9/05/16 ALSM 2: EXAM NOTES: WEIBULL DISTRIBUTION

1

Show it is a Distribution

y positive θ positive, exp is positive $\rightarrow$ positive

$\int_0^\infty \lambda \theta y^{\lambda-1} \exp[-\theta y^\lambda]\ dy$

Probability of Failure $= 1 - \exp[-\theta y^\lambda]$ $\qquad 1-\exp[-\theta y^\lambda]$

Probability of Survival $=$ ~~exp~~ $1 - F(y) = \exp[-\theta y^\lambda]$

Hazard function $h(y) = \frac{f(y \mid \lambda)}{S(y)} = \lambda \theta y^{\lambda-1}$ (accidental failure (rate), function of y)

Exponential case $\lambda = 1$ $h(y) = \theta \rightarrow$ not a function of y

$\hookrightarrow$ independent of age "chance of survival same at 100 as at 100"

$\hookrightarrow$ lack of memory $\Rightarrow$ y not important

Show it is a member of exponential family of distributions.

$= \lambda \theta y^{\lambda-1} \exp[-\theta y^\lambda]$

$= \cancel{\exp [\log \lambda \theta y^{\lambda-1}}$

$= \exp[\log(\lambda \theta y^{\lambda-1}) + (-\theta y^\lambda)]$

$= \exp[\log(y)^{\lambda-1} + \log(\lambda\theta) - \theta y^\lambda]$

$= \exp\ \lambda-1 \log(y) + \log \lambda\theta - \theta y^\lambda$

$b(\lambda\ a(y)\quad c(\lambda)\quad d(y)$

For exp distribution with $F(y) = 1 - \exp(-\lambda y)$ the median survival time is $\log(2)/\theta$

$\int_0^\infty \lambda \theta y^\lambda \exp[-\theta y^\lambda]\, dy$

$\int_0^\infty \lambda \mu \exp[-\mu]$ (μ

$\cancel{\mu = \theta y^\lambda}$

$p(y|\lambda\theta) = \lambda \theta y^{\lambda-1} \exp[-\theta y^\lambda]$

$u = \theta y^\lambda \Leftrightarrow y = \left(\frac{\mu}{\theta}\right)^{1/\lambda}$

$\frac{u}{\theta} = y^\lambda \quad \Rightarrow \left(\frac{u}{\theta}\right)^{1/\lambda} = y$

$\frac{du}{dy} = \lambda \theta y^{\lambda-1} \qquad du = \lambda \theta y^{\lambda-1} dy$

$dy = \frac{du}{\lambda \theta y^{\lambda-1}}$

$dy = \frac{du}{\lambda\theta \left(\left(\frac{\mu}{\theta}\right)^{1/\lambda}\right)^{\lambda-1}}$

$= \lambda\theta \left(\frac{u}{\theta}\right)^{\frac{\lambda-1}{\lambda}}$

$\mu = \theta y^\lambda \quad y = \left(\frac{u}{\theta}\right)^{1/\lambda}$

Suby in

$du = \theta\lambda y^{\lambda-1} dy \quad \Rightarrow dy = \frac{du}{\theta\lambda y^{\lambda-1}} = \frac{du}{\theta\lambda\left(\left(\frac{u}{\theta}\right)^{1/\lambda}\right)^{\lambda-1}} = \frac{du}{\theta\lambda\left(\frac{\mu}{\theta}\right)^{\frac{\lambda-1}{\lambda}}}$

$= \frac{du}{\theta\lambda} \frac{\mu}{\theta}^{(\frac{1}{\lambda}-1)}$

$\theta\lambda \frac{\theta^{\frac{\lambda-1}{\lambda}}}{\mu^{\frac{\lambda-1}{\lambda}}}$

$\int_0^\infty \frac{\lambda \theta y^\lambda \exp[-\theta y^\lambda] dy}{\mu} = \int_0^\infty \lambda\mu \exp[-\mu] \left[\cancel{\theta\lambda}\left(\frac{\mu}{\theta}\right)^{\frac{1-\lambda}{\lambda}}\right] \frac{du}{\theta\lambda}$

$\int_0^\infty (1/\theta)(1/\theta)^{1/\lambda - 1}\, u^{[(1/\lambda - 1) + 1]} \exp[-\mu]\, du$

$= (1/\theta)^{1/\lambda}\, \Gamma\left(\frac{1}{\lambda}+1\right)$