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i. $f(x) = -x$. $y = -x$ is ~~odd~~ even function.

$f(x) = -x$ for $-\pi \le x \le \pi$

Odd fourier series → $f(x) = \sum b_n \sin(nx)$.

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} -x \sin(nx)\, dx. \qquad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} -x\cos(nx)$$

$$b_n = \frac{1}{\pi} \left. \frac{nx\cos(nx) - \sin(nx)}{n^2} \right|_{x=-\pi}^{\pi}$$

$$\frac{n\pi\cos(n\pi)}{n^2} - \frac{nx\cos(nx)}{n^2} = 0$$

ii. $f(x) = x^2$. even ⇒ $f(-x) = f(x)$.

$F.S = a_0 + \sum a_n \cos nx$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx \qquad \frac{1}{2\pi}\int x^2$$

$$\frac{1}{2\pi} \left. \frac{x^3}{3} \right|_{-\pi}^{\pi} = \frac{\pi^2}{3}$$

$a_0 = 0$.

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx) \qquad \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\cos(nx)$$

$$\sum_{n=1}^{2} \frac{1}{\pi} \frac{(2 - n^2x^2)\cos(nx) + 2nx\sin(nx)}{n^3}$$

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$\cancel{b_n}^{2}$ a_n $= \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cos nx\,dx$ show calculations

$= \frac{1}{\pi}\left(\frac{(n^2x^2-2)\sin(nx) + 2nx\cos(nx)}{n^3} \Big|_{-\pi}^{\pi} \right)$

$= (-1)^n \frac{4}{n^2}$

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$F.S = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \frac{4}{n^2} \cos nx$

$= \frac{\pi^2}{3} - 4\cos x + \frac{4\cos 2x}{2^2} + - \frac{4\cos 3x}{3^2} + \dots$

i. $f(x) = -x$, is odd function

$f(x) = \sum b_n \sin nx$

$\frac{1}{\pi}\int_{-\pi}^{\pi} -x\sin(nx)$

$\frac{1}{\pi}$. $\frac{-nx\sin(nx) + \cos(nx)}{\cancel{n}^2}\Big|_{-\pi}^{\pi}$ $b_n = \frac{2\cos(n\pi)}{n}$

$\sin(n\pi) = 0$

$\frac{-\pi n \sin \pi n + \cos(\pi n)}{n^2} - \frac{-n - \pi\sin(n\pi) + \cos(n)}{n^2}$

its a multiply

$= \sum \frac{-\pi n \sin(\pi n) + \cos(\pi n) + n + \pi \cancel{\sin(n\pi)} - \cos(n\pi)}{n^2}$

$= -\pi \sum \frac{(-1)^n + n - (-1)^n}{n^2}$

$= \frac{-1+1+1}{1} + \frac{-1^2 + 2 - (-1)^2}{2^2} + \frac{-1^3 + 3 - (-1)^3}{3^2} \dots$

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Fourier Integral $f(x) = \begin{cases} 0 & \text{if } |x| > 1 \\ -x & \text{if } |x| < 1 \end{cases}$

$A(w) = \frac{1}{\pi} \int_{-\infty}^{-1} \int_{-1}^{1} \int_{1}^{\infty}$

$\frac{1}{\pi} \int_{-\infty}^{-1} 0 \cos(wx)dx + \int_{-1}^{1} -x \cos(wx)dx + \int_{1}^{\infty} 0 \cos(wx)dx$

$= \cancel{\frac{w\, x \sin(wx) + \cos(wx)}{\pi\, w^2}}\Big|_{x=-1}^{1}$

sorry

$\frac{w \sin w + \cos w}{w^2} - \frac{-w \sin(-w) + \cos(-w)}{(w)^2}$

$\frac{w \sin w + \cos w}{w^2} - \frac{w \sin(w) + \cos w}{w^2} = 0.$

sorry the $A(\omega) = 0$ as the function is odd

$B(w) = \frac{1}{\pi} \int_{-\infty}^{-1} \int_{-1}^{1} \int_{1}^{\infty} f(x) \sin(wx)dx.$

$\frac{1}{\pi} \int_{-\infty}^{-1} 0 \sin wx + \int_{-1}^{1} -x \sin(wx) + \int_{1}^{\infty} 0 \sin(wx)\,dx$

$0 \quad \frac{1}{\pi} \frac{w\, x \cos(wx) + \sin(wx)}{w^2}\Big|_{-1}^{1}$

$\frac{w \cos(w) - \sin w}{w^2} - \left[\frac{-w \cos(w) - \sin(-w)}{w^2}\right]$

$\cos(w) + \sin(w)$

$B(w) = \frac{-2 w \cos(w)}{w^2} = \frac{2}{\pi\omega}\left(\frac{-\sin(\omega)}{\omega} + \cos(\omega)\right)$

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$$f(x) = \int_{-\infty}^{\infty} \left[\underbrace{A(w)\cos(wx)}_{0} + B(w)\sin(wx)\right] dw$$

$$\int_{-\infty}^{\infty} \frac{-2w\cos(w)}{w^2} \sin(wx) \quad dw$$