Tutorial 6: MA1E01

Derivatives 2

1. For each of the following functions, compute dy/dx:

   (a) $y = \sin(1/x^2)$

   (b) $y = \frac{1+\csc(x^2)}{1-\cot(x^2)}$

   (c) $y = \cos(\cos(x))$

2. Recall that

$$\frac{d}{dx}(|x|) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

   Use this result and the chain rule to find

$$\frac{d}{dx}(|\sin x|)$$

   for non-zero x in the interval $(-\pi, \pi)$.

3. Find d^2y/dx^2 by implicit differentiation:

   (a) $2x^2 - 3y^2 = 4$

   (b) $xy + y^2 = 2$

   (c) $x \cos y = y$

4. A 17-ft ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5 ft/s, how fast will the top of the ladder be moving down the wall when it is 8 ft above the ground.

5. The side of a cube is measured with a possible percentage error of $\pm 2\%$. Use differentials to estimate the percentage error in the volume.

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1a. $y = \sin \frac{1}{x^2}$ $\quad u = \frac{1}{x^2}$

$y = \sin u$ $\quad dy = \cos u$

$u = -x^{-2}$

$\frac{du}{dx} = -2x^{-3}$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$\therefore \cos x \, (-2x^{-3})$

$= -\frac{2}{x^3} \cdot \cos \frac{1}{x^2}$

1b $y = \frac{1 + \csc(x^2)}{1 - \cot(x^2)}$ $\quad \csc(x) = \frac{1}{\sin x}$ $\quad \cot(x) = \frac{1}{\tan x} = \frac{\cos}{\sin x}$

$y = \frac{1 + \frac{1}{\sin x}}{1 - \frac{\cos x}{\sin x}}$

$= \frac{\sin x^2 + 1}{\sin x^2 - \cos x}$ $\quad \begin{matrix} u \\ v \end{matrix}$

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\frac{(\sin x^2 - \cos x)(+\cos x) - (\sin x^2 + 1)(\cos x + \sin x^2)}{(\sin x^2 \cos x)^2}$

$\frac{\cancel{\sin x^2 \cos x} \; \cancel{- \cos^2 x} \; \cancel{- \sin^2 x \cos x} \; \cancel{+ \sin^2 x^2} - \cos x - \sin x^2 + 1}{(\sin x - \cos x)^2}$

$\frac{-\cos x - \sin x^2 + 1}{(\sin x^2 - \cos x)^2}$

$\frac{[\sin x^2 - \cos x^2][2x \cos(x^2)] - [\sin(x^2) + 1][2x \cos(x^2) + 2x \sin(x^2)]}{[\sin(x^2) - \cos(x^2)]^2}$

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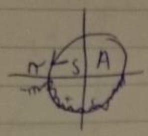
2.



$\sin x > 0 \quad 0 < x < \pi$

$\sin x < 0 \Rightarrow -\pi < x < 0$

$$\frac{dy}{dx} = \begin{cases} \cos x & 0 < x < \pi \\ -\cos x & -\pi < x < 0 \end{cases}$$

f	f'
y	$\frac{dy}{dx}$
y^2	$= 2y \frac{dy}{dx}$

3a $2x^2 - 3y^2 = 4$

$4x - 6y \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-4x}{-6y}$

$\frac{dy}{dx} = \frac{2x}{3y}$

$$\frac{d^2y}{dx^2} = \frac{3y(2) - 2x \cdot 3\frac{dy}{dx}}{9y^2}$$

$$= \frac{6y - 6x\frac{dy}{dx}}{9y^2}$$ Sub in $\frac{dy}{dx}$

$$\frac{6y - 6x\left(\frac{2x}{3y}\right)}{9y^2}$$

$$\frac{6y - \frac{4x}{y}}{9y^2}$$

$$\frac{6y^2 - 4x^2}{9y^3}$$

$$= \frac{2(3y^2 - 2x^2)}{9y^2}$$

$$-\frac{2(2x^2 - 3y^2)}{9y^2}$$ Sub in (4)

$$= \frac{-8}{9y^3}$$

3b $xy + y^2 = 2$

$x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(x+2y) = -y$

$\frac{dy}{dx} = \frac{-y}{(x+2y)}\ \begin{matrix}u\\v\end{matrix}$

$\frac{d^2y}{dx^2} = \frac{(x+2y)(-\frac{dy}{dx}) - (-y)(1+2\frac{dy}{dx})}{(x+2y^2}$

$\frac{-(x+2y)(\frac{-y}{x+2y}) + y(1+2(\frac{-y}{x+2y})}{(x+2y^2)^2}$

$\frac{+y + y + 2y\left(\frac{-y}{x+2y}\right)}{(x+2y^2)^2}$

$\cancel{2y+2y}\ \frac{2y(1\ \frac{-y}{x+2x})}{(x+2y^2)^2}$

3c $x\cos y = y$

$-\sin y \frac{dy}{dx} \overset{+\cos y}{=} \frac{dy}{dx}$

$\frac{d^2y}{dx^2} = (-x\sin y - 1)(\cos y \frac{dy}{dx}) - (\cos y)($

$-x\sin y \frac{dy}{dx} - \frac{dy}{dx} = 0$

$\frac{dy}{dx}(-x\sin y - 1) = \cos y$

$\frac{dy}{dx} = \frac{\cos y}{(-x\sin y - 1)} = \frac{\cos y}{(1+x\sin y)}$

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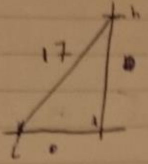
5.

4 

let h = position of top of ladder

l = position of bottom of ladder

Given $\frac{dl}{dt} = 5$ when $h=8$ $\frac{dh}{dt} = ?$

$17^2 = h^2 + l^2$

$\cancel{h^2 = 17^2 - l^2}$

$\cancel{h = \sqrt{17^2 - l^2}}$

$2h\frac{dh}{dt} + 2l\frac{dl}{dt} = 0$

$\frac{dh}{dt} = -\frac{2l\frac{dl}{dt}}{2h} \Rightarrow \frac{dh}{dt} = \frac{-5l}{h}$

$\left.\frac{dh}{dt}\right|_{h=8} \quad \frac{-5l}{8}$

$\frac{-5(15)}{8}$ ft/sec

$l^2 = 17^2 - h^2$

$l = \sqrt{17^2 - h^2}$

$l = \sqrt{17^2 - 8^2}$

$l = 15$

5. percentage error $= \frac{dx}{x}$

$x \Rightarrow \frac{dx}{x} = \pm 0.02$

want $\frac{dV}{V}$

$V = x^3$

$\frac{dV}{dx} = 3x^2$

$dV = 3x^2(dx)$

$\frac{dV}{V} = \frac{3x^2 dx}{V}$

$\frac{dV}{V} = \frac{3x^2 dx}{x^3}$

$\frac{3dx}{x}$

$= 3(\pm 0.02)$

$= \pm 0.06$

$= \pm 6\%$