3.

From left hand side inequality:

$-3/2 \le -C_s/9$ or $3/2 \geq C_s/9$

Thus $27/2 \geq C_s$ or $C_s \le 27/2 = 13.5$

For right hand side we have

$\frac{-C_s}{9} \le -7/10$ or $\frac{C_s}{9} \geq \frac{7}{10}$

Thus $C_s \geq 63/10$ or $C_s \geq 6.3$

Combining calculated limits we know have C_s which provides us with the range of optimality for the standard bag contribution

$6.3 \le C_s \le 13.5$

- Everything else unchanged, profit for standard bag can range from £6.3 to £13.5 and solution will remain optimal
- Profit contribution will change
- For delux bag with $C_s = 10 \Rightarrow 6.67 \le C_D \le 14.29$

- In case of vertical line, we will have one limit and the other will be ∞ eg $13.5 \le C_s < \infty$

Right hand side:

The change in the value of the optimal solution per unit increase in the right hand side of the constraint is called the dual value

Example, add 10 extra units to constraint and find new optimal solution. Find profit for solution and difference between old and new profit

difference $\div$ 10 is then the dual value

1. Goal Programming

Multi criterion decision making, within general framework of LP

Formulation and graphical solution

Example Nicb Advisors. Client has 80000 to invest and want to invest in two different stocks:

stock	price/share	return/share	riskindex/share
US oil	$25	$3	0.5
hub properties	$50	$5	0.25

US oil is 12% return while hub is 10%. Obviously wants to minimise risk aswell. He could buy 8000 of oil shares = 3200 shares at 0.5 = 1600 risk. or 10 shares and have risk of 0 risk will vary from 0 to 1600

Client wants risk index of 700 maximum. One goal of is to find Risk index under 700

- Another goal is to obtain an annual return of at least 9000
- Portfolio selection is multicriteria problem as cannot max return ~~with~~ with risk of 700

Goal programming will identify a portfolio that comes close to achieving both goals. Client must determine which goal is most important.

Primary goal (Priority level 1)
Goal 1: Find portfolio with risk index ≤ 700

Secondary goal (Priority level 2)
Goal 2: Find portfolio with annual return of atleast 9000

Primary goal is priority level 1, Secondary is priority level 2

2

Called preemptive priorities because decision maker is not willing to sacrifice any amount of achievement of the priority level 1 goal for any other priority level

Risk index of 700 is target value for priority level 1 goal and return of 9000 is target value for priority level 2.

Developing the constraints and Goal eq^n

Decision variables : U = number U.S. oil shares
H = number hub properties shares

$25U + 50H \leq 80000$ funds available

Goal equation one for risk index.

$$0.5U + 0.25H = 700 + d_1^+ - d_1^-$$

d_1^+ amount by which portfolio risk exceeds target of 700
d_1^- amount by which portfolio risk index is less than the target value of 700

- d_1^+ and d_1^- known as deviation variables
- Deviation variables allow for the possibility of not meeting the target value exactly
- Example, U=2000 = 2000(0.5) = 1000. $d_1^+ = 300$ reflects risk index exceeded by 300
- Bring deviation variables to LHS

$$0.5U + 0.25H - d_1^+ + d_1^- = 700$$

The value on the right hand side of goal equation is target value for goal. LHS consists of (1) A function that defines the amount of goal achievement in terms of decision variables (eg $0.5U + 0.25H$)
(2) Deviation variables representing the difference between the target value for the goal and the level achieved

Goal eqn for funding goal:

3

Annual return $= 3U + 5H$.

d_2^+ = amount by which annual return is greater than target goal

d_2^- = amount by which annual return is lower than target goal

$$3U + 5H = 9000 + d_2^+ - d_2^-$$
$$3U + 5H - d_2^+ + d_2^- = 9000$$

Developing an objective function with preemptive priorities.

- The objective function in a goal programming model calls for minimizing a function of the deviation variables
- GP problems are solved by treating priority level 1 goal (P_1) first in an objective function.
- Idea is to find a solution that comes closest to satisfying P_1 goal.
- This solution is then modified by solving a problem with O.F involving P_2.
- Revisions in solution are only allowed if they do not hinder achievement of the P_1 goal
- One LP must be solved for each P level

- First formulate OF for P_1, risk level under 700. Underachieving target is not a concern but over it will not do. Thus O.F. should minimise d_1^+ to stay under 700

P_1 problem: Min d_1^+

ST:	$25U + 50H$		≤ 80000	funds available
	$0.5U + 0.25H - d_1^+ + d_1^-$		$= 700$	P_1
	$3U + 5H$	$-d_2^+ + d_2^-$	$= 9000$	P_2

$U, H, d_1^+, d_1^-, d_2^+, d_2^- \geq 0$

Graphical Solution Procedure:

- Similar to LP graph but separate solution for each P level.
- Because decision variables are non negative we consider only that portion of graph where $U \geq 0$ and $H \geq 0$.

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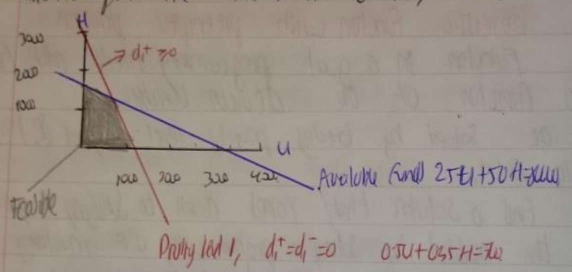
- Start by identifying all solution points for available funds: $25U + 50H \leq 80000$
- Objective for P_1 is to minimise d_1^+, when P_1 is met exactly $d_1^+ = 0$ and $d_1^- = 0$. goal eqⁿ is: $0.5U + 0.25H = 700$
- Graph shows solution that satisfies available funds and P_1 goal.

- We have solved P_1 goal. Alternative optimal solutions are possible, all solution points have risk index or less, hence $d_1^+ = 0$



- P_2 goal is return of 9000. If value exceeds it, it is ok, values under not acceptable want to minimise value of d_2^-
- Because goal 2 is priority level 2, solution to P_2 must not degrade orig optimal solution to P_1

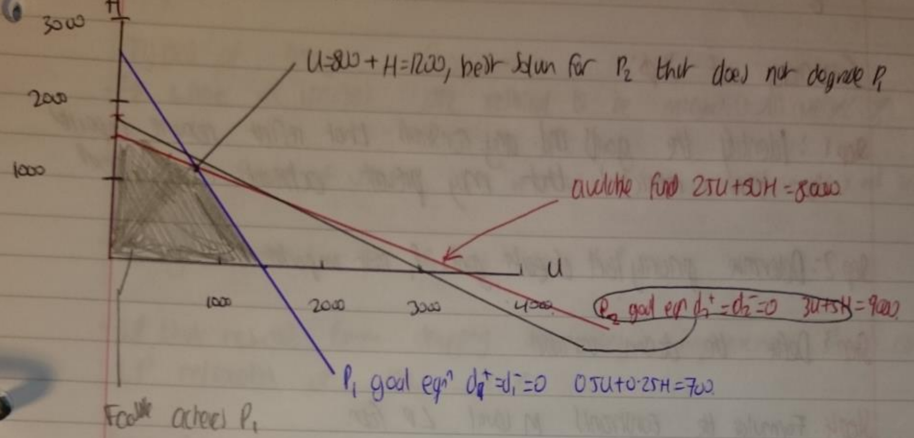
P_2 problem: Min d_2^-

$$
\begin{array}{llll}
\text{ST:} & 25U + 50H & \leq 8000 & \text{Funds available} \\
& 0.5U + 0.25H - d_1^+ + d_1^- & = 700 & P_1 \text{ goal} \\
& 3U + 5H - d_2^+ + d_2^- & = 9000 & P_2 \text{ goal} \\
& d_1^+ \quad d_1^- & = 0 & \text{Maintain achievement of } P_1 \text{ goal} \\
& U, H, d_1^+, d_1^-, d_2^-, d_2^+ \geq 0 & &
\end{array}
$$

P_2 LP differs in two ways:
- OF. minimises amount by which annual return level 2 goal.
- Constraint added to ensure no amount of achievement of P_1 goal is sacrificed.

Goal equation P_2 is $3U + 5H - d_2^+ + d_2^- = 9000$
reduce when $d_2^-, d_2^+ = 0$ to $3U + 5H = 9000$ on next graph

5. GOAL PROGRAMMING



- We cannot consider any solution that will degrade P_1.
- U=800 H=1200 comes closest to satisfying P_2 when P_1 is solved
- Annual return is $3(800) + 5(1200) = 8400$
- Satisfying both goals is impossible
- Best solution underachieved goal 2 by $d_2^- = 9000 - 8400 = 600$

Recommend 800 U shares, 1200 H shares. Risk index ≤ 700 has been achieved. P_2 not achieved. annual return = 8400.

In writing the overall objective function for portfolio selection problem we must write the OF in a way that reminds us of the preemptive priorities. We write OF as:

$$\min \quad P_1(d_1^+) + P_2(d_2^-)$$

Priority level P_1 and P_2 are not numerical weights on deviation variables, but simply labels to remind us of the priority level for goals

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Summary of Steps:

Step 1: Identify the goals and any constraints that reflect resource capacity or other restrictions that may prevent achievement of the goals.

Step 2: Determine priority level of each goal. P_1 most important etc

Step 3: Define the decision variables

Step 4: Formulate the constraints in usual LP form.

Step 5: For each goal develop a goal eq^n with the RHS specifying the target value for the goal. Decision variables d_i^+ and d_i^- are included in each goal eq^n to reflect the possible deviations above or below the target value

Step 6: Write O.F. in terms of minimising a prioritised function of the deviation variables