2/13 2013 Exam Paper Q1

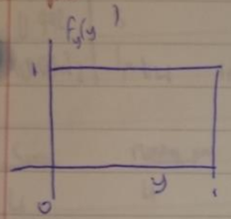
1 $E[y_p | y_e = 1] = 0 \cdot 0.25 \times 1 \times 0.5 + 2 \times 0.25 = 1$

$E[y^2_p | y_e = 1] = 0^2 \times 0.25 + 1^2 \times 0.5 + 2^2 \times 0.25 = 1.5$

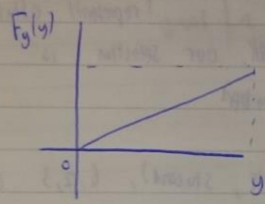
$Var[y_p | y_e = 1] = E[y_p^2 | y_e = 1] - E[y_p | y_e = 1]^2$

$1.5 - 1^2 = 0.5$

b pdf




cdf.




Generate y multiple times through RAND() resulting in $y_1, y_2 \ldots y_n$

For each calculate its cubed value $y_1^3, y_2^3 \ y_n^3$

$E[y^3]$ is long run average of $y_1^3, y_2^3 \ y_n^3$

i.e $\frac{1}{n} \sum_{i=1}^{n} y_i^3$

$E[y^3] = \int_{-\infty}^{\infty} y^3 f_y(y)\, dy = \int_0^1 y^3\, dy = \frac{y^4}{4}\Big|_{y=0}^{1} = \frac{1}{4}$

$P(Z \leq z) \quad P(y^3 \leq z) = P(y \leq z^{1/3}) = z^{1/3}$ if y uniform $y \sim U(0,1)$

c 3 chosen 6 possible

$P(A, B, C) = P(A) P(B|A) P(C|BA) \qquad P(A,B,C) = P(C|BA) P(B|A) P(A)$

$A \rightarrow$ no repeat $\qquad = 1 \times \frac{5}{6} \times \frac{4}{6} = \frac{20}{36} = \frac{5}{9}$

$B \rightarrow$ no repeat

$C \rightarrow$ no repeat

2

Alternatively $6^3$ possible selections of 3 students with replacement

of which $6 \times 5 \times 4$ of these involve no repetition so probability $\frac{6 \times 5 \times 4}{6 \times 6 \times 6}$

$U_1, U_2, U_3, U_4, U_5, U_6,$ in cos 1

0.6 0.4 0.3 0.8 0.9 0

represents a student.

take ordered rank, our selection is the student with smallest 3 random numbers

So in example, students 6, 2, 3 are chosen

D $P(Q_2) = \frac{4}{52} = \frac{1}{13}$

$P(Q_2 | \bar{Q_1}) P(\bar{Q_1}) + P(Q_2 | Q_1) P(Q_1)$

$\frac{4}{51} \times \frac{48}{52} + \frac{3}{51} \times \frac{4}{52} = \frac{1}{13}$

$P(Q_1 | Q_2) = \frac{P(Q_1 \text{ and } Q_2)}{P(Q_2)} = \frac{\frac{4}{52} \times \frac{3}{51}}{4/52} = \frac{3}{51}$

- Simulate repeated draws of 2 cards from a deck
- Consider proportion of those with a queen on second draw that had a queen on the first

i E

| First | Second | P |
|---|---|---|
| 6 | $\bar{6}$ | 5/36 |
| 6 | 6 | 1/36 |
| $\bar{6}$ | $\bar{6}$ | 25/36 |
| $\bar{6}$ | 6 | 5/36 |

$P(2\text{ 6's} | \text{at least } 1\ 6) = \frac{P(\text{at least } 1\ 6 | \text{two 6's}) P(\text{two 6's})}{P(\text{at least } 1\ 6)} = \frac{1/36}{11/36} = \frac{1}{11}$

10/12/13
3

P(second a six | first is a 6) = 1/6 more information know which one was a 6.

Q2. a First die                                                     Second die

| RAND | |
|---|---|
| 0.542 | 2 |
| 0.61 | 2 |
| 0.992 | 3 |
| 0.053 | 1 |

| die | p | cum |
|---|---|---|
| 1 | 1/3 | 1/3 |
| 2 | 1/3 | 2/3 |
| 3 | 1/3 | 1 |

| RAND | |
|---|---|
| 0.384 | 2 |
| 0.751 | 3 |
| 0.415 | 2 |
| 0.628 | 2 |

| Sum | running avg | variance |
|---|---|---|
| 4 | 4 | 0 |
| 5 | 4.5 | 0.5 |
| 5 | 4.67 | 0.33 |
| 5 | 4.25 | 0.92 |

$$\frac{(4-4.67)^2+(5-4.67)^2+(5-4.67)^2}{2}$$ mean (n-1)

b

| $S_2$ | $P(S_2)$ | $S_2^2$ |
|---|---|---|
| 2 | 1/9 | 4 |
| 3 | 2/9 | 9 |
| 4 | 3/9 | 16 |
| 5 | 2/9 | 25 |
| 6 | 1/9 | 36 |
| | 1 ✓ | |

$E(S_2) = 2/9 + 6/9 + 12/9 + 10/9 + 6/9 = 36/9 = 4$

$E(S_2^2) = 4/9 + 18/9 + 48/9 + 50/9 + 36/9 = 156/9 =$

$var = \frac{156}{9} - (4)^2 = 4/3$

One die $E(x) = 2$

$E(x^2) = 14/3$          $14/3 - 2^2 = 2/3$

$E(S_2) = 2E[x] = 2 \times 2 = 4$

$Var[S_2] = Var[x+x] = Var[x] + Var[x] = 2 \times 2/3 = 4/3$