Management Science 2 Sample Paper

4 A. Role of random numbers in simulation

To emulate random event

To generate other random ~~ee~~ variable

Capture stochastic nature of system

B PRNG ~~also known as~~ is an algorithm for generating a sequence of numbers that approximates the properties of random numbers

- Appears to be random but is not
- Exhibit statistical randomness while being generated by an entirely deterministic causal proc

C. Desirable properties of series of pseudo random numbers

- Uncorrelated (independent) sequences - sequences of random numbers should be serially uncorrelated
- Long Period - Generator should be of long period, ideally generator should not repeat, practically the repetition should occur only after the generation of a very large set of random numbers
- Uniformity - Sequence should be uniform and unbiased
- Efficiency - Generator should be efficient. Low overhead for computat (Easily calculated)

D Kolmogorov - Smirnoff test

Tests to see how well a hypothesized distribution $F(x)$ fits an empirical distribution function $F_n(x)$

Given an observed random sample $x_1, x_2 \ldots x_n$ an empirical distribution function $F_n(x)$ is the fraction of sample observations less than or equal to value of x

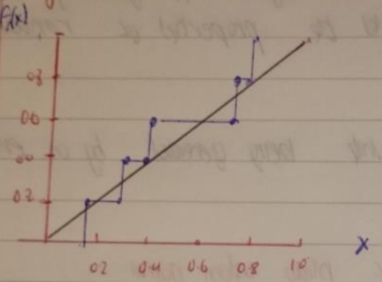
H₀ Null hypothesis → uniform dist.

4 E. Carry out KS test

Order random numbers in increasing size

x: 0.18 0.24 0.33 0.65 0.66

y: 0.2 0.4 0.6 0.8 1



Calculate $\frac{i}{N} - R(i)$ 0.02, 0.16, 0.27 0.15 0.34

Calculate $R(i) - \frac{(i-1)}{N}$ 0.18, 0.04 – 0.05 –

Set $D^+ = \max(\frac{i}{N} - R(i)) = 0.34$

Set $D^- = \max(R(i) - \frac{(i-1)}{N}) = 0.18$

Set $D = \max(D^+, D^-) = 0.34$

For $\alpha = 0.05$ $D_\alpha = 0.565 > D$, hence H_0 not rejected

Alternatives to Kolmogorov Smirnoff test

F. Chi-Square test - need significant sample size and high binsize

Q-Q plot - graphical aid

Histogram - graphical aid

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Q5. A. Inverse transform Algorithm

$R = F(x)$

$x = F^{-1}(R)$

Generate $r = \text{uniform}(0,1)$

Accept Sample $x = F^{-1}(r)$

B. Acceptance Rejection Algorithm

1. Generate proposal y from $g(x)$
2. Generate r from $u(0,1)$
3. if $r < \frac{f(y)}{c\,g(y)}$ accept y, otherwise go to 1.

$c = \text{Supremum}(x) \left(\frac{f(x)}{g(x)}\right)$

C. Inverse Transform vs Acceptance Rejection

I-T has 100% acceptance rate

When we cannot calculate inverse of function use A-R

D. Implement Generation for $u(\frac{1}{4},1)$

Special property: $x \sim u(0,1)$ $\frac{3x}{4} + \frac{1}{4} \sim u(\frac{1}{4},1)$

Generate $x \Rightarrow$ use $\frac{3x}{4} + \frac{1}{4}$ $x = 0.3$ $\rightarrow \frac{3(0.3)}{4} + \frac{1}{4}$

Inverse transform

$\wedge \sim (\frac{1}{4},1)$ $F(x) = \left(\frac{x-0.25}{0.75}\right)$ $R = F(x)$ $R = \frac{x-0.25}{0.75}$

$\Rightarrow x = 0.75R + 0.25$

Acceptance Rejection

$g(x) \sim u(0,1)$

$c = \text{Sup}(x) \left(\frac{f}{g}\right)$

$= \sup(x) \left(\frac{4/3}{1}\right) = 4/3$

$\frac{f}{cg} = \frac{4/3}{4/3} = \begin{cases} 1 & \text{if } \frac{1}{4} \le x \le 1 \\ 0 & x \le \frac{1}{4} \end{cases}$

Propose u from g,
Accept if (U_2 from $U(0,1)$
$U_1 \geq \frac{1}{4}$

E. Non-Stationary Poisson Process

Arrival rate of poisson process - counting process rate λ arrivals are at time stationary and independent increments, arrivals at time t are independent with arrival rate λt.
λ becomes function of time $\lambda(t) \rightarrow$
Cafe, restaurant or bank.

F. Thinning Algorithm For Simulating A Non Stationary Poisson Process

- Generate poisson arrival at faster rate, but accept only a proportion
- ① let λ^* be max arrival rate and set $t=0$.
② Generate $E \sim \exp(\lambda^*)$ and let $t = t+E$.
③. Generate U_1 from $U(0,1)$ and accept ~~if~~ E if $U_1 \le \frac{\lambda(t)}{\lambda^*}$

If reject, return to ②.

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Q 6 A. Three Prevalent World View on Simulation

Event Scheduling - concentrate on events and their effect on system state
Update time to next event etc

Process interaction - Concentrated on process - a process is a time sequenced list of events, activities and delays that define the life cycle of one entity as it moves through system
- Usually many processes are active simultaneously in model, and interaction can be complex. (follow the customer)

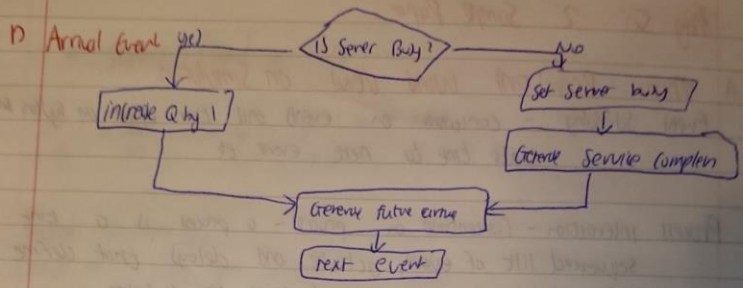
Activity Scanning - concentrates on activities of a model and that conditions that allow an activity to begin
- At each clock advance, the conditions for each activity are checked, and if conditions are true, then the corresponding activities begin

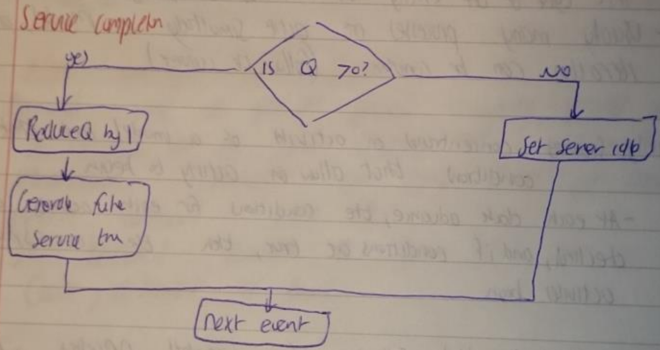
B System State Variables - Collection of variables necessary to record state of system

System Event - occurrences that change system state

Future Event List - list of event notices, ordered by time of occurrence

C M Exponential interval times
G Generic service time distribution
1 number of servers
S system capacity
∞ calling population





Customer	interarrival	arrival time	Service time	time s begins	waiting in Q time	service end time	time in system	idle time server
1	0	0	0·81	0	0	0·81	0·81	0
2	1·63	1·63	1·60	1·63	0	3·23	1·6	0 ·82
3	2·01	3·64	1·05	3·64	0	4·69	1·05	0·41
4	2·05	5·64	2·41	5·69	0	8·1	2·41	1
5	0·34	6·03	2·24	8·1	2·07	10·34	4·31	0
6	0·82	6·85	?	10·34	3·49	?	?	0

Time	Q length	Server busy
0	0	1
0·81	0	0
1·63	0	1
3·23	0	0
3·64	0	1
4·69	0	0
5·64	0	1
6·03	1	1
6·85	2	1

2/4/14 Many Scene Sample Paper Simulation

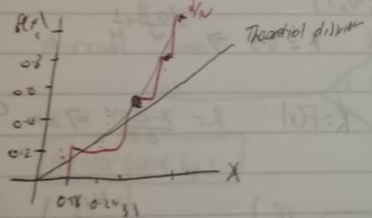
Q4 A. To emulate random events
To generate other random variable
Capture Stochastic nature of system

b Mechanism for generating strings of numbers
Deterministic.

c Easily calculated, long period, appear to be independent, uniform / space filling

D Test of uniformity for random numbers vs. ~~uniform~~ empirical cdf. Should be a straight line
Appear to follow uniform distribution
null hypothesis: uniform distribution

E



Order random numbers in incr. size x 0.18, 0.24, 0.33, 0.65, 0.66

$\frac{(i)}{N}$ $\frac{i}{5}$ y 0.2 0.4 0.6 0.8 1

$\frac{i}{N} - R_{(i)}$ 0.02 0.16 0.27 0.15 0.34

Max difference $\boxed{0.34}$

$\frac{i}{N} - R_{(i)}$ where positive

$R_{(i)} - \frac{i}{N}$ lower diff 0.18 0.04 – 0.05 –

$D = Max(\frac{i}{N} - R_{(i)}$ and $R_{(i)} - \frac{(i-1)}{n})$ = 0.34

F Chi-Square test - need sufficient sample size and right bin size
Q-Q plot - graphical aid
Histogram

2

QS4 A $R = F(x)$

$x = F^{-1}(R)$

Generate $r = \text{uniform}(0,1)$ Accept sample $x = F^{-1}(r)$

B 1. Generate proposal y from $g(x)$.
2. Generate r from $u(0,1)$
3. If $r < \dfrac{f(y)}{c\,g(y)}$ accept y, otherwise go to 1

$c = \sup_{f(x)} \left(\dfrac{f(x)}{g(x)} \right)$ supremum

Inverse transform:

C 100% acceptance rate,
When you can't calculate inverse of function - use A/R then

D. $X \sim U(0,1)$ $\frac{3x}{4} + \frac{1}{4} \sim U(\frac{1}{4}, 1)$

Generate $x \Rightarrow$ use $\frac{3x}{4} + \frac{1}{4}$ $x = 0.3 \Rightarrow \frac{3(0.3)}{4} + \frac{1}{4}$

OR / $x \sim (\frac{1}{4}, 1)$ $F(x)$ $\left(\frac{x - 0.25}{0.75} \right)$ $R = F(x)$ $R = \frac{x - 0.25}{0.75} \Rightarrow x = 0.75R + 0.25$

Inverse transform!

A/R: $g() \sim U(0,1)$ $c = \sup_x \left(\frac{f}{g} \right) = \sup_{(x)} \left(\frac{4/3}{1} \right) = \frac{4}{3}$

$\frac{f}{cg}$ $\frac{4/3}{4/3}$ $= \begin{cases} 1 & \text{if } \frac{1}{4} \leq x \leq 1 \\ 0 & \cancel{\text{otherwise}}\ x \leq \frac{1}{4} \end{cases}$

Propose U_1 from g, accept criteria: U_2 from $U(0,1)$ [also g], accept if $U_1 \geq \frac{1}{4}$

E. Arrival rate of poisson process - counting process rate λ arrivals are at the stationary and independent increments arrivals at time t are independent with arrival rate λt.
λ become function of time $\lambda(s)$ $\Rightarrow$ cafe - restaurant

F. Generate poisson arrivals at the fastest rate, but accept only a proportion ① let λ^* be max arrival rate and set $t = 0$. ② Generate $E \sim \exp(\lambda^*)$ and let $t = t + E$ ③ Generate U from $U(0,1)$ and accept E if $u \leq \dfrac{\lambda(t)}{\lambda^*}$ If reject $\Rightarrow$ return to ②

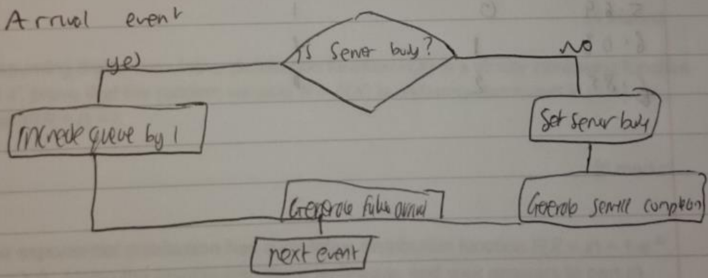
1/4/14 Mony Science Supplement Exam paper

Q6A Process interaction
Event Scheduling
Activity Scanning

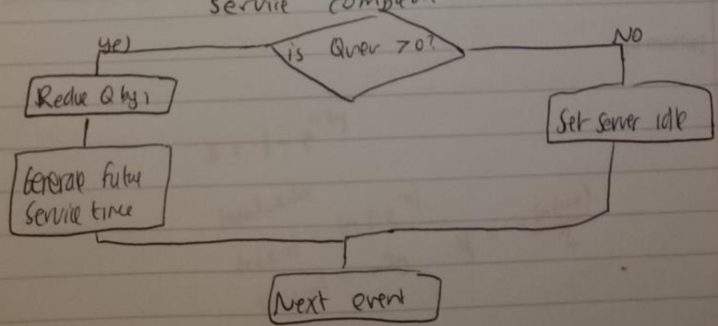
b i) S.S.V – collection of variables necessary to record state of system
SE. – instantaneous occurrences that change system state
FEL. – list of event notices ordered by time of occurrence

c M – exponential interarrival time
G – generic service time distribution
1 – number of servers
5 – system capacity
∞ – call population

d Arrival event



Service completion



6E. customer	inter-arrival	arrival-time	service-time	time serv begins	waiting in Q	Time serv ends	T in sys	idle time
1	0	0	0.81	0	0	0.81	0.81	0
2	1.63	1.63	1.60	1.63	0	3.23	1.6	0.82
3	2.01	3.64	1.05	3.64	0	~~1.05~~ 4.69	~~4.69~~ 1.05	0.41
4	2.05	5.69	2.41	5.69	0	8.1	2.41	1
5.	0.34	6.03	2.24	8.1	2.07	10.34	4.31	0
6.	0.82	6.85	?	10.34	3.49	?	?	0

Time	Q length	Server busy
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6.85	2	1