17/04/16 DECISIONS - AHP

Analytic Hierarchy Process

- AHP a structured way of ranking various options open to decision maker
- Requires a step by step process of pairwise comparison of options against each of objectives
- Objectives then compared and weighted
- Each objective can have a collection of subobjectives on which this process can be carried out, from where the 'hierarchy' originates.

Example:

- Grad with 3 job offers
- 4 criteria to base objective on: Salary, attractiveness of city/job, interesting work, close to family
- In a structured way, find weights for each of the 4 objectives (that) sum to 1. $w_1 \dots w_4$
- Similarly divide up a unit across each of the options for each of the objectives.
  So for objective j we have $s_{1j}, \dots s_{3j}$
- Then the score associated with each option may be generated by SW

<u>Pairwise Comparisons</u>

- Key to AHP is to consider pairwise comparisons of objectives, and of options.
- May result in inconsistencies (if $A > B$ and $B > C$ then $A > C$)
- Objectives are compared to each other (4 options)
  ↳ entries for more important items go in first.
- The entry in the matrix a_{ij} takes values with meanings:
  i 'as important' as $j \Rightarrow a_{ij} = 1$
  i 'strongly more important than' j $a_{ij} = 5$
  i 'Absolutely more important than' j $a_{ij} = 9$

<u>Complete the matrix</u>

- The rest of the matrix is completed
- The diagonals take value $= 1$
- The other off diagonal elements are completed using the constraint that $a_{ij} = 1/a_{ji}$

Normalise

- Normalise matrix so columns sum to one
- Done by dividing each entry by the total in the column
- This is called A_{norm}
- The weight vector, W, is the average of each row
  $W^T = [0.515, 0.038, 0.0437, 0.146]$ e.g.

Repeat with options

- For first objective salary, compare the options. This matrix is called $C_{1\,norm}$
- The numbers used as before, and $C_{1\,norm}$ is generated in the same way
- S_1 is then the vector of the subscores given by averaging each row

Scores and Consistency

- By calculation, show that job 1 scores 0.34, J2 0.38 and J3 0.25
- Clear through the weight and score method how each option differs
- Aw compared to w gives a check for consistency.
- The average of the elementwise ratios $- n$ divided by $(n-1)$ is denoted CI
- This should be small, $n=4 < 0.09$, $n=5 < 0.11$ .... Small comes from 10% of the index from a 'random' comparison matrix

17/04/16 DECISIONS - AHP

Company investing in 2 locations

initial pairwise comparison

	T	Q	A
T	1	4	5
Q	0.25	1	1
A	0.2	1	1
Sum	1.45	6	7

Standardise (divide by column total)

	T	Q	A	Average
T	0.69	0.66	0.71	0.69
Q	0.17	0.16	0.14	0.16
A	0.14	0.16	0.14	0.14

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Transport.	C	D
C	1	1/2
D	2	1
Sum	3	1.5

Transport	C	D	Average
C	1/3	1/3	0.33
D	2/3	2/3	0.66

Workforce	C	D
C	1	3
D	1/3	1
	4/3	4

Workforce	C	D	Average
C	3/4	3/4	0.75
D	1/4	1/4	0.25

Attractiveness	C	D
C	1	4
D	1/4	1
	5/4	5

Attractiveness	C	D	Average
C	4/5	4/5	0.8
D	1/5	1/5	0.2

For answer, for each town calculate weighted average of each objective from standardised initial matrix by standardised individual matrix

	T	Qu.	A
C	0.69 (1/3)	0.16 (0.75)	0.14 (0.8)
D	0.69 (.66)	0.16 (0.25)	0.14 (0.2)

	T	Q	A	Average
C	0.23	0.12	0.11	0.46
D	0.46	0.04	0.02	0.53

$\Rightarrow$ choose D

Consistency check

- for each row in initial matrix
  → multiply initial matrix row by the average of each row (or column) and divide by average for that row

$$\text{Consistency index } CI = \frac{(\text{average of consistency measures}) - \#\text{criteria}}{\#\text{criteria} - 1}$$

19/04/16 DECISIONS - AHP - Q4 XMAS EXAM

	T	P	A
T	1	1/2	2
P	2	1	6
A	1/2	1/6	1
Sum	3.5	10/6	9

Standardised:	T	P	A	Average
T	2/7	3/10	2/9	0.264
P	4/7	3/5	2/3	0.613
A	1/7	1/10	1/9	0.118

Transport	C	R
C	1	0.5
R	2	1
Sum	3	1.5

Standardised	C	R	Average
C	1/3	1/3	1/3
R	2/3	2/3	2/3

Parks	C	R
C	1	7
R	1/7	1
Sum	8/7	8

Standardised	C	R	Average
C	7/8	7/8	7/8
R	1/8	1/8	1/8

Amenities	C	R
C	1	1/3
R	3	1
Sum	4	4/3

Standardised	C	R	Average
C	1/4	1/4	1/4
R	3/4	3/4	3/4

A Weighting on each public transport: 0.264 parks: 0.613 amenities: 0.118

	T	P	A	Total
C	264/3000	4291/8000	59/2000	0.655
R	264/1500	613/8000	177/2000	0.345

C Hub is more favourable

0.8400

- Consistency $(1(0.264) + \frac{1}{2}(0.613) + 2(0.118))/0.264$ = ~~[illegible]~~ 3.01

$(2(0.264) + 1(0.613) + 6(0.118))/0.613$ = ~~[illegible]~~ 3.0142

$(\frac{1}{2}(0.264) + \frac{1}{6}(0.613) + 1(0.118))/0.118$ = 3.0056

CI = (Average − 3)/2 = ~~[illegible]~~ 0.00917

RI = 0.58.

C.Ratio = ~~[illegible]~~ 0.015811 ⇒ not consistent 20.1

- If perfectly consistent, then the consistency measure will equal n only therefore the CI will be equal to zero and so will C. ratio

- If this ratio is large, Saaty suggest >0.1 then we are not consistent enough and best thing to do is go back and redo the comparison

- Reflects the consistency of ones judgements

- Pairwise comparisons could be inconsistent because the cert is not purely mathematical, it is also psychological.
- It relies on the decision maker defining their preferences for the decision criteria, which may not be done in a reasonable or rational way

Alternative to AHP for MCDA "non compensatory"
- Trade off curves
- Goal programming - meet a number of objectives, when an ordering exists, GP can be used
- Problem is to get as close as possible to ~~each objective~~ a specified value for each objective.

01/05/16.

DECISIONS: REDDY AHP

XMAS EXAM Q4.

4A.

Initial	T	P	A
T	1	1/2	2
P	2	1	6
A	1/2	1/6	1
Total	3.5	10/6	9

column wise / row wise

Standardised	T	P	A	Average
T	2/7	3/10	2/9	0.269
P	4/7	3/5	2/3	~~0.~~ 0.613
A	1/7	1/10	1/9	0.118

Initial Transport	C	R
C	1	1/2
R	2	1
Total	3	1.5

T Std	C	R	Average
C	1/3	1/3	1/3
R	2/3	2/3	2/3

Parks	C	R
C	1	7
R	1/7	1
Total	8/7	8

Parks Std	C	R	Average
C	7/8	7/8	7/8
R	1/8	1/8	1/8

A	C	R
C	1	1/3
R	3	1
Total	4	4/3

A Std	C	R	Av
	1/4	1/4	1/4
	3/4	3/4	3/4

Weighting - Transport: 0.264 Parks: 0.613 Amenities: 0.118

	T	P	A	Total
C	264/3000	4241/8000	54/2000	0.655
R	264/1500	613/8000	197/2000	0.345

Chose C.

Consistency check:
Multiply row in original TPA matrix against Average and sum then divide by that row

$[1(0.269) + \frac{1}{2}(0.613) + 2(0.118)] / 0.269 = 3.02$
$[2(0.269) + 1(0.613) + 6(0.118)] / 0.613 = 3.03$
$[\frac{1}{2}(0.269) + \frac{1}{6}(0.613) + 1(0.118)] / 0.118 = \underline{3.01}$
9.06

Consistency index = ~~$\frac{\text{Average} - 3}{2} = \frac{9.06}{2} = 3.03$~~

$$\frac{[(3.02 + 3.03 + 3.01)/3] - 3}{2} = 0.01$$

Divide CI by RI $\quad 0.01/0.58 = 0.0172$
It is consistent as it lies below 0.1 ⇒ generally accepted consistency value

$(r_1 \times c_1)/c_{11}$
$(r_2 \times c_2)/c_{22}$
$(r_3 \times c_3)/c_{33}$
Sum these
Calculate consistency index = $\frac{\text{Average (of Sum)} - \text{number of products}}{\#\text{ products} - 1}$

Divide CI by RI
Values less than 0.1 are consistent

Analytic Hierarchy Process
- Structured way of ranking options available to decision maker
- Requires a step by step process of pairwise comparisons of opinions against each of the objectives
- Objectives then compared and weighted
- Each objective can have a collection of sub objectives on which this process can be carried out, from where the 'hierarchy' originated
- As important = 1 $\quad$ Strongly more important 4 or 5 $\quad$ Absolutely more important = 9

31/05/16

Why pairwise comparisons may be mutually inconsistent
- Based on observation that it is easier to rank the importance of 2 objects than it is for several objects
- Saaty's theorm states pairwise comparison is consistent if and only if there exists positive numbers
- Values are seldom consistent, so measures of inconsistency are needed
- People forget their prior assessments
- Error prone as it is humans making up the numbers - using human judgement.

AHP

The method attempts to, in a structured way, present a rational perspective of a decision problem, quantifying the elements involved in the decision, for relating these elements to the overall goal and for evaluating alternative options

The decision is broken down into a series of smaller criteria that can be compared to one another in terms of relevant importance. This can theoretically lead to inconsistency. A>B and B>C then A>C etc. Comparison value of 1 means equal importance. 9 means absolutely more important

- These values (which are usually taken from the decision maker) should then be normalised to sum to 1. This is done by dividing each value by the sum of the column. The weight of each decision criteria is then the sum of each of the rows

The options should then be compared according to each of the decision criteria, making a matrix for each criteria. The "score" for each job is found by multiplying the weight for each criteria by that job score in the criteria

The largest score indicates what the most rational decision would be, given the input data to the model. If, for some reason, the decision maker does not want to choose this model, then we know that they are not making the decision according to how they think. They have either wrongly compared priorities

of the importance of the crime. This method gives us insight into the [illegible]
[illegible] mental