MHS Tutorial Set 3 Week 4 DAVID WEITBRECHT 12300644

$\frac{8.5}{10}$

i. $u = (1, 2, -1)$

$\vec{u} = (1, 2, -1)$ $\qquad \vec{w} = k(1, 2, -1)$

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = k \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$w_1 = k$

$w_2 = 2k$

$w_3 = -k$ ← parametric eq^n of line

ii. System of linear eq^n for line above

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = k \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$w_1 = k$

$w_2 = 2k$

$w_3 = -k$

$\cancel{w_2 = 2(w_1)}$ $\qquad w_3 = -w_1$

$w_3 = -k$ $\qquad \cancel{k = \frac{w_2}{2}}$

$$\begin{cases} \cancel{w_1 = k} \\ w_2 = 2w_1 \\ w_3 = -w_1 \end{cases}$$

eq^n of line

iii. $u = (1, -1, -2)$ $\qquad v = (-2, 0, 1)$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = k_1 \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} + k_2 \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$$

$x = k_1 - 2k_2$

$y = -k_1$

$z = -2k_1 + k_2$

$\frac{4}{4}$

$k_1 = y$ $\qquad k_2 = z + 2k_1$

$x = k_1 - 2k_2$

$x = y - 2(z + 2k_1)$

$x = y - 2z + 2k_1$ $\qquad (k_1 = y)$

$x = y - 2z + 2y$

$\boxed{x = 3y - 2z}$ eq^n for plane

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Q2

i. $(a, 2a, 0)$ $\vec{v} = \begin{pmatrix} a \\ 2a \\ 0 \end{pmatrix}$ $\vec{w} = \begin{pmatrix} b \\ 2b \\ 0 \end{pmatrix}$

$\vec{v} + \vec{w} = \begin{pmatrix} a+b \\ 2a+2b \\ 0 \end{pmatrix} = \begin{pmatrix} c \\ 2c \\ 0 \end{pmatrix}$ $c = a+b$ ✓

$\vec{v} \in S \overset{?}{\Rightarrow} k\vec{v} \in S$

$\vec{v} \begin{pmatrix} a \\ 2a \\ 0 \end{pmatrix}$ $k\vec{v} = \begin{pmatrix} ka \\ 2ka \\ 0 \end{pmatrix} = \begin{pmatrix} c \\ 2c \\ 0 \end{pmatrix}$ work for $c \in k$

it is a subspace ✓ ✓

ii. $(2, 3, a)$ $\vec{v} = \begin{pmatrix} 2 \\ 3 \\ a \end{pmatrix}$ $\vec{w} = \begin{pmatrix} 2 \\ 3 \\ b \end{pmatrix}$

$\vec{v} + \vec{w} = \begin{pmatrix} 4 \\ 9 \\ a+b \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ c \end{pmatrix}$ does not hold ✓ $4 \neq 2$, $9 \neq 3$

$c = ?$ $c = a+b$

$\vec{v} \begin{pmatrix} 2 \\ 3 \\ a \end{pmatrix}$ $k\vec{v} = \begin{pmatrix} 2k \\ 3k \\ ka \end{pmatrix} = \begin{matrix} 2 \\ 3 \\ c \end{matrix}$ $2k = 2$, $3k = 3$, $ka = c$ work if $k = 1$

NOT A SUBSPACE ✓

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1ii. $\vec{v} = \begin{pmatrix} 2b \\ -b \\ a \end{pmatrix}$ $\vec{w} = \begin{pmatrix} 2d \\ -d \\ c \end{pmatrix}$

$\begin{pmatrix} 2b + 2d \\ -b - d \\ a + c \end{pmatrix} = \begin{pmatrix} 2e \\ -e \\ f \end{pmatrix}$ $e = b + d$, $e = b + d$, $f = a + c$ works ✓

$\vec{v} = \begin{pmatrix} 2b \\ -b \\ a \end{pmatrix}$ $k\vec{v} = \begin{pmatrix} 2kb \\ -kb \\ ka \end{pmatrix} = \begin{pmatrix} 2c \\ -c \\ d \end{pmatrix}$ $kb = c$ ✓, $-kb = -c$ ✓, $ka = d$ ✓

works ✓

It is a subspace of $\mathbb{R}^3$

3/3

3i. v_1 (1,2,1) v_2 (2,4,2) v_3 (-1,0,0)

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = k_1 \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + k_2 \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix} + k_3 \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 2 & 4 & 0 & 0 & 1 & 0 \\ 1 & 2 & 0 & 0 & 0 & 1 \end{array}\right)$$

$k_1 + 2k_2 - k_3 = x$

$2k_1 + 4k_2 = y$

$k_1 + 2k_2 = z$

$k_1 = z - 2k_2$

$2z - 4k_2 + 4k_2 = y$

$2z = y$

$z - 2k_2 + 2k_2 - k_3 = x$

$\cancel{z - k_3 = x}$

$k_1 + 2k_2 = z$

$k_3 = -x + z$

$0 = y - 2z$

$z =$ $z - k_3 = x$

$2z = y$

$z = k_1 + 2k_2$

$y = 2z$ so doesn't span $\mathbb{R}^3$.

$z =$

we have a zero on LHS

Does ~~not~~ have span on space $\mathbb{R}^3$ ✗

Does span all $\neq 0$ all have some k value $\neq 0$

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3 ii. $\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = k_1 \begin{pmatrix} 1 \\ 0 \\ 2 \\ 1 \end{pmatrix} + k_2 \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + k_3 \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix} + k_4 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 2 \end{pmatrix}$

$k_1 + k_2 + k_3 = x$

$0 = y$

$2k_1 - k_2 + 2k_3 + k_4 = z$

$k_1 + 2k_4 = t \qquad k_1 = t - 2k_4$

$~~2t - 4k_4 - k_2 + 2k_3 + k_4 = z~~$

$~~2t - 3k_4 - k_2 = z~~$

$1c_1 + 1c_2 + 1c_3 + 0c_4 = 1d_1$

$0c_1 + 0c_2 + 0c_3 + 0c_4 = 1d_2 \qquad \boxed{\Rightarrow 0 = d_2 \Rightarrow \text{Doesn't Span}}$

$2c_1 - 1c_2 + 2c_3 + 1c_4 = 1d_3$

$c_1 + 0c_2 + 0c_3 + 2c_4 = 1d_4$

$1c_1 + 2c_4 = 1d_4$

$d_2 = 0 \Rightarrow$ cannot have zero on LHS for c value, span is not consistent with $\mathbb{R}^4$

1.5/3