

2/10/13 MA 2E01 PROBLEM SET 1

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7/10

1 a $r(t) = (t^2-1)i + \sqrt{t}j - t^3k$

domain of $(t^2-1) = (-\infty, \infty)$

domain of $\sqrt{t} = [0, \infty)$

domain of $-t^3 = (-\infty, \infty)$

$D(r) = [0, \infty)$ ✓

b $r(t) = e^t i + t^2 j + \ln(t-2)k$

domain of $e^t = (-\infty, \infty)$

domain of $t^2 = (-\infty, \infty)$

domain of $\ln(t-2) = (2, \infty)$

$D(r) = (2, \infty)$ ✓

2 $r(t) = 2ti + (1-t^2)j - \ln t k$

A. $\frac{dr}{dt} = 2i - 2tj - \frac{1}{t}k$ ✓

b Norm = $\sqrt{(2)^2 + (-2t)^2 + (-\frac{1}{t})^2}$

$\sqrt{4 + 4t^2 + \frac{1}{t^2}}$

$2t + \frac{1}{t}$ ✓

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2c. Unit tangent vector $\mathbf{T} = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

$$= \frac{2\mathbf{i} - 2t\mathbf{j} - \frac{1}{t}\mathbf{k}}{2t + \frac{1}{t}}$$

✓ (A)

3. $\mathbf{r}(t) = 2t\mathbf{i} + (1-t^2)\mathbf{j} - \ln tk$

$$\frac{2\mathbf{i} - 2t\mathbf{j} - \frac{1}{t}\mathbf{k}}{\sqrt{4t^2 - 4t^2 - \frac{1}{t^2}}} \quad P_0(2, 0, 0)$$

$$F(x, y, z) = 2t, 1-t^2, \ln t$$

$$\nabla F = \left(\frac{1}{t}, -2t, \frac{1}{t} \right)$$

$$\nabla F(2, 0, 0) = (2, 0, 0)$$

Tangent plane: $2(x-2) + 0(y-0) + 0(z-0) = 0$

Normal line: $(2, 0, 0) + t(2, 0, 0) = (2+2t, 0, 0)$ ✗

Position + direction

$$\mathbf{R}(t) = \mathbf{r}_0 + \mathbf{v}_0 T$$

point (2, 4, 0) equal sign when $t=1$

$$2t\mathbf{i} + (1-t^2)\mathbf{j} - \ln(t)\mathbf{k} + 2\mathbf{i} - 2t\mathbf{j} - \frac{1}{t}\mathbf{k}$$

$$t=1 \Rightarrow 2\mathbf{i} + 0\mathbf{j} - 0\mathbf{k} + 2\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}$$

$$(2+2)\mathbf{i} + (0-2)\mathbf{j} + (0-1)\mathbf{k}$$

$$(2+2)\mathbf{i} + (0-2)\mathbf{j} + (0-1)\mathbf{k}$$

$$2\mathbf{i} + (2-2)\mathbf{j} - 1\mathbf{k}$$

✓ (2)

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4 a

arc length over $[1, 4]$ $e^{2t}i + e^{2t}\sin(\pi t)j + e^{2t}\cos(\pi t)k$

$$\int_1^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt \quad \int_1^4 \left\| \frac{dr}{dt} \right\| dt$$

$$\int_1^4 \sqrt{(2e^{2t})^2 + (-t e^{2t})^2 + (-e^{2t}(2t+1))^2} dt \quad \int_1^4 \left(4e^{4t} + t^2 e^{4t} + e^{4t}(2t+1)^2 \right)^{1/2} dt$$

$$\int_1^4 \sqrt{10} e^{4t} dt = \frac{\sqrt{10}}{4} e^{4t} \Big|_1^4 = \frac{\sqrt{10}}{4} (e^{16} - e^4)$$

$$e^{2t} \sin(\pi t)$$

$$\sin(\pi t)$$

$$b. \frac{dr}{dt} \Big|_{t=0} = \frac{\sqrt{(2e^0)^2 + (-e^0)^2 + (-e^0)^2}}{\sqrt{45}} \Big|_0^0 = \frac{\sqrt{6}}{\sqrt{45}}$$

$$s = \int_0^t \sqrt{45} dt = \sqrt{45} t \Big|_0^t = \sqrt{45} t$$

$$r(s) = e^{2t}i + e^{2t}\sin(\pi t)j + e^{2t}\cos(\pi t)k$$