1/10 ~~T~~DECISIONS - UTILITY

TRAVEL INSURANCE EXAMPLE

	Nothing 0.87	Phone 0.07	Laptop 0.03	P+L 0.02	Everything 0.01	Expected
No	3000	2700	2500	2200	2000	2938
AA	3000 -50	2800	2850	2850	2800	2850
BB	2930	2880	2880	2880	2850	29255
CC	2980	2680	2930	2930	2680	29535

Dominant - one choice that is better than the rest? → NO.
If risk neutral should pick CC option → highest expected value

$U(x) = 1 - e^{-\frac{\cancel{x}}{1000}}$

Utility	Nothing .87	Phone .07	Laptop .03	P+L .02	Everything .01	Expected utility
No	$1-e^{-3}$	$1-e^{-2.7}$	$1-e^{-2.5}$	$1-e^{-2.2}$	$1-e^{-2}$	0.9459
AA	$1-e^{-2.950}$	$1-e^{-2.8}$	$1-e^{-2.85}$	$1-e^{-2.85}$	$1-e^{-2.85}$	0.9469
BB	$1-e^{-2.93}$	$1-e^{-2.88}$	$1-e^{-2.88}$	$1-e^{-2.88}$	$1-e^{-2.85}$	0.9462
CC	$1-e^{-2.98}$	$1-e^{-2.68}$	$1-e^{-2.93}$	$1-e^{-2.93}$	$1-e^{-2.68}$	0.9456

AA is best option here with highest utility of 0.9469, marginally

Fair gamble - Same expected value is bet
Unfair gamble - Expected value of a loss in gamble

- A risk averse person will <u>never</u> accept a fair gamble
- A risk prone/loving person will always accept a fair gamble
- A risk neutral person will be indifferent towards a fair gamble

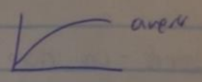
• Given the choice between earning the ~~money~~ sure amount of money through a gamble or certainty:

- Risk averse will opt for certainty
- Risk prone will opt for gamble
- Risk neutral is indifferent

Why does risk averse reject fair gamble? because her marginal utility of money diminishes

⇒ your gain in utility from another 1 euro is less than your loss in utility from loss of 1

- Your MU diminishes, you are risk averse



DECISIONS - UTILITY   CHRISTMAS QUESTION

A.

	Survived $\frac{19}{20}$	Damaged $\frac{1}{20}$	combined
Insure	1500 000 - x	1500 000 - x	1500 000 - x
Don't insure	1500 000	1250 000	1487500

Available actions: insure or do not insure

States of nature: House is not damaged and house is damaged

Consequence: 4 values in the table

B. Risk neutral = 12500

$U(x) = 1 - e^{(-x/1{,}000\,000)}$

Utility	not Damaged $\frac{19}{20}$	Damaged $\frac{1}{20}$	
Insure	$\frac{19}{20}\left(1-e^{-\frac{(1500\,000-x)}{1000\,000}}\right)$	$\frac{1}{20}\left(1-e^{-\frac{1500\,000-x}{1000\,000}}\right)$	$1-e^{-1.5+x}$
Don't Insure	$\frac{19}{20}\left(1-e^{-1.5}\right)$	$\frac{1}{20}\left(1-e^{-1.25}\right)$	0.7737

$0.7737 = 1 - e^{-1.5+x}$

$0.22629 = e^{-1.5+x}$

$\ln(0.22629) = -1.5 + \frac{x}{1000\,000}$

~~$x = \ln(0.22624) + 1.5$~~

$x = 14062.48355$

C. Risk premium = price willing to pay - risk neutral price

$14062 - 12500 = 1562$

D.

Utility	not damaged	damaged	Combined
Insure	$\frac{19}{20}\left(1-e^{-\frac{(1500\,000-x)}{1000\,000}}\right)$	$\frac{1}{20}\left(1-e^{-\frac{(1450\,000-x)}{1000\,000}}\right)$	$\frac{19}{20}\left(1-e^{\left(\frac{-1500\,000-x}{1000\,000}\right)}\right)+\frac{1}{20}\left(1-e^{\left(\frac{-1450\,000-x}{1000\,000}\right)}\right)$
Don't	$\frac{19}{20}\left(1-e^{-1.5}\right)$	$\frac{1}{20}\left(1-e^{-1.25}\right)$	0.7737

$0.7737 = \frac{19}{20}\left(1-e^{\frac{-1500\,000-x}{1000\,000}}\right) + \frac{1}{20}\left(1-e^{\frac{-1500\,000-x}{1000\,000}}\right)$

$0.22629 = \frac{19}{20}e^{\frac{-1500\,000+x}{1000\,000}} + \frac{1}{20}e^{\frac{-1500\,000-x}{1000\,000}}$

$-1.485437916 = \frac{19}{20}\left(\frac{-1500\,000+x}{1000\,000}\right) + \frac{1}{20}\left(\frac{-1450\,000+x}{1000\,000}\right)$

$-29.718 = 19(\;''\;) + (\;''\;)$

$$-24.71875 = -28.5 + 14\left(\frac{x}{10000}\right) - 1.45 + \left(\frac{x}{10000}\right)$$

$$0.23125 = 14\left(\frac{x}{10000}\right) + \frac{x}{10000}$$

$$11562.5 = x$$ = amount willing to pay

E How to calculate utility?

Bisection method:

- A most preferred stimulus and a least preferred stimulus are identified, and subsequently a midpoint stimulus is found that is equidistant from both extremes
- Exponential utility functions of the form $U(x) = 1 - e^{-x/R}$ are often suggested
  R - the risk tolerance, measures the degree of concavity of the utility function
- Takes into account law of diminishing returns

- Give people "would you rather" options
- Use a timboard for number of years to chose which option you prefer/value

15/4/16 DECISIONS - UTILITY

Calculate Expected Utility

	Good Product (0.8)	Bad Product (0.2)	
Inspect	0.9	0.5	$= 0.9(0.8) + 0.5(0.2) = 0.82$
Don't inspect	1.0	0	$= 1.0(0.8) + 0(0.2) = 0.8$

	1-0.000 0205	0.000 005	0.000 02	
	win 0	win 1000	win 100000	
Play	-1	1000-1	100000-1	0.7
Don't Play	0	0	0	0

Optimal option → Do not play, loss of 0.3

Satellite example

	Failure 0.05	Success 0.95	
insure	100-c	100-c	$= 1-e^{\frac{-(100-c)}{50}}$
don't insure	50	100	$= 1-e^{\frac{-50}{50}} + 1-e^{\frac{-100}{50}}$

$$1-e^{\frac{-(100-c)}{50}} = 1-e^{-1} + 1-e^{-2}$$

$$\frac{-100-c}{50} = -1 + -2 + \log(1)$$

$$-100-c = -3(50) + 50\log(1)$$

2

|04|16

48

	Crashes 0.05	Don't Crash 0.95	
Insure	100-s	100-s	100-s
Don't Insure	50	100	97.5

$\Rightarrow 100 - s = 97.5 \Rightarrow s = 2.5m$ as fair price

With Utility	Crashes 0.05	Don't 0.95	
Insure	$0.05\left(1-e^{-\frac{(100-s)}{50}}\right)$	$0.95\left(1-e^{-\frac{(100-s)}{50}}\right)$	$1-e^{-\frac{(100-s)}{50}}$
Don't	$0.05(1-e^{-1})$	$0.95(1-e^{-2})$	$0.05(1-e^{-1})+0.95(1-e^{-2})$

$1-e^{-\frac{(100-s)}{50}} = 0.05(1-e^{-1})+0.95(1-e^{-2})$

$1-e^{-\frac{(100-s)}{50}} = 0.853$

$0.147 = e^{-\frac{(100-s)}{50}}$

$\log(0.147) = -\frac{(100-s)}{50}$

$50\log(0.147) = -100+s$

$s = 50\log(0.147)+100$

$\ln(1) - (-(100-s)/50) = \ln(0.853)$

$\frac{100-s}{50} = \ln(0.853) - \ln(1)$

$-(100-s)/50 = \ln(0.853)$

$-100+s = -0.1589\,(50)$

$-2 + s/100 = -0.1589/(10)$

$\frac{s}{100} =$

$-e^{-\frac{(100-s)}{50}} = -0.147$

$e^{-\frac{(100-s)}{50}} = 0.147$

$-(100-s) = -1.917$

$-100+s = -1.917(50)$

$+\ s = 4.13$

- Company willing to pay 4.12136 million to insure
- Risk premium = $4.13 - 2.5 = 1.62$

15/04/16 DECISIONS

Business woman investment

	Successful 0.1	Unsuccessful 0.9	
Invest	1100 000	−100 000	0.1(1100,000) + 0.9(−100,000) = 20 000
Don't Invest	0	0	0

- If risk neutral → invest.

Utility of $1-e^{\frac{-x}{1000\,000}}$

	Success 0.1	Failure 0.9	
Invest	$1-e^{-1.1}$ ~~3.004~~ 0.667	$1-e^{0.1}$ −0.105	$0.1(1-e^{-1.1}) + 0.9(1-e^{0.1})$ = −0.02795
Don't Invest	0	0	0

→ Should not invest.

→ The business woman is risk averse because the risk neutral decision says to invest ~~anyway~~ even though her utility decision says not to invest.

Extra Questions 2

	Bear Market 0.4	Bull Market 0.6	
Invest A	8000	15 000	8000(0.4) + 15000(0.6) = 12200
Invest B	10500	11 500	10500(0.4) + 11500(0.6) = 11100

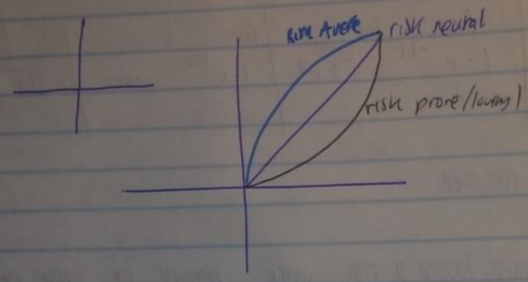
= invest in A

Extra Question 3

	lose ~~0.5~~ 0.52	win 0.48	
		100	$= 0.48(100) = 48$
roulette	0		$= 1(50) = 50$
no roulette	50	50	

- Do not take part in game

- Yes, not going to go to the match either way, may as well go



- No likely to lose money

Extra Question Slide 5.

	destroyed 0.05	not destroyed 0.95	
Insure	80-5	80-5	= 80-5
Don't Insure	40	80	= 78

$80 - s = 78 = 2m$ ask about 2m for insurance

Utility	Destroyed	Not	
Insure	$0.05(1-0.5e^{-\frac{(80-s)}{20}})$	$0.95(1-0.5e^{-\frac{(80-s)}{20}})$	$1-0.5e^{-\frac{(80-s)}{20}}$
Don't	$0.05(1-0.5e^{-40/20})$ ~~0.04773~~	$0.95(1-0.5e^{-80/20})$	

~~[illegible]~~ $0.48791 = 1 - 0.5e^{-\frac{(80-s)}{20}}$

$-0.0121 = -0.5e^{-\frac{(80-s)}{20}}$

15/04/16 DECISIONS UTILITY 5

$0.0242 = e^{-\frac{80-S}{20}}$

$-3.7214 = -4 + \frac{S}{20}$

$5.5719 = S$

Charge 5.57m to insure

ii. Half the risk

	Destroyed 0.05	Survives 0.95	
Insure	80-S	80-S	80-S
Don't	60	80	79 S=1

Utility	Destroyed	Survives	
Insure	$0.05(1-0.5e^{-\frac{(80-S)}{20}})$	$0.95(1-0.5e^{-\frac{(80-S)}{20}}$	$1-0.5e^{-\frac{(80-S)}{20}}$
Don't	$0.05(1-0.5e^{-3}$	$0.95(1-0.5e^{-4})$	0.9900

$0.99 = 1 - 0.5e^{-\frac{(80-S)}{20}}$

$-0.01 = -0.5e^{\frac{-80+S}{20}}$

$0.02 = e^{\frac{-80+S}{20}}$

$-3.912 = -4 + \frac{S}{20}$

$1.7595 = S$ = in-between amount.

- Marginal utility of money decreases → diminishing returns - concave utility func.

01/05/16

Exam Paper Example from Dr. North

$u(x) = 1 - 0.5e^{-x/20} - 0.5e^{-x/200}$ x in million

Utility	destroyed 0.05	survives 0.95
insure	$0.05(1-0.5e^{-(60+x)/20} - 0.5e^{-(60+x)/200})$	$0.95(1-0.5e^{-(60+x)/20} - 0.5e^{-(60+x)/200})$
don't		

$(1 - 0.5e^{-5/20} - 0.5e^{-5/200}) = 0.05(1-0.5e^{-\frac{(60+5)}{20}} =$

$0.05(1-0.5e^{-5/20} - 0.5e^{-5/200}) + 0.95(1-0.5e^{-(60+5)/20} - 0.5e^{-\frac{(60+5)}{200}}) = (1-e^{\frac{-60}{20}} - 0.5e^{\frac{-60}{200}})$

$= (0.9502 - 1)^2$

$0.05(e^{-5/20} + e^{-5/200}) + 0.95(e^{\frac{-(60+5)}{20}} + e^{\frac{-(60+5)}{200}}) = 0.09957$

$0.05\frac{(-5)}{20} + 0.05\frac{(-5)}{200} + 0.95\left(\frac{-60+5}{20}\right) + 0.95\left(\frac{-60+5}{200}\right) = -2.30685$

$0.5(-x) + 0.05(-x) + 0.95(-60+x) + 9.5(-60+x) = -461.37$

$9.9x - 627 = -461.37$

$x = 104.93$

Reassurance?

Insuring the first deal and insuring the second deal both have the same utility value of 0.604 if the minimum premiums are used.

- However the price they are willing to charge in the second deal is substantially less.
- This is because the deal is considered less risky
- This demonstrates how the company is risk averse.
- They do not need to charge as high a premium on the second deal because they are "reassured" by their assets at 60m which comfortably cover the gamble

01/05/16

DECISIONS: REDDY UTILITY

(CHRISTMAS) EXAM 2015

3A

	Delayed 1/20	Not Delayed 19/20	
Insure	1500 000 − x	1500 000 − x	1500 000 − x
Don't Insure	1250 000	1500 000	$\frac{1}{20}(1250000) + \frac{19}{20}(1500000)$

$$1500\,000 - x = \frac{1}{20}(1250\,000) + \frac{19}{20}(1500\,000)$$

$$1500\,000 - x = 62500 + 1425000$$

$x = 125000$ − amount willing to pay if risk neutral

Available Actions: Insure or don't insure

States of Nature: The house is delayed, house is not delayed

Consequences: The four boxes

B

Utility	Delayed 1/20	Not Delayed 19/20	Total
Insure	$1-e^{(-1500000+x)/1000000}$	$1-e^{(x-1500000)/1000000}$	$1-e^{(x-1500000)/1000000}$
Don't Insure	$1-e^{(-1250000)/1000000}$	$1-e^{(-1.5)}$	$\frac{1}{20}(1-e^{-1.25}) + \frac{19}{20}(1-e^{(-1.5)})$

$$\cancel{1} - e^{(x-1500000)/1000000} = \cancel{\tfrac{1}{20}} - \tfrac{1}{20}e^{-1.25} + \cancel{\tfrac{19}{20}} - \tfrac{19}{20}e^{-1.5}$$

$$e^{(x-1500000)/1000000} = \frac{1}{20}e^{-1.25} - \frac{19}{20}e^{-1.5}$$

$$\frac{x-1500000}{\cancel{1000000}} = \cancel{\tfrac{1}{20}(\ldots)} - \cancel{\ldots\tfrac{19}{20}}$$

$$\cancel{x-1500000} = \cancel{\ldots}$$

$$\cancel{0.014325}$$

$$e^{(x-1500000)/1000000} = 0.226248842$$

$$\frac{x-1500000}{1000000} = -1.48184$$

$$x - 1500000 = -1488848.623$$

$$x = 11151.377$$

$x = £11151.38$ willing to pay

C Calculate Risk Premium

RP = Price will to pay − fair price

14101.38 − 12500 = 1601.38 − Risk premium

D Excess of 50000

	Deltoryd $\frac{1}{20}$	Not deltoryd $\frac{19}{20}$	Total
Inke	$1-e^{-1.45+x}$	$1-e^{-1.5+x}$	$\frac{1}{20}(1-e^{-1.45+x}) + \frac{19}{20}(1-e^{-1.5+x})$
Don't Inke	$1-e^{-1.25}$	$1-e^{-1.5}$	$\frac{1}{20}(1-e^{-1.25}) + \frac{19}{20}(1-e^{-1.5})$

$\frac{1}{20}(1-e^{-1.45+x/1000000}) + \frac{19}{20}(1-e^{-1.5-x/1000000}) = 0.226248842$

$\frac{1}{20}(-1.45+\frac{x}{1000000}) + \frac{19}{20}(-1.5+\frac{x}{1000000}) = \cancel{1485898.843} \quad -1.48184$

$= -1485898.843$

$-72500 + \frac{x}{20} + (-1425000) + \frac{19}{20}x = -1485898.843$

$x = 1497500 - 1485898.843$

$x = 11651.157$ = price will to pay

E Estimating individual utilities

- Preference based measures − play a central role in these evaluations
- They allow patients to describe the impact of ill health and also value these descriptions with a single "utility" score or term
- A value of 1 represents full health, 0 represents the value of dead and negatives values (if defined by PBM) represent states worse than dead
- Utility scores can then be calculated for Quality Adjusted Life Years QALY
- Value of health can be derived from people's preference
- Health economists generally support the idea that the amount of utility ~~between~~ ~~two possible~~ an individual gains from something can be observed in their choices

DECISIONS: READY: UTILITY

Probability Premium: the extra odds a decision maker needs to accept the risk, given their utility function
p = value where game is fair
P = probability of success for which gamble is acceptable
Probability Premium $= P - p$

Risk Premium: the difference between what a decision maker will pay to insure a risk and the expected loss associated with that risk

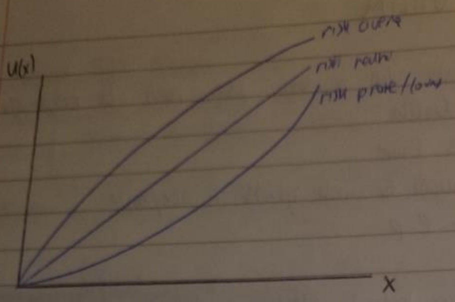
Utility Estimation: Normally a function $1 - e^{\frac{-x}{R}}$ is used where R represents the "risk" tolerance or concavity of the utility function. The higher the risk tolerance, the less risk averse an individual is

Risk premium - Maximum amount a person is prepared to pay to avoid the gamble

Three Attitudes to Risk
Risk Averse, neutral, prone/loving
- A fair gamble allows you to receive the same amount of money through 2 distinct ways: gambling or not gambling.
- Fair gamble - when the sum of the bet is equal to the expected return
- Risk averse person will never accept a fair gamble
- Risk loving person will always accept a fair gamble
- Risk neutral person will be indifferent towards a fair gamble

Given the choice between earning the same amount of money through a gamble or through certainty
- Risk averse → certainty, Risk loving → gamble, risk neutral → indifferent

- Why does risk averse person reject the fair gamble?
Because their marginal utility of money diminishes


U(x)
X
risk averse
risk neutral
risk prone/lover