205

(a) $y''(t) + y(t) = t + \delta(t-1)$ $y(0)=1$ $y'(0)=0$

$s^2 Y(s) - s - y'(0) + Y(s) = \frac{1}{s^2} + e^{-s}$ $\delta(t-1) \to e^{-s}$

$(s^2+1)Y(s) = \frac{1}{s^2} + e^{-s} + s$

$Y(s) = \frac{1}{s^2(s^2+1)} + \frac{e^{-s}}{(s^2+1)} + \frac{s}{s^2+1}$

$y(t) = t - \sin(t) + u(t-1)\sin(t-1) + \cos(t)$

$u(t-a) \to \frac{e^{-as}}{s}$

$\int_0^\infty \delta(t-a)\,dt = 1$

$\int_0^\infty f(t)\,\delta(t-a)\,dt = f(a)$

$\mathcal{L}[\delta(t-a)] = \int_0^\infty e^{-st}\delta(t-a)\,dt = e^{-as}$

$\mathcal{L}^{-1}(e^{-as}) = \delta(t-a)$

202 $y'' + 9y = -9u(t-\pi) + 6\delta(t-2\pi)$ $y(0)=1$ $y'(0)=0$

$s^2\mathcal{L}(y) - sy(0) - y'(0) + 9\mathcal{L}(y) = 6e^{-2\pi s} - 9\frac{e^{-\pi s}}{s}$

$(s^2+9)\mathcal{L}(y) = s + 6e^{-2\pi s} - \frac{9e^{-\pi s}}{s}$

$\mathcal{L}(y) = \frac{s}{s^2+9} + \frac{6e^{-2\pi s}}{s^2+9} \quad \frac{-9e^{-\pi s}}{s(s^2+9)}$

$\mathcal{L}(y) = \frac{s}{s^2+9} + \frac{6e^{-2\pi s}}{s^2+9} + \frac{se^{-\pi s}}{s^2+9} - \frac{e^{-\pi s}}{s}$

$\cos(3t) + 2u(t-2\pi)\sin(3t) \quad - u(t-\pi)\cos(3t) \quad - u(t-\pi)$

$$y'' + 4y = + 8u(t-\pi) - 8\delta(t-3\pi) \qquad y(0) = 2 \quad y'(0) = 0$$

$$[s^2 Y - sy(0) - y'(0)] + bY = R(s)$$

$$s^2 Y - 2s + 4Y = \frac{8e^{-\pi s}}{s} - 8e^{-3\pi s}$$

$$s^2 Y + 4Y = \frac{8e^{-\pi s}}{s} - 8e^{-3\pi s} + 2s$$

$$Y = \frac{8e^{-\pi s}}{s(s^2+4)} - \frac{8e^{-3\pi s}}{(s^2+4)} + \frac{2s}{(s^2+4)}$$

$$\mathcal{L}^{-1}(Y) =$$

$$2e^{-\pi s}\frac{(4)}{s(s^2+4)} - (4e^{-3\pi s})\frac{(2)}{s^2+2^2} + 2\cos 2t$$

$$2e^{-\pi s}\left(\frac{1}{s} - \frac{s}{s^2+2^2}\right) - 4e^{-3\pi s}\frac{(2)}{s^2+2^2} + (2)\frac{s}{s^2+2^2}$$

$$\mathcal{L}^{-1}\left(\frac{1}{s}\right) = 1 \qquad \mathcal{L}^{-1}\left(\frac{s}{s^2+2^2}\right) = \cos 2t \qquad \mathcal{L}^{-1}\left(\frac{2}{s^2+2^2}\right) = \sin 2t.$$

$$\mathcal{L}^{-1}(e^{-as}F(s)) = u(t-a)f(t-a) \qquad \mathcal{L}^{-1}\left(\frac{e^{-as}}{a}\right) = u(t-a).$$

$$2u(t-\pi) - 2\cos 2(t-\pi)u(t-\pi) - \cancel{4u(t-3\pi)}$$

$$-4\sin 2(t-3\pi)u(t-3\pi) + 2\cos 2t$$

2010 06 $\quad y'' + 4y = 4u(t-\pi) - 4\delta(t-3\pi) \quad y(0) = 1 \quad y'(0) = 0$

$$s^2Y - sy(0) - y'(0) + 4Y = \frac{4e^{-\pi s}}{s} - 4e^{-3\pi s}$$

$$Y(s^2+4) = \frac{4e^{-\pi s}}{s} - 4e^{-3\pi s} + s$$

$$Y = \frac{4e^{-\pi s}}{s(s^2+4)} \quad \frac{-4e^{-3\pi s}}{s^2+2^2} + \frac{s}{s^2+2^2}$$

$$e^{-\pi s}\left(\frac{1}{s} - \frac{s}{s^2+2^2}\right) - 2e^{-3\pi s}\left(\frac{2}{s^2+2^2}\right) + \frac{s}{s^2+2^2}$$

$$(1) \cancel{u(t-\pi)(t-\pi)} \quad -4\cos 2\, u(t-\pi)(t-\pi) \quad + \cos 2t$$

$$u(t-\pi) - \cos 2(t-\pi)u(t-\pi) - 2\sin 2(t-3\pi)u(t-\pi) + \cos 2t.$$

240 MATHS 1 Q1

1A. $f(x,y,z) = \sqrt{y^2 - \sin(x+2z)}$ $\quad P(0,2,0)$

$$\frac{df}{dx} = \frac{1}{2}\frac{-\cos(x+2z)}{\sqrt{y^2-\sin(x+2z)}}\Bigg|_{(0,2,0)} \qquad \frac{-1}{4}$$

$$\frac{df}{dy} = \frac{1}{2}\frac{2y}{\sqrt{y^2-\sin(x+2z)}} \qquad \frac{4}{4} = 1$$

$$\frac{df}{dz} = \frac{1}{2}\frac{-\cos(x+2z)(2)}{\sqrt{y^2-\sin(x+2z)}} = \frac{-2}{4} = \frac{1}{2}$$

$\nabla f = \left(\frac{-1}{4}, 1, \frac{1}{2}\right)$

$\|\nabla f\| = \left(\frac{-1}{4}\right)^2 + (1)^2 + \left(\frac{1}{2}\right)^2 \qquad \frac{\sqrt{21}}{4}$

$u = \frac{\nabla f}{\|\nabla f\|} = \frac{-1/4}{\sqrt{21}/4},\ \frac{1}{\sqrt{21}/4},\ \frac{1/2}{\sqrt{21}/4}$

$u = \left(\frac{\sqrt{21}}{21}, \frac{4}{\sqrt{21}}, \frac{2}{\sqrt{21}}\right)$

B. Decrease most rapidly at $-u \Rightarrow \left(-\frac{\sqrt{21}}{21}, \frac{-4}{\sqrt{21}}, \frac{-2}{\sqrt{21}}\right)$

C. Rate of change in direction of $\pm u$ is equal to $\pm\|\nabla f\|$
$= \pm\frac{\sqrt{21}}{4}$ respectively

2010 MATHS 1 Q5.

Q 5. A Rectangular (x, y, z) Spherical $(\rho \sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)$

Jacobian $\rho^2 \sin\phi$

B $x^2+y^2+z^2=r^2$

$$\int_0^{2\pi}\int_0^{\pi/2}\int_0^{r} r^2 \sin\phi \; dr\, d\phi\, d\theta$$

$$\frac{r^3}{3}\int_0^{2\pi}\int_0^{\pi/2} \sin\phi \; d\phi\, d\theta$$

$$\frac{r^3}{3}\int_0^{2\pi} -\cos\phi \Big|_0^{\pi/2}$$

$$\frac{r^3}{3}\int_0^{2\pi} d\theta$$

$$= 2\pi\left(\frac{r_2^3-r_1^3}{3}\right)$$

ii. $\Gamma(x,y,z) = \dfrac{e^{-x^2 y^2+z^2}}{\sqrt{x^2+y^2+z^2}}$ $\quad x^2+y^2+z^2=\rho^2$

$$\frac{e^{-\rho^2}}{\rho} \quad \int_0^{2\pi}\int_0^{\pi/2}\int_2^{3} \rho e^{-\rho^2}\sin\phi \; d\rho\, d\phi\, d\theta.$$

$$\int_0^{2\pi}\int_0^{\pi/2} \frac{e^{-\rho^2}}{2} \sin\phi \Big|_2^3$$

$$-\left(\frac{e^{-9}-e^{4}}{2}\right)\int_0^{2\pi}\int_0^{\pi/2} \sin\phi \; d\phi\, d\theta$$

$$\left(\frac{e^{4}-e^{-9}}{2}\right)\int_0^{2\pi} -\cos\phi\Big|_0^{\pi/2} d\theta$$

$$\left(\frac{e^{4}-e^{-9}}{2}\right)\int_0^{2\pi} d\theta$$

$$\pi\left(e^{4}-e^{-9}\right) = \text{mass}$$