23/04/15.

FORECASTING

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4 A. Stationary - That the mean and variance are stationary - constant.
Usually stationary in TS with no trend or seasonalities

B. Autoregressive uses previous values to predict new value
Linear regression uses a new observation to predict the new value using all of the existing data
link is $X_i = X_{i-1}$

C. Trend, seasonality, error/noise

D. MAPE - Mean absolute percent error $= 100 \frac{1}{n} \left| \frac{(y_i - F_i)}{y_i} \right|$

$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_i - F_i)^2} = \sqrt{\frac{SSE}{n}}$

E. $AIC = -2Log(L) + 2m$ Akaike
$BIC = -2Log(L) + m\log n$ Bayes
- rewards model fit/accuracy while penalizing number of parameters

F. Also known as DES or multiplicative/additive to cater for a TS with seasonality and ~~[illegible]~~ and trend. takes trend, level and seasonality into account

G. look at data to see repeating patterns.
From repeating patterns seasonality can be identified

Q5 Ai. AR - Auto regressive

ii. If we define $x_i = y_{i-1}$, this is called the lagged series x_i is only defined for $i = 2 \dots n$
- AR model is then $y_i = \phi_0 + \phi_1 x_i + \varepsilon_i$ which is a linear regression model
- Can fit this model by doing a linear regression of the series against the lagged series

5A. iii $\varepsilon_t \sim N(0, \sigma^2)$

ε_i and ε_j independent if $i \neq j$.

iv Least squares algorithm

$\theta = (X^T X)^{-1} X^T y$

$$\underset{\theta}{\begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix}} \quad \underset{X}{\begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}} \quad \begin{bmatrix} y_1 \\ y_2 \\ y_n \end{bmatrix} \quad \underset{\varepsilon}{\begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{bmatrix}}$$

B i Errors are used as explanatory variables. We are using the weighted errors or previous fitted values to create our new fitted value

ii No we cannot. The terms $(\psi_1 ..)$ are not linear, hence LSE will not work equation non linear wrt ψ

iii $\mathbb{E}[y_t] = \mathbb{E}[\psi_1 \varepsilon_{t-1} + \psi_0 + \varepsilon_t]$

$= \psi_1 \mathbb{E}[\varepsilon_{t-1}] + \mathbb{E}[\psi_0] + \mathbb{E}[\varepsilon_t]$

$= 0$

$Var = \mathbb{E}[y_t - \mathbb{E}[y_t]]^2$

$= \mathbb{E}[\psi_1 \varepsilon_{t-1} + \varepsilon_t]^2$

$= \mathbb{E}[\psi_1^2 \varepsilon_{t-1}^2 + 2\varepsilon_t \psi_1 \varepsilon_{t-1} + \varepsilon_t^2]$

$= \psi_1^2 \mathbb{E}[\varepsilon_{t-1}^2] + 2\psi_1 \mathbb{E}[\varepsilon_t \varepsilon_{t-1}] + \mathbb{E}[\varepsilon_t^2]$

$= \psi_1^2 \sigma^2 + \sigma^2$

$= (1 + \psi_1^2)\sigma^2$

C i. ACF - auto correlation function measures correlation between y_t and y_{t-k}

$$\frac{\sum_{t=k+1}^{n} (y_t - \bar{y})(y_{t-k} - \bar{y})}{\sum_{t=1}^{n} (y_t - \bar{y})^2}$$

Used to choose an AR or MA model

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5 (i) PACF at lag k is the auto correlation between y_t and y_{t+k} that is not accounted for by lags 1 through $k-1$.
- found by fitting a linear regression model of y_t for k lags
- the $\hat{\phi}_k$ value is the pacf value

(ii) First plot: spike in ACF at 1, pacf 1,2,3 $\Rightarrow MA(1)$
Second: spikes in acf at 1,2,3, in pacf at 1 $\Rightarrow AR(1)$.

In B, $\phi < 0$ because ~~sign~~ spike in pacf is negative
In A, spike is positive so $\theta > 0$ ACF.

6 A i. Autoregressive order 1 AR(1)
I(1) integrated difference
MA(2) moving average of order 2.
$AR(2)_4$ seasonal auto regressive order 2
$I(1)_4$ seasonal integrated differencing order 1
$MA(1)_4$ " moving average of order 1
4 - seasonality of 4

ii. $(1-\phi_1 B)(1-\beta_1 B^4 - \beta_2 B^8)(1-B)^1(1-B^4)^1 = c + (1-\psi_0 B - \psi_1 B^2)(1-\theta_1 B^4)\varepsilon_t$

iii ~~$(1-\beta_1 B^4 - \beta_2 B^8)$~~ " without B

B i $y_{n+1} = \phi_0 + \phi_1 y_n + \varepsilon_{n+1}$
$y_{n+1} = \phi_0 + \phi_1 y_n \pm 2$ with $s^2 = \frac{\sum_{i=2}^{n} \varepsilon_i^2}{n-3}$

ii $y_{n+1} = \hat{\phi}_0 + \hat{\phi}_1 y_n \pm 2s$

iii $y_{n+k} = \hat{\phi}_0 \left[\sum_{i=1}^{k} \phi_1^{i-1}\right] + \phi_1^k y_n + \sum_{i=1}^{k} \phi_1^{i-1} \varepsilon_{n+k-i-1}$

iv. $y_{n+h} = \phi_0 \left(\Sigma \phi_1^{i-1} \right) + \phi_1^h y_n \pm 2\sigma \sqrt{\sum_{i=1}^{h} \phi_1^{2(i-1)}}$

SES - no trend no seasonal

$F_1 = y_1 \qquad 0 < \alpha < 1$

$F_{t+1} = F_t + \alpha (y_t - F_t)$

then $F_{n+h} = F_{n+1} \qquad \forall\, h \geq 1$

B. $SSE = \sum_{i=1}^{n} (y_t - F_t)^2$

$MAPE = 100 \frac{1}{n} \left(\frac{y_t - F_t}{y_t} \right)$

or $RMSE = \sqrt{\frac{\sum_{i=1}^{n} (y_t - F_t)^2}{n}} = \sqrt{\frac{SSE}{n}}$

Choose α with lowest value

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4. d. trend Seasonality noise

5 A. AR - Auto Regressive

C. Assumptions on ε_t in linear regression and AR(1) model.

$\varepsilon_t \sim N(0, \sigma^2)$ for all t.
$\varepsilon_i, \varepsilon_T$ independent if $i \neq T$.

iv. estimate ϕ_0, ϕ_1

Use least squared algorithm estimation

$y_2 = \phi_0 + \phi_1 y_2 + \varepsilon_t$
$y_3 = \phi_0 + \phi_1 y_3 + \varepsilon_3$
$y_n = \phi_0 + \phi_1 y_{n-1} + \varepsilon_n$

$$\Rightarrow \begin{bmatrix} y_2 \\ y_3 \\ \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & y_1 \\ 1 & y_2 \\ \vdots & \\ 1 & y_{n-1} \end{bmatrix}^T \begin{bmatrix} \phi_0 \\ \phi_1 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

$$y = X\phi + \varepsilon$$

$\Rightarrow \hat{\phi} = (X^TX)^{-1} X^T y$ → best model estimate / fitting value of ϕ_0 ϕ_1

b. Why called Moving Average?

$y_2 = \psi_1 \varepsilon_1 + \psi_0 + \varepsilon_2 \Rightarrow \varepsilon_2 = y_2 \psi_1 \varepsilon_1 - \psi_0$
$y_3 = \psi_1 \varepsilon_2 + \psi_0 + \varepsilon_3 \Rightarrow \psi_1 (y_2 - \psi_1 \varepsilon_1 - \psi_0) + \psi_0 + \varepsilon_3$
$\Rightarrow \psi_1 y_2 - \psi_1^2 \varepsilon_1 - \psi_0 \psi_1 + \psi_0 + \varepsilon_3$
$\varepsilon_3 = y_3 - \psi_1 y_2 + \psi_1^2 \varepsilon_1 + \psi_0 \psi_1 - \psi_0$

$\ldots y_n = \psi_1 y_{n-1} - \psi_1^2 y_{n-2} - \psi_1^3 y_{n-3} + \ldots + \varepsilon_n$

$|\psi_1| < 1 \Rightarrow \psi_1 > \psi_1^2 > \psi_1^3 > \ldots$ weighted average of past values

more weight weight placed on average of more recent values

c. No we can't least squared only works for linear equation.
The equation is non linear.

iii. $y_t = \phi_1 \varepsilon_{t-1} + \varepsilon_t$

$\varepsilon_t \sim N(0, \sigma^2)$
$E[\varepsilon_t] = 0$
$Var[\varepsilon_t] = \sigma^2$

$E[y_t] = E(\phi_1 \varepsilon_{t-1} + \varepsilon_t)$
$= \phi_1 E[\varepsilon_{t-1}] + E[\varepsilon_t]$
$\phi_1(0) + 0 \quad = 0$

$Var[y_t] = E[(y_t) - E(y_t))^2] = E[y_t^2] = E[(\phi_1 \varepsilon_{t-1} + \varepsilon_t)^2]$
$E[\phi_1^2 \varepsilon_{t-1}^2 + 2\phi\varepsilon_{t-1}\varepsilon_t + \varepsilon_t^2]$
$= \phi_1^2 E[\varepsilon_{t-1}^2] + 2\phi_1 E[\varepsilon_{t-1}\varepsilon_t] + E[\varepsilon_t^2]$
$= \phi_1^2 \sigma^2 + 2\phi_1(0) + \sigma^2$
$= \sigma^2(1+\phi_1)^2$

$Var[\varepsilon_t] = E[\varepsilon_t^2] = \sigma^2$

(ii) Identify AR(1) and MA(1)

First plot spike in acf at 1, pacf spike at 1,2,3
First plot is MA(1)

Second plot spike in acf at 1,2,3 ~~p~~pacf spike at lag 1,

(iii) Comment on the sign

In B, $\phi_1 < 0$ because spike in pacf negative
In A, spike in pacf $\phi_1 > 0$ spike in acf positive

Q6 Write in with backshift operator

ARIMA $(1,1,2)(2,1,1)_4$

$$\underset{AR(1)}{(1-\phi_1 B)} \underset{SAR_4(2)}{(1-\beta_1 B^4 - \beta_2 B^8)} \underset{I(1)}{(1-B)^1} \underset{SI_4(1)}{(1-B^4)^1} = C + \underset{MA(2)}{(1-\psi_1 B - \psi_2 B^2)} \underset{SMA_4(1)}{(1-\Theta_1 B^4)}\varepsilon_t$$

SES - No trend, no seasonality
Box MC, RSC SES Sum of squared error.