Tutorial 10: MA1E01

Applications of the Definite Integral in Geometry

1. Find the area of the region enclosed between $y = 2x^2 - 1$ and $y = 2x + 3$ and on the sides by $x = -2$ and $x = 3$.

2. A solid is generated by revolving the region enclosed by $x = (y-2)^2$ and $x = 4$ about the x-axis, find the volume.

3. The region between $y = x^2$ and $y = x^3$ over the interval $[0, 1]$ is revolved about the y-axis. Find the volume.

4. Find the arclength of the curve $x = \frac{1}{8}y^4 + \frac{1}{4}y^{-2}$ from $y = 1$ to $y = 4$.

5. Find the area of the surface generated by revolving the curve $y = \sqrt{4 - x^2}$ over the interval $[-1, 1]$ about the x-axis.

1. $2x^2 - 1 = 2x + 3$

$2x^2 - 2x - 4 = 0$

$(x-2)(x+1) = 0$

$x = 2, \ x = -1$

$-2, \ 3$

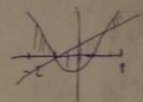
$[-2,-1] \ [-1,2] \ [2,3]$

$[-2,-1]$ $y = 2x^2 - 1$

-2 $2(4) - 1 = 7$

0 -1

3 17



$A = \int_{-2}^{-1} [f(x) - g(x)]dx + \int_{-1}^{2} [g(x) - f(x)]dx + \int_{2}^{3} [f(x) - g(x)]dx$

$\int_{-2}^{-1} [2x^2 - 2x - 4]dx + \int_{-1}^{2} [-2x^2 + 2x + 4]dx \ \int_{2}^{3} [2x^2 - 2x - 4]dx$

$\frac{2x^3}{3} - x^2 - 4x \Big|_{-2}^{-1} + \frac{-2x^3}{3} + x^2 + 4x \Big|_{-1}^{2} + \frac{2x^3}{3} - x^2 - 4x \Big|_{2}^{3}$

$= \frac{49}{3}$

2. $\int y \, dy$

$x \in [0, 4]$

$x = (y-2)^2$

$\pm\sqrt{x} = y - 2$

$y = 2 \pm \sqrt{x}$

$f(x)$ $g(x)$

$y = 2 + \sqrt{x}$ or $y = 2 - \sqrt{x}$

seperate regions

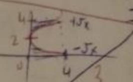
~~$A = \int_0^4 (f(x) - g(x))$~~

$V = \int \pi (f^2 x - g^2(x))$

$V = \pi \int_0^4 (4 + 4\sqrt{x} + x - (4 - 4\sqrt{x} + x)) dx$

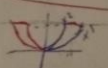
$8\pi \int_0^4 x^{\frac{1}{2}} \, dx$

$= \frac{128\pi}{3}$



3.

	$y=x^2$	$y=x^3$
$x=0$	0	0
$x=1$	1	1
$x=\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
$x=\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$



$\int f(y) - g(y) \, dy$

$y = x^2$ $y = x^3$

$x = y^{1/2}$ $x = y^{1/3}$

x goes $[0, 1]$

sub in points: y goes $[0, 1]$

$\int_0^1 \pi (f^2 y - g^2 y) dy$

$\pi \int_0^1 y^{2/3} - y$

$= \frac{\pi}{10}$

4. $x = \frac{1}{8}y^4 + \frac{1}{4}y^{-2}$

$x = f(y)$

$y \in [c, d]$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$\frac{dx}{dy} = \frac{1}{2}y^3 - \frac{1}{2}y^{-3}$$

$$\left(\frac{dx}{dy}\right)^2 = \frac{1}{4}y^6 - \frac{1}{2}y^0 + \frac{1}{4}y^{-6}$$

$$\frac{1}{4}(y^6 - 2 + y^{-6})$$

$$L = \int_1^4 \sqrt{1 + (\tfrac{1}{4})(y^6 - 2 + y^{-6})}\, dy$$

$$\int_1^4 \sqrt{\tfrac{1}{4}y^6 + \tfrac{1}{2} + \tfrac{1}{4}y^{-6}}\, dy$$

$$\int_1^4 \sqrt{\tfrac{1}{4}(y^6 + 2 + y^{-6})}\, dy$$

$$\int_1^4 \sqrt{\tfrac{1}{4}}\sqrt{y^6 + 2 + y^{-6}}$$

$$\int_1^4 \tfrac{1}{2}\sqrt{y^6 + 2y^3y^{-3} + y^{-6}}$$

$$\int_1^4 \tfrac{1}{2}\sqrt{(y^3 + y^{-3})^2}\, dy$$

$$\int_1^4 \tfrac{1}{2}(y^3 + y^{-3})$$

$$= \frac{2055}{64}$$

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5. $y = \sqrt{4 - x^2}$

$= (4 - x^2)^{1/2}$

$x \in [-1, 1]$

$$\frac{dy}{dx} = \frac{1}{2}(4 - x^2)(-2x)$$

$$= \frac{-x}{\sqrt{4 - x^2}}$$

$$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\int_{-1}^1 2\pi \sqrt{4 - x^2}\sqrt{1 + \frac{x^2}{4 - x^2}}\, dx$$

$$\int_{-1}^1 2\pi \sqrt{4 - x^2}\sqrt{\frac{4 - x^2 + x^2}{4 - x^2}}\, dx$$

$$\int_{-1}^1 2\pi \sqrt{4 - x^2}\sqrt{\frac{4}{4 - x^2}}\, dx$$

$$\int_{-1}^1 2\pi \sqrt{4 - x^2}\,\frac{\sqrt{4}}{\sqrt{4 - x^2}}\, dx$$

$$= \int_{-1}^1 2\pi\, 2\, dx$$

$$\int_{-1}^1 4\pi\, dx$$

$$= 8\pi \text{ units}^2$$