[illegible] Applied Probability

x	1	2	3	4	5	6
$P(x)$	1/6	1/6	1/6	1/6	1/6	

$$E[X] = \sum_{i=1}^{6} i\,P(X=i) = \sum_{i=1}^{6} i\cdot\frac{1}{6} = \frac{1}{6}\sum_{i=1}^{6} i$$

$$= \frac{1}{6}(7\times 3) = \frac{7}{2} = 3.5$$

$$\sum_{i=1}^{100} i = 101\times 50$$

$$E[X^2] = \sum_{i=1}^{6} i^2 P(X=i) = \sum_{i=1}^{6} \frac{i^2}{6} = \frac{1}{6}\sum_{i=1}^{6} i^2$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \Rightarrow \frac{1}{6}\left[\frac{6(6+1)(12+1)}{6}\right]$$

$$= \frac{7\times 13}{6} = \frac{91}{6} = 15.1666$$

$$E[X-\mu] = E[X]-\mu = \mu-\mu = 0$$

$$E[(X-\mu)^2] = E[X^2]-\mu^2 = E[X^2] - E[X]^2$$

$$= \frac{91}{6} - \left[\frac{7}{2}\right]^2$$

$$= \frac{91}{6} - \frac{49}{4} = \frac{364-294}{24} = \frac{70}{24} = \frac{35}{12} = 2.91666$$

Applied Probability ① 29/10/15

Let $E[x] = \mu$

$m = \frac{1}{n} \sum_{i=1}^{n} x_i$ ← average after n trial.

let $d = \mu - m$ $E[d] = E[m-\mu] = E[\frac{1}{n} \sum_{i=1}^{n} x_i]$
$= \frac{1}{n} n E[x_i] - \mu = \frac{n}{n}\mu - \mu = 0$

Average difference will be 0 if done loads of times
expectation of linear sum is linear sum of expectation

Aside

$Var[ax+b] = E[(ax+b - E(ax+b))^2]$
$= E[(ax + b - aE[x] - b)^2]$
$= E[a^2(x - E[x])^2] = a^2 E[(x - E(x))^2]$
$= a^2 Var[x]$

$E[ax+b] = aE[x] + b$

Variance D

$Var[d] = Var[m-\mu] = Var[m] = Var[\frac{1}{n} \sum_{i=1}^{n} x_i] = \frac{1}{n^2} Var[\sum_{i=1}^{n} x_i]$
$\underset{x_i \text{ independent}}{=} \frac{1}{n^2} n Var[x_i] = \frac{Var[x_i]}{n}$

$SD[d] = \sqrt{Var[d]} = \frac{SD[x_i]}{\sqrt{n}} \propto \frac{1}{\sqrt{n}}$

2.

Pay stake S if win get back fS, p chance of winning

Expected net pay-off

$$p(fS-S) + (1-p)(-S) > 0 \Rightarrow pfS - S > 0$$
$$\Rightarrow pf > 1$$

want positive bet (o/w dont take part)

Start with V_0

V_n = amount after n trials

Bet a fraction α each time

$$R_K \begin{cases} f & \text{if win} \\ 0 & \text{o/w} \end{cases}$$

$$V_1 = (1-\alpha + \alpha R_1)V_0$$
$$V_2 = (1-\alpha + \alpha R_2)V_1 = (1-\alpha + \alpha R_2)(1-\alpha + \alpha R_1)V_0$$
$$V_n = (1-\alpha + \alpha R_1)(1-\alpha + \alpha R_2)\dots(1-\alpha + \alpha R_n)V_0$$

Express as $V_n = V_0 e^{nG_n}$ for approximate function f^n G_n (known as growth factor)

$$\Rightarrow G_n = \frac{1}{n}\log\left(\frac{V_n}{V_0}\right) \qquad \frac{V_n}{V_0} = \frac{(1-\alpha+\alpha R_1)(1-\alpha+\alpha R_2)\dots(1-\alpha+\alpha R_n)\cancel{V_0}}{\cancel{V_0}}$$

$$\log(a \times b) = \log(a) + \log(b)$$

$$\Rightarrow G_n = \frac{1}{n}\left[\log(1-\alpha+\alpha R_1) + \log(1-\alpha+\alpha R_2) + \dots \log(1-\alpha+\alpha R_n)\right]$$

all independent and identically distributed

$$\lim_{n\to\infty} G_n = \underset{\text{long run}}{\text{average}}\left[\log(1-\alpha+\alpha R)\right] \Rightarrow = E\left[\log(1-\alpha+\alpha R_n)\right] \quad R = \begin{cases} f, p \\ 0, p-1 \end{cases}$$

$$= p\log(1-\alpha+\alpha f) + (1-p)\log(1-\alpha) = \text{long run growth factor}$$

29/10/13. Applied probability

differentiate with respect to α to maximise k.
Take derivative and equate to 0.

$$g'(\alpha) = \frac{p(f-1)}{(1-\alpha+\alpha f)} - \frac{(1-p)}{(1-\alpha)} = 0$$

$$\Rightarrow \frac{p(f-1)}{(1-\alpha+\alpha f)} = \frac{(1-p)}{(1-\alpha)} \Rightarrow (1-\alpha)p(f-1) = (1-p)(1-\alpha+\alpha f)$$

$$\rightarrow \cancel{p\alpha f}.\ pf - \cancel{p} - \cancel{\alpha pf} + \cancel{\alpha p} = \alpha + \alpha f - \cancel{p} + \cancel{p\alpha} - \cancel{p\alpha f}$$

$$pf = 1 - \alpha + \alpha f$$

$$pf = 1 + \alpha f - 1$$

$$\frac{pf-1}{f-1} = \alpha \qquad p = 0.4,\ f = 3 \quad [pf > 1]$$

$$\Downarrow$$

$$\alpha^* = \frac{\frac{4}{10} \times 3 - 1}{3-1} = \frac{1}{10}$$

Correlation $= \dfrac{cov(x,y)}{\sqrt{var(x)\ var(y)}}$

3 questions, equal marks best 2 questions taken

2013 Q1

(Y_p, Y_e)	Probability	cumulative
(0,0)	0.2	0.2 ← 0.1563 ⇒ (0,0).
(0,1)	0.1	0.3
(0,2)	0.1	0.4
(1,0)	0.1	0.5 ← 0.6291 ⇒ 1,1
(1,1)	0.2	0.7
(1,2)	0.1	0.8
(2,0)	0	0.8
(2,1)	0.1	0.9
(2,2)	0.1	1

could had (Y_e, Y_p) order would change

Marginal

Y_p	Probability
0	0.3
1	0.4
2	0.3

conditional given $1 = Y_e$

$Y_e=1$	Y_p	
	$=0$	0.25
	1	0.5
	2	0.25

$p(Y_p|Y_e=1) = \dfrac{p(Y_p \cap Y_e=1)}{p(Y_e=1)} \leftarrow 0.4$ = divide row value by 0.4 to standard

Expected value: $Y_p = \Sigma\ (0)(0.3)+1(0.4)+2(0.3) = 1$ $E[Y_p^2] = 1.6$

$Y_e = \Sigma\ (0)(0.4)+1(0.4)+2(0.2) = 0.8$

$1.6-1^2 = 0.6 = var[Y_p]$

LAB 3 Workings

Geometric $(1-p)^{x-1} \quad p \qquad x = 1, 2, 3 \ldots n$

$L(p) = (1-p)^0 p \quad (1-p)^1 p \quad (1-p) p^2 \quad \ldots \quad (1-p)^{n-1} p$

$= p^n (1-p)^{\sum x_i - n}$ LIKELIHOOD

Log:

$\ln L = n \ln p + (-n + \sum x_i) \ln(1-p)$

Log likelihood from part (a) $\rightarrow$ $\ln L = n \ln p + (-n + \sum_1^n x_i) \ln(1-p)$

Derivate with respect to p:

$$\frac{d \ln L}{dp} = \frac{n}{p} - \frac{(-n + \sum_1^n x_i)}{(1-p)} = 0$$

Equate to 0 and solve:

$$\frac{n}{p} - \frac{(-n + \sum_1^n x_i)}{(1-p)} = 0$$

$$\frac{n}{p} = \frac{-n + \sum x_i}{1-p}$$

$p(-n + \sum x_i) = n - np$

$p(-n + \sum x_i + n) = n$

$p(\sum x_i) = n$

$$p = \frac{n}{\sum x_i} = \frac{1}{\bar{x}}$$

Each X has probability:

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$L = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}} \times \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}} \times \dots \times \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_n-\mu)^3}{2\sigma^2}}$$

Log likelihood

$$Log(L) = \ln\left(2\pi\sigma^{2^{-n/2}} \exp\left(-\frac{1}{2\sigma^2} \sum_i^n (x_i-\mu)^2 \right)\right)$$

$$\ln\left(2\pi\sigma^{2^{-n/2}}\right) \quad \ln\left(\exp\left(\frac{-1}{2\sigma^2} \sum_i^n (x_i-\mu)^2 \right)\right)$$

$$= -\frac{n}{2} Log\, 2\pi\sigma^2 \quad - \frac{1}{2\sigma^2} \sum_i^n (x_i-\mu)^2$$

$$= -\frac{n}{2} \ln(2\pi) - \frac{n}{2} (\sigma^2) - \frac{1}{2\sigma^2} \sum_i^n (x_i-\mu)^2$$

Probability 1

Slides 2

Random digits
- have values in {1, 2, ... n}
- Occur with equal frequency (uniform) in long run
- Unpredictable

- Generating a random password, use looking up or char with random number.
- What about restrictions?

Rejection algorithm:
- Use simple generator
- Reject until condition is satisfied
- For example atleast one number chosen

Alternatively:

Shuffle
- Generate first from 0...9
- Generate second from complete list and third...
- Randomly re-order

Conditional algorithm
- Nominate randomly one column
- Generate from 0, 1. 9 for that column
- Generate from complete list for other columns

Sampling without replacement
- For example choose 4 from list without any repeat
- Pull to index

Ranking ⇒ ties may occur ⇒ problem

Slides 3

$P(Y=y) = P_Y(y)$.

- Note that summaries of column(s) converge as n increase
- Discrete R.V: possible values are confined to explicit list e.g $Y \in \{0,1,2\}$

Challenge:

- Password, 1 Symbol from (0...9) and 5 from (A..Z)
- Solution: Rejection, based on list of 36
  OR: - generate 0..9 in column 1; A to Z in other columns
  - Re arrange randomly

Probability (by Gregory and carey)

- Total number of possibilities 26^6

Total number of acceptable = 26x25x24 x 23 x 22 x 21.

Acceptance rate: Theory : 54.66%

Birthday problem

- Take 23 random drawings of numbers from 1:365.
- Success if we have repetition
- Look at ratio of success to trials

Probability of not same bday = $\frac{365 \times 364 \times \dots \times (365-23+1)}{365^{23}}$

Answer = 1 - answer. 15 people $\Rightarrow$ 0.2529
20 $\Rightarrow$ 0.4414
23 $\Rightarrow$ .5073

For how would pay 10 $\frac{(1-0.5073)}{0.5073} = 9.71$.

Lottery problem

Same 6 numbers coming up 20 years late (6 out of 44)

Probability of no repeat in 1348816 = 0.7224

Hence probability of 0.2776 that the same combination of 6 numbers will be drawn 2 (or more) times in 3016

3

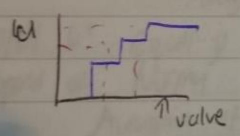
Applied Probability 1.

The long run -
- propn of duplic in n repetions
- Solve via system of replication and summary

For a password of length 2 from alphabet with 4 characters tol converge to 0.25 (mean square error decline with n)

Law of large numbers
- Long run proportions converge to theoretical probability
- Long run averages converge to "expected value".
- Long run avg of RAND() = U is E(U) = 0.5

Non uniform Random numbers
Assign num 0 - 1025, 70% 1.7-1.2 20% 7 1.2 10% < 1.
Need a transform of RAND()



Read off value

If U in the ith interval then return i.
y axis goes to 1.

Example
- Draw random letters possible A B C D
Probabil. 0.1 0.3 0.1 0.5.
- In Excel. Draw U = RAND()
Lookup (U, Cum Prob, Poss)

Conditional Probability
P (Second Q given first Q) = $P(Q_2 | Q_1)$
→ Check by simulation: form sim: filter by Q on first draw

P (First Q given Second Q) = $P(Q_1 | Q_2)$ Calc by Simula

Use logic:
- Joint Simulation or conditional Simula
- Joint simulation impl $P(Q_1 | Q_2) = P(Q_2 | Q_1)$
- Confirm by Simula
- Calcula and informa

Discrete RV and probability Distribution in math notation:
- y is name for variable
- Probability distribu of y: - Possibilities
  - Probabilities $P(y=y)$ for all y

Example y is first letter in password
Possib are {A, B, C, D}
Probs $(y=y) = 0.25$ for $y = A, = B$ ...

$E[y] = \sum t P(y=y)$

5.

Probability 1 Slide ...

SLIDES 4.

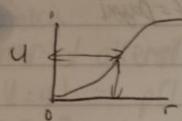
Continuous random variable

- CDF - cumulative density function

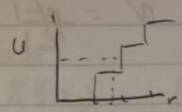
PDF - probability density function

General transformation : long run properties

~~Specially~~ Specify desired properties by $P(R \le r) = F(r)$ known as cumulative probability distribution function cdf.



$P(R \le r) = F(r)$



$P(R \le r) = F(r)$

Long run properties

Properties of U : Equally likely to be any value in $(0,1)$.

Average value of 0.5.

Properties of $3U+2$: Equally likely $(2,5)$

- Average of 3.5

Property U^2 : Equally likely $(0,1)$

Properties of Normal (RAND(), 0.8, 0.05)

Average of 0.8

95% within ± 0.1

Properties of U : Equally likely in $(0,1)$

By Symmetry $E(U) = 0.5$

$P(U \le u) = u$ for $0 < u < 1$

Properties of $R = 3U+2$: Equally likely in $(2,5)$

By Symmetry $E(U) = 3.5$

$P(R \le 3u+2) = u$ for $0 < u < 1$

$\Rightarrow P(R \le r) = \frac{r-2}{3}$ for $2 < r < 5$.

$R = U^2$: $P(R \le u^2) = u$ for $0 < u < 1$

$P(R \le r) = \sqrt{r}$ for $0 < r < 1$

Properties of $R = g(u)$

- Not necessarily equally likely to be any value in $(g(0), g(1))$
- $P(R \le g(u)) = g(u)$ for $0<u<1$
- $P(R \le r) =$ function of r; namely value of u for which $r = g(u)$ or $u = g^{-1}(u)$

Slides 5.

Kelly bet formula: $a^* = \frac{pf-1}{f-1}$ p = prob f = payout

$P(A \text{ or } B) = P(A) + P(B)$ if mutually exclusive

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ and } B) = P(A|B)\,P(B)$

equivalently $P(A|B) = P(A \text{ and } B) / P(B)$

Special case $P(A \text{ and } B) = P(A)P(B)$ when statistically independent

$$P(Q_2) = P((\bar{Q_1} \text{ and } Q_2) \text{ or } (Q_1 \text{ and } Q_2))$$
$$P(\bar{Q_1} \text{ and } Q_2) + P(Q_1 \text{ and } Q_2)$$
$$P(Q_2|\bar{Q_1})\,P(\bar{Q_1}) + P(Q_2|Q_1)\,P(Q_1)$$
$$\frac{4}{51}\,\frac{48}{52} + \frac{3}{51}\,\frac{4}{52} = \frac{4}{52}$$

Bayes rule:
$$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{P(A|B)P(B)}{P(A|\bar{B})P(\bar{B}) + P(A|B)P(B)}$$

Alternatively
$$\frac{P(B|A)}{P(\bar{B}|A)} = \frac{P(A|B)}{P(A|\bar{B})} \times \frac{P(B)}{P(\bar{B})}$$

First ratio posterior odds, last is prior odds

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Probability 1 Slides Notes

Binomial

$X \sim B(n,p)$

- n independent trials, S succ with $P(S)=p$

$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$ $E[X]=np$ $Var[X] = np(1-p)$

Poisson approximation to $B(n,p)$ when n large and p small $\lambda = np$

Geometric

Number of attempts needed for success.

$P(Y=y) = p(1-p)^{y-1}$ $P(Y \le y) = 1-(1-p)^y$

$E[Y] = 1/p$ $Var[Y] = (1-p)/p^2$

Poisson

Application Y is unlimited count of number of "incidents" in time or area

$p(Y=y) = \frac{e^{-\lambda}\lambda^y}{y!}$ $E[Y] = Var[Y] = \lambda$

When n is large and p small. In the limit the binomial formulae with n, p simplify and coincide with Poisson formulae with $\lambda = np$

Thus $\binom{n}{y} p^y (1-p)^{n-y} \approx \frac{e^{-\lambda}\lambda^y}{y!}$

$Y \sim U(0,1)$ CDF: $P(Y \le y) = \begin{cases} 0 & y<0 \\ y & 0 \le y < 0 \\ 1 & 1<y \end{cases}$

$f(y) = \frac{dF(y)}{dy} = \begin{cases} 0 & y<0 \\ 1 & 0 \le y \le 1 \\ 0 & 1<y \end{cases}$

Exponential $\lambda=1$

$Y \sim Exp(\lambda=1)$ $Y > 0$

CDF $F(y) = 1-e^{-y}$ PDF $f(y) = e^{-y}$

Uniform $a \le y < b$ $F(y) = \frac{y-a}{(b-a)}$ $f(y) = \frac{1}{b-a}$

Exponential $y \sim Exp(\mu)$ $F(y) = 1 - e^{-y/\mu}$ $f(y) = \frac{e^{-y/\mu}}{\mu}$

Exponential $y \sim Exp(\lambda)$ $F(y) = 1 - e^{-\lambda y}$ $f(y) = \lambda e^{-\lambda y}$

1

TI STUMS

The law of large numbers and simulation

After seeing many heads in a row probability of tail goes up? No
events are independent - memoryless

Law of large numbers → in long run fraction of heads and tails will be the same but in short run not.

Relative frequency $\frac{\text{number of success}}{\text{number of trials}}$

The empirical law of large numbers → the relative frequency with which event A occurs will fluctuate less and less as time goes on, and will approach a limiting value as the number of repetitions increases without bound

Sample Space: set of all possible outcomes of an experiment.
Any subset of the sample space is called an event.

P(A) is the sum of the probabilities of the individual outcomes in the set A

$P(A) \geq 0$ for every event A
$P(A) = 1$ when A is equal to the sample space
$P(A \cup B) = P(A) + P(B)$ for disjoint events A and B

Events A and B are said to be disjoint when the subsets A and B have no common elements

Expected value Σ value × probability
- Weighted average of possible values X could take

Kelly bet $n^* = \frac{pF-1}{F-1}$ p = probability of win
F = payoff 2:1 etc

law of conditional probabilities:

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \dots + P(A|B_n)P(B_n)$$

$$P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i)$$

Binomial Distribution

- Two outcomes Success or fail
- p or $1-p$
- X - total number of successes in n independent trials of Bernoulli even

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k} \quad \text{for } k = 0, 1, \dots, n$$

$$E[X] = np$$

$$0 \times (1-p) + 0(1-p) + 1(p) + \dots = np$$

Poisson Distribution

In the case of very large number of bernoulli trials with a small probability of success the binomial dist gives way to the Poisson dist.

$$P(X=k) = \frac{e^{-\lambda}\lambda^k}{k!} \quad \text{for } k = 0, 1, \dots$$

$$E[X] = \lambda$$

for bernoulli approximation $\lambda = np$

Poisson Process

- This process is used to count events that occur randomly in time
- Consider poisson dist as customers arriving at a Q
  - the customers arrive one at a time
  - The number of arrivals during non overlapping time intervals are independent of each other
  - The number of arrivals during any given time interval has a poisson dist of which the expected value is proportional to the duration of the interval

3.
T(J(M)

Defining the arrival intensity of the poisson process by
α = expected value of number of arrivals during a given time interval of unit length

Then property C demands that for each $t > 0$, it is true that

P(k arrivals during a given time interval of duration t)
$$= \frac{e^{-\alpha t} (\alpha t)^k}{k!} \quad \text{for } k = 0, 1, \ldots$$

The number of arrivals in any interval (s, s+t) does not depend on the sequence of arrivals up to time s

In a poisson arrival process the number of arrivals during a given time interval is a discrete R.V, but the time between 2 successive arrivals can take on any positive value and is thus a continuous R.V.

P(time between 2 successive arrivals is greater than y) =
= P(during an interval of duration y there are no arrivals)
$= e^{-\alpha y}$ for each $y > 0$

Thus in a PAP with arrival intensity α, the time T between 2 successive arrivals has the probability distribution function
$$P(T \leq y) = 1 - e^{-\alpha y} \quad \text{for } y \geq 0$$

Known as exponential distribution with $E[x] = \frac{1}{\alpha}$

For every fixed point in time, the waiting period from that point until the first arrival after that point has the <u>same</u> exponential distribution as the interarrival times.

- this is memoryless property

Probability density function
- Function f(x) describing the normal curve is example of a pdf.
- Any non neg function for which the total area under the graph of the function equals 1 is called the PDF.
- Any pdf underlies a continuous r.v.

$$P(a \le X \le b) = \int_a^b f(x)\,dx$$

This definition parallels the definition $E(x) = \sum_x xP(X=x)$ for a discrete R.V X.

If a random variable X is normally distributed with parameters μ and σ, then for each two constants $a \neq 0$ and b the random variable $U = aX + b$ is normally distributed with parameters $a\mu + b$ and $|a|\sigma$

Variance is a measure of the spread of the random variable X around the expected value μ.

A measure for the spread that has the same dimension as the random variable X is the Standard Deviation $(\text{dollars})^2 \rightarrow \text{dollar}$

$$\sigma^2(aX+b) = a^2(\sigma^2(X))$$

Independent if $\mathrm{Cov}(X+Y) = 0$

Central limit theorem
The sum (or average) of a sufficiently large number of independent random variables approximately follows a normal dn.

$\mu = E[x]$ $\quad \sigma = \sigma[x]$

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TIJIMI

If $X_1, X_2 .. X_n$ are independently random variables each having the same distribution with expected value μ and S.D σ, then the sum $X_1+X_2+..+X_n$ has an approximately normal dist with expected value $n\mu$ and standard deviation $\frac{\sigma}{\sqrt{n}}$ when n is sufficiently large

- Holds true no matter what form the distribution of random variable X_k takes
- Essential that r.v X_k are independent

$E[X] = n\mu$ and $sd = \sigma/\sqrt{n}$

Bayes Rule

$P(\text{disease}) = 0.001$ $P(\text{No D}) = 0.999$

$P(\text{Positive} \mid \text{disease}) = 0.99$ $P(\text{Neg} \mid \text{disease}) = 0.01$

$P(\text{Positive} \mid \text{no disease}) = 0.02$ $P(\text{Neg} \mid \text{no disease}) = 0.98$

The posterior probability $P(\text{disease} \mid \text{positive})$ satisfies the relation

$$P(\text{disease} \mid \text{positive}) = \frac{P(\text{positive and disease})}{P(\text{positive})};$$

A repeated application of the definition of conditional probability gives:

$P(\text{positive and disease}) = P(\text{positive} \mid \text{disease})\,P(\text{disease})$

and $P(\text{positive}) = P(\text{positive and disease}) + P(\text{positive and no disease})$

$= P(\text{positive} \mid \text{disease})\,P(\text{disease}) + P(\text{positive} \mid \text{no disease})\,P(\text{no disease})$

$$\Rightarrow P(\text{disease} \mid \text{positive}) = \frac{0.99 \times 0.01}{0.99 \times 0.001 + 0.02 \times 0.999} = 0.0472$$

Conditional Probability

A conditional probability now reflects our knowledge of the occurrence of the event A given that event B has occurred

P(A) known as prior odds before knowing B

For any 2 events A and B with $P(B) > 0$ the conditional probability $P(A|B)$ is defined

$$P(A|B) = \frac{P(AB)}{P(B)}$$

$$\Rightarrow P(AB) = P(A|B)P(B)$$

Two events are independent if $P(AB) = P(A)P(B)$

Let A be an event that can only occur when one of mutually exclusive events $B_1 \dots B_n$ occurs then:

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) \dots P(A|B_n)P(B_n)$$

This rule is called Law of conditional probabilities

Bayes Rule in odds form

H - hypothesis true $\bar{H}$ - false

Before examining, assign prior probability $P(H)$ and $P(\bar{H}) = 1 - P(H)$

The updated value of the probability that H is true given the fact that E has occurred is denoted by $P(H|E)$

The posterior probability $P(H|E)$ satisfies:

$$\frac{P(H|E)}{P(\bar{H}|E)} = \frac{P(H)}{P(\bar{H})} \cdot \frac{P(E|H)}{P(E|\bar{H})}$$

In words Bayes rule in ~~[illegible]~~ odds form:

posterior odds = prior odds x likelihood ratio

7.

TTJIMS

Formula follows by twice applying the definition of conditional probability

$$P(H|E) = \frac{P(HE)}{P(E)} = P(E|H)\frac{P(H)}{P(E)}$$

Same expression holds for $P(\bar{H}|\bar{E})$: $P(E|\bar{H})\frac{P(\bar{H})}{P(E)}$

Dividing first by second results in odd form of bayes rule
Factor $P(H)/P(\bar{H})$ gives prior odds

PDF

$$P(X \le a) = \int_0^a f(x)\,dx$$

$$E[X] = \int_{-\infty}^{\infty} x f(x)\,dx$$

Uniform $f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a < x < b \\ 0 & \text{otherwise} \end{cases}$

$$F(x) = \frac{b-x}{b-a} \quad \text{for } a \le x < b$$

Exponential $f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$

$E[X] = \frac{1}{\lambda}$ $Var[X] = \frac{1}{\lambda^2}$ $F[x] = \begin{cases} 1-e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases}$