1

Applied Prob 1   2011 Exam Paper

1a Regular die → coin toss.

number 1, 2, 3 = heads   number 4, 5, 6 = tails

Toss coin 3 times only 6 different combinations available assigning 1-6.

001	010	100	111	110	011
1	2	3	4	5	6

i No. number heads the probability distribution is the probability of getting a head

N_6 - distribution of prob of getting a six.

N_H is Binomial

N_6 is uniform

B i uniform cdf $\begin{cases} 0 & \text{if } x<0 \\ x & \text{if } 0<x\leq 1 \\ 1 & \text{if } x>1 \end{cases}$

ii cdf is a mean + U(σ).

iii Derive value of NORMINV(RAND()) by $x = \phi^{-1}(u)$

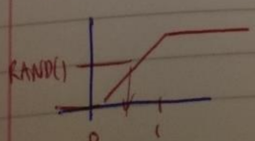
$P(X \leq x) = P(\phi^{-1}(u) \leq x) = P(U \leq \phi(x))$

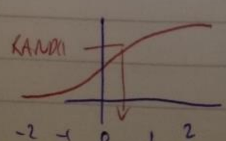
$= \phi(x) =$ normal cdf

$P(X \leq x) = F_X(x) = \Phi(x)$ $f_X(x) = \frac{d\Phi(x)}{dx} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$

Norminv (p) = value of x for which $\Phi(x) = p$

Eg $(u) = 0.594$, NORMINV(0.594) between 0.225 and 0.230.





2

C 1 2 3 4 5 a b c d e

Unconstrained, select U from RAND!

If $U < 0.1$ choose 1

else if $U < 0.2$ choose 2 etc

else $U < 1$ choose e

Reject if no number in password

Multi step - Choose 1st from 1,2,3,4,5.

choose next from 1,2.. e

Shuffle by choosing 5 values of U and rank

Efficiency - latter involve 10 calls of f(U)

Former has 5 calls to U some of which are ignored

$P(\text{rejection}) = (1/2)^5$

D - Choose 13 values of U

- Determine ranks for first 2 in the 52, have 6 and 12, hence choose 6H and QH (are U i, k is 13)

Similarly using 52 values of U, noting only Q now form table:

	Q_1	$\bar{Q_1}$	
Q_2	n_{11}	n_{12}	$n_{1\cdot}$
$\bar{Q_2}$	n_{21}	n_{22}	$n_{2\cdot}$
	$n_{\cdot 1}$	$n_{\cdot 2}$	n

Then $P(Q_1|Q_2) = \frac{n_{11}}{n_{1\cdot}}$ This is also $\frac{n_{11}/n}{n_{1\cdot}/n} = \frac{\text{rel freq both Q}}{\text{Rel freq 2nd Q}}$

Theory: $P(Q_1|Q_2) = \frac{P(Q_2|Q_1)P(Q_1)}{P(Q_2)} = P(Q_2|Q_1) = \frac{3}{51}$

Applied Probability 2011 Exam Prac

1 E. Condional dist of y for x=3.

1	2	3	4
$\frac{0.10}{0.24}$	$\frac{0.08}{0.24}$	$\frac{0.06}{0.24}$	$\frac{0.05}{0.24}$

$E(x) = 1\left(\frac{10}{24}\right) + 2\left(\frac{8}{24}\right) + 3\left(\frac{6}{24}\right) + 4\left(\frac{5}{24}\right) = \frac{64}{24} \approx 2.07$

Max	1	2	3	4
1	1	2	3	4
2	2	2	3	4
3	3	3	3	4
4	4	4	4	4

Max

Poss	1	2	3	4
Prob	0.03	0.16	0.37	0.44

$E(x) = 1(0.03) + 2(0.16) + 3(0.37) + 4(0.44) = 3.22$

2 A $\frac{5 \times 4 \times 3}{5^3}$ $p = 0.52$ that do share a birthday

if (a=b) or (b=c) or (c=a) True else False

3	1	1
0.594	0.051	0.156

Generate random num

0–0.2 → 1 0.2 → 0.4 2 etc

3 numbers made

Check condition

B No birthday in common described in terms of 1^{st} 2^{nd} 3^{rd} day

Equivalent to 1^{st} = any day And (2^{nd} not 1^{st}) and (3^{rd} not 1^{st} or 2^{nd})

4

2 c $n=2$ $\frac{5(4)}{5^2} = 0.8$

$=3$ $\frac{5(4)(3)}{5^3} = 0.48$

$=4$ $\frac{5(4)(3)(2)}{5^4} = 0.49$

$=5$ $\frac{5(4)(3)}{5^3} = 0.04$

$=6$ $6(5)(4)(\ldots) = 0$

Regulr yrs $\frac{365 - n+1}{365}$

d Pr (all different fm ys) $\left(\frac{364}{365}\right)^{74} = 0.816$

3a pdf : $F(y) = \begin{cases} 0 & y \le 0 \\ y & 0 < y < 1 \\ 0 & y > 1 \end{cases}$ cdf $\begin{cases} 0 & y < 1 \\ \frac{1}{b-a} & 0 < y \le 1 \\ \ldots & y > 1 \end{cases}$

Long run prop ≤ Specific numer value y by

$f(y)$ $\frac{dF(y)}{dy} = 1$ $0 < y \le 1$

B $E[y] = \int_0^1 y(f(y))dy$

$\frac{y^2}{2} = \frac{1}{2}$

$E[y^2] = \int_0^1 y^2 f_y$

$\frac{y^3}{3} = \frac{1}{3}$

$\frac{1}{3} - \left(\frac{1}{2}\right)^2 = \frac{1}{12} = \text{var}[y]$

C The random data have mean of 0.462 ⇒ good to or

Random data has st of 0.282 corresponds to $\sqrt{\text{var}} = \sqrt{\frac{1}{12}} = 0.288$

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3c $E[x] = E[x^2] = 1/3$

$E[x^2] = E[x^4] = \int_0^1 x^4 \, d(y) \, dy = \frac{1}{5}.$

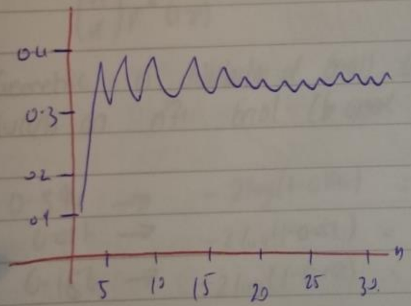
$\frac{1}{5} - \left(\frac{1}{3}\right)^2 = 4/45$

$G(x) = P(X \le x) = P(Y^2 \le x) = P(Y \le \sqrt{x}) = \sqrt{x}$

$G(0.5) = P(Y \le \sqrt{0.5}) = \sqrt{0.5} = 0.20$

$G(x) = \sqrt{x} \qquad g(x) = \frac{1}{2\sqrt{x}}$

D



Theory $\bar{X}_n : \frac{1}{n}(X_1 + X_2 + \dots X_n)$

$E[\bar{X}_n] = \frac{1}{n}(n\mu) = \mu =$ expected value of any of the X vars

$Var[\bar{X}_n] = \frac{1}{n^2}[n\sigma^2] = \frac{1}{n}\sigma^2 = \frac{1}{n}$ var of any of X vars

G CLT $\Rightarrow$ $\bar{X}_n \sim N(\mu, \frac{1}{n}\sigma^2)$ atleast approx

$\sim N(\mu, \frac{1}{3}\sigma^2)$