Maths Tutorial Set 6 Week 8 DAVID WEITBRECHT
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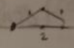
1 $\iint_R \sin \frac{1}{2}(x+y) \cos \frac{1}{2}(x-y)\, dA$

$$\int_0^1 \int_0^2 \sin\tfrac{1}{2}(x+y) \cos\tfrac{1}{2}(x-y)\, dx\, dy$$



6.5/10

polar coords $x = p\cos\theta$ $y = p\sin\theta$ ✗

$\tan\theta = \frac{y}{x}$

$\theta = \tan^{-1}(1)$

$$\int_0^1 \int_0^2 \sin\left(\frac{p\cos\theta + p\sin\theta}{2}\right) \cos\left(\frac{p\cos\theta - p\sin\theta}{2}\right)$$ ✗

$$\frac{1}{2} \sin(p\cos\theta + p\sin\theta) \cos(p\cos\theta - p\sin\theta)$$

$$\frac{1}{2} \int_0^{\pi/4} \int_0^{\sqrt{2}} \sin(p\sin\theta) + \sin(p\cos(\theta))\, p\, dp\, d\theta$$

$$\frac{1}{2} (\sin x \cos y + \cos x \sin y)(\cos x \cos y - \sin x \sin y)$$

$x = p\cos\theta$
$y = p\sin\theta$

$$[\sin(p\cos\theta)\cos(p\sin\theta) + \cos(p\cos\theta)\sin(p\sin\theta)][\cos(p\cos\theta)\cos(p\sin\theta) - \sin(p\cos\theta)\sin(p\sin\theta)]$$

$$(\cancel{\sin(p\cos\theta)\cos(p\sin\theta)})(\cos(p\cos\theta)\cos(p\sin\theta)) + ((\cancel{\sin p\cos\theta)(\cos p\sin\theta)})(\cancel{-\sin p\cos\theta})\sin p\sin\theta)]$$
$$+ (\cos(\cancel{p\cos\theta)\sin p\sin\theta})(\cancel{\cos p\cos\theta \cos p\sin\theta}) + (\cos p\cos\theta \sin p\sin\theta)(-\sin p\cos\theta \sin p\sin\theta)$$

$$\sin(p\cos\theta)\cos(p\cos\theta) \cos^2 p^2 \sin\theta + (-\sin^2 p^2 (\cos\theta)\cos(p\sin\theta)\sin(p\sin\theta))$$

$$\frac{1}{2} \int_0^{\pi/4} \int_0^{\sqrt{2}} \sin(p\sin\theta)\, p\, dp\, d\theta + \frac{1}{2} \int_0^{\pi/4} \int_0^{\sqrt{2}} \sin(p\cos\theta)\, p\, dp\, d\theta$$

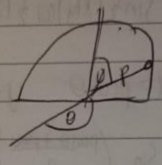
$u = p\cos\theta$ $dv =$

$\frac{du}{dp} = \sin\theta$

$du = \sin\theta\, dp$

$$\frac{1}{2} \int_0^{\pi/4} \int_0^{\sqrt{2}} \sin(u)$$

Q2 Find mall and cene of grauty of porhn of ball $x^2+y^2+z^2=4$ above the xp plae with denily $\delta(x,y,z) = \sqrt{x^2+y^2+z^2}$



Mall $= \iiint \delta x,y,z$

$\rho = \sqrt{x^2+y^2+z^2}$ $\quad \tan\theta = y/x$

$\rho = 2$ $\quad \theta = \tan^{-1}\frac{y}{x}$

$$\int_0^{\pi/2}\int_0^{\pi}\int_0^{2} \sqrt{\rho}\,(\rho^2 \sin\phi)\, d\rho\, d\theta\, d\phi$$

$\frac{\rho^2}{2}\Big|_0^2$

$\int_0^{\pi/2}\int_0^{\pi} 2 \; d\theta\, d\phi$

$2\theta\Big|_0^{\pi}$

$\int_0^{\pi/2} 2\pi\, d\phi$

$2\pi\phi\Big|_{0}^{\pi/2}$ $\quad 2\pi\frac{\pi}{2} = \pi^2 =$ mall

Center of gravity $= \bar{x}, \bar{y}, \bar{z} = \frac{M_x}{M}, \frac{M_y}{M}, \frac{M_z}{M}$

$M_x = \iiint \cancel{x\rho}\, \rho\, (\rho\sin\phi\cos\theta)(\rho^2\sin\phi)$ $\quad M_z = \int_0^{\pi/2}\int_0^{\pi}\int_0^{2} \rho z \sin\phi(o)\phi$

$\int_0^{\pi/2}\int_0^{\pi}\int_0^{2} \rho^2 \sin\phi\cos\theta \; d\rho\, d\theta\, d\phi$

$\int_0^{\pi/2}\int_0^{\pi} \frac{8}{3}\sin\phi\cos\theta \; d\theta\, d\phi$

$\frac{8}{3}\sin\phi \quad -\sin\Big|_0^{\pi}$

$\frac{8}{3}\sin\phi\,(0) = 0 \quad M_x = 0 \qquad \frac{M_x}{M} = 0 = \bar{x}$

Similarly $\bar{y}, \bar{z} = 0$ ✗ $\bar{z} \neq 0$

$0, 0, 0 =$ center of gravity

$\cancel{1.5}$
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Q2 contnued

$$M_z = \int_0^{\pi/2}\int_0^{\pi}\int_0^{2} \phi\, \rho \sin\phi \cos\theta \; d\rho\, d\theta\, d\phi \qquad z = \rho\cos\phi$$

$$\int_0^{\pi/2}\int_0^{\pi} \phi \frac{\rho^2}{2} \sin\phi \cos\theta \Big|_0^2 \; d\theta\, d\phi$$

$$\int_0^{\pi/2}\int_0^{2\pi} 2\phi \sin\phi \cos\theta \; d\theta\, d\phi$$

$$-2\phi \sin\phi \sin\theta \Big|_0^{2\pi}$$

$$\int_0^{\pi/2} 0 \; d\phi$$

$$= 0$$

$$-2\phi \sin\phi \; \cancel{\cos\theta} \Big|_0^{\pi/2}$$

$$-2\phi \cos\phi \Big|_0^{\pi/2}$$

density is ρ.

$$\int_0^{\pi/2}\int_0^{2\pi}\int_0^{2} \underbrace{\rho\cos\phi}_{z} \underbrace{(\sin\phi\cos\theta)}_{\text{density}} \underbrace{\rho^2 \sin\phi}_{\text{jacobian}} \; d\rho\, d\theta\, d\phi$$

$$\int_0^{\pi/2}\int_0^{2\pi}\int_0^{2} \rho^3 \sin^3\phi \cos\phi \cos\theta \; d\rho\, d\theta\, d\phi$$

$$\int_0^{\pi/2}\int_0^{2\pi}\int_0^{2} \frac{\rho^4}{4} \sin^3\phi \cos\phi \cos\theta$$ ×

$$\int_0^{\pi/2}\int_0^{2\pi} 4\sin^3\phi \cos\phi \cos\theta \; d\theta\, d\phi$$

$$4\sin^3\phi \cos\phi \; (-\sin\theta)\Big|_0^{2\pi}$$

$$\int_0^{\pi/2} 4\sin^3\phi \cos\phi$$

1.5/3

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Q. $\iint_R \sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$x = u+v$

$y = u-v$

$u = \frac{x+y}{2}$ $\qquad v = \frac{x-y}{2}$

$J \left\| \begin{matrix} \frac{dx}{du} & \frac{dx}{dv} \\ \frac{dy}{du} & \frac{dy}{dv} \end{matrix} \right\|$ $\qquad \frac{dx}{du}\frac{dy}{dv} - \frac{dx}{dv}\frac{dy}{du}$

$\frac{1}{2}\left(\frac{-1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ $= \left|\left(1(-1) - (1)(1)\right)\right|$

$-\frac{1}{4} \quad -\frac{1}{4} = -\frac{1}{2}$ $\qquad = |-2| = 2$

$\int_0^u \int_0^1 \sin(u)\cos(v)\, du\, dv$

$\frac{-1}{2} \int_0^1 -\sin v \sin u \quad du$

$\frac{-1}{2} \int_0^1 -\sin u \sin u$

$2 \quad \frac{-1}{2} \int_0^1 -\sin^2 u \quad du$

cant do this

$= \frac{-1}{2} (-\sin^2 u)(\cos u)$

$\frac{-1}{2} (-2 \sin u \cos u)$

$\sin u \cos u \Big|_0^1$

$= 1$

3.0 / 4

3 Maths Tutorial Week 8 Set 6

Find the conservative vector field F which is the gradient of the potential $\phi = x^2y + 2xz^3$

Calculate div F and curl F.

$$\nabla \phi = \frac{d\phi}{dx} i + \frac{d\phi}{dy} J + \frac{d\phi}{dz} K$$

$$= (2xy + 2z^3) i + (x^2) J + (6xz^2) K = \text{conservative vector field } F$$

$$\text{div } F = \frac{df}{dx} + \frac{dg}{dy} + \frac{dh}{dz}$$

$$\Rightarrow \quad 2y + 2z^3 + 0 + 12zx = \cancel{12zx+2}\ 2y + 2z^3 + 12zx$$

X

$$\text{Curl } F = \left(\frac{dh}{dy} - \frac{dg}{dz}\right) i + \left(\frac{df}{dz} - \frac{dh}{dx}\right) J + \left(\frac{dg}{dx} - \frac{df}{dy}\right) K$$

$$\begin{array}{ccc} i & J & K \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 2y+2z^3 & 0 & 12zx \end{array}$$

∂F? should be F.

$$\cancel{\left[\frac{d(12z)}{dy} - \frac{d0}{dx}\right] i + \left[\frac{d\,12z}{dx} - \frac{d(2)}{dz}\right] J + \left[\frac{d(0)}{dx} - \frac{d\,2}{dy}\right]}$$

$$\cancel{0i + 0J + 0K}$$

$$i\left[\frac{d\,12xz}{dy} - \frac{d\,0}{dx}\right] + \left[\frac{d\,12zx}{dx} - \frac{d\,2y+2z^3}{dz}\right] + \frac{d\,0}{dx} - \frac{d\,2y+2z^3}{dy}$$

$$[0] i \qquad [12z - 6z^2] J \quad [-2] K$$

$$= (12z - 6z^2) J - 2K = \text{Curl } F$$

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