2012 MATHS 1 EXAM PAPER

6 A If path independent $\frac{df}{dy} = \frac{dy}{dx}$

$4x - 2\pi^3 \sin(x)$ $\qquad$ $-4x - 6x$

$4x \quad +6y \quad \stackrel{?}{=} -(-4x - 6y)$

✓ path independent

B $\frac{d\phi}{dx} = (-2x^2 + 4xy + 2\pi^3\cos(x) + 3y^2)$ $\qquad$ $\frac{du}{dy} = -(-2x^2 - 6xy + 3\pi^3\sin(y))$

integrate WRT x.

$-\frac{2x^3}{3} + 2x^2y + 2\pi^3\sin(x) + 3xy^2 + c_1(y) = \phi$

where $c_1(y)$ dependent on y only

Diff wrt y:

$2x^2 + 6xy + \frac{d(c_1(y))}{dy} = -(-2x^2 - 6xy + 3\pi^3\sin y)$

$\frac{d(c_1(y))}{dy} = -3\pi^3\sin y$

integrate $c_1(y) = 3\pi^3\cos(y)$

$\Phi = -\frac{2x^3}{3} + 2x^2y + 2\pi^3\sin(x) + 3xy^2 + 3\pi^3\cos(y) + C$

where C any constant

C $\phi(\pi, \pi/2) - \phi(-\pi/2, \pi)$

$\left[-\frac{2\pi^3}{3} + \pi^3 + 2\pi^3\sin(\pi) + \frac{3\pi^3}{2}\right] - \left[\frac{\pi^3}{12} + -\pi^3 - 2\pi^3 - \frac{3\pi^3}{2} - 3\pi^3\right]$

$\left[\frac{11\pi^3}{6}\right] - \left[-\frac{89}{12}\pi^3\right]$

$\cancel{\frac{37\pi^3}{4}}$

$(\pi, \pi/2) \to (-\frac{\pi}{2}, \pi)$

$-\frac{2\pi^3}{3} + 2(\pi^2)(\frac{\pi}{2}) + 2\cancel{\pi^3 \sin\pi} + 3\pi(\frac{\pi}{2})^2 + 3\pi^3\cos\pi$

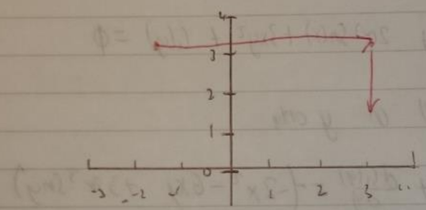
$-\frac{23}{12}\pi^3 - \left[\frac{-2\left[-\frac{\pi}{2}\right]^3}{3} + 2\left[\left[-\frac{\pi}{2}\right]^2(\pi)\right] + 2\pi^3\sin\left(-\frac{\pi}{2}\right) + 3\left(-\frac{\pi}{2}\right)(\pi^2) + 3\pi^3\cos\pi\right.$

$\left[\frac{\pi^3}{12} + \frac{\pi^3}{2} - 2\pi^3 - \frac{3}{2}\pi^3 - 3\pi^3\right]$

$-\frac{23}{12}\pi^3 - \left[-\frac{71}{12}\pi^3\right]$

$= 4\pi^3$

D



ii. C_1: $(-\frac{\pi}{2}, \pi)$ and (π, π) so $r_1 = (1-t)r_0 + t(r_1$ for $0 \le t \le 1$

$(1-t)(-\frac{\pi}{2}, \pi) + t(\pi, \pi)$

$= (\frac{3t\pi}{2} - \frac{\pi}{2}, \pi)$

C_2: (π, π) and $(\pi, \pi/2)$

$(\pi, \pi)(1-t) + t(\pi, \pi/2)$

$(\pi, \pi - t\frac{\pi}{2})$

1. $(\frac{3t\pi}{2} - \frac{\pi}{2}, \pi)$ $dy = 0$, no second part.

$-2\left[\frac{3t\pi}{2} - \frac{\pi}{2}\right]^2 + 4\left[\frac{3t\pi}{2} - \frac{\pi}{2}\right][\pi] + 2\pi^3\cos\left[\frac{3t\pi}{2} - \frac{\pi}{2}\right] + 3[\pi^2]$

$-2\left[\frac{9t^2\pi^2}{4} - \frac{6t\pi^2}{4} + \frac{\pi^2}{4}\right] + 4\left[\frac{3t\pi^2}{2} + \frac{\pi^2}{2}\right] + 2\pi^3\cos\left[\frac{3t\pi}{2}\right] + 3\pi^2$

$-\frac{9t^2\pi^2}{2} + 3t\pi^2 - \frac{\pi^2}{2} + 6t\pi^2 - 2\pi^2\cos\left[\frac{3t\pi}{2}\right] + 3\pi^2$

$\int_0^1 9t\pi^2 + \frac{5}{2}\pi^2 - \frac{9t^2\pi^2}{2} - 2\pi^2\cos\left[\frac{3t\pi}{2}\right]\ dt$

2012 Exam Paper Q6 Meth 1.

$$9\pi^2\frac{t^2}{2} + \frac{5}{2}t\pi^2 - \frac{9\pi^2}{2}\frac{t^3}{3} - \frac{2\cos\left(\frac{3\pi t}{2}\right)}{3\pi}\Big|_{t=0}^{1}$$

$$\left[\frac{9\pi^2}{2} + \frac{5}{2}\pi^2 - \frac{9\pi^2}{6}\right] - \left[+\frac{2\pi}{3\pi}\right]$$

$$\left(\frac{11}{2}\pi^2 - \frac{2}{3\pi}\right).$$

$$C_2 : \left(\pi, \frac{2\pi - t\pi}{2}\right)$$

$$+2x^2 + 6xy + 3\pi^3\sin(y).$$

$$\left[2(\pi^2) + 6(\pi)\left(\frac{2\pi - t\pi}{2}\right) + 3\pi^3\sin\left(\frac{2\pi - t\pi}{2}\right)\right] \frac{-\pi}{2}$$

$$-\frac{\pi}{2}\int_0^1\left[2\pi^2 + \frac{12\pi^2 - 6t\pi^2}{2} + 3\pi^3\sin\left(\frac{2\pi - t\pi}{2}\right)\right]$$

$$2t\pi^2 + 6t\pi^2 - \frac{3t^2\pi^2}{2} - \frac{3\pi^3}{2}\left[-\frac{2\cos\left(\frac{\pi t}{2}\right)}{\pi}\right]\Big|_0^1$$

$$\frac{-\pi}{2}\left[2\pi^2 + 6\pi^2 - \frac{3\pi^2}{2}\right] - \left[\frac{-3\pi^3}{2\pi}\right]$$

$$\frac{13}{2}\pi^2 + \frac{3}{2}\pi^2$$

$$\frac{-\pi}{2}\left[8\pi^2\right] = -\frac{8\pi^3}{2} = -4\pi^3$$

$$\frac{11\pi^2}{2} - \frac{2}{3\pi} \quad -4\pi^3$$

1.

2012 Maths 3 Exam Paper.

Q1 $y'' + 9y = -9u(t-\pi) + 6\delta(t-2\pi) \quad y(0)=1 \quad y'(0)=0$

$s^2 \mathcal{L}(y) - sy(0) - y'(0) + 9\mathcal{L}(y) = -\frac{9e^{-\pi s}}{s} + 6e^{-2\pi s}$

$(s^2+9)\mathcal{L}(y) = -\frac{9e^{-\pi s}}{s} + 6e^{-2\pi s} + s$

$$\mathcal{L}(y) = -\frac{9e^{-\pi s}}{s(s^2+9)} + \frac{6e^{-2\pi s}}{s^2+9} + \frac{s}{s^2+9}$$

partial fraction $e^{-\pi s}\left(\frac{s}{s^2+9} - \frac{1}{s}\right) + \frac{6e^{-2\pi s}}{s^2+9} + \frac{s}{s^2+9}$

$$\frac{e^{-\pi s}s}{s^2+9} - \frac{e^{-\pi s}}{s} + \frac{6e^{-2\pi s}}{s^2+9} + \frac{s}{s^2+9}$$

$u(t-\pi)\cos(3t-3\pi) - u(t-\pi) + 6u(t-2\pi)\sin(3t-6\pi) + \cos 3t$

$= \cos 3t + 2u(t-2\pi)\sin(3t) - u(t-\pi)\cos(x) - u(t-\pi)$

2a $F(x,y,z) = \sqrt{z^2 + x - y + 2\cos(3y-2x)}$

$\frac{\partial F}{\partial x} = \frac{1}{2}\frac{(1 - 2\sin(3y-2x)(-2))}{\sqrt{z^2+x-y+2\cos(3y-2x)}}$ at $(3,2,-1) = \frac{1}{4}$

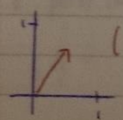
$\frac{\partial F}{\partial y} = \frac{1}{2}\frac{(-1 + 2\sin(3y-2x)(3))}{\sqrt{z^2+x-y+2\cos(3y-2x)}}$ at $(3,2,-1) = \frac{-1}{4}$

$\frac{\partial F}{\partial z} = \frac{1}{2}\frac{(2z)}{\sqrt{z^2+x-y+2\cos(3y-2x)}}$ at $(3,2,-1) = -\frac{1}{2}$

Divide by magnitude $\|F\|$ to get unit vector

$\|\vec{v}\| = \sqrt{\frac{1}{16} + \frac{1}{16} + \frac{1}{4}} = \frac{\sqrt{6}}{4}$

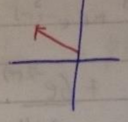
$u = \frac{\nabla F}{\|\nabla F\|} = \left(\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right)$ = unit vector

b  $\left(\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right)$ Projection onto xz plane, ignore y-coordinate

2

2c. Since u is direction which increases most rapidly, −u is decreasing rapidly ⇒ $u = \left(\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$

D. Point is $\left(-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$



E. Rate of change is equal to $\pm\|\nabla f\| = \pm\frac{\sqrt{6}}{4}$

3 A. Rewrite as $\frac{1}{3}\ln(3\cos(2x-y)+6x^2-6xy^2-y^3+31) - \ln(2)$

$$\frac{df}{dx} = \frac{1}{3}\cdot\frac{-3\sin(2x-y)(2)+12x^2-6y^2}{3\cos(2x-y)+6x^2-6xy^2-y^3+31} \quad \text{at } (1,2)$$

$$= -\frac{1}{2}$$

$$\frac{df}{dy} = \frac{1}{3}\cdot\frac{-3\sin(2x-y)(-1)-12xy-3y^2}{3\cos(2x-y)+6x^2-6xy^2-y^3+31} = -\frac{3}{2}$$

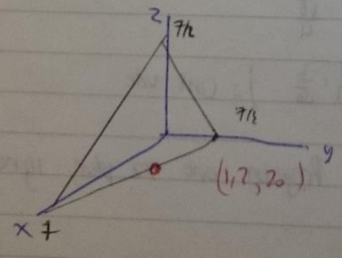
$$z_0 = f(1,2) \Rightarrow \frac{1}{3}\ln(3\cos(0)+5 - \ln(2)$$
$$\tfrac{1}{3}\ln(8) - \ln(2) = 0$$

$$z = z_0 + f_x(x-x_0) + f_y(y-y_0)$$
$$z = 0 + \frac{-1}{2}(x-1) + \frac{-3}{2}(y-2) \qquad z = \frac{-(x+3y-7)}{2}$$

B. Points of intersection

for x-axis	$y=z=0$ →	$x-7=0$	$(7,0,0)$
y-axis	$x=z=0$ →	$3y-7=0$	$(0,\frac{7}{3},0)$
z-axis	$y=x=0$ →		$(0,0,\frac{7}{2})$

C.



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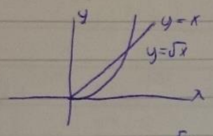
3D Parametric eq^n.
write in form $2z = -x - 3y + 7$
$x + 3y + 2z = 7$.
Normal line passes through $(1,2,0)$ with direction $(1,3,2)$
So $x = 1 + t$ $\quad y = 2 + 3t$ $\quad z = 0 + 2t$

E. Normal line should be perpendicular to tangent plane at given point

Q4 A Projection lies between $y = x$ and $y = \sqrt{x}$



B. Volume $= \int_0^1 \int_x^{\sqrt{x}} \frac{e^{1-x} - \sin(\pi x/2)}{1-x} y \, dy \, dx$

When it cut to inner integral → $\int_x^{\sqrt{x}} y = \frac{y^2}{2}\Big|_x^{\sqrt{x}} = \frac{x - x^2}{2} = \frac{x(1-x)}{2}$

$$\int_0^1 \frac{e^{1-x} - \sin(\pi x/2)}{(1-x)} \cdot \frac{(x)(1-x)}{2} \, dx$$

Integration by parts
$$\left[\left(-e^{1-x} + \frac{2}{\pi}\cos\frac{\pi x}{2}\right)\frac{x}{2}\right]_0^1 - \int_0^1 \left(-e^{1-x} + \frac{2}{\pi}\cos\frac{\pi x}{2}\right)\frac{1}{2}dx$$

$$= \left[\left(-e^{1-x} + \frac{2}{\pi}\cos\frac{\pi x}{2}\right)\left(\frac{x}{2}\right) - \left(e^{1-x} + \frac{4}{\pi^2}\sin\frac{\pi x}{2}\right)\cdot\frac{1}{2}\right]_0^1$$

$\frac{e}{2} - \frac{2}{\pi^2} - 1$

4C – Ignore

Q5A If path independent $\frac{df}{dy} = \frac{dg}{dx}$

$6y$

2

5b $\frac{d\varphi}{dx} = 3y^2 + 2\pi^3\cos x$ $\quad \frac{d\varphi}{dy} = 6xy - 3\pi^{-3}\sin y$

$\Phi = 3xy^2 + 2\pi^3\sin x + C_1(y)$

where $C_1(y)$ depends on y and y only

$\frac{d\varphi}{dy} = 6xy + \frac{dC_1(y)}{dy} = 6xy - 3\pi^3 \cancel{\cos} \sin y$

$\frac{dC_1(y)}{dy} = -3\pi^3 \sin y$

$C_1(y) = 3\pi^3\cos y$

$\Phi = 3xy^2 + 2\pi^3\sin x + 3\pi^3\cos y.$

c $\Phi(\pi, \pi/2) - \Phi(-\pi/2, \pi)$

$3\pi^3/4 - (-13\pi^3/2) = 29\pi^3/4$

D First line segment from $(-\pi/2, \pi)$ to (π, π) and its parametric eqn is

$r(t) = (t, \pi) \quad -\pi/2 \le t \le \pi$

Given $x = t$ and $y = \pi$

$L_1: \int_{-\pi/2}^{\pi} (3\pi^2 + 2\pi^3\cos t)\,dt = [3\pi^2 t + 2\pi^3\sin t]_{-\pi/2}^{\pi} = 13\pi^3/2$

Second: from (π, π) to $(\pi, \pi/2)$ $\quad r(t) = (\pi, t) \quad \pi \le t \le \pi/2$

$L_2: \int_{\pi}^{\pi/2} (6\pi t - 3\pi^3\sin t)\,dt = [3\pi t^2 + 3\pi^3\cos t]_{\pi}^{\pi/2} = 3\pi^3/4$

$L_1 + L_2 = 29\pi^3/4$

2012 MATHS 1 Q6

Q6 A. rectangular (x, y, z)

cylindrical $(\rho\cos\theta, \rho\sin\theta, z)$

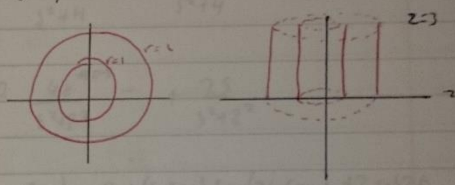
$x = \rho\cos\theta \qquad y = \rho\sin\theta$

b i. $x^2+y^2=1$ is cylinder centre $(0,0)$ radius 1

ii $x^2+y^2=4$ is ~~circle~~ cylinder centre $(0,0)$ radius 2.

iii $z=0$ is plane the (x,y) plane $\quad z=3$ is all xy plane

iv



v. $0 \le z \le 3 \qquad 0 \le \theta \le 2\pi \qquad 1 \le \rho \le 2$ $\quad$ ρd jaco

$$\int_0^{2\pi}\int_0^{3}\int_1^{2} \rho \, d\rho \, dz \, d\theta$$

$$\frac{\rho^2}{2}\Big|_1^2$$

$$\frac{3}{2}\int_0^{2\pi}\int_0^{3} dz\,d\theta$$

$$= \frac{9}{2}\int_0^{2\pi} d\theta$$

$$= 9\pi$$

vi mass $f(x,y,z) = e^{-(x^2+y^2+z)} \qquad e^{-\rho^2 - z}$

$$\int_0^{2\pi}\int_1^{2}\int_0^{3} \rho e^{-\rho^2} e^{-z} \, dz\,d\rho\,d\theta$$

$$\rho e^{-\rho^2} e^{-z}\Big|_0^3$$

$$\rho e^{-\rho^2}(1-e^{-3})$$

$(1-e^{-3}) \int_0^{2\pi}\int_0^{2} \rho e^{-\rho^2}\, d\rho\, d\theta$

$(1-e^{-3}) \int_0^{2\pi} \frac{\rho e^{-\rho^2}}{2}\Big|_0^{2}$

$(1-e^{-3}) \cdot \frac{(e^{-1}-e^{-4})}{2} \cdot 2\pi$

$= (1-e^{-3})^2 \cdot e^{-1}\pi$