

Math Tutorial Week 8 Start 6 DAVID WEITBRECHT 12300644

1. Distance between u and v

$$\langle u-v, u-v \rangle$$

$$\sqrt{1^2 + 0^2 + 2^2} = \sqrt{5} \quad \text{wrt standard dot product}$$

$$\sqrt{1^2 + 0^2 + 2^2} = \sqrt{5} \quad \text{distance between } u \text{ and } v$$

$$\text{angle} = \frac{u \cdot v}{\|u\| \|v\|} = \cos \theta$$

$$\frac{1 \cdot 0 + 0 \cdot 0 + 1 \cdot (-1)}{\|u\| \|v\|} = \frac{-1}{\sqrt{2}} = \cos \theta \quad \theta = 135^\circ$$

$$\text{length of } u = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}$$

With respect to Inner Product:

$$\|u\| = \sqrt{(u, u)} = \sqrt{u_1^2 + 2u_2^2 + 3u_3^2} = \sqrt{1^2 + 2(0)^2 + 3(1)^2} = \sqrt{4} = 2$$

Distance between u and $v \Rightarrow$

$$(1-0)^2 + 2(0-0)^2 + 3(1-(-1))^2$$

$$1 + 0 + 12$$

$$\sqrt{13} = \text{distance between } u \text{ and } v$$

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angle between u and v :

$$= \cos^{-1} \frac{(u, v)}{\|u\| \cdot \|v\|} \propto \frac{7}{\sqrt{2}} \cos^{-1} \frac{7}{\sqrt{2}} \quad \times$$

$$\langle u, v \rangle = 3 \quad \langle v, v \rangle = \sqrt{3}$$

2. If $u \cdot v = 0$ orthog

$$i. \quad 3 \cdot 0 + 0 \cdot -1 = 0 \Rightarrow \text{orthog.} \quad \checkmark$$

$$ii. \quad 0 \cdot -1 \cdot -1 + 0 \cdot 1 \cdot -1 + -1 \cdot 0 \cdot 0 = 0 \quad \times$$

\Rightarrow orthogon

$$iii. \quad 1 \cdot 0 \cdot 0 + 0 \cdot \frac{3}{5} \cdot \frac{4}{5} + 0 \cdot \frac{-4}{5} \cdot \frac{3}{5} = 0 \Rightarrow \text{orthogon} \quad \times$$

$$\text{check that } u_1 \cdot u_2 = u_2 \cdot u_3 = u_3 \cdot u_1 = 0$$

3. Coordinates of v relative to orthogonal basis

$$(2, -1, -3) \cdot (1, 0, 0) = \cancel{2} \cdot \frac{1}{\sqrt{1}} = 2 \quad \checkmark$$

$$(2, -1, -3) \cdot (0, 2, -3) = \frac{7}{\sqrt{14}} \quad \checkmark$$

$$(2, -1, -3) \cdot (0, 3, 2) = \frac{-9}{\sqrt{13}} = \cancel{2} \cdot \frac{7}{\sqrt{13}} \quad \checkmark$$

$$\left(2, \frac{7}{\sqrt{13}}, \frac{-9}{\sqrt{13}} \right)$$

$$ii. \quad (-1, -1, 1) \cdot (1, 0, 0) = -1/\sqrt{2}$$

$$(-1, -1, 1) \cdot (0, 2, -3) = -5/\sqrt{13}$$

$$(-1, -1, 1) \cdot (0, 3, 2) = 1/\sqrt{13} = (-1, -5/\sqrt{13}, 1/\sqrt{13})$$

$$\left(\frac{3}{3} \right)$$

$$\left(-1, -5/\sqrt{13}, 1/\sqrt{13} \right)$$