7/05/16 ALSM 2 EXAM NOTES: EXPONENTIAL DISTRIBUTION

- times between events in a poisson proc
- Time until failure
- Times y are non-negative $y \in \mathbb{R}^+$
- Some subjects may survive beyond the study and their failure is not observed - in this case data is "censored"

$P(y|\theta) = \lambda e^{-\lambda y} \quad y \in \mathbb{R}^+ \quad \theta \in \mathbb{R}^{+*}$

Distribution?

Always positive (+)(+)

Integrates to 1? $\int_0^\infty \lambda e^{-\lambda y} dy$

$= \lambda \frac{-e^{-\lambda y}}{\lambda}$

$= \left[-e^{-\lambda y}\right]_0^\infty \quad [0 - (-1)]$

$= 0 + 1 = 1$ ✓

Expectation?

$\mathbb{E}[x] = \int_0^\infty y \lambda \exp(-\lambda y) dy$ Choosing $y = \lambda x$

$= [-x \exp(-\lambda x)]_0^\infty + \int_0^\infty \exp(-\lambda x) dx$ (integrating by parts)

$= (0 - 0) + \left[\frac{-1}{\lambda} \exp(-\lambda x)\right]_0^\infty$

$= 0 + (0 + \frac{1}{\lambda})$

$= 1/\lambda$

Value that maximises

$\frac{d\, p(y|\theta)}{d(\theta)} = \frac{d\, (\lambda \exp[-\lambda y])}{d(\theta)} = \exp[-\lambda y] + \lambda(-y \exp(-\lambda y))$

$= \exp(-\lambda y) - \lambda y \exp(-\lambda y)$

$\exp(-\lambda y)[1 - \lambda y] = 0$

$\lambda = 1/y$

where y is the mean time of failure

Members of exponential family?

$\exp[\log(\lambda e^{-\lambda y})]$ $\quad \exp \log(\lambda) \log \exp(-\lambda y)$

$\exp[\ y \log \lambda - \lambda y]$ $\quad = \log(\lambda) - \lambda y$

Survival Function

Probability of failure $P(0 \le t \le y) = \int_0^y p(t|\theta)dt = \int_0^y \lambda \exp(-\lambda y)\, dy$

$= [\lambda \frac{1}{\lambda} \exp(-\lambda y)]_0^y$

$= 1 - \exp(-\lambda y) = F(y)$

$S(y) = 1 - F(y) = 1 - (1 - \exp(-\lambda y))$

$= \exp(-\lambda y)$

At 0 is same probability of failure as survival

- When $F(y) = S(y) = 0.5$ called the median point of survival

Hazard Function

- Prop $h(y) = \frac{P(y \le t \le y + \delta y)}{P(y \le t)}$ chance of failure between time y and $y+t$ / chance surviving beyond time (y) $S(y)$

$h(y)$ = density/survival $= \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$

- Chance of someone dying in the next minute, given a survival rate and given you have not died already
- Hazard function is a constant with respect to time - memoryless property
- When y_i is censored, use $S(y_i)$ instead of pdf $(p(y_i|\theta_i))$

δ_i an indicator variable, 0 if y_i is censored, 1 if it is N

$L(\theta) = \prod_{i=1}^{N} p(y_i|\theta_i)^{\delta_i} S(y_i)^{1-\delta_i}$

exponential $= \prod_{i=1}^{N} [\theta \exp(-\theta y)]^{\delta_i} [\lambda e^{-\lambda y}]^{1-\delta_i}$

ALSM 2

Show it is

y positive

$\int_0^\infty \lambda$

Probabil

Probabil

Hazar

Expo