Maths 3 Q2's

2013 Q2 a $P(2,0)$ $f(x,y) = \sin\left(\frac{x^3y^2+\pi}{xe^y+1}\right)$

$\vec{\nabla} f = \frac{df}{dx}\vec{i} + \frac{df}{dy}\vec{j}$

$\frac{df}{dx} = \cos\left(\frac{x^3y^2+\pi}{xe^y+1}\right) \frac{(xe^y+1)(3x^2y^2) - (x^3y^2+\pi)(e^y)}{(xe^y+1)^2}$

$P(2,0)$ $\cos\left(\frac{\pi}{3}\right)\left(\frac{0-\pi}{9}\right)$ $\cos\left(\frac{\pi}{3}\right)\left(\frac{-\pi}{9}\right)$ $\frac{1}{2}\left(\frac{-\pi}{9}\right) = \frac{-\pi}{18}$

$\frac{df}{dy} = \cos\left(\frac{x^3y^2+\pi}{xe^y+1}\right) \frac{(xe^y+1)(2x^3y) - (x^3y^2+\pi)(xe^y)}{(xe^y+1)^2}$

at $P(2,0)$ $\cos\left(\frac{\pi}{3}\right)\left(\frac{3(0) - \pi(2)}{9}\right) = \frac{1}{2}\left(\frac{-2\pi}{9}\right) = +\frac{\pi}{9}$

$\vec{\nabla} = \frac{-\pi}{18}\vec{i} \quad -\frac{\pi}{9}\vec{j}$

B Rate at which f is change $= \|\vec{\nabla}\|$

$= \sqrt{\left(\frac{-\pi}{18}\right)^2 + \left(\frac{-\pi}{9}\right)^2} = \frac{\sqrt{5}}{18}\pi$

C decrease at $\frac{\pi}{18}i \; \cancel{+} \frac{\pi}{9}j$

$\frac{\pi}{18} \Big/ \frac{\sqrt{5}}{18}\pi \; i + \frac{\pi}{9} \Big/ \frac{\sqrt{5}\pi}{18} \; j$

$= \frac{\sqrt{5}}{5} i \quad \cancel{+} - \frac{2\sqrt{5}}{5} j.$

D Eq^n of plane $= z = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

$z = \sin\left(\frac{\pi}{3}\right) + \left(-\frac{\pi}{18}\right)(x-2) + \frac{\pi}{9}(y-0)$

$z = \sqrt{3}/2 \quad -\frac{\pi}{18}x \quad +\frac{\pi}{9} \quad +\frac{\pi}{9}y$

$18z = -\pi x \quad +2\pi y \quad +9\sqrt{3} \quad +2\pi$

2

2012 Q2 $f(x,y,z) = \sqrt{z^2 + x - y + 2\cos(3y - 2x)}$ $\quad P(3,2,-1)$

$\frac{df}{dx} = \frac{1}{2} \frac{1 - 4\sin(3y-2x)}{\sqrt{z^2 + x - y + 2\cos(3y-2x)}}$ at $P(3,2,-1)$

$\frac{1}{2\sqrt{4}} = \frac{1}{4}$

$\frac{df}{dy} = \frac{1}{2} \frac{-1 + 6\sin(3y-2x)}{\sqrt{z^2 + x - y + 2\cos(3y-2x)}} = \frac{-1}{2\sqrt{4}} = \frac{-1}{4}$

$\frac{df}{dz} = \frac{1}{2} \frac{2z}{\sqrt{z^2 + x - y + 2\cos(3y-2x)}} = \frac{-1}{2}$

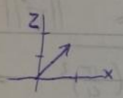
$\|\vec{\partial}\| = \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{-1}{4}\right)^2 + \left(\frac{-1}{2}\right)^2} = \frac{\sqrt{5}}{8}$

unit vector $= \frac{1}{4} / \frac{\sqrt{5}}{8} \quad -\frac{1}{4} / \frac{\sqrt{5}}{8} \quad -\frac{1}{2} / \frac{\sqrt{5}}{8}$

$\frac{2\sqrt{5}}{5} i \quad -\frac{2\sqrt{5}}{5} j \quad -\frac{4\sqrt{5}}{5} k$

0.89 $\qquad$ -1.7

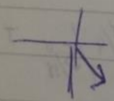
b Sketch on xz, ignore y coord



c Change sign : $\frac{-2\sqrt{5}}{5} i + \frac{2\sqrt{5}}{5} j + \frac{4\sqrt{5}}{5} k$

-only $\quad$ +ve

D on xy ignore z.



E rate of change = norm = $\sqrt{5}/8$

3

2011 Q2 $F(x,y,z) = \sqrt{y^2 - \sin(3x-2z)}$ $P(2,-1,3)$

$\frac{dF}{dx} = \frac{1}{2}\frac{-3\cos(3x-2z)}{\sqrt{y^2-\sin(3x-2z)}}$ $\frac{-3}{2\sqrt{1}} = -\frac{3}{2}$

$\frac{dF}{dy} = \frac{1}{2}\frac{2y}{\sqrt{y^2-\sin(3x-2z)}}$ $\frac{-2}{2\sqrt{1}} = -1$

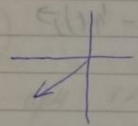
$\frac{dF}{dz} = \frac{1}{2}\frac{+2\cos(3x-2z)}{\sqrt{y^2-\sin(3x-2z)}}$ $= \frac{2}{2} = 1$

$\Rightarrow -\frac{3}{2}\vec{i} - 1\vec{j} + 1\vec{k}$

divide by mag $\sqrt{(-3/2)^2+(-1)^2+(1)^2} = \frac{\sqrt{17}}{2}$

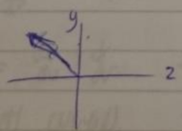
unit vector $= \frac{-3\sqrt{17}}{17}\vec{i} - \frac{2}{\sqrt{17}}\vec{j} + \frac{2}{\sqrt{17}}\vec{k}$

b. xy



c. $\frac{3\sqrt{17}}{17}\vec{i} + \frac{2}{\sqrt{17}}\vec{j} - \frac{2}{\sqrt{17}}\vec{k}$

D



E. rate of change $= \sqrt{17}/2$

2

2010 Q2 $z = \ln \sqrt{\frac{2x^2+y^2}{3}}$ $P(2,-1,0)$

$\frac{d}{dx} \frac{1}{2} \ln(2x^2+y^2)\big|_{(2,-1)} = \frac{2x}{x^2+y^2}\big|_{(2,-1)} = \frac{4}{9}$

$\frac{d}{dy} = z|_{(2,-1)} = \frac{y}{x^2+y^2}\big|_{(2,-1)} = -\frac{1}{9}$

$z = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

$z = \frac{4}{9}(x-2) - \frac{1}{9}(y+1)$

$z = \frac{4}{9}x - \frac{1}{9}y - 1$

Normal line:

$r(t) = r_0 + t(-f_x(x_0,y_0)\vec{i} - f_y(x_0,y_0)\vec{j} + \vec{k})$

$r = 2\vec{i} - \vec{j} + t(-\frac{4}{9}\vec{i} + \frac{1}{9}\vec{j} + \vec{k})$

$= (2-\frac{4}{9}t)\vec{i} - (1-\frac{1}{9}t)\vec{j} + t\vec{k}$