Maths 2 - 2012

1a $y = \cos^{-1}x$

$\cos y = x$

$-\sin y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{-\sin y} \Rightarrow \frac{1}{-\sin(\cos^{-1}x)}$

$= \frac{-1}{\sqrt{\sin^2 y}}$ $\sqrt{\sin^2 y} = \sqrt{1-\cos^2 y}$ $= \sqrt{1-x^2}$

$\frac{-1}{\sqrt{1-x^2}}$

b i $\lim_{x\to\infty} \frac{\ln(x)}{x} = \frac{\infty}{\infty}$

L'Hospital rule $f(x) = \ln x$ $f'(x) = \frac{1}{x}$ $\frac{\frac{1}{x}}{1} = \frac{1}{x}$

$g(x) = x$ $g'x = 1$

$\lim_{x\to\infty} \frac{1}{x} = \frac{1}{\infty} = 0$ ✓

ii $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{e^x-1}\right)$ $\frac{1}{0} - \frac{1}{0} = \infty - \infty$

RE combine $\frac{e^x-1-x}{xe^x-x}$ $f(x) = e^x-1-x$ $f'x = e^x-1$

$g(x) = xe^x - x$ $g'x = x(e^x)+(e^x-1)$

$\underset{x\to 0}{} = \frac{1-1}{0+1-1} = \frac{0}{0}$

$f(x) = e^x-1$ $f'(x) = e^{x}$ $x\to 0 = \frac{1}{1+1} = \frac{1}{2}$ ✓

$g(x) = xe^x + e^x - 1$ $g'(x) = xe^x + e^x + e^x$

$\frac{e^x}{2e^x\ xe^x}$

iii. $\lim_{x \to \pi^-} x \sin\left(\frac{\pi}{x}\right)$

$\frac{\sin\left(\frac{\pi}{x}\right)}{1/x}$ $f'(x) = \frac{\pi\left(\frac{-1}{x^2}\right)\cos\frac{\pi}{x}}{-1/x^2}$

$\pi\cos\frac{\pi}{x}$

$x \to \pi$ $\pi\cos\frac{\pi}{\pi} = 1\pi = 3.14$

c $\sinh x$ = odd func $\frac{e^x - e^{-x}}{2}$ $\cosh(u) = \frac{e^x + e^{-x}}{2}$

$d\sinh x = \frac{1}{2}\left(e^x - (-1e^{-x})\right)$

$= \frac{1}{2}\left(e^x + e^{-x}\right.$

$= \frac{e^x + e^{-x}}{2} = \cosh$

A ~~hyperbolic function is any~~ function of a
It is any group of function of an angle expressed as a relationship between the distance of a point on a hyperbola to the origin and to the coordinate axes

2 ai $\int x^2 \ln(x)\,dx$ $u = \ln x$ $du = \frac{1}{x}dx$

$\int \frac{x^2 u}{\frac{1}{x}}$

$= \int x^3 u \cancel{du}$ $\cancel{u = x^3}$ $dv = u\,du$

$du \cancel{= 3x^2 dx}$ $v = 1$

2012 maths 2

2ai $\int \underset{x^2 \ln(x)}{u\,dv} = uv - \int v\,du$

$u = \ln(x)$ $du = \frac{1}{x}dx$ $dv = x^2dx$ $v = \frac{x^3}{3}$ $du = 2x\,dx$

$\ln(x)\left(\frac{x^3}{3}\right) - \int \frac{x^3}{3}\,\frac{1}{x}dx$

$\frac{x^3\ln(x)}{3} - \frac{1}{3}\frac{x^3}{3}$

$\frac{x^3\ln(x)}{3} - \frac{x^3}{4}$

$\frac{x^3}{3}\left(\ln(x) - \frac{1}{3}\right)$

$u = x^2$ $dv = \ln x\,dx$
$du = 2x\,dx$ $v = \frac{1}{x}$

$= x^2\left(\frac{1}{x}\right) - \int \frac{2x}{x}dx$

$x - 2\int dx$

$x - 2x$

$= -x + C$

ii $\int_0^{\pi} 3x^2\cos\left(\frac{x}{2}\right)$

$\cos u$ $u = \frac{1}{2}x$ $du = \frac{1}{2}$

$u = 3x^2$ $dv = \cos\left(\frac{x}{2}\right)dx$

$du = 6x\,dx$ $v = -2\sin\left(\frac{x}{2}\right)$

$3x^2\left(-2\sin\left(\frac{x}{2}\right)\right) - \int -2\sin\left(\frac{x}{2}\right)6x$

$3x^2\left(-2\sin\left(\frac{x}{2}\right)\right) + 12\int \sin\left(\frac{x}{2}\right)x$

12

$\sin\left(\frac{x}{2}\right)x$

$u = x$ $dv = \sin\left(\frac{x}{2}\right)dx$

$du = dx$ $v = 2\cos\left(\frac{x}{2}\right)$

$2x\cos\left(\frac{x}{2}\right) - 2\int\cos\left(\frac{x}{2}\right)$

$3x^2\left(-2\sin\left(\frac{x}{2}\right)\right) + 12\left(2x\cos\left(\frac{x}{2}\right) - 4\sin\frac{x}{2}\right)$

$2\sin\left(\frac{x}{2}\right)$

2aii

$3\int x^2\cos(\frac{x}{2})$

$u = x^2$ $\quad dv = \cos(\frac{x}{2})dx$

$du = 2x\,dx$ $\quad v = \int \cos(\frac{x}{2})dx$

$= 2\sin(\frac{x}{2})$

$3\left[x^2(2\sin\frac{x}{2}) - \int 2\sin(\frac{x}{2})2x\,dx\right]$

$3\left[2x^2\sin\frac{x}{2} - 4\int \sin(\frac{x}{2})x\right]$

$u = x$ $\quad dv = \sin(\frac{x}{2})dx$

$du = dx$ $\quad v = -2\cos(\frac{x}{2})$

$-2x\cos(\frac{x}{2}) - 2\int\cos\frac{x}{2}dx$

$\left[6x^2\sin\frac{x}{2} - 48\sin(\frac{x}{2}) + 24x\cos\frac{x}{2} \cancel{-2x\cos(\frac{x}{2})}\right]_0^{\pi} -$

$6\pi^2 \quad -48 \quad + \cancel{24\pi}$

$= 6\pi^2 - 48.$

Maths 2 2012

2aiii $\int \frac{x^3+2x^2+1}{(x+3)^2(x^2+1)}$

$$\frac{A}{(x+3)} + \frac{B}{(x+3)^2} + \frac{Cx+D}{x^2+1}$$

$$\frac{A(x+3)(x+3)(x^2+1) + B(x+3)(x^2+1) + (Cx+D)(x+3)(x+3)(x+3)}{(x+3)(x+3)(x+3)(x^2+1)}$$

$A(x^3+x \quad +3x^2+3) + B(x^2+1) + (x+D\ (x^2+3x+3x+9)$

$Ax^3 + 3Ax^2\ Ax+3A + Bx^2+B + Cx^3 + 6Cx^2 + 9Cx$
$+ Dx^2 + 6Dx + 9D$

$x^3[A+C] + x^2[3A+B+6C+D] + x[A \quad +9C+6D] + [3A+B+9D]$

$A+C=1$
$A=1-C$

$3A+B+6C+D=2$
$B+3c+D \ -1$

$A+9C+6D=0$
$8c+6D=-1$

$3A+B+9D=1$
$B-3c+9D=-2$

$-B+3c+D=-1$
$B-3c+9D=-2$

$2B+10D=-3$
8

$-6c+8D-1 \quad \times 8$
$8c+6D=-1 \quad \times 6$

$-48c+48D=-8$
$48c+48D=-6$

$96D=-14$
$D=\frac{-7}{48}$

2b $\frac{dy}{dx}+4y=x^2e^{-4}$

$\int 4dx$
$=4x$

$e^{4x}\frac{dy}{dx}+4y(e^{4x}) = x^2e^{-4}e^{4}$

$\int e^{4x}dy \quad = \int x^2$

$e^{4x}y = \frac{x^3}{3}$

$y = \frac{x^3}{e^{4x}} + ce^{-4x}$ ✓

3(i) Integral test

Mam 2012

$$\int_1^\infty \frac{x^2}{2x^2+3x} \qquad \frac{x^2}{x(2x+3)}$$

$$\frac{a}{2x+3} + \frac{b}{x} \qquad \int_1^\infty \frac{x}{2x+3} < \frac{1}{5} + \int_1^\infty \frac{x}{2x+3}$$

$u = 2x+3$

$du = 2dx$

~~$2bx + 3b + ax$~~

$$= \frac{1}{4}\left(2x - 3\log(2x+3) + 3\right) + c$$

$$1 = \frac{1}{4}\left(2 - 3\log 5 + 3\right) \qquad \infty \text{ diverge}$$

$$\frac{1}{4}(5 - 3\log 5)$$

~~0.72 converge~~

b $\sum_{n=0}^{\infty} \frac{8}{9^n}$ $a = 8$ $r = \frac{1}{9}$ $\frac{ar}{1-r}$ $\frac{8/9}{1-1/9}$ $\frac{8/9}{8/9} = 1$

$$\frac{8}{1-1/9} = 9 \quad \frac{8}{8/9} \; 9$$

$$S = \frac{1}{9}\left(8 + \frac{8}{9} + \frac{1}{9^2}\right)$$

$$S = \frac{1}{9}(8+S)$$

$$9S = 8 + S$$

$$8S = 8$$

$$S = 1$$

Maths 2 2012

2id $\int \frac{1}{u} + \int \frac{1}{x^2+2x+3}$

$\ln u + \int \frac{1}{(x+1)^2+2} dx$

$\ln |x^2+2x+3| + \int \frac{1}{u^2+3}$

$= \ln |x^2+2x+3| + \tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right) + C$

2ie $\int_0^1 x^{-\frac{1}{2}}$ improper integral

$\lim_{e \to 0^+} \int_e^1 \frac{1}{\sqrt{x}} dx = \lim_{e \to 0^+} (2\sqrt{x})_e^1$

evaluate integral

$\lim_{e \to 0^+} (2 - 2\sqrt{e}) = 2$

2ii a $x \frac{dy}{dx} = y^2$

$\frac{1}{y^2} \frac{dy}{dx} = \frac{1}{x}$

$\int \frac{1}{y^2} dy = \ln x + C$

$-\frac{1}{y} = \ln x + C$

$y = \frac{1}{\ln x + C}$

b $\frac{dy}{dx} + 3y = e^{-3x}$ $\qquad \int 3\,dx = 3x = I(t)$

$e^{3x}\frac{dy}{dx} + e^{3x}3y = e^{-3x}e^{3x}$

$\frac{d(e^{3x}y)}{dx} = 1$

$e^{3x}y = x + C$

2012

Q3A i) $\sum_{n=1}^{\infty} \frac{n^2}{(2n+1)!}$

Ratio test $a_n = \frac{n^2}{(2n+1)!}$ $a_{n+1} = \frac{(n+1)^2}{(2n+3)!}$

$\frac{a_{n+1}}{a_n} = \frac{(n+1)^2}{(2n+3)!} \cdot \frac{(2n+1)!}{n^2}$ $= \frac{n+1}{2n^2 2n+3}$

$\frac{(n+1)^2 \; (2n+1)!}{(\cancel{2n+1})(2n+2)(2n+3)\, n^2}$ $\frac{(n^2+2n+1)/(2n+1)!}{\cancel{(2n+3)!\;(n^2)}}$ $\frac{1}{2} \; \frac{n+1}{n^2(2n+3)}$

$= 0$

$\frac{(n+1)^2}{(2n+2)(2n+3)\, n^2}$ $n \to \infty$ $\cancel{\frac{\frac{n^2}{n^2} + \frac{2n}{n^2} + \frac{1}{n^2} + \frac{2n}{n^2} + \frac{1}{n^2}}{\left(\frac{2n}{n^2} + \frac{3}{n^2}\right) \; \frac{n^2}{n}}}$ $= 1$ cannot say

$\frac{n^2}{n^4}$ $n \to \infty = 0$ converge < 1

ii $\sum_{n=1}^{\infty}$ $\frac{n^2}{2n^2+3n} \cdot \frac{2(n+1)^2+3(n+1)}{(n+1)^2}$

$\cancel{n^2(2n^{-2}+3n^{-1})}$

$\frac{n}{2n^2+3n}$ $\frac{2(n+1)+3}{1}$

$\frac{2(n+1)+3}{2n+3}$ $\frac{2n+5}{2n+3}$

$n \to \infty$ $= \frac{2}{2} = 1$

$\frac{n^2}{2n^2+3n} \cdot \frac{2(n+1)^2+3(n+1)}{(n+1)^2}$

$\frac{n^2}{2n^2+3n}$ $\frac{2n^2+2n+2+3n+3}{n^2+2n+2}$ $\frac{2n^4+2n^3+2n^2+3n^3+3n^2}{2n^4+4n^3+4n^2+3n^3+6n^2+6n}$

$= \frac{2n^4+5n^3+5n^2}{2n^4+7n^3+10n^2+6n}$ $\frac{2n^3+5n^2+5n}{2n^3+7n^2+10n+6}$

$\frac{2}{2} = 1$

Maths 2 2012

Q3c i. $\sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$

Integral ω $\int_1^{\infty} \frac{1}{x^{5/2}}$

$\int_1^{\infty} x^{-5/2}$

$\cancel{\frac{-5}{2} x^{-3/2}}$

$\frac{-2}{3x^{3/2}}$

$\frac{-2}{3(\infty)^{3/2}} + \frac{2}{3(1)} = \frac{2}{3} < 1$ converg

0

$= \int_2^{\infty} \frac{-4}{2 x^{3/2}}$

$\frac{-5}{2} \quad \frac{1}{\infty} = $ bond

$= 1 + \frac{2}{3} = \frac{5}{3}$

$\frac{-5}{2} \quad -0 \quad \frac{-5}{2}$ converg

bound = $a_n \leq a_1 + \int_1^{\infty} f(x)dx$

$1 + \frac{-5}{2} = \frac{-3}{2}$

$a_n \leq \frac{-3}{2}$

$u = n+1 \quad du = dx$

ii $\sum_{n=1}^{\infty} \frac{1}{(n+1)^2}$

$\int_2^{\infty} \frac{1}{u^2} du$

diverged

$-1u^{-1} = \frac{-1}{u}$

$\left[\frac{-1}{2}\right] - \left[\frac{-1}{\infty}\right]$

$= +\frac{1}{2}$ converg

$a_n \leq a_1 + (-1)$

$a_n \quad \frac{1}{2} + \frac{1}{4}$

$= \quad a_n \leq +\frac{3}{4}$

2012 2 Mam

a

$3x - 3y + 3x = 7$

$92x - y = 3$

$73x - 5y - z = 7$

$$R \left(\begin{array}{ccc|c} 3 & -3 & 3 & 7 \\ 92 & -1 & 0 & 3 \\ 73 & -5 & -1 & 7 \end{array}\right)$$

$R \div 3$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & \frac{7}{3} \\ -92 & +1 & 0 & -3 \\ -73 & +5 & 1 & 7 \end{array}\right) \quad R+92$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & \frac{7}{3} \\ 0 & +93 & 0 & 89 \\ 0 & +78 & +74 & 80 \end{array}\right) \quad \begin{array}{c} \\ \div 93 \\ -78 \end{array}$$

$$\begin{array}{c} x \\ y \\ z \end{array} \left(\begin{array}{ccc|c} 1 & -1 & 1 & 7/3 \\ 0 & 1 & 0 & 89/93 \\ 0 & 0 & -4 & 2 \end{array}\right)$$

$$\begin{array}{c} x \\ y \\ z \end{array} \left|\begin{array}{ccc|c} 1 & -1 & 1 & 7/3 \\ 0 & 1 & 0 & 89/93 \\ 0 & 0 & 1 & -2/4 \end{array}\right.$$

$z = -2/4$

$y = 89/93$

$x - 89/98 + -2/4 = 7/3$

$x = \frac{550}{147}$ one solun

4A. $\begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & 2 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 3 & 1 & 2 & 0 & 0 & 1 \end{array}\right)$$

$R_2 - 2R_1$ $$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -2 & 0 \\ 3 & 1 & 2 & 0 & 0 & 1 \end{array}\right)$$

$R_3 - 3R_1$ $$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & -2 & 0 \\ 0 & -2 & -1 & 0 & -3 & 1 \end{array}\right)$$

$R_1 + R_2$ $$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & -1 & 0 \\ 0 & -1 & 1 & 1 & -2 & 0 \\ 0 & -2 & -1 & 0 & -3 & 1 \end{array}\right)$$

$R_3 - 2R_2$ $$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & -1 & 0 \\ 0 & -1 & 1 & 1 & -2 & 0 \\ 0 & 0 & -3 & -2 & 1 & 1 \end{array}\right)$$

$R_2(-1)$ and $R_3(-\frac{1}{3})$ $$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & -1 & 0 \\ 0 & 1 & -1 & -1 & 2 & 0 \\ 0 & 0 & 1 & 2/3 & -1/3 & -1/3 \end{array}\right)$$

$R_1 - 2R_3$ $$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/3 & -1/3 & 2/3 \\ 0 & 1 & -1 & -1 & 2 & 0 \\ 0 & 0 & 1 & 2/3 & -1/3 & -1/3 \end{array}\right)$$

$R_2 + R_3$ $$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/3 & -1/3 & 2/3 \\ 0 & 1 & 0 & -1/3 & 5/3 & -1/3 \\ 0 & 0 & 1 & 2/3 & -1/3 & 1/3 \end{array}\right)$$

4B

4b $\left(\begin{array}{ccc|c} -1/3 & -1/3 & 2/3 & 1 \\ -1/3 & 5/3 & -1/3 & 2 \\ 2/3 & -1/3 & -1/3 & 3 \end{array}\right)$

$$\begin{array}{l} x = \\ y = \\ z = \end{array} \begin{pmatrix} -1/3 - 1/3 + 2/3 \\ -2/3 + 10/3 - 2/3 \\ 6/3 - 3/3 - 3/3 \end{pmatrix} \begin{array}{c} 0 \\ 6/3 \\ 0 \end{array}$$

$$\begin{pmatrix} -1/3 & -2/3 & +6/3 \\ -2/3 & +10/3 & -3/3 \\ 2/3 & -2/3 & -3/3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \begin{array}{l} = x \\ = y \\ = z \end{array}$$

c. $BA = I$

Let $Ax = 0 \Rightarrow BAx = 0$ $x = 0$ hence A^{-1} exist

Then $BAA^{-1} = I A^{-1} \rightarrow B = A^{-1} \rightarrow AB = I$

d. If A is an nxn matrix then A is invertible if equal to

i $Ax = 0$ $x = 0$

ii. A can be row reduced to I

iii. A can be written as product of elementary matrices

iv $Ax = b$ has a solution x for all b

v $Ax = b$ has unique solution x for all b

vi $\det A \neq 0$

4b $Ax = B$

Inverse of a matrix is another matrix A^{-1} such that $A^{-1}A = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

determinant of A is only for square matrix

$\det(A)$ = amount A stretched a vector

$\det(A) < 1$ = shrinks vector

$\det(A) > 1$ = lengthens vector

$\det(A) = 1$ only rotated

$m \times n$ square matrix

m = row n = column

$m = n = 2$

Input and out have the same dimension (number of components)

c $\det(A) \neq 0$

$Ax = 0$ has only one solution $x = 0$

$Ax = B$ has only one solution

columns of A are linear independent

rows of A are linearly independent (rows aren't multiples of each other)

A is a product of elementary matrices

2ai. $\int \overset{u}{x^2} \overset{dv}{\ln(x)} dx$ $\quad u = x^2$ $\quad dv = \ln(x)dx$

$v = \int dv = \int \ln(x)dx = \frac{1}{x}$

$\int u\,dv = uv - \int v\,du$ $\quad \frac{du}{dx} = 2x$

$du = 2x\,dx$ $\quad v = \frac{1}{x}$

$x^2\left(\frac{1}{x}\right) - \int \frac{2x}{x} dx$

$x - \int 2dx\,54$

$= 4x$

ii. $\int \overset{u}{3x^2} \overset{dv}{\cos\left(\frac{x}{2}\right)} dr$

$u = 3x^2$ $\quad dv = \cos\left(\frac{x}{2}\right)dx$

$\frac{du}{dx} = 6x$ $\quad v = \int dv = \int \cos\left(\frac{x}{2}\right)dx$

$= +2\sin\left(\frac{x}{2}\right)$

$du = \frac{6x}{dx}$ $\quad v = +2\sin\left(\frac{x}{2}\right)$

$\int u\,dv = uv - \int v\,du$

$\int 3x^2 \cos\left(\frac{x}{2}\right)dx = 3x^2\left(2\sin\frac{x}{2}\right) - \int 2\sin\left(\frac{x}{2}\right)\frac{6x}{dx}dx$

$6x^2 \sin\frac{x}{2} - \cancel{\int 12x \sin\left(\frac{x}{2}\right)} - 48\sin\left(\frac{x}{2}\right) - 24x\cos\left(\frac{x}{2}\right)$

$6\left((x^2-8)\sin\left(\frac{x}{2}\right)\right) - 24x\cos\left(\frac{x}{2}\right)\Big|_0^{\pi}$

$6\left((\pi^2-8)\sin\frac{\pi}{2}\right) - 24\pi\cos\frac{\pi}{2} - [0 - 0\ 0]$

$6\pi^2\frac{\sqrt{2}}{2} - 48\frac{\sqrt{2}}{2} - 24\pi\frac{\sqrt{2}}{2}$

$3\sqrt{2}\pi^2 - 24\sqrt{2} - 12\pi\sqrt{2}$

Maths 2 2a2

2 aii) $\int \frac{x^3+2x^2+1}{(x+3)^2(x^2+1)}$

$\frac{a}{(x+3)} + \frac{b}{(x+3)^2} + \frac{cx+d}{x^2+1} = x^3+2x^2+1$

$a(x^2+6x+9)(x^2+1) + b(x+3)(x^2+1) + c(x+3)(x+3)(x+3)$

$\cancel{ax^4+ax^2} \cancel{a(x+3)(x^2+1)+}$

$a x^3 \cancel{+ax+3ax^2+3a+}$

$a(x+3)(x^2+1) + b(x^2+1) + c^{x+d}(x+3)^2$

$(ax+3x)(x^2+1) + bx^2+b + c(x^2+6x+9)$

$\cancel{ax^3+ax+3x^3+3x+bx^2+b+cx^2+6cx+9c}$

$ax^3+3x^3 + bx^2+cx^2 + ax+3x+6cx + b+9c$

$(a+3)x^3 + (b+c)x^2 + (a+3+6c)x + (b+9c) = x^3+2x^2+0x+1$

$a+3=1 \quad b+c=2 \quad a+3+6c=0 \quad b+9c=1$

$a=-2 \quad b=13/6 \quad -2+3+6c=0 \quad -b+c=2$

$c=\frac{-1}{6} \quad 8c=-1$

$ax^3+ax+3x^3+3x+bx^2+b+cx^3+6cx^2+9cx \quad c=\frac{-1}{8}$

$+dx^2+6dx+9d$

$ax^3+3x^3+cx^3+bx^2+6cx^2+dx^2+ax+3x+9cx+6dx+b+9d$

$(a+3+c)x^3 + (b+6c+d)x^2 + (a+3+9c+6d)x + (b+9d)$

$a+3+c=1 \quad b+6c+d=2 \quad a+3+9c+6d=0 \quad b+9d=1$

$b+6c+d=2 \quad a+3+9c+6d=0$

$b+9d=1$

(5) $6c-8d=1$

$c=1-3-a$

$c=-2-a \rightarrow a+-18c-18a+6d=-3$

$-17a+8c+6d=-3$

Maths 2 2012

$y = \cos^{-1} x \qquad x = \cos y$

$\cos x = y \qquad \frac{dy}{dx} = \sin y \frac{dy}{dx}$

1 a $\frac{dy}{dx} = \frac{d \cos^{-1}(x)}{d \cos(x)} \cdot \frac{d\cos(x)}{dx}$

$-\cos(x) \quad (-\sin x)$

$\frac{1}{\sin y} = \frac{dy}{dx} = \frac{1}{\sin(\cos^{-1}x)}$

bi $\lim_{x \to \infty} \frac{\ln x}{x} \quad \frac{\ln(\infty)}{(\infty)} = \frac{\infty}{\infty} \qquad f(x) = \ln x \quad f'x = \frac{1}{x}$

$g(x) = x \quad g'x = 1$

$\frac{1/x}{1/1} = \frac{1}{x} \quad \lim_{x \to \infty} = \frac{1}{\infty} = 0$

ii $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right) \quad \frac{1}{0} - \frac{1}{0} \quad \infty - \infty$

$\frac{e^x - 1 - x}{x(e^x - 1)} \qquad f'x = \frac{e^x - 1}{x(e^x) + (e^x - 1)} \underset{\to 0}{x} \frac{1-1}{0+1-1} = \frac{0}{0}$

$f(x) = e^x - 1$

$g(x) = xe^x + e^x - 1$

$\frac{e^x}{xe^x + e^x + e^x} \quad \frac{1}{1+1} = \frac{1}{2}$

iii $\lim_{x \to \pi^-} \quad x \sin\left(\frac{\pi}{x}\right) \qquad \frac{\sin\left(\frac{\pi}{x}\right)}{1/x}$

$\pi \sin\left(\frac{\pi}{\pi}\right) \quad \pi \sin(1)$

x^{-1}

$\frac{\cos\left(\frac{\pi}{x}\right)(\pi) \frac{-1}{x^2}}{-1/x^2}$

$\cos\left(\frac{\pi}{x}\right)(\pi) = 1$

C. $\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$

$\frac{d \sinh x}{dx} = \frac{1}{2} \frac{d(e^x - e^{-x})}{dx} = \frac{1}{2} \; e^x - (-1)e^{-x}$

$= \frac{e^x + e^{-x}}{2} = \cosh x.$

2012 Maths 2

Q2A (i) $\int x^2 \ln(x) dx$

$u = \ln(x)$

$\frac{du}{dx} = \frac{1}{x}$ $du = \frac{1}{x} dx$

$\int \frac{x^2}{1/x} u \, du$

$dv = x^2$

$v = \frac{x^3}{3}$

$\int x^3 u \, du$

$\int u dv = uv - \int v du$

$\ln(x) \frac{x^3}{3} - \int \frac{x^3}{3} \frac{1}{x} dx$

$\frac{1}{3} \int x^2$

$\frac{\ln(x) x^3}{3} - \frac{1}{3}\left(\frac{x^3}{3}\right)$

$= \frac{\ln(x) x^3}{3} - \frac{3x^3}{9}$

(ii) $\int u dv = uv - \int v du$

$u = 3x^2$

$du = 6x dx$

$dv = \cos\left(\frac{x}{2}\right) dx$

$v = \int \cos\left(\frac{x}{2}\right)$

$\frac{\sin\left(\frac{x}{2}\right)}{2}$

$u = x$

$du = dx$

$dv = \sin\left(\frac{x}{2}\right)$

$v = \frac{-\cos\left(\frac{x}{2}\right)}{2}$

$3x^2 \left(\frac{\sin\left(\frac{x}{2}\right)}{2}\right) - \int \frac{\sin\left(\frac{x}{2}\right)}{2} 6x \, dx$

$- 3 \int x \sin\left(\frac{x}{2}\right)$

$-\frac{x \cos\left(\frac{x}{2}\right)}{2} - \frac{1}{2}\left(\frac{-\sin\left(\frac{x}{2}\right)}{2}\right)$

$3x^2 \left(\frac{\sin\left(\frac{x}{2}\right)}{2}\right) - 3\left(\frac{-x \cos\left(\frac{x}{2}\right)}{2} + \frac{1}{4}\sin\left(\frac{x}{2}\right)\right)$

$3\pi^2 \left(\frac{1}{2}\right) - 3\left(0 + \frac{1}{4}\right)$

$\frac{3\pi^2}{2} - \frac{3}{4} - \left[+3\left(\frac{+\pi}{2}\right)\right.$

$= \frac{3\pi^2}{2} - \frac{3}{4} - \frac{3\pi}{2}$

iii. $\dfrac{x^3+2x^2+1}{(x+3)^2(x^2+1)}$

$$\frac{ax+b}{(x^2+1)} + \frac{c}{(x+3)} + \frac{d}{(x+3)^2}$$

$\cancel{ax+b(x+3)^2(x^2+1)+c}$

$$\frac{ax+b\cancel{(x+3)}(x+3)^2 + c(x^2+1)\overset{(x+3)}{\cancel{(x+3)}^2} + d\cancel{(x+3)}(x^2+1)}{(x^2+1)(x+3)^2}$$

$ax+b(x^2+6x+9) + \cancel{c(x^2+1)}\ 1$
$+c(x^3+3x^2+x+3) + d(x^2+1)$

$[ax^3+6ax^2+9ax+bx^2+6bx+9b] + cx^3+3cx^2+cx+3c+dx^2+d$

$x^3[a+c] + x^2[6a+b+3c+d] + x[9a+6b+c] + [9b+3c+d]$

$a+c=1$ $\quad 6a+b+3c+d=2 \quad 9a+6b+c=0 \quad 9b+3c+d=0$

$6a+b+3c+d=2 \quad \times 4$
$9a+6b+c=6 \quad \times 6$

$54a+9b+27c+9d=18$
$-54a+36b+6c=36$

(5) $-27b+21c+9d=18$
$-81b+27c+9d=0$
(6) $9b+3c+d=0 \quad \times 4$

$-108b-6c=18$
(7) $-108b-6c=-18$

$9(1-c)+6b+c=0$
$9-9c+6b+c$
$6b-8c=-9 \quad 6$

(7) $-108b-6c=-18 \quad 8.$

$36b-48c=-54$
$+864b-48c=-144$

$824b=90$

Maths 2 2012

2b): $\frac{dy}{dx} + 4y = x^2 e^{-4x}$

i. $p(t) = 4$

$\int pt = 4x$

ii. Integrating factor $I(x) = 4x$

iii. multiply across by $e^{i(x)}$ e^{4x}

$\rightarrow e^{4x}\frac{dy}{dx} + 4y\, e^{4x}_{y} = x^2 \cancel{e^{-4x}}\, \cancel{e^{4x}}$

$= x^2$

$\int \frac{d\, e^{4x} y}{dt} = \int x^2$

$e^{4x} y = \frac{x^3}{3} + c$

$y = \frac{\frac{x^3}{3} + c}{e^{4x}}$

$\frac{x^3 e^{-4x}}{3} + ce^{-4x}$