CPSC 121 - MATHEMATICAL PROOFS AND SEQUENTIAL CIRCUITS

Note the definition: $f(n) \in O(g(n)) \equiv \exists c \in \mathbb{R}_+, \exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N}, n \geq n_0 \rightarrow f(n) \leq cg(n)$

Problem 1. Given that $f(n) = n^3 + 2n^2 + 96$, prove that $f(n) \in O(n^3)$.

Problem 2. Given that $f(n) = n^3$ and $g(n) = 9988n^2$, prove that $f(n) \notin O(g(n))$.

Problem 3. Prove that for every distinct pair of real numbers, there exists a third real number that is strictly between the first two numbers.

Problem 4. We define the set $[0, 1) \subset \mathbb{R}$ as $[0, 1) = \{x \in \mathbb{R} | 0 \leq x < 1\}$

(1) Prove that $[0, 1)$ has no maximum (largest) element.

(2) Prove that 1 is the least upper bound of $[0, 1)$. For a number $s \in \mathbb{R}$ to be the least upper bound of a set $A \subset \mathbb{R}$, it must:

  (a) Be an upper bound of the set A, specifically, it must satisfy: $\forall x \in A, x \leq s$
  (b) Be the smallest (minimum) upper bound. If S is the set of upper bounds of A, s is the least upper bound of A if $\forall s' \in S, s \leq s'$

Problem 5. The following is a very incomplete sequential circuit whose output should be the average (more specifically, the floor of the average) of the 8-bit values that were present at the input every time the clock ticked. Complete the circuit so it behaves according to this description.

Hint: you will need two adders, two registers, a clock, and maybe some other simpler components (gates, constants, etc). The divider divides the top input by the bottom input and outputs the floor of the quotient. Do not worry about its behaviour when the bottom input is 0.