1/12/13 Week 11
PROBLEM SET 9 DAVID LIEMBRECHT 12300644

1. Express rectangular coords in terms of spherical coords

$x = \rho \sin\varphi \cos\theta \qquad y = \rho\sin\varphi\sin\theta \qquad z = \rho\cos\varphi$

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1b $\iint_S F \cdot n \, dS = \iiint_G \operatorname{div} F \, dv$

$\operatorname{div} F = \frac{d(x^3 - e^y)}{dx} + \frac{d(y^3 + \sin z)}{dy} + \frac{d(z^3 - xy)}{dz}$

$3x^2 + 3y^2 + 3z^2 = \operatorname{div} F$

$\iiint 3x^2 + 3y^2 + 3z^2 \, dV$

$\int_0^{\pi} \int_0^{\pi/2} \int_0^{2} \left(3(2\sin\varphi\cos\theta)^2 + 3(2\rho\sin\varphi\sin\theta)^2 + 3(2\cos\varphi)^2\right) 2^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$

$12\rho^2\sin^2\varphi\cos^2\theta + 12\rho^2\sin^2\varphi\sin^2\theta + 12\rho^2\cos^2\varphi$

$12\rho^2\sin^2\varphi + 12\rho^2\cos^2\varphi$

$12\rho^2 \quad /4$

$\int_0^{\pi} \int_0^{\pi/2} \int_0^{2} 3\rho^2(\rho^2\sin\varphi)$

$3\rho^4\sin\varphi \, d\rho \, d\varphi \, d\theta$

$\int_0^{\pi} \int_0^{\pi/2} \frac{3\rho^5}{5}\sin\varphi \Big|_{\rho=0}^{2} d\varphi \, d\theta$

$\int_0^{\pi} \int_0^{\pi/2} \frac{96}{5}\sin\varphi \, d\varphi \, d\theta$

$\frac{96}{5} \int_0^{2\pi} -\cos\varphi \Big|_{\varphi=0}^{\varphi=\pi/2}$

$[0 - -1]$

$\frac{96}{5}(2\pi) = \frac{192\pi}{5}$

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2(ii) Find curl of F $\nabla \times F$

$$\begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ (xz^2 - xy) & (3x - yz - x^2z) & (yz - 4x) \end{vmatrix}$$

$= \frac{d}{dx}(xz^2)$

$3x - y^2 - x^2z$

$= i\left[\frac{d(yz-4x)}{dy} - \frac{d(3x-yz-x^2z)}{dz}\right] - \left[\frac{d(yz-4x)}{dx} - \frac{d(xz^2-xy)}{dz}\right]j + \left[\frac{d(3x-yz-x^2z)}{dx} - \frac{d(xz^2-xy)}{dy}\right]k$

$-x$ not -1

$[z - (-y - x^2)]i + [-4 - (2xz)]j + [3 - 2xz - (-1)]k$

$[z + y + x^2]i + [4 + 2xz]j + [4 - 2xz]k$ $= (\text{curl } F)$

Since plane is oriented upward this indicates a positive direction

eq^n of $x + y + z = 1 \Rightarrow z = g(x,y) = 1 - x - y$

$n\,dS = (-f_x, -f_y, 1) \Rightarrow (1,1,1)$

$\vec{i} + \vec{j} + \vec{k}$

Region get eq^n by plugging in $z = 0 \Rightarrow y = -x + 1$

Based on ranges $\Rightarrow$ $0 \le x \le 1$ $0 \le y \le -x + 1$

$\int_C \vec{F} \cdot d\vec{r} = \iint [z + y - x^2]i + [4 + 2xz]j + [4 - 2xz]k \cdot (\vec{i} + \vec{j} + \vec{k})$

$\iint z + y - x^2 + 4 + \cancel{2xz} + 4 - \cancel{2xz}\ dA$

$\iint 8 + y + x^2 + z\ dA$ but $z = 1 - x - y$

$\iint 8 + y - x^2 + (1 - x - y)\ dA$

$\iint 9 + x^2 - x\ dA$

$= \int_0^1 \int_0^{-x+1} \cancel{8}\ 9 + x^2 - x\ (8 + x^2)\,dy\,dx$

$\int_0^1 9y + \cancel{9}x^2 - \cancel{9}x \Big|_{y=0}^{-x+1} dx$

$\int_0^1 9(-x+1) + \cancel{9}(-x+1)x^2 - x(-x+1)\ dx$

$\cancel{\int_0^1 -9x+9+x^3-x^2+x^2-x \quad dx}$

$\cancel{\int_0^1 -10x+9+x^3} \quad dx$

$\int_0^1 -9x+9-x^3+x^2+x^2-x \quad dx$

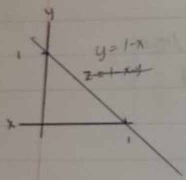
$\int_0^1 -10x+9-x^3+2x^2 \quad dx$

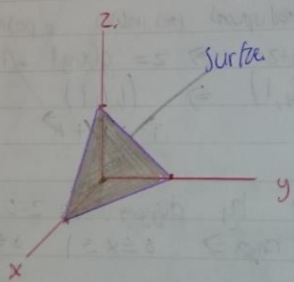
$\frac{-10x^2}{2} + 9x - \frac{x^3}{2} + \frac{2x^3}{3} \Big|_{x=0}^{1}$

$\left[-\frac{10}{2} + 9 - \frac{1}{2} + \frac{2}{3}\right] - [0] = \frac{25}{6}$

$\frac{2}{3}$

2(i) Sketch





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Q3 $\oint_C F \cdot dr$

$z = 1-x-y$ $\qquad \frac{(1,1,1)}{\|\nabla F\|} = 1,1,1$

$n = (1,1,1)$

$xz^2 - xy + \cancel{3x} - \cancel{yz} - x^2 z + \cancel{yz} - \cancel{4x}$

$xz^2 - x - xy - x^2 z \qquad z = (1-x-y)$

$x(1-x-y)^2 - xy - x - x^2(1-x-y)$

$x^3 + 2x^2y - 2x^2 + xy^2 - 2xy + x - xy - x \quad \cancel{-x^2 + x^3 + x^2y}$

$\cancel{2x}$

$\cancel{xy^2 + x^2y}$

$3x^2y + 2x^3 - 3xy - 3x^2 - x + xy^2$

$$\int_0^1 \int_0^{-x+1} \frac{3x^2y^2}{2} + \cancel{2x} \; 2x^3y - \frac{3xy^2}{2} - 3x^2y - xy + \frac{xy^3}{3} \Big|_{y=0}^{-x+1} \quad (1-x)$$

$$\frac{3x^2(1-2x+x^2)}{2} + 2x^3(1-x) - \frac{3x(1-2x+x^2)}{2} - 3x^2(1-x) - x(1-x)$$

$$\frac{3x^2-6x^3+3x^2}{2} + 2x^3 - 2x^4 \; \frac{-3x+6x^2-3x^3}{2} - 3x^2 + 3x^3 - x + x^2 \; \frac{-x^4+3x^3-3x^2+x}{3}$$

$$\frac{9x^2 - 18x^3 + 9x^2 + 12x^3 - 12x^4 \; - 9x + 18x^2 - 9x^3 - 18x^2 + 18x^3 - 6x + 6x^2 - 2x^4 + 6x^3 - 6x^2 + 3x}{6}$$

$$\int_0^1 18x^2 + 9x^3 - 14x^4 - 12x \; dx$$

$$\frac{18x^3}{3} + \frac{9x^4}{4} - \frac{14x^5}{5} - \frac{12x^2}{2} \Big|_{x=0}^{1} \qquad \frac{-11}{20}$$

$\frac{7}{3}$

X $\leftarrow$ $\oint F \cdot dr$ is parameterised in terms of 1 variable, it's a line integral

some sort of area integral.