1. 2012 MANG. SCI. PAPER 3 Q3 DAVID LIETBRECHT.

3(A) i. Essential Components of linear Programming

- Has an objective function which is a linear function of the decision variables of the problem, which must be maximised or minimised

- Has a set of constraints, each of which is a linear equality/inequality of the decision variables. ($\leq, \geq, =$)

- Has n variables. Each term has a coefficient which can be positive negative or zero

- Each variable has a highest power of 1.
- A variable cannot be multiplied by another variable

ii. Simplex Method

- Can be used on Linear Problems that are difficult to solve graphically i.e, when there are more than 2 decision variables

- Solves Linear Program using Gauss-Jordan matrices algorithm along with use of surplus variables an artificial variables

iii. General approach for Linear Programming formulation

For 2/3 variables this method is applicable, else see Simplex:
- Identify and list all the decision variables in the problem
- Derive a linear function in terms of these variables, which when maximised or minimised meets the objective of the exercise

- Identify all the items that constrain the problem (i.e. availability of labour and goods)

2

- Express these constraints in the form of linear equalities/inequalities of the decision variables

Simplex

- Do above steps with additional steps:
- Change to standard form:
- All ~~constraints must be in form of~~ (≤)
- ~~If (≥) inequality~~
- RHS values must be positive
- If (≤) constraint add slack variable S_I.
- If (=) constraint add artificial variable a_I
- If (≥) constraint add slack and artificial variable $-S_I$ and a_I.
- If min problem, multiply O.F. by (-1) to change to max problem

iv. Primal

- This entails solving the linear program via simplex with the objective function and constraints unchanged
- The "original" problem

Dual

- This entails changing the formulation of the linear program.
- Fundamental property of the primal/dual relationship is that the optimal solution to either the primal or dual provide the same answer.
- If primal is harder to solve we choose dual.

- The dual of a max primal problem is a min problem in canonical form

- When primal has n decision variables, the dual will have n constraints.

3 2012 MANG SCI PAPER 3 Q3 DAVID LIEBRECHT

First constraint of dual is associated with variable x_1 and 2nd with variable x_2 and so on

- When primal has m constraints the dual will have m decision variables. Dual variable U_1 associated with first primal constraint, dual variable U_2 with 2nd constraint and so on

- The right hand sides of the primal constraints become the objective function coefficients in dual

- The objective function coef's of primal become RHS of dual
- Constraint coef's of the ith primal variable become the coef in the ith

B Formulate:

x_1 = Model 1
x_2 = Model 2

Max $Z = 200x_1 + 280x_2$ Profit
ST: $20x_1 + 25x_2 \leq 4000$ steel
$40x_1 + 100x_2 \leq 2000$ Man.
$60x_1 + 40x_2 \leq 1600$ assembly
$x_1, x_2 \geq 0$

Mins? lbs

①

Basic	var	x_1	x_2	s_1	s_2	s_3		
		200	280	0	0	0		
s_1	0	20	25	1	0	0	4000	$\frac{4000}{25} = 160$
s_2	0	40	100	0	1	0	2000	$\frac{2000}{100} = 20$
s_3	0	60	40	0	0	1	1600	1600/40 = 4
Z_j		0	0	0	0	0	0	
$C_j - Z_j$		200	280	0	0	0		

x_2 enters, S_2 exits wrt $\frac{0}{0}$

$r_2 \div 100$
$r_1 - 25R_2$
$r_3 - 40R_2$

Why not use graphical method?

2.

Basis	Value	X_1	X_2	S_1	S_2	S_3		ratio
		200	280	0	0	0		
S_1	0	10	0	1	-0.25	0	39500	$\frac{39500}{10}$ 3950
X_2	~~200~~ 280	0.4	1	0	0.01	0	20	$\frac{20}{0.4}$ = 50
S_3	0	44	0	0	-0.4	1	800	800/44 = 200/11
Z_j		~~112~~ 112	0	0	28	0	5600	
$C_j - Z_j$		~~88~~ 88	0	0	-28	0		

X_1 enters, S_3 exits

$R_3 \div 44$ $\quad$ $R_2 - 0.4R_3$ $\quad$ $R_1 - 10R_3$

3.

Basis	Value	X_1	X_2	S_1	S_2	S_3	
		200	280	0	0	0	
S_1	0	0	0	1	-7/44	-5/22	39318.2
X_2	280	0	1	0	3/220	-1/110	140/11
X_1	200	1	0	0	-1/110	1/44	200/11
Z_j		200	280	0	+2	+2	7200
$C_j - Z_j$		0	0	0	-2	-2	

Solution found: $X_1 = 200/11$ $\quad S_2 = 0$

$X_2 = 140/11$ $\quad S_3 = 0$

$S_1 = 39318.2$

Value $= 280(140/11) + 200(200/11) = 7200$

opt 424,000

$x_1 = 1000$

$x_2 = 800$

b ii Shadow Prices

Shadow/dual price is the economic gain/benefit from having one more unit available in a binding constraint.

$S_2 \rightarrow$ Constraint 2 is 2, for extra unit of Mon time, profit +2

$S_3 \rightarrow$ Constraint 3 is 2, for extra unit of assembly, profit +2.

C1 – 8.8 C2 – 0.6 C3 – 0

5.

2012 MANG SCI PAPER 3 Q3 DAVID WETTBRECHT

b(iii) Coefficient Range)

$X_1 \Rightarrow \quad +\frac{1}{110}c_1 - \frac{3}{220}(280) \leq 0 \qquad -\frac{1}{40}c_1 + \frac{280}{110} \leq 0$

$c_1 \leq 420 \qquad c_1 \geq 112$

$112 \leq c_1 \leq \cancel{420}$ 224

$X_2 \Rightarrow \quad -\frac{3}{220}c_2 + \frac{200}{110} \leq 0 \qquad \frac{1}{110}c_2 - \frac{200}{44} \leq 0$

$c_2 \geq 400/3 \qquad c_2 \geq 500$

$\cancel{400/3}$ 280 $\leq c_2 \leq 500$

B (iv) Right hand side range).

- Also known as range of feasibility
- Predicts the change in value of objective function corresponding to a unit change in a b_i
- Show range for right hand side value after it has be increased /decreased as much as it can while remaining optimal

For Constraint 1

S_1 old + new

$$\begin{bmatrix} 39318.2 \\ 140/11 \\ 200/11 \\ 720 \end{bmatrix} + \Delta b_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad 39318.2 + \Delta b_1 \geq 0 \quad \Delta b_1 \geq -39318.2$$

$b_1 = 40000$

$40000 - 39318.2 \leq \Delta b_1 \leq \infty$

$681.8 \leq \Delta b_1 \leq \infty$

$7500/11 \leq \Delta b_1 \leq \infty$

$30\,000 \leq \cancel{40\,000} \leq 40909.091$

7 6

For Constraint 2

$$\underbrace{\begin{bmatrix} 39318.2 \\ 140/11 \\ 200/11 \\ 7200 \end{bmatrix}}_{\text{old}} + \Delta b_2 \underbrace{\begin{bmatrix} -7/44 \\ 3/220 \\ -1/110 \\ 2 \end{bmatrix}}_{\text{new}}$$

$39318.2 + \Delta b_2(-7/44) \geq 0$ $\qquad \Delta b_2 \leq 247142.97$

$140/11 + \Delta b_2(3/220) \geq 0$ $\qquad \Delta b_2 \geq -2800/3$ $\qquad -933.33$

$200/11 + \Delta b_2(-1/110) \geq 0$ $\qquad \Delta b_2 \leq 2000$

$7200 + \Delta b_2(2) \geq 0$ $\qquad \Delta b_2 \geq -3600$

$b_2 = 2000$ $\qquad 2000 - \frac{2800}{3} \leq \Delta b_2 \leq 2000 + 2000$

$3200/3 \leq \Delta b_2 \leq 4000$

$114,285.714 \leq 120000 \leq 160,000$

For Constraint 3

$$\begin{bmatrix} 39318.2 \\ 140/11 \\ 200/11 \\ 7200 \end{bmatrix} + \Delta b_3 \begin{bmatrix} -5/22 \\ -1/10 \\ 1/44 \\ 2 \end{bmatrix}$$

$39318.2 - 5/22\,\Delta b_3 \geq 0$ $\qquad \Delta b_3 \leq 173000.08$

$140/11 - 1/10\,\Delta b_3 \geq 0$ $\qquad \Delta b_3 \leq 1400$

$200/11 + 1/44\,\Delta b_3 \geq 0$ $\qquad \Delta b_3 \geq -800$

$7200 + 2\,\Delta b_3 \geq 0$ $\qquad \Delta b_3 \geq -3600$

$-800 \leq \Delta b_3 \leq 1400$ $\qquad b_3 = 1600$

$1600 - 800 \leq b_3 \leq 1400 + 1600$

$800 \leq b_3 \leq 4000$

$92000 \leq 96000 \leq \infty$

B(v) Should you purchase additional steel

- According to solution, most certainly not. ✓
- Currently 39318.2 kg of stell unused
- Extra stell will add no profit to operation
- Look for extra manufacturing or assembly time
- Steel is worthless in this situation

✓

7.

2012 MANG. SCI PAPER 3 Q3 DAVID WETTBRECHT.

B(vi). - I am in favour of introducing overtime.
- Each hour of overtime assembly time is work £2 of extra profit per every extra hour of assembly time got.

- As long as hours introduced result in total hours between 800 and 4000 hours the solution will remain optimal

- I would not pay more than £2 per hour otherwise profit will decrease

B (vii) Here we are considering changes in objective function coefficient of X

From our sensitivity analysis, coefficient of X_1 can range between 112 and 420 and still have an optimal solution

Value of solution will be $\left(\frac{200}{11}\right)(175) + \left(\frac{140}{11}\right)(280) = 6745.45$ profit

B (viii). Manufacturing ⇒ Constraint 2 → can range between $\frac{5200}{3}$ and 4000 and still have an optimal solution

- 2000 original hours + 500 new hours is 2500 hours.
- lies within interval
- Shadow price will remain the same as it is inside interval

1

2012 MANG SCI PAPER 3 Q3 CORRECTION.

		X_1	X_2	S_1	S_2	S_3	
[illegible]	value	200	280	0	0	0	
X_1	200 c_1	1	0	$0.1c_1$	$-1/40$	0	1000
X_2	280 c_2	0	1	-0.04	$0.02c_2$	0	800
S_3	0	0	0	-4.4	0.7	1	4000
Z_J		$200c_1$	280	8.8	0.6	0	424000
$C_J - Z_J$		0	0	-8.8	-0.6	0	

Solution $X_1 = 1000$ $X_2 = 800$ $S_3 = 4000$, value $= 424000$

ii. Shadow price represent the extra value added per unit increase in availability of binding contract.

$S_1 \Rightarrow$ binding $\Rightarrow$ value $= 8.8$

$S_2 \Rightarrow$ binding $\Rightarrow$ value $= 0.6$

$S_3 \Rightarrow$ non binding $\Rightarrow$ value $= 0$

Value is how much profit will be increased by.

iii. Coefficient ranges.

$X_1 \Rightarrow$ $-0.1c_1 + 11.2 \leq 0$ $\quad \frac{+1}{40}c_1 - 5.6 \leq 0$

$0.1c_1 \geq 11.2$ $\quad c_1 \leq 224$

$c_1 \geq 112$

$112 \leq c_1 \leq 224$

$X_2 \Rightarrow$ $0.04c_2 - 20 \geq 0$ $\quad 5 - 0.02c_2 \geq 0$

$c_2 \geq 500$ $\quad c_2 \leq 250$

$250 \leq c_2 \leq 500$

2.

iv. Right hand Side range

- Also known as range of feasibility
- Predicts the change in value of the objective function corresponding to a unit change in a constraint
- Shows the range for right hand Side values after it had been increased or decreased as much as it can while remaining optimal

For constraint one

$$\begin{array}{c} \text{old} \\ \begin{bmatrix} 100 \\ 800 \\ 4000 \\ 42400 \end{bmatrix} \end{array} + \Delta b_1 \begin{array}{c} \text{new} \\ \begin{bmatrix} 0.1 \\ -0.04 \\ -4.4 \\ 8.8 \end{bmatrix} \end{array}$$

$100 + \Delta b_1 0.1 \geq 0$ $\quad \Delta b_1 \geq -1000$

$800 - \Delta b_1 0.04 \geq 0$ $\quad \Delta b_1 \leq 20000$

$4000 - 4.4 \Delta b_1 \geq 0$ $\quad \Delta b_1 < \frac{1000}{11}$ $\quad 909.09$

$42400 + 8.8 \Delta b_1 \geq 0$ $\quad \Delta b_1 \geq -4.7045.4?$

$$-1000 \leq \Delta b_1 \leq \frac{10000}{11}$$

$$4000 - 1000 \leq \Delta b_1 \leq 4000 + \frac{1000}{11}$$

$$39000 \leq \Delta b_1 \leq 4909.09$$

Constraint 2.

$$\begin{bmatrix} 1000 \\ 800 \\ 4000 \\ 42400 \end{bmatrix} + \Delta b_2 \begin{bmatrix} 0.25 \\ 0.01 \\ \cancel{0}.4 \\ 0.6 \end{bmatrix}$$

$100 - 0.25 \Delta b_2 \geq 0$ $\quad \Delta b_2 \leq 400$

$800 + 0.01 \Delta b_2 \geq 0$ $\quad \Delta b_2 \geq -80000$ $\quad >$

$40000 \cancel{+} - 0.4 \Delta b_1 \geq 0$ $\quad \Delta b_1 \leq 100000$

$424000 + 2.8 \Delta b_1 \geq 0$ $\quad \Delta b_1 \geq$ $\cancel{-151428}$ -706660 $\quad >$

$-80000 \cancel{400} \leq \Delta b_2 \leq 4000$ $\quad b_2 = 120000$

$40000 \leq \Delta b_2 \leq \cancel{120000}\ 160000$

3. CORRECTION

Constraint 2

$$\begin{bmatrix} 1000 \\ 800 \\ 4000 \\ 424000 \end{bmatrix} \Delta b_2 \begin{bmatrix} -1/40 \\ 0.02 \\ 0.7 \\ 0.6 \end{bmatrix}$$

$1000 - \frac{1}{40}\Delta b_2 \geq 0$ $\quad \Delta b_2 \leq 40000$

$800 + 0.02\Delta b_2 \geq 0$ $\quad \Delta b_2 \geq -40000$

$4000 + 0.7\Delta b_2 \geq 0$ $\quad \Delta b_2 \geq -57142$

$424000 + 0.6\Delta b_2$ $\quad \Delta b_2 \geq -706666$

$\frac{400000}{7}$

$120000 - 57142 \leq \Delta b_2 \leq 120000 + 40000$

$114286 \leq \Delta b_2 \leq 160000$

Constraint 3

$$\begin{bmatrix} 1000 \\ 800 \\ 4000 \\ 424000 \end{bmatrix} + \Delta b_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$4000 + \Delta b_3 \geq 0$ $\quad \Delta b_3 \geq -4000$

$96000 - 4000 \leq \Delta b_3 \leq \infty$

$92000 \leq \Delta b_3 \leq \infty$

v. Steel = constraint 1
Should purchase steel, worth 8.8 @ in profit per unit increase but at the moment it will fit [illegible]

vi. Assembly = constraint 3 ⇒ overtime is worthwhile

vii. already optimal solution ⇒ within range of costs
result is 399000 profit

viii. Second constraint ⇒ we are allowed to increase it by 1
Shadow price will remain the same