DECISION ANALYSIS

Action you can take - small med large

State of nature - what competitor can do small med larg → 9 possibilities

Decision table

Actions	States of nature: small	medium	large	Security level
small	12000	10000	4000	max 4000
medium	10000	12000	2000	2000
large	18000	12000	0	0
best consequence	18000	12000	4000	

utilities → numbers

← consequences of utilities

Decision Criteria

1. Wald's maximum - "better safe than sorry" - Run small campaign - action - with security level - 4000

2. Hurwitz optimism-pessimism index $\alpha \in [0,1]$, $\alpha = \frac{1}{2}$

optimism-pess index $= \alpha \times \text{sec.Lev} + (1-\alpha) \times \text{Opt level}$

Action large campaign with opt-pess interval = 9000

$\alpha = 1$ = pessimistic
$\alpha = 0$ optomistic

	Optimism level	Optimism/Pessimism index $\alpha = \frac{1}{2}$	Average
small	12000	8000	8667
med	12000	7000	8000
larg	18000	9000 max	10000

3. Laplace's principle of insufficient reason - Action: Large with an average value of 10000

4. Savage's minimax regret

Tables of regrets:

	Small	medium	large	worse regret
Small	6000	2000	0	6000
Medium	8000	0	2000	8000
Large	0	0	4000	4000

Action - Large

regret = best consequence for state of nature - utility

810
170

	very good	good	average	bad	very bad	Security lvl	opt lvl	opt pol index
voluntary redundancy	300	200	140	110	100	100	300	240
early retirement	200	180	160	140	120	120	200	176
Hiring freeze	150	150	150	150	150	150	150	150

	Average
voluntary	170
early re	160
hirin	150

Walds maximum → select = 150 = hiring freeze = Action

Hurwitz → $\alpha = 0.3$ 90 30 210 36 160 45 105

Action = voluntary redundancy = 240

Lapalaces principle of Insufficient reason Action = voluntary redundancy = 170

Savage ~~minimum regret~~ minimax regret

	v good	good	average	bad	v. bad	worst reget
voluntary	0 in good	0	20	40	50	50 minimum
early retire	100	20	0	10	30	100
hiring	150	50	10	0	0	150
	300	200	160	150	150	

Action voluntary - best consequence for state of nature (50)

9/10/12 DECISION ANALYSIS II
TUTORIAL ① DAVID WEITBRECHT 12300644

states of nature

ACTIONS: Company	worse	no change	better	Security level	Optimum level	Opto-pessim index	Av. avg	
Amalgamated:	-3m	2	8	-3	8	2·5	7/3	2·3
Allied	-1	4	8	-1	8	3·5	11/3	3·6
National	-3	1	12	-3	12	4·5	10/3	3·3
	~~-7~~ -1 ~~-8~~	~~7~~ 4	~~28~~ 12					

Decision Criteria:

1 Wald's maximin criterion = Choose Allied max security level of -1m

2 Hurwicz's optimism-pessimism index = $\alpha \times$ security level $+ (1-\alpha) \times$ optimal level

Answer choose National with op/pes/index of +4·5M

3 Savage's minimax regret criterion

	worse	no change	better	worst regret	
Amalgamated	-2	2	4	0	-2
Allied	-0	0	4		0
National	-4 -2	3	0	0	-2

~~Amalgamated or National~~ Allied

4 Laplace's principle of insufficient reason Action = Allied = 11/3

9/10/12. Management Science

1. coin & die

P(head and 6)

P(head) x P(6 | head)

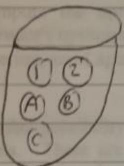
independent event!

P(head) x P(6)

$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

2 no replacement

2 choice ball not returned



P(number and letter)

NOT INDEPENDANT

= p (number) x P(~~number~~ letter | number)

$\frac{2}{5} \times \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$

P(letter and number) = p(letter) x P(number | letter)

$= \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$

$P(x \text{ and } y) = P(x) \times P(y|x) = P(x) \times P(x|y)$

15/10/12 P(either number then letter) or (letter and the number)

= P(number and letter) + p(letter and number) − p(number and letter and letter and number)

3/10 + 3/10 − [=0]

= 6/10 = 3/5

Sample Space

(1, 2)	(2, 1)	(A, 1)	(B, 1)	(C, 1)
(1, A)	(2, A)	(A, 2)	(B, 2)	(C, 2)
(1, B)	(2, B)	(A, B)	(B, A)	(C, A)
(1, C)	(2, C)	(A, C)	(B, C)	(C, B)

TUTORIAL 2 DECISION ANALYSIS

		state of nature							
	Scheme:	v.bad	bad	average	good	v.good	Security level	optimism lev	opt/pes index
	voluntary red	100	110	140	200	300	100	300	240	170
action	early retirement	120	140	160	180	200	120	200	176	160
	hiring freeze	150	150	150	150	150	150	150	150	150
		150	150	160	200	300				

1. Walds' maximum criterion = 150 = hiring freeze 150 security level

2. Hurwicz's optimism/pessim index = $\alpha \times$ security level + $(1+\alpha) \times$ optimism level
choose voluntary redundancy optimism pessim index of 24.

3. Savages minimax regret criterion =

	v.bad	bad	averag	good	v.good	worst regret
voluntary	50	40	10	0	0	50
early	30	10	0	20	100	100
freeze	0	0	10	50	150	150

Action - choose voluntary redundancy = worst regret of 50

4. Laplace's principle of insufficient reason
Action = voluntary redundancy = 170 average

DECISION ANALYSIS II
TUTORIAL 3

DAVID WEITBRECHT 12300644

manufacturing site:	favourable	unfavourable	neither	Security level	optimism level	opt-pes index	Average
China	5	5.5	6	~~5~~ 6	5	5.5	5.5
Indonesia	4.5	7	4.8	~~4.5~~ 7	4.5	5.75	5.43
Morocco	4	6	5.5	4 6	4	5	5.16
	4	5.5	4.8				

1. Walds "minimax" = Action = ~~Morocco~~ indonesia Security level of 4million china or morocco

2. Hurwic's optimism pessimism index = $\alpha \times$ sec level $+ (1-\alpha) \times$ opt level
= Moroco = 5M

3. Savage minimax regret

	State of nature			
	fav	unfavour	neither	worst regret.
china	1	0	1.2	1.2
indonesia	0.5	1.5	0	1.5
Moroco	0	0.5	0.7	0.7

Action = Morocco with 0.7

4. Laplace's principle of insufficient reason
Choose Moroco with average of 5.16

22/10/12 Decision Analysis 4 - Value of Information

Action	S	M	L
S	12000	10000	~~4000~~ 4000
M	1000	12000	~~20000~~ 2000
L	18000	17000	0
Probability	0.5	0.3	0.2

states of nature — competitors

E.V. Small = 12000(0.5) + 1000(0.3) + 4000(0.2) = ~~[illegible]~~ 9800

E.V. Medium 1000(0.5) + 12000(0.3) + 2000(0.2) = 9000

E.V. Large = 12600 optimal decision

Optimal decision is to run a large campaign EV(decision) = 12600

EV given Perfect Information (PI) = 18000 x 0.5 + 12000 x 0.3 + 4000 x 0.2
= 13400

EV of PI = EV given PI - EV given no info
= 13400 - 12600
= 800 pay no more than that for inside info

Property decision

action	economy G	B
apartment	50000	30000
office	10000	-40000
Warehouse	30000	10000
	0.6	0.4

EV apartment - 50000 x 0.6 + 30000 x 0.4 = 42000

EV. office - 100000 x 0.6 + (-40000) 0.4 = 44000

EV. Warehouse - 30000 x 0.6 + 10000 x 0.4 = 22000

Optimal decision = 44000 => office

EV. given P.I 100000 x 0.6 + 30000 x 0.4
60000 + 12000 = 72000

TUTORIAL 2: PROBABILITY THEORY

DAVID WEISBRECHT 1720064

1 P(B|A) = 0·1 × 0·04 = 0·004

P(B not faulty / A lost) 0·1 × 0·96 = 0·096

$\frac{P(A) + P(B|A)}{P(B)}$

2a P(not defective / passed test) = 0·95 × 0·97 or 0·95 × 0·05

0·9215 ~~0·3395~~ + 0·0475

= ~~0·387 = 38·7% chance~~ 0·909 92·15%

b P(defective | passed test) 0·97 × 0·05 or 0·05 × 0·05

0·0485 + ~~0·0025~~

= ~~0·051 = 5·1% chance~~ 4·85%

3 accident prone ·200 — accident 0·3, no accident 0·7

not prone ·800 — acc 0·1, no acc 0·9

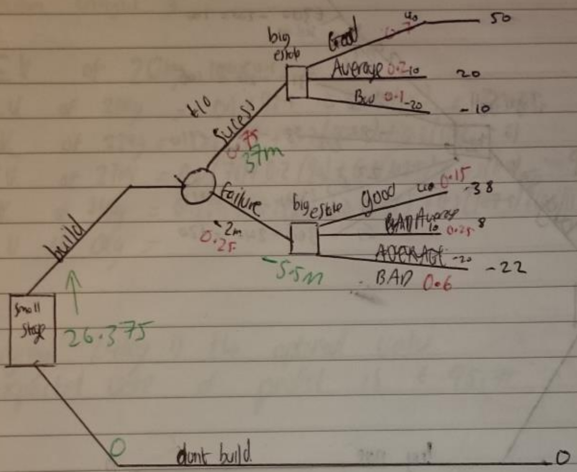
Probability of accident = 0·2 × 0·3 + 0·8 × 0·1

0·06 + 0·08 = 0·14 = 14% chance

P(no accident / not accident prone) 0·8 · 0·9 = 0·72 = 72% chance

Decision analysis DAVID WEITBRECHT 12300644

1.



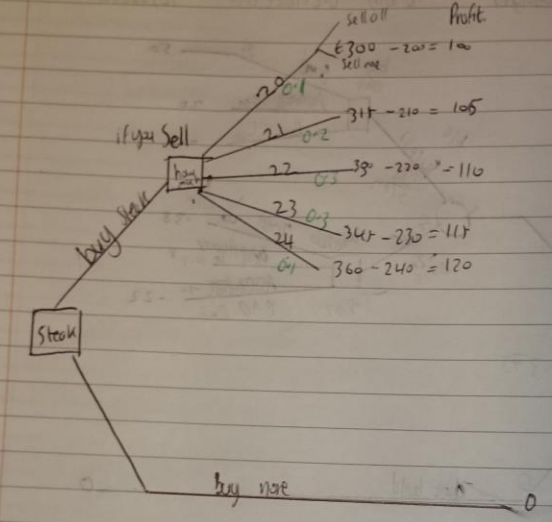
E.V. of (big estate | Pilot Success) = 50×0.7 + 20×0.2 + (−20) 0.1
35 + 4 + (−2) = 37 million

E.V. of (big estate | pilot failure) 38×0.15 + 0.25(8) + 0.6(−22)
5.7 + 2 −13.2 = −5.5 million

E.V. build Small Stage = 0.25(−5.5m) + 0.75(37)
−1.375 + 27.75 = 26.375 million

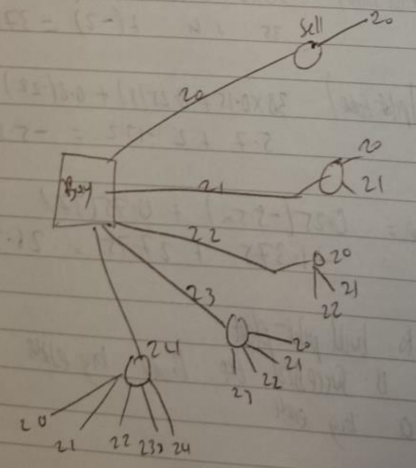
Optimal decision is to build pilot plant
If Small Stage is successful then Build big estate
else dont build a big estate

E.V. of Optimal decision = 37 million



Sell all
Profit.
£300 - 200 = 100
Sell none
20
0.1
21
0.2
315 - 210 = 105
if you Sell
22
0.3
330 - 220 = 110
23
0.3
345 - 230 = 115
24
0.1
360 - 240 = 120
buy stall
Stock
buy none
0

E.V.



Sell
20
20
Buy
21
20
21
22
20
21
22
23
20
21
22
21
24
20
21
22
23
24

22/10/ DAVID WELTBRECHT 12300644

Decision analysis 3

E.V of 20kg = 100×0.1 = 10

E.V of 21kg = 0.1×92.5 + 0.2(105) = 30.25.

E.V. of 22kg = 0.1(85) + 0.2(97.5) + 0.3(110) = 61

E.V. of 23kg = 0.1(76) + 0.2(90) + 0.3(102.5) + 0.3(115) = 90.85

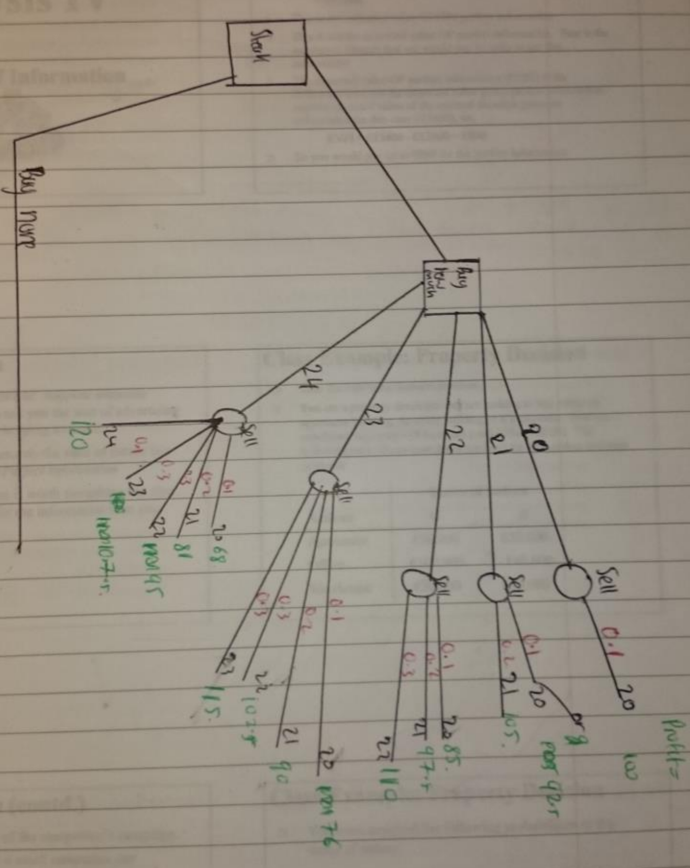
E.V. of 24kg = 0.1(68) + 0.2(81) + 0.3(95) + 0.3(107.5) + (0.1)(120) = 95.75.

E.V of 0kg = 0

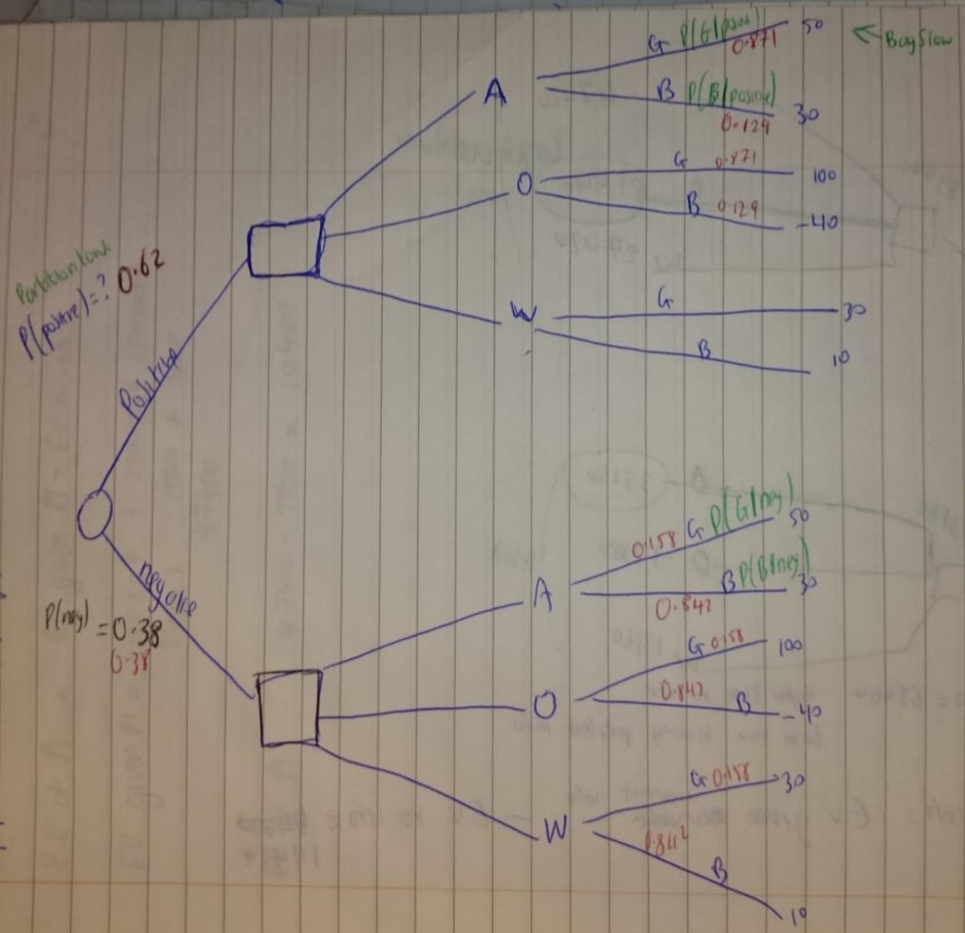
Buying 24kg is the optimal value

Expected value of profit is £ 95.75

Decision Analysis III   12300646   DAVID WEITBRECHT



30/10/12 decision Analysis IV



Given

① $P(g) = 0.6$ $P(B) = 0.4$

② $P(\text{positive}|G) = 0.4$

③ $P(\text{negative}|G) = 0.1$

$P(\text{Positive}|B) = 0.2$

$P(\text{negative}|B) = 0.8$

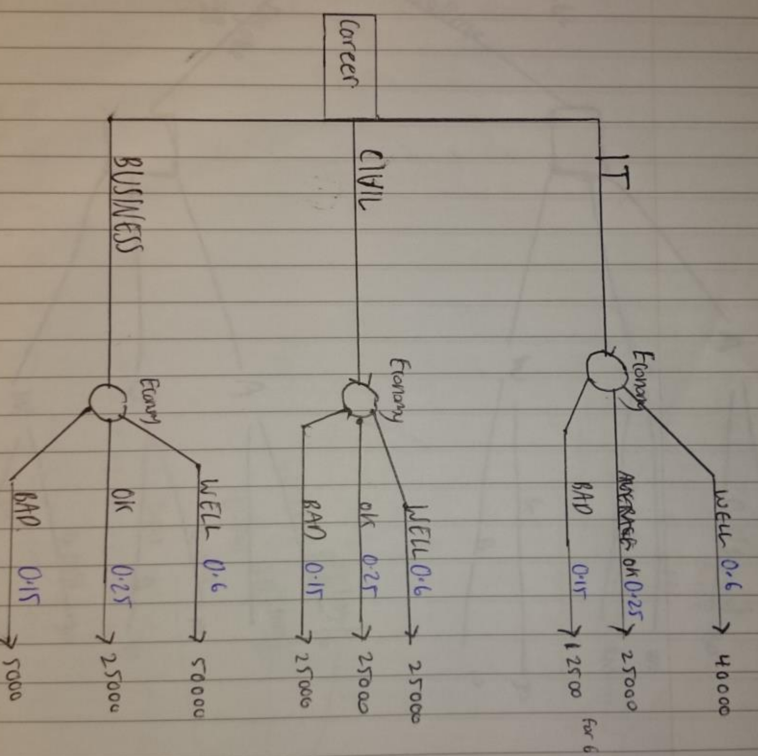
Wrong

EV given imperfect info =

EV of imperfect info = ?

30/10/12

DECISION ANALYSIS IV 12300644 DAVID WEITBRECHT.



E.V of IT = 40000X0.6 + 25000X0.25 + 12500X0.15 = 32125

E.V of CIVIL = 25000x0.6 + 25000x0.25 + 25000x0.15 = 25000

E.V of BUS = 50000x0.6 + 25000x0.25 + 5000x0.15 = 37000

Optimal action - Go for Setting up your own business.

EVPI = Economy goes well = Set up own business with 50000 economy

Decision Analysis IV 12300[illegible] DAVID WEITBRECHT

1. E.V of PI = EV given PI - EV no info

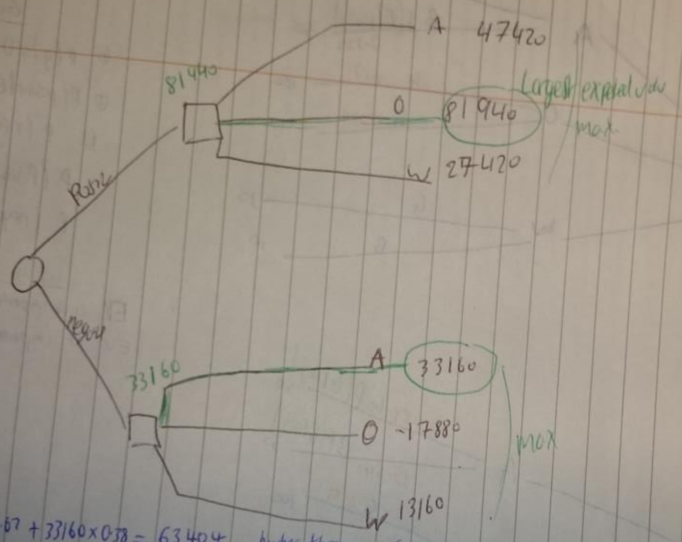
EV given PI = 4000 x 0.6 + 25000 x 0.6 + [illegible]

2400 + 15000 + 30000

47400

EV of PI = 47400 - 37000 = 10400

EV given imperfect information =



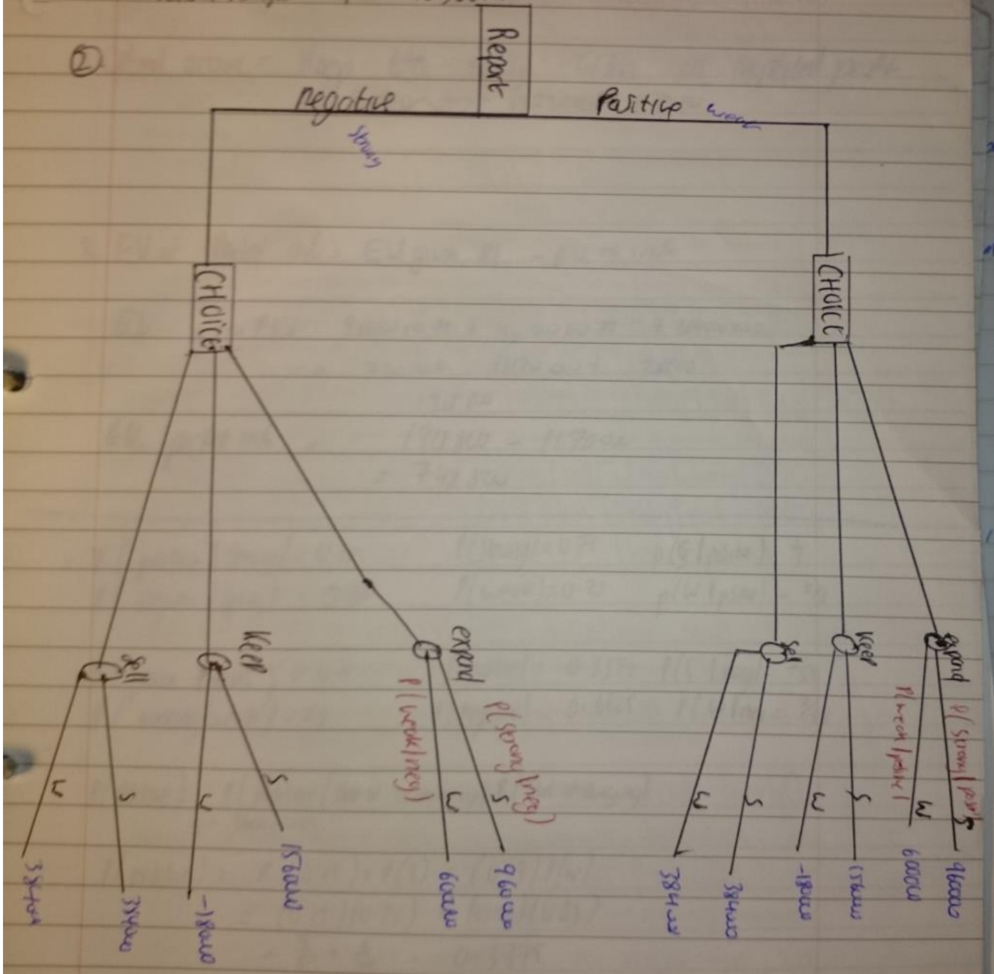
P(pos) = 0.62

$81940 \times 0.62 + 33160 \times 0.38 = 63404$ higher than no info
lower than having perfect info

E.V. of imperfect info = E.V. given imperfect info − E.V. no info = ~~44000~~ 19404

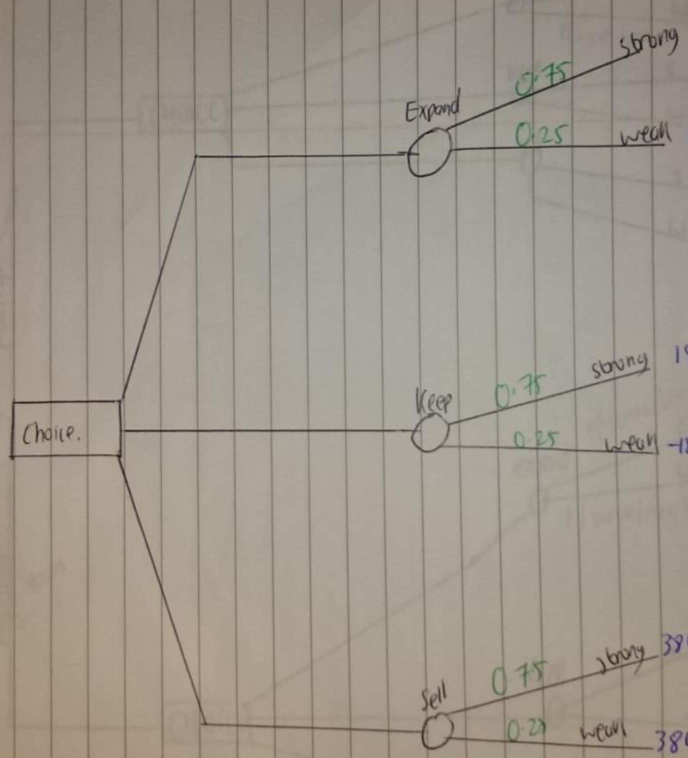
Decision Analysis 4 12300644

②



30/10/12 Decision Analysis IV 12300644 DAVIDWEITBRECHT

(2)



960000 x 0.75 = 720000
600000 x 0.25 = 150000
EV expand - 870000

1560000 x 0.75 = 1170000
-180000 x 0.25 = -45000
EV. Keep - 1125000

384000 x 0.75 = 288000
384000 x 0.25 = 96000
EV. Sell - 384000

3

30/10/12. Decision Analysis IV 12300644 DAVID WEITBRECHT

1. Optimal action = Keep the factory with an expected profit of £ ~~1170000~~ ~~1125000~~ 1170000

2. EV of Perfect info = EV given PI − EV no info

EV given PI = 960000 × 0.75 + 1560000 × 0.75 + 384000 × 0.75
= 720000 + 1170000 + 28800
= 1918800

EV perfect info = 1918800 − 1170000
= 748800

---

i. P(positive | Strong) = 0.15   P(Strong) = 0.75   P(S | positive) = $\frac{1}{3}$
P(negative | Strong) = 0.85   P(weak) = 0.25   P(W | positive) = $\frac{2}{3}$

P(positive | weak) = 0.9   P(positive) = 0.3375   P(S | neg) = $\frac{51}{53}$
P(~~bore~~neg | weak) = 0.1   P(negative) = 0.6625   P(W | neg) = $\frac{2}{53}$

P(positive) = P(positive | State of economy = y) P(State of economy = y)
y = Strong, weak

P(positive) = P(0.15) × P(S) + (0.9) P(W)
= (0.15)(0.75) + (0.9)(0.25)
= $\frac{9}{80} + \frac{9}{40}$ = 0.3375.

Bays law   P(x and y) = P(x) P(y|x) = P(y) P(x|y)

$$P(\text{Strong}|\text{positive}) = \frac{P(\text{Strong})\,P(\text{positive}|\text{Strong})}{P(\text{positive})}$$

$$\frac{0.75(0.15)}{0.3375} = \frac{1}{3}$$

P(S | negative)

$P(\text{S} | \text{negative}) = \dfrac{P(S)\,P(\text{neg} | \text{strong})}{P(\text{negative})}$

strong / neg

$P(Y|X) = \dfrac{P(Y)\,P(X|Y)}{P(X)}$

$\dfrac{P(\text{strong})\,P(\text{neg}|\text{str})}{P(\text{neg})}$

$\dfrac{0.75 \times 0.85}{0.6625}$

$\dfrac{0.75\,(0.85)}{0.6625} = \dfrac{51}{53}$

EV. (expand | positive) = $\frac{1}{3}(960000) + \frac{2}{3}(600000)$ = 320000 + 400000 = 720000
EV. (keep | positive) = $\frac{1}{3}(1560000) + \frac{2}{3}(-110000)$ = 400000
EV. (sell | positive) $\frac{1}{3}(384000) + \frac{2}{3}(384000)$ = 384000

EV. (expand | positive) = $\frac{51}{53}(960000) + \frac{2}{53}(600000)$ = 946415
EV. (keep | positive) $\frac{51}{53}(1560000) + \frac{2}{53}(-180000)$ = 1433207
EV. (sell | positive) $\frac{51}{53}(384000) + \frac{51}{53}(384000)$ = 384000

EV given imperfect info = 720000 × 0.3375 + 1433207 × 0.6625
949500 + 243000 = 1192500
EV given no information = 1170000

Optimal decision given imperfect information:
- If consultant say economy will be positive expand.
- If consultant say economy will be negative keep.

EV given imperfect info = ~~information~~ 1192500.
EV given no info = 1170000

EV of imperfect info = EV given imperfect − EV given no info
1192500 − 1170000
= 22500

I would not pay 50 000 − information only worth 22500

6

12300644 Decision Analysis V.

① Ⓐ $U(x) = 1 - e^{-0.0004x}$

$x = 40000 \quad = 1 - e^{-4} \quad = 0.98168$

$= 25000 \quad = 1 - e^{-2.5} \quad = 0.91792$

$= 12500 \quad = 1 - e^{-1.25} \quad = 0.71349$

$= 50000 \quad = 1 - e^{-5} \quad = 0.99326$

$= 5000 \quad = 1 - e^{-0.5} \quad = 0.39346$

EU(IT) = $0.6(0.98168) + (0.25)(0.91792) + (0.15)(0.71349) = 0.925545$

EU(civil) = $0.6(0.91792) + (0.25)(0.91792) + (0.15)(0.91792) = 0.91792$

EU(Business) = $0.6(0.99326) + (0.25)(0.91792) + (0.15)(0.39346) = 0.884453$

Optimal decision is IT sector

Has expected utility of 0.925545

Ⓑ $U(ex) = 1-e^{0.0001x}$

$e^{0.0001x} - 1$

$x = 0 \quad = 0$

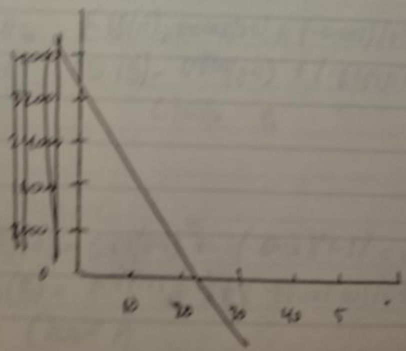
$x = 8000 = 1.2255$

$x = 16000 = 3.953$

$x = 24000 = 10.023$

$x = 32000 = 23.532$

$x = 40000 = 53.599$

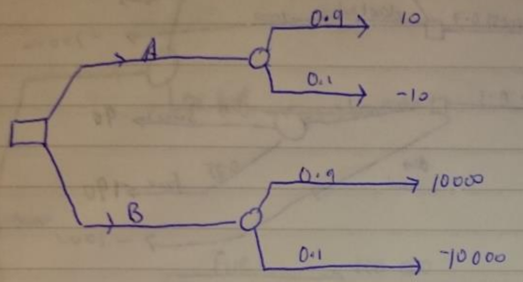


11/1c 2. Marginski

risk adverse utility

Utility function: $U(x) = 1-e^{-rx}$ 9013414

$r = \frac{1}{2500} = 0.0004$



Expected Utility

1 $U(10) = 1-e^{-0.0004(10)}$ $E.U(A) = 0.004(0.9) + (-0.004)(0.1) = \underline{0.0032}$

$1-e^{-0.004} = 0.004$ $E.U(B) = (0.9817)(0.9) + (-53.60)(0.1) = -4.4763$

$U(-10) = -0.004$

$U(10000) = 1-e^{-0.0004(10000)} = 0.9817$

$U(-10000)$ $1-e^{4}$ -53.60 Maximise the expected Utility

choose A

2. $r = \frac{1}{5000} = 0.0002$ $U(x) = 1-e^{-rx}$

$U(10) = 1-e^{-0.0002(10)} = 0.002$ $E.U(A) = 0.0002(0.9) + (-0.002)(0.1) = 0.0016$

$U(-10)$ $1-e^{0.002} = -0.002$ $EU(B) = 0.864(0.9) + (-6.389)(0.1) = \underline{0.1387}$

$U(10000) = 1-e = 0.864$ Choose B

$U(-10000) = -6.389$

3. $r = \frac{1}{1000} = 0.001$ $EU(A) = 0.01(0.9) + (-0.01)(0.1) = 0.008$

$U(10) = 0.01$ $EU(B) = 0.99995(0.9) + (-22025.07)(0.1) = -2201.6$

$U(-10) = -0.01$ Choose A

$U(10000) = 0.99995$

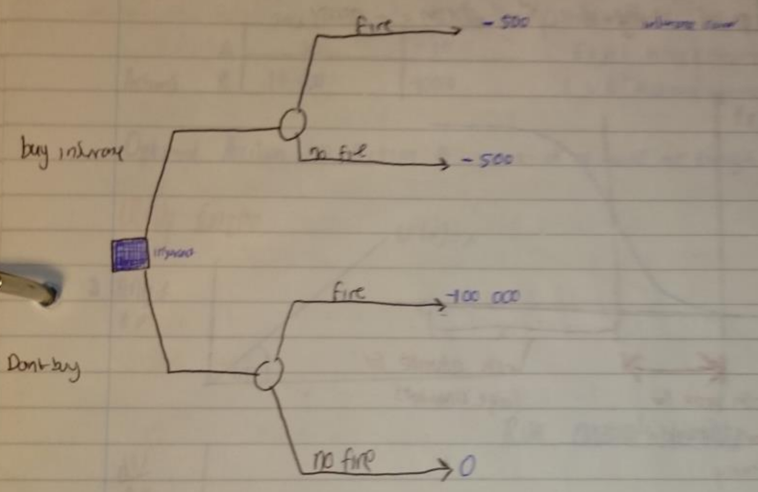
$U(-10000) = -22025.5$

large value of r - the larger our risk adversense.

(17)

13/11/12 Mang Science Decision Analysis V

Why we buy insurance



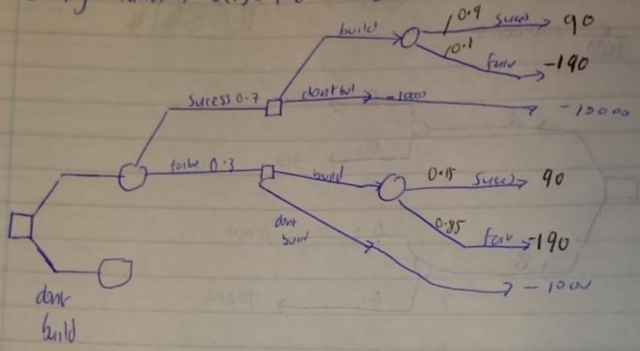
$P(\text{Fire}) = \frac{1}{1000} = 0.0001$
$p(\text{no fire}) = 0.9999$

Risk neutral (45° line)
$U(x) = x$
E.V. Buy = $-500 \times 0.0001 + (-500)(0.9999) = -500$
E.V. Dont buy = $-100000(0.0001) + (0)(0.9999) = -10$

Optimal decision is 'dont buy', Expected value = -10

Risk Adverse
$U(x) = 1 - e^{-rx}$ where $r = \frac{1}{10000}$
$U(-500) = -0.051$ $\quad E(U)_{\text{Buy}} = (-0.051)(0.0001) + (-0.051)(0.9999) = -0.051$
$U(-100000) = -22025$ $\quad E(U)(\text{not buy}) = (-22025)(0.0001) + (0)(0.9999) = -2.2025$
$U(0) = 0$ $\quad$ chose action with largest utility (Buy)

Utility function $U(x) = 1 - e^{-rx}$ where $r = 0.000\ 000\ 05$



1

12/11/12 Week 8 Decision Analysis 5.

A Decision problem. (1)

		SN1 (0.9)	SN2 (0.1)
	A	10	- 10
Actions.	B	10,000	-10000

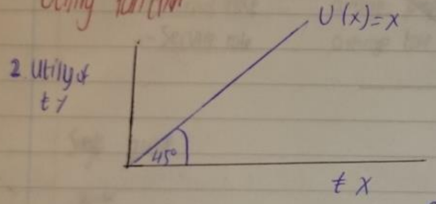
States of nature.

E.V.(A) 10×0.9 + (-10)0.1 = 8

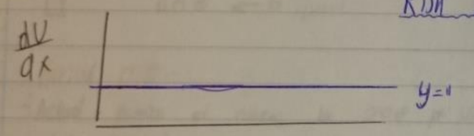
E.U.(B) 10000×0.9 + (-10000)(0.1) = 8000

Optimal decision is Action B - most of us would not though

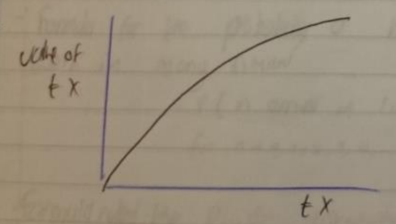
Utility function



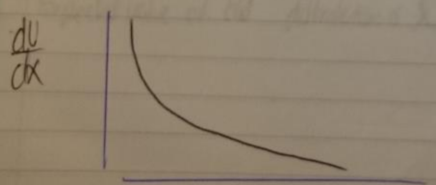
Risk neutral Utility



(2)



Risk adverse



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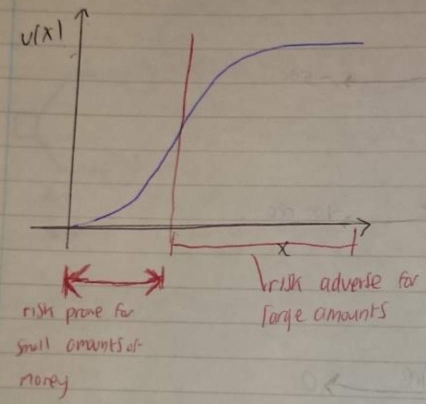
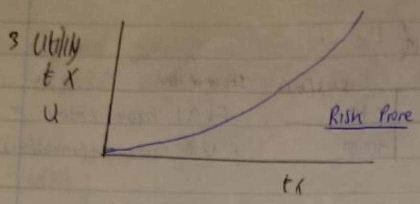
2.

Optimil decision = buy insurance
expected utility of optimal decision = −0.051

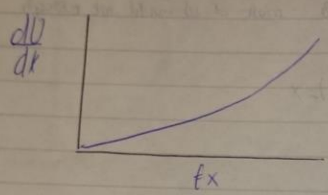


3



could be e^x because $\frac{dy}{dx}$ is e^x

tangent line + and increasing



Slope positive and increasing