# Tutorial 5: MA1E01

## Derivatives 1

1. Find the equation of the tangent line to the curves
   (a) $f(x) = 2x^2 - x + 1$ at $x = 1$,
   (b) $f(x) = 3/x^2$ at $x = -1$,
   (c) $f(x) = 2x^3 + 1$ at $x = 2$.
2. Show that
   $$f(x) = \begin{cases} x^2 + 2 & x \leq 1 \\ x + 2 & x > 1 \end{cases}$$
   is is continuous but not differentiable at $x = 1$. Sketch the graph of $f(x)$.
3. Compute the derivatives of the following functions:
   (a) $f(x) = 7x^{-6} - 5\sqrt{x}$
   (b) $f(x) = x^{48} + 2x^{24} + 3x^{16} + 4x^{12}$
   (c) $f(x) = \left(\frac{3x+2}{x}\right)(x^{-5} + 1)$
   (d) $f(x) = \frac{4x+7}{x^2-1}$
4. Prove that $y = 1/x$ and $y = 1/(2 - x)$ intersect at right angles.

$y = y$

$\frac{1}{x} = \frac{1}{2-x} \Rightarrow 2 - x = x$

$x = 1$

$f'(x) = \frac{-1}{x^2}$ $\quad f'_2(x) = \frac{2}{(2-x)^2}$

$$\cancel{15(x^2+y^2)^2 = 284(x^2-y^2)} \quad (4,1)$$

$$\cancel{30(x^2+y^2)(2x+2y\frac{dy}{dx}) = 284(2x-2y\frac{dy}{dx})}$$

$$3(x^2+y^2)^2 = 25(x^2-y^2); \quad 2,1$$

$$6(x^2+y^2)(2x+2y\frac{dy}{dx}) = 25(2x-2y\frac{dy}{dx})$$

$$6(4+1)(4+2\frac{dy}{dx}) = 25(4-2\frac{dy}{dx})$$

$$120+60\frac{dy}{dx} = 100-50\frac{dy}{dx}$$

$$60\frac{dy}{dx}+50\frac{dy}{dx} = 100-120$$

$$\frac{dy}{dx}(110) = -20$$

$$\frac{dy}{dx} = -\frac{2}{11}$$

$$2(x^2+y^2)^2 = 25(x^2-y^2) \quad 3,1$$

3, 1

$$4(x^2+y^2)(2x+2y\frac{dy}{dx}) = 25(2x-2y\frac{dy}{dx})$$

$$4(9+1)(6+2\frac{dy}{dx}) = 25(6-2\frac{dy}{dx})$$

$$40(6+2\frac{dy}{dx}) = 150-50\frac{dy}{dx}$$

$$240+80\frac{dy}{dx} = 150-50\frac{dy}{dx}$$

$$\frac{dy}{dx}(80+50) = 150-240$$

$$\frac{dy}{dx} = \frac{-90}{130}$$

2-11-12 Maths Tutorial 5

1 a. $f(x) = 2x^2 - x + 1$ at $x=1$

$4x - 1$ $x=1$

$4-1 = 3$ ✓

b $f(x) = \frac{3}{x^2}$ $x=-1$

$3x^{-2}$

$-6x^{-3} = \frac{-6}{x^3} = 6$ ✓

c. $f(x) = 2x^3 + 1$ $x=2$

$6x^2$

$6(2^2) = 24$ ✓

2 $x^2 + 2$

$2x$ $x=1 = 2$

$\lim_{x \to 1^-} (x^2+2)$ $1^2+2 = 3$

continuous if limit equal at left and right

$\lim_{x \to 1^+} (x+2)$ $1+2=3$

lim is the same function is continual

two sided limit exist

3a $f(x) = 7x^{-6} - 5\sqrt{x}$

$-42x^{-7}$ $\cancel{-\frac{1}{2}}$ $\cancel{25}$ $\frac{-5}{2\sqrt{x}}$ ✓

b $x^{48} + 2x^{24} + 3x^{16} + 4x^{12}$

$48x^{47} + 48x^{23} + 48x^{15} + 48x^{11}$ ✓

$u \quad v$ multiply across by $x^{n}$

3c $\left(\frac{3x+2}{x}\right)(x^{-5}+1)$ $\quad d\left(\frac{3x+2}{x}\right)\frac{g}{h}$

$\frac{du}{dx}.v + \frac{dv}{dx}.u$ $\quad \frac{3(x) - (3x+2)}{x^2}$ $\quad \frac{2}{x^2}$

$-\frac{2}{x^2}(x^{-5}+1) + (-5^{-6})\left(\frac{3x+2}{x}\right)$

$\frac{-2x^{-5}+1}{x^2}$ $\quad (-5^{-6})\frac{(3x+2)}{x}$

$\frac{-2x^{-5}+1}{x^2}$ $\quad \frac{-15x^{-5}-10x^{-6}}{x}$ $\quad (3x+2)(-6x^{-7}-x^{-2})+(3)(x^{-6}+x^{-1})$

$-2x^{-7}+x^{-2}$ $\quad -15x^{-6}-10x^{-7}$

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3d $\frac{4x+7}{x^2-1}$ $\frac{g}{h}$ $\quad \frac{4(x^2-1)-(2x)(4x+7)}{(x^2-1)^2}$

$\frac{4x^2-4 \quad -8x^2-14x}{(x^2-1)^2}$ $\quad \frac{-4x^2-14x-4}{(x^2-1)^2}$

4 Slopes should be perpendicular.

$-x^{-1}+(2-x)^{-1}$

$(2-2x)^{-1}=0$

$\frac{1}{x} = x^{-1}$ $\quad y=\frac{1}{2-x}\frac{g}{h}$ $\quad x=1 \quad y=1$

$(2-x)^{-1}$

$\frac{dy}{dx} = -1x^{-2} = \frac{-1}{x^2}$ $\quad \frac{dy}{dx} = -1(2-x)$ $\quad \frac{-(-1)(1)}{(2-x)^2} = \frac{1}{(2-x^2)}$

$-2+x$ $\quad x=1 \quad \frac{1}{2-1} = 1 \quad y=1$

$x=1 \quad y=-1$ $\quad x=1 \quad y=-1$

$m = \frac{1}{(2-x)^2}$

$\frac{-1}{x^2} \cdot \frac{1}{(2-x)^2}$ $\quad \frac{-1}{x^2(2-x)^2}$

$x=1$ $\quad \frac{-1}{1} \cdot \frac{1}{1} = -1$ slopes are perpendicular