MA1E02 Tutorial Sheet 1.

January 28th 2013

Questions Evaluate the following limits:

1.
$$\lim_{x \to 0} \frac{e^{2x} - 1}{\sin(x)} \quad (1)$$

2.
$$\lim_{x \to 0} x \tan\left(\frac{1}{x}\right) \quad (2)$$

3.
$$\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right) \quad (3)$$

4.
$$\lim_{x \to 0^+} (\cos(x))^{\frac{1}{x^2}} \quad (4)$$

$2^x = 3$

$\ln 2^x = \ln 3$

$x \ln 2 = \ln 3$

$x = \frac{\ln 3}{\ln 2}$

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1. $\lim_{x \to 0} \frac{e^{2x}-1}{\sin x}$ $\qquad \frac{e^0-1}{\sin 0}$ $\frac{1-1}{0} = \frac{0}{0}$

$f(x) = e^{2x}-1 \quad f'x = 2e^{2x}$ $\qquad \lim_{x \to 0} \frac{2e^{2x}}{\cos x}$

$g(x) = \sin x \quad g'x = \cos x$

$\frac{2e^0}{\cos 0} = \frac{2}{1} = 2.$

2. $\lim_{x \to 0} x\tan(\frac{1}{x})$ $\qquad \frac{x \cdot \sin \frac{1}{x}}{\cos \frac{1}{x}}$

$0(\tan(\frac{1}{0})) =$

$f(x) = x\sin\frac{1}{x} \qquad f'x = x \cdot \cos(\frac{1}{x})(-x^{-2}) + \sin\frac{1}{x}$

$g(x) = \cos x \qquad 0 = -\sin(\frac{1}{x})(-x^{-2})$

$\lim_{x \to 0} \frac{\tan(\frac{1}{x})}{\frac{1}{x}}$

$\cancel{f(x) = \tan\frac{1}{x} \quad f'(x) = \tan^2}(\frac{x-1}{x^2})$

$g(x) = \cancel{\frac{1}{x} \quad g(x)} = \frac{x-1}{x^2}$

$\frac{f'(x)}{g'(x)} \qquad \frac{\frac{-1}{x}\cos(\frac{1}{x}) + \sin(\frac{1}{x})}{\frac{1}{x^2}\sin(\frac{1}{x})}$

$\lim_{x \to 0} \frac{\tan(\frac{1}{x})}{1/x}$ $\qquad f(x) = \tan(\frac{1}{x}) \quad f'(x) = \sec^2(\frac{1}{x})(-x^{-2})$

$g(x) = 1/x \quad g'(x) = -x^{-2}$

$\frac{f(x)}{g'(x)} = \frac{\sec^2(\frac{1}{x})\cancel{(-x^{-2})}}{\cancel{-x^{-2}}} = \sec^2(\frac{1}{x}) = \frac{\sin^2(\frac{1}{x})}{\cos^2(\frac{1}{x})} = \infty$

3 $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$ $\qquad$ $\lim \frac{e^0 - 0 - 1}{0} = \frac{1-1}{0} = \frac{0}{0}$

$\lim_{x \to 0} \frac{e^x - 1 - x}{x(e^x - 1)}$

$f(x) = e^x - x - 1 \qquad f'(x) = e^x - 1$

$g(x) = x(e^x - 1) \qquad g'(x) = x(e^x) + (e^x - 1)$

$\lim_{x \to 0^+} \frac{e^x - 1}{xe^x + e^x - 1}$ $\qquad$ $g'(0) = 0 + 1 - 1 = 0$ go again

$f''(x) = e^x$

$g''(x) = xe^x + e^x + e^x = e^x(x+2)$

$\lim_{x \to 0} \frac{e^x}{e^x(x+2)} \qquad \frac{e^0}{e^0(0+2)} \qquad \frac{1}{1(2)} = \frac{1}{2}$

4 $\lim_{x \to 0^+} (\cos(x))^{\frac{1}{x^2}}$ $\qquad$ $\lim_{x \to 0} \frac{1}{x^2} \log(\cos x) = \frac{\log 1}{0} = \frac{0}{0}$

L'hopital rule

$f(x) = \log \cos x \qquad f'(x) = \frac{1}{\cos x} \cdot (-\sin x) = \frac{-\sin x}{\cos x}$

$g(x) = x^2 \qquad g'(x) = 2x$

$\frac{f'x}{g'x} = \frac{-\sin(x)}{2x\cos(x)} \overset{x \to 0}{=} 0$ on both

$f''x = -\cos x$

$g''(x) = 2x(-\sin x) + 2\cos x$ $\qquad$ $\frac{f''x}{g''(x)} = \frac{-\cos x}{-2x\sin x + 2\cos x} = \frac{-1}{2}$

limit of $\log (\cos x)^{\frac{1}{x^2}}$

limit of $(\cos x)^{\frac{1}{x^2}} = e^{-\frac{1}{2}}$