1

2012 MANG SCI PAPER 3 Q4 DAVID WEINBRECHT

4(a) Assignment Problem

Origin \ Destination	1	2	3	4	5
1	5.5	1.6	9.4	5.2	6.8
2	5.0	0.4	5.2	3.6	0.8
3	6.3	10.8	10.6	8.8	10
4	5.6	2.4	5.6	4.8	2.4
5	5.5	6.8	12.0	10	5.2
6	5.5	2.0	11.2	5.6	7.2

i. Minimise: $5.5X_{11} + 1.6X_{12} + 9.4X_{13} + 5.2X_{14} + 6.8X_{15}$
$+ 5.0X_{21} + 0.4X_{22} + 5.2X_{23} + 3.6X_{24} + 0.8X_{25}$
$+ 6.3X_{31} + 10.8X_{32} + 10.6X_{33} + 8.8X_{34} + 10X_{35}$
$+ 5.6X_{41} + 2.4X_{42} + 5.6X_{43} + 4.8X_{44} + 2.4X_{45}$
$+ 5.5X_{51} + 6.8X_{52} + 12.0X_{53} + 10X_{54} + 5.2X_{55}$
$+ 5.5X_{61} + 2.0X_{62} + 11.2X_{63} + 5.6X_{64} + 7.2X_{65}$

ST: $X_{11} + X_{12} + X_{13} + X_{14} + X_{15} \leq 1$
$X_{21} + X_{22} + X_{23} + X_{24} + X_{25} \leq 1$
$X_{31} + X_{32} + X_{33} + X_{34} + X_{35} \leq 1$
$X_{41} + X_{42} + X_{43} + X_{44} + X_{45} \leq 1$
$X_{51} + X_{52} + X_{53} + X_{54} + X_{55} \leq 1$
$X_{61} + X_{62} + X_{63} + X_{64} + X_{65} \leq 1$

$X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + X_{61} = 1$
$X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + X_{62} = 1$
$X_{13} + X_{23} + X_{33} + X_{43} + X_{53} + X_{63} = 1$
$X_{14} + X_{24} + X_{34} + X_{44} + X_{54} + X_{64} = 1$
$X_{15} + X_{25} + X_{35} + X_{45} + X_{55} + X_{65} = 1$

All vars $\{0,1\}$

2

A(ii)

Supplier → destination

Manufacturer (origin)	1	2	3	4	5	6
1	614	660	534	680	590	630
2	536	568	624	616	616	560
3	692	664	620	680	720	744
4	608	632	584	664	680	632
5	640	576	606	616	584	664

Dummy Row (SERIOUS ERROR)

A(iii) Hungarian Method

Subtract smallest value from each row first.

Supplier → destination

		1	2	3	4	5	6
-534	1	80	126	0	146	56	96
-536	2	0	32	88	80	80	24
-620	3	72	44	0	60	10	124
-584	4	24	48	0	80	96	48
-576	5	64	0	30	40	8	88

(origin: manufacturer)

Subtract smallest value from column with out a zero

		1	2	3	4	5	6
	1	80	126	0	106	48	72
	2	0	32	88	40	72	0
	3	72	44	0	20	42	100
	4	24	48	0	40	88	24
	5	64	0	30	0	0	64

(origin: manufacturer)

Subtracted 40 from C4, 8 from C5, 24 from C6

⇒ cross out zeros in as little lines as possible
- need a minimum of 5 lines, only have 3

-20 is smallest value not covered by line
⇒ Subtract 20 from each uncovered row.
⇒ Then add 20 to each covered row.

3 2021 MANG SCI PAPER 3 Q4 DAVID WEITBRECHT

		1	2	3	4	5	6
Man	1	60	106	20	86	28	52
origin	2	0	32	18	40	72	0
	3	52	24	0	0	72	80
	4	04	28	0	20	68	24
	5	64	0	50	0	0	64

⇒ (req'd) cut zeros ↗
⇒ Still less than 5.

4 is smallest entry not covered
- Subtract 4 from each uncovered row
- Then add 4 to each covered row

		1	2	3	4	5	6
Man	1	56	102	20	86	24	48
origin	2	0	32	112	44	72	0
	3	48	20	0	0	68	76
	4	0	24	0	20	64	20
	5	64	0	54	4	4	68

⇒ (req'd) cut zeros ↗
20 is smallest entry uncovered. Subtract 20 from each uncovered row
add 20 to each covered column

		1	2	3	4	5	6
Man	1	36	82	(0)	66	4	28
origin	2	0	32	12	44	72	(0)
	3	48	20	0	(0)	68	76
	4	(0)	24	0	20	64	20
	5	64	(0)	54	4	4	68

Solution Found

4

For Supple 1. We manufacture: 3
2 6
3. 4
4. 1
5. 2

Total cost of Shipment + cost per kg included is.

534 + 560 + 680 + 608 + 576 = €2958 cost

M1 → 55
M2 → 51
M3 — 53
M4 — 56
M5 — 52
(M1) — 54

Dummy

M4/S1 Total Cost = Purchase Cost = 40,000 × 25.2
Trans. = 40,000 × 6.5
T.C = 1,228,000

etc.,

5. 2012 MANG SCI PAPER 3 Q4 DAVID WETTBRECHT

B(i) $\text{Min } Z = 7x_{11} + 3x_{12} + 5x_{13} + 5x_{14} + 5x_{21} + 5x_{22} + 7x_{23} + 6x_{24} + 8x_{31} + 6x_{32} + 6x_{33} + 5x_{34} + 6x_{41} + x_{42} + 6x_{43} + 4x_{44}$

ST:
$x_{11} + x_{12} + x_{13} + x_{14} \leq 340$
$x_{21} + x_{22} + x_{23} + x_{24} \leq 150$
$x_{31} + x_{32} + x_{33} + x_{34} \leq 120$
$x_{41} + x_{42} + x_{43} + x_{44} \leq 190$

and:
$x_{11} + x_{21} + x_{31} + x_{41} = 210$
$x_{12} + x_{22} + x_{32} + x_{42} = 280$
$x_{13} + x_{23} + x_{33} + x_{43} = 170$
$x_{14} + x_{24} + x_{34} + x_{44} = 170$

all $x_{ij} \geq 0$

B(ii) VAM

- Vogel's approximation method
- Attempts to identify the greater advantage of a cell over the next best in its row or column
- This method generally gives a close to optimum starting solution and is superior to Least Cost method and North West Corner method

Not asked for steps just to use it.

Steps in VAM

- Calculate a penalty for each row and column by subtracting the smallest cost element in the row or column from the next smallest cost from the same row or column
- Identify the row with greatest penalty. Allocate as much as possible to the cell with least cost in this row or column
- Adjust free demand and supply accordingly
- When all rows and columns filled and totals are zero → Solution found

6

Penalties for VAM 1

row 1 5-3 = 2 | column 1 6-5 = 1
row 2 6-5 = 1 | 2 7-1 = 2
row 3 6-5 = 1 | 3 6-5 = 1
row 4 4-1 = 3 | 4 5-4 = 1

	1	2	3	4	Supply
1.	7	3	5	5	340
2.	5	5	7	6	150
3.	8	6	6	5	120
4.	6	1 (190)	6	4	190 - 190 = 0
Demand	210	250 - 190 = 60	170	170	

Penalties for VAM 2

row 1 5-3 = 2 | colum 1 6-5 = 1
2 6-5 = 1 | 2 5-3 = 2
3 6-5 = 1 | 3 6-5 = 1
4 Satisfied | 4 5-4 = 1

	1	2	3	4	Supply
1	7	3 (60)	5	5	340 - 60 = 280
2	5	5	7	6	150
3	8	6	6	5	120
4	6	1 (190)	6	4	0
Demand	210	60 - 60 = 0	170	170	

Penalties for VAM 3

row 1 7-5 = 2 | colum 1 6-5 = 1
2 6-5 = 1 | 2 Satisfied
3 6-5 = 1 | 3 6-5 = 1
4 Satisfied | 4 5-4 = 1

7. 202 MANG. SCI PAPER 3 Q4 DAVID WEINBRENT

	1	2	3	4	Supply
1	7	3(60)	5(170)	5	280-170=110
2	5	5	7	6	150
3	8	6	6	5	120
4	6	1(190)	6	4	0
Demand	210	0	170-170=0	170	

Penalite) VAM (4)

Row 1 ~~7-5=2~~ Colum 1 6-5=1
2 6-5=1 2 Satisfied
3 6-5=1 3 Satisfied
4 Satisfied 4 5-4=1

	1	2	3	4	Supply
1	7	3(60)	5(170)	5(110)	110-110=0
2	5	5	7	6	150
3	8	6	6	5	120
4	6	1(190)	6	4	0
Demand	210	0	0	170-110=60	

Penalite) VAM (5)

Row 1 Satisfied Colum 1 6-5=1
2 6-5=1 2 Satisfied
3 6-5=1 3 Satisfied
4 Satisfied 4 5-4=1

Choose arbitrarily

	1	2	3	4	Supply
1	7	3(60)	5(170)	5(110)	0
2	5(150)	5	7	6	150-150=0
3	8	6	6	5	120
4	6	1(190)	6	4	0
Demand	210-150=60	0	0	60	

8.

Penalties VAM G

row 1 Satisfied
2 Satisfied
3 6-5=1
4 Satisfied

column 1 7-6=1
2 Satisfied
3 Satisfied
4 5-4=1

	1	2	3	4	Supply
1	7	3(60)	5(70)	5(110)	0
2	5(150)	5	7	6	0
3	8	6	6	5(60)	120-60=60
4	6	1(190)	6	4	0
Demand	60	0	0	60-60=0	

Next is row 3, Solution is

	1	2	3	4	Supply
1	7	3(60)	5(170)	5(110)	340
2	5(150)	5	7	6	150
3	8(60)	6	6	5(60)	120
4	6	1(190)	6	4	190
Demand	210	250	170	170	

Current Solution:

$5(150) + 8(60) + 3(60) + 1(190) + 5(170) + 5(110) + 5(60)$

$= 3300$ cost.

Now solve using MODI METHOD

9

2012 MANG SI PAPER 3 Q4 DAVID WEMBRECHT

(1)

		1 $K_1=8$	2 $K_2=3$	3 $K_3=5$	4 $K_4=5$	Supply
$R_1=0$	1	60 (7)	60 (3)	170 (5)	110-60 (5)	340
$R_2=-3$	2	150 (5)	(5)	(7)	(6)	150
$R_3=0$	3	60-60 (8)	(6)	(6)	60+60 (5)	120
$R_4=-2$	4	(6)	140 (1)	(6)	(4)	140
Demand		210	250	170	170	

Calculation

1→1 $7-0-8=-1$	3→2 $6-3+3=6$	4→3 $6-5+2=3$
4→1 $6+2-8=0$	2→3 $7-5+3=5$	2→4 $6-7+3=4$
2→2 $5+3-3=5$	3→3 $6-5-0=1$	4→4 $4--2-5=1$

(2)

		1 $K_1=7$	2 $K_2=3$	3 $K_3=5$	4 $K_4=5$	Supply
$R_1=0$	1	60 (7)	60 (3)	170 (5)	50 (5)	340
$R_2=-2$	2	150 (5)	(5)	(7)	(6)	150
$R_3=0$	3	(8)	(6)	(6)	120 (5)	120
$R_4=-2$	4	(6)	190 (1)	(6)	(4)	190
Demand		210	250	170	170	

Calculation

3→1 $8-0-7=1$	3→2 $6-0-3=3$	4→3 $6+2-5=3$
4→1 $6+2-7=1$	2→3 $7-5-2=4$	2→4 $6-5--2=3$
2→2 $5--2+3=10$	3→3 $6-5-0=1$	4→4 $4--2-5=1$

Solution found, all values positive, no savings to be made

Cost =

$60(7)+150(5)+60(3)+190(1)+170(5)+50(5)+120(5)=3240$

1. 2012 MANGSCI PAPER3 Q4 CORRECTION

Assignment problem — demand minimization

		1	2	3	4	5	D	Dummy
origin	1	5·50	1·60	9·40	5·20	6·50	0	
Source	2	5·0	0·4	5·2	3·6	0·8	0	
	3	6·3	10·8	10·6	8·8	10·0	0	
	4	5·60	2·4	5·6	4·8	2·4	0	
	5	5·50	6·8	12·0	10	5·2	0	
	6	5·50	2·0	11·2	5·6	7·2	0	

Minimize $5.5X_{11} + 1.6X_{12} + 9.4X_{13} + 5.2X_{14} + 6.8X_{15} + 0X_{10}$
$+ 5.0X_{21} + 0.4X_{22} + 5.2X_{23} + 3.6X_{24} + 0.8X_{25} + 0X_{20}$
$+ 6.3X_{31} + 10.8X_{32} + 10.6X_{33} + 8.8X_{34} + 10.0X_{35} + 0X_{30}$
$+ 5.6X_{41} + 2.4X_{42} + 5.6X_{43} + 4.8X_{44} + 2.4X_{45} + 0X_{40}$
$+ 5.5X_{51} + 6.8X_{52} + 12.0X_{53} + 10X_{54} + 5.2X_{55} + 0X_{50}$
$+ 5.5X_{61} + 2.0X_{62} + 11.2X_{63} + 5.6X_{64} + 7.2X_{65} + 0X_{60}$

Subject to: Supply

$X_{11} + X_{12} + X_{13} + X_{14} + X_{15} + X_{10} = 1$
$X_{21} + X_{22} + X_{23} + X_{24} + X_{25} + X_{20} = 1$
$X_{31} + X_{32} + X_{33} + X_{34} + X_{35} + X_{30} = 1$
$X_{41} + X_{42} + X_{43} + X_{44} + X_{45} + X_{40} = 1$
$X_{51} + X_{52} + X_{53} + X_{54} + X_{55} + X_{50} = 1$
$X_{61} + X_{62} + X_{63} + X_{64} + X_{65} + X_{60} = 1$

Demand must be supplied

$X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + X_{61} = 1$
$X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + X_{62} = 1$
$X_{13} + X_{23} + X_{33} + X_{43} + X_{53} + X_{63} = 1$
$X_{14} + X_{24} + X_{34} + X_{44} + X_{54} + X_{64} = 1$
$X_{15} + X_{25} + X_{35} + X_{45} + X_{55} + X_{65} = 1$
$X_{16} + X_{26} + X_{36} + X_{46} + X_{56} + X_{66} = 1$

All $X \in \{0, 1\}$ only

3 2012 MAUG SCI PAPER 3 Q4 ... CORRECTION

2

A (ii) Total cost of supplying manufactured covered

		1	2	3	4	5	6	Source supply
Destination	1	614	660	534	680	590	630	534
Man.	2	536	568	624	616	616	580	536
	3	692	664	620	680	720	744	620
	4	608	632	584	664	680	632	584
	5	640	576	606	616	584	664	576
	D	0	0	0	0	0	0	0

iii. Hungarian method

Subtract smallest value from each row.

-Then subtract smallest value from column containing no zeros.

Source (Supplier)

		1	2	3	4	5	6
Destination	1	80	126	0	146	56	96
Man.	2	0	32	88	80	80	24
	3	72	44	0	60	100	124
	4	24	48	0	80	96	48
	5	64	0	30	40	8	88
	D	0	0	0	0	0	0

Cross out zeros is as few lines as possible

-Not 0 Solution

-four lines used.

- Subtract smallest value which is uncrossed by a line from each row (8)

- Add 8 to each covered column

3 2012 NAMUSO PAPER 3 Q4 CORRECTION

		Source - Supplier					
		1	2	3	4	5	6
2 Destination	1	88	126	0	132	48	88
	2	0	32	88	72	72	16
	3	72	44	0	52	42	116
	4	24	48	0	72	88	40
	5	64	0	30	32	0	80
	D	20	20	20	0	0	0

- 16 is smallest uncovered
- Subtract 16 from each uncovered row
- Add to each covered column

		Source - Supplier					
		1	2	3	4	5	6
3 Destination	1	80	110	0	116	32	72
	2	0	16	88	56	56	0
	3	72	28	0	36	26	100
	4	24	32	0	56	72	24
	5	80	0	46	32	0	80
	D	36	20	36	0	0	0

24 i) Smallest - Subtract from uncovered row, add to covered column

		1	2	3	4	5	6
Destination	1	66	86	0	92	8	48
	2	0	16	112	56	56	0
	3	48	4	0	12	2	76
	4	0	8	0	32	48	0
	5	80	0	72	32	0	80
	D	36	20	60	0	0	0

- 2 i) Smallest.
- Subtract 2 from each uncovered row
- Add to each covered column

4.

		1	2	3	4	5	6
Destr	1	64	84	[0]	90	6	46
man	2	0	16	114	56	56	0
	3	46	8	0	15	[0]	74
	4	0	8	2	32	48	0
	5	80	[0]	74	32	0	80
	D	16	20	62	[0]	0	0

Solution found

Destination (manufacturer)		Source (supplier)	Cost
1		3	534
2	1 or 6 24	1	536
3		5	720
4	1 or 6	6	632
5		2	576
D		4	0

Total = 2998

For (2 and 4) they could be (1 or 6) but cost still 2998

2012 Q4

Assignment Problem

origin	Destination 1	2	3	4	5
1	5.5	1.6	9.4	5.2	6.8
2	5.0	0.4	5.2	3.6	0.8
3	6.3	10.8	10.6	8.8	10
4	5.6	2.4	5.6	4.8	2.4
5	5.5	6.8	12.0	10	5.2
6	5.5	2.0	11.2	5.6	7.2

Supply ① $5.5X_{11} + 1.6X_{12} + 9.4X_{13} + 5.2X_{14} + 6.8X_{15}$

Objective function Min

$5.5X_{11} + 1.6X_{12} + 9.4X_{13} + 5.2X_{14} + 6.8X_{15} + 5.0X_{21} + 0.4X_{22} + 5.2X_{23} + 3.6X_{24}$
$+ 0.8X_{25} + 6.3X_{31} + 10.8X_{32} + 10.6X_{33} + 8.8X_{34} + 10X_{35}$
$+ 5.6X_{41} + 2.4X_{42} + 5.6X_{43} + 4.8X_{44} + 2.4X_{45}$
$+ 5.5X_{51} + 6.8X_{52} + 12.0X_{53} + 10X_{54} + 5.2X_{55}$
$+ 5.5X_{61} + 2.0X_{62} + 11.2X_{63} + 5.6X_{64} + 7.2X_{65}$

ST: Supply ① $5.5X_{11} + 1.6X_{12} + 9.4X_{13} + 5.2X_{14} + 6.8X_{15} = 25.20$

② $5.0X_{21} + 0.4X_{22} + 5.2X_{23} + 3.6X_{24} + 0.8X_{25} = 28.0$

③ $6.3X_{31} + 10.8X_{32} + 10.6X_{33} + 8.8X_{34} + 10X_{35} = 20.4$

④ $5.6X_{41} + 2.4X_{42} + 5.6X_{43} + 4.8X_{44} + 2.4X_{45} = 28.4$

⑤ $5.5X_{51} + 6.8X_{52} + 12.0X_{53} + 10X_{54} + 5.2X_{55} = 24.0$

⑥ $5.5X_{61} + 2.0X_{62} + 11.2X_{63} + 5.6X_{64} + 7.2X_{65} = 26.0$

ST. $X_{11} + X_{12} + X_{13} + X_{14} + X_{15} \leq 1$

$X_{21} + X_{22} + X_{23} + X_{24} + X_{25} \leq 1$

$X_{31} + X_{32} + X_{33} + X_{34} + X_{35} \leq 1$

$X_{41} + X_{42} + X_{43} + X_{44} + X_{45} \leq 1$

$X_{51} + X_{52} + X_{53} + X_{54} + X_{55} \leq 1$

$X_{61} + X_{62} + X_{63} + X_{64} + X_{65} \leq 1$

$$
\begin{aligned}
&X_{11} + X_{21} + X_{31} + X_{41} + X_{51} = 1\\
&X_{12} + X_{22} + X_{32} + X_{42} + X_{52} = 1\\
&X_{13} + X_{23} + X_{33} + X_{43} + X_{53} = 1\\
&X_{14} + X_{24} + X_{34} + X_{44} + X_{54} = 1\\
&X_{15} + X_{25} + X_{35} + X_{45} + X_{55} = 1
\end{aligned}
$$

If (column) 1 and supply	1	2	3	4	5	6
1	614	660	534	680	590	630
2	536	568	456			
3	692	664	520			
4	608	632	504			
5	640	576	456			

Supplier

Manufacturer	1	2	3	4	5	6
1	614	660	534	680	590	630
2	536	568	624	616	616	560
3	692	664	620	680	720	744
4	608	632	584	664	680	632
5	640	576	600	616	584	664

Subtract smallest from each row

		1	2	3	4	5	6
-534	1	80	126	0	146	56	96
-536	2	0	32	88	80	80	24
-620	3	72	44	0	60	100	124
-584	4	24	48	0	80	96	48
-576	5	64	0	30	40	8	88

Subtract smallest from each column

0	0	1	2	3	4	5	6
0	2	1 80	126	0	106	48	72
0	3	2 0	32	88	40	72	0
40	4	3 72	44	0	20	92	100
8	5	4 24	48	0	40	88	24
24	6	5 64	0	30	0	0	64

242 04

20 is smallest entry not covered by the
Subtract 20 from each uncovered row
Then add 20 to each covered colum.

	①	②	③	④	⑤	⑥
①	60	106	20	86	28	52
②	0	32	108	40	72	0
③	52	24	20	0	72	80
④	04	28	60	20	68	24
⑤	64	0	50	0	0	64

4 0 Smallest entry not cov
Subtrat 4 from each uncovd row.
Then add 4 to each covd colm

	①	②	③	④	⑤	⑥
①	56	102	20	86	24	48
②	0	32	112	44	72	0
③	48	20	0	0	68	76
④	0	24	0	20	64	20
⑤	64	0	54	4	4	68

24 Smallest entry. Subtrat 20 from each uncovd row
add 24 to each covered colum.

	①	②	③	④	⑤	⑥
①	36	82	0	66	04	28
②	0	32	112	44	72	0
③	48	20	0	0	68	76
④	0	24	0	20	64	20
⑤	64	0	54	4	4	68

SOLUTION FOUND

For Supplier ① use manufacter: 3
② 6
③ 4
④ 1
⑤ 2

cost = 534 + 560 + 680 + 608 + 576 = 2958

2012 Q4

bi. Min $Z = 7X_{11} + 3X_{12} + 5X_{13} + 5X_{14} + 5X_{21} + 5X_{22} + 7X_{23} + 6X_{24} + 8X_{31} + 6X_{32} + 6X_{33} + 5X_{34} + 6X_{41} + 11X_{42} + 6X_{43} + 4X_{44}$

ST: $X_{11} + X_{12} + X_{13} + X_{14} \leq 340$

$X_{21} + X_{22} + X_{23} + X_{24} \leq 150$

$X_{31} + X_{32} + X_{33} + X_{34} \leq 120$

$X_{41} + X_{42} + X_{43} + X_{44} \leq 190$

also $X_{11} + X_{21} + X_{31} + X_{41} = 210$

$X_{12} + X_{22} + X_{32} + X_{42} = 250$

$X_{13} + X_{23} + X_{33} + X_{43} = 170$

$X_{14} + X_{24} + X_{34} + X_{44} = 170$

ii. VAM

- Vogel's approximation method.
- Attempts to identify the greatest advantage of a cell over the next best in its row or column
- This method generally gives a close to optimum starting solution and is superior to LCM and NWC method

Steps in VAM

- Calculate a penalty for each row and column by subtracting the smallest cost element in the row or column from the next smallest cost from the same row or column

- Identify the row with the greatest penalty. Allocate as much as possible to the cell with the least cost in this row or column

- Adjust the for demand and supply (cabal) and cross out the row or column where demand or supply is entirely used up.
- If both row and the column have no supply or demand left, only cross out one
- If all column and rows have been satisfied then we have an initial solution otherwise recalculate penalties for unused row + column

Penalties for VAM

row 1 5-3 = 2
row 2 6-5 = 1
row 3 6-5 = 1
row 4 4-1 = 3

column 1 6-5 = 1
column 2 3-1 = 2
column 3 6-5 = 1
column 4 5-4 = 1

	①	②	③	④	Supply
①	7	3	5	5	340
②	5	5	7	6	150
③	8	6	6	5	120
④	6	1(190)	6	4	190-190=0
Demand	210	250-190	170	170	

Penalties VAM ②

row 1 5-3 = 2
row 2 6-5 = 1
row 3 6-5 = 1
row 4 Satisfied

column 1 6-5 = 1
column 2 5-3 = 2
column 3 6-5 = 1
column 4 5-4 = 1

	①	②	③	④	Supply
①	7	3(60)	5	5	340-60
②	5	5	7	6	150
③	8	6	6	5	120
④	6	1(190)	6	4	0
Demand	210	60-60	170	170	

Penalties VAM ③

row 1 7-5 = 2
row 2 6-5 = 1
row 3 6-5 = 1
row 4 Satisfied

column ① 6-5 = 1
② Satisfied
3 6-5 = 1
4 5-4 = 1

3 vam

2012 q4

	1	2	3	4	Supply
1	7	3(60)	5(170)	5	280 - 170
2	5	5	7	6	150
3	8	6	6	5	120
4	6	1(190)	6	4	0
Demand	210	0	170-170	170	

Penalty VAM (4)

row 1 7-5 = 2

2 6-5 = 1

3 6-5 = 1

4 Solved

colum 1 6-5 = 1

2 Solved

3 Solved

4 5-4 = 1

	1	2	3	4	Supply
1	7	3(60)	5(170)	5(110)	110 - 110
2	5	5	7	6	150
3	8	6	6	5	120
4	6	1(190)	6	4	0
Demand	210	0	0	170-110	

Penalty Vam (5)

row 1 Solved

2 6-5 = 1

3 6-5 = 1

4 Solved

colum 1 6-5 = 1

2 Solved

3 Solved

4 5-4 = 1

choose arbitrary

	1	2	3	4	Supply
1	7	3(60)	5(70)	5(110)	0
2	5(150)	5	7	6	150 - 150
3	8	6	6	5	120
4	6	1(190)	6	4	0
Demand	210-150	0	0	60	

Penalty VAM ⑥

row 1 Satisfied
2 Satisfied
row 3 6-5 = 1
4 Satisfied

column 1 7-6 = 1
2 Satisfied
3 Satisfied
4 5-4 = 1

	1	2	3	4	Supply
1	7	3 (60)	5 (70)	5 (110	0
2	5 (150)	5	7	6	0
3	8	6	6	5 (60)	120-60
4	6	1 (140)	6	4	0
Demand	60	0	0	60-60	

Penalty VAM ⑦

row 1 Satisfied
2 8 ~~8~~ 6 = 2
3 Satisfied
4 Satisfied

c 1. ~~7~~ 6 = 1
2. Satisfied
3. Satisfied
4. Satisfied

least to initial problem:

	1	2	3	4	Supply
1	7	3 (60)	5 (170)	5 (110)	340
2	5 (150)	5	7	6	150
3	8 (60)	6	6	5 (60)	120
4	6	1 (140)	6	4	140
Demand	210	250	170	170	

New sole using MODI METHOD

Solve = 5(150) + 8(60) + 3(60) + 1(140) + 5(170) + 5(110) + 5(60) = 3300

2012 604

(1)

		$K_1=8$		$K_2=3$		$K_3=5$		$K_4=5$		
		1		2		3		4		Supply
$R_1=0$	1	60	7	60	3	170	5	110-60	5	340
$R_2=-3$	2	150	5		5		7		6	150
$R_3=0$	3	60-60	8		6		6	60+60	5	120
$R_4=-2$	4		6	190	1		6		4	190
	Demand	210		250		170		170		

$1 \to 1$ $7-8=-1$ $3 \to 2$ $6-3+3=6$ $4 \to 3$ $6-5+2=3$

$4 \to 1$ $6+2-8=0$ $2 \to 3$ $7-5+3=5$ $2 \to 4$ $6-5+3=4$

$2 \to 2$ $5+3-3=5$ $3 \to 3$ $6-5-0=1$ $4 \to 4$ $4--2-5=1$

(2)

		$K_1=7$		$K_2=3$		$K_3=5$		$K_4=5$		
		1		2		3		4		Supply
$R_1=0$	1	60	7	60	3	170	5	50	5	340
$R_2=-2$	2	150	5		5		7		6	150
$R_3=0$	3		8		6		6	120	5	120
$R_4=-2$	4		6	190	1		6		4	190
	Demand	210		250		170		170		

$3 \to 1$ $8-0-7=1$ $3 \to 2$ $6-0-3=3$ $4 \to 3$ $6--2-5=3$

$4 \to 1$ $6+2-7=1$ $2 \to 3$ $7-5--2=4$ $2 \to 4$ $6-5--2=3$

$2 \to 2$ $5--2+3=4$ $3 \to 3$ $6-5-0=1$ $4 \to 4$ $4--2-5=1$

Solution found

$60(7)+150(5)+60(3)+190(1)+170(5)+50(5)+120(5)=3240$