12/12 Week 11 Dynamic Programming 1

Knapsack problem

Stage 1.

weight available	no of A	weight	Profit
6	3	6	195
5	2	4	130
4	2	4	130
3	1	2	65
2	1	2	65
1	0	0	0
0	0	0	0

Stage 2.

weight available	no of B	weight B	Profit	Avail weight	Best Sol' stage 1	Total pro.
6.	2	3	160	0	0	160
	0	0	0	6	195	195.
5.	1	3	80	2	65	145
	0	0	0	5	130	130
4.	1	3	80	1	0	80
	0	0	0	4	130	130
3.	1	3	80	0	0	80
	0	0	0	3	65	65
2	0	0	0	2	65	65
	0	0	0	2	65	65
1	0	0	0	1	0	0
0	0	0	0	0	0	

2.

Stage 2:

weight	possible [B]	weight [B]	profit	Add weight (Stage 1)	profit (S1)	Total Pft
6	2	6	160	0	0	160
	1	3	80	3	65	145
	0	0	0	6	195	195
5	1	3	80	2	65	145
	0	0	0	5	130	130
4	1	3	80	1	0	80
	0	0	0	4	130	130
3	1	3	80	1	0	80
	0	0	0	3	65	65
2	0	0	0	2	65	65
1	0	0	0	1	0	0
0	0	0	0	0	0	0

Stage 3:

weight	possible number [C]	weight [C]	Profit	Add weight	Best Profit 2	Profit
6	6	6	180	0	0	180
	5	5	150	1	0	150
	4	4	120	2	65	185
	3	3	90	3	80	170
	2	2	60	4	130	190
	1	1	30	5	145	175
	0	0	0	6	195	195

Best profit is 0 C's 0
0 B's 0
3 A's 195

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Dynamic Programming (1)

reps per region. Polar, temperate, equator

Stage 1 - Polar

Number of reps available	no. allocated to region	profit
4	4	40
3	3	33
2	2	22
1	1	10
0	0	0

STAGE 2 Polar and Temperate:

Number of reps available	no. allocated Stage Temp	profit	Available for Stage 1	profit	Total profit
4	4	46	0	0	46
	3	36	1	10	46
	2	24	2	22	46
	1	9	3	33	42
	0	0	4	40	40
3	3	36	0	0	36
	2	24	1	10	34
	1	9	2	22	31
	0	0	3	33	33
2	2	24	0	0	24
	1	9	1	10	19
	0	0	2	22	22
1	1	9	0	0	9
	0	0	1	10	10
0	0	0	0	0	0

Stage 3 Equatorial

Solid rep available	number rep to Equ	profit	rep1 available s2	profit stage 2	total profit
4	4	38	0	0	38
	3	30	1	10	40
	2	20	2	24	44
	1	13	3	36	49
	0	0	4	46	46

max profit 49
1 equatorial
3 tempered
0 polar

10/12/12 Management Scien Dynamic Programming 2

Resource Allocation Problem.

Decision Variables: Region & North, South, West

D_r - number of Sales rep to allocate to each region $r \in$ regions

Objective Function:

profit: $F(D_{North}, D_{South}, D_{West})$

number of rep	north	south	sales
0	0	0	2
1	7	9	6
2	12		10
3	20	18 15	15

Profit table

$\mathbb{N}$ x region → $\mathbb{N}$

from natural numbers

partial function

to natural numbers

$\mathbb{N}$ = set of all natural numbers

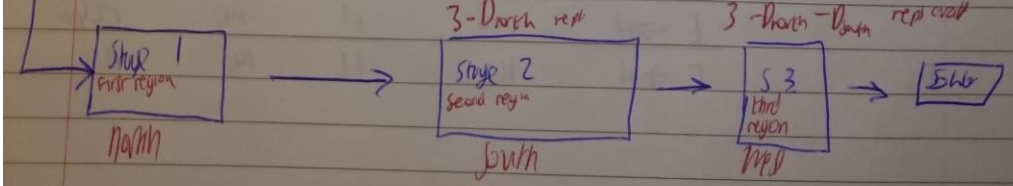
$|A \times B|$ ← multiplication of sets

$|A|$ ... multiplication of numbers

CONSTRAINTS: $D_{North} + D_{South} + D_{West} \leq 3$

$D_r \in \mathbb{N}$

3 reps (number available)



Shortest Path Problem:

Stages: Number of steps away from Frankfurt
Stage 1 = {G, H} Stage 2 = {D, E, F} Stage 3 = {A, B, C} Stage 4 = Frankfurt

State: a location in the stage eg. If we are at stage 3 a state will be A, B, C

Decision at each stage: Route to take from current stage (i.e. location) to previous stage

Objective function: Distance of chosen route from current state to previous stage

Stage 1 - 1 Step from Frankfurt

State (location of each)	Decision (route to take)	Objective function (distance)
G	G → Frankfurt	130
H	H → Frankfurt	140

Stage 2 - 2 Steps from Frankfurt

State (location of each)	Decision (route to take)	Objective function (distance)	State of Stage 1	Dist. S1	Total Distance
D	D → G	150	G	130	280
E	E → G	120	G	130	250
	E → H	100	H	140	240
F	F → H	110	H	140	250

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Available Crates	Number of Route A	Profit
4	4	525
3	3	500
2	2	450
1	1	350
0	0	0

Available crates	Number Route B	Total	Profit	Available	Number Stage 1	profit	Total profit
4	4	4	~~525~~ 650	0	0	0	~~650~~
	3	3	450	1	~~3~~ 1	350	800
	2	2	250	2	2	450	700
	1	1	100	3	3	500	600
	0	0	0	4	4	650	650
3	3	3	450	0	0	0	450
	2	2	250	1	1	350	600
	1	1	100	2	2	450	550
	0	0	0	3	3	500	500
2	2	2	200	0	0	0	250
	1	1	100	1	1	350	450
	0	0	0	2	2	450	450
1	1	1	100	0	0	0	100
	0	0	0	1	1	350	350
0	0	0	0	0	0	0	0

Available Crates	Number C.	total C.	Profit.	Available	Stage 2	prof	total profit
4	4	4	600	0	0	0	600
	3	3	475	1	1	350	825
	2	2	300	2	2	450	750
	1	1	225	3	3	600	825
	0	0	0	4	4	800	800

3 route C + 1 route B or 1 route C + 2 route B + 1 route A

3.

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Stage 3 3 Steps from Grandfurt.

State location of truck	Decision Route to take	Objective function Dollars for S2	State of Stage 2	Best Dollars Stage 2	Total Dollars
A	A → D	100	D	280	380
B	B → D	70	D	280	350
B	B → E	150	E	240	390
B	B → F	100	F	250	350
C	C → E	160	E	240	400
C	C → F	140	F	250	390

Stage 4 : 4 Steps from Frankfurt (Paris)

State location	decision Route to use	Objective function Dollars from S3	State of S3	Best Dollars Stage 3	Total Dollars
Paris	P → A	100	A	380	480
	P → B	110	B	350	460
	P → C	80	C	390	470

G → E → B → Rail

G → D → B → Paris

Dynamic Programming Tutorial

Available weight	number of items	weight	profit
7	1	4	10
6	1	4	10
5	1	4	10
4	1	4	10
3	0	0	0
2	0	0	0
1	0	0	0
0	0	0	0

Stage 2

Available weight	number of items	weight	profit	Available weight	profit of stage 1	Total Profit
7	3	6	13.5	1	0	13.5
	2	4	9	3	0	9
	1	2	4.5	5	10	14.5
	0	0	0	7	10	10
6	3	6	13.5	0	0	13.5
	2	4	9	2	0	9
	1	2	4.5	4	10	14.5
	0	0	0	6	10	10
5	2	4	9	1	0	9
	1	2	4.5	3	0	4.5
	0	0	0	5	10	10
4	2	4	9	0	0	9
	1	2	4.5	2	0	4.5
	0	0	0	2	0	0
3	1	2	4.5	1	0	4.5
	0	0	0	3	0	0
2	1	2	4.5	0	0	4.5
	0	0	0	2	0	0
1	0	0	0	1	0	0
0	0	0	0	0	0	0

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Stage 3	Available Weight	Number ID	weight	Profit	Actual weight	profit (2)	Total profit
	7	2	6	14	1	0	14
		1	3	7	4	9	16
		0	0	0	7	14.5	14.5
	6	2	6	14	0	0	14
		1	3	7	3	4.5	11.5
		0	0	0	6	14.5	14.5
	5	1	3	7	2	4.5	11.5
		0	0	0	5	10	10
	4	1	3	7	1	0	7
		0	0	0	4	9	9
	3	1	3	7	0	0	7
		0	0	0	3	4.5	4.5
	2	0	0	0	2	4.5	4.5
	1	0	0	0	1	0	0
	0	0	0	0	0	0	0

Stage 2	Available weight	Number ID	Weight	Profit	Actual weight	profit (3)	Total Profit
	7	7	7	14	0	0	14
		6	6	12	1	0	12
		5	5	10	2	4.5	14.5
		4	4	8	3	7	15
		3	3	6	4	9	15
		2	2	4	5	11.5	15.5
		1	1	2	6	14.5	16.5
		0	0	0	7	16	16

Optimal Decision

Item ID	Item ID	Item ID	Item ID
1	0	2	4