DW.

2013 MANG SCI 11 Q6 DAVID WETTBRECHT.

Consider the vector field $F(x,y) = (2xy+y^3)i + (x^2+3xy^2+2y)$

a Show that F is a conservative vector field

$f = 2xy+y^3$ $g = x^2+3xy^2+2y$

If conservative $\frac{df}{dy} = \frac{dg}{dx}$

$\frac{df}{df} = 2x+3y^2$ $\frac{dg}{dx}$ $2x+3y^2$

$\frac{df}{dy} = \frac{dg}{dx}$ ∴ conservative

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b Find a potential function $\phi(x,y)$.

AS F is conservative potential function exists

We have

$\frac{d\phi}{dx} = 2xy+y^3$ and $\frac{d\phi}{dy} = x^2+3xy^2+2y$

To progress, integrate first eq^n with respect to x

$\int \frac{d\phi}{dx} dx = \int(2xy+y^3)\, dx = \frac{2x^2y}{2} + xy^3 + C_1(y)$

$\phi = x^2y + xy^3 + C_1(y)$

where $C_1(y)$ depends on y only and is therefore a constant with respect to integration in x.

2.

We differentiate this expression for ϕ with respect to y to equate it to our second eqn

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$$\frac{d\phi}{dy} = \frac{d}{dy}\left(x^2y + xy^3 + G(y)\right) = x^2 + 3xy^2 + \frac{d[G(y)]}{dy}$$

and so $x^2 + 3xy^2 + \frac{d[G(y)]}{dy} = x^2 + 3xy^2 + 2y$

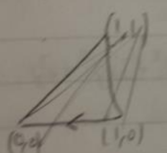
$$\frac{dG}{dy} = 2y$$

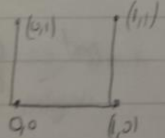
Integrating $G = y^2 + C$ where C is any constant.

Potential field is $\phi = x^2y + xy^3 + y^2 + C$. 6

c Let C be the curve that consists of three line segments, one from $(0,1)$ to $(0,0)$, one from $(0,0)$ to $(1,0)$ and one from $(1,0)$ to $(1,1)$ compute:

$$\int_C (2xy + y^3)\,dx + (x^2 + 3xy^2 + 2y)\,dy$$





$C_1: (1-t)(0,1) + t(0,0) = (0, 1-t)$

$C_2: (1-t)(0,0) + t(1,0) = (t, 0)$

$C_3: (1-t)(1,0) + t(1,1) = (1, t)$

$$C_1: \int_{C_1} \left[(2x(t)y(t) + y(t)^3)\frac{dx}{dt} + (x(t)^2 + 3x(t)y(t)^2 + 2y(t))\frac{dy}{dt}\right]$$

$$\int_{C_1} \left[(2(0)(1-t) + (1-t)^3)\frac{dx}{dt} + (0 + 3(0)(1-t)^2 + 2(1-t))\cdot -1\right]$$

$$\int_{C_1} (0 + (1-t)^3)(0) + [2 - 2t(-1)]\,dt$$

$$\int_0^1 (-2 + 2t)\,dt$$

$$-2t + t^2\Big|_0^1 = -2 + 1 = -1 = \boxed{-1}$$

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C cont.

$C_2 : \int_0^1 \left[(2x(t)y(t) + y(t)^3)\frac{dx}{dt} + (x(t)^2 + 3x(t)y(t)^2 + 2y(t))\frac{dy}{dt} \right] dt$

$(t,0)$ $\quad (t(0) + 0^3)1 \quad + \quad [t^2 + 3t(0) + 0]0$

$0 \quad + \quad 0$

$= 0$

$C_3 : (1,t) \quad = \int_0^1 \left[(2x(t)y(t) + y(t)^3)\frac{dx}{dt} + (x(t)^2 + 3x(t)y(t)^2 + 2y(t))\frac{dy}{dt} \right] dt$

$\int_0^1 \left[[2(t) + t^3]0 + (1 + 3(1)(t)^2 + 2(t))1 \right] dt$

$\int_0^1 \left[0 \quad 1 + 3t^2 + 2t \right] dt$

$\int_0^1 5t \, dt$

$\left. \frac{5t^2}{2} \right|_{t=0}^{1} = 2$

B

$= \frac{5}{2} + 0 + 1 = \frac{7}{2} = \quad$ ans

2

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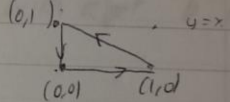
D. Let D be the curve that consists of three line segments; one from (0,0) to (1,0) and one from (1,0) but (0,1) Use green's theorm to compute: $\int_D xy\,dx + x^2y^3\,dy$



$f(x,y) = xy \qquad g(x,y) = x^2y^3$

line from (1,0) to (0,1) has eq^n $y = 1-x$ which will be upper limit for the y integral

0, 1.

$\frac{-1}{1} \quad -1$

$y - 1 = -1(x-0)$

$y - 1 = -x$

$y + x = 1$

$y = 1 - x$

$\int_D xy\,dx + x^2y^3\,dy = \iint_R \left[\frac{d}{dx}(x^2y^3) - \frac{d}{dy}(xy) \right] dA$

$\int_0^1 \int_0^{1-x} 2xy^3 - x \quad dy\,dx$ 2.

$= \int_0^1 \left[\frac{2xy^4}{3} - xy \right]_{y=0}^{y=1-x} dx$

$\int_0^1 \frac{2x(1-x)^3}{3} - x(1-x) \quad dx$

$\int_0^1 2x \frac{(-x^3 + 3x^2 - 3x + 1)}{3} - x - x^2 \quad dx$

$\int_0^1 \frac{-2x^4 + 6x^3 - 6x^2 + 2x - 3x - 3x^2}{3} \quad dx$

$\frac{1}{3} \int_0^1 -2x^4 + 6x^3 - 9x^2 - x + 1 \quad dx$

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$$\frac{1}{3}\left[-\frac{2x^5}{5} + \frac{6x^4}{4} - \frac{9x^3}{3} - \frac{x^2}{2} + x\right]_{x=0}^{1}$$

$$\frac{1}{3}\left[-\frac{2}{5} + \frac{6}{4} - \frac{9}{3} - \frac{1}{2} + 1\right]$$

$$= -\frac{7}{15}$$

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$-\frac{3}{20} \rightarrow +\frac{3}{20}$

area must be positive.

$\frac{17}{20}$

2013 MA1NG X1 PL Q6 CORRECTION

Q6 a) If vector field is conservative it is also path independent

conservative if $\frac{df}{dy} = \frac{dg}{dx}$

$F = 2xy + y^3$ $\qquad$ $g = (x^2 + 3xy^2 + 2y)$

$\frac{df}{dy} = 2x + 3y^2$ $\qquad$ $\frac{dg}{dx} = 2x + 3y^2$

$\frac{df}{dy} = \frac{dg}{dx} \Rightarrow$ it is conservative and path independent

b) As F is conservative a potential function exists

$\frac{d\phi}{dx} = 2xy + y^3$ and $\frac{d\phi}{dy} = x^2 + 3xy^2 + 2y$

$\phi = \int (2xy + y^3)\,dx$

$\phi = x^2y + xy^3 + C_1(y)$ where $C_1(y)$ is ~~a constant~~ depend on y and is therefore a constant with respect to integration in x

Differentiate with respect to y to equal

$\frac{d\phi}{dy} = x^2 + 3xy^2 + \frac{d\,C_1(y)}{dy}$

$\Rightarrow x^2 + 3xy^2 + 2y = x^2 + 3xy^2 + \frac{d[C_1(y)]}{dy}$ 2.

$2y = \frac{d[C_1(y)]}{dy}$

integrate $C_1(y) = y^2$

$\Rightarrow \phi = x^2y + xy^3 + y^2 + C$ where C is any constant

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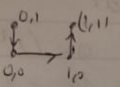
Use potential function

$\phi(1,1) - \phi(0,1)$

$(1)^2(1) + (1)(1)^3 + (1)^2 - [(0)^2(1) + (0)(1)^3 + (1)^2]$

$3 - 1 = 2$

via line integrals.



From $(0,1)$ to $(0,0)$ C_1: $(1-t)(0,1) + t(0,0) = (0, 1-t)$

$(0,0)$ to $(1,0)$ $(1-t)(0,0) + t(1,0) = (t, 0)$

$(1,0)$ to $(1,1)$ $(1-t)(1,0) + t(1,1)$ $(1,t)$

C_1: $[2x(t)y(t) + y(t)^3 \frac{dx}{dt}] + (x(t)^2 + 3x(t)y(t)^2 + 2(y(t))) \frac{dy}{dt}$

$\frac{dx}{dt} = 0$ $\quad \times \quad + [0 + 3(0)(1-t)^2 + 2(1-t)](-1)$

$[0 + 0 + 2 - 2t]$

$\int_0^1 -2 + 2t \, dt$

$= -2t + t^2 \Big|_{t=0}^{1}$

$-2 + 1 = \boxed{-1}$

C_2: $[2x(t)y(t) + y(t^3)] \frac{dx}{dt} + [x(t)^2 + 3x(t)y(t)^2 + 2y(t)] \frac{dy}{dt}$

$\frac{dx}{dt} = 0$ $\qquad$ $t^2 \cancel{+ 3(t)(0) + 2(0)}$ $\quad \cancel{\frac{dy}{dt}} = 0$

$\cancel{[t^2 + 0 + 0]1}$

$1[2(t)(0) + y(0)]$ $\qquad \cancel{\int_0^1 t^2 \, dt}$

$1[0]$ $\qquad \frac{t^3}{3}\Big|_{t=0}^{1}$

$= 1$

$= 0.$

C_3: $\frac{dx}{dt} = 0. \Rightarrow [x(t)^2 + 3x(t)y(t)^2 + 2y(t)$

$[1 + 3(1)(t^2) + 2t]1$

$1 + 3t^2 + 2t$

$\int_0^1 1 + 3t^2 + 2t \, dt$

$t + \frac{3t^3}{3} + \frac{2t^2}{2} \Big|_{t=0}^{1}$

$1 + 1 + 1 = 3$

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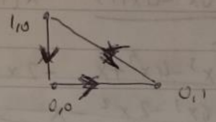
Add togete $C_1 + C_2 + C_3$

$-1 + 0 + 3 = 2$

D) Use green theorm



$\oint xy\,dx + x^2y^3\,dy$

line from $(1,0)$ to $(0,1)$ $\frac{y_2-y_1}{x_2-x_1}$ $\frac{1-0}{-1} = -1 = \text{slo}$

$y - y_1 = m(x - x_1)$

$y - 0 = -1(x-1)$

$y + x = 1 \quad y = 1-x$

$\iint_R \left[\frac{d}{dx}(x^2y^3) - \frac{d}{dy}(xy) \right] dA$

$\int_0^1 \int_0^{1-x} 2xy^3 - x \quad dy\,dx$

$\int_0^1 \frac{2xy^4}{4} - xy \Big|_{y=0}^{1-x}$

$\int_0^1 \frac{x(1-x)^4}{2} - x(1-x) \quad y$

$\int_0^1 \frac{x - 2x^2 + 6x^3 + x^5}{2} - x + x^2 \quad dx$

$(1-x)(1-x)$

$1 - x - x + x^2$

$(1-2x+x^2)^2 \quad (1-2x+x^2)$

$1 - 2x + x^2 + 4x^2 - 2x^3$

$+ x^2 - 2x^3 + x^4$

$1 - 2x + 6x^2 + x^4$

$\cancel{x}\ \cancel{-2x^2} + 6x^3 + x^5 \cancel{-2x} + \cancel{2x^2}$

$-x + 6x^3 + x^5 \quad dx$

$-\frac{x^2}{2} + \frac{6x^4}{4} + \frac{x^6}{6} \Big|_{x=0}^{1}$

$-\frac{1}{2} + \frac{6}{4} + \frac{1}{6} = \frac{7}{6}$

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Q5 A Express Cartesian coordinates in terms of cylindrical coordinates

$(1-x)^4$

$= (1-2x+x^2)^2$

$= x^4-4x^3+6x^2-4x+1$

$x\frac{(x^4-4x^3+6x^2-4x+1)}{2} - x(1-x) \qquad -x+x^2$

$-\frac{x^2}{2}+\frac{x^3}{3}$

$\cancel{x^5-4x^4+6x^3-4x^2+x \quad -2x+2x^2}$

$\cancel{x^5-4x^4+6x^3-2x^2-x} \; dx$

$\cancel{\frac{x^6}{6}-\frac{4x^5}{5}+\frac{6x^4}{4}-\frac{2x^3}{3}-\frac{x^2}{2}}\Big|_{x=0}^{1}$

$\cancel{\frac{1}{2}\left(\frac{x^6}{6}-\frac{4x^5}{5}+\frac{6x^4}{4}-\frac{4x^2}{2}+x\right)-\frac{2x^2}{2}+\frac{x^3}{3}}$

$\cancel{-\frac{1}{15} \quad -\frac{2}{3}}$

$\cancel{\frac{x^5}{2}-2x^3+3x^2-x^2+\frac{x}{2}}$

$x^5-4x^4+6x^3-4x^2+x \quad \div 2$

$\frac{x^5}{2}-2x^4+3x^3-2x^2+\frac{x}{2} \quad -x+x^2$

$\frac{1}{2}\frac{x^6}{6}-\frac{2x^5}{5}+\frac{3x^4}{4}-\frac{2x^3}{3}+\frac{1}{2}\frac{x^2}{2}-\frac{x^2}{2}+\frac{x^3}{3}$

$= \frac{-3}{20}$ area cannot be negative $\Rightarrow \frac{3}{20}$