

Time Series

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Time Series

$$X_t = \alpha_1 X_{t-1} + \epsilon_t$$

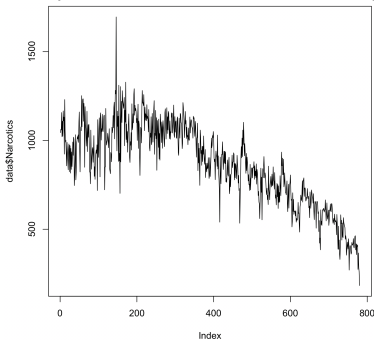
- ▶ t represents the time.

Time Series

$$X_t = \alpha_1 X_{t-1} + \epsilon_t$$

- ▶ t represents the time.
- ▶ Example: Crime related to Narcotics in Chicago

```
data = read.table("data.txt", header = TRUE)  
data$Narcotics  
plot(data$Narcotics, type="l")
```



Time Series

$$X_t = \alpha_1 X_{t-1} + \epsilon_t$$

- ▶ Another example: ?AirPassengers
data(AirPassengers)
plot.ts(AirPassengers)

What do we see in this time series? Trends? Seasonal trends?
Increasing Variance?

Time Series

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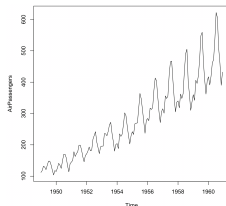
- ▶ Another example: ?AirPassengers

```
data(AirPassengers)
```

```
plot.ts(AirPassengers)
```

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Increasing Variance?

```
abline(reg=lm(AirPassengers ~ time(AirPassengers)))
```



Time Series

- ▶ Let's take a look at the cycles of this series

```
boxplot(AirPassengers cycle(AirPassengers))
```

Box plot across months gives us a sense on seasonal effect

Time Series

- ▶ With the dependence in the past, using the simple model

$$X_t = \alpha_1 X_{t-1} + \epsilon_t$$

may not be appropriate, since the series does not depend only on the previous day, but it depends on seasons ago!

Time Series

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- ▶ General model

$$X_t = \alpha_1 X_{t-1} + \dots + \alpha_p X_{t-p} + \epsilon_t$$

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$$X_t = \alpha_1 X_{t-1} + \dots + \alpha_p X_{t-p} + \epsilon_t$$

- ▶ This is called AR(d) - Auto Regressive of order p

Time Series

- ▶ Example: Random Walk with Drift

$$X_t = \delta + X_{t-1} + \epsilon_t$$

with $X_0 = 0$ and ϵ_t white noise (Gaussian).

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with $X_0 = 0$ and ϵ_t white noise (Gaussian).

- ▶ Let's generate this in R.
- ▶ Note that $X_t = \delta t + \sum_{j=1}^t \epsilon_j$

Time Series

- ▶ Example: Signal in Noise

$$X_t = 2 \cos \left(2\pi \frac{(t + 15)}{50} \right) + \epsilon_t$$

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$$X_t = \beta_1 \epsilon_{t-1} + \dots + \beta_q \epsilon_{t-q} + \epsilon_t$$

- ▶ This is called MA(p) - Moving Average of order q

Time Series

- ▶ Can we combine $AR(p)$ with $MA(q)$?

Time Series

- Can we combine AR(p) with MA(q)?

ARMA(p,q)

$$X_t = \alpha_1 X_{t-1} + \dots + \alpha_p X_{t-p} + \beta_1 \epsilon_{t-1} + \dots + \beta_q \epsilon_{t-q} + \epsilon_t$$

Time Series

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Time Series

- ▶ Properties

Time Series

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Mean Function of a Moving Average Series

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Time Series

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Time Series

- ▶ **Definition:** The autocovariance function is defined as

$$\gamma(s, t) = \text{cov}(X_t, X_s) = E((X_t - \mu_t)(X_s - \mu_s))$$

- ▶ If $t = s$, the autocovariance reduces to the (assumed finite) variance

$$\gamma(s, t) = \text{cov}(X_t, X_t) = \text{Var}(X_t)$$

Time Series

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- ▶ Recall White Noise $X_t = \epsilon_t$.
- ▶ So

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$$\gamma(s, t) = \text{cov}(\epsilon_t, \epsilon_s) = \begin{cases} \sigma_\epsilon^2 & \text{if } s = t, \\ 0 & \text{otherwise} \end{cases}$$

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$$\gamma(s, t) = \text{cov}(X_t, X_s) = E((X_t - \mu_t)(X_s - \mu_s))$$

- ▶ Consider MA: $X_t = (1/3)\epsilon_{t-1} + (1/3)\epsilon_t + (1/3)\epsilon_{t+1}$.
- ▶ So

$$\gamma(t, s) = \text{cov}((1/3)(\epsilon_{t-1} + \epsilon_t + \epsilon_{t+1}), (1/3)(\epsilon_{s-1} + \epsilon_s + \epsilon_{s+1}))$$

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$$\begin{aligned}\gamma(t, s) &= \text{cov}((1/3)(\epsilon_{t-1} + \epsilon_t + \epsilon_{t+1}), (1/3)(\epsilon_{s-1} + \epsilon_s + \epsilon_{s+1})) \\ &= \begin{cases} (3/9)\sigma_\epsilon^2 & \text{if } s = t, \\ (2/9)\sigma_\epsilon^2 & \text{if } |s - t| = 1 \\ (1/9)\sigma_\epsilon^2 & \text{if } |s - t| = 2 \\ 0 & \text{otherwise} \end{cases}\end{aligned}$$

Time Series

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Time Series

- **Definition** A **strictly stationary** time series is one for which the probabilistic behavior of every collection of values

$$\{x_{t_1}, x_{t_2}, \dots, x_{t_k}\}$$

is identical to that of the time shifted set

$$\{x_{t_1+h}, x_{t_2+h}, \dots, x_{t_k+h}\}$$

Time Series

- ▶ **Definition** A **weakly stationary** time series is a finite variance process such that
 - (i) the mean value function is constant and does not depend on time t , and
 - (ii) the autocovariance function, $\gamma(s, t)$ depends on s and t only through their difference $|s - t|$.

Time Series

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 - (i) the mean value function is constant and does not depend on time t , and
 - (ii) the autocovariance function, $\gamma(s, t)$ depends on s and t only through their difference $|s - t|$.People use the term **stationary** to mean weakly stationary.

Time Series

- **Definition** The autocovariance function of a stationary time series will be written as

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$$\rho(h) = \frac{\gamma(t+h, t)}{\sqrt{\gamma(t, t)\gamma(t+h, t+h)}} = \frac{\gamma(h)}{\gamma(0)}$$

Time Series

- ▶ **Example** Stationarity of White Noise
- ▶ Mean = 0 and

$$\gamma_{\epsilon}(h) = \text{cov}(\epsilon_{t+h}, \epsilon_t) = \begin{cases} \sigma_{\epsilon}^2 & \text{if } s = t, \\ 0 & \text{otherwise} \end{cases}$$

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$$\gamma(t, s) = \text{cov}((1/3)(\epsilon_{t-1} + \epsilon_t + \epsilon_{t+1}), (1/3)(\epsilon_{s-1} + \epsilon_s + \epsilon_{s+1}))$$

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Time Series

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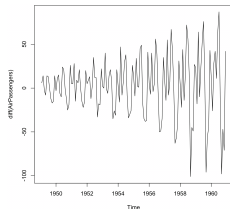
Time Series

- ▶ How do I make my time series stationary? Differencing

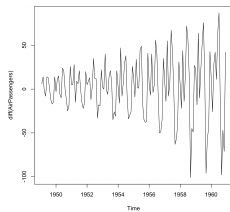
Time Series

- How do I make my time series stationary? Differencing

```
library(tseries)  
plot.ts(AirPassengers)  
plot.ts(diff(AirPassengers))
```

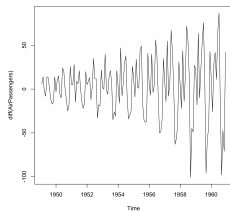


Time Series



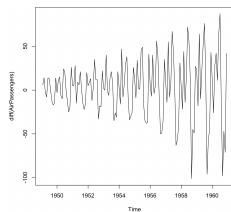
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Time Series



- Variance is increasing!
- To fix increasing variance use transformation: log

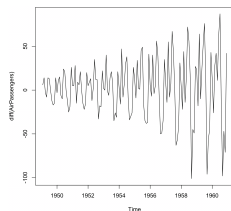
Time Series



- ▶ Variance is increasing!
- ▶ To fix increasing variance use transformation: log

```
logdata = log(AirPassengers)
plot.ts(diff(logdata))
```

Time Series



- ▶ Variance is increasing!
- ▶ To fix increasing variance use transformation: log

```
logdata = log(AirPassengers)
plot.ts(diff(logdata))
```

Now, check for stationarity:

```
adf.test(diff(log(AirPassengers)), alternative="stationary", k=0)
```

p-value is small \Rightarrow stationary!

Time Series

- **Definition** The **sample autocovariance** function is defined as

$$\hat{\gamma}(h) = \frac{1}{n} \sum_{t=1}^{n-h} (x_{t+h} - \bar{x})(x_t - \bar{x})$$

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- ▶ **Definition** The **sample autocorrelation** function is defined, analogously as

$$\hat{\rho}(h) = \hat{\gamma}(h)/\hat{\gamma}(0)$$

Time Series

- **Definition** An autoregressive model of order p , abbreviated $AR(p)$, is of the form

$$X_t = \phi_1 X_{t-1} + \dots + \phi_p X_{t-p} + \epsilon_t,$$

where X_t is stationary, $\epsilon_t \sim N(0, \sigma_\epsilon^2)$, and ϕ_1, \dots, ϕ_p are constants.

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- **Definition** The moving average model of order q , or $MA(q)$ model, is defined to be

$$X_t = \theta_1 \epsilon_{t-1} + \dots + \theta_q \epsilon_{t-q} + \epsilon_t,$$

where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$, and ϕ_1, \dots, ϕ_p are constants.

Time Series

- **Definition** A time series X_t is ARMA(p , q) if it is stationary and

$$X_t = \phi_1 X_{t-1} + \dots + \phi_p X_{t-p} + \theta_1 \epsilon_{t-1} + \dots + \theta_q \epsilon_{t-q} + \epsilon_t,$$

Time Series

The Partial Autocorrelation Function (PACF)

Time Series

- **Definition** The partial autocorrelation function (PACF) of a stationary process X_t is

$$\phi_{11} = \text{corr}(X_{t+1}, X_t) = \rho(1)$$

$$\phi_{hh} = \text{corr}(X_{t+h} - \hat{X}_{t+h}, X_t - \hat{X}_t) \text{ for } h \geq 2$$

Time Series

Consider the PACF of the AR(1) process given by $x_t = \phi x_{t-1} + w_t$, with $|\phi| < 1$. By definition, $\phi_{11} = \rho(1) = \phi$. To calculate ϕ_{22} , consider the regression of x_{t+2} on x_{t+1} , say, $\hat{x}_{t+2} = \beta x_{t+1}$. We choose β to minimize

$$E(x_{t+2} - \hat{x}_{t+2})^2 = E(x_{t+2} - \beta x_{t+1})^2 = \gamma(0) - 2\beta\gamma(1) + \beta^2\gamma(0).$$

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Taking derivatives with respect to β and setting the result equal to zero, we have $\beta = \gamma(1)/\gamma(0) = \rho(1) = \phi$. Next, consider the regression of x_t on x_{t+1} , say $\hat{x}_t = \beta x_{t+1}$. We choose β to minimize

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$$E(x_t - \hat{x}_t)^2 = E(x_t - \beta x_{t+1})^2 = \gamma(0) - 2\beta\gamma(1) + \beta^2\gamma(0).$$

This is the same equation as before, so $\beta = \phi$. Hence,

$$\begin{aligned}\phi_{22} &= \text{corr}(x_{t+2} - \hat{x}_{t+2}, x_t - \hat{x}_t) = \text{corr}(x_{t+2} - \phi x_{t+1}, x_t - \phi x_{t+1}) \\ &= \text{corr}(w_{t+2}, x_t - \phi x_{t+1}) = 0\end{aligned}$$

by causality. Thus, $\phi_{22} = 0$. In the next example, we will see that in this case, $\phi_{hh} = 0$ for all $h > 1$.

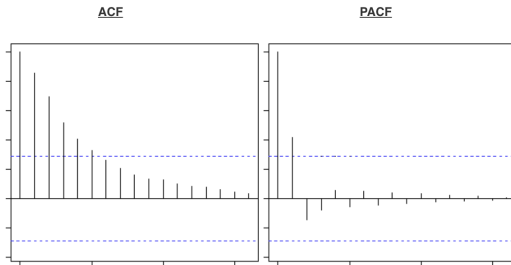
Time Series

- ▶ Which model should I use? AR, MA, or ARMA?

Time Series

- ▶ Which model should I use? AR, MA, or ARMA? Check ACF and PACF

If they look like this:



Use an AR(2)

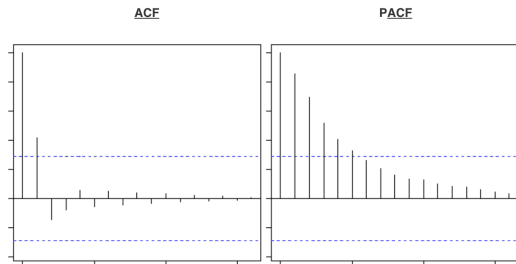
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If they look like this:



Use an MA(2)

Time Series

- ▶ Choose ARMA when both ACF and PACF tail off

Time Series

	$AR(p)$	$MA(q)$	$ARMA(p, q)$
ACF	Tails off	Cuts off after lag q	Tails off
PACF	Cuts off after lag p	Tails off	Tails off

Time Series

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logdata = log(AirPassengers)
acf(diff(logdata))
pacf(diff(logdata))
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Time Series

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 2. If there are seasonal patterns, we can also include that in the command ARIMA

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- ▶ ARIMA(p, d, q)

where d is the number of differences necessary

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 1. We can use the differentiation inside the command ARIMA
 2. If there are seasonal patterns, we can also include that in the command ARIMA
- ▶ ARIMA(p, d, q)

where d is the number of differences necessary

?arima

Time Series

- `fit <- arima(log(AirPassengers), c(0, 1, 1), seasonal = list(order = c(0, 1, 1), period = 12))`

```
pred <- predict(fit, n.ahead = 36)
```

```
ts.plot(AirPassengers, 2.718^pred$pred, log = "y", lty = c(1,3))
```

Time Series

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```
pred <- predict(fit, n.ahead = 36)
```

```
ts.plot(AirPassengers, 2.718^pred$pred, log = "y", lty = c(1,3))
```

```
tsdiag(fit, gof.lag = 30)
```

Time Series

- ▶ How about predicting the future observations with confidence bounds?

Time Series

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```
pm <- forecast(fit, h=10*12)
```

Time Series

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```
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```

```
plot(pm)
```

Time Series

- ▶ How do we decide which model to use? $ARIMA(p, d, q)$
- ▶ How far in the past should we look? What's p , q , and d ?

Time Series

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- ▶ Create a Table of AIC or BIC using $BIC(\text{fit})$ or $AIC(\text{fit})$

Time Series

- ▶ Create a Table of AIC or BIC using BIC(fit) or AIC(fit)

- ▶ Example:

```
library(timeSeries)
```

```
?USDCHF
```

Time Series

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```
adf.test(diff(USDCHF)[-1], alternative="stationary", k=0)
```

Time Series

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we know that the differenced series is stationary. Now create the table for different MA and AR

Time Series

- ▶ Create a Table of AIC or BIC using BIC(fit) or AIC(fit)

- ▶ Example:

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```
?USDCHF
```

```
adf.test(diff(USDCHF)[-1], alternative="stationary", k=0)
```

we know that the differenced series is stationary. Now create the table for different MA and AR

```
fit <- arima(USDCHF, c(0, 1, 0))
```

```
AIC(fit)
```

```
-634191.6
```

	MA(0)	MA(1)	MA(2)	MA(3)
AR(0)	-634191.6	-634213.6	-634219.3	-634218.4
AR(1)	-634214.1	-634211.9	-634217.3	-634216.4
AR(2)	-634219.4	-634217.3	-634215.3	-634214.4
AR(3)	-634218.3	-634216.3	-634214.3	-634212.3

Time Series

- ▶ Your turn

Run a time series analysis with the Robberies data

Time Series

Exercises:

<https://www.r-exercises.com/2018/06/06/intro-to-time-series-analysis-part-1/>

<https://www.r-exercises.com/2018/06/20/intro-to-time-series-analysis-part-2-exercises/>

<https://www.r-exercises.com/2017/04/10/forecasting-time-series-exploration-exercises-part-1/>