MATHS SET 7 WEEK 8

14/11/12 DAVID WEITBRECHT 12300644

1 a $F(x,y) = (x^2y - \frac{1}{2}y^2)i + (\frac{1}{3}x^3 - xy + 1)j$ conservative?

$\frac{8.5}{13}$

If conservative $\frac{df}{dy}$ should equal $\frac{dg}{dx}$

$$\frac{d(x^2y - \frac{1}{2}y^2)}{dy} = x^2 - y$$

$$> \frac{df}{dy} = \frac{dg}{dx} \Rightarrow \text{conservative}$$

1

$$\frac{d(\frac{1}{3}x^3 - xy + 1)}{dx} = x^2 - y$$

b $\frac{d\phi}{dx} = (x^2y - \frac{1}{2}y^2)$ $\quad \frac{d\phi}{dy} = (\frac{1}{3}x^3 - xy + 1)$

$\frac{x^3y}{3} - \frac{1}{2}y^2x + C(y)$

$$\int \frac{d\phi}{dx} = \int (x^2y - \frac{1}{2}y^2)\,dx \Rightarrow \frac{x^3y}{3} + G_1(y) = \phi$$

$G_1(y)$ depends on y only and is therefore a constant with respect to integration of x.

Now differentiate both for ϕ with respect to y

$$\rightarrow \frac{d}{dy}\left(\frac{x^3y}{3} + G_1(y)\right) = \frac{x^3}{3} + \frac{dG_1}{dy} - xy$$

$$\frac{\cancel{x^3}}{\cancel{3}} + \frac{dG_1}{dy} = \cancel{\frac{1}{3}x^3} - xy + 1$$

$$\frac{dG_1}{dy} = -xy + 1$$

$$G_1(y) = \frac{-xy^2}{2} + y + C$$

Therefore potential $\phi = \frac{x^3y}{3} + G_1(y)$

1

$$\phi = \frac{x^3y}{3} - \frac{xy^2}{2} + y + C$$

1.c. $\varphi(x_1, y_1, z_1) - \varphi(x_0, y_0, z_0)$

$\varphi = \frac{x^3 y}{3} - \frac{xy^2}{2} + y$

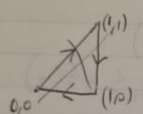
$\overset{x,y}{(1,1)} \quad \overset{x_0\ y_0}{(0,0)}$

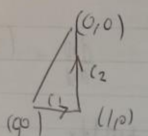
$\Rightarrow \left[\frac{(1)^3(1)}{3} - \frac{(1)(1)^2}{2} + (1)\right] - \left[\frac{(0)^3(0)}{3} - \frac{(0)(0)^2}{2} + 0\right]$

$\frac{1}{3} - \frac{1}{2} + 1 \quad - [0]$

$= \frac{5}{6}$ ✓ 1

d





$C_1: r(t) = (1-t)(0,0) + (1,0) \quad = (t, 0)$
$C_2: r(t) = (1-t)(1,0) + (1,1) \quad = (1, t)$
$C_3: r(t) = (1-t)(1,1) + (0,0) \quad = (1-t, 1-t)$

On C_1 $y'(t) = 0$ so we immediately see

$\int_{C_1} (x^2 y - \frac{1}{2}y^2)\,dx + (\frac{1}{3}x^3 - xy + 1)\,dy = 0.$ ✓

On C_2 $x'(t) = 0$ $y'(t) = 1$

$\int_{C_2} ((x^2y - \frac{1}{2}y^2)dx + (\frac{1}{3}x^3 - xy + 1)dy) = \int_{C_2}$

1/2

no C3. ✗

On C_3. $x'(t) = -1$ $y'(t) = -1$

$= \int_0^1 (1-t)^2(1-t) - \frac{1}{2}(1-t)^2 \frac{d}{dt}(1-t) + \int_0^1 \frac{1}{3}(1-t)^3 - (1-t)(1-t) \frac{d}{dt}(1-t) \; dt$

$= \int_0^1 (1-t)^3 - \frac{(1-t)^2}{2}(-1)\,dt + \int_0^1 \frac{(1-t)^3}{3} - (1-t)^2 \; d$

$\int_0^1 2\left(\frac{1-t}{1}\right) dt \quad + \int_0^1 1 - t - 3 \; dt$

$\int_0^1 2 - 2t + 1 - t - 3 \; dt$

$\int_0^1 -3t \; dt \quad = \frac{-3t^2}{2}\Big|_0^1 = \frac{-3}{2}$

3

Maths Tutorial Week 8 Set 7. 12300644

1 AUD WETTBURTT?

2a If independent of path $\cancel{\int_C F dr = \phi(r(b)) - \phi(r(a))}$

$= \cancel{\phi(x_1, y_1, z_1) - \phi(x_0, y_0, z_0)} = 0$

$\int_{(1,-1)}^{(3,2)} (y-2x)dx + (x+4y^3)dy$

If vector field is conservative = path independence

conservative if $\frac{df}{dy} = \frac{dg}{dx}$

$\frac{d(y-2x)}{dy} = 1$

$\frac{d(x+4y^3)}{dx} = 1$

$\Rightarrow \frac{df}{dy} = \frac{dg}{dx}$ = conservative $\Rightarrow$ path independence

1

b $\frac{d\phi}{dx} = (y-2x)$ $\frac{d\phi}{dy} = (x+4y^3)$

$\int \frac{d\phi}{dx} = \int (y-2x)dx$

$= xy - x^2 + C_1(y) = \phi$

diffn ϕ with respect to y.

$\frac{d}{dy}(xy - x^2 + C_1(y)) = \cancel{x} + \frac{d(C_1(y))}{dy} = \cancel{x} + 4y^3$

$\frac{d(C_1(y))}{dy} = 4y^3$

integrate $C_1 = y^4 + C$

$\phi = xy - x^2 + C_1(y)$

$\Rightarrow \phi = xy - x^2 + y^4 + C$

$\phi(x,y) = xy - x^2 + y^4 + C$

1

2c $\phi(x_1,y_1,z_1) - \phi(x_0,y_0,z_0)$

$\phi = xy - x^2 + y^4$

$\begin{pmatrix} x_1 & y_1 \\ 3, & 2 \end{pmatrix}$ $\begin{pmatrix} x_0 & y_0 \\ 1 & -1 \end{pmatrix}$

$[3(2) - (3)^2 + (2)^4] - [(1)(-1) - (1)^2 + (-1)^4]$

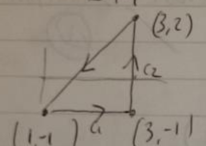
$[6 - 9 + 16] \quad - [-1 - 1 + 1]$

$13 \quad - [-1]$

$= 14$

1

d



1-t   -1+4
+3t   -t
1-2t   -1

$C_1 : \vec{r}(t) = (1-t)(1,-1) + t(3,-1)$ $\overset{1-t+3t \quad -1+t-t}{(1+2t, -1)}$ $3-3t+3t \quad -1+t+t$

$C_2 : \vec{r}(t) = (1-t)(3,-1) + t(3,2)$ $(3, \ 3t-1)$

no C_3 x $C_3 : \vec{r}(t) = (1-t)(3,2) + t(1,-1)$ $(3-2t, \ 2-3t)$

$x'(t) = 2 \quad y'(t) = 0$

$C_1 = \int (y-2x)dx + (x+4y^3)dy$

$= \int_0^1 [-1 - 2(1+2t)](2) + (1+2t+4(-1)^3)0$

$\cancel{\int_0^1 (-1-4-8t)dt = \int_0^1 (-5-8t)dt = -5t - 4t^2 |_0^1 \ -5-4} = -9$

$2\int_0^1 (-1-2-4t)dt$

$2\int_0^1 (-3-4t)dt$

$2\int_0^1 -3t - 2t^2 = 2[-3-2] = -10$

$\int (y-2x)dx$

$(-1) - 2(1+2t) \ \frac{d}{dt}(1+2t)$

take $|-10| = 10$
because in lower quadrant.

$-1 - 2 - 4t$

$-3 - 4t \ (2)$

no C_3!!

DAVID WEMBRELL 12300644

C_2: $\int (y - \cancel{2}x)\,dx + (x+4y^3)\,dy$ $x'' = 0.$

$\int_0^1 (3) + 4(3t-1)^3 \frac{d(3t-1)}{dt}\, dt$

$[3 + 12t - 4]\ (3)$ ✗ $\frac{1.5}{2}$

$[-1 + 12t]3$

$3\int_0^1 -1 + 12t\ dt$

$= 3\ \ -t + 6t^2 \Big|_{t=0}^{1}$

$3[-1+6] = 15.$

C_3: $\int (y-2x)\,dx + (x+4y^3)\,dy$

$[(2-3t) - 2(3-2t)][-2] + [(3-2t) + 4(2-3t)^3][-3]\ dt$

$[2-3t-6+4t]-2 + [\cancel{3-2t}\ 5(3\ (3-2t)+4(2-3t)^3][-3]\ dt$

no C3!!

$-2\int_0^1 [-4+t]$

$= -2\left[-4t + \frac{t^2}{2}\right]\Big|_0^1$ $\quad -3\left(-27t^4 + 72t^3 - 73t^2 + 35t \Big|_0^1\right)$

$-3[-27 + 72 - 73 + 35]$

$-2\left[-4 + \frac{1}{2}\right]$ $\quad -3[7]$

$-2[-3.5]$ $\quad = |-21| = 21$

$= 7$ $\quad 21$

$= 15 + 7 + 21 = 43 = \text{area}$