Applied Prob 2 2012 Exam Paper

4 A $u = \frac{558}{6} = 93$

$\text{variance} = \sum (x_i - \bar{x})^2 \quad (94-93)^2 + (85-93)^2 + (99-93)^2 + (104-93)^2 + (86-93)^2 + (90-93)^2$

$\frac{280}{n-1} \quad \frac{280}{5} = 56$

$s = \sqrt{56} = 2\sqrt{14}$

$\bar{x} \pm t(0.025, 5)\frac{s}{\sqrt{n}}$

$93 \pm 2.015 \frac{2\sqrt{14}}{5}$

$93 - 6.743, \quad 93 + 6.743$

$(86.257, \quad 94.74)$ 95% CI

B $\frac{(n-1)s^2}{\chi^2(0.505, n-1)}, \quad \frac{(n-1)s^2}{\chi^2(1-0.505, n-1)}$

$\frac{4(\sqrt{56})}{6.635}, \quad \frac{4(\sqrt{50})}{0.554}$

$\cancel{0.554 \; 6.635} \qquad 0.554$

$\cancel{(4.511, \quad 54.031)}$ $\quad (33.76, \; 404.332)$

C

Digit	value	Digit	
1	94	4	104
5	86	4	104
6	90	5	86
1	94	2	85
6	90	3	99
1	94	3	99

D - Use bootstrap for simulation
- Do bootstrap multiple time for large number of times
- Calculate average/mean and s.t for each sample
- To create 95% CI Rank means/variance and select 2.5% percentile and 97.5% percentile, this generate 95% CI.

This interval will generally be wider as it

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is possible to have a sample with all of the smallest numbers / largest, thus interval will be larger.

G - Again, re-do boot strap sampling
- For each sample calculate the proportion of days vechicles > 95 and divide by 6
- Rank the data
- Choose 2.5 percentile and 97.5 percentile to create 95% CI
95% of data lies between 2.5, and 97.5 th percentile

5 A. Probability of passing the test is p, thus probability of failure is 1-p as there are only two outcomes success or failure

$P(\text{pass first time}) = p$
$P(\text{pass second time}) = (1 \text{ failure}, 1 \text{ pass}) = p(1-p)$
$P(\text{pass } 3^{rd} \text{ time}) = (2 \text{ failures, then pass}) = p(1-p)^2$
.... Success on k^{th} trial $= p(1-p)^{k-1}$

B 1 2 1 4 3 1 1 2 3 1
$p \quad p(1-p) \quad p \quad p(1-p)^3 \quad p(1-p)^2 \quad p \quad p \quad p(1-p) \quad p(1-p)^2 \quad p$

$L = \prod_{\text{sum prob}} p\, p(1-p)\, p\, (1-p)^3\, p(1-p)^2\, p\, p\, p(1-p)\, p(1-p)^2 p$
$= p^{10}(1-p)^9$

C $\text{Log}(L) = \text{Log}\, p^{10}(1-p)^9$
$\text{Log}\, p^{10} + \text{Log}(1-p)^9$
$10\,\text{Log}(p) + 9\,\text{Log}(1-p)$

$\frac{dL}{dp} = 10\frac{1}{p} + \frac{9}{1-p} = 0 \Rightarrow$ at max $= 0$

$\frac{10}{p} = \frac{+9}{1-p}$

~~$10(1-p) = 9p$~~
~~$10 - 10p = 9p \quad p = 10$~~

$10(1-p) = 9p$
$10 - 10p = 9p$
$p = \frac{10}{19}$

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D $H_0: p = 0.5$ vs $H_1\ p \neq 0.5$

s^2 for geometric $= \frac{1}{n-1}\frac{(1-p)}{p}$

$t_{calc} = \dfrac{0.5 - \frac{10}{19}}{s/\sqrt{n}}$

$\dfrac{\frac{9}{19}}{9(\frac{10}{19})} = 1/10$

$\dfrac{0.5 - 10/19}{0.1/\sqrt{10}}$

$= -0.8321$

$t_{critical}(0.975, 9) = 2.262$

$|t| < t_{critical}$

No evidence against H_0 $\Rightarrow$ fail to reject

E It could be reasoned that the data is normally distributed as most of the values lie within $\pm 2\sigma$ of the mean, there seem to be no outliers either.

For small sample sizes this test could be appropriate as by the central limit theorem

F From the bootstrap, the 95% CI is (2.5 percentile, 97.5th percentile)

$(0.333, 0.769)$

The value of 0.5 lies within this mean, we fail to reject H_0 that $p = 0.5$.

A.i $Var(y) = \cancel{E(ax+b)^2}\ E[y^2] + E[y]^2$

$E[(ax+b)^2] - E[(ax+b)]^2$

$E[a^2x^2] + E[2ax] + E[b^2] - E[ax]^2 + E[b]^2$

$a^2 E[x^2] + 2ab E[x] + b^2 - (a^2 E[x]^2 + 2ab E[x] + b^2)$

$= a^2 E[x^2] - a^2 E[x]^2$

$= a^2 Var[x]$

ii. correlation $= \dfrac{Cov(x,y)}{Var(x)\,Var(y)}$

$Cov(x,y) = E[xy] - E(x)E(y)$

$E(x(ax+b)) - E(x)E(ax+b)$

$E[ax^2+bx] - E(x)(aE[x]+b)$

$aE[x^2] + bE[x] - aE[x]^2 - bE[x]$

$= aE[x^2] - aE[x]^2$

$= aVar[x]$

So correlation $\rho = \dfrac{aVar[x]}{\sqrt{a^2Var[x]\,Var[x]}} = \dfrac{a}{aVar[x]} = \dfrac{a\,Var[x]}{a\,Var[x]}$

$= \dfrac{a}{a}$ if $a>0$ $\rho = 1$.

B i.

x,y	0,-1	0,0	0,1	1,-1	1,0	1,1	2,-1	2,0	2,1
$p(x+y)=0$?	X	✓	X	✓	X	X	X	X	X

0,0 and 1,-1 $0.1 + 0.25$

$= 0.35$.

ii. for $x \Rightarrow$

$x =$	0	1	2
$p(x) =$	0.3	0.4	0.3

for $y \Rightarrow$

$y =$	-1	0	1
$p(y) =$	0.55	0.25	0.2

iii. conditional of $X|y=0$

$x =$	0	1	2
$p(x) =$	$\frac{0.1}{0.25}$	$\frac{0.05}{0.25}$	$\frac{0.1}{0.25}$

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iii. $U_{x|y=y}$ $\mu_x + p \frac{\sigma_x}{\sigma_y}(y - U_y)$ | $\sigma^2_{x|y=y}$ $(1-p^2)\sigma_x^2$

$(1-0.75^2)(1)$

$= 7/16$

$-1 + 0.75 \frac{1}{1}(1-2)$

$-1 - 0.75$

$U_{x|y=y}$ -1.75

$X \sim (-1.75, \sqrt{7}/4)$

$p(-2 \leq X \leq -1 \mid y=1)$

$\frac{-2 - -1.75}{\sqrt{7}/4} \leq x \leq \frac{-1 - -1.75}{\sqrt{7}/4}$

$-0.25/\sqrt{7}/4 \leq x \leq 0.75/\sqrt{7}/4$

$-\sqrt{7}/7 \leq x \leq$

$-0.377 \leq x \leq 1.13$

0.8413 $(1 - 0.6331)$ $= 0.4744$

iv. Distribution of $-x+3y$.

$Z \sim N(-U_x + 3U_y, -\sigma_x^2 + 3\sigma_y^2)$

$Z \sim N(7, (\sqrt{2})^2)$. is distribution of $-x+3y$

S.

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6 B iv. Covariance $= E(xy) - E(x)E(y)$.

$E(x) = 0(0.3) + 1(0.4) + 2(0.3) = 1$

$E(y) = -1(0.55) + 0(0.25) + 1(0.2) = -0.35$.

$E(xy) = -1(0)(0.2) + -1(0.25)(1) + -1(2)(0.1)$

0 row $0(0.1)(0) + 0(0.05)(1) + 0(0.1)(2)$

$1(0)(0) + 1(0.1)(1) + 1(0.1)(2) = -3/20$

v. Correlation $= \dfrac{\text{Covariance}(x,y)}{\sqrt{var(x)\, var(y)}}$.

$var(x)$ $E(x^2) = 8/5$ $\quad 8/5 - 1^2 = 3/5$

$var(y)$ $E(y^2) = 3/4$ $\quad 3/4 - \cancel{\ } = 251/400$

$$\frac{-3/20}{\sqrt{\frac{3}{5}\left(\frac{251}{400}\right)}} = -\frac{100}{251} \neq 0 \Rightarrow \text{relationship exist.}$$

or ~~p(x = -1, y~~ $p(y=-1, x=0) = 0.2$

$p(y=-1)\,p(x=0)$ $\quad 0.55 \times 0.3 = 0.165 \neq 0.2$

Not independent

C $\mu_x = -1$ $\quad \mu_y = 2$ $\quad \sigma_x^2 = 1$ $\quad \sigma_y^2 = 1$ $\quad p = 0.75$.

i. $y \sim N(2, 1^2)$

ii. $p(0 \leq x \leq 1.5)$

$$\frac{0 - -1}{1} \leq x \leq \frac{1.5 - -1}{1}$$

$1 \leq x \leq 2.5$.

$0.9953 - 0.8413 = 0.154$