1. ◦ # Q4 Evolutionary Selection Experiment Report

1. Experimental Methodology

1.1 Experimental Setup

Initial Population Configuration:
A mixed population consisting of AllCooperate, AllDefect, TitForTat, GrimTrigger, and PAVLOV was used, with an equal initial distribution:

Generation	ALLC	ALLD	TFT	GRIM	PAVLOV
0	0.200	0.200	0.200	0.200	0.200

1.2 Evolutionary Mechanism

Replicator Dynamics

```
for (int gen = 0; gen < config_.generations; gen++) {
    history.push_back(populations);

    if (gen == config_.generations - 1) break;

    auto fitness = calculateFitness(populations, config_.rounds, config_.repeats);
    updatePopulations(populations, fitness);
}
```

```
std::map<std::string, double> SimulatorRunner::calculateFitness(
    const std::map<std::string, double>& populations, int rounds, int repeats) {

    std::map<std::string, double> fitness;

    for (const auto& strat_i : strategies_) {
        std::string name_i = strat_i->getName();
        double pop_i = populations.at(name_i);

        if (pop_i < 1e-6) {
            fitness[name_i] = 0.0;
            continue;
        }

        double total_fitness = 0.0;

        for (const auto& strat_j : strategies_) {
            std::string name_j = strat_j->getName();
            double pop_j = populations.at(name_j);

            if (pop_j < 1e-6) continue;

            double avg_score = playMultipleGames(strat_i, strat_j, rounds, repeats);
            total_fitness += avg_score * pop_j;
        }

        fitness[name_i] = total_fitness;
    }

    return fitness;
```

```cpp
void SimulatorRunner::updatePopulations(
    std::map<std::string, double>& populations,
    const std::map<std::string, double>& fitness) {

    double avg_fitness = 0.0;
    for (const auto& [name, pop] : populations) {
        avg_fitness += fitness.at(name) * pop;
    }

    if (avg_fitness < 1e-9) {
        std::cerr << "Warning: Average fitness is too low, skipping update.\n";
        return;
    }

    std::map<std::string, double> new_populations;
    for (const auto& [name, pop] : populations) {
        new_populations[name] = pop * (fitness.at(name) / avg_fitness);
    }

    populations = new_populations;
```

1.3 Experimental Parameters

1. **Noise-free environment**: ε = 0.0
2. **Noisy environment**: ε = 0.100000
3. **Evolutionary generations**: 50 generations

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2. Results

2.1 Noise-Free Environment Evolution (ε = 0.0)

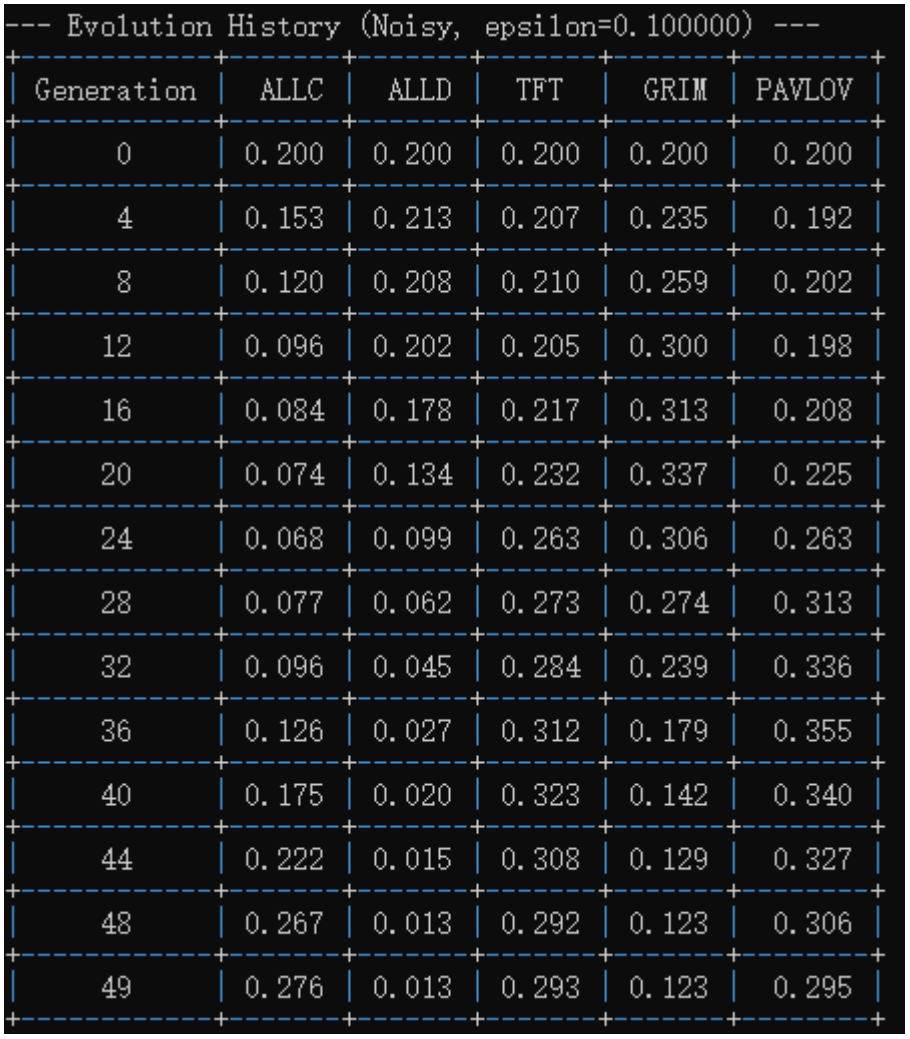
--- Evolution History (Noise-Free, epsilon=0.0) ---

Generation	ALLC	ALLD	TFT	GRIM	PAVLOV
0	0.200	0.200	0.200	0.200	0.200
4	0.202	0.053	0.248	0.248	0.248
8	0.205	0.009	0.262	0.262	0.262
12	0.205	0.001	0.264	0.264	0.264
16	0.205	0.000	0.265	0.265	0.265
20	0.205	0.000	0.265	0.265	0.265
24	0.205	0.000	0.265	0.265	0.265
28	0.205	0.000	0.265	0.265	0.265
32	0.205	0.000	0.265	0.265	0.265
36	0.205	0.000	0.265	0.265	0.265
40	0.205	0.000	0.265	0.265	0.265
44	0.205	0.000	0.265	0.265	0.265
48	0.205	0.000	0.265	0.265	0.265
49	0.205	0.000	0.265	0.265	0.265

Key Observations:

1. **Rapid Elimination Phase (Generations 0-16)**:
   - ALLD declined sharply: 20% → 5.3% (Gen 4) → 0.9% (Gen 8) → 0% (Gen 16)
   - TFT, GRIM, and PAVLOV grew synchronously to ~26.5%
   - ALLC increased slightly to 20.5%
2. **Stable Equilibrium Phase (Generations 16-49)**:
   - A stable four-strategy equilibrium formed
   - Final distribution: TFT (26.5%), GRIM (26.5%), PAVLOV (26.5%), ALLC (20.5%)
   - ALLD completely extinct

2.2 Noisy Environment Evolution (ε = 0.10)



--- Evolution History (Noisy, epsilon=0.100000) ---

Generation	ALLC	ALLD	TFT	GRIM	PAVLOV
0	0.200	0.200	0.200	0.200	0.200
4	0.153	0.213	0.207	0.235	0.192
8	0.120	0.208	0.210	0.259	0.202
12	0.096	0.202	0.205	0.300	0.198
16	0.084	0.178	0.217	0.313	0.208
20	0.074	0.134	0.232	0.337	0.225
24	0.068	0.099	0.263	0.306	0.263
28	0.077	0.062	0.273	0.274	0.313
32	0.096	0.045	0.284	0.239	0.336
36	0.126	0.027	0.312	0.179	0.355
40	0.175	0.020	0.323	0.142	0.340
44	0.222	0.015	0.308	0.129	0.327
48	0.267	0.013	0.292	0.123	0.306
49	0.276	0.013	0.293	0.123	0.295

Key Observations:

1. **Initial Fluctuation Phase (Generations 0-20)**:
   - GRIM showed abnormal growth: 20% → 33.7% (peak at Gen 20)
   - ALLD first increased then decreased: 20% → 21.3% (Gen 4) → 13.4% (Gen 20)
   - ALLC continuously declined: 20% → 7.4%
   - TFT remained relatively stable
   - PAVLOV grew slowly
2. **Strategy Reversal Phase (Generations 20-36)**:
   - GRIM declined sharply: 33.7% → 17.9%
   - PAVLOV surged rapidly: 22.5% → 35.5%
   - TFT grew steadily: 23.2% → 31.2%
   - ALLC began to rebound: 7.4% → 12.6%
   - ALLD continued to decline: 13.4% → 2.7%
3. **New Equilibrium Phase (Generations 36-49)**:
   - TFT leading: 29.3%
   - PAVLOV second: 29.5%
   - ALLC significantly recovered: 27.6%
   - GRIM maintained low position: 12.3%

- ALLD near extinction: 1.3%

4. **Key Differences from Noise-Free Environment**:
   - GRIM transformed from dominant to marginal strategy (26.5% → 12.3%)
   - ALLC recovered from marginal to mainstream (20.5% → 27.6%)
   - PAVLOV maintained competitiveness (26.5% → 29.5%)
   - TFT performed robustly (26.5% → 29.3%)
   - Evolution process was more turbulent, experiencing significant strategy transitions

2.3 Comparative Summary

Final Population Distribution Comparison:

Strategy	Noise-Free (%)	Noisy (%)	Change
ALLC	20.5	27.6	+34.6%
ALLD	0.0	1.3	+1.3%
TFT	26.5	29.3	+10.6%
GRIM	26.5	12.3	-53.6%
PAVLOV	26.5	29.5	+11.3%

The noisy environment significantly altered evolutionary dynamics, giving forgiving strategies (ALLC, PAVLOV) an advantage, while severely weakening the strict retaliatory strategy (GRIM).

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3. Result Analysis

3.1 Strategy Dynamics in Noise-Free Environment

Theoretical Basis for TFT/GRIM/PAVLOV Coexistence

The experimental results show that three reciprocal strategies achieved perfect equilibrium (each at 26.5%), reflecting the "reciprocal alliance" phenomenon in evolutionary game theory:

These three strategies maintain perfect cooperation when playing against each other, obtaining the same high payoff, and therefore cannot replace one another, forming a **neutrally stable equilibrium**.

Rapid Elimination Mechanism of ALLD

ALLD became completely extinct within 16 generations for the following reasons:

1. **Disadvantage Against Reciprocal Strategies**:
   - ALLD vs TFT: First round T=5, then forever P=1 → average payoff ≈ 1.2
   - ALLD vs GRIM: First round T=5, then forever P=1 → average payoff ≈ 1.2
   - ALLD vs PAVLOV: Alternating T/P or locked at P → average payoff ≈ 1.5-3
2. **Positive Feedback Loop**:

- ALLD proportion decreases → cooperative environment improves → reciprocal strategies' payoffs increase → ALLD further eliminated

The Mystery of ALLC's Continued Survival

Although ALLC has no defensive capability, it stably maintains at 20.5% because:

- After ALLD is eliminated, ALLC only faces cooperators
- ALLC vs TFT/GRIM/PAVLOV: Always cooperate → payoff = R = 3
- Same payoff as dominant strategies, therefore not eliminated

3.2 Strategy Dynamics in Noisy Environment

A 10% noise rate fundamentally changed strategy performance:

Impact on Strict Strategies (GRIM Collapse):

GRIM plummeted from 26.5% to 12.3% (down 53.6%) due to its "never forgive" characteristic:

TFT's Moderate Resilience

TFT increased from 26.5% to 29.3%, performing excellently:

1. **Short-Term Memory Advantage**:
   - Only remembers the most recent round, limiting the impact of misjudgment
   - After a misoperation, only triggers one round of retaliation, then can resume cooperation

PAVLOV's Forgiving Mechanism

PAVLOV maintained 29.5%, becoming one of the optimal strategies in noisy environments:

ALLC Proportion Increase

ALLC surged from 20.5% to 27.6% (increase of 34.6%), becoming the third largest strategy:

1. **Value of Unconditional Forgiveness**:
   - Does not retaliate due to misoperations
   - Provides "reset" opportunities for all strategies
   - Reduces overall conflict level in the population
2. **Ecological Niche Expansion**:
   - Noise-free: Only safe in ALLD-free environment
   - Noisy: Reciprocal strategies hurt each other → ALLC's relative payoff increases

ALLD's Persistent Survival

ALLD was not completely extinct (retained 1.3%) for the following reasons:

1. **"Smokescreen" Provided by Noise**:
   - Reciprocal strategies weakened by internal conflicts
   - ALLD can profit short-term in the chaos
2. **Exploitation of ALLC**:
   - ALLC proportion increased to 27.6% → ALLD's food source increased