Applied Prob 1 2012 Exam Paper

Q 1 a Probability p of success and $1-p$ of failure

Success is given by $p(1-p)$

For example success of rolling a six $=1$
failure (other numbers) $=0$

Binomial is an extension of bernoulli with
- n independent trials
- Same probability of success/failure for each trial
- Success / failure only
- K success in n trials

$$\binom{n}{k} p^k (1-p)^{n-k}$$

- Geometric is distribution of trials until first success $p(1-p)^n$
success on $n+1^{th}$ trial (b option)

B
$0.594 \rightarrow -2\log(1-0.594) = 0.78294$ These three numbers are the
$0.051 \rightarrow -2\log(1-0.051) \therefore 0.04540$ transform
$0.150 \rightarrow -2\log(1-0.150) \therefore 0.14116$

Cdf is defined as $p(X \leq x) = F(x)$
For uniform $F(x) = x$ for $0 \leq x \leq 1$.
For exponential $f(x) = \lambda e^{-\lambda x}$

$y = -2\log_e(1-z)$ is the inverse transform of exponential cdf. Using random uniform numbers, this will create numbers from an exponential distribution
The mean value here is 2 which is also expected value, thus long run average will tend toward 2

2

C Rejection algorithm - generate the 3 letters, if there is Duplication, reject the word

Ranks - Choose 6 random numbers in $U(0,1)$, get the ranks for each, then select 1, 2, 1st second and third from ranks

A key issue with rejection algorithm is the frequency of rejection. If this is too high the method is ineffective. The sequence of accepts/rejects is a binary bernoulli sequence. The time between rejections is a random variable with geometric dist. Calculates probability of n trials until success at $n^{th}+1$ trial.

D Key is $P(B|A)P(A) = P(A|B)P(B)$
Random numbers can be used to simulate K, K, Q, K... In this sense the first card determines the probability of drawing the second, there is a forward "causal flow".

If A stands for first card is K and B for second card is King:
$P(A) = 4/8$ $P(B|A) = 3/7$ $P(\bar{A}) = 4/8$ $P(A \text{ and } B) = \frac{4}{8}\frac{3}{7}$
$P(B) = 4/8 \Rightarrow P(A|B) = 3/7 = P(B|A)$ $= P(B \text{ and } A)$

That is, given many pairs of (1st card, 2nd card) and focusing only on those pairs for which 2nd card is K, it will be observed that the proportion of those for which 1st card is also K will be 3/7, but now this info flows backward.

E A first X then Y simulation is first choose X from marginal distribution then choose Y from the conditional distribution

Marginal X dist is (0.22, 0.26, 0.29, 0.23) random uniform (0-0.22) gives X = 1 etc.

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1 Ead Now $x=1$, prob for y's are ($0.03/0.22$, $0.05/0.22$ etc).
the second random number will select the y value

$E[x \mid y=3] = [0.07(1) + 0.06(2) + 0.06(2) + 0.02(4)] / \text{Sum of } y=3$
$= 2.14287.$

$Var[x \mid y=3] = E[x^2 \mid y=3] - E[x \mid y=3]$

$0.07(1)^2 + 0.06(2)^2 + 0.06(3^2) + 0.02(4^2)/(0.21)$
$1.17/0.21$
$= 39/7 - 15/7 = 24/7 = 3.4285.$

2 A $p = 0.2$ $q = 0.25.$

b Probability Distribution of T is list of possibilities and probabilities.
$P(T=1) = p.$
$P(T=2) = p(1-p)$
$P(T=3) = p(1-p)^2$
$P(T=t) = p(1-p)^{t-1}$ for $t = 1, 2, 3 \ldots$
$P(T=1) = 0.2,$ $P(T=2) = 0.8 \times 0.2.$
The cdf is $P(T \le 1) = 1-0.8,$ $P(T \le 2) = 1-0.8^2$ etc.
The distribution is discrete the cdf is a step function

C This is the geometric distribution, expected value is $1/p = 5$ for

D $\lambda = 5.$ $1 - e^{-y/\mu}$ $1 - e^{-y/5}$

$P(T) = 1:$ $1 - e^{-1/5}$
$T = 2$ $1 - e^{-2/5}$
$T = 3$ $1 - e^{-3/5}.$
Probability density function is the derivative of the cdf

As it is continuous, the pdf is a smooth curve

E Theory suggests that when p and q are quite similar, there will be great congestion
If e.g. p/q $p=0.99$ $E[S] = 0.99(1-0.99)^{-1} \approx 100$
Similarly the probabilities for any specific i are very small, all with a common cdf.

Note that $\lim_{p \to 1} \left(\frac{p}{1-p}\right) \to \infty$ is unchanged

3A From table, standardising μ and σ^2 we get $P(T<12) = \phi\left(\frac{12-15}{3}\right)$ $\phi(-1) = 0.16$ $P(T>18) = 0.16$
$P(13<T<17) = \phi\left(\frac{17-15}{3}\right) - \phi\left(\frac{13-15}{3}\right) = 0.5$

The cdf cannot be computed, so instead we use the standardised tables of probability to compute the probabilities

B Norminv (number, mean, variance)
~~Calculates the probability~~

Norminv can be achieved by "going in" to the body of the tables of normal probabilities NORMDIST and heading to the margin (Margin to body gives $P(Y<1.64) = 0.9495$, body to margin gives RAND = 0.9495, leading to $Y=1.64$).

Simulation is via Norminv (U) where U are taken from table of random numbers (0, 1). T values can be generated by 15+3Y

C Simulate values for components A, B, C.
Use system life = Max (C, min (A,B))

5.

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Q3

0.594	0.051	0.156	16.782.	15.153	15.468	$\rightarrow$	16.782
0.817	0.802	0.696	17.451	17.406	17.08		17.451
0.040	0.094	0.088	15.77	15.282	15.264		15.77

D Avrg $\frac{1}{3}(T_A+T_B+T_C) \sim N(15, \frac{3^2}{3})$ $P(\text{avg} \le a) = \phi\left(\frac{a-15}{\sqrt{3}}\right)$

$(T_3 \le 12) = A_1\ ((T_A \le 12 \text{ or } T_B \le 12))$ and $(T_C \le 12)$

$P(T_3 \le 12) = (P(T_A \le 12) + P(T_B \le 12) - P(T_A < 12)P(T_B \le 12))\ P(T_C \le 12$