MA1E02 Tutorial Sheet 3.

Week 5 2013

Questions Evaluate the following integrals:

1. $$\int x^{3/2} \ln(x) dx \quad (1)$$

2. $$\int \frac{\ln(x)^2}{x^3} dx \quad (2)$$

3. $$\int x^2 \sin(ax) dx \quad (3)$$

4. $$\int x e^{x/2} dx \quad (4)$$

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3. $\int x^2 \sin(ax)\,dx$

$\int u\,dv = uv - \int v\,du$

$u = x^2$ $\qquad \int dv = \int \sin(ax)\,dx$

$du = 2x$ $\qquad = -\frac{1}{a}\cos(ax)$

$du = 2x\,dx$

$\int u\,dv = x^2\left(-\frac{1}{a}\cos(ax)\right) + \frac{1}{a}\int \cos(ax)\,2x\,dx$

$= -\frac{x^2}{a}\cos(ax) + \frac{2}{a}\int x\cos(ax)\,dx$

(I'm by parts)

$u = x$ $\qquad \int dv = \int \cos(ax)\,dx = \frac{1}{a}\sin(ax) = v$

$du = dx$

$\int u\,dv = \frac{1}{a}x\sin(ax) - \frac{1}{a}\int \sin(ax)$

$= \frac{1}{a}x\sin(ax) - \frac{1}{a}\left(-\frac{1}{a}\cos(ax)\right)$

$\int x^2\sin(ax)\,dx = -\frac{1}{a}x^2\cos(ax) - \frac{2}{a^2}\sin(ax) + \frac{2}{a^3}\cos(ax) + C$

4. $\int x e^{x/2}\,dx$

$u = e^{x/2}$ $\qquad u = x$ $\qquad \int dv = e^{x/2}\,dx$

$\frac{du}{dx} = \frac{1}{2}\cdot e^{x/2}$ $\qquad du = dx$ $\qquad v = 2e^{x/2}$

$\int u\,dv = uv - \int v\,du$

$= 2xe^{x/2} - 2\int e^{x/2}) + C$

$= 2xe^{x/2} - 4e^{x/2} + C$

$= 2e^{x/2}(x-2) + C$

2. $\int \frac{\ln(x)^2}{x^3}\,dx = \int \ln(x)^2 \cdot \frac{1}{x^3}\,dx$

$\int u\,dv = uv - \int v\,du$

$u = \ln(x)^2$

$\frac{du}{dx} = 2 \cdot \ln(x) \cdot \frac{1}{x}$

$= \frac{2}{x}\ln(x)$

$\int dv = \int \frac{1}{x^3}\,dx$

$= \int x^{-3}$

$= \frac{1}{-2}x^{-2}$

$= -\frac{1}{2}x^{-2}$

$\Downarrow$

$\int u\,dv = uv - \int v\,du$

$= (\ln x)^2\left(-\frac{1}{2}x^{-2}\right) - \int\left(-\frac{1}{2}x^{-2}\right)\frac{2}{x}\ln x\,dx$

$= -\frac{1}{2}\frac{(\ln x)^2}{x^2} + \int \underbrace{\frac{\ln x}{x^3}\,dx}$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$ $du = \frac{1}{x}dx$

$\int dv = \int x^{-3}$ $v = -\frac{1}{2}x^{-2}$

$uv - \int v\,du = \ln x\left(-\frac{1}{2}\cdot x^{-2}\right) + \frac{1}{2}\int$

$= -\frac{1}{2}\frac{\ln x}{x^2} - \frac{1}{4}\frac{1}{x^2}$

$= -\frac{1}{2}(\ln x)^2 + \frac{1}{2}\frac{\ln x}{x^2} - \frac{1}{8}\cdot\frac{1}{x^2} + C$

$= -\frac{1}{2x^2}\left(x^2(\ln x)^2 - \ln x + \frac{1}{4}\right) + C$

Maths Tutorial 3. Week 5

1. $\int x^{3/2} \ln(x) dx$

$u(x) \cdot v(x)$

uv

$\frac{d(u.v)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$\int \frac{d(u.v)}{dx} dx$

$u.v = \int u \frac{dv}{dx} dx + \int v \frac{du}{dx} dx$

$= \int u\,dv + \int v\,du.$

$\int u\,dv = u.v - \int v\,du$

$\Rightarrow \int x^{3/2} \ln(x) dx$

$u = \ln x \quad \frac{du}{dx} = \frac{1}{x} \quad du = \frac{1}{x}.dx$

$\int dv = \int x^{3/2} dx \quad v = \frac{1}{5/2} x^{5/2} = \frac{2x^{5/2}}{5}$

$\int u\,dv = \ln x . \frac{2}{5} - \frac{2}{5} \int x^{5/2}$

$= \frac{2}{5} \ln x . x^{5/2} - \frac{2}{5}\left(\frac{2}{5} x^{5/2}\right) + C$