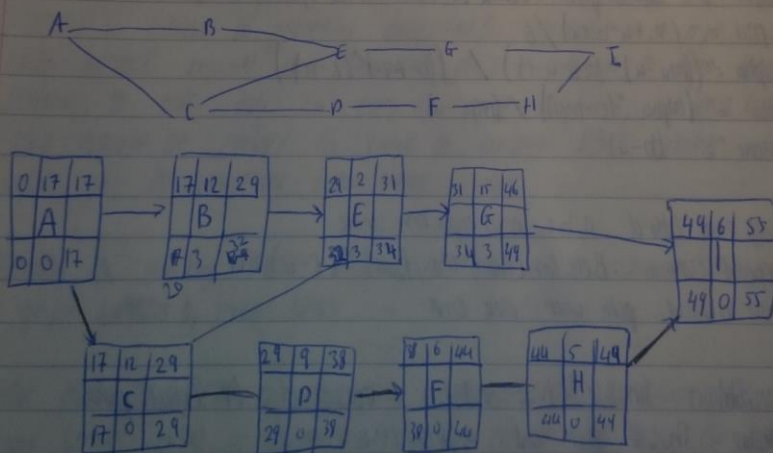


21/04/16 DECISION

### CPM

- Given deterministic activity times, it is possible to describe a project as a network, and find a critical path through the network
- Set up network with dummy nodes so that activities do not finish at the same place
- Do a forward and backward pass
- Should be clear, in practice that activities do not have fixed times known a priori. CPM is appropriate for deterministic situations



A → C → D → F → H → I

Start time | duration | finish time  
node

Start time | Start | latest finish time

Activity path with the 0's

22/04/16

## PERT

- Programme Evaluation and Review Technique
- Key output is time to completion of project
- Time can be uncertain
- Used to estimate time for part of project
- It is a distribution taking, min, most likely, and max
- It is "smoother" than a triangle
- In fact, it is a Beta distribution
- $PERT(a,b,c) = \text{Beta}(\alpha, \beta) * (c-a) + a$

### Parameterisation

- If  $\alpha$  and  $\beta$  are given (where the PERT is not symmetric) by:

$$\mu = (a + 4b + c) / 6$$

$$\alpha = ((\mu - a) * (2b - a - c)) / ((b - \mu) * (c - a))$$

$$\beta = (\alpha * (c - \mu)) / (\mu - a)$$

$$\text{Variance} = (b - a)^2$$

Set up excel table of  $a, b, c, \mu, \alpha, \beta, \text{variance}$

$$\text{Expected time} = \text{Beta.inv}(\text{Rand}, \alpha, \beta) * (c - a) + a$$

Create circular path with Risk Solver

For probabilities: Simulate 1000 or 10000 versions of the critical path

- calculate shortest path and expected time
- Use proportions to calculate expected times

- In reality each of these completion times are random variables that each have some probability distribution of.

- First step in project scheduling is to establish the tasks that need to be completed and the order in which they can be done. We must know which task or pre-requisite of others beginning others. A network diagram of the tasks should be drawn and the critical path for tasks must be identified.

22/04/16 DECISIONS

3

PERT...

The critical path through a project network is one, where, should the duration of any activity ~~increase~~ on the path increase; the total time taken to complete the project would increase. This is based off estimated completion times for each task. Activities that are not on the critical path can be said to have float. This is the time that the activity can be delayed without affecting the overall completion time of the project. Activities on the critical path have 0 float.

Up to this point, no deviation from estimated task times has been considered. In reality, however, tasks will not take exactly as long as they have been predicted to.

Because of this uncertainty must be modelled. As with the next step in the PERT method is to establish the probability distribution for the tasks in the project. Each of these "variables" requires three estimated pieces of information to form their individual distribution; the most likely time taken, the minimum time taken, and maximum time taken. These distributions are combined to create the complete PERT distribution which can consider all the uncertainty without using a project.

Ultimately the resulting PERT distribution can be used to estimate the probabilities of project completion by certain times.

Another name for PERT is dependency diagrams. This is because they easily show which tasks are dependent on one another. It shows the tasks in a logical order and indicates which tasks cannot be started or completed without the start or completion of other tasks.

There are four types of dependencies.

1. Finish Start: Task B cannot be started until task A is complete - e.g. foundation set before concrete. - standard pre-requisite we deal with mostly
2. Finish-Finish: Task B cannot be completed until task A is e.g. roof laid before final
3. Start-Start: B can't start before A does (Sorting material can't be before unloading)
4. Start-Finish: B can't finish until A starts (paving start can't start until road start complete)



Arrows between activities indicate a dependency between tasks outlined in a PERT network chart.

### Spreadsheet

- PERT can be used exclusively for modelling expert estimates.
- Have estimates for min, likely, and max time
- Smaller version of the triangular distribution
- Three defined parameters needed: mu (heavily weighted average of our range of estimates), Alpha, beta

By creating a table of all the activities and their respective 3 estimates, we can apply the above formulae to find mu, alpha and beta.

- Once this is done we have all the information we need to simulate a set of each activity, generating a random selection from the Beta distribution of the activities.
- The BETADIST(probability, alpha, beta, min, max) function in excel can do this.
- By passing RAND() to the probability argument, we can obtain our random from from the distribution.

- Do this in a row for each task; we then have our simulated time for each task.

- Can use the max function to determine which is the critical path from the simulated time.

- Can allow us to select the latest finish time for dependent activities.

- Adding our simulated activity time to the latest completion time gives us the completion time of an activity with dependencies.

- Sequentially summing masses of dependencies is simpler and more efficient way of incorporating them into spreadsheet models.

- PERT assumes a distribution of values for the time taken to perform a task instead of a fixed time.

- CPM activity oriented PERT is task oriented.

### DECISIONS CPM, PERT

- Given deterministic activity times, it is possible to determine a project as a network and find a critical path through the network.

earliest start time | Duration | earliest finish time

latest start time | Slack | latest finish time

Standard CPM of analyzing project assume that the estimated time of a task completion will be the actual time of task completion.

PERT - Programable Evaluation and Review technique attempt to consider uncertainties in our estimates or how long tasks will take. In reality, each of these completion times are random variables that each have some probability distribution.

The first step in project scheduling is to establish the tasks that need to be completed and the order in which they can be done. We must know which tasks are prerequisites for beginning others. A network diagram of a task should be drawn and the critical path for tasks must be identified.

The critical path through a <sup>project</sup> network is one where, should the duration of any activity on the path increase, the total time taken to complete the project would increase. This is based off estimated completion times for each task. Activities that are not on the critical path can be said to have "Slack". This is the time that the activity can be delayed without affecting the overall completion time of the project. Activities on a critical path have 0 slack.

Up to this point, no deviation from estimated task times has been considered. In reality, however, tasks will not take exactly as long as they have been predicted.

Because of this uncertainty must be modelled. Next step in PERT method is to estimate the probability distribution for the time in the project. Even if these 'variables' require three estimated pieces of info to form the individual distributions: the most likely time taken, the minimum time estimated to occur and the maximum time taken. These data are combined to create the complete PERT dist, which can consider all of the uncertainties contained within a project.

Ultimately the resulting PERT dist can be used to estimate the probability of project completion by certain time.

4 types of dependencies:

1. Finish Start - task B cannot be started until task A is completed (Random day - your course)
2. Finish Finish - Task B cannot be completed until task A is completed (your exam - have finished)
3. Start Start - B can't start before A starts - starting materials - material data
4. Start Finish - B can't finish before A starts - previous shift can end until new shift starts

PERT dist used exclusively for modelling expert estimates. We have estimates for min, max and most likely expected at  $PERT(a, b, c)$ . It is a smooth version of the triangular distribution, which also takes these 3 parameters.

There are 3 derived parameters that are also included:

1. The mean  $\mu = (a + 4b + c) / 6$  Heavily weighted average of our range of estimates
2. Alpha  $\alpha = \frac{(\mu - a)(2b - a - c)}{(b - \mu)(c - \mu)}$
3. Beta  $\beta = \alpha \mu \cdot ((c - \mu) / (\mu - a))$
4. Variance  $= (b - a)^2$

By creating a table of all the activities and their 3 respective estimates, we can apply the above formulae to find  $\mu, \alpha, \beta$ .



1/5/2016

## DECISION: PERT

Once this is done, we have all the info needed to simulate a run for each activity, generating a random selection from the beta distribution of the activity. The BetaDist (probability, upper, lower, min, max) function in excel can do this. By putting RAND() into the probability argument, we can obtain our random selection from the distribution.

Doing this for each row we have our simulation time for each task. The MAX() function can help us determine which dependent activities take the longest and therefore activity is on the critical path. If D depends on B, C, being finished, we must find the time at which the last of B & C is finished, this is the max and should be summed to the prior dependent time for A.

Adding our simulated activity time of the activity to this total completion time gives us the completion time of the activity with dependant.

- sequentially summing maxes of dependant is the simplest and most effective way of incorporating them into spreadsheet model.

CPM and PERT are both concerned with project scheduling and management. A primary difference between the two is that CPM does not model uncertainty.

- PERT allows a distribution of (var) for the time taken to perform a task instead of single value estimate.

- CPM is activity oriented, PERT is node oriented (time taken to reach node gives the start time of the next dependent activity).

