1 2011 MANG SCI PAPER 3 Q4 DAVID LIEMPREGH

4 A i. Min Z = $6x_{11} + 8x_{12} + 10x_{13} + 7x_{21} + 11x_{22} + 11x_{23} + 4x_{31} + 5x_{32} + 12x_{33}$

ST: Include dummy supply constraint with value of 50.

$x_{11} + x_{12} + x_{13} \leq 150$

$x_{21} + x_{22} + x_{23} \leq 175$

$x_{31} + x_{32} + x_{33} \leq 275$

$x_{41} + x_{42} + x_{43} \leq 50$ dummy

$x_{11} + x_{21} + x_{31} + x_{41} = 200$ ✓

$x_{12} + x_{22} + x_{32} + x_{42} = 150$

$x_{13} + x_{23} + x_{33} + x_{43} = 300$

all vars ≥ 0

ii. VAM — Use least cost or question to explain NWC or VAM

- Vogel's approximation method.
- Attempts to identify greatest advantage of a cell over the next best cell in a row or column
- Generally yields a close to optimum starting solution, supposed to Least cost method and North West Corner

~~VAM Penalties~~ 1.

Starting table

	W	C	K	Supply
D	6	8	10	150
L	7	11	11	175
G	4	5	12	275
Dum	0	0	0 (50)	50 - 50 = 0
Demd	200	150	300 - 50 = 250	

Penalties

row 1: 8 - 6 = 2
2: 11 - 7 = 4
3: 5 - 4 = 1
4: 0 - 0 = 0

column 1: 4 - 0 = 4
2: 5 - 0 = 5
3: 11 - 0 = 11

2

	W	C	K	Supply
D	6	8	10	150
L	7 (175)	11	11	175-175 = 0
G	4	5	12	275
Dum	0	0	0 (50)	0
Demand	200-175 = 25	150	~~Zero~~ 250	

Penalty) VAM 2

row 1 8-6 = 2 colum 1 ~~6-4~~ 7-4 = 3

2 11-7 = 4 2 ~~5-0=5~~ 8-5 = 3

3 5-4 = 1 3 11-11 = 1

4 Satisfied

3

	W	C	K	Supply
D	6	8	10	150
L	7 (175)	11	11	0
G	4	5 (150)	12	275-150 = 125
Dum	0	0	0 (50)	0
Demand	25	150-150 = 0	250	

Penalty) VAM 3

row 1 8-6 = 2 colum 1 6-4 = 2

2 Satisfied 2 8-5 = 3

3 5-4 = 1 3 11-10

4 Satisfied

4

	W	C	K	Supply
D	6	8	10	150
L	7 (175)	11	11	0
G	4 (25)	5 (150)	12	125-25 = 100
Dum	0	0	0 (50)	0
Demand	25-25 = 0	0	250	

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Penalty VAM 4

row 1 8-6=2 column 1 6-4=2
2 satisfied 2 satisfied
3 12-4 = 8 3 12-11=1
4 satisfied

Only two possible entries left in third column will be:
10(150)
11
12(100)
0(50)

Initial Solution:

	Wexford	Cavan	Kerry	Supply
Dublin	6	8	10(150)	150
Limerick	7(175)	11	11	175
Galway	4(25)	5(150)	12(100)	275
Dummy	0	0	0(50)	50
Demand	200	150	300	

(cost = 7(175) + 4(25) + 5(150) + 10(150) + 12(100) + 0(50) = 4775.

Alternate Method Least Cost Method
- Allocate as much as possible to the cell with the lowest cost.
- Adjust row and total columns accordingly.
- From remaining cell which have some free demand and free supply, pick cell with lowest cost, and assign as much as possible to cell
- Repeat until all of supply and demand is used

4

USE MODI To Solve

①

		$K_1=2$ W		$K_2=3$ C		$K_3=10$ K		Supply
$R_1=0$	D		6		8	150	10	150
$R_2=5$	L	175 −100	7		11	+100	11	175
$R_3=2$	G	25+100	4	150	5	100−100	12	275
$R_4=-10$	Dum		0		0	50	0	50
Demand		200		150		300		

D → W 6−2−0 = 4 L → C 11−5−3 = 3

Dum → W 0−2−−10 = 8 Dum → C 0−−10−3 = 7

D → C 8−0−3 = 5 L → K 11−10−5 = −4

②

		W $K_1=6$		C $K_2=3$		K $K_3=10$		Supply
$R_1=0$	D	75	6		8	150+75	10	150
$R_2=1$	L	75−75=0	7		11	100−75=25	11	175
$R_3=2$	G	125	4	150	5	0	12	275
$R_4=-10$	Dum		0		0	50	0	50
Demand		200		150		300		

D → W 6−6−0 = 0 L → C 11−3−1 = 7

Dum → W 0−−10−6 = 4 Dum → C 0−−10−3 = 7

D → C 8−0−3 = 5 G → K 12−10−2 = 0

Alternate optimal solution

current cost = 4375.

③

		W $K_1=6$		C $K_2=7$		$K_3=10$ K		Supply
$R_1=0$	D	75	6		8	225	10	150
$R_2=1$	L		7		11	25	11	175
$R_3=-2$	G	125	4	150	5		12	275
$R_4=-10$	Dum		0		0	50	0	50
Demand		200		150		300		

L → W 6−6=0 L → C 11−1−7 = 3

Dum → W 4−−2−6=0 0−−10+6 = 4 Dum → C 0−−10−7=3

D → C 8−7 = 1 G → K = 12+0−−2 = 4

current cost = 4225

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5. Go back to (2) and pick G→K instead of D→W

	W	C	K	Supply
D	6	8	150 10	150
L	75+100 =175 7	11	100−100=0 11	175
G	125−100 =25 4	150 5	0 100 12	275
Dum	0	0	50 0	50
Demd	200	150	300	

G→K

Value = 175(7) + 25(4) + 150(5) + 150(11) + 100(12) + 50(0) = 4775.

Second Optimal Solution is optimal with cost of 4775.

(iv) - This information can be incorporated into final optimal solution
- Assign an extremely large cost M to the box from Galway to Cavan (current value = 5)

(v) - Yes it will change
- Given the significant increase in cost from Galway to Cavan the solution will no longer be optimal
- Saving will be available using MODI method
- Iterate via MODI method to obtain new solution
- What is likely to happen is, Dublin and Limerick will supply Cavan

(vi) - Firstly it need to increase supply
- The largest cost is from Delivering to Dublin
- Company should try and minimise these transportation cost
- Maintain current solution or improve by decreasing transport cost

6.

(bi) $\text{Min } Z = 4.3X_{11} + 5.2X_{12} + 3.8X_{13} + 4.3X_{14} + 5.9X_{15} + 2.8X_{16}$
$+ 4.2X_{21} + 5.5X_{22} + 4X_{23} + 4.5X_{24} + 5.6X_{25} + 3.2X_{26}$
$+ 4.6X_{31} + 5.3X_{32} + 3.9X_{33} + 4.2X_{34} + 5.3X_{35} + 2.6X_{36}$
$+ 5X_{41} + 5X_{42} + 4.2X_{43} + 4.4X_{44} + 5X_{45} + 3.1X_{46}$
$+ 4.7X_{51} + 5.9X_{52} + 3.7X_{53} + 3.9X_{54} + 4.8X_{55} + 3X_{56}$

ST: $X_{11} + X_{12} + X_{13} + X_{14} + X_{15} + X_{16} = 1$
$X_{21} + X_{22} + X_{23} + X_{24} + X_{25} + X_{26} = 1$
$X_{31} + X_{32} + X_{33} + X_{34} + X_{35} + X_{36} = 1$
$X_{41} + X_{42} + X_{43} + X_{44} + X_{45} + X_{46} = 1$
$X_{51} + X_{52} + X_{53} + X_{54} + X_{55} + X_{56} = 1$

AND

$X_{11} + X_{21} + X_{31} + X_{41} + X_{51} \leq 1$
$X_{12} + X_{22} + X_{32} + X_{42} + X_{52} \leq 1$
$X_{13} + X_{23} + X_{33} + X_{43} + X_{53} \leq 1$
$X_{14} + X_{24} + X_{34} + X_{44} + X_{54} \leq 1$
$X_{15} + X_{25} + X_{35} + X_{45} + X_{55} \leq 1$
$X_{16} + X_{26} + X_{36} + X_{46} + X_{56} \leq 1$

All vars $\{0,1\}$

ii. Hungarian Method

Initial table:

		1	2	3	4	5	6
G	1	4.3	5.2	3.8	4.3	5.9	2.8
	2	4.2	5.5	4	4.5	5.6	3.2
	3	4.6	5.3	3.9	4.2	5.3	2.6
	4	5	5	4.2	4.4	5	3.1
	5	4.7	5.9	3.7	3.9	4.8	3

Subtract lowest value from each row
Subtract smallest entry in (sum) from all column

Need dummy

Be

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	R1		2		3		4		5		6	
G 1	1·5	0·5	2·4	0·5	1·0	0·1	1·5	0·6	3·1	1·3	0	0
2	1·0	0	1·2	0·3	0·8	0·1	1·3	0·4	2·4	0·6	0	0
3	2·0	1·0	2·7	0·8	1·3	0·6	1·6	0·7	2·7	0·9	0	0
4	1·9	0·9	1·9	0	1·1	0·4	1·3	0·4	1·9	0·1	0	0
5	1·7	0·7	2·9	1	0·7	0	0·9	0	1·8	0	0	0
	-1		-1·9		-0·7		-0·4		-1·8			

-2·8
-3·2
-2·6
-3·1
-3

Cross off zeros using minimum amount of horizontal and vertical lines, done in green out of blue zeros

Only have 4 lines, reattempt

⇒ ~~Satisfies minimum lines~~ needed (5)

~~Multiple solutions available~~

~~4·2 + 5 + 3·7~~

Subtract smallest entry not covered by line from each (4) 0·3 uncovered row and add it to each covered column

R	1	2	3	4	5	6
G 1	0·2	0·2	0	0·3	1·0	0
2	0	0·3	0·1	0·4	0·6	0
3	0·7	0·5	0·3	0·4	0·6	0
4	0·9	0	0·4	0·4	0·1	0
5	0·7	1	0	0	0	0

5 lines ✓ solution found

Two solutions

4·2 + 5 + 3·8 + 3·9 + 2·6 = 19·5 or 4·2 + 5 + 3·8 + 4·8 + 2·6 = 20·4

Girlchile

1	3
2 to row ⇒	1
3	6
4	2
5	4

Solution

A2 → T1 ✓ T6
A4 → T2 ✓ NOT
A1 → T3 ✓ DONE
A5 → T4 ✓
A3 → T6 ✓

2011 MANG SCI PAPER 3 Q 4B CORRECTION

4B (i) Minimise : $4.3X_{11} + 5.2X_{12} + 3.8X_{13} + 4.3X_{14} + 5.9X_{15} + 2.9X_{16}$
$+ 4.2X_{21} + 5.5X_{22} + 4.0X_{23} + 4.5X_{24} + 5.6X_{25} + 3.2X_{26}$
$+ 4.6X_{31} + 5.3X_{32} + 3.9X_{33} + 4.2X_{34} + 5.3X_{35} + 2.6X_{36}$
$+ 5.0X_{41} + 5X_{42} + 4.2X_{43} + 4.4X_{44} + 5.0X_{45} + 3.1X_{46}$
$+ 4.7X_{51} + 5.4X_{52} + 3.7X_{53} + 3.9X_{54} + 4.8X_{55} + 3.0X_{56}$

Subject to: Add dummy getter D

$X_{11} + X_{12} + X_{13} + X_{14} + X_{15} + X_{16} = 1$
$X_{21} + X_{22} + X_{23} + X_{24} + X_{25} + X_{26} = 1$
$X_{31} + X_{32} + X_{33} + X_{34} + X_{35} + X_{36} = 1$
$X_{41} + X_{42} + X_{43} + X_{44} + X_{45} + X_{46} = 1$
$X_{51} + X_{52} + X_{53} + X_{54} + X_{55} + X_{56} = 1$
$X_{01} + X_{02} + X_{03} + X_{04} + X_{05} + X_{06} = 1$

and $X_{11} + X_{21} + X_{31} + X_{41} + X_{51} + X_{01} = 1$
$X_{12} + X_{22} + X_{32} + X_{42} + X_{52} + X_{02} = 1$
$X_{13} + X_{23} + X_{33} + X_{43} + X_{53} + X_{03} = 1$
$X_{14} + X_{24} + X_{34} + X_{44} + X_{54} + X_{04} = 1$
$X_{15} + X_{25} + X_{35} + X_{45} + X_{55} + X_{05} = 1$
$X_{16} + X_{26} + X_{36} + X_{46} + X_{56} + X_{06} = 1$

4B (ii) Solve Using Hungarian Method

Subtract smaller value from each row

Then subtract smallest value from each column

	R 1	2	3	4	5	6	
Grade 1	1·5	2·4	1·0	1·5	3·1	0	− 2·8
2	1·0	2·5	0·8	1·3	2·4	0	− 3·2
3	2·0	2·7	1·3	1·6	2·7	0	− 2·6
4	1·9	1·9	1·1	1·5	2·9	0	− 3·1
5	1·7	2·4	0·7	0·9	1·8	0	− 3
D	0	0	0	0	0	0	

2

Subtract smallest uncovered value from each uncovered row
Add value to each covered column ⇒ 0.8

	1	2	3	4	5	6
Enter 1	0.8	1.7	0.3	0.8	2.5	0
2	0.3	1.6	0.1	0.6	1.7	0
3	1.3	2.0	0.6	0.9	2.0	0
4	1.2	1.2	0.4	1.6	2.2	0
5	1.0	2.2	0	0.2	1.1	0
D	0	0	0	0	0	0.7

Subtract 0.2 from uncovered row, add to covered column

	1	2	3	4	5	6
E 1	0.6	1.5	0.3	0.6	2.3	0
2	0.1	1.4	0.1	0.4	1.5	0
3	1.1	1.8	0.6	0.7	1.8	0
4	1.0	1.0	0.4	1.4	2.0	0
5	0.8	2.0	0	0	1.9	0
6	0	0	0	0	0	0.9

Subtract 0.1 from uncovered row, add to covered column

	1	2	3	4	5	6
E 1	0.5	1.4	0.2	0.5	2.2	0
2	0	1.3	0	0.3	1.4	0
3	1.0	1.7	0.5	0.6	1.7	0
4	0.9	0.9	0.3	1.3	1.9	0
5	0.8	2.0	0	0	1.9	0.1
6	0	0	0	0.1	0	1.0

Subtract 0.2 ⇒

3. 2011 MANG SCI PAPER 3 Q4B CORRECTION.

	1	2	3	4	5	6
G 1	0·3	1·2	0	0·3	2·0	0
2	0	1·3	0	0·3	1·4	0·2
3	0·8	1·5	0·3	0·4	1·5	0
4	0·7	0·7	0·1	1·2	1·7	0
5	0·8	2·0	0	0	1·9	0·3
D	0	0	0	0	0	1·2

Subtract 0·3 ⇒

	1	2	3	4	5	6
G 1	0	0·9	0	0	1·7	0
2	0	1·3	0·3	0·3	1·4	0·5
3	0·5	1·2	0·3	0·1	1·2	0
4	0·4	0·4	0·1	0·9	1·4	0
5	0·8	2·0	0·3	0	1·9	0·6
D	0	0	0·3	0	0	1·5

Subtract 0·1

	1	2	3	4	5	6
G 1	0·1	0·9	0	0·1	1·7	0·1
2	0	1·2	0·2	0·3	1·3	0·5
3	0·5	1·1	0·2	0	1·1	0
4	0·4	0·3	0	0·9	1·3	0
5	0·8	1·9	0·2	0	1·8	0·6
D	0·1	0	0·3	0	0	1·6

Subtract 0·3

	1	2	3	4	5	6	G	D	cost
G1	0·1	0·6	0	0·1	1·4	0·1	1	3	3·8
2	0	0·9	0·2	0·3	1·0	0·5	2	1	4·2
3	0·5	0·8	0·2	0·1	0·8	0	3	6	2·6
4	0·9	0	0	0·9	1·0	0	4	2	5
5	0·8	1·6	0·2	0	1·5	0·6	5	4	3·9
6	0·4	0	0·6	0·4	0	1·9	D	5	0
									19·5

6.

MANG SC PAPER 3 2011 Q4

4 (i) $\text{Min } Z = 6x_{11} + 8x_{12} + 10x_{13} + 7x_{21} + 11x_{22} + 11x_{23} + 4x_{31} + 5x_{32} + 12x_{33}$

Introduce a dummy source to make up shortfall of supply of 50.

ST: $x_{11} + x_{12} + x_{13} \leq 150$

$x_{21} + x_{22} + x_{23} \leq 175.$

$x_{31} + x_{32} + x_{33} \leq 275$

Dummy: $x_{41} + x_{42} + x_{43} \leq 50$

and $x_{11} + x_{21} + x_{31} + x_{41} = 200$

$x_{12} + x_{22} + x_{32} + x_{42} = 150$

$x_{13} + x_{23} + x_{33} + x_{43} = 300$

ii. Northwest Corner:

- Starting at top left corner allocate the least of the row supply or the column demand to the cell
- Subtracted the amount allocated to the cell from the row and column total.
- If the column total is now zero, move to next cell on the right. If it is the row total that is zero, move down to cell below
- Allocate an amount to the new cell as in step (1) and repeat until all demand and supply is used

	Wexford	Cavan	Kerry	Supply
Dublin	6 (150)	8	10	150 − 150 = 0
Limerick	(7)	11	11	175
Galway	4	5	12	275
Dummy	0	0	0	50
Demand	200 − 150 = 0	150	300	

move down now

2.

	Wexford	Cavan	Kerry	Supply	
D	6(150)	8	10	0	
L	7(50)	(11)	11	175-50	move to next column
G	4	5	12	275	
Dummy	0	0	0	50	
Demand	50-50=0	150	300		

	Wexford	Cavan	Kerry	Supply
D	6(150)	8	10	0
L	7(50)	11(125)	11	125-125
G	4	(5)	12	275
Dummy	0	0	0	50
Demand	0	150-125	300	

	Wexford	cavan	Kerry	Supply
D	6(150)	8	10	0
L	7(50)	11(125)	11	0
G	4	5(25)	(12)	275-25
Dummy	0	0	0	50
Demand	0	25-25=0	300	

	W	C	K	Supply
D	6(150)	8	10	0
L	7(50)	11(125)	11	0
G	4	5(25)	12(250)	250-250
Dummy	0	0	(0)	50
Demand	0	0	300-250	

	W	C	K	Supply
⇒ D	6(150)	8	10	150
L	7(50)	11(125)	11	175
G	4	5(25)	12(250)	275
Dummy	0	0	0(50)	50
Dem	200	150	300	

INITIAL SOLUTION.

3

3(ii) Alternate method - Least Cost Method

- Allocate as much as possible to the cell with the least cost.
- Adjust row and column totals like Northwest corner method
- From the remaining cells whose column has some free demand and whose row has some free supply select one with lowest cost.
- Repeat until all has been allocated.

iii. Modified Distribution Modi method

		$K_1 = 6$	$K_2 = 10$	$K_3 = 17$	
		W	C	K	Supply
$R_1 = 0$	D	150 ~~150~~	6 8 10		150
$R_2 = 1$	L	50 ~~200~~	7 11 11		175
$R_3 = -5$	G		125 4 5 12		275
$R_4 = -17$	Dum		25 0 0	250 0 50	50
	Demand	200	150	300	

G → W $4 - -5 - 6 = 3$

Dum → W $0 - -5 - 6 = -1$

D → C $8 - 0 - 10 = -2$

Dum → C $0 - -17 - 10 = 7$

D → K $10 - 0 - 17 = -7$

L → K $11 - 1 - 17 = -5$