WEEK 5

28/10/15 DECISION MAKING

Decision making a 'social process' - Lomy Phillips

	Good Summer	Bad Summer	
Sell	3	3	3
Chipper	11	10	10
icecream	20	5	5
	20	10	

maximin - choose highest value of worst outcome - chipper.

minmax regret -

Regret table - how much you lose out on on worse option compared to best option in that row/column

			max regret
Chipper	$\frac{20-11}{}$ = 9	0	-9
icecream	0	=5	-5

Choose icecream.

	GS p=0.75	B=0.25		Probability
Chips	11	10	10.75	p+10
Icecream	20	8	16.25	15p+5

$p+10 = 15p+5$

$p = \frac{5}{14}$

Choice switched here

	θ_1 everything OK 0.8	θ_2 Substandard found 0.2	
		c_{12} 0.5	0.82
d_1 Inspect	c_{11}: 0.9		0.8
d_2 Don't inspect	c_{21}: 1.0	c_{22}: 0.0	
	0.8	0.2	

Utility between 0,1
Choose highest expected utility value

Example 2.

First row $0.6p + 0.8(1-p)$ $\quad 0.6p + 0.8 - 0.8p$

First row: $\quad -0.2p + 0.8$

Second row: $1(p) + 0(1-p) = p$ $\quad$ ~~Switch at p=0.88~~

$p = 0.8 - 0.2p \quad 1.2p = 0.8 \quad p = \frac{0.8}{1.2} = \frac{2}{3}$ switches over decision at that value

B A 9 7 Q
H S d Lowest

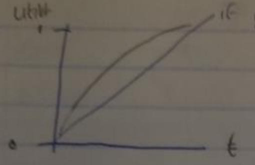
		outcome		
	0.999 975	0.000005	0.000 02	
	Win nothing	Win prize 1	Win Prize 2	
Choice: Play £1	~~0~~	1 000 000 (0.5)	10 000 $\times 0.00002$ 0.2	0.7
Don't play	0	0	0	0

Optimal action → do not play loss of −0.3 if you play. idk 0.48

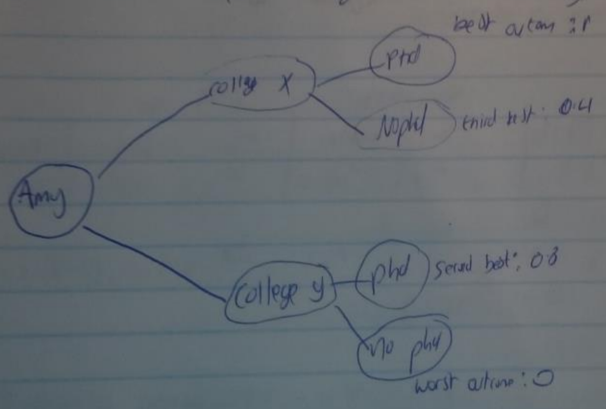
02/10/15

DECISIONS - CALCULATING UTILITY

Utility example - finding point of indifference - when option changes



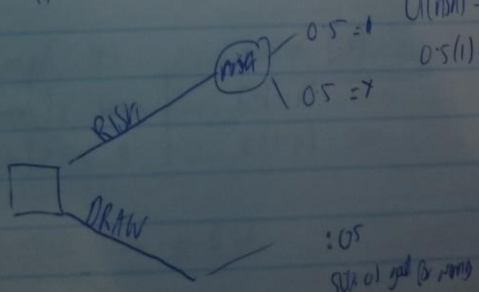
Element of risk aversion
Different answers for same question ⇒ hour/day



$U(X) = P(\text{Grad}|X) + P(\text{Not}|X)\,U(\text{not})$

$U(X)$ " "

College Example

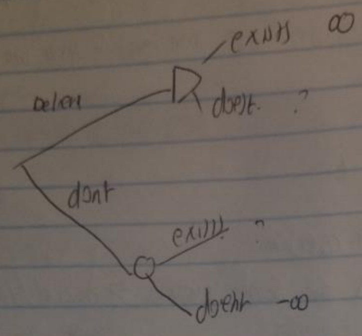


$U(\text{risk}) = U(\text{draw})$

$0.5(1) + 0.5(0) = U(\text{draw}) = X$

$X = 0.5$

Pascal - basically invented decision theory
- Pascal's wager.



03/11/15 DECISIONS

MONEY - Utility and Wealth

	up (P)	down (1-p)	
Invest	c + 1000	~~c+1000~~ c - 1000	6000p + 4000(1-p) = 2000p + 4000 = 5000
don't invest	c	c	

Func = $1-e^{\frac{-x}{r}}$ $\quad 1-e^{\frac{-6000}{2000}}$ $\quad 1-e^{\frac{-4000}{2000}}$

r = 2000 $\quad 1-e^{-5000/2000}$ $\quad 1-e^{-5000/2000}$

c = 5000

If risk neutral → only invest if p was ≥ 0.5

Solve for p, p = 0.5

	up (P)		down (1-P)		
invest	u(6000)	$1-e^{-3}$	u(4000)	$1-e^{-2}$	$p(1-e^{-3}) + (1-e^{-2})(1-p)$
don't	u(5000)	$1-e^{-5/2}$	u(5000)	$1-e^{-5/2}$	$1-e^{-5/2}$

$u(6000) = 1-e^{\frac{-6000}{2000}} = 1-e^{-3}$

Solve for p.

p = 0.372 ≈ 38%

Probability premium = 0.5 - 0.38 = 0.12

Difference between fair bet and your risk utility value

Insurance Example:

	destroyed (P)		survived (1-P)		
Insure	~~(c-h-m)~~ (c-m) p	5000-m	c-m(1-p)	5000-m	5000-m
Don't insure	(c-h) p	0	c(1-p)	5000	5000-5000p = 4950

if c = 5000 and h = 5000 and p = 0.01

	Burn 0.01	Survive 0.99	
insure	$1-e^{-\frac{5000-m}{2000}}$	$1-e^{-\frac{5000-m}{2000}}$	$1-e^{-\frac{(5000-m)}{2000}}$
don't	0	$1-e^{-5000/2000}$	$0.99(1-e^{-5/4}) = 0.2192$

equate them $m = \frac{2000\,(1-e^{-})}{e^{-5/4}}$

Some number close 50

Say $M=75$, highest fair price is 50, then risk premium is 25

Example:

	up $\overset{0.55}{(p)}$	down $\overset{0.45}{(1-p)}$	
invest	$(5000+100)p$ $5000+0.2(i)$	$\overset{5000-0.2(i)}{(5000-100)(1-p)}$	$\overset{600}{5000}(0.55) + \overset{400}{5000}(0.45) = 5000$
don't	$5000p$ 5000	$5000(1-p)$ 5000	5000

$p=0.55$ $\quad i = \frac{1}{1000}$

Utility	up	down	
invest	$1-e^{-\frac{5000(+0.2i)}{1000}}$	$1-e^{-\frac{(5000-0.2i)}{1000}}$	$\overset{0.55}{(1-e^{-\frac{5000+0.2i}{1000}})} + \overset{0.45}{[1-e^{-\frac{(5000-0.2i)}{1000}}]}$
don't	$1-e^{-5000/1000}$	$1-e^{-5000/1000}$	$1-e^{-5000/1000}$

equate ↑ solve for i

Let $i=1000$ $\quad 0.55(1-e^{-5}\cdot e^{-1/5}) + 0.45\,(1-e^{-5}e^{\frac{1}{5}})$

compare against $1-e^{-5}$

Exercise II (million)

	attack 0.05	no attack 0.95	
insure	$(40mil-4mil+S)$ S	42m+S	S+38million
don't insure	40m	40m	40m

↑ need 2m or more

	attack 0.05	no attack 0.95	
insure	$1-e^{-S/25}$	$1-e^{-(42m)/25}$	$0.05(1-e^{-S/m}) + 0.95\,(1-e^{-(40m+S)/25})$
don't	$1-e^{-40/25}$	$1-e^{-40/25}$	$1-e^{-38m}$

S will be more than 2 million

↑ equate and find S

→ 4.509 million to insure

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4 B S = amount of insurance

	Crashes 0·05	Doesn't 0·95	
Insure	$100 - S$	$100 - S$	$100 - S$
Don't Insure	50	100	97·5

$100 - S = 97{\cdot}5$ $S = 2{\cdot}5$ million = fair price

Utility.

	Crashes	Doesn't	Total
Insure	$1 - e^{-(100-S)/50}$	$1 - e^{-(100-S)/50}$	$1 - e^{-(100-S)/50}$
Don't Insure	$1 - e^{-50/50}$	$1 - e^{-100/50}$	$0{\cdot}05(1-e^{-50}) + 0{\cdot}95(1-e^{-100})$
Don't Insure	0.632	0.864	0·853

Solver Solution $1 - e^{-(100-S)/50} = 0{\cdot}853$

$S = 4{\cdot}121336$

i Company willing to pay 4·12136 million to insure

ii. Risk premium $= 4{\cdot}12 - 2{\cdot}5 = 1{\cdot}62$

iii $1 - e^{-247/r} = 0{\cdot}05(1 - \exp^{-200/r}) + 0{\cdot}95(1 - \exp^{-250/r})$

$0 + 247/r = 0{\cdot}05(200/r) + 0{\cdot}95(250/r)$

$247r = 10r + 237{\cdot}5r$

$$1 - e^{-247/r} = 0.05\left(1 - e^{-200/r}\right) + 0.95\left(1 - e^{-250/r}\right)$$

$$1 - e^{-247/r} = 0.05 - 0.05e^{-200/r} + 0.95 - 0.95e^{-250/r}$$

$$-e^{-247/r} = -0.05e^{-200/r} - 0.95e^{-250/r}$$

$$\cancel{-247/r} = \cancel{-0.05(200/r) - 0.95(250/r)}$$

$$\cancel{-10/r \quad -237.5/r}$$

$$\ln(-247) + \ln(r) = 0.05\ln(200) + 0.05\ln(r) + 0.45(-250) + 0.45(\ln r)$$

$$-0.009154 = 0.05\ln(r) + 0.95(\ln r) - \ln(r)$$

$$= 0$$

Plug in value 132 → 0.846 difference = 0.00000574

133 → 0.00005741

134 → 0.0000411478

$$e^{-247/r} = 0.05e^{-200/r} + 0.95e^{-250/r}$$

$$\cancel{1} - e^{-247/r} = 0.05\left(\cancel{1} - e^{-200/r}\right) + 0.95\left(\cancel{1} - e^{-250/r}\right)$$

$$e^{-247/r} = 0.05\,e^{-200/r} + 0.95\,e^{-250/r}$$

16/11/15 DECISIONS

3(i)

	~~[illegible]~~ nothing 0.84	Phone 0.07	Laptop 0.03	P+L 0.02	Everything 0.01	Other 0	
No insurance	1000	700	500	300	0	?	960
AA							
BB							
CC							

	Nothing 0.87	Phone 0.07	Laptop 0.03	Phone+Laptop 0.02	Everything 0.01	Expected
No insurance.	3000	2700	2500	2200	2000	[illegible]
AA	2950	2850	2850	2850	2850	[illegible]
BB	~~[illegible]~~ 2930	2780	2780	2780	2780	[illegible]
CC	2980	~~[illegible]~~ ~~[illegible]~~ 2680	~~[illegible]~~ 2930	~~[illegible]~~ ~~[illegible]~~ 2630	~~[illegible]~~ ~~[illegible]~~ 2630	[illegible]

no insur	~~[illegible]~~ 2938 expected value
AA	~~[illegible]~~ 2937 ← correct
BB	~~[illegible]~~ 2923.5
CC	~~[illegible]~~ Changed 2926

IF risk neutral ⇒ choose A

Correct.

	Nothing 0.87	Phone 0.07	Laptop 0.03	P+L 0.02	Everything 0.01	
No insure	$1-e^{-3}$	$1-e^{-2.7}$	$1-e^{-2.5}$	$1-e^{-2.2}$	$1-e^{-2}$	0.9459
AA	$1-e^{-2.95}$	$1-e^{-2.85}$	$1-e^{-2.85}$	$1-e^{-2.85}$	$1-e^{-2.85}$	0.9469
BB	$1-e^{-2.93}$	$1-e^{2.78}$	$1-e^{2.7}$	$1-e^{-2.78}$	$1-e^{-2.78}$	0.9462
CC	$1-e^{-2.98}$	$1-e^{-2.68}$	$1-e^{-2.93}$	$1-e^{-2.6[illegible]}$	$1-e^{-2.[illegible]3}$	0.9456

Important to take total amount into consideration when using a utility function!!

Dominant ⇒ better under every circumstance

/11/15 DECISIONS

1. Example Question

	Success 0.1	Fail 0.9		
Invest	1200 000	0	120 000	invest ✓
Don't	100 00	100 00	100 000	

If risk neutral will invest.

Utility function = $1-e^{\frac{-x}{1000000}}$

	Success 0.1		Fail 0.9		
Invest	$1-e^{-1.2}$	0.698	0	0	0.698(0.1) + 0(0) = 0.0698
Don't	$1-e^{-0.1}$	0.095	$1-e^{-0.1}$	0.095	0.095(0.1) + 0.095(0.9) = 0.095

Should ~~[illegible]~~ not invest,

She is risk adverse ⇒ decision says not to invest even though normal expected value says to invest.

2

	Destroyed 0.05	Safe 0.95	
Insure	$40-40+y$	$40+y$	$0.05y+0.95(40+y)$
Don't	40m	40m	40m

$0.05y + 38 + 0.95y = 40_m$

$y = 2$ million anything over → make profit.

	Destroyed 0.05	Safe 0.95	
Insure	$1-0.5e^{-y/20}$	$1-0.5e^{-40-y/20}$	$0.05(1-0.5e^{-y/20})+0.95(1-0.5e^{\frac{-40-y}{20}})$
Don't	$1-0.5e^{-40/20}$	$1-0.5e^{-40/20}$	$1-0.05e^{-2}$

$\cancel{0.05} - 0.025e^{-y/20} + \cancel{0.95} - 0.025e^{\frac{-40-y}{20}} = \cancel{1} - 0.5e^{-2}$

$+0.025e^{-y/20} \quad + 0.025e^{-40-y/20} = \cancel{+0.5e^{-2}}$

$0.025(-y/20) \quad + \overset{0.475}{\cancel{0.025}}\left(\frac{-40-y}{20}\right) = 0.06766$

$-0.00125y \quad -\overset{0.475}{\cancel{0.00125}}y - 0.05 = 0.06767$

$-0.0025y \quad = 0.7267$

$0.47625 \quad = 2.09$

$y = 2.09$

Answer $y = 5.45$

If they take 20m risk value is 2

	Destroyed	Safe
Insure	$20+x$	$40+x$
Don't	40	40

Value = 1.6 million

Cheaper to use 2 different companies at 2m each