Tutorial 8: MA1E01

Applications of the Derivative

1. Find the absolute maximum and minimum values of f on the given interval, and state where those values occur:

   (a) $f(x) = 2x^3 + 3x^2 - 12x$ on $[1, 4]$

   (b) $f(x) = (x^2 + x)^{2/3}$ on $[-2, 3]$

   (c) $f(x) = x - 2\sin x$ on $[-\pi/4, \pi/2]$

   (d) $f(x) = |6 - 4x|$ on $[-3, 3]$

   (e) $f(x) = x^2 - x - 2$ on $(-\infty, \infty)$

   (f) $f(x) = x^3 - 9x + 1$ on $(-\infty, \infty)$

2. A closed rectangular container with a square base is to have a volume of $2000\,cm^3$. It costs twice as much per square centimeter for the top and bottom as it does for the sides. Find the dimensions of the container of least cost.

3. The equation $f(x) = x^5 + x^4 - 5 = 0$ has one real solution. Use the Intermediate Value Theorem to obtain an initial estimate of the solution, then apply Newton's method to approximate the root to 5 decimal places.

4. Apply Rolle's Theorem and use the fact that

$$\frac{d}{dx}(3x^4 + x^2 - 4x) = 12x^3 + 2x - 4$$

   to show that the equation $12x^3 + 2x - 4 = 0$ has at least one solution in the interval $(0, 1)$.

5. For the function $f(x) = \sqrt{25 - x^2}$, verify that the hypothesis of the Mean-Value Theorem is satisfied on the interval $[-5, 3]$ and find all values of c in that interval that satisfy the conclusion of the theorem.

6. The function $s(t) = t^4 - 4t^2 + 4$ describes the position of a particle moving along a coordinate line, where s is in metres and t is in seconds. Analyze the motion of the particle by determining the intervals over which the particle is moving in the positive or negative direction, the times when the particle has stopped and the intervals over which the particle is speeding up or slowing down. Give a schematic picture of the motion.

3

Maths Tutorial 8

3. $F(x)$ continuous $\forall x$

$f(0) = -5 < 0$

$f(2) =$ positive

$\therefore \exists$ ~~root~~ there exists in interval $(0,2)$

$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$

4 $12x^3 + 2x - 4 = 0$

$6x^3 + x - 2 = 0$

$6x\left(\frac{x^2}{6} + 1\right) - 2 = 0$ $\quad 6x\left(\frac{x^2}{6}+1\right) = 2$

$6x = 0$ $\quad \frac{x^2}{6} + 1 = 0$ $\quad 6x = 2$ or $\left(\frac{x^2}{6}+1\right) = 2$

$x = 0$ $\quad x^2 = -6$ $\quad x = \frac{1}{3}$ $\quad x^2 + 6 = 12$

$x^2 = 6$

$x = \pm\sqrt{6}$

$f(x) = 3x^4 + x^2 - 4x$

$f'(x) = 12x^3 + 2x - 4$

$f(0) = 0$ cuts x-axis

$f(1) = 0$

there is a c inside $(1,0)$

$f'(c) = 0$

5. Mean value theorem $f(x)$ continuous on $[a,b]$ and differentiable on (a,b)

there exists a $c \in (a,b)$ slope is equal to slope of ab

$f(x) = \sqrt{25-x^2}$ no discontinuity on $[-5,3]$ is cts

$f'(x) = \frac{-x}{\sqrt{25-x^2}}$ not defined when $\sqrt{25-x^2} = 0$

$25 - x^2 = 0$

$x = \pm 5$ on $(-5,3)$ ± 5 not members on interval

$\therefore$ by MVT there exists a c s.t. $f'(c) = \frac{f(b)-f(a)}{b-a}$ $\quad f(a) = 0$, $f(b) = 4$

$\frac{4-0}{3--5} = \frac{1}{2}$

$\frac{-x}{\sqrt{25-x^2}} = \frac{1}{2}$

$-2x = \sqrt{25-x^2}$

$4x^2 = 25 - x^2$ $\quad 5x^2 = 25$ $\quad x^2 = 5$ $\quad x = \pm\sqrt{5}$

$c = \pm\sqrt{5}$

6. $S(t) = t^4 - 4t^2 + 4$

$v(t) = s'(t) = 4t^3 - 8t$

$a(t) = s''(t) = 12t^2 - 8$

$S(t) = 0 \Rightarrow 4t^3 - 8t = 0$

$4t(t^2 - 2) = 0$

$4t(t - \sqrt{2})(t + \sqrt{2}) = 0$

$t = 0, \quad t = \sqrt{2}, \quad t = \cancel{-\sqrt{2}}$ no minus time

$s''(t) = 0 \qquad 12t^2 - 8 = 0$

$3t^2 - 2 = 0$

$t^2 = \frac{2}{3}$

$t = \pm\sqrt{\frac{2}{3}}$ no minus time

$s(0) = 4$

$s'(0) = 0$

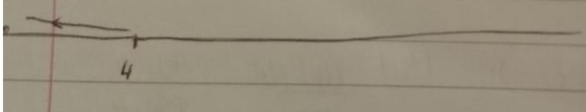
$s''(0) = -8$

a becoming positive at $+\sqrt{\frac{2}{3}}$

velocity will become positive at $\sqrt{2}$

$S(\sqrt{2}) = 4 - 8 + 4 = 0$

$s\left(\sqrt{\frac{2}{3}}\right) = \frac{4}{9} - \frac{8}{3} + 4 = 1$



11/12 Maths Tutorial 8

$\cancel{4x\cos x = x \quad x > 0}$ $\quad\quad$ $\cancel{\pm \quad 0.0001}$

$4\cos x \ \cancel{-x = 0}$ $\quad\quad$ $x > 0$

$\frac{dy}{dx} = \cancel{4\sin x - 1}$

1a $f(x) = 2x^3 + 3x^2 - 12x$

$f'x = 6x^2 + 6x - 12 = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2 \quad x = 1$

$x = 1, -2, 4$

$F(1) = -7$

$f(-2) = 20$ $\quad$ abs max at $x = 4$

$f(4) = 128$ $\quad$ abs min $= x = 1$

b. $(x^2+x)^{2/3}$

$f'(x) = \frac{2}{3}(x^2+x)^{-\frac{1}{3}} + 2x + 1$

$\frac{2}{3(x^2+x)^{1/3}} + 2x + 1 = 0$

$2 \cancel{+ (2x+1)(3(x^2}$

$\frac{2(2x+1)}{3(x^2+x)^{1/3}}$ $\quad\quad$ $2(2x+1) = 0$

$4x + 2 = 0$

not defined at $3(x^2+x)^{1/3} = 0$ $\quad\quad$ $4x = -2$

$(x^2+x)^{1/3} = 0$ $\quad\quad$ $x = \frac{-2}{4}$

$(x^2+x) = 0$ $\quad$ ~~xna~~ $\quad\quad$ $\boxed{x = -\frac{1}{2}}$

$x(x+1)$

$x = 0 \quad x = -1$ $\quad\quad$ $x = \frac{-1}{2}, -2, 3$

[illegible] critical points

$f(-1) =$

$f(0) =$

$f(\frac{-1}{2}) =$

$f(-2) =$

$f(3) =$

week 10

MATH1 Tutorial 8

1c $x - 2\sin x$

$f'(x) = 1 + 2\cos x = 0$

$2\cos x = -1$

$\cos x = \frac{-1}{2}$

$x = \frac{2}{3}\pi$ (120°)

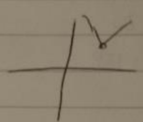
$-\frac{\pi}{4}$, $\frac{2\pi}{3}$ $\frac{\pi}{2}$

$f(-\frac{\pi}{4}) = 0.628$

$f(\frac{2\pi}{3} = 0.684$

d. $f(x) = |6-4x|$ on $[-3, 3]$

$\begin{cases} 6-4x & 6-4x \geq 0 \\ -6+4x & 6-4x < 0 \end{cases}$



$6-4x=0$ not differentiable cannot draw tangent on curve

$\therefore x = \frac{3}{2}$ $x = \frac{3}{2}, -3, 3$

e $f(x) = x^2 - x - 2$

$f'x = 2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$ absolute min

no absolute maximum.

f. No absolute max or minimum tends to infinity

2. $V = x^2 h = 2000$

p = price per cm²

$2x^2(2p) + 4xh(p)$

$4px^2 + 4xhp$

$h = \frac{2000}{x^2}$

$4px^2 + 4\cancel{x}p.\frac{(2000)x^{-2}}{\cancel{x}\,x}$

$f'(x)\ 8px - 4p\frac{(2000)}{x^2} = 0$

~~$8p\,x(x^2) - 8p(2000) = 0$~~

~~$x^3 = 2000$~~

$-8x^3 - 8000$

$x^3 = 1000$

$x = 10cm$