1

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3.(A) Identify the steps in the BALAS algorithm.

Formulate problem by:

- Minimising objective function by multiplying be -1 if it is a max problem

- All coef of min O.F. must be positive. If they are negative replace variable X_J with $(1-X_J)$ and drop the resulting constant

- If $X_J = (1-X_J)$ was used, substitute it into the constraints
- Constraint must be all in form of $\le$
- If not; multiply both sides by -1 and swap equality

- Start with solution were all variable $=0$ Check for feasibility

- Pick X_J in O.F with smallest coef and assign value of 1 to it leaving other X_J at 0.
- If feasible you have found solution

- If not, pick second lowest X_J and set remaining to 0. Check for feasibility, if not repeat step until you find feasibility

- Keep iterating until solution is found
- Remember that if you substitute $X_J = 1-X_J$ you must enter value in at end to get solution

2

(B) Formulate

Max $90x_1 + 40x_2 + 10x_3 + 37x_4$

ST:

$C_1: 15x_1 + 10x_2 + 10x_3 + 15x_4 \leq 40$

$C_2: 20x_1 + 15x_2 + 0 + 10x_4 \leq 50$

$C_3: 20x_1 + 20x_2 + 0 + 10x_4 \leq 40$

$C_4: 15x_1 + 5x_2 + 4x_3 + 10x_4 \leq 35.$

all vars $\{0,1\}$

1. Change O.F. to minimisation by multiplying by (-1)

$-90x_1 - 40x_2 - 10x_3 - 37x_4$

Coefficients must be positive, Sub in $x_j = (1-x_j)$

$-90(1-x_1) - 40(1-x_2) - 10(1-x_3) - 37(1-x_4)$

$90x_1 + 40x_2 + 10x_3 + 37x_4 \quad -177$

Drop the constants

2. Substitute $x_j = 1 - x_j$ into 4 constraints, no need to change equalities:

$C1: 15(1-x_1) + 10(1-x_2) + 10(1-x_3) + 15(1-x_4) \leq 40$

(1) $-15x_1 - 10x_2 - 10x_3 - 15x_4 \leq -10$

$C2: 20(1-x_1) + 15(1-x_2) + 0 + 10(1-x_4) \leq 50.$

(2) $-20x_1 - 15x_2 \quad -10x_4 \leq 5$

$C3: 20(1-x_1) + 20(1-x_2) + 0 + 10(1-x_4) \leq 40$

(3) $-20x_1 - 20x_2 \quad -10x_4 \leq -10$

$C4: 15(1-x_1) + 5(1-x_2) + 4(1-x_3) + 10(1-x_4) \leq 35$

(4) $-15x_1 - 5x_2 - 4x_3 - 10x_4 \leq 1$

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Which gives us:

Min $90x_1 + 40x_2 + 10x_3 + 37x_4$

ST: (1) $-15x_1 \quad -10x_2 \quad -10x_3 \quad -15x_4 \leq -10$

(2) $-20x_1 \quad -15x_2 \quad -10x_4 \leq 5$

(3) $-20x_1 \quad -20x_2 \quad -10x_4 \leq -10$

(4) $-15x_1 \quad -5x_2 \quad -4x_3 \quad -10x_4 \leq 1$

$N = \{x_1, x_2, x_3, x_4\}$ variables with value 0

$T = \{\ \}$ variables with value 1

$Z = 0$ O.F. value

$S = \{-10, 0, -10, 0\}$ amount of infeasibility

Choose $x_3 \Rightarrow$

$N = \{x_1, x_2, x_3\}$

$T = \{x_3\}$

$Z = 10$

$S = \{0, 15, -10, 0\}$ NOT FEASIBLE

Choose $x_4 \rightarrow$

$N = \{x_1, x_2, x_3\}$

$T = \{x_4\}$

$Z = 37$

$S = \{0, 0, 0, 0\}$ SOLUTION

$x_1 = 0 \quad x_2 = 0 \quad x_3 = 0 \quad x_4 = 1$

Change back from $x_T = (1 - x_T)$

$X_1 = 1 - x_1 = 1 - 0 = 1 = x_1$

$X_2 = 1 - x_2 = 1 - 0 = 1 = x_2$

$X_3 = 1 - x_3 = 1 - 0 = 1 = x_3$

$X_4 = 1 - x_4 = 1 - 1 = 0 = x_4$

SOLUTION: $90(1) + 40(1) + 10(1) + 37(0) = 140$ value

4.

3(C)

Formulate: Max $10x+8y$

ST: $x+y \le 10$

$5x+y \le 20$

$x, y \ge 0$

Basis	Value	x_1 10	x_2 8	s_1 0	s_2 0		
s_1	0	1	1	1	0	10	$\frac{10}{1}=10$
s_2	0	5	1	0	1	20	$\frac{20}{5}=4$
Z_j		0	0	0	0	0	
C_j-Z_j		10	8	0	0		

Why not use graphical method?

x_1 enters, s_2 leaves. $r_2 \div 5$ $r_1 - r_2$

Basis	Value	x_1 10	x_2 8	s_1 0	s_2 0		
s_1	0	0	0.8	1	-0.2	6	$\frac{6}{0.8}$
x_1	10	1	0.2	0	0.2	4	$\frac{4}{0.2}$
Z_j		10	2	0	2	40	
C_j-Z_j		0	6	0	-2		

x_2 enters, s_1 leaves ... $r_1 \times \frac{5}{4}$ $r_2 - \frac{1}{5} r_1$

Basis	Value	x_1 10	x_2 8	s_1 0	s_2 0	
x_2	8	0	1	1.25	-0.25	7.5
x_1	10	1	0	-0.25	0.25	2.5
Z_j		10	8	7.5	0.5	85
C_j-Z_j		0	0	-7.5	-0.5	

SOLUTION FOUND

$x_1 = 2.5$

$x_2 = 7.5$

Value $= 2.5(10) + 7.5(8) = 85$.

Integer Sol. required.

$Z = 85$, $x_1 = 2$, $x_2 = 8$

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ii. Shadow Prices

Constraint 1 : 7.5

Constraint 2 : 0.5

Not asked for any of this but correct

iii. O.F coefficient Ranges

X_1:

$+0.25c_1 - 8(1.25) \le 0$ and $-0.25c_1 + 0.25(8) \le 0$

$c_1 \le 40$ & $c_1 \geq 8$

$8 \le c_1 \le 40$ ✓

X_2: $-1.25c_2 + 0.25(10) \le 0$ and $+0.25c_2 - 0.25(10) \le 0$

$c_2 \geq 2$ and $c_2 \le 10$

$2 \le c_2 \le 10$ ✓

iv. Right hand Side Ranges

Constraint 1:

$$\begin{bmatrix} 7.5 \\ 2.5 \\ 85 \end{bmatrix} + \Delta b_1 \begin{bmatrix} 1.25 \\ -0.25 \\ 7.5 \end{bmatrix}$$

$7.5 + \Delta b_1(1.25) \geq 0$ $\quad \Delta b_1 \geq -6$

$2.5 - \Delta b_1(0.25) \geq 0$ $\quad \Delta b_1 \le 10$

$85 + \Delta b_1(7.5) \geq 0$ $\quad \Delta b_1 \geq -63.75$.

$-6 \le \Delta b_1 \le 10$ $\quad b_1 = 10$

$10-6 \le b \le 10+6$

$4 \le b \le 20$ ✓

Constraint 2:

$$\begin{bmatrix} 7.5 \\ 2.5 \\ 85 \end{bmatrix} + \Delta b_2 \begin{bmatrix} -0.25 \\ 0.25 \\ 0.5 \end{bmatrix}$$

$7.5 - 0.25\Delta b_2 \geq 0$ $\quad \Delta b_2 \le 30$

$2.5 + 0.25\Delta b_2 \geq 0$ $\quad \Delta b_2 \geq -10$

$85 + 0.5\Delta b_2 \geq 0$ $\quad \Delta b_2 \geq -16$

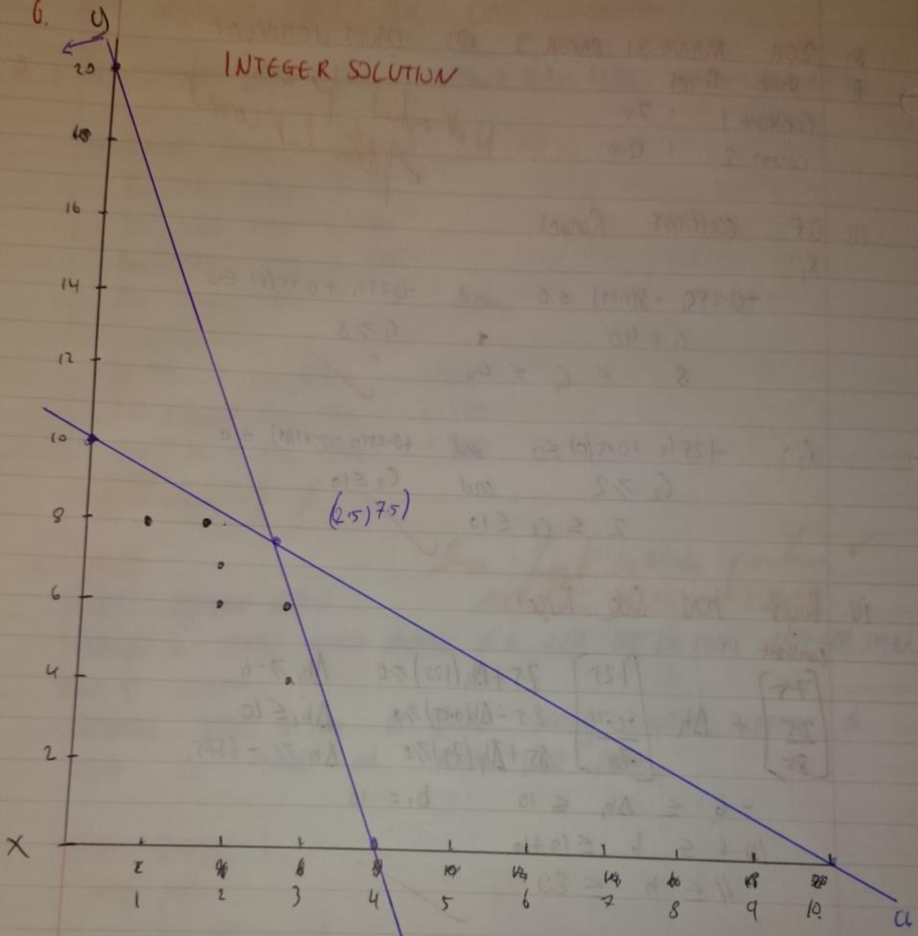
$-10 \le \Delta b_2 \le 30$

$20-10 \le b_2 \le 30+20$

$10 \le b_2 \le 50$ ✓

6.

INTEGER SOLUTION



C1: $X+y \leq 10$. $x=0, y=10$ $y=0$ $x=10$

C2: $5x+y \leq 20$ $x=0$ $y=20$ $y=0$ $x=4$

Points: (3,4) profit = 62

(2,7) = 76

(2,8) = 84

(1,8) = 74

(0,10) = 80

Optimal integer solution = (2,8) profit = 84

2010/2011 $\boxed{3}$ Q3

3 a Formulate problem by:

- Minimising objective function by multiplying by -1 if it is a max problem
- All coefs of min O.F. must be positive. If negative replace variable x_J with $(1-x_J)$ and drop the resulting constant.
- If $x_J = (1-x_J)$ was used, substitute it into the constraints
- Constraints must be all in form of $\leq$
If not; multiply both sides by -1 and swap equality
- Start with solution where all variables = 0.
Check for feasibility
- Pick x_J in O.F. with smallest coef and assign value of 1 to it leaving other x_J at 0.
- If feasible you have found solution
- If not, pick second lowest x_J and set remaining to 0.
Check for feasibility, if not repeat step until you find feasibility
- Keep iterating until solution is found
- Remember that if you substituted $x_J = 1 - x_J$ you must enter value in at end to get proper solution

2

b max $90x_1 + 40x_2 + 10x_3 + 37x_4$

ST:

$C_1: 15x_1 + 10x_2 + 10x_3 + 15x_4 \leq 40$

$C_2: 20x_1 + 15x_2 + 0 + 10x_4 \leq 50$

$C_3: 20x_1 + 20x_2 + 0 + 10x_4 \leq 40$

$C_4: 15x_1 + 5x_2 + 4x_3 + 10x_4 \leq 35.$

1. Change O.F. to minimisation by multiplying by (-1)

$-90x_1 - 40x_2 - 10x_3 - 37x_4$

coeff's must be positive, $x_J = (1 - x_J)$

$-90(1-x_1) - 40(1-x_2) - 10(1-x_3) - 37(1-x_4)$

$90x_1 + 40x_2 + 10x_3 + 37x_4 \quad -1\cancel{77}$

drop the constant

2. Substitute $x_J = 1 - x_J$ into 4 constraints, no need to change equality

$C_1: 15(1-x_1) + 10(1-x_2) + 10(1-x_3) + 15(1-x_4) \leq 40$

① $-15x_1 - 10x_2 - 10x_3 - 15x_4 \leq -10$

$C_2: 20(1-x_1) + 15(1-x_2) + 0 + 10(1-x_4) \leq 50$

② $-20x_1 - 15x_2 \quad -10x_4 \leq 5$

$C_3: 20(1-x_1) + 20(1-x_2) + 0 + 10(1-x_4) \leq 40$

③ $-20x_1 - 20x_2 \quad -10x_4 \leq -10$

$C_4: 15(1-x_1) + 5(1-x_2) + 4(1-x_3) + 10(1-x_4) \leq 35$

④ $-15x_1 - 5x_2 - 4x_3 - 10x_4 \leq 1$

3 2010/2011 $\boxed{3}$ Q3

Which gives us:

$$\text{Min} \quad 90x_1 + 40x_2 + 10x_3 + 37x_4$$

ST:
- ⓪ $-15x_1 - 10x_2 - 10x_3 - 15x_4 \leq -10$
- ① $-20x_1 - 15x_2 - 10x_4 \leq 5$
- ③ $-20x_1 - 20x_2 - 10x_4 \leq -10$
- ④ $-15x_1 - 5x_2 - 4x_3 - 10x_4 \leq 1$

$N = \{x_1\, x_2\, x_3\, x_4\}$ variable value $= 0$

$T = \{\ \}$ variable value $= 1$

$Z = 0$ O.F value

$S = \{-10, 0, -10, 0\}$ amount of infeasibility

Choose $x_3 \rightarrow$

$N = (x_1, x_2, x_4)$ variable value $= 0$

$T = \{x_3\}$ variable value $= 1$

$Z = 10$

$S = \{0, 15, -10, 0\}$ amount of infeasibility X

Choose $x_4 \rightarrow$

$N = (x_1, x_2, x_3)$ variable value $= 0$

$T = \{x_4\}$ variable value $= \phi$

$Z = 37$

$S = \{0, 0, 0, 0\}$ SOLUTION

$\downarrow$ $x_1 = 0$ $x_2 = 0$ $x_3 = 0$ $x_4 = 1$

$\swarrow$

$x_1 = 1 - x_1 = 1 - 0 = 1 = x_1$

$x_2 = 1 - x_2 = 1 - 0 = 1 = x_2$

$x_3 = 1 - x_3 = 1 - 0 = 1 = x_3$

$x_4 = 1 - x_4 = 1 - 1 = 0 = x_4$

$90(1$ … $) +$ … 140 = solution

4

Max $10x + 8y$

ST: $x + y + s_1 = 10$

$5x + y + s_2 = 20$ $\quad x, y \geq 0$

Basis	C_B	x 10	y 8	s_1 0	s_2 0		
s_1	0	1	1	1	0	10	$\frac{10}{1} = 10$
s_2	0	5	1	0	1	20	$\frac{20}{5} = 4$
Z_j		0	0	0	0	0	
$C_j - Z_j$		10	8	0	0		

Want $\begin{smallmatrix}0\\1\end{smallmatrix}$ row2 ÷ 5

row1 − row2

Basis	C_B	x 10	y 8	s_1 0	s_2 0		
s_1	0	0	4/5	1	−1/5	6	$\frac{6}{0.8} = 7.5$
x	10	1	1/5	0	1/5	4	$\frac{4}{0.2} = 20$
Z_j		10	2	0	2	40	
$C_j - Z_j$		0	6	0	−2		

Want $\frac{1}{0}$ row1 × 5/4

row2 − 1/5 row1

Basis	C_B	x 10	y 8	s_1 0	s_2 0	
y	8	0	1	5/4	−1/4	15/2
x	10	1	0	−1/4	1/4	5/2
Z_j		10	8	10 − 4 7.5	0	[illegible] 84
$C_j - Z_j$		0	0	−7.5 −10 + 4	0	

Optimal Solution $C_j - Z_j$ row are all 0 or negative

5. $\boxed{3}$ 2010/2011 Q3 Maysc

c i. Optimal solution = $x = 2.5$

$y = 7.5$

ii. shadow price $x = 10$

$y = 8.$

iii. X coefficient $-10 + c_1 \leq 0$ and $2 - c_1 \leq 0$

$c_1 \leq 10$ $c_1 \geq 2$

$2 \leq c_1 \leq 10$

y cof. $-c_2 - 2.5 \leq 0$ $+c_2 - 2.5 \leq 0$

$c_2 \geq -2.5$ $c_2 \leq 2.5$

$-2.5 \leq c_2 \leq 2.5$

iv. RHS old column 1

$$\begin{bmatrix} 7.5 \\ 2.5 \\ 85 \end{bmatrix} + \Delta b_1 \begin{bmatrix} 5/4 \\ -1/4 \\ 7.5 \end{bmatrix}$$

$7.5 + \frac{5}{4}\Delta b_1 \geq 0$ $\Delta b_1 \geq -6$

$2.5 + \frac{1}{4}\Delta b_1 \geq 0$ $\Delta b_1 \leq 10$

$85 + \Delta b_1 7.5 \geq 0$ $\Delta b_1 \geq -\frac{34}{3}$

$-6 \leq \Delta b_1 \leq 10$

$10 - 6 \leq \Delta b_1 \leq 10 + 10$

$4 \leq \Delta b_1 \leq 20.$ $\boxed{C_1}$

RHS C_2

$$\begin{bmatrix} 7.5 \\ 2.5 \\ 85 \end{bmatrix} + \Delta b_2 \begin{bmatrix} -1/4 \\ 1/4 \\ 0 \end{bmatrix}$$

$7.5 - \frac{1}{4}\Delta b_2 \geq 0$ $\Delta b_2 \leq 30$

$2.5 + \frac{1}{4}\Delta b_2 \geq 0$ $\Delta b_2 \geq -10$

$85 + 0\Delta b \geq 0$ —

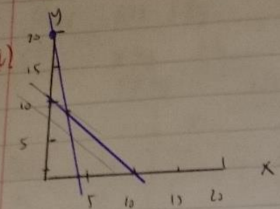
$-10 \leq \Delta b_2 \leq 30$

$20 - 10 \leq \Delta b_2 \leq 20 + 30$

$10 \leq \Delta b_2 \leq 50.$ $\boxed{C_2}$

6.

c (ii)



(2.5, 7.5)

(1

$10x + 8y = \frac{80}{2}$

$x=0 \quad y=\frac{1}{4}$

$y=0 \quad x=\frac{8}{10}$

4, 8

(4, 6) (4, 8)

(3, 5) (3, 6) (3, 7)

30+40   2, 6,   2, 7,   2, 8   1, 9,   0, 10

70   (20+48 68)   (76)   (84)   82   80

(2, 8) integer