ELONEX 1376 0206 cm

2016 MANG SCI PAPER 1 QC 063

6 - A. PMF.

The probability mass function p(x) of a DISCRETE random variable X represents the probability that X takes the value x as a function of x. That is:

$p(x) = p(X=x)$ for all values of x.

The probability of the event (X=x) is thus defined as the sum of the probabilities of the individual outcomes for which X takes on value x).

Geometric Distribution

Geometric distribution is discrete. It is number of events needed until success.

$P(X=n) = (1-p)^{n-1} p$

Here we want to be successful after n trials. 1-p represents the probability of failure n-1 means we have n-1 failures before success on the n^{th} trial.

~~Cumulative Probability Distribution~~

~~The cumulative probability distribution $F(x_0)$ of a random variable X represents the probability that X does not exceed the value x_0 as a function of x_0:~~

~~$F(x_0) = P(X \leq x_0)$~~

~~Where the function is evaluated at all values of~~ x_0

Cumulative Distribution Function

The cumulative distribution function F(x) for a continuous random variable X expresses the probability that X does not exceed the value X as a function of x.

$F(x) = P(X \leq x)$

4 2

Probability of a range using a cumulative distribution function

Let X be a continuous random variable with a cumulative distribution function F(x) and let a and b be two possible values of X, with a < b. The probability that X lies between a and b is as follows:

$P(a \le X < b) = F(b) - F(a)$

For a continuous random variable it does not matter whether we write ≤ or < because the probability that X is precisely equal to b is 0

Probability Density function

Let X be a continuous random variable, and let x be any number lying in the range of values for the random variable. The probability density function f(x) of the random variable is a function with following properties:

- f(x) > 0 for all values of x
- Area under curve = 1
- Suppose that this density function is graphed. Let a and b be two possible values of random variable X, with a < b. Then the probability that X lies between a and b is the area under the probability density function between these points

P($P(a \le X < b) = \int_a^b f(x)\,dx$

- The CDF $F(x_0)$ is the area under the probability density function f(x) up to x_0

$F(x_0) = \int_{x_m}^{x_0} f(x)\,dx$

where x_m is minimum value of r.v X

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Exponential Distribution

- Restricted to random variables with positive values and its distribution is not symmetric

$y \sim \exp(\mu)$ where μ is mean parameter

pdf : $f(y) = \frac{1}{\mu} e^{-y/\mu}$

cdf = $1 - e^{-y/\mu}$

As you can see pdf is cdf differentiated

6 b ~~Characterised by the Poisson Distribution, the number of occurrence or successes of a certain event~~ in a given ~~continuous interval such as time~~

Characterised by the exponential distribution. Estimates/calculates time until the next arrival.

~~The exponential random variable $(t > 0)$ has a probability density function $f(t) = \lambda e^{-\lambda t}$ for $t > 0$ where λ is the mean number of independent arrivals per time unit t, is the number of time until until the next arrival~~

~~The CDF is $1 - e^{-\lambda t}$ for $t > 0$~~ mean = $\frac{1}{\lambda}$

~~for us $\mu/\lambda = 0.02$~~

~~$0.02 e^{-0.02 \cdot 0}$~~

$1 - 0.02 = 0.98$

4

Geometric ~~n-1 failure then~~ on nth success

$= (1-p)^{n-1} p$

$(0.98)^0 \, 0.02^1 \quad = 0.02$

Exponential, models time until first success

modelled by $1 - e^{-\lambda x}$

- where λ is the rate parameter which is 0.02 in our case
- X is the time to the next success

6 C Poisson

- characterised as the number of occurrences or success of a certain event in a given continuous interval such as time

Assumptions

- an interval is divided into a very large number of equal subintervals so that the probability of the occurrence of an event in any subinterval is small.
- the probability of the occurrence of an event is constant for all subintervals
- can be no more than one occurrence in a subinterval.
- occurrences independent

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad \text{for } x = 0, 1, 2 \ldots$$

$P(x)$ = probability of x success over a given time interval λ.
λ = expected number of success per unit time

$$P(x) = \frac{e^{-0.02} \, 0.02^x}{x!} \quad \text{per hour.}$$

$$\frac{e^{-20} \, 20^x}{x!} \quad x \text{ is in hours}$$

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C Normal Approx.

$$pdf = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}$$

Where y is the possible value and μ is the mean and is 20 in our calc.

σ^2 represents the standard deviation

D (1) $\hat{p} = \frac{14}{1000} = 0.014$

$$s.e. = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.014(1-0.014)}{1000}} = 0.003715.$$

$0.014 \pm 2(0.003715)$

$(0.00656, 0.0214)$

Success rate of 0.02 lies within the interval

(2) $\hat{p} = \frac{25}{1000} = 0.025$

$$s.e. = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad \sqrt{\frac{0.025(1-0.025)}{1000}} = 0.00493710$$

$0.025 \pm 2(0.00493710)$

0.025 ± 0.009874

$(0.0200629, 0.034874)$

does not contain 0.020

Q E. No this should have no implication.

It is assumed that each trial is independent of each other trial.

It is based on the binomial random variable where there is an underlying assumption that each trial is independent.

The sample voted must be independent of each other.

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5 A.

X(Spots) =	1	2	3	4	5	6	Σ
x^2 =	1	4	9	16	25	36	
p(x) =	1/6	1/6	1/6	1/6	1/6	1/6	
E[x] =	1/6	1/3	1/2	2/3	5/6	1	7/2 = 3.5.
E[x^2] =	1/6	2/3	1.5	8/3	25/6	6	

$E[x^2] = \frac{1}{6}\times 1 + 4\times\frac{1}{6} + 9\times\frac{1}{6} + 16\times\frac{1}{6} + 25\times\frac{1}{6} + 36\times\frac{1}{6} = 91/6$

Variance $= E[x^2] - E[x]^2$

$91/6 - \left[\frac{7}{2}\right]^2 \quad 91/6 - 49/4 = \frac{35}{12}$

B. x = 2 3 4 5 6 7 8 9 10 11 12.

p(x) 1/36 2/36 3/36 4/36 5/36 6/36 5/36 4/36 3/36 2/36 1/36

x^2

p(x)

E[x] = 32/3.

one dice E[x] = 3.5

2 = 7.

Variance one $= \frac{35}{12}\times 2 = \cancel{\frac{840}{144}} \quad \frac{35}{6}$

Average. 1 3/2 2 5/2 6/2 7/2 8/2 9/2 10/2 11/2 12/2

= 7/2

Varian $= \frac{35}{6}\div 2 = \frac{35}{12}$

C