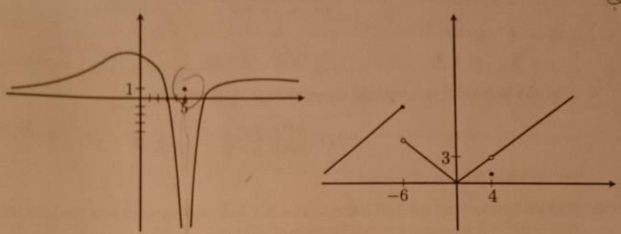
Tutorial 2: MA1E01

Limits

1. Consider the functions graphed below.



(a) For the first graph, find

$-\infty$ $\quad -\infty \quad -\infty$

$$\lim_{x\to5^-} f(x), \quad \lim_{x\to5^+} f(x), \quad \lim_{x\to5} f(x), \quad f(5).$$

(b) For the second graph above, what values of x_0 does the limit $\lim_{x\to x_0} f(x)$ not exist? -6

2. By evaluating the function at the x-values $x = \pm0.25, \pm0.1, \pm0.001, \pm0.0001$, make a conjecture about the limit (if it exists)

$$\lim_{x\to0} \frac{\sqrt{x+1}-1}{x}$$

3. Compute the following limits:

(a)

$2(2^5)-9(2^2)+7$

$$\lim_{x\to2} 2x^5 - 9x^2 + 7$$

(b)

$$\lim_{x\to6} \frac{x-6}{x^2-36} \qquad \frac{(x-6)}{(x-6)(x+6)} \quad \frac{1}{x+6} = \frac{1}{12}$$

(c)

$$\lim_{x\to9} \frac{x-9}{\sqrt{x}-3} \qquad \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(\sqrt{x}-3)} \quad \frac{\sqrt{x}+3}{1} = \frac{\sqrt{9}+3}{1} = 6$$

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2

(d)

$$\lim_{x\to\infty}\frac{x-7}{x^2+2x-3}$$

(e)

$$\lim_{x\to-\infty}\frac{\sqrt{16x^2-3}}{x+3}$$

(f)

$$\lim_{x\to\infty}\left(\sqrt{x^2-3}-x\right)$$

4. Find the horizontal and vertical asymptotes of the function

$$f(x)=\frac{3x^2-x-2}{2x^2+x-3}$$

and sketch a graph of the curve.

5. Find the largest open interval centered at $x=1$ such that for each x in the interval, $f(x)=3x^2+2$ is within $\epsilon=0.1212$ units of $f(1)=5$.

6. From the formal definitions of limits (the "epsilon-delta" formalism), prove the following limits:

(a)

$$\lim_{x\to0}\frac{2x^2+x}{x}=1$$

(b)

$$\lim_{x\to2}f(x)=5,\quad\text{where}\quad f(x)=\begin{cases}9-2x, & x\neq2\\ 49, & x=2\end{cases}$$

(c)

$$\lim_{x\to-\infty}\frac{4x-1}{2x+7}=2$$

$-\epsilon < f(x) < \epsilon$ if $-\delta < x < \delta$

$\lim_{x\to0}\frac{2x^2+x}{x}=1$

$\left|\frac{2x^2+x}{x}-1\right|<\epsilon \quad 0<|x|<\delta$

$\Leftrightarrow \left|\frac{2x^2+x-x}{x}\right|<\epsilon \quad 0<|x|<\delta$

$\Leftrightarrow \left|\frac{2x^2}{x}\right|<\epsilon \quad 0<|x|<\delta$

$\Leftrightarrow |2x|<\epsilon \quad 0<|x|<\delta$

$\Leftrightarrow |x|<\frac{\epsilon}{2} \quad 0<|x|<\delta$

$x-2<\frac{22}{1}$

$\delta=\frac{22}{1}$

$y=(x+2)^4$

$\sqrt[4]{y}=x+2$

$|f(x)-L|<\epsilon$ if $0<|x-a|<\delta$

$|11x-6-11|<\epsilon$

$|11x-22|<0.5$

$y=2x^3+5x+13$