Travelling Salesman

Travelling Salesman visits all the nodes in a network, known as Hamilton's algorithm

Chinese postman travels along all the arcs - Euler's algorithm

Hamiltonian Algorithm

	A	B	C	D
A	–	4	6	9
B	4	–	9	3
C	6	9	–	7
D	9	3	7	–

Solution Methods

- Linear programming will give an optimal solution
- Enumeration will also give an optimal solution but there are $(N-1)!$ solutions, so for 6 city problem there are 120 options.

- Heuristics - rules of thumb or good solutions that are not necessarily an optimum solution but could be

Heuristics

- Nearest neighbour
- Nearest Insertion

Linear programming

Formulate as LP

$$\text{Min } Z = 4x_{AB} + 6x_{AC} + 9x_{AD} + 4x_{BA} + 9x_{BC} + 3x_{BD}$$
$$+ 6x_{CA} + 9x_{CB} + 7x_{CD} + 9x_{DA} + 3x_{DB} + 7x_{DC}$$

Entering Constraints

$x_{BA} + x_{CA} + x_{DA} = 1$ going into A
$x_{AB} + x_{CB} + x_{DB} = 1$
$x_{AC} + x_{BC} + x_{DC} = 1$
$x_{AD} + x_{BD} + x_{CD} = 1$

Leaving Constraints

$x_{AB} + x_{AC} + x_{AD} = 1$ leaving A.
$x_{BA} + x_{BC} + x_{BD} = 1$ "
$x_{CA} + x_{CB} + x_{CD} = 1$ "
$x_{DA} + x_{DB} + x_{DC} = 1$ "

All variables $\{0,1\}$

Solution (1)

$x_{AC} = 1$, $x_{BD} = 1$ $x_{CA} = 1$ $x_{DB} = 1$
Total distance = 18
Two tours A – C – A and B – D – B

Solution (2)

Additional constraints required.
Eliminate A – C – A by adding:
$x_{AC} + x_{CA} \le 1$
$x_{AC} = x_{CD} = x_{DB} = x_{BA} = 1$
New distance = 20

Nearest Neighbour Heuristic

	A	B	C	D	E	F
A	–	13	12	18	7	14
B	13	–	21	26	15	25
C	12	21	–	11	6	4
D	18	26	11	–	12	14
E	7	15	6	12	–	9
F	14	25	4	14	9	–

A → E → C → F → D → B → A = 70

	A	B	C	D	E	F
A	–	13	12	18	7	14
B	13	–	21	26	15	25
C	12	21	–	11	6	4
D	18	26	11	–	12	14
E	7	15	6	12	–	9
F	14	25	4	14	9	–

13 + 7 + 6 + 4 + 14 + 26 = 70

Other Solutions

A E C F D B A = 70
B A E C F D B = 70
C F E A B D C = 70
D C F E A B D = 70
E C F A B D E = 75
F C E A B D F = 70

Multiple Solution (not necessarily optimum)

Nearest Insertion

	A	B	C	D
A	–	4	6	9
B	4	–	9	3
C	6	9	–	7
D	9	3	7	–

A → A leaves B, C and D to be included

Min {AB, AC, AD} Min (4, 6, 9)
= 4 = AB
A – B – A, C and D not included

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(4) ~~An~~ A - B - A
Min {AC, BC, BD, AD}
Min {6, 9, 3, 9}
= 3
= BD

A - D - B A = 9 + 3 + 4 = 16 OR
A - B - D - A = 4 + 3 + 9 = 16 Choose Either

(5) A - D - B - A C to be included
Min {AC, DC, BC}
A - C - D - B - A = 6 + 7 + 3 + 4 = 20 OR
A - D - C - B - A = 9 + 7 + 9 + 4 = 29 OR
A - D - B - C - A = 9 + 3 + 9 + 6 = 27

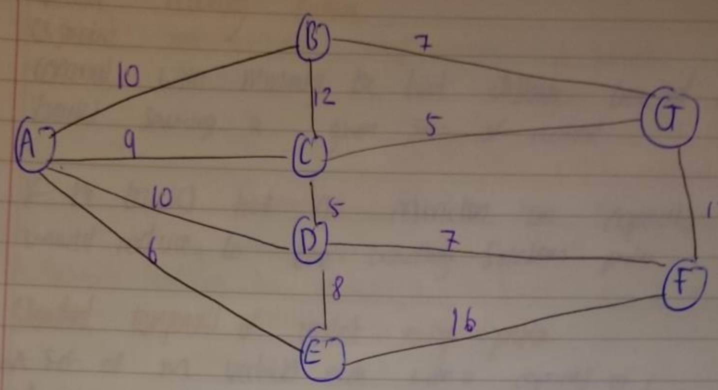
Best is A - C - D - B - A = 6 + 7 + 3 + 4 = 20

- Above was for a symmetrical problem
- If problem is not symmetrical must look at total length to be added
- e.g. if we have A - B - A, when adding C look at length of A - C - B and compare with B - C - A

Chinese Postman / Euler Algorithm

Solution method
- If there are NO arcs of ODD degree then the solution is just the sum of all the arcs
- Identify the nodes of odd degree
- Pair them in the shortest way (see all NCr pairs possible)
- Add extra arcs to network
- Calculate length of tour

5.



Solution (1)

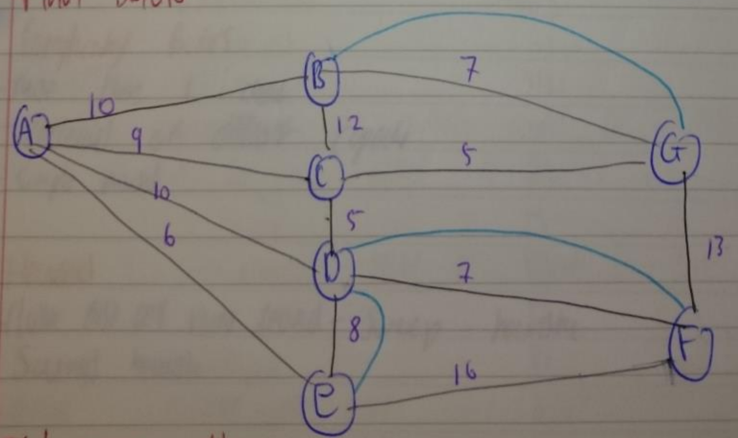
Total length of arcs = 108

Node	A	B	C	D	E	F	G.
Degree	4	3	4	4	3	3	3

Nodes of odd degree B E F G.

These must be paired

Final Solution



Total minimum distance:

Original dist + extra added arcs

108 + 7 + 7 + 8

108 + 22 = 130

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Vechile Routing Problem

Objective

concerned with minimising the total distance travelled by a fleet of trucks servicing a given set of customers

If the trucks had no restriction on capacity then problem would reduce to the travelling Salesman problem

Standard components of vechile routing problem

- A set of m vechicles each with a capacity of c
- A set of n customers each with requirements q_i where $i = 1, 2, \dots$
- A single depot from which all tours start and finish
- A distance matrix giving the distance between all customers

Formulation

OF. for VRP is to find a set of tours, one for each vechicle so that the sum of all tour lengths is minimised

- It is assumed that the total quantity demanded or supplied to all customers will not exceed the vechicle capacity

Complicating factors

- more than 1 depot
- Vechicles of different capacity
- Simple model

Heuristics

- Cluster first and route second - Sweep heuristic
- Savings heuristic

SWEEP HEURISTIC - Cluster first, Route Second

First cluster the customers into routes having regard for vechicle capacity and customer demand/supply

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- Use TSP method to find a "good" vechicle tour through the customers in the cluster

Sweep heuristic (1)

- Locate depot and all customers on graph.
- Create a sweep hand at the depot and pointed horizontally to right
- Rotate the sweep hand anti-clockwise. All customers included in the sweep will be allocated to cluster 1

- Stop sweep hand when inclusion of next customer would cause vechicle capacity to be exceeded
- Restart the sweep hand to form the remaining cluster
- Use one of the TSP methods to generate a tour through the clusters

Example:

- Trucks capacity 45 tonnes
- Depot located at point (35,35)

Customer	Quantity	Location (x,y)
1	10	40,50
2	14	55,20
3	5	20,50
4	16	30,60
5	23	30,25
6	19	10,80
7	17	15,60
8	18	45,10
9	6	65,20

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Vechicle Routin

Solution (1) cluster 1

Customer	Quantity	Cumulate
1	10	10
4	16	26
7	17	43

Solution (2) cluster 2

Customer	quantity	cumulate
3	5	5
6	19	24

Solution (3) cluster 3

Customer	Quantity	Cumulate
5	23	23
8	18	41

Solution 4 - cluster 4

Customer	Quantity	Cumulate
2	19	19
9	6	25

SOLUTION (5) - NOTE

Total required is 133 which could be delivered on 3 trucks but as we see we used 4 trucks as a heuristic was used

TO use 3, we would have to divide some cell

Savings Heuristic
- Groups customers into pairs.
- Pairs are then ranked
- Then combined together until the best set of routes is found

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Method

- Like nearest insertion for TSP, Start with one truck per pallet
- Distance for two will be: $2D_{0i} + 2D_{0J}$
- Joining two gives the following Saving: $S_{iJ} = D_{0i} + D_{0J} - D_{iJ}$

Steps in algorithm

- Calculate Savings for all pairs of customers : $S_{iJ} = D_{0i} + D_{0J} - D_{iJ}$
- Order the Savings into descending order.
- Starting at the top of the list try to combine customers onto the same route
- Repeat until all possible Savings are made

Example:

Trucks 15 tonne capacity and 6 customers:

1	4
2	10
3	4
4	3
5	35
6	5.

Distance matrix

	0	1	2	3	4	5	6
0	–	24	19	20	27	16	12
1	24	–	17	31	44	36	23
2	19	17	–	16	24	35	25
3	20	31	16	–	15	34	28
4	27	44	24	15	–	40	37
5	16	36	35	34	40	–	11
6	12	23	25	28	37	11	–

Solution (1)

- Savings joining 1 and 2 is $24 + 14 - 17 = 26$
- Savings joining 2 and 3 $= 14 + 20 - 16 = 22$
- Continue for other pairs and sort in descending order to save as much as possible

5 Vechicle Routing

Solution 2

i	j	s
1	2	26
1	3	13
1	4	7
1	5	4
1	6	13
2	3	23
2	4	17
2	5	0
2	6	6
3	4	32
3	5	2
3	6	4
4	5	3
4	6	2
5	6	17

Solution 3 Ordered

i	j	s
3	4	32
1	2	26
2	3	23
2	4	17
5	6	17
1	3	13
1	6	13
1	4	7
2	6	6
1	5	4
3	6	4
4	5	3
3	5	2
4	6	2
2	5	0

Solution 4

Join 3 and 4 ? yes
Join 1 and 2 ? yes
Join 2 and 3 ? No combined route exceeds capacity as 2 already joined to 1
Join 2 and 4 ? Both already joined
Join 5 and 6 ? Yes

Solution 5 1

Three routes
1 depot - 3 - 4 depot
2 depot - 1 - 2 depot
3 depot - 5 - 6 depot

Total required is 29t which could be delivered on 2 trucks but we used 3 trucks as heuristic was used

To use 2 we would have to divide some loads