

4/2/13

MA1E02 Tutorial Sheet 2.

Week 4 2013

Questions Evaluate the following integrals:

1.

$$\int x \cos(x^2) dx \quad (1)$$

2.

$$\int x^2 \sqrt{x^3 + 1} dx \quad (2)$$

3.

$$\int x^2 e^{x^3} dx \quad (3)$$

4.

$$\int \frac{e^x}{(1 - e^x)^2} dx \quad (4)$$

$$\int \frac{e^x}{u^2} \frac{du}{-e^x}$$

$$\Rightarrow \int \frac{e^x}{-e^x u^2} du$$

$$\Rightarrow \int \frac{1}{-u^2} du$$

$$\Rightarrow \int -u^{-2} du$$

$$\Rightarrow -\int u^{-2} du = -\left[\frac{1}{-1} u^{-1} \right] + C$$

$$\Rightarrow \frac{1}{u} + C$$

$$= \frac{1}{1 - e^x} + C$$

$$u = 1 - e^x$$

$$\frac{du}{dx} = -e^x$$

$$du = dx(-e^x)$$

$$dx = \frac{du}{-e^x}$$

Week 4
4/2/13 Maths Tutorial 2

L I A T E
 $\int \frac{1}{x} dx = \ln|x| + C$
 $\int x^n dx = \frac{x^{n+1}}{n+1} + C$
 $\int e^x dx = e^x + C$
 $\int \sin x dx = -\cos x + C$
 $\int \cos x dx = \sin x + C$

1. $\int x \cos(x^2) dx$ $u = x^2$
 $\frac{du}{dx} = 2x$
 $du = 2x dx$
 $dx = \frac{du}{2x}$

$\int x \cos(u) \left(\frac{du}{2x}\right)$
 $\int \frac{\cos(u)}{2} du$

$\frac{1}{2} \int \cos(u) du$
 $\frac{1}{2} \sin(u) + C$
 $= \frac{1}{2} \sin(x^2) + C$

2. $\int x^2 \sqrt{x^3+1} dx$

$u = \sqrt{x^3+1}$
 $\frac{du}{dx} = \frac{1}{2}(x^3+1)^{-\frac{1}{2}}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x^3+1}}$
 $\frac{3x^2}{2\sqrt{x^3+1}}$

$\int x^2 u dx$
 $\int x^2 u \frac{2\sqrt{x^3+1}}{3x^2} dx$

$3x^2 dx = 2\sqrt{x^3+1} du$
 $dx = \frac{2\sqrt{x^3+1}}{3x^2} du$

$\int u \frac{2\sqrt{x^3+1}}{x^2} du$

$$2. \int x^2 \sqrt{x^3+1} \, dx \quad u = x^3+1$$

$$\frac{du}{dx} = 3x^2$$

$$dx \cdot 3x^2 = du$$

$$\frac{du}{3x^2} = dx$$

$$\int x^2 \sqrt{u} \frac{du}{3x^2}$$

$$= \frac{1}{3} \int u^{1/2} du$$

$$\frac{1}{3} \cdot \frac{1}{3/2} u^{3/2} + C$$

$$\frac{2}{9} \sqrt{u^3} + C$$

$$\frac{2}{9} \sqrt{(x^3+1)^3} + C$$

$$3. \int x^2 e^{x^3} \, dx \quad u = x^3 \quad du = dx \cdot 3x^2$$

$$\frac{du}{dx} = 3x^2 \quad dx = \frac{du}{3x^2}$$

$$\int x^2 e^u \frac{du}{3x^2}$$

$$\frac{1}{3} \int e^u du$$

$$\frac{1}{3} e^u + C$$

$$\frac{1}{3} e^{x^3} + C$$