C Use fundamental theorem of line integral to find value of integral when $F(x,y) = \nabla\phi(x,y)$

$$\int_C F(x,y)\cdot dr = \int \nabla\phi(\cdot dr) = \phi(x_1, y_1) - \phi(x_0, y_0)$$

$(-\pi, 0) \quad (\pi/2, \pi)$

$$-(-\pi)^3 + \pi^3 \sin(-\pi) + 2(0^2)(-\pi) + 3\pi^3\cos(0) - \left[-\left(\tfrac{\pi}{2}\right)^3 + \pi^3\sin\left(\tfrac{\pi}{2}\right) + 2(\pi^2)\left(\tfrac{\pi}{2}\right) + 3\pi^3\cos(\pi)\right]$$

$0 \quad 0$

$$\left[\pi^3 + 3\pi^3\right] - \left[-\frac{\pi^3}{8} + \pi^3 + \pi^3 - 3\pi^3\right]$$

$$4\pi^3 - \left[-\tfrac{7}{8}\pi^3\right]$$

$$4\pi^3 + \tfrac{7}{8}\pi^3$$

$$\frac{39\pi^3}{8}$$

$$-\left(\frac{\pi}{2}\right)^3 + \pi^3\sin\left(\frac{\pi}{2}\right) + 2(\pi^2)\left(\frac{\pi}{2}\right) + 3\pi^3\cos(\pi)$$

$$\frac{-\pi^3}{8} + \pi^3 + \pi^3 - 3\pi^3 = -\frac{9}{8}\pi^3$$

$$-\frac{9}{8}\pi^3 \quad -4\pi^3 = \frac{-41\pi^3}{8}.$$

D r_0 to r_1 = $(1-t)r_0 + t(r_1)$

$C_1\ (-\pi, 0) \Rightarrow (\pi/2, 0) : (1-t)(-\pi, 0) + t\left(\tfrac{\pi}{2}, 0\right) \cdot \left(-\pi + \pi t + \tfrac{t\pi}{2}, 0\right)$

$C_2 : (\pi/2, 0) \Rightarrow (\pi/2, \pi) = (1-t)(\pi/2, 0) + t(\pi/2, \pi) = (\pi/2, t\pi)$

$C_1\ \frac{dx}{dt} = \pi + \frac{\pi}{2} = \frac{3\pi}{2} \qquad \frac{dy}{dt} = 0$ no need for second part $\left[-\pi + \pi t + \frac{t\pi}{2}, 0\right]$

$$\left[-3\left(-\pi + \frac{3t\pi}{2}\right)^2 + \pi^3\cos\left(-\pi + \frac{3t\pi}{2}\right) + 2(0)^2\right]\frac{3\pi}{2}$$

$$\left[-3\left(\pi^2 + \frac{9t^2\pi^2}{4} - 3t\pi^2\right) + \pi^3\cos\left(-\pi + \frac{3t\pi}{2}\right)\right]\frac{3\pi}{2}$$

$$\left[-3\pi^2 - \frac{27t^2\pi^2}{4} + 9t\pi^2 + \pi^3\cos\left(-\pi + \frac{3t\pi}{2}\right)\right]\frac{3\pi}{2}$$

$\frac{8}{20}$

$$\frac{3\pi}{2}\left[-3\pi^2 t - \frac{27\pi^2}{4}\frac{t^3}{3} + \frac{9t^2\pi^2}{2} + \left(-\frac{\sin(\frac{3\pi}{2})}{2}\right)3\pi\right] \quad u = \frac{3\pi t}{2} \quad du = \frac{3\pi}{2}\, dt$$

$$\frac{3\pi}{2}\int \cos(u)\, du$$

$$\frac{3\pi}{2}\sin\left(\frac{3\pi}{2}\right)$$

$$\frac{3\pi}{2}\left[-3\pi^2 t - \frac{27\pi^2}{4}\frac{t^3}{3} + \frac{9t^2\pi^2}{2} - \frac{3\pi}{2}\sin\left(\frac{3\pi}{2}\right)\Big|_{t=0}^{1}\right.$$

$$\frac{3\pi}{2}\left[\left[-3\pi^2 - \frac{27\pi^2}{12} + \frac{9\pi^2}{2} + \frac{3\pi}{2}\right] - \left[\frac{3\pi}{2}\right]\right]$$

$$\frac{3\pi}{2}\left[\frac{-3}{4}\pi^2 + \frac{3\pi}{2}\right] - \frac{9\pi^2}{4}$$

$$\cancel{= \frac{-3}{4}\pi^2}$$

$$\frac{3\pi}{2}\left[-3\pi^2 - \frac{9\pi^2}{4} + \frac{9\pi^2}{2} + \pi^3\left(\frac{2}{3\pi}\right)\right] -$$

$$\frac{3\pi}{2} \quad \cancel{} \quad \frac{-1}{12} = -\frac{1}{8}\pi^3$$

$$C_2: \frac{dx}{dt} = 0 \quad \frac{dy}{dt} = \pi \quad \left(\pi/2,\ t\pi\right)$$

$$-3\pi^3\sin(t\pi) - 4\left(\frac{\pi}{2}\right)(t\pi) \quad dt$$

$$\int_0^1 -3\pi^3 \sin t\pi - 2t\pi^2 \quad dt$$

$$-3\pi^3\left(-\frac{\cos(t\pi)}{\pi}\right) + \frac{t^2\pi^2}{1}\Big|_{t=0}^{1}$$

$$\pi \cdot \left[\left[\frac{-3\pi^3}{\pi} + \pi^2\right] - \left[\frac{3\pi^3}{\pi}\right]\right]$$

$$\left[-2\pi^2 - 3\pi^2\right]\pi$$

$$= -5\pi^3$$

$$-5\pi^3 - \frac{\pi^3}{8} = -\frac{41}{8}\pi^3$$

1 2010 MANG SCI PAPER 1 Q1 DAVID WEITBRECHT $\frac{8}{20}$

1 a Expess rectangular coordinates in term of spherical coordinates

Rectangular : x, y, z

Spherical $x = p\sin\phi\cos\theta$ $y = p\sin\phi\sin\theta$ $z = p\cos\phi$

For 5 marks a diagram would be needed 2

Bi $x^2+y^2+z^2 = r^2$ Compute volume of ball of radius R

$0 \le \theta \le 2\pi$
$0 \le \phi \le \pi$
$0 < p < p\cos\phi$
R ?

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{r} r^2\sin\phi \; dp\,d\phi\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi}\left[\frac{r^3}{3}\right]_0^r \sin\phi \; d\phi\,d\theta$$

$$= \frac{1}{3}\int_0^{2\pi}\int_0^{\pi} r^3\sin\phi \; d\phi\,d\theta$$

$$= \frac{r^3}{3}\int_0^{2\pi} -\cos\phi\Big|_{\phi=0}^{\pi} \; d\theta$$

$$\frac{2r^3}{3}\int_0^{2\pi} d\theta$$

$$= \frac{2r^3}{3}(2\pi) = \frac{4\pi r^3}{3}$$

6

ii Find the mass of the solid enclosed between the sphere $x^2+y^2+z^2 = 4$ and $x^2+y^2+z^2 = 9$ if density is $\delta(x,y,z) = \frac{e^{-(x^2+y^2+z^2)}}{\sqrt{x^2+y^2+z^2}}$

Volume $r=3$ $-r=2$

$$\frac{4}{3}(3)^3\pi - \frac{4}{3}(2)^3\pi = 36\pi - \frac{32}{3}\pi = \frac{76}{3}\pi$$

2

$\int_0^{2\pi}\int_0^{\pi}\int_2^3 \quad d\rho\, d\phi\, d\theta$

Mass?

Please detail next steps

$$M = \int_0^{2\pi}\int_0^{\pi}\int_2^3 \delta(\rho,\theta,\phi)\, \cancel{d\rho\, d\phi}\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

$$\delta(\rho,\theta,\phi) = \cancel{\delta(x,y)} \cdot \frac{e^{-\rho^2}}{\rho}$$

$$\Rightarrow M = \int_0^{2\pi}\int_0^{\pi}\int_2^3 \frac{e^{-\rho^2}}{\rho}\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_2^3 \rho e^{-\rho^2} \sin\phi\, d\rho\, d\phi\, d\theta$$

Similar to 2011 ~~exam~~

$$\int_2^3 \rho e^{-\rho^2} d\rho \qquad u = \rho^2 \Rightarrow du = 2\rho\, d\rho$$

$\rho = 2 \Rightarrow u = 4$

$\rho = 3 \Rightarrow u = 9$

$$= \frac{1}{2}\int_4^9 e^{-u} du$$

$$= -\frac{1}{2}(e^{-9} - e^{-4}) = \frac{1}{2}(e^{-4} - e^{-9})$$

$$\int_0^{\pi} \sin\phi\, d\phi = -\cos\phi \Big|_0^{\pi} = \cancel{} =$$

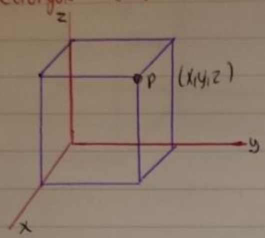
$$= -[(-1) - (+1)] = 2.$$

$$\int_0^{2\pi} d\theta = 2\pi$$

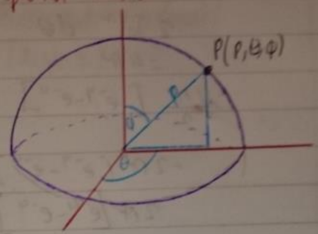
$$\Rightarrow M = 2\pi(e^{-4} - e^{-9})$$

2010 MANGSCI PAPER 1 Q1 CORRECTION

1 a Rectangular coordinates



Spherical coordinates



$P(x, y, z)$

$\rho \geq 0, \quad 0 \leq \theta \leq 2\pi \quad 0 \leq \varphi \leq \pi$

Bi. Compute volume of ball of radius r.

$x^2 + y^2 + z^2 = r^2$

$0 \leq \theta \leq 2\pi$

$0 \leq \phi \leq \pi$

$0 \leq \rho \leq r$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{r} r^2 \sin\varphi \; d\rho\, d\varphi\, d\theta$$

$$\int_0^{2\pi}\int_0^{\pi} \frac{r^3}{3}\Big|_{r=0}^{r} \; d\phi\, d\theta$$

$$\frac{r^3}{3}\int_0^{2\pi} -\cos\phi \Big|_{\phi=0}^{\phi=\pi} \; d\theta$$

$$\frac{2r^3}{3}\int_0^{2\pi} d\theta$$

$$= \frac{4\pi r^3}{3}$$

ii.

$$\iiint_G \delta(x, y, z) \qquad \delta(x, y, z) = \delta(\rho, \theta, \varphi) = \frac{e^{-\rho^2}}{\rho}$$

$$\int_0^{2\pi}\int_0^{\pi}\int_2^{3} \frac{e^{-\rho^2}}{\rho}\rho^2 \sin\varphi \; d\rho\, d\varphi\, d\theta$$

Solve $\int_2^3 \frac{\rho e^{-\rho^2}}{\cancel{\rho}}$ $\qquad u = \rho^2 \quad du = 2\rho\, d\rho$

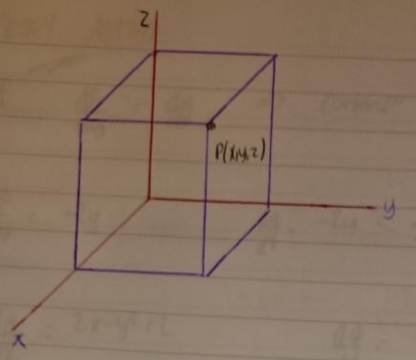
$\rho = 2 \quad u = 4$

$\rho = 3 \quad u = 9$

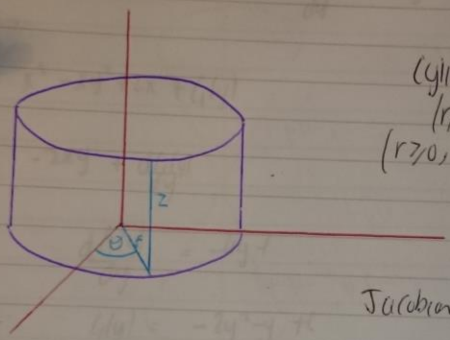
$\cancel{\int_4^9 \frac{e^{-u}}{2\rho}} \qquad \int_2^3 \rho e^{-\rho^2} d\rho$

$$\int_4^9 e^{-u}\, du$$

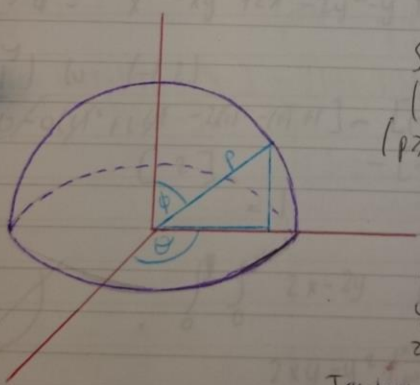
$$= -\frac{1}{2}\left[e^{-9} - e^{-4}\right]$$



Rectangular Cordinates
(x, y, z)



Cylindrical coordinates
(r, θ, z)
$(r \geq 0,\ 0 \leq \theta \leq 2\pi)$
$x = r\cos\theta$
$y = r\sin\theta$
$z = z$
Jacobian is r



Spherical coordinates
(ρ, θ, ϕ)
$(\rho \geq 0,\ 0 \leq \theta \leq 2\pi,\ 0 \leq \phi \leq \pi)$

$x = \rho\sin\phi\cos\theta$
$y = \rho\sin\phi\sin\theta$
$z = \rho\cos\phi$
Jacobian is $\rho^2 \sin\phi$

$\int \sin\phi = -\cos\phi$
$\int \cos\phi = \sin\phi$
$d\cos\phi = -\sin u$
$d\sin\phi = \cos\theta$