MA2E01: Problem Set 4

Due at the end of the tutorial, 22-24 October.

1. Using a double integral, find the volume under the surface

$$z = x^2 \cos y + x \sin 2y\,,$$

for the region given by $R = \{(x, y) : 0 \le x \le 2, 0 \le y \le \pi/2\}$.

2. Find the volume below the surface

$$z = e^{2x}\,,$$

above the region $R = \{(x, y) : 0 \le x \le 1, x^2 \le y \le x\}$. Sketch R.

3. Use a double integral to find the volume of the solid in the first octant[1] bounded by the surface

$$3x + y + 2z = 4\,,$$

which is a plane. Follow the steps outlined below:

(a) Sketch the solid and its projection onto the xy-plane.

(b) Use the projection to find the limits of integration.

(c) Perform the integration to find the volume.

4. Consider the solid above the z-axis bounded by the solid

$$x^2 + y^2 + z^2 = 16\,,$$

and outside the region $x^2 + y^2 = 4$.

[1]The first octant is the region $x, y, z \ge 0$.

(a) Identify the solid. **Hint:** Identify all the objects that bound it.

(b) Sketch the solid and its projection onto the xy-plane.

(c) Using polar coordinates, calculate the volume of the solid.

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Maths Tutorial

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1 $z = x^2\cos y + x\sin 2y$

$R = \{(x,y) : 0 \le x \le 2 \quad 0 \le y \le \frac{\pi}{2}$

$$\int_0^{\pi/2}\int_0^2 x^2\cos y + x\sin 2y \frac{dz}{dx}\frac{dz}{dy}$$

$$\int_0^{\pi/2}\left[\int_0^2 x^2\cos y + x\sin 2y \frac{dz}{dx}\right]\frac{dz}{dy}$$

$$\int_0^{\frac{\pi}{2}}\left[\frac{x^3}{3}\cos y + \frac{x^2}{2}\sin 2y \Big|_{x=0}^{2}\right]\frac{dz}{dy}$$

$$\int_0^{\frac{\pi}{2}}\frac{8}{3}\cos y + 2\sin 2y \frac{dz}{dy}$$

$$\frac{8}{3}\sin y \; \cancel{+} - 2(\cos 2y)\Big|_{y=0}^{y=\frac{\pi}{2}}$$

$$\frac{8}{3}\sin y - \cos(2y)\Big|_{y=0}^{y=\frac{\pi}{2}}$$

$$-\left((0 - 1) - \left(\frac{8}{3}(1) - (-1)\right)\right)$$

$$-1 - (\tfrac{8}{3}+1$$

$$-1 - \tfrac{11}{3}$$

$$= \cancel{} + \frac{14}{3} = \text{Volume}$$

how??

2

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2. $z = e^{2x}$

$R = \{(x,y) : 0 \le x \le 1 \quad x^2 \le y \le x$

$\frac{e^{2xy}}{2}$ $3ye^{2xy}$ $y \cdot e^{2x}$ $1e^{2x}$

$\int_0^1 \int_{x^2}^{x} e^{2x} \frac{dz}{dy} \frac{dz}{dx}$

$4xe^{2x^2}$

$= \cancel{e^{2xy} \Big|_{y=x^2}^{x}}$ ye^{2x} $\frac{4e^{2x^2}}{4}$

$\cancel{e^{2x x} - e^{2x x^2}}$

$= \cancel{\int_0^1 e^{2x^2} - e^{2x^3} \frac{dz}{dx}}$

$= \cancel{\frac{e^{2x^2}}{4} - \frac{e^{2x^3}}{6} \Big|_{x=0}^{x=1}}$

$\frac{e^2}{4} - \frac{e^2}{6}$

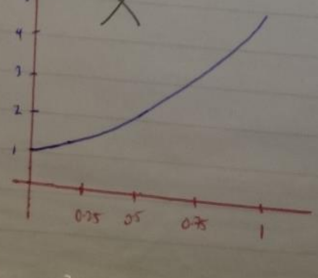
$= ye^{2x} \Big|_{y=x^2}^{x}$

$\int_0^1 xe^{2x} - x^2e^{2x} \frac{dz}{dx}$

$= -\frac{1}{2} e^{2x} (x-1)^2 \Big|_{x=0}^{1}$ how??? cant just work from above

$-\frac{1}{2}e^2 (1-0)^2 - \left(-\frac{1}{2}e^0 (0-1)^2\right)$

$-\frac{e^2}{2} + \frac{1}{2}$ = volume $\frac{1}{2}$



3.

Tom DAVID WEITBRECHT 12300044 Set 4 week 5.

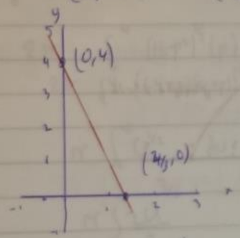
3. $3x + y + 2z = 4$ $\qquad$ $2z = 4 - 3x - y$

$z = 2 - \frac{3}{2}x - \frac{y}{2}$

$\cancel{x=0}$ $z = 0$ $\qquad$ $3x + y = 4$

$x=0 \quad y=4 \qquad y=0 \quad x = 4/3$

$(0,4) \qquad (3/4, 0)$



$3x + y = 4$

$y = 4 - 3x$

b limits $\qquad 0 \le x \le 4/3 \qquad 0 \le y \le 4 - 3x$

$$\int_0^{4/3} \left[\int_0^{4-3x} 2 - \frac{3}{2}x - \frac{y}{2} \, dz/dy \right] dz/dx$$

xy not x+y

$$= \int_0^{4/3} \left[2y - \frac{3}{2}(x+y) - y^2/4 \right] \Big|_{y=0}^{4-3x}$$

$$\left[8 - 6x - \frac{3}{2}(4 - 3x + x) - \frac{(4-3x)^2}{4} \right] - \left[0 - \frac{3}{2}x - 0 \right]$$

$$8 - 6x - \frac{3}{2}(4 - 2x) - \frac{(16 - 24x + 9x^2)}{4}$$

$$\cancel{8} - 6x - \cancel{6} + 3x - \cancel{4} - 6x + \frac{9}{4}x^2$$

$$\int_0^{4/3} -2 - 12x + \frac{9}{4}x^2 - \frac{3}{2}x \, dz/dx$$

$$\int_0^{4/3} -2 - \frac{21}{2}x + \frac{9}{4}x^2 \, dz/dx$$

$$-2x - \frac{21x^2}{4} + \frac{9x^3}{12} \Big|_{x=0}^{4/3}$$

$$-2(4/3) - \frac{21(4/3)^2}{4} + \frac{9(4/3)^3}{12} - 0$$

$$-8/3 - \frac{28}{3} + \frac{16}{9} = -\frac{92}{..} =$$

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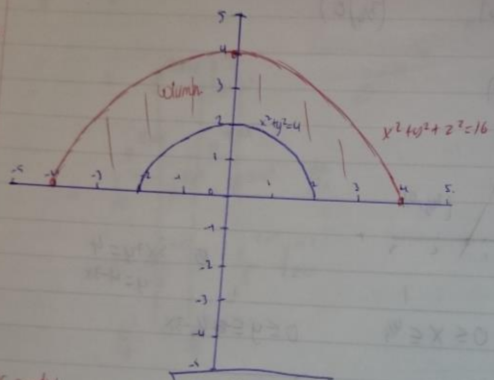
4a. $x^2+y^2+z^2=16$ $\quad z \geq 0 \quad z=\sqrt{16-x^2-y^2}$

outside $x^2+y^2=4$

Sphere with radius 4 centre (0,0)

x^2+y^2 is circle centre (0,0) radius 2

b)



c. Polar coordinates

$z=\sqrt{16-x^2-y^2}$ $\quad z \geq 0 \quad x=\rho\cos\theta \quad y=\rho\sin\theta$

$z=\sqrt{16-\rho^2\cos^2\theta-\rho^2\sin^2\theta}$

$=\sqrt{16-\rho^2}$ $\quad \rho \frac{dz}{d\rho} \frac{dz}{d\theta}$

$\int d\rho \int d\theta$ not $\frac{\partial z}{\partial\theta} \frac{\partial z}{\partial\rho}$

that's where J comes from.

$2\int_0^{2\pi}\int_2^4 (16-\rho^2)^{1/2} \, dz/d\rho \; dz/d\theta$

$u=16-\rho^2$

$\int_0^4 \frac{2(16-\rho^2)^{3/2}}{3}$

$\frac{du}{d\rho}=-2\rho$

$du=-2\rho\, d\rho$

$\frac{du}{-2\rho}=d\rho.\rho$

$2\left(\frac{-1}{2}\right)\int^{2\pi/2}\int_2^4 \sqrt{u}\, du\, d\theta$

$=\int_0^{\pi}\int_4^2 \frac{2u^{3/2}}{3}$

$\int_0^{\pi}\int_0^2 \frac{2(16-\rho^2)^{3/2}}{3}$ $\quad \frac{2(12)^{3/2}}{3}-\frac{2(0)^{3/2}}{3}$

$=27.71$

$=\frac{2(27.71)\pi}{3}$ = volume

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04/10/13

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4c. $z = \sqrt{16 - x^2 - y^2}$ $\quad x = p\cos\theta \quad y = p\sin\theta$

$z = \sqrt{16 - p^2}$

$\int_2^4 \int_0^{2\pi} (\sqrt{16-p^2})(p) \, \frac{dz}{d\theta} \frac{dz}{dp}$

$= 2\pi \int_2^4 (16-p^2)^{1/2} (p) \, \frac{dz}{dp}$ $\quad u = 16 - p^2$

$2\pi \int_2^4 ((4-p)(4+p))^{1/2} (p) \, dz/dp$ $\quad \frac{du}{dp} = -2p$ $\quad \frac{du}{-2} = dp.p$

$\pi \int_2^4 u^{1/2} \, du$

$\pi \int_2^4 \frac{2u^{3/2}}{3}$

$-\pi \left(\frac{2(16-4^2)^{3/2}}{3} - \left[\frac{2(16-2^2)^{3/2}}{3} \right] \right)$

$0 - \frac{2(12)^{3/2}}{3}$

$= \frac{2}{3}(12)^{3/2} \pi$

87.06236948 ✓ $\frac{3}{3}$

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JAKO WETTERBERG MATH TUTORIAL WEEK 1

4c. $z = \sqrt{16 - x^2 - y^2}$ $\quad x = p\cos\theta \quad y = p\sin\theta$

$z = \sqrt{16 - p^2}$

$\int_2^4 \int_0^{2\pi} (\sqrt{16-p^2})(p) \, \frac{dz}{d\theta} \frac{dz}{dp}$

$= 2\pi \int_2^4 (16-p^2)^{1/2} (p) \, \frac{dz}{dp}$ $\quad u = 16 - p^2$

$2\pi \int^4 ((4-p)(4+p))^{1/2} (p) \, dz/dp$ $\quad \frac{du}{dp} = -2p$

$\frac{du}{-2} = dp\,p$

$\pi \int_2^4 u^{1/2} \, du$

$\pi \int_2^4 \frac{2u^{3/2}}{3}$

$-\pi \left(\frac{2(16-4^2)^{3/2}}{3} - \left[\frac{2(16-2^2)^{3/2}}{3} \right] \right)$

$0 \quad - \frac{2(12)^{3/2}}{3}$

$= \frac{2}{3}(12)^{3/2}\pi$

87.06236448 ✓

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