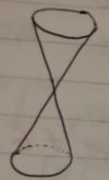
2013 MANG. SCIENCE PAPER 1 Q5. DAVID LIETBRECHT

Consider the solid G which is bounded by the surface $z = \sqrt{x^2+y^2}$ from below and the surface $x^2+y^2+z^2=8$ from above

A. What kind of a surface does the equation $z=\sqrt{x^2+y^2}$ describe?

This represents an infinite cone with origin (0,0,0)



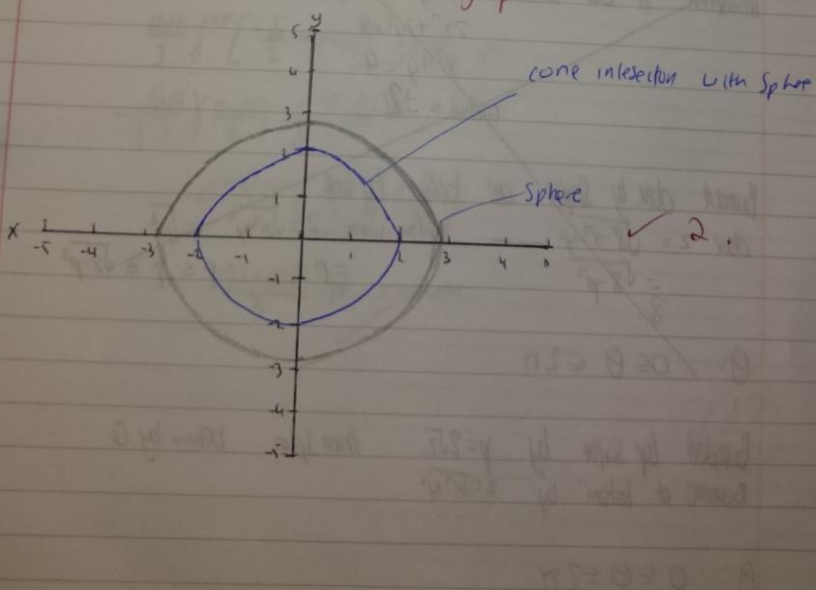
✓ 2.

B. What kind of surface does the equation $x^2+y^2+z^2=8$ describe?

Describes a sphere with radius $\sqrt{8} = 2\sqrt{2}$ ✓

2.

C. Sketch the solid G onto the xy plane



✓ 2.

DW

2

D. Use cylindrical coordinate to set up an integral for the volume of G, You do not need to evaluate the integral

cylindrical $x = p\cos\theta$ $y = p\sin\theta$ and z.

$0 \le z \le 2$ $p \le z \le \sqrt{8-\rho^2}$

$0 \le \theta \le 2\pi$

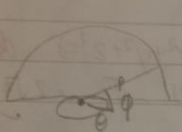
$0 \le p \le \sqrt{8-p^2}$ $0 \le \rho \le 2\sqrt{2}$.

$$\int_0^{2\pi}\int_0^{2\sqrt{2}}\int_p^{\sqrt{8-p^2}} p \, dp \, dz \, d\theta$$

switch

3.

E. Use Spherical coordinates to compute volume of G



$x = p\sin\phi\cos\theta$ $y = p\sin\phi\sin\theta$ $z = p\cos\phi$ $p^2\sin\phi \, dp \, d\phi \, d\theta$

$x^2+y^2+z^2 = p^2$

Intersection of two surface = $x^2+y^2+x^2+y^2=8$

$2x^2+2y^2=8$

$x^2+y^2=4$

radius $= \pm 2$

Bounded above by sphere and below by cone

above $z = \sqrt{8-x^2-y^2}$ below with $z = \sqrt{x^2+y^2} = \sqrt{p^2}$

$= \sqrt{8-p^2}$ $= p$ $p \le \rho \le \sqrt{8-p^2}$

θ: $0 \le \theta \le 2\pi$

Bounded top surface by $p = 2\sqrt{2}$ from sphere below by 0

Bounded at bottom by $z = \sqrt{x^2+y^2}$

θ: $0 \le \theta \le 2\pi$

3. DAVID LIEMBRECHT Q5. 2013

E. bottom $z = \sqrt{x^2+y^2}$

$$\rho\cos\varphi = \sqrt{\rho^2\sin^2\varphi\cos\theta + \rho^2\sin^2\varphi\sin^2\theta}$$
$$= \sqrt{\rho^2(\sin^2\varphi\cos^2\theta + \sin^2\varphi\sin^2\theta)}$$
$$= \sqrt{\rho^2\sin^2\varphi}$$

$\rho\cos\varphi = \rho\sin\varphi$

$\cos\varphi = \sin\varphi$

So $\varphi = \frac{\pi}{4}$ only place wher $\cos\varphi = \sin\varphi$ at 45°. ✓

So $0 \le \rho \le 2\sqrt{2}$ $\quad 0 \le \varphi \le \frac{\pi}{4}$ $\quad 0 \le \theta \le 2\pi$

$$V = \int_0^{2\pi}\int_0^{\pi/4}\int_0^{2\sqrt{2}} \rho^2\sin\varphi \; d\rho\, d\varphi\, d\theta$$ ✓

$$= \int_0^{2\pi}\int_0^{\pi/4} \sin\varphi \left[\frac{\rho^3}{3}\right]_{\rho=0}^{2\sqrt{2}} d\varphi\, d\theta$$

$$\frac{16\sqrt{2}}{3}\int_0^{2\pi}\int_0^{\pi/4} \sin\varphi \; d\varphi\, d\theta$$ ✓

$$\frac{16\sqrt{2}}{3}\int_0^{2\pi} \left[-\cos\varphi\right]_{\varphi=0}^{\varphi=\pi/4} d\theta$$

$$\frac{16\sqrt{2}}{3}\int_0^{2\pi}\left[-\frac{\sqrt{2}}{2} - (-1)\right] d\theta$$ ✓

$$- \left(\frac{16\sqrt{2}}{3}\right)\left(\frac{2-\sqrt{2}}{2}\right)\int_0^{2\pi} d\theta$$

$$= -2\pi\left(\frac{16\sqrt{2}}{3}\right)\left(\frac{2-\sqrt{2}}{2}\right)$$

$$= \frac{-32+32\sqrt{2}}{3}\pi = \text{volume}$$ ✓

8

17/20

DW.

2012 MANG SCI II Q6 DAVID WETTBREAD

2013 MANG&CI PAPER I Q5 CORRECTION.

i. Represents an infinite cone with centre (0,0,0)

ii. Represents a sphere with origin (0,0,0) and radius $2\sqrt{2}$

$x^2+y^2+(\sqrt{x^2+y^2})^2=8$

$2x^2+2y^2=8$

$x^2+y^2=4$

$r=2$ = intersection of solid

iii. $x=\rho\cos\theta$ $\quad y=\rho\sin\theta$ $\quad z=z$

$0 \leq z \leq \sqrt{8-\rho^2}$ bounded below and above

$\sqrt{x^2+y^2} \leq z \leq \sqrt{8-x^2-y^2}$ $\qquad x^2+y^2=\rho^2$

$\rho \leq z \leq \sqrt{8-\rho^2}$

$0 \leq \theta \leq 2\pi$

$0 \leq \rho \leq 2\sqrt{2}$

$$\int_0^{2\pi}\int_0^{2\sqrt{2}}\int_\rho^{\sqrt{8-\rho^2}} \rho \, dz\, d\rho\, d\theta$$

iv. $x=\rho\cos\theta\sin\phi$ $\quad y=\rho\sin\phi\sin\theta$ $\quad z=\rho\cos\phi$

$0\leq\theta\leq 2\pi$ full rotation

$0\leq\rho\leq 2\sqrt{2}$

$\sqrt{x^2+y^2} < \rho\cos\phi < \sqrt{8-x^2-y^2}$

$\sqrt{\rho^2\cos^2\theta\sin^2\phi+\rho^2\sin^2\phi\sin^2\theta} < \rho\cos\phi < \sqrt{8-\rho^2\cos\theta^2\sin\phi^2-\rho^2\sin^2\phi\sin^2\theta}$

$\sqrt{\rho^2\sin^2\phi} < \rho\cos\phi < 8-\rho^2 \cancel{\cos}\sin\phi^2$

$\rho\sin\phi \qquad \sqrt{8(1-\sin^2\phi^2)}$

$\rho\sin\phi < \rho\cos\phi < 2\sqrt{2}\cos\phi$

$\sin\phi < \cos\phi < \cos\phi$

equal at $\phi=\pi/4 = 45°$

$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{2\sqrt{2}} \rho^2 \sin\phi \; d\rho\, d\phi\, d\theta$$

$$\int_0^{2\pi}\int_0^{\pi/4}\left[\frac{\rho^3}{3}\right]_{\rho=0}^{2\sqrt{2}} \sin\phi \; d\phi\, d\theta$$

$$\frac{16\sqrt{2}}{3}\int_0^{2\pi}\int_0^{\pi/4} \sin\phi \; d\phi\, d\theta$$

$$\frac{16\sqrt{2}}{3}\int_0^{2\pi} -\cos\phi\Big|_{\phi=0}^{\pi/4}$$

$$\frac{16\sqrt{2}}{3}\left(\frac{2-\sqrt{2}}{2}\right)\int_0^{2\pi} d\theta$$

$$= 2\pi\left(\frac{16\sqrt{2}}{3}\right)\left(\frac{2-\sqrt{2}}{2}\right)$$

$$\frac{-32+32\sqrt{2}\pi}{3} = \text{volume}$$