25/3/14 Solutions to Assignment Meng Xu

Q1. Acceptance - Rejection

$f(x) = \frac{2}{\sqrt{2\pi}} e^{-x^2/2}$

$g(x) = \lambda e^{-\lambda x}$

$h(x) = \frac{f(x)}{g(x)} = \frac{2}{\lambda\sqrt{2\pi}} e^{\left(-\frac{x^2}{2} + \lambda x\right)}$

Maximised when exponent is maximised

$\frac{d\left(\frac{-x^2}{2} + \lambda x\right)}{dx} = -x + \lambda$

$-x + \lambda = 0$

$\Rightarrow x = \lambda$

$c = \text{Supremum}\left(\frac{f(x)}{g(x)}\right) \frac{2}{\lambda\sqrt{2\pi}} e^{\frac{\lambda^2}{2}} = c(\lambda)$

want c minimised, so diff wrt λ

$\frac{dc(\lambda)}{d\lambda} = \frac{-2}{\lambda^2\sqrt{2\pi}} e^{\lambda^2/2} + \frac{2}{\sqrt{2\pi}} e^{\lambda^2/2}$

Set to 0 $\Rightarrow e^{\lambda^2/2}\left(1 - \frac{1}{\lambda^2}\right) = 0$

$\Rightarrow 1 - \frac{1}{\lambda^2} = 0 \quad 1 = \frac{1}{\lambda^2} \Rightarrow \lambda^2 = 1$

$\lambda = 1 \quad (\lambda > 0)$

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Q2 $f(x) = 12x^2(1-x)$

$g(x) = 1$ for $0 \leq x \leq 1$ uniform

$\frac{f(x)}{g(x)} = 12x^2(1-x) = 12x^2 - 12x^3$

Maximise: differentiate $24x - 36x^2 = 0$

$2x - 3x^2 = 0$

$x = 0$ or $x = 2/3$

Maximum when $x = 2/3$ min at $x = 0$

$c = 12\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right) = \frac{16}{9}$ $(48/27)$

Algorithm

1. Generate u_1 from $U(0,1)$ (proposal)
2. Generate U_2 from $U(0,1)$ (acceptance probability)
3. Accept u_1 if $U_2 \leq 12(u_1)^2(1-u_1) \times \frac{9}{16}$

Otherwise reject and start again

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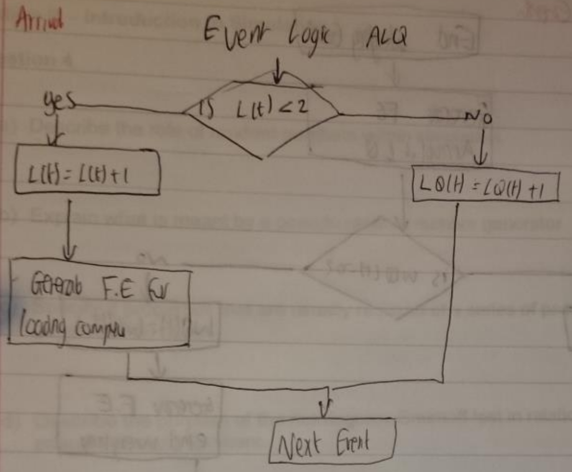
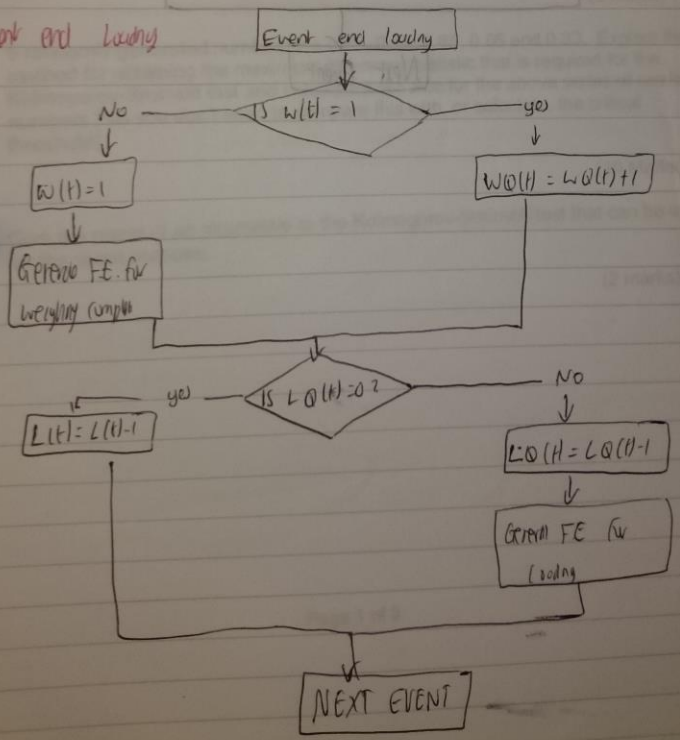
25/3/14 Solutions to Assignment Mony Sc

Q 3 Arrival



Event end Loading



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LIVE!

End Weighing Event.