18/3/14

1 DAVID WEITBRECHT 12300644 Sheet 8 MI1S1

7.5/13

Q1(i). $A\bar{v} = \lambda\bar{v}$

$p(\lambda) = \det[\lambda I - A]$

$\det\left[\begin{pmatrix}-2 & 0\\ -1 & 2\end{pmatrix} - \lambda\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\right]$

$\det\left[\begin{pmatrix}-2-\lambda & 0\\ -1 & 2-\lambda\end{pmatrix}\right] = (-2-\lambda)(2-\lambda) - (-1)(0)$

$-4 + 2\lambda - 2\lambda + \lambda^2$

$= \lambda^2 - 4$

$(\lambda - 2)(\lambda + 2) = 0$

$\lambda = 2$ or $\lambda = -2$. eigen values

$\overset{A}{\begin{pmatrix}-2 & 0\\ -1 & 2\end{pmatrix}}\overset{\bar{v}}{\begin{pmatrix}v_1\\ v_2\end{pmatrix}} = \overset{\lambda\bar{v}}{2\begin{pmatrix}v_1\\ v_2\end{pmatrix}}$

$\left[\begin{pmatrix}-2 & 0\\ -1 & 2\end{pmatrix} - \begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}\right]\begin{pmatrix}v_1\\ v_2\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}$

$\begin{pmatrix}-4 & 0\\ -1 & 0\end{pmatrix}\begin{pmatrix}v_1\\ v_2\end{pmatrix} = \begin{matrix}0\\ 0\end{matrix}$

no info on v_2 can be anything

$-4v_1 = 0$ $v_1 = 0$ ~~$v_2 = 0$~~

$-v_1 = 0$

$\bar{v} = (0, 1)$. eigen vector

$\begin{pmatrix}-2 & 0\\ -1 & 2\end{pmatrix}\begin{pmatrix}v_1\\ v_2\end{pmatrix} - 2\begin{pmatrix}v_1\\ v_2\end{pmatrix}$

$\left[\begin{pmatrix}-2 & 0\\ -1 & 2\end{pmatrix} - \begin{pmatrix}-2 & 0\\ 0 & -2\end{pmatrix}\right]\begin{pmatrix}v_1\\ v_2\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}$

$\begin{pmatrix}0 & 0\\ -1 & 4\end{pmatrix}\begin{pmatrix}v_1\\ v_2\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}$

$-v_1 + 4v_2 = 0$

$v_1 = +4v_2$

$v_1 = 1$ $v_2 = -4$

$\bar{v}$ $(1, +4)$

is eigen vector

2 DAVID WETTBRECHT 12300644

$A\bar{v} = \lambda \bar{v}$

$p(\lambda) = \det[A - \lambda I]$

$$\det\left[\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & -2 & -3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]$$

$$\det\left[\begin{pmatrix} 1-\lambda & 1 & 1 \\ 0 & 2-\lambda & 3 \\ 0 & -2 & -3-\lambda \end{pmatrix}\right]$$

$$(1-\lambda)\left[(2-\lambda)(-3-\lambda) - (-2)(3)\right] - 1\left[(0)(-3-\lambda) - (3)(0)\right] + 1\left[(0-2) - (2-\lambda)(0)\right]$$

0 $\quad$ 5

$1-\lambda\left[-6 - 2\lambda + 3\lambda + \lambda^2 + 6\right]$

$1-\lambda\left[\lambda^2 + \lambda\right]$

$\lambda = 1$ $\qquad \lambda^2 + \lambda = 0$

$\lambda(\lambda+1) = 0$

$\lambda = 0$ or $\lambda = -1$

$\lambda = 1$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & -2 & -3 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \lambda \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

$$\left[\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & -2 & -3 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & -2 & -4 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$V_2 + V_3 = 0$

$V_2 + 3V_3 = 0$

$-2V_2 - 4V_3 = 0$

$V_2 = -V_3 \rightarrow 2V_3 = 0$

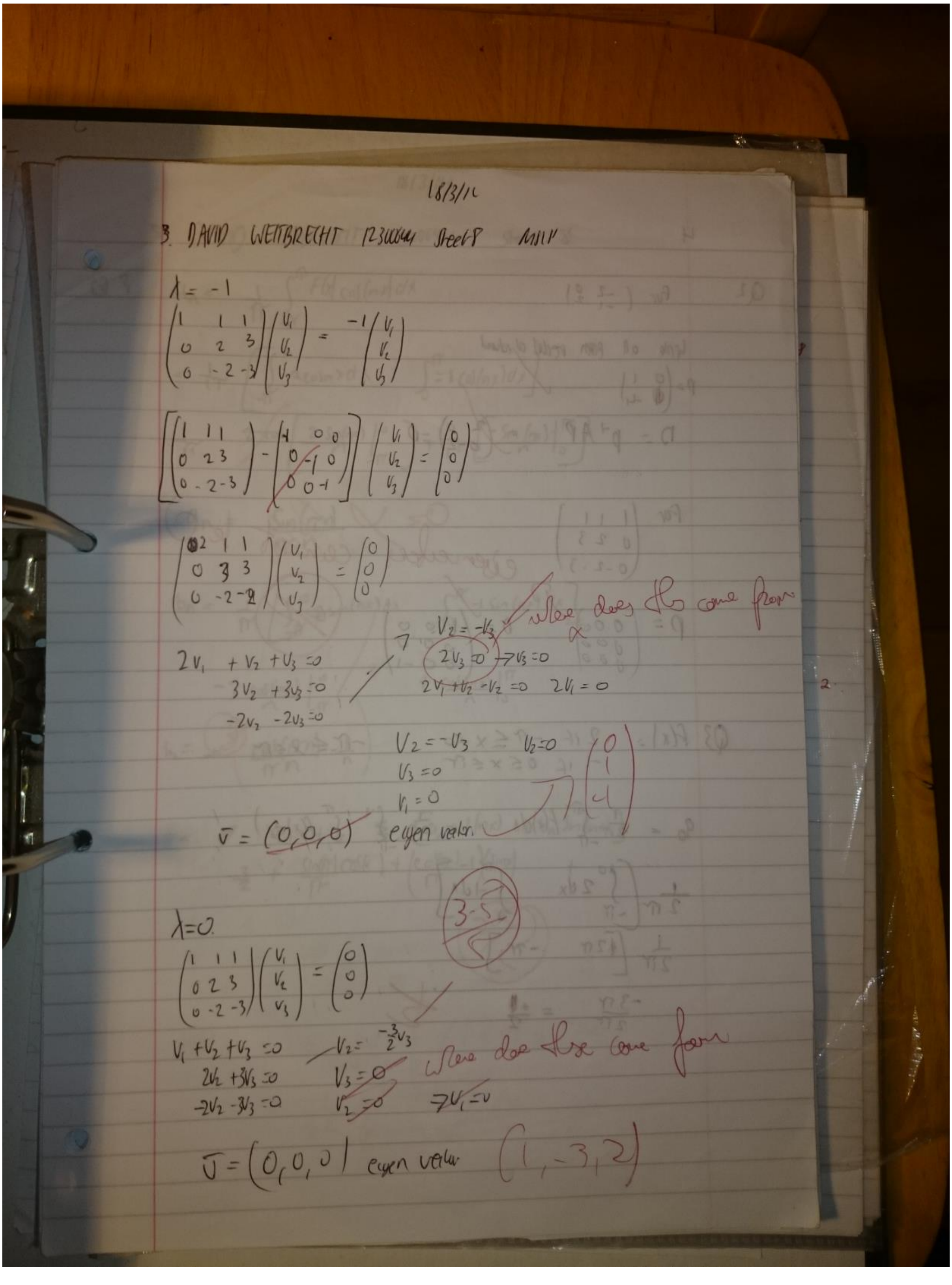
$V_3 = 0$

~~$V_1 = 0$~~ again

$V_2 = 0$

v1 can be anything

$\bar{v} = (0, 0, 0)$



18/3/16

3. DAVID WETTBRECHT 123... Sheet 8 MSV

$\lambda = -1$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & -2 & -3 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = -1\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$$\left[\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & -2 & -3 \end{pmatrix} - \underset{\lambda}{\cancel{\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}}}\right]\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 1 \\ 0 & 3 & 3 \\ 0 & -2 & -2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$2v_1 + v_2 + v_3 = 0$

$3v_2 + 3v_3 = 0$

$-2v_2 - 2v_3 = 0$

$V_2 = -V_3$

$2V_3 = 0 \rightarrow V_3 = 0$

$2V_1 + V_2 - V_2 = 0 \quad 2V_1 = 0$

where does this come from

$V_2 = -V_3 \quad V_2 = 0$

$V_3 = 0$

$V_1 = 0$

$\bar{v} = \cancel{(0,0,0)}$ eigen vector.

$$\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

3-5

$\lambda = 0$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & -2 & -3 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$V_1 + V_2 + V_3 = 0$

$2V_2 + 3V_3 = 0$

$-2V_2 - 3V_3 = 0$

$V_2 = -\frac{3}{2}V_3$

$\cancel{V_3 = 0}$

$\cancel{V_2 = 0}$

$\Rightarrow V_1 = 0$

Where does these come from

$\bar{v} = (0, 0, 0)$ eigen vector $(1, -3, 2)$

Q2 For $\begin{pmatrix} -2 & 0 \\ -1 & 2 \end{pmatrix}$

Write all eigen vectors as column

$p = \begin{pmatrix} 0 & 1 \\ 0 & -4 \end{pmatrix}$ ✓

$D = p^{-1}AP \quad = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix} = D$ ✓

For $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & -2 & -3 \end{pmatrix}$

eigenvectors cannot be 0

$p = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ ✓ $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

4/5

Q3. $f(x) = \begin{matrix} 2 \text{ if } -\pi \le x < 0 \\ -1 \text{ if } 0 \le x \le \pi \end{matrix} \qquad -\pi \le x \le \pi.$

$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx \quad = \frac{1}{2\pi}\int_{-\pi}^{0} \int_{0}^{\pi} f(x)dx)$

$\frac{1}{2\pi}\left[\int_{-\pi}^{0} 2\,dx \quad \int_{0}^{\pi} -1\,dx\right]$

$\frac{1}{2\pi}\left[+2\pi \quad -\pi\right]$

$\frac{-3\pi}{2\pi} = \frac{+1}{2}$

1/2

18/3/14

5. DAVID WETTBRECHT 12300641 Sheet 8

Q 3 $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$

$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 2\cos nx\,dx \quad \int_{0}^{\pi} -1\cos(nx)\,dx\right]$

$\frac{1}{\pi} \quad \cancel{2\cos}\left[\frac{2\sin(nx)}{n}\Big|_{-\pi}^{0}\right] + \frac{1}{\pi}\left[-\frac{\sin(nx)}{n}\Big|_{0}^{\pi}\right]$

$a_n = \cancel{\frac{\sin(\pi n)}{\pi n}} = 0$

$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} 2\sin(nx)\,dx \quad \int_{0}^{\pi} -1\sin(nx)\,dx\right]$

$-\frac{2\cos(nx)}{n}\Big|_{-\pi}^{0} \qquad + \frac{\cos(nx)}{n}\Big|_{0}^{\pi}$

$b_n = -3\left(\frac{\cos(\pi n)}{\pi n} - \frac{1}{n}\right)$

$= \cancel{a}\ a_0 + \sum\left[a_n\cos(nx) + b_n\sin(nx)\right]$

$-\frac{3}{2} + \frac{\sin(\pi)}{\pi}\cos(x) + \left(\frac{\cos(\pi)}{\pi} - 1\right)(\sin x) + \ldots$

2.5

2