1 2013 MANG SCI PAPER 3 Q2 DAVID LEMORECHI

Q2 A Differentiate between the Euler and Hamiltonian algorithm

Travelling Salesman visit all the nodes in a network - known as the Hamiltons algorithm -

Find shortest closed path or circuit that visit every edge of a (connected) undirected graph

Chinese postman travell along all the arc of a network - known as eulers algorithm

B i. Formulate the delivery problem the company face as a linear programming problem

$$\begin{aligned}\text{Min } Z = {} & 13_{12} + 21_{13} + 9_{14} + 7_{15} + 18_{16} + 20_{17} + 15_{18} \\ & + 13_{21} + 9_{23} + 18_{24} + 12_{21} + 26_{26} + 23_{27} + 11_{28} \\ & + 21_{31} + 9_{32} + 26_{34} + 17_{35} + 25_{36} + 19_{37} + 10_{38} \\ & + 9_{41} + 18_{42} + 26_{43} + 7_{45} + 16_{46} + 15_{47} + 9_{48} \\ & + 7_{51} + 17_{52} + 17_{53} + 7_{54} + 9_{56} + 11_{57} + 8_{58} \\ & + 18_{61} + 26_{62} + 25_{63} + 16_{64} + 9_{65} + 6_{67} + 10_{68} \\ & + 20_{71} + 23_{72} + 19_{73} + 15_{74} + 11_{75} + 6_{76} + 5_{78} \\ & + 15_{81} + 11_{82} + 10_{83} + 9_{84} + 8_{85} + 10_{86} + 5_{87}\end{aligned}$$

Subject to: entering constraints:

$$\begin{aligned}& X_{21} + X_{31} + X_{41} + X_{51} + X_{61} + X_{71} + X_{81} = 1 \\ & X_{12} + X_{32} + X_{42} + X_{52} + X_{62} + X_{72} + X_{82} = 1 \\ & X_{13} + X_{23} + X_{43} + X_{53} + X_{63} + X_{73} + X_{83} = 1 \\ & X_{14} + X_{24} + X_{34} + X_{54} + X_{64} + X_{74} + X_{84} = 1 \\ & X_{15} + X_{25} + X_{35} + X_{45} + X_{65} + X_{75} + X_{85} = 1 \\ & X_{16} + X_{26} + X_{36} + X_{46} + X_{56} + X_{76} + X_{86} = 1 \\ & X_{17} + X_{27} + X_{37} + X_{47} + X_{57} + X_{67} + X_{87} = 1 \\ & X_{18} + X_{28} + X_{38} + X_{48} + X_{58} + X_{68} + X_{78} = 1\end{aligned}$$

2.

leaving Constraint

$x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} = 1$

$x_{21} + x_{23} + x_{24} + x_{25} + x_{26} + x_{27} + x_{28} = 1$

$x_{31} + x_{32} + x_{34} + x_{35} + x_{36} + x_{37} + x_{38} = 1$

$x_{41} + x_{42} + x_{43} + x_{45} + x_{46} + x_{47} + x_{48} = 1$

$x_{51} + x_{52} + x_{53} + x_{54} + x_{56} + x_{57} + x_{58} = 1$

$x_{61} + x_{62} + x_{63} + x_{64} + x_{65} + x_{67} + x_{68} = 1$

$x_{71} + x_{72} + x_{73} + x_{74} + x_{75} + x_{76} + x_{78} = 1$

$x_{81} + x_{82} + x_{83} + x_{84} + x_{85} + x_{86} + x_{87} = 1$

all vars $\{0,1\}$

Nearest Neighbour - Euler.

Start at A on pick nearest neighbour (lowest node)

1 ✓ 2 ✓ 3 ✓ 4 ✓ 5 ✓ 6 ✓ 7 ✓ 8 ✓

1 → 5 → 4 → 8 → 7 → 6 → 3 → 2

1. 1 → 5 → 4 → 8 → 7 → 6 → 3 → 2 → 1 = 81
2. 2 → 3 → 5 → 1 → 8 → 7 → 6 → 4 → 2 = 9+17+7+15+5+6+16 = 93
3. 3 → 2 → 8 → 7 → 6 → 5 → 1 → 4 → 3 = 9+11+5+6+9+7+9+26 = 82
4. 4 → 5 → 1 → 2 → 3 → 8 → 7 → 6 → 4 = 7+7+13+9+12+5+6+16 = 73
5. 5 → 1 → 4 → 8 → 7 → 6 → 3 → 2 → 5 = 7+9+9+12+6+25+9+11 = 82
6. 6 → 7 → 8 → 5 → 4 → 1 → 2 → 3 → 6 = 6+5+8+7+9+13+4+21 = 82
7. 7 → 8 → 5 → 1 → 4 → 6 → 3 → 2 → 7 = 7+9+7+14+16+25+9+23 = 102
8. 8 → 7 → 6 → 5 → 1 → 4 → 2 → 3 → 8 = 5+6+9+7+9+18+9+10 = 73

Multiple solutions, but not necessarily optimum

There are $(n-1)!$ solution (5040) routes so we cannot be certain that the routes above are anywhere near optimal

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Nearest ~~neighbour~~ Insertion

1 → 1 leaves (2, 3, 4, 5, 6, 7, 8) to be included

1. Min (12, 13, 14, 15, 16, 17, 18) = min {13, 21, 9, 7, 18, 24, 15}
   = 1 → 5.
   1 → 5 → 1 (2, 3, 4, 6, 7, 8) to be included

2. Min (12, 13, 14, 16, 17, 18, 52, 53, 54, 56, 57, 58) min (13, 21, 9, 18, 24, 15, 12, 17, 7, 9, 11, 8)
   5 → 4 = 7.
   1 → 5 → 4 → 1 7 + 7 + 9 = 23 OR
   1 → 4 → 5 → 1 9 + 7 + 7 = 23. (make either.

3. 1 → 5 → 4 → 1 (2, 3, 6, 7, 8) to be included
   Min (12, 13, 16, 17, 18, 52, 53, ~~54~~, 56, 57, 58, 42, 43, 46, 47, 48)
   Min (13, 21, 18, 20, 15, 12, 17, 9, 11, 8, 18, 26, 16, 15, 9)
   5 → 8 = 8.
   1 → 8 → 5 → 4 → 1 15 + 8 + 7 + 9 = 39
   or 1 → 5 → 8 → 4 → 1 7 + 8 + 9 + 9 = 33

4. 1 → 5 → 8 → 4 → 1 (2, 3, 6, 7) to be included.
   Min (13, 16, 17, 52, 53, 56, 57, 82, 83, 86, 87, 42, 43, 46, 47)
   (21, 18, 20, 12, 17, 9, 11, 11, 10, 10, 5, 18, 26, 9, 11)
   8 → 7.
   ⇒ 1 → 5 → 7 → 8 → 4 → 1 = 7 + 11 + 7 + 9 + 9 = 41
   1 → 5 → 8 → 7 → 4 → 1 = 7 + 8 + 5 + 15 + 9 = 44

5. 1 → 5 → 7 → 8 → 4 → 1 (2, 3, 6) to be included
   Min (12, 13, 16, 52, 53, 56, 72, 73, 76, 82, 83, 86, 42, 43, 46)
   Min (13, 21, 18, 12, 17, 9, 23, 19, 16, 11, 10, 10, 18, 26, 16)
   7 → 6
   ⇒ 1 → 5 → 8 → 7 → 6 → 4 → 1 = 7 + 8 + 5 + 6 + 16 + 9 = 51
   1 → 5 → 8 → 6 → 7 → 4 → 1 = 7 + 8 + 10 + 6 + 15 + 9 = 55.

4

1 → 5 → 8 → 6 → 7 → 4 → 1 (2,3) to be included

min (12,13, 52,53, 82,83, 62,63, 72,73, 42,43)

min (13,21, 12,17, 11,19, 26,25, 23,19, 18,26)

8 → 3

1 → 5 → 8 → 3 → 6 → 7 → 4 → 1 = 7 + 8 + 10 + 25 + 6 + 15 + 9 = 80

⇒ 1 → 5 → 3 → 8 → 6 → 7 → 4 → 1 = 7 + 17 + 10 + 10 + 6 + 15 + 9 = 74

Insert 2.

min (12, 52, 82, 83, 62, 72, 42)

min 13, 12, 11, 9, 26, 23, 18)

3 → 2

1 → 5 → 3 → 2 → 6 → 7 → 4 → 1 = 7 + 17 + 9 + 26 + 6 + 15 + 4 = 84

1 → 5 → 2 → 3 → 6 → 7 → 4 → 1 = 7 + 12 + 9 + 25 + 6 + 15 + 9 = 83

Not optimal - could of started at different number, also 504 routes available

iii. If it were necessary to deliver along all roads use Euler's algorithm

- Add up total length of arcs
- For each node state amount of arcs from node
- Nodes with odd degree of arcs (1, 3 ...) must be paired
- ie if four nodes have arc 3 pairs, calculate distance between nodes by using shortest distance from node to node

$^{n}C_{R}$

- Once find, add these new pairs to solution. Add these added pairs distance to total distance to give final solution

There are $^{8}C_{2}$ possible solutions

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Using Saving's heuristic

Capacity of 30 tonne

6 8 32+33-15 = 52 ✓

7 8 16+35-25 = 26 ✓

Calculate Saving's

i	J	S
1	2	10+20-10 = 20 ✓
1	3	10+25-40 = -5
1	4	10+18-20 = 18 ✓
1	5	10+30-45 = -5
1	6	10+32-33 = 12 ✓
1	7	10+16-20 = 6 ✓
1	8	10+35-22 = 23 ✓
2	3	20+25-40 = 5 ✓
2	4	20+18-15 = 23 ✓
2	5	20+30-30 = 20 ✓
2	6	20+32-20 = 32 ✓
2	7	20+16-25 = 11 ✓
2	8	20+35-10 = 45 ✓
3	4	25+20-18-20 = 23 ✓
3	5	25+20+30-20 = 35 ✓
3	6	25+30+32-30 = 27 ✓
3	7	25+10+16-10 = 31 ✓
3	8	25+36+35-36 = 24 ✓
4	5	18+30-15 = 33 ✓
4	6	18+32-10 = 40 ✓
4	7	18+16-10 = 24 ✓
4	8	18+35-12 = 41 ✓
5	6	30+32-12 = 50 ✓
5	7	30+16-15 = 31 ✓
5	8	30+35-22 = 43 ✓
6	7	32+16-18 = 30 ✓

Top ranking

i.	J.	S
6	8	52
2	8	45
5	6	50
5	8	43
4	8	41
4	6	40
3	5	55
4	5	33
2	6	32
3	7	31
5	7	31
6	7	30
3	6	27
7	8	26
4	7	24
3	8	24
3	4	23
2	4	23
1	8	23
1	2	20
2	5	20
1	4	18
1	6	12
2	7	11
1	7	6
2	3	5
1	3	-5
1	5	-5

1 ✓ 2 ✓ 3 ✓ 4 ✓ 5 ✓ 6 ✓ 7 ✓ 8 ✓

Join 6 and 8? yes c=17.

Join 2 and 8. no capacity Full

5 and 6? yes c=24 ✓ — ~~5-6 c=24~~

5 and 8? yes c=31 — 8-5-6 c=31

4 and 8? no capacity exceed ~~3-7 c=25~~

4 and 6? ~~yes c=33~~ NO. — 3-7-4 = c=25+9=34

3 and 5? no Full 1-2 = c=35

4 and 5? NO Full

2 6 ? ~~no full~~ NO full

3 7 ? yes c=25 —

5 7 ? no full

6 7 ? ~~yes c=~~ NO full

3 6 ? ~~already done~~ NO full

7 8. ? ~~already done~~ NO full

4 7 ? ~~Again~~ yes. —

3 8 ? full ✓

3 4 ? ~~Done~~ Done ✓

2 4 ? full ✓

1 8 ? Full ✓

1 2 ? yes c=30

Solution found (1) D-8-5-6-D = 7+14+10 = 31

(2) 4-6 D-3-7-4-D = 13+12+9 = 34

D-1-2-D = 11+19 = 30

3 trucks 6) per drive