Tutorial 3: MA1E01

Continuity

1. Suppose f and g are continuous functions such that

$$f(3) = -7 \qquad \text{and} \qquad \lim_{x\to 3}[f(x) - 2g(x)] = 4.$$

Find

$[-7 - 2g(x)] = 4$ $\quad 2g(x) = -11$ $\quad g(x) = \frac{-11}{2}$ $\quad g(3) = \frac{11}{2}$

   (a) $g(3)$ -6

   (b) $\lim_{x\to 3} g(x)$

2. Find the values of x for which f is not continuous:

   (a)
$$f(x) = 5x^7 + 6x^4 + 2x - 3$$
continuous

   (b)
$$f(x) = \sqrt{x-2}$$
not CTS for $x - 2 < 0$

   (c)
$$f(x) = \frac{2x}{x^2 - 2x}$$
$x^2 - 2x < 0$

   (d)
$$f(x) = \frac{x^2 + 2x + 19}{|x| + 19}$$
continuous

   (e)
$$f(x) = \begin{cases} 2x + 3 & x \leq 4 \\ 7 + 16/x & x > 4 \end{cases}$$
continual. $f(4) = 11$ $\quad \lim_{x\to 4} f(x) = 11$ and 11 contains one sided limits equal each other.

3. Find the value of k such that f is continuous everywhere,

take as 2 seperate functions

$$f(x) = \begin{cases} kx^2 \text{ cts} & x \leq 2 \\ 2x + k \text{ cts} & x > 2 \end{cases}$$

what about $x = 2$

$\lim_{x\to 2^-} f(x) \stackrel{?}{=} \lim_{x\to 2^+} f(x)$

$4k \stackrel{?}{=} 4 + k$

not equal

If continual must equal

$4k = 4 + k$

$3k = 4$

$k = \frac{4}{3}$

Substitute in

$= \frac{16}{3}$ and $\frac{16}{3}$

$f(2) = \frac{4}{3}(2^2) = \frac{16}{3}$

4. Show that $f(x) = x^3 + x^2 - 3x - 1$ has a root in the interval $[1, 2]$. Approximate this root to 2 decimal places.

5. Prove, using the Intermediate Value Theorem, that if $p(x)$ is a polynomial of odd degree, then the equation $p(x) = 0$ has at least one real solution.

$p(x) = C_n x^n + C_{n-1} x^{n-1} + C_1 x + C_0$ example $2x^3 + 6x + 1 = C_3 x^3 + C_1 x + C_0$ example $n = 3$ = degree of polynomial

① $C_n > 0$ $\quad p(-\infty) = C_n(-\infty)^n + C_{n-1}(-\infty)^{n-1} \ldots$
$= -\infty$ always negative

$p(\infty) = C_n(\infty)^n + C_{n-1}(\infty)^{n-1} \ldots$
$= +\infty$ always positive

② $C_n < 0$ $\quad p(-\infty)$ $\quad C_n(-\infty)^n = \infty$
$p(\infty)$ $\quad C_n(\infty)^n = -\infty$

McM tutorial 3

a $g(3) =$

1. $f(3) = -7$ $\lim_{x \to 3} \left(f(x) - 2g(x) \right) = 4$

$-7 - 2g(3) = 4$

$2g(3) = -11$ g is cts

$g(3) = -\frac{11}{2}$ = limit ✓

9.125.

4.

x	1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
$f(x) - 2$			-1.013	-0.400	$\frac{1}{8}$					5.

8.4 - 0.1

lie between 1.4 1.5.

1.4	1.41	1.42	1.43	1.44	1.45	1.46	1.47	1.48	1.49	1.5
-0.496					-0.19875		-0.0720	0	0.058 049	$\frac{1}{8}$

lied between

1.48 1.481 $f(1.485) = 0.02498417$

-0.007808 $f(x) = 0.0250$

∃ there exists

$-1, 1$ $-1 + 9 + 2 - 4 \stackrel{?}{=} 0$

-1

$-1 + 9 + 2 - 9$

$+1$

1

$1 + 9 - 2 - 4$

-1