Mang Sci 2013 Exam Paper

1 A. Transportation

Deals with the determination of a minimum cost plan for transporting a single commodity type from a number of sources to a number of destinations

Transhipment

Are transportation problems in which a shipment may move through intermediate nodes (transhipment nodes) before reaching a particular destination node

Assignment

Variation of transportation problem where there are equal number of origins and destinations and each origin supply and destination demand is unity. As a consequence, the quantity allocated must be either zero or one

B Min: $90x_{11} + 75x_{12} + 75x_{13} + 80x_{14} + 35x_{21} + 85x_{22} + 55x_{23} + 65x_{24} + 125x_{31} + 95x_{32} + 90x_{33} + 105x_{34} + 45x_{41} + 110x_{42} + 95x_{43} + 115x_{44}$

Subject to:

$x_{11} + x_{12} + x_{13} + x_{14} = 1$

$x_{21} + x_{22} + x_{23} + x_{24} = 1$

$x_{31} + x_{32} + x_{33} + x_{34} = 1$

$x_{41} + x_{42} + x_{43} + x_{44} = 1$

$x_{11} + x_{21} + x_{31} + x_{41} = 1$

$x_{12} + x_{22} + x_{32} + x_{42} = 1$

$x_{13} + x_{23} + x_{33} + x_{43} = 1$

$x_{14} + x_{24} + x_{34} + x_{44} = 1$

all x 0 or 1

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bii) Solve using hungarian method

- Subtract smallest number in each row from all elements in row:
- Subtracting smallest number in each column of resulting matrix from all other element in column

	1	2	3	4	
1	15	0	0	5	(-75)
2	0	50	20	30	(-35)
3	35	5	0	15	(-90)
4	0	65	50	70	(-45)
	(-0)	(-0)	(-0)	(-5)	

↓

	1	2	3	4
1	15	0	0	0
2	0	50	20	25
3	35	5	0	10
4	0	65	50	65

Not optimal : only 3 lines → Subtract smallest uncovered number from every other uncovered number in table
Add number to intersection of lines (-5)

	1	2	3	4
1	20	0	5	0
2	0	45	20	20
3	35	0	0	5
4	0	60	50	60

Not optimal → subtract 20 from each uncovered number
add to intersection

	1 ✓	2 ✓	3	4
1 ✓	40	$\boxed{0}$	5	$\boxed{0}$
2	$\boxed{0}$	25	$\boxed{0}$	$\boxed{0}$
3	55	$\boxed{0}$	$\boxed{0}$	5
4 ✓	$\boxed{0}$	40	30	40

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3 Mary Sci 2013 Exam Paper

4 lines ⇒ Solution found

Job	team		or Job	team	
1	2	75	1	4	80
2	4	65	2	3	55
3	3	90	3	2	95
4	1	45	4	1	45
cost: 275.			cost:	275.	

Both optimal solution

C Use Savings Heuristic:

Saving = $D_{0i} + D_{0j} - D_{ij}$

i	J	ij	saving			ij				Ranks:		Size
1	2	12	+	6	-	14	=	4	✓	1	6	25
1	3	12	+	21	-	24	=	9	✓	4	6	24
1	4	12	+	10	-	17	=	5	✓	3	6	20
1	5	12	+	12	-	9	=	15	✓	5	6	20
1	6	12	+	18	-	5	=	25	✓	3	5	17
2	3	6	+	21	-	15	=	12	✓	1	5	15
2	4	6	+	10	-	18	=	-2	✓	2	3	12
2	5	6	+	12	-	7	=	1	✓	2	6	12
2	6	6	+	18	-	12	=	12	✓	4	5	11
3	4	21	+	10	-	25	=	6	✓	1	3	9
3	5	21	+	12	-	16	=	17	✓	3	4	6
3	6	21	+	18	-	19	=	20	✓	1	4	5
4	5	10	+	12	-	11	=	11	✓	1	2	4
4	6	10	+	18	-	4	=	24	✓	2	5	1
~~5~~	~~5~~	~~12~~	~~+~~	~~12~~						2	4	-2.
5	6	12	+	18	-	10	=	20	✓			

problem

Find

Capacity of truck 20.

1	2	3	4	5	6
11 ✓	12	8 ✓	6	10 ✓	9. ✓

Join 1, 6? yes capacity = 11+9 = 20 ✓
Join 4, 6? No capacity exceeded
Join 3, 6? No capacity exceeded
Join 5, 6? No capacity exceed
Join 3, 5? Yes 8 + 10 = 18
Join 1, 5? Cant.
Join 2, 3? No - capacity exceeded
Join 2, 6? Cant - capacity exceed
Join 4, 5? Cant capacity exceed
Join 1, 3? Cant capacity exc
Join 3, 4? No - C.E.
Join 1 4? No - C.E.
Join 1, 2? No - cant
Join 2, 5? Yes capacity = 12+6 = 18 ✓

Solution found

Truck 1, 6 capacity = 20
Truck 3, 5 capacity = 18
Truck 2, 5 capacity = 18

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3 Mgmt Sci 2013 Exam Paper

4 lines ⇒ Solution found

Job	team		or Job	team	
1	2	75	1	4	80
2	4	65	2	3	55
3	3	90	3	2	95
4	1	45	4	1	45
cost:	275.		cost:	275.	

Both optimal solution

C Use Savings Heuristic:

Saving = $D_{0i} + D_{0j} - D_{ij}$

i	J	iJ Saving	ij			Ranks:		Sav
1	2	12 + 6	- 14 =	4	✓	1	6	25
1	3	12 + 21	- 24 =	9	✓	4	6	24
1	4	12 + 10	- 17 =	5	✓	3	6	20
1	5	12 + 12	- 9 =	15	✓	5	6	20
1	6	12 + 18	- 5 =	25	✓	3	5	17
2	3	6 + 21	- 15 =	12	✓	1	5	15
2	4	6 + 10	- 18 =	-2	✓	2	3	12
2	5	6 + 12	- 7 =	1	✓	2	6	12
2	6	6 + 18	- 12 =	12	✓	4	5	11
3	4	21 + 10	- 25 =	6	✓	1	3	9
3	5	21 + 12	- 16 =	17	✓	3	4	6
3	6	21 + 18	- 19 =	20	✓	1	4	5
4	5	10 + 12	- 11 =	11	✓	1	2	4
4	6	10 + 18	- 4 =	24	✓	2	5	1
~~5~~	~~5~~	~~12 + 12~~				2	4	-2.
5	6	12 + 18	- 10 =	20	✓			

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Q2 A Linear:

Identify decision variables first then objective function and the constraints

For goal programming, we identify any "hard" constraints, then the goal and any constraints on achieving the goals, then then priority of the constraints, the the deviation variables and list of all objective function
- Try to minimise deviation above/below target value

B Maximise: $190x_1 + 170x_2 + 155x_3$

where x_1 = Product 1, x_2 = Product 2, x_3 = Product 3

Subject to:

$3.5x_1 + 5.2x_2 + 2.8x_3 \leq 500$ First process hours available

$1.2x_1 + 0.8x_2 + 1.5x_3 \leq 240$ Second process H.A.

$40x_1 + 55x_2 + 20x_3 \leq 6500$ raw material.

Bi. Standard form:

$$\max\ 190x_1 + 170x_2 + 155x_3 + 0S_1 + 0S_2 + 0S_3$$

ST: $3.5x_1 + 5.2x_2 + 2.8x_3 + 1S_1 = 500$

$1.2x_1 + 0.8x_2 + 1.5x_3 + 1S_2 = 240$

$40x_1 + 55x_2 + 20x_3 + 1S_3 = 6500$

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		x_1	x_2	x_3	s_1	s_2	s_3		
Basis	CB	190	170	155	0	0	0	RHS	ratio
s_1	0	3.5	5.2	2.8	1	0	0	500	3.5/500 0.007
s_2	0	1.2	0.8	1.5	0	1	0	240	0.005
s_3	0.	40	55	20.	0	0	1	6500	0.00611
Z_j		0	0	0	0	0	0	0.	
$C_j - Z_j$		190	170	155	0	0	0		

X_1 in, S_2 out.

Biii. - After identify entering and leaving variables:
- Divide pivot row by number so pivot value is now 1.
- Subtract ~~pivot r~~ multiple of pivot row from other rows in the table such that all other values in pivot column are zero.
- Update table
- Check if RHS are still all positive
- Calculate next entering and leaving variables and repeat above process until all $C_j - Z_j$ values are ≤ 0, solution has been reached for

3A. i: Shadow Price:

Also known as dual price, this is the amount of change in the optimum solution if one more unit of a binding constraint was available

ii: Sensitivity analysis of Objective function coefficients:

This is the ~~rate~~ amount by which the value of a single objective coefficient can increase or decrease under the condition that the solution remains optimal

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3A iii. Feasible Solution Space

The area created by the constraints in the LP, where the optimal solution lies. This can be non-existent (over constrained), a single point, a line or a polygon or an unbounded area

iv. Standard Format.

The Format a LP must be converted to, to use Simplex method.

- ($\leq$) constraints have a slack variable added.
- ($\geq$) constraints have a slack variable subtracted and an artificial variable added
- ($=$) constraints have an artificial variable added

All RHS values must be positive, if not, multiply inequality by -1 and reverse inequality symbol

Slack variables have O.F cost value of zero, Artificial variables have value of M, where M is infinitely small value

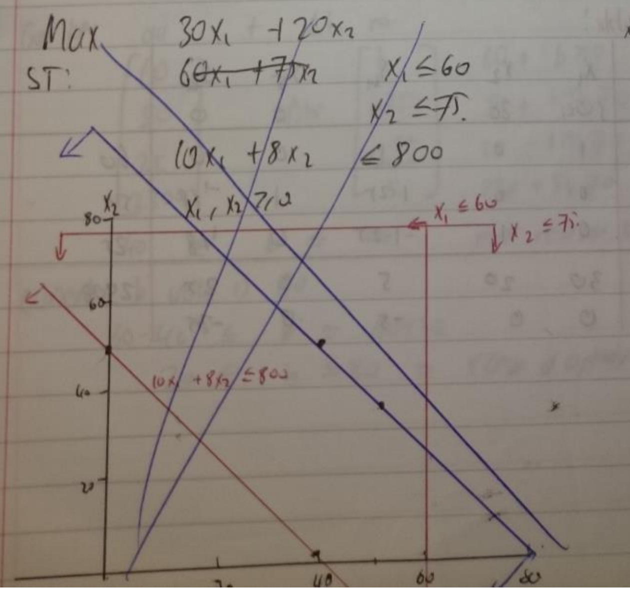
3B Max $30x_1 + 20x_2$

ST: ~~$60x_1 + 75x_2$~~ $x_1 \leq 60$

$x_2 \leq 75$.

$10x_1 + 8x_2 \leq 800$

$x_1, x_2 \geq 0$



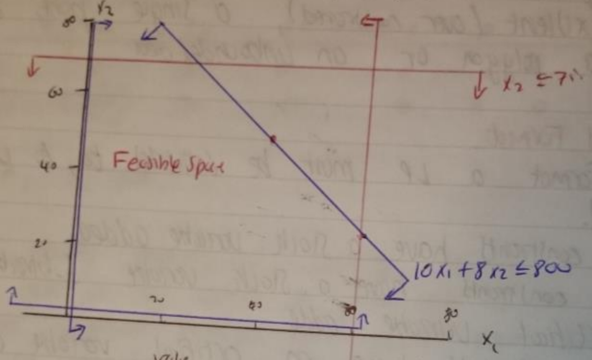
8
Max $30x_1 + 20x_2$

ST: $x_1 \le 60$

$x_2 \le 75$

$10x_1 + 8x_2 \le 800$ $\quad x_1, x_2 \geq 0$

3 Bii



Soln:		value
$(0,0)$	:	0
$(60,0)$	:	1800
$(60,25)$	:	2300
$(20,75)$	:	2100
$(0,75)$	:	1500

Optimal Soln: $(60,25)$: 2300

Biii Simplex table:

Basis	C_B	x_1	x_2	S_4	S_5	S_6	
		30	20	0	0	0	
x_1	30	1	0	1	0	0	60
S_4	0	0	0	1.25	1	-1/8	50
x_2	20	0	1	-1.25	0	1/8	25
Z_j		30	20	5	0	2.5	2900
$C_j - Z_j$		0	0	-5	0	-2.5	

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23 cont Biii. O.F coef range of optimality

$25 - c \le 0$

$c \ge 25$. $25 \le c \le +\infty$

~~$25 \le c$ $c = 30$~~

~~$30 + 25$~~

~~$55 \le c \le +\infty$~~

Biv. Second coef.

~~[illegible]~~ $-30 + 1.25c \le 0$

$-1/8\, c \le 0$

$c \ge 0$ and $c \le 24$

current value = 20

$0 \le c \le 24$

B V. Shadow prices are Z_j values for S_4, S_5, S_6

Constraint one : +5 binding

Constraint 2 : 0 non binding

Constraint 3 : 2.5 binding

B V. From table old + Δb_i new

$$\begin{bmatrix} 60 \\ 50 \\ 25 \\ 2300 \end{bmatrix} + \Delta b_i \begin{bmatrix} 1 \\ 1.25 \\ -1.25 \\ 5 \end{bmatrix}$$

$60 + b \ge 0$ $b \ge 60$

$50 + 1.25b \ge 0$ $b \ge 40$

$25 - 1.25b \ge 0$ $b \le 20$

$2300 + 5b \ge 0$ $b \le 460$

$40 \le b \le 20$ most restrictive value

current b value is 60

$60 - 40 \le b \le 60 + 20$

$20 \le b \le 80$ = range of optimality

Constraint 2:

$$\begin{bmatrix} 60 \\ 50 \\ 25 \\ \hline 2300 \end{bmatrix} + b_2 \begin{bmatrix} 0 \\ 1 \\ \underline{0} \\ 0 \end{bmatrix}$$

$50 + b \geq 0 \quad b \geq 50$

value is 75.

range is $25 \leq b \leq +\infty$.

Constraint 3:

old + change new

$$\begin{bmatrix} 60 \\ 50 \\ 25 \\ \hline 2300 \end{bmatrix} \quad b_3 \begin{bmatrix} 0 \\ -1/8 \\ 1/8 \\ \hline 2.5 \end{bmatrix}$$

$50 - \frac{1}{8}b \geq 0 \quad b \leq ~~400~~ \ 400$

$25 + \frac{1}{8}b \geq 0 \quad b \geq 200$

$2300 + 2.5b \geq 0 \quad b \geq -920$

current b value is 800

$800 - 200 \leq b \leq 800 + 400$

$600 \leq b \leq 1200$