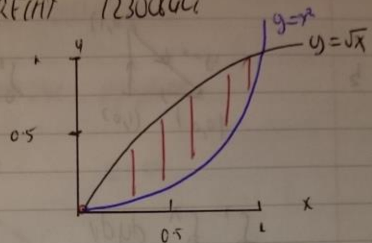
Maths Tutorial   DAVID WEITBRECHT   12300864



1a

$y = x^2$	$x = y^2$
$x = \sqrt{y}$	$y = \sqrt{x}$
$y = x^2$
$(0,0)$	$(0,0)$
$(1,1)$	$(1,1)$

limits are $(0,0)$ and $(1,1)$

$\frac{5.5}{10}$

b

$\frac{d(x^2y)}{dx} = 2xy$

$f(x,y) = (x^2y)$ $g(x,y) = (y + xy^2)$

$\frac{d(x^2y)}{dx} = 2xy$ $\frac{d}{dy}(y+xy^2) = 1 + 2xy$

$= \oint_C x^2y\,dx + (y + xy^2)\,dy$

$= \iint_R \left[\frac{d}{dx}(g) - \frac{df}{dy}\right] dA$

$= \int_0^1 \int_{0\,x^2}^{\sqrt{x}} y^2 - x^2 \quad dy\,dx$

$= \int_0^1 \left(xy^2 - \frac{x^3}{3}\right)\Big|_{x=0}^{1} \,dy$

$= \int_0^1 y^2 - \frac{1}{3} \,dy$

$= \frac{y^3}{3} - \frac{1}{3}y \quad dy$

$\frac{1}{3} - \frac{1}{3} = 0$

$\int_0^1 \frac{y^3}{3} - yx^2 \Big|_{y=0}^{y=\sqrt{x}}$ x^2

$\int_0^1 \frac{(\sqrt{x})^3}{3} - \sqrt{x}(x^2)$

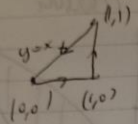
$\int_0^1 \frac{1}{3}x^{3/2} - x^{5/2} \,dx$

$\left[\frac{1}{3}\frac{2}{5}x^{5/2} - \frac{2}{7}x^{7/2}\right]_{x=0}^{1}$

$\frac{2}{15} - \frac{2}{7} = \frac{16}{105}$

$\frac{1.5}{2}$

b



$\int_0^1 \int_0^x dy\,dx.$

$\frac{x^2}{2}$

$\int_0^1 \int_0^x dy\,dx$
$= \int_0^1 y\big|_0^x \,dx$

$\int_0^1 x\,dx$
$= \int_0^1 \frac{x^2}{2}\big|_{x=0}^1$

$= \frac{1^2}{2} = \frac{1}{2}$ = area

$\frac{1}{2}$ supposed to use Greens theorem and evaluate using line integrals, not use standard

$\iint dA$ = Area

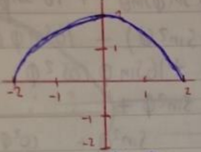
26/11/13

DAVID WEMBRECHT 12300644 MA113 set 8

3a $F(x,y,z) = xi + yj + 2z k$

Surface σ portion of xy plane above xy-plane

$x^2 + y^2 = 4$ circle. radius 2 $z = 0$



$$n = \frac{\frac{dr}{du} \times \frac{dr}{dv}}{\left\| \frac{dr}{du} \times \frac{dr}{dv} \right\|}$$

this describes a sphere not a parabola.

3b We need derivates

$$r(\theta,\phi) = 2\sin\phi\cos\theta i + 2\sin\phi\sin\theta j + 2\cos\phi, \ k$$

$$r_\phi = 2\cos\phi\cos\theta i + 2\cos\phi\sin\theta j - 2\sin\phi k$$

$$r_\theta = -2\sin\phi\sin\theta i + 2\sin\phi\cos\theta j$$

$$r_\theta \times r_\phi = \begin{vmatrix} i & j & k \\ 2\cos\phi\cos\theta & 2\cos\phi\sin\theta & -2\sin\phi \\ -2\sin\phi\sin\theta & 2\sin\phi\cos\theta & 0 \end{vmatrix}$$

$$= 4\sin^2\phi\cos\theta i + 4\sin^2\phi\sin\theta j + 4\sin\phi\cos\phi k$$

$$\sqrt{(4\sin^2\phi\cos\theta)^2 + (4\sin^2\phi\sin\theta) + 4\sin\phi\cos\phi}$$

$$\sqrt{16\sin^4\phi\cos^2\theta + 16\sin^4\phi\sin^2\theta + 16\sin^2\phi\cos^2\phi}$$

$$\sqrt{16\sin^4\phi(\cos^2\theta + \sin^2\theta) + 16\sin^2\phi\cos^2\phi}$$

$$\sqrt{16\sin^4\phi + 16\sin^2\phi\cos^2\phi}$$

$$\sqrt{16\sin^2\phi(\sin^2\phi + \cos^2\phi)}$$

$$\sqrt{16\sin^2\phi(1)}$$

$$= 4\sin\phi$$

$$\frac{4\sin^2\phi\cos\theta i}{4\sin\phi} + \frac{4\sin^2\phi\sin\theta}{4\sin\phi} + \frac{4\sin\phi\cos\phi}{4\sin\phi}$$

$$= \sin\phi\cos\theta i + \sin\phi\sin\theta j + \cos\theta k$$

D.W.

Flux $\bar{\Phi} = \iint F \cdot \left(\frac{dr}{d\phi} \times \frac{dr}{d\theta}\right) dA$

$\cancel{2}$ $2\sin\phi\cos\theta i + 2\sin\phi\sin\theta + 4\cos\phi k \cdot 4\sin^2\phi\cos\theta i + 4\sin^2\phi\sin^2\theta + 4\sin\phi\cos\phi$

$8\sin^3\phi\cos^2\theta + 8\sin^3\phi\sin\theta^2 + 16\sin\phi\cos^2\phi$

$8\sin^3\phi(\cos^2\theta + \sin^2\theta) + 16\sin\phi\cos^2\phi$

$8\sin^3\phi + 16\sin\phi\cos^2\phi$

$\cancel{8 \quad \sin^2\phi +}$

$\cancel{\frac{1}{2}\sin\phi} \quad \sin^2\phi + \cos^2\phi$

$8\sin^3\phi + 16\sin\phi\cos^2\phi$

$\int_0^{2\pi}\int_0^{\pi/2} 8\sin^3\phi + 16\sin\phi\cos^2\phi \, d\phi \, d\theta$

$-\frac{2}{3}$

$u = \cos\phi \quad du = -\sin(\phi)dx$

$8\int \sin^3(\phi) \quad + 16\int \sin\phi\cos^2\phi$

$8\int \sin^3\phi \quad + 16\int u^2 du$

$= 8\int \sin^3(x)dx \quad - \frac{16u^3}{3}$

$\int_0^{2\pi} \frac{8}{3}\cos^3\phi - 8\cos(\phi) - \frac{16\cos\phi^3}{3} \Big|_{\phi=0}^{\phi=\pi/2} d\theta$

$\left(\frac{8}{3} - 8 - \frac{16}{3}\right) - (0)$

$\int_0^{2\pi} +\frac{32}{3} \, d\theta$

$= \frac{32}{3}\pi$ = Flux of F across σ

right idea, wrong ~~parameter~~ normal vector.

also wrong integration region,

we are integrating over the surface of the circle, not the surface of a sphere.

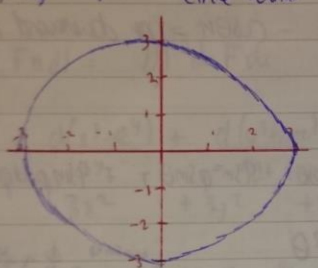
1.5/3

DAVID WEITBRECHT Set 8 12300644

4a $F(x,y,z) = yj + k.$

σ $z = x^2 + y^2$ below $z = 9$ oriented down

$x^2 + y^2 = 9$ circle radius 3



b Downward normal vector = $\dfrac{\frac{dr}{du} \times \frac{dr}{dv}}{\left\| \frac{dr}{du} \times \frac{dr}{dv} \right\|}$ see problems as question 3.

write in cylindrical terms $r(\theta,\phi) = 3\sin\phi\cos\theta i + 3\sin\phi\sin\theta + 3\cos\phi$

$r_\phi = 3\cos\phi\cos\theta i + 3\cos\phi\sin\theta j - 3\sin\phi k$

$r_\theta = -3\sin\phi\sin\theta i + 3\sin\phi\cos\theta j.$

$$r_\theta \times r_\phi = \begin{vmatrix} i & j & k \\ 3\cos\phi\cos\theta & 3\cos\phi\sin\theta & -3\sin\phi \\ -3\sin\phi\sin\theta & 3\sin\phi\cos\theta & 0 \end{vmatrix}$$

$= 9\sin^2\phi\cos\theta i + 9\sin^2\phi\sin\theta j + 9\sin\phi\cos\phi k$

magnitude = $\sqrt{(9\sin^2\phi\cos\theta)^2 + (9\sin^2\phi\sin\theta)^2 + (9\sin\phi\cos\phi)^2}$

$\sqrt{81\sin^4\phi\cos^2\theta + 81\sin^4\phi\sin^2\theta + 81\sin^2\phi\cos^2\phi}$

$\sqrt{81\sin^4\phi(\cos^2\theta + \sin^2\theta) + 81\sin^2\phi\cos^2\phi}$

$\sqrt{81\sin^4\phi(1) + 81\sin^2\phi\cos^2\phi}$

$\sqrt{81\sin^2\phi(\sin^2\phi + \cos^2\phi)}$

$\sqrt{81\sin^2\phi(1)}$

$= 9\sin\phi$

$\frac{9\sin^2\varphi\cos\theta i}{9\sin\varphi} + \frac{9\sin^2\varphi\sin\theta j}{9\sin\varphi} + \frac{9\sin\varphi\cos\varphi k}{9\sin\varphi}$

$\sin\varphi\cos\theta i + \sin\varphi\sin\theta j + \cos\theta k =$

$-\sin\varphi\cos\theta i - \sin\varphi\sin\theta j - \cos\theta k =$ unit downward normal vector

c $Flux = \iint F\cdot\left(\frac{dr}{d\varphi}\times\frac{dr}{d\theta}\right)$

$3\sin\varphi\sin\theta \cdot 9\sin^2\varphi\cos\theta i + 9\sin^2\varphi\sin\theta j + 9\sin\varphi\cos\varphi k$

$3\sin$

$27\sin^3\varphi\sin^2\theta$ $u = \sin^3\varphi$ $\frac{du}{d\theta} = 3\sin\varphi\sin\theta$

$\int_0^{2\pi}\int_0^{\pi} 27\sin^3\varphi\sin^2\theta \, d\varphi \, d\theta$

$27\sin^2\theta\int_0^{2\pi}\int_0^{\pi}\sin^3\varphi \, d\varphi \, d\theta$

$27\sin^2\theta\int_0^{2\pi}\frac{1}{12}(\cos(3\varphi)-9\cos(\varphi))\Big|_0^{\pi} d\theta$

$\frac{1}{12}(1-9)$

$\frac{-8}{12}(27)\int_0^{2\pi}\sin^2\theta \, d\theta$

$18\left[\frac{1}{2}(x) - \frac{1}{4}\sin 2x\right]_0^{2\pi}$

$18\left[\pi - \frac{1}{4}(0)\right]$

$= 18\pi$ = flux of F across S

1.5

3