1/10/12

Management Science

Linear programming - method to solve a particular problem

Simplex method - decision variable, objective function, constraint

Paint problem   interior   exterior

profit   £300 per to   £200pt.

raw material a.   1 tonne of interior required 2t of rmA and 1t of rmB.

raw material b.   1 tonne of exterior required 1t of rmA and 2t of rmB

Atmost 6t of rmA per week

8t of rmB per week

Sell atmost 3.5t of exterior per week

$$\begin{array}{ccc} \text{real world} & \xrightarrow{\text{build}} & \text{maths model} \\ & & \downarrow \text{analysis} \\ \text{real world} & \xleftarrow{\text{interpret}} & \text{maths result} \end{array}$$

① Decision variable:

x = number of tonnes of interior

y = number of tonnes of exterior.

what we want to max/minimise

② Objective function: profit (x,y) = linear function $300x + 200y$

max/min it

③ Constraints on Decision Variable

① $2x + y$ = 6 of raw material a ≤ 6

② $x + 2y$ = 8 of raw material B ≤ 8

⑤ → value of y is ≤ 3.5

③ → $x \geq 0$

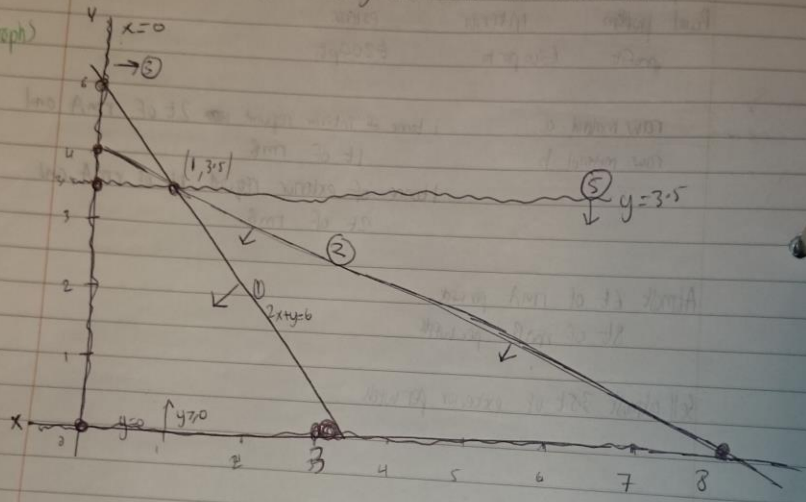
④ → $y \geq 0$

condition

2

requires all expressions to hold to be true
graphical analyses only works with two decision variables

Graphs



① $2x + y = 6$
$x = 0 \rightarrow y = 6 \quad (0, 6)$
$y = 0 \rightarrow x = 3 \quad (3, 0)$
let $x = 0$ and $y = 0$ $\quad 0 \leq 6$ cond true

② $x + 2y = 8$
$y = 0 \quad x = 8 \quad (8, 0)$
$x = 0 \quad y = 4 \quad (0, 4)$
let $x = 0$ and $y = 0$ $\quad 0 \leq 8$ true

Simultaneous equations,
② $x + 2y = 8$ $\qquad x + 7 = 8$
⑤ $y = 3.5$ $\qquad x = 1$
② and ① $\qquad x + 2y = 8 \qquad 2x + 4y = 16 \qquad 3y = 10$
$2x + y = 6 \qquad -2x - y = -6 \qquad y = 10/3$

① and $\quad x \cdot 2$: $-4x - 2y = -12$
$x + 2y = 8$
$-3x = -4$
$x = 4/3 = 1\tfrac{1}{3}$

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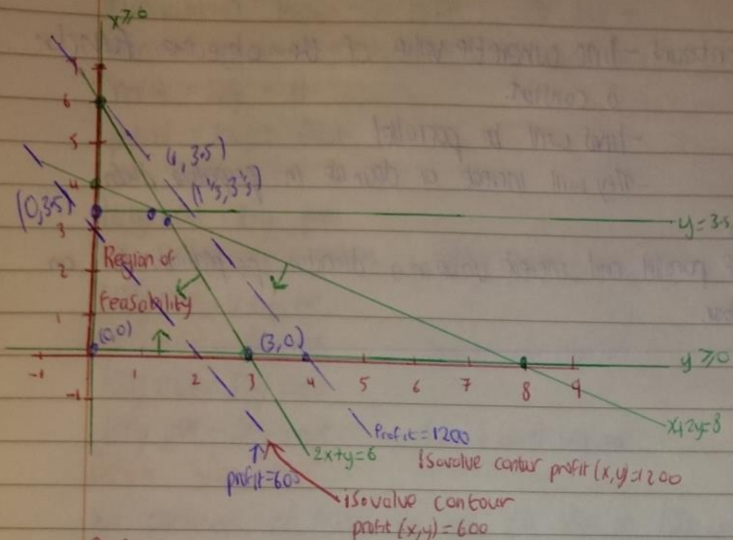
3. MANAGEMENT SCIENCE



Constraints:

1. $2x+y \leq 6$
2. $x+2y \leq 8$
3. $y \leq 3.5$
4. $x \geq 0$
5. $y \geq 0$

Profit: $300x+200y$

Claim: The max or min value of the objective function which will satisfy all of the constraints will be at one of the corners of the region of feasibubility.

Corner ~~regn~~ feasible regn	value of profit (object function)	(x,y)
(0,0)	0	0
(3,0)	900	900
(0,3.5)	700	700
($1\frac{1}{3}$,$3\frac{1}{3}$)	400 + 666 = $1066\frac{2}{3}$	$1066\frac{2}{3}$
(1,3.5)	300 + 700 = 1000	1000

Where is profit 600?

$300x+200y=600$

$3x+2y=6$ $x=0, y=3$ (0,3)

$y=0$ $x=2$ (2,0)

Profit (x,y) = 1200

$300x+200y=1200$

$3x+2y=12$ $x=0$ $y=6$ (0,6)

$y=0$ $x=4$ (4,0)

Isovalue contours - line where the value of the objective function is constant.
- lines will be parallel
- they will increase or decrease in perpendicular direction

value contour
iscocontours are parallel and increase value in a direction perpendicular to an isovalue contour.

5. Linear programming

Minimisation Problem - Diet problem   ght Lite bat 2.

minimum requirement of combination of Foods meat + potatoes

meat = 100g - €1

potatoes = 100g = 40cent

100g meat = 25g protein

100g potatoes = 15g protein   75g protein needed

100g meat = 48mg iron

100g potatoes = 6mg iron   48mg iron required

100g meat = 30g carb

100g potatoes = 30g carb   120g carbs required

Decision Variable:

x = amount of meat to eat per day in 100g unit

y = amount of potatoes " " "

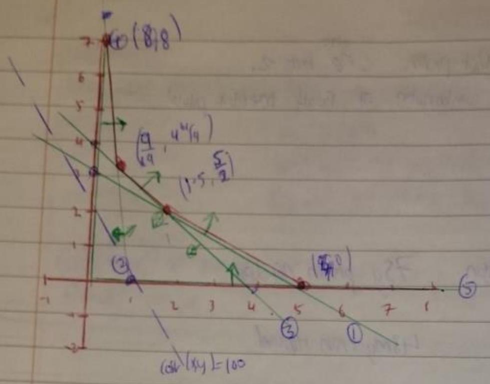
Objective function:

cost (x, y) $100x + 40y$ = cost in cent   or $x + \frac{4}{10}y$ = euro (minimise it)

Constraints:

1. $25x + 15y \geq 75$ (protein)
2. $48x + 6y \geq 48$ (iron)
3. $30x + 30y \geq 120$
4. $x \geq 0$
5. $y \geq 0$

Require all constraints to hold through



1. $25x+15y \geq 75$ — $x=0$ $y=5$ 0,5 — $y=0$ $x=3$ 3,0

2. $48x+6y \geq 48$ — $x=0$ $y=8$ — $y=0$ $x=1$

3. $3x+3y \geq 12$ — $x=0$ $y=4$ — $y=0$ $x=4$

③ and ①

$25x+15y \geq 75$

$3x+3y \geq 12$ ×5

$25x+15y = 75$

$15x+15y \geq 60$

$10x = 15$

$x = 1.5$

$3(1.5)+3y=12$

$3y = 5.5$

$y = \frac{5.5}{3} = \frac{11}{3}$

② and ③

$48x+6y=48$

$x+y=4$ ×6

$48x+6y=48$

$-6x-6y=-24$

$42x = 24$

$x = \frac{1}{2}$

Corners of feasible region	value of objective function (cost)(cent)
(0,8)	320
(9/19, 4 4/19)	215.7...
(1.5, 2.5)	150 + 100 = 250
(4, 0)	400
$100x+40y$	

Unbounded Solution:

Objective function: $2x + y$ (Maximise)

Constraints: $x - y \leq 10$ (1)

$2x \leq 40$ (2)

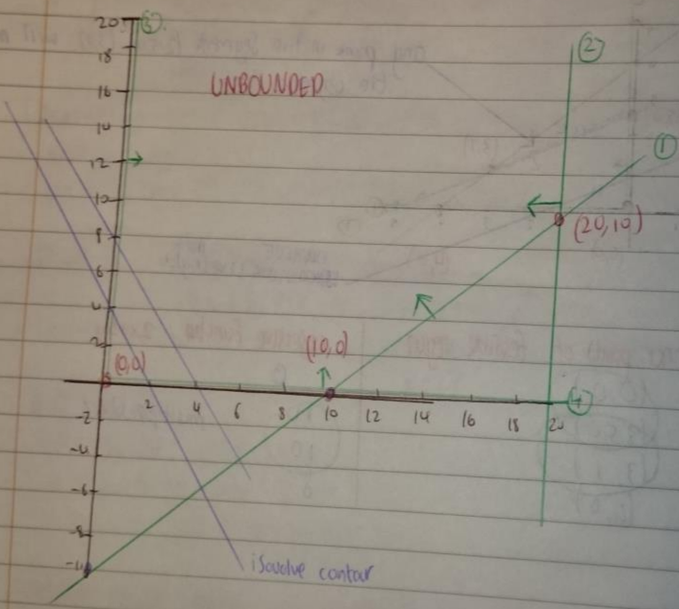
$x \geq 0$ (3)

$y \geq 0$ (4)

$x - y = 10$

$x = 0 \quad y = -10$

$y = 0 \quad x = 10$



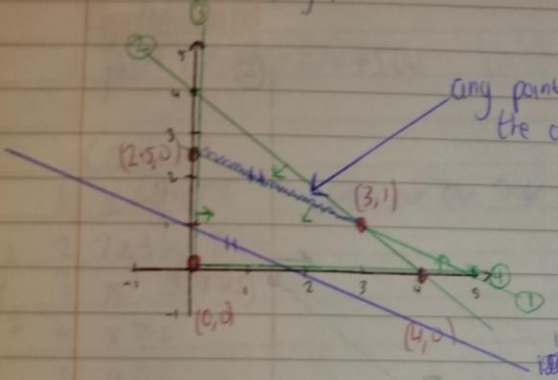
2/11/12 Linear Programming

Multiple Solutions:

Decision variables : x, y

Objective function : profit(x, y) = 2x + 4y   Maximise

Constraints
1. $x + 2y \leq 5$
2. $x + y \leq 4$
3. $x \geq 0$
4. $y \geq 0$



Corner points of feasible region	objective function = 2x+4y
(0,0)	0
(2.5,0)	10
(3,1)	10
(4,0)	8

multiple solution

1/2 | 1

Cornors of feasible region	value of objective function
~~(0, 5)~~	~~250~~
(5, 0)	50
(2, 3)	170
(5, 0)	50

$10x + 50y$

Management Science
Tutorial linear Programming ①. 6½/10 12300646

DAVID WEITBRECHT

Tailor problem

Decision Variable

x = number of trousers to make

y = number of Jackets to make

Objective function

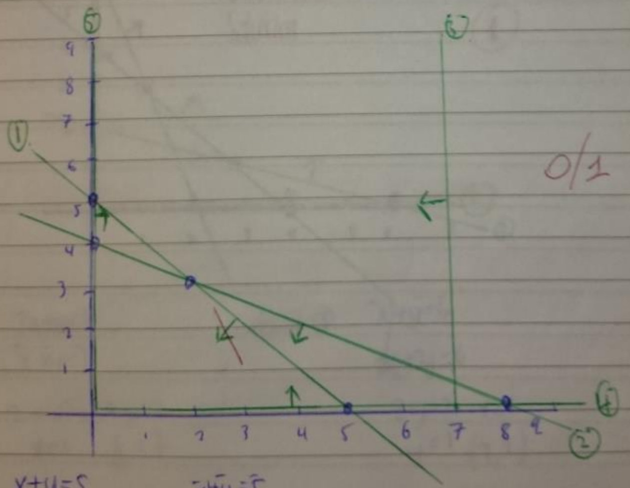
profit (x,y) = 10x+50y

Constraints

1. ~~x+y ≤ 5~~ (little more than 5 made)
2. 2x+4y ≤ 16
3. x ≤ 7
4. x ≥ 0
5. y ≥ 0

½/1

x+y ≤ 5 | 2x+4y=16
x=0 y=5 | x=0 y=4
y=0 x=5 | y=0 x=8



0/1

① and ④ x+y=5
2x+4y=16

x+y=5
x+2y=8

y=3 → 3+x=5
x=2

(2,3)

Tutorial 2

12300646
DAVID WEITBRECHT

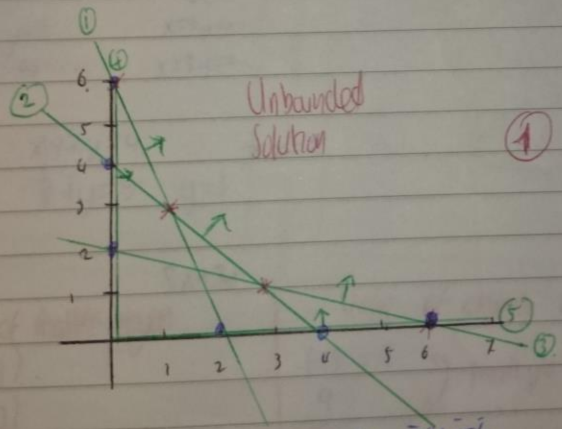
Decision variable
x
y
Objective function
20x + 16y

Constraint
1. 3x + y ≥ 6
2. x + y ≥ 4
3. 2x + 6y ≥ 12
4. x ≥ 0
5. y ≥ 0

3x + y = 6
x=0 y=6
y=0 x=2

x+y=4
x=0 y=4
y=0 x=4

2x+6y=12
x + 3y = 6
x=0 y=2
y=0 x=6



1 + 2. 3x + y = 6
−x − y = −4
2x = 2 → y = 3
x = 1 (1,3)

2 + 3. −x + y = 4
x + 3y = 6
2y = 2 x = 3
y = 1 (3,1)

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①

Corners of feasible region	value of objective function
(3, 1)	60+16 = 76
(1, 3)	20+48 = 68
(0, 6)	96
(6, 0)	120

$20x+16y$

Man. Sci.

Tutorial (3)

12300644

[illegible]

Decision vars

x

y

Maximise

Objective function:

$3x+3y$

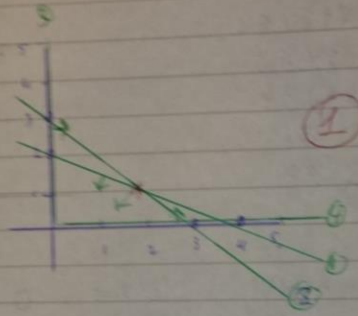
Constraints

1 $x+2y \le 4$

2 $x+y \le 3$

3 $x \geq 0$

4 $y \geq 0$



1 $x+2y=4$

$x=0 \quad y=2$

$y=0 \quad x=4$

2 $x+y=3$

$x=0 \quad y=3$

$x=3 \quad y=0$

(2) + (1) $x+2y=4$

$x+y=3 \quad y=1 \quad x=2$

$3x+3y$

Corners of feasible region	Value of objective function
(2, 1)	6+3 = 9
(3, 0)	9
(0, 2)	6
(0, 2)	6

multiple solutions

1/2/2

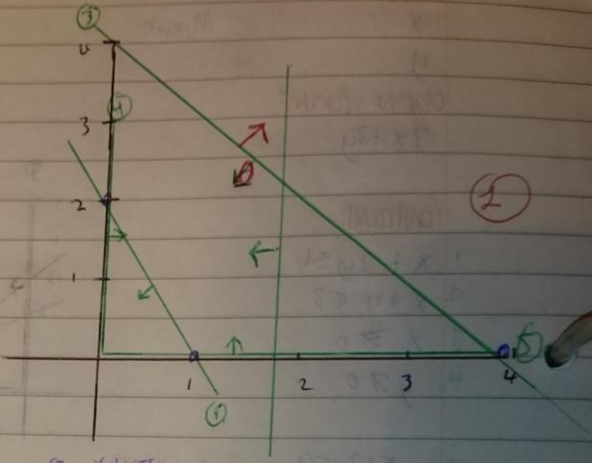
④ Tutorial

Decision variable: x, y

Objective function: $x + 5y$

Constraints:

1. $x + 2y \leq 2$
2. $3x \leq 5$
3. $x + y \geq 4$
4. $x \geq 0$
5. $y \geq 0$



1. $x+2y=2$
   $x=0\ y=1$ 0,1
   $y=0\ x=2$ 2,0

2. $3x=5$
   $x=5/3$

③ $x+y=4$
   $x=0\ y=4$
   $y=0\ x=4$

Infeasible