22/04/15

FORECASTING

EXAM PAPER 2013

Q1 A - require(fma) load fma package
- tsdisplay(eggs) display eggs time series as well as ACF and PACF
- HW() call Single Exponential Smoothing algorithm SSE
- DES SSE ...

B First algorithm is better ⇒ lower SSE

$SSE = \sqrt{\frac{1}{2}\Sigma(y_i - F_t)^2}$ Minimum char for value of α

Could use MAPE : $\left| \frac{1}{n} \Sigma \left| \frac{e_t}{y_t} \right| \right|$

C SES Algorithm

Initialise $F_1 = y_1$ choose $0 < \alpha < 1$

Forecasts: $F_{t+1} = F_t + \alpha(y_t - F_t)$

Until no more observation left then $F_{t+h} = F_{t+1}$ $\forall h \geq 1$

D It would not work ⇒ data has a trend

E Function tries multiple values for an ARIMA model with changing parameters p,d,q and selects the model with lowest AIC/BIC

F. Differencing can help remove the trend in the data and give a stationary mean

G. $(1-B)y_t = C + \varepsilon_t$

H AIC or BIC which reward model fit but penalise the complexity of a model

2A Trend - Upward or downward — General upward trend
Seasonality - Repeating value — no obvious seasonality
Error / noise

B $\text{ARIMA}\overset{p\,d\,q}{(2,1,2)}\overset{P\,D\,Q}{(1,0,1)}_{\overset{s}{4}}$

$$(1-\phi_1 B-\phi_2 B^2)(1-\beta_1 B^4)(1-B)^1(1-B^4)^1 = c + (1-\omega_1 B-\omega_2 B^2)(1-\theta B^4)\,\varepsilon_t$$

C. $\varepsilon_i \sim N(0,\sigma^2)$
ε_i and ε_j independent if $i \neq j$.

D Yes, trend has been removed, now stationary in mean
No spikes on either ACF or PACF.

E i Some ts are not stationary in mean and variance
Amplitude increasing over time indicative of non stationary mean
Try a mathematical transform such as $\log$ of $\sqrt{x}$ or $\sqrt[3]{x}$ to make the variance stationary

ii Yes, trend has been removed by ARIMA now
Variance has been removed by log function, as shown by the residuals in TS plot.

iii Use AIC or BIC
- Reward a better fitting model
- Penalise complexity of model
- Choose lowest AIC or BIC value to select model

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3 A AR - autoregressive

B $y_t = \phi_0 + \phi_1 y_{i-1}$

C We have to determine ϕ_0, ϕ_1 the weights
$\varepsilon_t \sim N(0, \sigma^2)$
$\varepsilon_i\ \varepsilon_T$ are independent if $i \neq s$.

D An AR(1) model is a linear regression model with two parameters
Define $x_i = y_{i-1}$ $\Rightarrow y_i = \phi_0 + \phi_1 x_i + \varepsilon_i$

E Can use least squares algorithm

$$\hat{\phi} = (X^T X)^{-1} X^T y$$

$$\begin{array}{cccc} y & X & \phi & \varepsilon_i \\ \begin{bmatrix} y_1 \\ y_2 \\ y_n \end{bmatrix} & \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \\ 1 & x_n \end{bmatrix} & \begin{bmatrix} \phi_0 \\ \phi_1 \end{bmatrix} & \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{bmatrix} \end{array}$$

F. $y_{n+1} = \hat{\phi}_0 + \hat{\phi}_1 y_n + \varepsilon_{n+1}$
$y_{n+1} = \hat{\phi}_0 + \hat{\phi} y_n \pm 2s$ where $s^2 = \frac{\sum_{i=1}^{n} \varepsilon_i^2}{n-3}$

G $y_{n+2} = \hat{\phi}_0 + \hat{\phi}_1 y_{n+1} + \varepsilon_{n+2}$
$= \hat{\phi}_0 + \hat{\phi}_1 (\phi_0 + \hat{\phi}_1 y_n + \varepsilon_{n+1}) + \varepsilon_{n+2}$
$= \hat{\phi}_0 + \hat{\phi}_1 \hat{\phi}_0 + \hat{\phi}_1^2 y_n + \hat{\phi} \varepsilon_{n+1} + \varepsilon_{n+2}$

$\mathbb{E}[y_{n+2}] = \hat{\phi}_0 + \hat{\phi}_1 \hat{\phi}_0 + \hat{\phi}_1^2 y_n$

$\text{Var}[y_{n+2}] = \mathbb{E}[(\hat{\phi}_1 \varepsilon_{n+1} + \varepsilon_{n+2})^2] = \hat{\phi}^2 \underbrace{\mathbb{E}[\varepsilon_{n+1}^2]}_{=s^2} + 2\hat{\phi}_1 \underbrace{\mathbb{E}[\varepsilon_{n+1}\varepsilon_{n+2}]}_{=0} + \underbrace{\mathbb{E}[\varepsilon_{n+2}^2]}_{=s^2}$

$= \hat{\phi}_1 s^2 + s^2$

$y_{n+2} = \hat{y}_{n+2} \pm 2s\sqrt{(\hat{\phi}_1^2 + 1)}$

H. We know $y_{n+1} = \hat{\phi}_0 + \hat{\phi}_1 y_n \pm 2s$

$$y_{n+2} = \hat{\phi}_0 + \hat{\phi}_0\hat{\phi}_1 + \hat{\phi}_1^2 y_n \pm 2s\sqrt{(\hat{\phi}_1^2+1)}$$

$$\begin{aligned} y_{n+3} &= \hat{\phi}_0 + \hat{\phi}_1 (y_{n+2}) + \varepsilon_{n+3} \\ &= \hat{\phi}_0 + \hat{\phi}_1 (\hat{\phi}_0 + \hat{\phi}_1 y_{n+1} + \varepsilon_{n+2}) + \varepsilon_{n+3} \\ &= \hat{\phi}_0 + \hat{\phi}_1 (\hat{\phi}_0 + \hat{\phi}_1 (\hat{\phi}_0 + \hat{\phi}_1 y_n + \varepsilon_{n+1}) + \varepsilon_{n+2}) + \varepsilon_{n+3} \\ &= \underbrace{\hat{\phi}_0 + \hat{\phi}_1\hat{\phi}_0 + \hat{\phi}_1^2\hat{\phi}_0 + \hat{\phi}_1^3 y_n}_{\text{forecast}} + \underbrace{\hat{\phi}_1^2\varepsilon_{n+1} + \hat{\phi}_1\varepsilon_{n+2} + \varepsilon_{n+3}}_{\text{error}} \end{aligned}$$

$$y_{n+3} = \hat{\phi}_0 + \hat{\phi}_1\hat{\phi}_0 + \hat{\phi}_1^2\hat{\phi}_0 + \hat{\phi}_1^3 y_n \pm 2s\sqrt{1+\hat{\phi}_1^2+\hat{\phi}_1^4}$$

$$\Rightarrow y_{n+k} = \hat{\phi}_0\left(\sum_{i=1}^{k}\hat{\phi}_1^{i-1}\right) + \hat{\phi}_1^k y_n + \sum_{i=1}^{k}\hat{\phi}^{i-1}\varepsilon_{n+k-i+1}$$

implying $y_{n+k} = \hat{\phi}_0\left(\sum_{i=1}^{k}\hat{\phi}_1^{i-1}\right) + \hat{\phi}_1^k y_n \pm 2s\sqrt{\sum_{i=1}^{k}\hat{\phi}_1^{2(i-1)}}$

I. Differencing removed the trend within the data

AR(P)$_s$ removed the seasonality within the data

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4 A require(fma) get package fma
tsdisplay plot timeseries + acf and pacf.
HoltWinters(exp...) SES model and return SSE
" DES and return SSE value

B SSE of SES was lower choose SES better fit.

D S HW would not work, downward trend but no seasonality

E remove trend.

F. $(1-B)y_t = c + \varepsilon_t$

G AIC, BIC

Q5. a. trend seasonality error

b Upward trend, non stationary mean or variance.

C $\varepsilon_t \sim N(0, \sigma^2)$ $\varepsilon_i \neq \varepsilon_j$ $i \neq j$

D yes, trend, seasonality removed

E i make stationary in variance

ii yes.

iii AIC, BIC

bolgerdoe tcdie.