

20/02/14 Applied Probability Question Sheet
 Submit project Friday 08/03/14 Friday lecture

①A 95% Confidence Interval:

$$\mu = 136.9 \quad s^2 = 4811.8 \quad n = 47$$

$$\mu \pm t_{0.025, 46} \frac{s}{\sqrt{n}}$$

$$136.9 \pm 2.01 \left(\frac{\sqrt{4811.8}}{\sqrt{47}} \right)$$

$$= 136.9 \pm 20.33$$

1B 90% CI for variance

$$\frac{(n-1)s^2}{\chi^2_{0.05, n}}, \frac{(n-1)s^2}{\chi^2_{0.95, n}}$$

$$\left(\frac{46(4811.8)}{62.8}, \frac{46(4811.8)}{31.4} \right) = (3525, 7049)$$

1C Hypothesis test.

$$H_0: \mu = 155 \quad H_1: \mu \neq 155 \quad 2\text{-tail test}$$

95% CI

$$T_{\text{calc}} = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{136.9 - 155}{\sqrt{4811.8}/\sqrt{47}} = -1.78 \text{ calc.}$$

Tabulated $T_{0.95, 46}$

critical

$$\text{Reject } H_0 \text{ if } |t|_{\text{calc}} > t_{0.025, 46} = 2.01$$

Not $> 2.01 \Rightarrow$ do not reject H_0

1.cii.

2 AP

$$H_0: \mu = 155$$

$$H_1: \mu < 155$$

1-tailed test

$$t_{calc} = -1.79 \text{ at before}$$

$$\text{Reject } H_0 \text{ if } t_{calc} < -t_{0.05, 40} = -1.68$$

\Rightarrow Accept Reject H_0 .

Note: If $H_1: \mu > 155$ then reject H_0 if $t_{calc} > t_{0.05, 40}$

Q2. 2-Sample t-test.

Before

$$\bar{x}_1 = 14.9$$

$$s_1^2 = 228.2$$

$$n_1 = 477$$

After

$$\bar{x}_2 = 16.4$$

$$s_2^2 = 172.9$$

$$n_2 = 502$$

$$H_0: \mu_{\text{before}} = \mu_{\text{after}}$$

$$H_1: \mu_{\text{before}} < \mu_{\text{after}}$$

$$\text{combined } s^2: \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{476(228.2) + 501(172.9)}{979-2} = 199.8$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s \sqrt{\frac{1}{n} + \frac{1}{m}}} = \frac{14.9 - 16.4}{\sqrt{199.8 \left(\frac{1}{477} + \frac{1}{502} \right)}} = -3.82$$

One tailed test 1% Significance

$$\text{Reject } H_0 \text{ if } t < -t_{0.01, 977} = -2.37$$

\Rightarrow Do not reject H_0 , if $H_1: \mu_{\text{before}} > \mu_{\text{after}}$ we would reject H_0 if $t > t_{\text{critical}}$.
So reject H_0 .

20/02/14 3. Applied Probability Question sheet.

Q2 correct
eg. rule has decreased mean length of play

Q3. Paired T-test.

(All data is paired: before and after score)

$$\bar{d} = 2.25 \quad (\text{after} - \text{before})$$

$$s_d^2 = 3.78$$

$H_0: \mu_{i1} = \mu_{i2}$ where $\mu_{i1} = \mu_{i2}$ $\mu_{i1} = \text{mean}$
point total before trials
 $\mu_{i2} = \text{mean after trials}$

$$H_1: \mu_{i1} \neq \mu_{i2}$$

Assume 5% or two tailed test

$$t = \bar{d} / s_d / \sqrt{n} = 2.25 / \sqrt{3.78} / \sqrt{20} = 5.17$$

Reject H_0 if $|t| > t_{0.025, 19} = 2.09$

$t_{\text{calc}} > t_{\text{critical}}$

\Rightarrow Reject H_0 .

Q4.A Proportion test

\hat{p} our estimate for $P(\text{death is in 3 months before birthday})$
 $= 60/747 = 0.0803$

$$99\% \text{ CI: } \hat{p} \pm t_{0.005, 746} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.0803 \pm 2.38 * \sqrt{\frac{0.0803(0.9197)}{747}}$$

4. Applied Probability
 0.0803 ± 0.0223
 $= (0.0580, 0.1026)$

4B We would expect $\frac{3}{12} = 0.25$

4C $H_0: p = 0.25$
 $H_1: p < 0.25$

One tailed test 5% level of significance

$$t = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.0803 - 0.25}{\sqrt{\frac{0.25(0.75)}{747}}} = -10.7$$

$t_{calc} = -10.7$

Reject H_0 if $t < -t_{0.05, 746} = -1.96$
 $-10.7 < -1.96$

Reject H_0 .