5/2/14 3. Math)

Set of vectors $\{\vec{v}_1, \dots, \vec{v}_n\}$ is linearly dependent if there is a relation $k_1\vec{v}_1 + \dots + k_n\vec{v}_n = 0$
with $(k_1 \dots k_n) \neq \vec{0}$
not all k_i are 0.

$$\vec{v}_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 3 \\ -1 \end{pmatrix} \quad \vec{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} \quad \vec{v}_3 = \begin{pmatrix} 0 \\ 1 \\ -2 \\ 0 \\ 3 \end{pmatrix}$$

$$k_1 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 3 \\ -1 \end{pmatrix} + k_2 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} + k_3 \begin{pmatrix} 0 \\ 1 \\ -2 \\ 0 \\ 3 \end{pmatrix} = 0$$

$k_2 = 0$ ✓ all k_i are zero $\Rightarrow$ Linearly independent

$k_3 = 0$ ✓

$k_1 - 2k_3 = 0$

$3k_1 = 0$ ✓

$-k_1 - k_2 + 3k_3 = 0$

27/1/14 Math Tutorial 2 Week 3

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Q1 i $\begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$

ii $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$

iii $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & -1 & -1 & 1 \\ -1 & 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{pmatrix}$

iv $T(x_1, x_2) = (-x_1, x_2) = \begin{pmatrix} -1, & 0 \\ 0, & 1 \end{pmatrix}$

v $T(x_1, x_2, x_3) = \quad x_3, \quad x_1 - x_2, \quad x_1 + 4x_2 + x_3, \quad -2x_2, \quad x_3, x_1$

$$\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{pmatrix} 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 4 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \end{array} \begin{pmatrix} x_1 \\ \cancel{y}_2 \\ \cancel{x_3} \end{pmatrix} =$$

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Applied 2 week 3

i. $\begin{pmatrix} 0 & -1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ $\begin{pmatrix} 0(1) + (-1)(-2) \\ 1(1) + (-2)(-2) \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

ii. $\begin{pmatrix} 0 & 1 & -1 \\ 4 & 0 & -2 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ $\begin{matrix} 0(-1) + 1(1) + -1(0) \\ 4(-1) + 0(1) + -2(0) \end{matrix}$

$= \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

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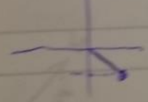
iii. Doesn't work, need three values in the x matrix

Q3 i. reflection of $(2,-1)$ $\begin{pmatrix} -1, & 0 \\ 0, & -1 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 2(-1), -1(0) \\ 2(0), -1(1) \end{pmatrix}$ $= (-2, -1)$

ii. 

$x=1$
$y=0$

$(2,-1)$ $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ α $(2(0) + -1(0),\ 2(0) + -1(1))$ $\cancel{(\ ,\ )}$ $(-1, 0)$

α

iii. $(2,-1)$ $\begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $(2\cos\frac{\pi}{4} + -1\sin\frac{\pi}{4},\ -2\sin\frac{\pi}{4} + -1\cos\frac{\pi}{6})$

$= \left(\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right)$

iv. rotation around z-axis
$(2,-1,-1)$ $\begin{pmatrix} \cos\frac{\pi}{6}, & -\sin\frac{\pi}{6} & 0 \\ \sin\frac{\pi}{6}, & \cos\frac{\pi}{6} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$

$2\cos\frac{\pi}{6} - \sin\frac{\pi}{6} - 1(0),\ -2\sin\frac{\pi}{6} - \cos\frac{\pi}{6},\ -1(1)$

$= \left(-\frac{1+2\sqrt{3}}{2}, -\frac{2-\sqrt{3}}{2}, -1\right)$

1.5/4

α