Blank

2.4 d)i & ii. **2 / 2**

✓ **+ 2 pts** **Correct: i. 7 & ii. 8**

- 0.5 pts i. Simplest form, but inadequate work shown

- 0.5 pts i. Not simplest form, but significant work shown

- 1 pts i. Incorrect or Blank

- 0.5 pts ii. Simplest form, but inadequate work shown

- 0.5 pts ii. Not simplest form, but significant work shown

- 1 pts ii. Incorrect or Blank

2.5 e)i & ii. **2 / 2**

✓ **+ 2 pts** **Correct: i. n^(5/4) & ii. 1/9**

- 0.5 pts i. Simplest form, but inadequate work shown

- 0.5 pts i. Not simplest form, but significant work shown

- 1 pts i. Incorrect or Blank

- 0.5 pts ii. Simplest form, but inadequate work shown

- 0.5 pts ii. Not simplest form, but significant work shown

- 1 pts ii. Incorrect or Blank

QUESTION 3

divisibility 9 pts

3.1 domains **1 / 2**

- **0 pts** Correct
- **1 pts** Incorrect/Blank domain for c (should be INTEGERS)

✓ **- 1 pts Incorrect/Blank domain for n (should be NATURAL NUMS)**

- **2 pts** No Submission

3.2 proof 3 / 3

✓ **- 0 pts Correct proof**

- **1 pts** Minor incorrect statement
- **1 pts** Minor leap without justification
- **2 pts** Attempted proof but missing crucial parts
- **3 pts** Incorrect or Blank

3.3 application 4 / 4

✓ **- 0 pts Correct proof (by rearrangement or induction)**

- **2 pts** No conversion to 20^n - 1
- **2 pts** Improper inductive proof/application of theorem
- **4 pts** Incorrect or Blank

QUESTION 4

magic 12 pts

4.1 5x5 square 3 / 3

✓ **- 0 pts Correct**

- **1 pts** 1 square wrong
- **3 pts** Incorrect or Blank

4.2 column sum 3 / 3

✓ **- 0 pts Correct: $n(n^2 + 1)/2$**

- **2 pts** Left in sigma notation
- **3 pts** Incorrect or Blank

4.3 group sum 3 / 3

✓ **- 0 pts Correct: $k(k^2 + 1)/2$**

- **2 pts** Left in sigma notation
- **3 pts** Incorrect or Blank

4.4 group sum proof 3 / 3

✓ **- 0 pts Correct**

- **1 pts** Minor mistake
- **2 pts** Major leap without justification
- **3 pts** Incorrect or Blank

QUESTION 5

asymptotic 12 pts

5.1 a) 4 / 6

✓ **+ 2 pts Correct: f(n) Θ(g(n))**

+ **1 pts** O: correct values for c (2) and n_0 (9)
+ **1 pts** O: shows that f(n) < c*g(n)

✓ **+ 1 pts Ω: correct values for c (1) and n_0 (1)**

✓ **+ 1 pts Ω: shows that f(n) > c*g(n)**

+ **0 pts** Incorrect or Blank
+ **6 pts** Correct use of limits
+ **0 pts** I cannot read this

5.2 b) 4 / 6

✓ **+ 2 pts Correct: f(n) Ω(g(n))**

+ **1 pts** Ω: acceptable values for c and n_0
+ **1 pts** Ω: shows that f(n) > c*g(n)

✓ **+ 1 pts O: correct negation/contradiction of definition**

✓ **+ 1 pts O: shows that for any choice of c and n_0 f(n) > c*g(n)**

+ **0 pts** Incorrect or Blank
+ **1 pts** Significant attempt at using limits
+ **4 pts** Correct use of limits

1 please assign pages properly next time or your work will not be graded

2 this is not a proof

QUESTION 6

fun times 16 pts

6.1 a) return 1 / 1

✓ **- 0 pts Correct: 2**

- **1 pts** Incorrect or Blank

6.2 a) run time 1 / 1

✓ **- 0 pts Correct: O(n)**

- **1 pts** Incorrect or Blank

6.3 b) return 2 / 2

✓ - **0 pts** **Correct: ceil(3lgn)^2**

- **1 pts** Missing ceiling function
- **2 pts** Incorrect or Blank

6.4 b) run time 2 / 2

✓ - **0 pts** **Correct: O(log n)**

- **2 pts** Incorrect or Blank

6.5 c) return 2 / 2

✓ - **0 pts** **Correct: 8*floor(n/16)*[floor(n/16) + 1]**

- **1 pts** Missing floor functions
- **2 pts** Incorrect or Blank

6.6 c) run time 2 / 2

✓ - **0 pts** **Correct: O(n^2)**

- **2 pts** Incorrect or Blank

6.7 d) return 2 / 2

✓ - **0 pts** **Correct: floor(sqrt(n)/4)**

- **1 pts** Missing floor function
- **2 pts** Incorrect or Blank

6.8 d) run time 0 / 2

- **0 pts** Correct: O(n^(3/2))

✓ - **2 pts** **Incorrect or Blank**

6.9 e) run time 2 / 2

✓ - **0 pts** **Correct: O(n^3)**

- **2 pts** Incorrect or Blank

1. (2 points) Using 140 characters or less, post a synopsis of your favorite movie to the course piazza space under the "HW1 tell me something!" notice, so that your post is visible to everyone in the class, and tagged by #HW1num1. Also, use Piazza's code-formatting tools to write a *private* post to course staff that includes at least 5 lines of code. It can be code of your own or from a favorite project—it doesn't even have to be syntactically correct—but it must be formatted as a code block in your post, and also include the tag #HW1num1. (Hint: Check http://support.piazza.com/customer/portal/articles/1774756-code-blocking). Finally, please write the two post numbers corresponding to your posts here:

Favorite Movie Post (Public) number:	243, 311
Formatted Code Post (Private) number:	473, 313

2. (16 points) In this problem, you will be a math detective! Some of the symbols and functions may not be familiar to you, but with a little digging, reading, and observing, you'll be able to figure them out. Your task is to simplify each of the following expressions as much as possible, **without using a calculator (either hardware or software)**. Do not approximate. Express all rational numbers as improper fractions. You may assume that n is an integer greater than 1. Show your work in the space provided, and write your final answer in the box.

   (a) i. $3 + 6 + 9 + 12 + \cdots + 3n$

Answer for (a.i):	$(3/2)n(n+1)$

We can use induction to prove $S_n = \sum_{i=1}^{n} i$ = 1+...+n = n(n+1)/2
Base case where n=1:
LHS: 1 = 1
RHS: 1(1+1)/2 = 1
For inductive hypothesis, assuming n=n case is true.
For inductive step, ie n=n+1 case:
LHS: 1+...+n+n+1 = S_n+(n+1)
RHS: (n+1)(n+2)/2 = (1/2)((n+1)n+(n+1)2) = S_n+(2n+2)/2 = S_n+(n+1)
Therefore, we have proved $\sum_{i=1}^{n} i$ = 1+...+n = n(n+1)/2 is true.

Therefore we can say that:
$= 3 + ... + 3n$
$= 3(1 + ... + n)$
$= (3/2)n(n+1)$

1 piazza posts 2 / 2

- ✓ **- 0 pts** Correct
- **- 1 pts** Missing one post
- **- 2 pts** Missing both posts

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Therefore, we have proved $\sum_{i=1}^{n} i$ = 1+...+n = n(n+1)/2 is true.

Therefore we can say that:
$= 3 + ... + 3n$
$= 3(1 + ... + n)$
$= (3/2)n(n + 1)$

2.1 a)i & ii. **4 / 4**

✓ **+ 4 pts** **Correct: i. 3(n+1)n/2 & ii. n(2n+5)**

- **- 1 pts** i. Simplest form, but inadequate work shown
- **- 1 pts** i. Not simplest form, but significant work shown
- **- 2 pts** i. Incorrect or Blank
- **- 1 pts** ii. Simplest form, but inadequate work shown
- **- 1 pts** ii. Not simplest form, but significant work shown
- **- 2 pts** ii. Incorrect or Blank

(b) i. $\displaystyle\sum_{r=2}^{\infty}\left(\frac{3}{5}\right)^r$

Answer for (b.i):	$\frac{9}{10}$

We can prove that $S_n = \sum_{r=0}^{n}(a)^r = a^0 + ... + a^n$, by saying that:
$aS_n = a^1 + ...a^{n+1}$
$aS_n - S_n = a^{n+1} - a^0$
$S_n = (a^{n+1} - 1)/(a-1)$

Additionally, if n $= \infty$, and a is less than 1, we can take the limit of the summation to infinity to solve:
$= lim_{n->\infty} S_n$
$= lim_{n->\infty} S_n$
$= lim_{n->\infty}(a^{n+1} - 1)/(a-1)$
$= -1/(a-1)$
$= 1/(1-a)$

Therefore we can say that:
$= \sum_{r=2}^{\infty}\left(\frac{3}{5}\right)^r$
$= \sum_{r=0}^{\infty}\left(\frac{3}{5}\right)^r - (\frac{3}{5})^0 - (\frac{3}{5})^1$
$= -1/(\frac{3}{5} - 1) - (\frac{3}{5})^0 - (\frac{3}{5})^1$
$= \frac{5}{2} - \frac{8}{5}$
$= \frac{9}{10}$

ii. $\displaystyle\sum_{r=(-2)}^{n}\left(\frac{1}{n}\right)^r$

Answer for (b.ii):	$\dfrac{n^{-n} + n^3 - 2n}{1-n}$

$= \sum_{r=(-2)}^{n}\left(\frac{1}{n}\right)^r$
$= \sum_{r=0}^{n}\left(\frac{1}{n}\right)^r + (\frac{1}{n})^{-2} + (\frac{1}{n})^{-1}$
$= ((\frac{1}{n})^{n+1} - 1)/(\frac{1}{n} - 1) + (\frac{1}{n})^{-2} + (\frac{1}{n})^{-1}$
$= (n^{(-n-1)} - 1)/(\frac{1-n}{n}) + n^2 + n$
$= \dfrac{n^{-n} - n}{1-n} + n^2 + n$
$= \dfrac{n^{-n} + n^3 - 2n}{1-n}$

2.2 b)i & ii. 2 / 4

+ **4 pts** Correct: i. 9/10 & ii. (n^3-(1/n)^n)/(n-1)
- **1 pts** i. Simplest form, but inadequate work shown
- **1 pts** i. Not simplest form, but significant work shown
- **2 pts** i. Incorrect or Blank
- **1 pts** ii. Simplest form, but inadequate work shown
- **1 pts** ii. Not simplest form, but significant work shown

✓ **- 2 pts ii. Incorrect or Blank**

(c) i. $\sum_{k=1}^{n} k2^{n-k}$

Answer for (c.i):	$2^n - (3 + \dfrac{n}{2})$

Say $S_n = \sum_{k=1}^{n} \dfrac{k}{2^k}$, we can use a proof method similar to b.i to simplify:

$S_n = \frac{1}{2^1} + \frac{2}{2^2} ... + \frac{n}{2^n}$
$S_n/2 = \frac{1}{2^2} + \frac{2}{2^3} ... + \frac{n}{2^{n+1}}$
$S_n - S_n/2 = (\frac{1}{2^1} + \frac{2}{2^2} ... + \frac{n}{2^n}) - (\frac{1}{2^2} + \frac{2}{2^3} ... + \frac{n}{2^{n+1}})$
$S_n - S_n/2 = \sum_{i=2}^{n-1} (\frac{1}{2})^i + \frac{1}{2} - \frac{n}{2^{n+1}}$
$S_n - S_n/2 = \sum_{i=0}^{n} (\frac{1}{2})^i - (\frac{1}{2})^n - (\frac{1}{2})^0 - (\frac{1}{2})^1 + \frac{1}{2} - \frac{n}{2^{n+1}}$

Using the equation derived in b.i, we can say that $\sum_{i=0}^{n} (\frac{1}{2})^i = \dfrac{\frac{1}{2}^{n+1} - 1}{\frac{1}{2} - 1}$, and further simplify:

$$S_n - S_n/2 = \frac{\frac{1}{2}^{n+1} - 1}{\frac{1}{2} - 1} - (\frac{1}{2})^n - (\frac{1}{2})^0 - (\frac{1}{2})^1 + \frac{1}{2} - \frac{n}{2^{n+1}}$$
$$S_n = 2(\frac{\frac{1}{2}^{n+1} - 1}{\frac{1}{2} - 1} - (\frac{1}{2})^n - (\frac{1}{2})^0 - (\frac{1}{2})^1 + \frac{1}{2} - \frac{n}{2^{n+1}})$$
$$S_n = 2(-(\frac{1}{2})^{n+1} + 2 - (\frac{1}{2})^n - (\frac{1}{2})^0 - (\frac{1}{2})^1 + \frac{1}{2} - \frac{n}{2^{n+1}})$$
$$S_n = 1 - (3 + \frac{n}{2})(\frac{1}{2})^n$$

Therefore, we can solve this question as:
$= \sum_{k=1}^{n} k2^{n-k}$
$= \sum_{k=1}^{n} k\dfrac{2^n}{2^k}$
$= 2^n(1 - (3 + \dfrac{n}{2})(\dfrac{1}{2})^n)$
$= 2^n - (3 + \frac{n}{2})$

ii. $\prod_{k=0}^{n} (1 + a^{2^k})$ (Hint: Try multiplying by $(1 - a)$.)

Answer for (c.ii):	$(1 - a^{2^{n+1}})/(1 - a)$

We can use induction to prove that $\prod_{k=0}^{n} (1 + a^{2^k}) = (1 - a^{2^{n+1}})/(1 - a)$

For base case where n = 0:
LHS: $\prod_{k=0}^{0} (1 + a^{2^k}) = (1 + a^{2^0}) = 1 + a$
RHS: $(1 - a^{2^{0+1}})/(1 - a) = (1 - a^2)/(1 - a) = (1 + a)(1 - a)/(1 - a) = 1 + a$
For induction hypothesis, we assume n=n case is true.
For induction step, we prove n=n+1 case:
LHS: $\prod_{k=0}^{n+1} (1 + a^{2^k}) = (1 + a^{2^{n+1}}) \prod_{k=0}^{n} (1 + a^{2^k}) = (1 + a^{2^{n+1}})(1 - a^{2^{n+1}})/(1 - a)$
RHS: $(1 - a^{2^{(n+1)+1}})/(1 - a) = (1 + a^{2^{n+1}})(1 - a^{2^{n+1}})/(1 - a)$

2.3 c)i & ii. 2 / 4

+ **4 pts** Correct: i. 2^(n+1) - n - 2 & ii. (1-a^(2^n+1))/(1-a)

- **1 pts** i. Simplest form, but inadequate work shown

- **1 pts** i. Not simplest form, but significant work shown

✓ - **2 pts** **i. Incorrect or Blank**

- **1 pts** ii. Simplest form, but inadequate work shown

- **1 pts** ii. Not simplest form, but significant work shown

- **2 pts** ii. Incorrect or Blank

(d) i. $7^{333} \bmod 10$

Answer for (d.i):	7

$= 7^{333} \bmod 10$
$= 7^{4*83+1} \bmod 10$
$= ((2401 \bmod 10)^{83} \bmod 10) * 7 \bmod 10$
$= (1 \bmod 10) * 7 \bmod 10$
$= 7$

ii. $16^{333} \bmod 14$

Answer for (d.ii):	8

$= 16^{333} \bmod 14$
$= (16 \bmod 14)^{333} \bmod 14$
$= 2^{333} \bmod 14$
$= 2^{9*37} \bmod 14$
$= (512 \bmod 14)^{37} \bmod 14$
$= 8^{37} \bmod 14$
$= 8^{3*12+1} \bmod 14$
$= (512 \bmod 14)^{12} \bmod 14 * (8^1 \bmod 14)$
$= 8^{13} \bmod 14$
$= 8^{3*4+1} \bmod 14$
$= (512 \bmod 14)^{4} \bmod 14 * (8^1 \bmod 14)$
$= 8^{4+1} \bmod 14$
$= 8^{3+2} \bmod 14$
$= (512 \bmod 14)^{1} \bmod 14 * (8^2 \bmod 14)$
$= 8^{1+2} \bmod 14$
$= 512 \bmod 14$
$= 8$

We could also notice that $8^n \bmod 14$ is always 8.

2.4 d)i & ii. **2 / 2**

- ✓ **+ 2 pts** **Correct: i. 7 & ii. 8**
- **- 0.5 pts** i. Simplest form, but inadequate work shown
- **- 0.5 pts** i. Not simplest form, but significant work shown
- **- 1 pts** i. Incorrect or Blank
- **- 0.5 pts** ii. Simplest form, but inadequate work shown
- **- 0.5 pts** ii. Not simplest form, but significant work shown
- **- 1 pts** ii. Incorrect or Blank

(e) i. $32^{(\log_2 n)/4}$

Answer for (e.i):	$n^{5/4}$

$$= 32^{(\log_2 n)/4}$$
$$= 2^{5(\log_2 n)/4}$$
$$= n^{5/4}$$

ii. $\dfrac{\log_{512} n}{\log_2 n}$

Answer for (e.ii):	1/9

$$= \frac{\log_{512} n}{\log_2 n}$$
$$= \frac{\frac{\log_2 n}{\log_2 512}}{\log_2 n}$$
$$= \frac{1}{\log_2 512}$$
$$= 1/9$$

2.5 e)i & ii. **2 / 2**

- ✓ **+ 2 pts** **Correct: i. n^(5/4) & ii. 1/9**
- **- 0.5 pts** i. Simplest form, but inadequate work shown
- **- 0.5 pts** i. Not simplest form, but significant work shown
- **- 1 pts** i. Incorrect or Blank
- **- 0.5 pts** ii. Simplest form, but inadequate work shown
- **- 0.5 pts** ii. Not simplest form, but significant work shown
- **- 1 pts** ii. Incorrect or Blank

3. (9 points)

   (a) (2 points) Fill in the blanks:
   Theorem: For any $c \in \underline{\mathbb{Z}}$, and for any $n \in \underline{2^m, m \in \mathbb{Z}^+}$, there exists a $k \in \mathbb{Z}$ so that $c^n - 1 = k \cdot (c-1)$.

   (b) (3 points) Prove the theorem from the previous part.

   We'll use induction to prove this:

   Base case where m = 0, n = 2^0 = 1:
   LHS = $c^n - 1 = c^1 - 1 = 0$
   RHS = $k \cdot (c-1) = k \cdot (1-1) = k \cdot 0$
   Since any number multiplied by 0 is 0, k can be any number, thus there exist at least one k where it is an integer.

   As induction hypothesis, we'll assume that $A_{tru} = c^{n_{tru}} - 1 = k_{tru} \cdot (c-1)$ is always true for m = m, n = 2^m case.

   For inductive step, we'll say $m_{ind} = m_{tru} + 1, n_{ind} = 2^{m_{ind}} = 2^{m_{tru}+1}$. From this, we can simplify the LHS:
   LHS = $c^{n_{ind}} - 1 = c^{2^{m_{tru}+1}} - 1 = (c^{2^{m_{tru}}} + 1)(c^{2^{m_{tru}}} - 1) = (c^{2^{m_{tru}}} + 1)A_{tru}$
   Then, we can equate LHS and RHS to isolate k_{ind} as:
   $k_{ind} = (c^{2^{m_{tru}}} + 1)A_{tru}/(c-1) = (c^{2^{m_{tru}}} + 1)k_{tru}$

   Since we know that c, k_{tru}, m_{tru} are integers, we can say that k_{ind} has to be integer since any integer multiplied, added, or powered by any integer always result in an integer.

   □

   (c) (4 points) Prove that $5^n \cdot 2^{2n} - 1$ is divisible by 19.

   Before proving this, we first have to constrain n as any positive integer, since integer (such as 5 and 2) are not guaranteed to result in an integer if powered by a fraction or a negative number.

   We can simplify the statement further by simplifying the equation:
   $= 5^n \cdot 2^{2n} - 1$
   $= 5^n \cdot 2^{2^n} - 1$
   $= 5^n \cdot 4^n - 1$
   $= 20^n - 1$
   Therefore, the statement we're trying to prove is:

   $\forall n \in \mathbb{Z}^+, (20^n - 1) \mod 19 = 0$

   After this constraint, we can use induction to prove this:

3.1 domains **1 / 2**

- **- 0 pts** Correct
- **- 1 pts** Incorrect/Blank domain for c (should be INTEGERS)
- ✓ **- 1 pts Incorrect/Blank domain for n (should be NATURAL NUMS)**
- **- 2 pts** No Submission

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LHS = $c^n - 1 = c^1 - 1 = 0$
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Since any number multiplied by 0 is 0, k can be any number, thus there exist at least one k where it is an integer.

As induction hypothesis, we'll assume that $A_{tru} = c^{n_{tru}} - 1 = k_{tru} \cdot (c-1)$ is always true for m = m, n = 2^m case.

For inductive step, we'll say $m_{ind} = m_{tru} + 1, n_{ind} = 2^{m_{ind}} = 2^{m_{tru}+1}$. From this, we can simplify the LHS:
LHS = $c^{n_{ind}} - 1 = c^{2^{m_{tru}+1}} - 1 = (c^{2^{m_{tru}}} + 1)(c^{2^{m_{tru}}} - 1) = (c^{2^{m_{tru}}} + 1)A_{tru}$
Then, we can equate LHS and RHS to isolate k_{ind} as:
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3.2 proof 3 / 3

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- 3 pts Incorrect or Blank

3. (9 points)

(a) (2 points) Fill in the blanks:
Theorem: For any $c \in \underline{\mathbb{Z}}$, and for any $n \in \underline{2^m, m \in \mathbb{Z}^+}$, there exists a $k \in \mathbb{Z}$ so that $c^n - 1 = k \cdot (c-1)$.

(b) (3 points) Prove the theorem from the previous part.

We'll use induction to prove this:

Base case where m = 0, n = 2^0 = 1:
LHS = $c^n - 1 = c^1 - 1 = 0$
RHS = $k \cdot (c-1) = k \cdot (1-1) = k \cdot 0$
Since any number multiplied by 0 is 0, k can be any number, thus there exist at least one k where it is an integer.

As induction hypothesis, we'll assume that $A_{tru} = c^{n_{tru}} - 1 = k_{tru} \cdot (c-1)$ is always true for m = m, n = 2^m case.

For inductive step, we'll say $m_{ind} = m_{tru} + 1, n_{ind} = 2^{m_{ind}} = 2^{m_{tru}+1}$. From this, we can simplify the LHS:
LHS = $c^{n_{ind}} - 1 = c^{2^{m_{tru}+1}} - 1 = (c^{2^{m_{tru}}} + 1)(c^{2^{m_{tru}}} - 1) = (c^{2^{m_{tru}}} + 1)A_{tru}$
Then, we can equate LHS and RHS to isolate k_{ind} as:
$k_{ind} = (c^{2^{m_{tru}}} + 1)A_{tru}/(c-1) = (c^{2^{m_{tru}}} + 1)k_{tru}$

Since we know that c, k_{tru}, m_{tru} are integers, we can say that k_{ind} has to be integer since any integer multiplied, added, or powered by any integer always result in an integer.

□

(c) (4 points) Prove that $5^n \cdot 2^{2n} - 1$ is divisible by 19.

Before proving this, we first have to constrain n as any positive integer, since integer (such as 5 and 2) are not guaranteed to result in an integer if powered by a fraction or a negative number.

We can simplify the statement further by simplifying the equation:
$= 5^n \cdot 2^{2n} - 1$
$= 5^n \cdot 2^{2^n} - 1$
$= 5^n \cdot 4^n - 1$
$= 20^n - 1$
Therefore, the statement we're trying to prove is:

$\forall n \in \mathbb{Z}^+, (20^n - 1) \mod 19 = 0$

After this constraint, we can use induction to prove this:

Base case, where n = 0:
$= 20^n - 1$
$= 20^0 - 1$
$= 1 - 1$
$= 0$
0 mod anything is 0, therefore the statement is true for base case.

For induction hypothesis, we'll assume that for $A_{tru} = 20^{n_{tru}} - 1$, $A_{tru} \mod 19 = 0$ is always true for $n = n_{tru}$.

For induction step, we'll say that $n = n_{ind} = n_{tru} + 1$. We can substitute n_{ind} into the equation and simplify as:
$= 20^n - 1$
$= 20^{n_{ind}} - 1$
$= 20^{n_{tru}+1} - 1$
$= 20 \cdot 20^{n_{tru}} - 1$
$= (19 + 1) \cdot 20^{n_{tru}} - 1$
$= (19 \cdot 20^{n_{tru}}) + (20^{n_{tru}} - 1)$
$= 19 \cdot 20^{n_{tru}} + A_{tru}$

Since we know that any term that is a multiple of 19 will result in 0 if we apply mod 19, the first term of the equation will return 0 is mod 19 is applied. Since we assumed that A_{tru} will return 0 if mod 19 is applied, we can say that the second term of the equation will also return 0 is mod 19 is applied. As a property of mod operation, if the sum of mod result of additive terms is less than the mod divisor, then the mod result of sum of the two additive terms is equals to the sum of mod result of additive terms.

Therefore, we can say that $19 \cdot 20^{n_{tru}} + A_{tru} = 0$, since $19 \cdot 20^{n_{tru}} = 0$ and $A_{tru} = 0$, prove the inductive step, and conclude that the statement is true.

□

3.3 application 4 / 4

✓ **- 0 pts** **Correct proof (by rearrangement or induction)**

- 2 pts No conversion to 20^n - 1

- 2 pts Improper inductive proof/application of theorem

- 4 pts Incorrect or Blank

4. (12 points)

   (a) (3 points) Complete the 5-by-5 magic square below so that every row, every column, and every diagonal have the same sum, and so that each integer from 1 through 25 is used exactly once.

19	25	8	11	2
13	1	17	24	10
22	9	15	3	16
5	18	21	7	14
6	12	4	20	23

   (b) (3 points) Give an expression for the column sum of an n-by-n magic square.

Answer for (b):	$n(1+n^2)/2$

   (c) (3 points) Consider an arrangement of the positive integers, grouped as shown, so that the kth group has k elements: $(1), (2,3), (4,5,6), (7,8,9,10), \ldots$. Find a formula for the sum of the k numbers in the kth group.

Answer for (c):	$k(1+k^2)/2$

   (d) (3 points) Prove that your formula from the previous part is correct.

We define x as the mean of the set A, which has $[1...k^2]$. If we overlay k-th group numbers on A, we can conveniently notice that (observation 0) all the numbers in a k-th group belongs in set A in the same order, and centered inside A. I arbitrarily pick 3 and 4 as sample odd and even numbers to illustrate this property.

1	2	3
4	5	6
7	8	9

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

We can notice that (observation 1) if any pair of numbers is symmetric about a number, the mean of those two numbers is the original number. For example, 4 and 6 is symmetric about 5, since 4 is 1 less than 5 and 6 is 1 more than 5. This is the definition of central tendency. We also notice that (observation 2) a mean of any number is itself. For example, mean of 5 is 5. We also notice that (observation 3) if any number of sets has the same mean, the mean of the sum of the sets are

4.1 5x5 square **3 / 3**

- ✓ **- 0 pts** **Correct**
- **- 1 pts** 1 square wrong
- **- 3 pts** Incorrect or Blank

4. (12 points)

   (a) (3 points) Complete the 5-by-5 magic square below so that every row, every column, and every diagonal have the same sum, and so that each integer from 1 through 25 is used exactly once.

19	25	8	11	2
13	1	17	24	10
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Answer for (b):	$n(1+n^2)/2$

   (c) (3 points) Consider an arrangement of the positive integers, grouped as shown, so that the kth group has k elements: $(1), (2, 3), (4, 5, 6), (7, 8, 9, 10), \ldots$. Find a formula for the sum of the k numbers in the kth group.

Answer for (c):	$k(1+k^2)/2$

   (d) (3 points) Prove that your formula from the previous part is correct.

   We define x as the mean of the set A, which has $[1...k^2]$. If we overlay k-th group numbers on A, we can conveniently notice that (observation 0) all the numbers in a k-th group belongs in set A in the same order, and centered inside A. I arbitrarily pick 3 and 4 as sample odd and even numbers to illustrate this property.

1	2	3
4	5	6
7	8	9

1	2	3	4
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13	14	15	16

   We can notice that (observation 1) if any pair of numbers is symmetric about a number, the mean of those two numbers is the original number. For example, 4 and 6 is symmetric about 5, since 4 is 1 less than 5 and 6 is 1 more than 5. This is the definition of central tendency. We also notice that (observation 2) a mean of any number is itself. For example, mean of 5 is 5. We also notice that (observation 3) if any number of sets has the same mean, the mean of the sum of the sets are

4.2 column sum 3 / 3

- ✓ - **0 pts** **Correct: n(n^2 + 1)/2**
- - **2 pts** Left in sigma notation
- - **3 pts** Incorrect or Blank

4. (12 points)

(a) (3 points) Complete the 5-by-5 magic square below so that every row, every column, and every diagonal have the same sum, and so that each integer from 1 through 25 is used exactly once.

19	25	8	11	2
13	1	17	24	10
22	9	15	3	16
5	18	21	7	14
6	12	4	20	23

(b) (3 points) Give an expression for the column sum of an n-by-n magic square.

Answer for (b):	$n(1+n^2)/2$

(c) (3 points) Consider an arrangement of the positive integers, grouped as shown, so that the kth group has k elements: $(1), (2,3), (4,5,6), (7,8,9,10), \ldots$. Find a formula for the sum of the k numbers in the kth group.

Answer for (c):	$k(1+k^2)/2$

(d) (3 points) Prove that your formula from the previous part is correct.

We define x as the mean of the set A, which has $[1...k^2]$. If we overlay k-th group numbers on A, we can conveniently notice that (observation 0) all the numbers in a k-th group belongs in set A in the same order, and centered inside A. I arbitrarily pick 3 and 4 as sample odd and even numbers to illustrate this property.

1	2	3
4	5	6
7	8	9

1	2	3	4
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9	10	11	12
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We can notice that (observation 1) if any pair of numbers is symmetric about a number, the mean of those two numbers is the original number. For example, 4 and 6 is symmetric about 5, since 4 is 1 less than 5 and 6 is 1 more than 5. This is the definition of central tendency. We also notice that (observation 2) a mean of any number is itself. For example, mean of 5 is 5. We also notice that (observation 3) if any number of sets has the same mean, the mean of the sum of the sets are

4.3 group sum **3 / 3**

- ✓ **- 0 pts** **Correct: k(k^2 + 1)/2**
- **- 2 pts** Left in sigma notation
- **- 3 pts** Incorrect or Blank

4. (12 points)

(a) (3 points) Complete the 5-by-5 magic square below so that every row, every column, and every diagonal have the same sum, and so that each integer from 1 through 25 is used exactly once.

19	25	8	11	2
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Answer for (c):	$k(1+k^2)/2$

(d) (3 points) Prove that your formula from the previous part is correct.

We define x as the mean of the set A, which has $[1...k^2]$. If we overlay k-th group numbers on A, we can conveniently notice that (observation 0) all the numbers in a k-th group belongs in set A in the same order, and centered inside A. I arbitrarily pick 3 and 4 as sample odd and even numbers to illustrate this property.

1	2	3
4	5	6
7	8	9

1	2	3	4
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13	14	15	16

We can notice that (observation 1) if any pair of numbers is symmetric about a number, the mean of those two numbers is the original number. For example, 4 and 6 is symmetric about 5, since 4 is 1 less than 5 and 6 is 1 more than 5. This is the definition of central tendency. We also notice that (observation 2) a mean of any number is itself. For example, mean of 5 is 5. We also notice that (observation 3) if any number of sets has the same mean, the mean of the sum of the sets are

4.4 group sum proof 3 / 3

- ✓ **- 0 pts** **Correct**
- **- 1 pts** Minor mistake
- **- 2 pts** Major leap without justification
- **- 3 pts** Incorrect or Blank

the same as the individual means. For example, mean of (7,10) and mean of (8,9) is 8.5, same as mean of the individual sets.

Therefore, in the case where k is even, we can pair the min and max of that set with each other, and pick the next min and the next max until all numbers are paired. Since all pairs are symmetric about the same center (due to the set being incremental by the same amount), we can say that all their mean are the same by observation 1. We can say that the mean of the set as a whole is the same as the individual means by observation 3. Because observation 0 (k-th number set is centered inside A) is true, we can say that their mean is the same as A's, defined as x, since we can apply the same pairing technique to A.

The same pairing technique can be applied to cases where k is odd, by adding a pair of itself at the center of k-th numbers set and A numbers set. The mean of that pair is itself by observation 2, therefore not altering our conclusion from the even case.

We can notice that 1 and n^2 are symmetric about x, therefore we can define x as the mean of 1 and n^2 by observation 1. To put that in math:
$x = (1 + k^2)/2$

Since the mean is a number where the sum of residual (ie member - mean) is zero, we can say that the sum of any set is a multiple of its mean and its size. In other word:
$x_1 - \bar{x} + ... + x_k - \bar{x} = 0$
$x_1 + ... + \bar{x} = k\bar{x}$

Therefore, we can say that sum of k-th group numbers set is the multiple of its mean, x, which is also defined as $(1 + k^2)/2$. Therefore, we can conclude that:
Sum of k-th group $= kx = k(1 + k^2)/2$

$\square$

5.1 a) **4 / 6**

- ✓ **+ 2 pts** **Correct: f(n) ∈ Θ(g(n))**
- **+ 1 pts** O: correct values for c (2) and n_0 (9)
- **+ 1 pts** O: shows that f(n) < c*g(n)
- ✓ **+ 1 pts** **Ω: correct values for c (1) and n_0 (1)**
- ✓ **+ 1 pts** **Ω: shows that f(n) > c*g(n)**
- **+ 0 pts** Incorrect or Blank
- **+ 6 pts** Correct use of limits
- **+ 0 pts** I cannot read this

5. (12 points) Indicate for each of the following pairs of expressions $(f(n), g(n))$, whether $f(n)$ is O, Ω, or Θ of $g(n)$. Prove your answers. Note, if you choose O or Ω, you must also show that the relationship is not Θ.

   (a) $f(n) = n \log n$, and $g(n) = 3n \log n - n \log(n^2 + 2)$.

Answer for (a):	$f(n) = \Theta(g(n))$

To prove $f(n) = \Theta(g(n))$, we must prove $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.

1. To prove $f(n) = O(g(n))$, we say there is a positive c and n_0 such that f(n) is less than or equal to c*g(n) for all n more than or equal to n_0.
This case, we can pick c = 2, n_0 = 100.
f(n) = 100log(100) = 200
c*g(n) = (3(100)log(100)-100(log(10000 + 2))*2 = 399.98
Since both f(n) and g(n) tends to positive infinity from n_0 without any n where instantaneous speed of f(n) or g(n) equals 0, we can say that $f(n) = O(g(n))$ for this chosen c and n_0.

or

Choose c = 100 n_0 = 2

$nlog(n) \leq$ 300nlog(n) - 100nlog($n^2 + 2$)

$0 \leq$ 299nlog(n) - 100nlog($n^2 + 2$)

This will always be true because the term will always be positive for $n > 2$

2. To prove $f(n) = \Omega(g(n))$, we say there is a positive c and n_0 such that f(x) is more than or equal to c*g(x) for all n more than or equal to n_0.
This case, we can pick c = 1, n_0 = 100.
f(n) = 100log(100) = 200
c*g(n) = 3(100)log(100)-100(log(10000 + 2) = 199.991
Since both f(n) and g(n) tends to positive infinity from n_0 without any n where instantaneous speed of f(n) or g(n) equals 0, we can say that $f(n) = \Omega(g(n))$ for this chosen c and n_0.

Or
Choose c = 1/3 and n_0 = 2
$nlog(n) \geq$ nlog(n) - (1/3)nlog($n^2 + 2$)
$0 \geq$ -(1/3)nlog($n^2 + 2$)

This inequality will always hold true because $-(1/3)nlog(n^2 + 2)$ will always be negative for $n \geq 2$

□

5.2 b) **4 / 6**

- ✓ + **2 pts** Correct: f(n) Ω(g(n))
- + **1 pts** Ω: acceptable values for c and n_0
- + **1 pts** Ω: shows that f(n) > c*g(n)
- ✓ + **1 pts** O: correct negation/contradiction of definition
- ✓ + **1 pts** O: shows that for any choice of c and n_0 f(n) > c*g(n)
- + **0 pts** Incorrect or Blank
- + **1 pts** Significant attempt at using limits
- + **4 pts** Correct use of limits

1 please assign pages properly next time or your work will not be graded

2 this is not a proof

(b) $f(n) = \frac{1}{2}n^3$, and $g(n) = n^2 + 4n + 37$.

Answer for (b):	$f(n) = \Omega(g(n))$

To prove $f(n) = \Omega(g(n))$, we can pick c = 1, $n_0 = 1000$.
f(n) = $(1/2) * (1000^3) = 5 * 10^8$
c*g(n) = $1000^2 + 4000 + 37 = 1.004037 * 10^6$
Since both f(n) and g(n) tends to positive infinity from n_0 without any n where instantaneous speed of f(n) or g(n) equals 0, we can say that $f(n) = \Omega(g(n))$ for this chosen c and n_0.

However, $f(n) = O(g(n))$ is not true because for any c and n_0 we pick, there will be n such that n is more than or equal to n_0 and f(n) is more than or equal to c*g(n).
To prove this, we can pick any arbitrary c and n_0, and pick n as $2(cn_0 + 6)$.
1. n is always more than n_0 since $2(cn_0 + 6)$ is always more than n_0 where c and n_0 are positive real numbers.
2. We can substitute n into the equations:
f(n) = $(2(cn_0+6))^3/2 = 4((cn_0)^3 + 18(cn_0)^2 + 108(cn_0) + 216) = 4c^3n_0^3 + 72c^2n_0^2 + 432cn_0 + 864$
c*g(n) = $c((2(cn_0 + 6))^2 + 4(2(cn_0 + 6)) + 37) = 4c^3n_0^2 + 56c^2n_0 + 229c$
We can substitute f(n) and c*g(n) into the equation $f(n) \leq c*g(n)$ to check if $f(n) = O(g(n))$ is true or not:
$f(n) \leq c * g(n)$
$4c^3n_0^3 + 72c^2n_0^2 + 432cn_0 + 864 \leq 4c^3n_0^2 + 56c^2n_0 + 229c$
Very obviously, however, we can notice that each terms of f(n) is more than or equals to each terms of c*g(n). In other words:
$4c^3n_0^3 \geq 4c^3n_0^2$
$72c^2n_0^2 \geq 56c^2n_0$
$432cn_0 \geq 229c$
$864 \geq 0$
Therefore, the inequality is not true, and we conclusively proved that there exists n where $f(n) = O(g(n))$ is false for any c and n_0.

□

6.1 a) return 1 / 1

✓ **- 0 pts** **Correct: 2**

- 1 pts Incorrect or Blank

6. (16 points) For each C++ function below, give the tightest asymptotic upper bound that you can determine for the function's **runtime**, in terms of the input parameter. Where prompted, also compute the return value of the function. Express both runtime and return value in simplest terms. For all function calls, assume that the input parameter $n \geq 1$.

(a)

```
int coffee(int n) {
   int s = n * n;
   for (int q = 0; q < n; q++)
      s = s - q;
   for (int q = n; q > 0; q--)
      s = s - q;
   return s + 2;
}
```

Return value for (a):	2

Running time of (a):	O(n)

(b)

```
int tea(int n) {
   int r = 0;
   for (int i = 1; i < n*n*n; i = i * 2)
      r++;
   return r * r;
}
```

Return value for (b):	$\lceil log_2(n^3) \rceil^2$

Running time of (b):	$O(log(n))$

6.2 a) run time **1 / 1**

✓ **- 0 pts** **Correct: O(n)**

- 1 pts Incorrect or Blank

6. (16 points) For each C++ function below, give the tightest asymptotic upper bound that you can determine for the function's **runtime**, in terms of the input parameter. Where prompted, also compute the return value of the function. Express both runtime and return value in simplest terms. For all function calls, assume that the input parameter $n \geq 1$.

(a)

```
int coffee(int n) {
   int s = n * n;
   for (int q = 0; q < n; q++)
      s = s - q;
   for (int q = n; q > 0; q--)
      s = s - q;
   return s + 2;
}
```

Return value for (a):	2

Running time of (a):	O(n)

(b)

```
int tea(int n) {
   int r = 0;
   for (int i = 1; i < n*n*n; i = i * 2)
      r++;
   return r * r;
}
```

Return value for (b):	$\lceil log_2(n^3) \rceil^2$

Running time of (b):	$O(log(n))$

6.3 b) return **2 / 2**

- ✓ **- 0 pts** **Correct: ceil(3lgn)^2**
- **- 1 pts** Missing ceiling function
- **- 2 pts** Incorrect or Blank

6. (16 points) For each C++ function below, give the tightest asymptotic upper bound that you can determine for the function's **runtime**, in terms of the input parameter. Where prompted, also compute the return value of the function. Express both runtime and return value in simplest terms. For all function calls, assume that the input parameter $n \geq 1$.

(a)

```
int coffee(int n) {
  int s = n * n;
  for (int q = 0; q < n; q++)
    s = s - q;
  for (int q = n; q > 0; q--)
    s = s - q;
  return s + 2;
}
```

Return value for (a):	2

Running time of (a):	O(n)

(b)

```
int tea(int n) {
  int r = 0;
  for (int i = 1; i < n*n*n; i = i * 2)
     r++;
  return r * r;
}
```

Return value for (b):	$\lceil log_2(n^3) \rceil^2$

Running time of (b):	$O(log(n))$

6.4 b) run time **2 / 2**

✓ **- 0 pts** **Correct: O(log n)**

- 2 pts Incorrect or Blank

6. (16 points) For each C++ function below, give the tightest asymptotic upper bound that you can determine for the function's **runtime**, in terms of the input parameter. Where prompted, also compute the return value of the function. Express both runtime and return value in simplest terms. For all function calls, assume that the input parameter $n \geq 1$.

(a)

```
int coffee(int n) {
   int s = n * n;
   for (int q = 0; q < n; q++)
      s = s - q;
   for (int q = n; q > 0; q--)
      s = s - q;
   return s + 2;
}
```

Return value for (a):	2

Running time of (a):	O(n)

(b)

```
int tea(int n) {
   int r = 0;
   for (int i = 1; i < n*n*n; i = i * 2)
      r++;
   return r * r;
}
```

Return value for (b):	$\lceil log_2(n^3) \rceil^2$

Running time of (b):	$O(log(n))$

6.5 c) return 2 / 2

✓ - 0 pts Correct: 8*floor(n/16)*[floor(n/16) + 1]

- 1 pts Missing floor functions

- 2 pts Incorrect or Blank

(c)

```
int mocha(int n) {
   int r = 0;
   for (int i=0; i<=n; i = i+16)
       for (int j=0; j<i; j++)
           r++;
   return r;
}
```

Return value for (c):	$8 \cdot \lfloor n/16 \rfloor \cdot (\lfloor n/16 \rfloor + 1)$

Running time of (c):	$O(n^2)$

(d)

```
int espresso(int n) {
    int j=0;
   for (int k = 16; coffee(k) * mocha(k) - k <= n; k+=16) {
       j++;
      cout << "I am having so much fun with asymptotics!" << endl;
   }
    return j;
}
```

Return value for (d):	$\lfloor \sqrt{n/16} \rfloor$

Running time of (d):	$O(n^3)$

6.6 c) run time 2 / 2

✓ - 0 pts Correct: O(n^2)

- 2 pts Incorrect or Blank

(c)

```
int mocha(int n) {
  int r = 0;
  for (int i=0; i<=n; i = i+16)
     for (int j=0; j<i; j++)
        r++;
  return r;
}
```

Return value for (c):	$8 \cdot \lfloor n/16 \rfloor \cdot (\lfloor n/16 \rfloor + 1)$

Running time of (c):	$O(n^2)$

(d)

```
int espresso(int n) {
   int j=0;
  for (int k = 16; coffee(k) * mocha(k) - k <= n; k+=16) {
     j++;
    cout << "I am having so much fun with asymptotics!" << endl;
  }
   return j;
}
```

Return value for (d):	$\lfloor \sqrt{n/16} \rfloor$

Running time of (d):	$O(n^3)$

6.7 d) return 2 / 2

✓ **- 0 pts** **Correct: floor(sqrt(n)/4)**

- 1 pts Missing floor function

- 2 pts Incorrect or Blank

(c)

```
int mocha(int n) {
  int r = 0;
  for (int i=0; i<=n; i = i+16)
     for (int j=0; j<i; j++)
        r++;
  return r;
}
```

Return value for (c):	$8 \cdot \lfloor n/16 \rfloor \cdot (\lfloor n/16 \rfloor + 1)$

Running time of (c):	$O(n^2)$

(d)

```
int espresso(int n) {
   int j=0;
  for (int k = 16; coffee(k) * mocha(k) - k <= n; k+=16) {
     j++;
    cout << "I am having so much fun with asymptotics!" << endl;
  }
   return j;
}
```

Return value for (d):	$\lfloor \sqrt{n/16} \rfloor$

Running time of (d):	$O(n^3)$

6.8 d) run time **0 / 2**

- **- 0 pts** Correct: O(n^(3/2))
- ✓ **- 2 pts Incorrect or Blank**

(e)

```
int latte(int n) {
    int j = 0;
    for (int k = 0; k < coffee(k) * n; k++)
        j = j + 2;
   return espresso(mocha(j));
}
```

Running time of (e):	$O(n^3)$

6.9 e) run time 2 / 2

✓ **- 0 pts** **Correct: O(n^3)**

- 2 pts Incorrect or Blank