2013 Probability 1 Exam Paper

Q1 A

(y_p, y_e)	$P(y_p, y_e)$	CDF	CDF RESULT
(0,0)	0.2	0.2	[0, 0.2]
(0,1)	0.1	0.3	[0.2, 0.3]
(0,2)	0.1	0.4	[0.3, 0.4]
(1,0)	0.1	0.5	[0.4, 0.5]
(1,1)	0.2	0.7	[0.5, 0.7]
(1,2)	0.1	0.8	[0.7, 0.8]
(2,0)	0	0.8	[0.8, 0.8]
(2,1)	0.1	0.9	[0.8, 0.9]
(2,2)	0.1	1	[0.9, 1]

$\therefore$ Simulation with provided random vars

$0.1563 \rightarrow (0,0)$ $\quad$ $0.6291 \rightarrow (1,1)$

Marginal Probability

y_p	0	1	2
	0.3	0.4	0.3

y_e	0	1	2
	0.4	0.4	0.2

$E[x] = \sum x P(X=x)$

$E[y_p] = 0(0.3) + 1(0.4) + 2(0.3) = 1$

Bayes Theorem

$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$

$Var[x] = E[x^2] - \{E[x]\}^2$

$Var[y_p] = 1.6 - 1^2 = 0.6$

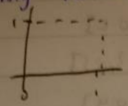
$P(y_p | y_e = 1) = \frac{P(y_e = 1 \text{ and } y_p)}{P(y_e = 1)}$

$E[x^2] = \sum x^2 P(X=x)$

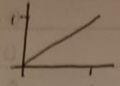
$E[y_p^2] = 0^2(.3) + 1^2(0.4) + 2^2(0.3) = 1.6$

y_p $(y_e = 1)$	Individual Probs	
0	0.1/0.4	= 0.25
1	0.2/0.4	= 0.5
2	0.1/0.4	= 0.25

B PDF = Probability Density Function



CDF = Cumulative Distribution Function



Rand() Should have equal Prob of falling at any value

$E[y^3] = \int y^3 dy = \frac{y^4}{4}\Big|_0^1 = 1/4 = 0.25$

Event identity for CDF:

↳ Simulate to get a large number of y's: $y_1, y_2 \dots y_n \sim U(0,1)$

↳ Calculate $y_1^3, y_2^3 \dots y_n^3$

↳ Estimate y^3 by $\sum y_i^3 / N$ (average of simulation)

↳ $P(Z \le z) = P(y^3 \le y) = P(Y \le y^{1/3}) = y^{1/3}$

C $P(A,B,C) = P(A)P(B|A)P(C|A,B)$

P(No duplicates) $= 1 \times P(2^{nd}\ ND \mid 1^{st})\ P(3^{rd}\ ND \mid 1^{st}, 2^{nd})$

$1 \times 5/6 \times 4/6 = 5/9$

General ranking values of RAND()

person	A	B	C	D	E	F
random prob	0.2	0.35	0.5	0.1	0.9	0.7
rank	5	4	3	6	1	2

D $P(2^{nd} = Q)$ = Uses the law of total probability

$P(Q_2 \mid Q_1)P(Q_1) + P(Q_2 \mid \bar{Q}_1)P(\bar{Q}_1)$

$(3/51 \times 4/52) + (4/51 \times 48/52) = 1/13$

$P(Q_1 \mid Q_2) = \frac{P(Q_2 \mid Q_1)P(Q_1)}{P(Q_2)} = \frac{3/51 \times 4/52}{1/13} = 3/51$

To Simulate, follow the steps

↳ draw 2 cards

↳ do this n times

↳ Find proportion of times first was Q, given second was Q

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Q2

Rep	rand	1st die	rand	2nd Die	Sum	Avge	Var
1	0.542	2	0.384	2	4	4	–
2	0.61	2	0.751	3	5	4.5	0.3
3	0.942	3	0.435	2	5	4.33	0.5
4	0.053	1	0.628	2	3	4.25	0.3

Variance here is calculated by $Var = \frac{\sum(x_i - \bar{x})^2}{n}$

B

S_2	$P(S_2)$
2	1/9
3	2/9
4	3/9
5	2/9
6	1/9

$P(1^{st}=1 \,\&\, 2^{nd}=1) = P(1^{st}=1)P(2^{nd}=1) = \frac{1}{3}\times\frac{1}{3} = \frac{1}{9}$

$E(S_2) = 2(1/9)+3(2/9)+4(3/9)+5(2/9)+6(1/9) = 4$

$Var(S_2) = E(S_2^2) - E(S_2)^2 = 100/9$

C $P(S_{n+1} = s) = P(S_n = s-x)\,P(X_{n+1} = x)$

$S_3 = \{3, 4 \dots, 9\}$

$P(S_3 = 5) = P(S_2 = 2)P(X_3 = 3) + P(S_2 = 3)P(X_3 = 2) + P(S_2 = 4)P(X_3 = 1)$ $= 1/9(1/3) + 2/9(1/3) + 3/9(1/3) = 2/9$

D $E(S_{10}) = 20$ $\quad Var(S_{10}) = 2/3 \sim 2/3 \cdot 10$

$E[aX] = E[\frac{1}{10}\sum x_i]$ $\quad Var[\bar{x}] = Var[\frac{1}{10}\sum x_i] = (\frac{1}{10})^2 Var \sum x_i$

$\frac{1}{10} E(\sum x_i) = \frac{1}{10}(20) = 2$ $\quad = \frac{1}{100} \cdot 20/3 = 1/15$

E $P(1.7 \le \bar{x} \le 2.3) = P(x < 2.3) - P(x < 1.7)$ where $z = \frac{x-\mu}{\sigma}$

$P(\bar{x} < 2.3) = P(Z < \frac{2.3-2}{\sqrt{1/15}}) = P(Z < 1.16)$

A 95% CI is an interval (a, b) which we are 95% that the true quantity of mean lies within

Defined as $\bar{x} \pm 2\frac{\sigma}{\sqrt{n}}$

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Q3 $P(0,1,2)$: P(C beats A & C beats B and B beats A)
: P(C beats A) P(C beats B) P(B beats A)

$P(0,2,1)$ P(B beats A) P(B beats C) P(C beats A)

NA		
0	0.24	P(lose to B) P(lose to C) . 0.3 x 0.8
1	0.62	
2	0.14	P(beats B) P(beats C) 0.7 x 0.2

$(2-1)^2 + (0-1)^2 + (1-1)^2$

$E[NA]^2 = 0^2(0.24) + 1^2(0.62) + 2^2(0.14) = 1.18$

$Var[NA] = 1.18 - 0.9^2 = 0.37$

$\frac{1}{2} \cdot t^{-1} = \frac{2}{3}$

$(2-1)$

	210	201	120	111	102	021	012
P	~~0.384~~ 0.056	0.084	0.024	0.26	0.336	0.096	0.144
V	2/3	2/3	2/3	0	2/3	2/3	2/3

Odds of p... - Suppose we have an event occuring with probability p then the odds of it happening are $p/1-p$.

$$\frac{P(\text{win at H})}{P(\text{lose at H})} = h \cdot \frac{P(\text{win away})}{P(\text{lose away})}$$

$$\frac{0.7}{0.3} = \frac{2 \cdot (p)}{1-p} \qquad 0.7(1-p) = 0.3(2)(p) \qquad p = -0.54$$

- E(V) minimised when teams are equally skilled (eg P(win h)-vi for average and two is home down i.e. $h=1$
- E(V) maxed when teams are unequally skilled