Exam Paper Laplace Transfo

2018 Q1 $y''(t) + y(t) = t + \delta(t-1) \qquad y(0) = 1 \quad y'(0) = 0$

$$s^2 Y - sy(0) - y'(0) + Y = \frac{1}{s^2} \qquad e^{-s}$$

$$Y(s^2+1) - s = \frac{1}{s^2} + e^{-s}$$

$$Y = \frac{1}{s^2(s^2+1)} + \frac{e^{-s}}{s^2+1} - \frac{s}{s^2+1}$$

$$\cancel{\frac{Ax + Bx}{s^2}}$$

$$Y \quad \frac{1}{s^2} - \frac{1}{s^2+1} + e^{-s}\frac{1}{s^2+1} - \frac{s}{s^2+1}$$

$$\mathcal{L}^{-1}[Y] = t - \sin t \quad u(t-1)\sin(t-1) \quad - \cos t$$

$$\cancel{t - 2\sin t + u(t-1)\sin(t-1)}$$

$$t - \sin t + u(t-1)\sin(t-1) + \cos(t)$$ 2

2012 6 $y'' + 9y = -9u(t-\pi) + 6\delta(t-2\pi) \qquad y(0) = 1 \quad y'(0) = 0$

$$s^2 Y - s + 9Y = \frac{-9e^{-\pi s}}{s} + 6e^{-2\pi s}$$

$$Y(s^2+9) = -\frac{9e^{-\pi s}}{s} + 6e^{-2\pi s} + s$$

$$Y = \frac{-9e^{-\pi s}}{s(s^2+9)} + \frac{6e^{-2\pi s}}{s^2+9} + \frac{s}{s^2+9}$$

$$-e^{-\pi s}\left(\frac{1}{s} - \frac{s}{s^2+3^2}\right) + 2e^{-2\pi s}\frac{3}{s^2+3^2} + \frac{s}{s^2+9}$$

$$-u(t-\pi)(1 - \cos 3(t-\pi)) + 2u(t-2\pi)(\sin 3(t-2\pi)) + \cos 3t$$

2011 Q1 $y'' + 4y = 8u(t-\pi) - 8\delta(t-3\pi) \quad y(0) = 2 \quad y'(0) = 0$

$$s^2 Y - 2s + 4Y = \frac{-8e^{-\pi s}}{s} - 8e^{-3\pi s}$$

$$Y(s^2+4) = \frac{-8e^{-\pi s}}{s} - 8e^{-3\pi s} + 2s$$

$$Y = \frac{-8e^{-\pi s}}{s(s^2+4)} \quad \frac{-8e^{-3\pi s}}{s^2+4} + \frac{2s}{s^2+4}$$

$$2e^{-\pi s}\frac{4}{s(s^2+4)} - 4e^{-3\pi s}\frac{2}{s^2+2^2} + 2\frac{s}{s^2+2^2}$$

$$2e^{-\pi s}\left(\frac{1}{s} - \frac{s}{s^2+2^2}\right) - 4e^{-3\pi s}\frac{2}{s^2+2^2} + 2\frac{s}{s^2+2^2}$$

$$2u(t-\pi)\left[1 - \cos(3(t-\pi))\right] - 4u(t-3\pi)(\sin 2(t-3\pi)) + 2\sin 2t$$

2010 Q6 $y'' + 4y = 4u(t-\pi) - 4\delta(t-3\pi) \quad y(0) = 1 \quad y'(0) = 0$

$$s^2 Y - s + 4Y = \frac{4e^{-\pi s}}{s} - 4e^{-3\pi s}$$

$$Y(s^2+4) = \frac{4e^{-\pi s}}{s} - 4e^{-3\pi s} + s$$

$$Y = \frac{4e^{-\pi s}}{s(s^2+4)} - \frac{4e^{-3\pi s}}{s^2+4} + \frac{s}{s^2+4}$$

$$e^{-\pi s}\left(\frac{1}{s} - \frac{s}{s^2+2^2}\right) - e^{-3\pi s}\frac{4}{s^2+4} + \frac{s}{s^2+2^2}$$

$$u(t-\pi)(1 - \cos 2(t-\pi)) - u(t-3\pi)(\sin 2(t-3\pi)) + \cos 2t$$

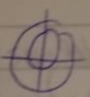
ii Q6 rectangle x, y, z

Spherical $x = \rho \sin\phi \cos\theta$ $y = \rho \sin\phi \sin\theta$ $z = \rho \cos\phi$ jacobian $\rho^2 \sin^2\phi$

$x^2 + y^2 + z^2 = 1$ = sphere of radius 1, center 0,0,0

$x^2 + y^2 + z^2 = 4$ sphere of radius 2 center 0,0,0

$z = 0$ is plane of $z = 0$



$$\int_1^2 \int_0^{\pi/2} \int_0^{2\pi} \rho^2 \sin\phi \quad d\theta \, d\phi \, d\rho$$

$$2\pi \int_0^{\pi/2} \int_1^2 \rho^2 \sin\phi \quad d\rho \, d\phi$$

$$\frac{\rho^3}{3}\Big|_1^2$$

$$\left(\frac{8}{3} - \frac{1}{3}\right)(2\pi) \quad \int_0^{\pi/2} \sin\phi$$

$$-\cos\phi \Big|_0^{\pi/2}$$

$$(1)(2\pi)\left(\frac{7}{3}\right) = 14\pi/3$$

vi. $\delta(x, y, z) = \dfrac{e^{-(\rho^2 \sin^2\phi \cos^2\theta + \rho^2 \sin^2\phi \sin^2\theta + \rho^2 \cos^2\phi)}}{\sqrt{\quad}}$

$$\int_0^{2\pi} \int_0^{\pi/2} \int_1^2 \frac{e^{-\rho^2}}{\rho} \rho^2 \sin\phi \quad d\rho \, d\phi \, dt.$$ let $u = \rho^2$ $du = -2\rho$

$$\int \int \int_1^2 \rho e^{-\rho^2} \quad d\rho \, d\phi \, d\theta$$

$$\int_1^2 -\frac{1}{2} e^u \, du \quad \int_0^{\pi/2} \sin\phi \int_0^{2\pi} d\theta$$

$$(2\pi)(1) \left(-\frac{1}{2}\right)\left[e^{-\rho^2}\right]_1^2$$

$$(-\pi)\left(e^{-4} - e^{-1}\right) = \text{mult}$$

2010 Q5. find x, y, z

Spherical $x = \rho \sin\phi \cos\theta$ $y = \rho \sin\phi \sin\theta$ $z = \rho \cos\phi$ $J_{ac} = \rho^2 \sin\phi$

$x^2 + y^2 + z^2 = r^2$ $\int_0^{2\pi} \int_0^{\pi/2} \int_0^r \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

$\left(\frac{r^3}{3}\right) (1)(2\pi) = \frac{2\pi r^3}{3}$

$\int_0^{2\pi} \int_0^{\pi/2} \int_2^3 \frac{e^{-r^2}}{r} r^2 \sin\phi$

let $u = -r^2$

$du = -2r$

$\int_0^{2\pi} d\theta \int_0^{\pi/2} \sin\phi \int_2^3 r e^{-r^2}$

$-\frac{1}{2} \int_2^3 e^u \, du$

$(2\pi)(1) \left(-\frac{1}{2}\right) \left[e^u\right]_2^3$

$(\pi)(e^3 - e^{-2})$