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Maths Tutorial 4 Week 5 13/2/14 12300844

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1. i. $(0,1)$ $(1,-2)$

$$a\begin{pmatrix}0\\1\end{pmatrix} + b\begin{pmatrix}1\\-2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

$b = 0$

$a - 2b = 0$ $(b=0) \Rightarrow a=0$

$a = b = 0$

$\Rightarrow$ linearly independent

ii. $$a\begin{pmatrix}0\\-1\\1\end{pmatrix} + b\begin{pmatrix}1\\1\\0\end{pmatrix} + c\begin{pmatrix}2\\0\\2\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$b + 2c = 0$ $= 2c = -b \Rightarrow$

$-a + b = 0$ $-a+b=0$

$a + 2c = 0$ $a-b=0 \Rightarrow \boxed{b=a} \Rightarrow -a+a=0$ $0=0$ ✓

~~a=0~~ ✓

end up with eqns $\Rightarrow$ $k_1 + 2c_3 = 0$ $2c_1 + 2c_2 - c_3 = 0$

$k_2 + 2c_3 = 0$

$0 = 0$ $\Rightarrow$ linearly dependent

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iii. $$a\begin{pmatrix}0\\0\\-1\\0\\0\end{pmatrix} + b\begin{pmatrix}1\\2\\1\\1\\1\end{pmatrix} + c\begin{pmatrix}1\\2\\0\\1\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix}$$

$b + c = 0$ $c = -b \Rightarrow$ $c = -b$ $\Rightarrow c = -a$

$2b + 2c = 0$ $0 = 0$ $-a = -b$

$-a + b = 0$ $-a+b=0$

$b + c = 0$ $b + c = 0$ $0=0$ ✓

$b + c = 0$ ✓

$\Downarrow$

linearly dependent.

Q2 Basis if $\vec{v} \neq \vec{0}$
if independent and spans V

i. Dimension is 1 ⇒ must have 1 vector.

$k_1 \begin{pmatrix} -1 \\ -2 \end{pmatrix} = 0$

$-k_1 = 0$ $k_1 = 0$
$-k_2 = 0$ $k_2 = 0$ independent ⇒ linearly independent and basis

~~Only Span $\mathbb{R}^1$~~ not $\mathbb{R}^2$ Not basis

ii. $(0,-1)$, $(1,-2)$ Dimension $= \mathbb{R}^2$
Spans $\mathbb{R}^2$

Independent?

$k_1 \begin{pmatrix} 0 \\ -1 \end{pmatrix} + k_2 \begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$k_2 = 0$ $k_2 = 0$ independent
$-k_1 - 2k_2 = 0$ $k_1 = 0$

it is a basis of $\mathbb{R}^2$

iii. Dimension $= \mathbb{R}^2$
~~Spans $\mathbb{R}^2$~~ doesn't

Independent? $k_1 \begin{pmatrix} -2 \\ 2 \end{pmatrix} + k_2 \begin{pmatrix} 3 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-2k_1 + 3k_2 = 0$ $3k_2 = 2k_1$ ⇒ $2k_1 - 2k_1 = 0$ $3k_2 = 2k_1$
$2k_1 - 3k_2 = 0$ $0 = 0$

not linearly independent
⇒ Not a basis for $\mathbb{R}^2$

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iv. Dimension = $\mathbb{R}^2$

3 - vectors $\Rightarrow$ ~~do not span $\mathbb{R}^2$~~ they span $\mathbb{R}^3$ as dependent

$\Rightarrow$ not a basis of $\mathbb{R}^2$

they do span

~~not~~ possess or: do not as are 2D vectors

v. Dimension = $\mathbb{R}^4$

3 - vectors $\Rightarrow$ does not span $\mathbb{R}^4$

$\Rightarrow$ Not a basis of $\mathbb{R}^4$ ✓

vi. Dimension = $\mathbb{R}^3$

~~Spans $\mathbb{R}^3$~~ ✓ do not span $\mathbb{R}^3$ as they are dependent

Independent?

$$k_1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + k_2 \begin{pmatrix} 0 \\ 3 \end{pmatrix} + k_3 \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$k_1 \quad -2k_3 = 0$ $\qquad k_1 = 2k_3 \Rightarrow$

$k_2 + k_3 = 0$ $\qquad k_2 = -k_3$

$k_1 \quad -2k_3 = 0$ $\qquad 0 = 0$ ✓ ✓

Not linearly independent $k_1 = 2k_3$ and $k_2 = -k_3$

Not a basis ✓

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Q2 $\vec{V} = K_1\vec{V_1} + K_2\vec{V_2}$

$$\begin{pmatrix}1\\-1\end{pmatrix} = K_1\begin{pmatrix}1\\1\end{pmatrix} + K_2\begin{pmatrix}-1\\2\end{pmatrix}$$

$K_1 - K_2 = 1$ $\quad K_1 = 1 + K_2 \Rightarrow 1 + K_2 + 2K_2 = -1$

$K_1 + 2K_2 = -1$ $\quad 3K_2 = -2$

$K_2 = \frac{-2}{3}$

$\Rightarrow K_2 = \frac{-2}{3}$ $\quad K_1 - \left(\frac{-2}{3}\right) = 1$

$K_1 = \frac{1}{3}$

$\left(\frac{1}{3}, \frac{-2}{3}\right)$ non standard coordinates

ii. $\vec{V} = K_1\vec{V_1} + K_2\vec{V_2} + K_3\vec{V_3} + K_4\vec{V_4}$

$$\begin{pmatrix}1\\0\\2\\-1\end{pmatrix} = K_1\begin{pmatrix}1\\0\\1\\0\end{pmatrix} + K_2\begin{pmatrix}1\\1\\0\\0\end{pmatrix} + K_3\begin{pmatrix}0\\0\\1\\0\end{pmatrix} + K_4\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

$K_1 + K_2 + K_4 = 1$ $\quad K_2 = 0$

$K_2 = 0$ $\quad K_4 = -1$

$K_1 + K_3 = 2$

$K_4 = -1$

$K_1 + K_2 = 2$ $\quad \Rightarrow K_1 = 2 - K_2 \Rightarrow 2 - K_2 + K_3 = 2$

$K_1 + K_3 = 2$

$\boxed{K_2 = K_3}$

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but $K_2 = 0$

$K_3 = 0$

$K_4 = -1$

$K_1 = 2$