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10/3/14 Maths Sheet 7 Week 9 DAVID LWENBRECHT 12300644 MIJI

$\frac{8.5}{10}$

Q 1 i $U_1 = (-1, 0)$ $U_2 = (1, -3)$

$\bar{V}_1 = U_1 = (-1, 0)$

$\bar{V}_2 = U_2 - \frac{\langle (-1,0), (1,-3) \rangle}{\| -1, 0 \|^2} (-1, 0)$

$(1, -3) - \frac{(-1)}{1} (-1, 0)$

$(1, -3) + (-1, 0)$

$\bar{V}_2 = (0, -3)$

$\bar{V}_1 = (-1, 0)$ orthogonal basis of $\{U_1, U_2\}$

$V_2 = (0, -3)$

D.W.

2

Q 1(ii) $u_1 = (1,0,-1)$ $u_2 = (1,0,0)$ $u_3 = (1,2,-1)$

$v_1 = (1,0,-1)$

$v_2 = (1,0,0) - \frac{\langle v_1, u_2 \rangle}{\|v_1\|^2} v_1$

$(1,0,0) - \frac{\langle (1,0,-1)(1,0,0) \rangle}{\|\sqrt{1^2+-1^2}\|^2} (1,0,-1)$

$(1,0,0) - \frac{1}{2}(1,0,-1)$

$\bar{v}_2 = (\frac{1}{2}, 0, \frac{1}{2})$

$v_3 = (1,2,-1) - \frac{\langle (1,2,-1),(1,0,-1) \rangle}{\|1,0,-1\|^2}(1,0,-1) - \frac{\langle (1,2,-1)\cdot(\frac{1}{2},0,\frac{1}{2}) \rangle}{\|\frac{1}{2},0,\frac{1}{2}\|^2}(\frac{1}{2},0,\frac{1}{2})$

$(1,2,-1) \quad -\frac{2}{2}(1,0,-1)$

$\frac{0}{\angle 0}$

$\bar{v}_3 = (0, \cancel{0}^{2}, 0)$

$v_1 = (1,0,-1)$
$v_2 = (\frac{1}{2},0,\frac{1}{2})$ = orthogonal basis of $\{u_1, u_2, u_3\}$
$\bar{v}_3 = (0, \cancel{0}_{2}, 0)$

$v_3$ is not a basis vector

$\frac{3.5}{4}$

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Q2

$$\underset{A}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}} \underset{\bar{x}}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = \underset{\bar{b}}{\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}}$$

$$\underset{A^T}{\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}} \underset{\bar{A}}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}} \underset{\bar{x}}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = \underset{A^T}{\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}} \underset{\bar{b}}{\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}}$$

$$\begin{pmatrix} 2 & -1 & +1 \\ -1 & 3 & -2 \\ 1 & -2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

$$\begin{cases} 2x - y + z = 1 \\ -x + 3y - 2z = 1 \\ x - 2y + 2z = -1 \end{cases} \Rightarrow y = 0$$

$x=1,\ y=0,\ z=-1$

$\cancel{2x - z = 1 \quad -x - 2z}$

$2x - z = 1$     $\cancel{2x} - z = 1$     $-5z = 3$

$-x - 2z = 1 \quad \times 2$     $-\cancel{2x} - 4z = 2$     $z = -3/5$

$\frac{z^5}{3}$

$y = 0 \quad z = -3/5$

$2x + 3/5 = 1$

$2x = 2/5$

$x = 1/5$

$y = 0$
$z = -3/5$ = Solution
$x = 1/5$.

DW

Q3 4

i. $\begin{pmatrix} 0 & \hat{2} \\ -3 & 0 \end{pmatrix}$ $\quad \begin{pmatrix} 0 & 2 \\ -3 & 0 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \lambda \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$P(\lambda) = \det[A - \lambda I]$

$\det\left[\begin{pmatrix} 0 & 2 \\ -3 & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right]$

$\det\begin{bmatrix} -\lambda & 2 \\ -3 & -\lambda \end{bmatrix}$

$= -\lambda(-\lambda) \cancel{+} (2)(-3)$

$\lambda^2 \cancel{+}6 \quad \lambda^2 + 6$

$P(\lambda) = \lambda^2 = +6 = \text{characteristic polynomial}$

$\cancel{\lambda = \pm\sqrt{6}}$

~~e-values~~

ii. A.

$\begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & -2 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \lambda \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

$P(\lambda) = \det[A - \lambda I]$

$= \det\left[\begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & -2 \\ 0 & 0 & -1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}\right]$

$= \det\begin{bmatrix} 1-\lambda & 2 & -1 \\ 0 & 3-\lambda & -2 \\ 0 & 0 & -1-\lambda \end{bmatrix}$

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3 ii Continued

$1-\lambda\left[(3-\lambda)(-1-\lambda)-0(2)\right] - 2\left[0\cancel{(-1-\lambda)}+2(0)\right] - 1\left[0(0)-\cancel{0(3-\lambda)}\right]$

0 0

$1-\lambda\ [-3-3\lambda+\lambda+\lambda^2]$

$(1-\lambda)\ [\cancel{\lambda^2}+\lambda^2-2\lambda-3]$

$\lambda^2-2\lambda-3\ -\lambda^3+2\lambda^2+3\lambda$

$p(\lambda) = -\lambda^3+3\lambda^2+\lambda-3$

= characteristic polynomial

iii $\begin{pmatrix}0&2&1\\1&0&2\\0&-2&1\end{pmatrix}\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix} = \lambda\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$

$p(\lambda) = \det\left[A-\lambda I\right]$

$\det\left[\begin{pmatrix}0&2&1\\1&0&2\\0&-2&1\end{pmatrix} - \begin{pmatrix}\lambda&0&0\\0&\lambda&0\\0&0&\lambda\end{pmatrix}\right]$

$= \det\left[\begin{pmatrix}-\lambda&2&1\\1&-\lambda&2\\0&-2&1-\lambda\end{pmatrix}\right]$

2.5 / 3

$-\lambda\left[-\lambda(1-\lambda)-2(-2)\right] - 2\left[1(1-\lambda)-2(0)\right] - 1\left[1(-2)-\cancel{(-\lambda)(0)}\right]$

$-\lambda\ [\lambda^2+4] \quad -2[1-\lambda] \quad -1[-2]$

$-\lambda^3-4\lambda \quad -2+2\lambda \quad +2$

$p(\lambda) = -\lambda^3-2\lambda$ = characteristic polynomial