CPSC 121 - PREDICATE LOGIC I SOLUTIONS

Problem 1. Let $K(x)$ be the statement "x can speak Klingon" and let $M(x)$ be the statement "x knows the computer language Malbolge (*mah-leh-bol-djeh*)". Express each of the following sentences in terms of $K(x), M(x)$, quantifiers and logical connectives. The domain for quantifiers, denoted by S, consists of all students at your school.

(1) There is a student at your school who can speak Klingon and who knows Malbolge.
(2) There is a student at your school who can speak Klingon but doesn't know Malbolge.
(3) Every student at your school can speak Klingon or knows Malbolge.
(4) No student at your school can speak Klingon or knows Malbolge.
(5) Students who know Malbolge do not speak Klingon.

Solution.

(1) $\exists x \in S,\ K(x) \wedge M(x)$
(2) $\exists x \in S,\ K(x) \wedge \sim M(x)$
(3) $\forall x \in S,\ K(x) \vee M(x)$
(4) $\sim \exists x \in S,\ K(x) \vee M(x)$
(5) $\forall x \in S,\ M(x) \rightarrow \sim K(x)$

Problem 2. For these questions, translate English sentences to predicate logic, and translate predicate logic statements to (naturally sounding) English. Use the following domain and predicates:

- A: the domain of all animals
- C(x): x is a cheetah
- T(x): x is a turtle
- P(x): x is a pigeon
- R(x,y): x runs faster than y
- F(x): x can fly
- B(x): x is blue
- G(x): x is green
- E(x,y): x wants to eat y

a) $\exists x \in A, G(x) \wedge P(x)$
Solution: There exists a green pigeon.
b) $\exists x \in A, P(x) \wedge G(x)$
Solution: There exists a green pigeon.
c) $\exists x \in A, P(x) \rightarrow G(x)$
Solution: There is an animal that if it is a pigeon then it is green. Note, this statement is trivially true if there is a non-pigeon in the set of animals, A, even though there may be no green pigeons.
d) $\forall x \in A, P(x) \rightarrow G(x)$
Solution: All pigeons are green.
e) $\forall x \in A, G(x) \rightarrow P(x)$
Solution: All green animals are pigeons.
f) $\forall x \in A, G(x) \wedge P(x)$
Solution: All animals are green pigeons.
g) $\exists x \in A, \exists y \in A, (E(x, y) \wedge R(x, y))$
Solution: There is an animal that wants to eat another animal and can run faster than that animal.
h) Cheetahs run faster than turtles.
Solution: $\forall x \in A, \forall y \in A, (C(x) \wedge T(y)) \rightarrow R(x, y)$
i) There is a turtle that runs faster than some cheetah.
Solution: $\exists x \in A, \exists y \in A, C(x) \wedge T(y) \wedge R(y, x)$

j) There are no blue cheetahs.
Solution: $\sim (\exists x \in A, B(x) \wedge C(x))$

Or equivalently, $\forall x \in A, \sim (B(x) \wedge C(x))$

k) No turtle can outrun a cheetah.
Solution: $\sim (\exists x \in A, \exists y \in A, T(x) \wedge C(y) \wedge R(x, y))$

l) Cheetahs that want to eat all blue pigeons can fly.
Solution: $\forall x \in A, (C(x) \wedge (\forall y \in A, (P(y) \wedge B(y)) \rightarrow E(x, y))) \rightarrow F(x)$

m) Flying blue turtles want to eat green cheetahs and can run faster than pigeons.
Solution: $\forall x \in A, (B(x) \wedge F(x) \wedge T(x)) \rightarrow (\forall y \in A, (G(y) \wedge C(y)) \rightarrow E(x, y)) \wedge (\forall z \in A, P(z) \rightarrow R(x, z))$