24/9/13 Management Science.

Linear programming Problem

x = bowls
y = mugs

p = 4x + 5y
x+y ≤ 40
4x + 3y ≤ 120
y ≤ 10
x and y ≥ 0

xy = 0 x = 40
y = 0 x = 30

Draw Graph →

- A binding constraint is one which passes through the optimum solution (Scarce resource)

- Non binding, does not pass through optimum solution

25/9/13 SIMPLEX METHOD

All constraints + equality with non neg h

Assignment.
Monday 3pm Computer Science Monday 7th oct

Solve LP graphically:
- feasible solution area
- optimum sol
- ranges for objective function co-efficient
- shadow prices
- Right hand side ranges

Maximise z = 2x + y
x ≤ 4
x - y ≤ 0
x + y ≤ 10
y ≥ 0
x ≥ 0

Minimise z = 8x + 6y
x + y ≥ 20
x ≥ 5
x ≤ 12
y ≥ 6
y ≤ 10
x, y ≥ 0

8/10/13 Management Science

T R P

$R < 50$ *

$T > 10$

$100000t + 18000R + 40000P$ ✓

$2000t + 300R + 600P \leq 18200$ budget ✓

$T \leq 10$ ✓

$R \leq 20$ ✓

$P \leq 10$ ✓

$-0.5T + 0.5R - 0.5R \leq 0$

$0.4T - 0.1R - 0.1P \geq 0$

Min $z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$

$x_1 + x_6 \geq 5$

$x_1 + x_2 \geq 6$

$x_2 + x_3 \geq 10$

$x_3 + x_4 \geq 7$ all variables ≥ 0 and integer valued

$x_4 + x_5 \geq 4$

$x_5 + x_6 \geq 6$

24/10/13 Mang Science

1.

	W	S	Amer	Sur
Dublin	180^{9}	20^{11} -1	+1 ^{8}	200
Cork	^{6}	120^{15} +1	30^{10} -1	150
Galway	^{7}	^{6}	100^{6}	100
D	180	140	130	

$D^{ublin} \rightarrow A^{mer}$ adding 1 unit in A cost 8.

cost	savings
8	10
15	11
23	21 = +2 not save money

2.

	W	S	A	Sur
D	180^{9} -1	20^{11} +1	^{8}	200
C	+1 ^{6}	120^{15} -1	30^{10}	150
G	^{7}	^{6}	100^{6}	100
D	180	140	130	

$C \rightarrow W$ cost. saving

	cost	saving
6	6	9
	11	15
	17	24 = Saving 7

3.

	W	S	A	Sur
D	180^{9} -1	20^{11} +1	^{8}	200
C	^{6}	120^{15} -1	30^{10} +1	150
G	+1 ^{7}	^{6}	100^{6} -1	100
D	180	140	130	

$G \rightarrow W$.

cost	saving
7	9
11	15
10	6
28	30 Saving 2

4.

	W	S	A	Sr
D	180 ^{9}	20 ^{11}	^{8}	200
C	^{6}	120 − 1 ^{15}	30 + 1 ^{10}	150
G	^{7}	+1 ^{6}	100 − 1 ^{6}	100
D	180	140	130	

G → S cost 6 + 10 = 16; saving 6 + 11 = 21 ∴ saving of 5.

Choose best saving C → W with 7.

	W	S	A	Sr
D	180 − 120 ^{9}	20 + 120 ^{11}	^{8}	200
C	+120 ^{6}	120 − 120 ^{15}	30 ^{10}	150
G	^{7}	^{0}	100 ^{6}	100
D	180	140	130	

0 cost change for cell gives several optimum solutions

1.

23/10/13 Management Science DAVID WEITBRECHT 123006UU

1. x_1 = unfinished table
x_2 = finished table
x_3 = unfinished chair
x_4 = finished chair

$Z_{max} = 70x_1 + 140x_2 + 60x_3 + 110x_4$
wood $= 40x_1 + 40x_2 + 30x_3 + 30x_4 \leq 40000$
time $= 2x_1 + 5x_2 + 2x_3 + 4x_4 \leq 6000$

Max profit = 146.67
$x_4 = 1333.33$, $x_1, x_2, x_3 = 0$

Shadow price for constraint 1: 1/30
constraint 2: 0

Right hand side range		current RHS	Min RHS	Max RHS
	constraint 1:	40000	0	45000
	constraint 2:	6000	5333.33	∞

Changes in the objective function coefficients:

Variable	current cost	min cost	max cost
x_1	70	$-\infty$	440/3
x_2	140	$-\infty$	440/3
x_3	60	$-\infty$	100
x_4	110	105	$+\infty$

DW

24/10/13

2 Product 1 $= x_1$
2 $= x_2$
3 $= x_3$

$$\text{Max } Z = 50x_1 + 20x_2 + 20x_3$$

$$x_3 = 20$$

$$9x_1 + 3x_2 + 5x_3 \leq 500$$
$$5x_1 + 4x_2 + 0x_3 \leq 350$$
$$5x_1 + 0x_2 + 2x_3 \leq 150$$

Max profit = 2700

$x_1 = 22$ $x_2 = 60$ $x_3 = 20$

$50(22) + 20(60) + 20(20) = 2700$

Shadow price Constraint 1 = 0
Constraint 2 = 0
Constraint 3 = -5
Constraint 4 = -5

Right hand side range

constraint	current RHS	min RHS	Max RHS
row1	20	0	800/29
row2	500	478	+∞
row3	350	110	1138/3
row4	150	40	3590/21

Changes in objective function coeff

Variable	current coeff	min coeff	max coeff
x_1	50	25	+∞
x_2	20	0	40
x_3	20	-∞	+∞

25/10

/13. Management Science Ass 2 DAVID WEITBRECHT R3006W

3. Plant 1 = X_1 $\quad$ Min z = $1500x_1 + 1000x_2 + 900x_3 + 700x_4$

2 = X_2 $\quad$ labour $2x_1 + 3x_2 + 4x_3 + 5x_4 \leq 3300$

3 = X_3 $\quad$ m/unit $3x_1 + 4x_2 + 5x_3 + 6x_4 \leq 4000$

4 = X_4 $\quad$ $X_3 \geq 400$

units $X_1 + X_2 + X_3 + X_4 = 1000$

Solution Min cost = 1160000

$X_1 = 400$ $\quad X_2 = 200$ $\quad X_3 = 400$ $\quad X_4 = 0$

$1500(400) + 1000(200) + 900(400) + 700(0) = 1,160,000$

Shadow price: constraint 1 : 0

2 : 500

3 : 400

Right hand side ranges:

	current RHS	Min RHS	max RHS
row 1	3300	3000	+∞
row 2	4000	3800	4300
row 3	1000	900	1066.7
row 4	400	0	500

Changes in objective function coefficient

variable	current coef	Min coef	Max coef
X_1	1500	1150	+∞
X_2	1000	-∞	1200
X_3	900	500	+∞
X_4	700	0	+∞

DW

4 Max = $3x_1 + 5x_2$

$x_1 \leq 4$

$2x_2 \leq 12$

$3x_1 + 2x_2 \leq 18$

$x_1 \geq 0$

$x_2 \geq 0$

Solution: 36

$x_1 = 2$ $x_2 = 6$

$3(2) + 5(6) = 36$

Shadow price: constraint 1: 0

2: -1.5

3: -1

4: 0

5: 0

Right hand side ranges:

row	current RHS	min RHS	Max RHS
1	4	2	$+\infty$
2	12	6	18
3	18	12	24
4	0	$-\infty$	2
5	0	$-\infty$	6

Changes in objective function coefficients:

variable	current	min	max
x_1	3	0	7.5
x_2	5	2	$+\infty$

30/10/13 Management Science.

		W ($v_1=9$)	S ($v_2=11$)	A ($v_3=6$)	D demand
	$u_1=0$ D	180 [9] −120	20 [11] +120	[8]	200
supply	$u_2=4$ C	+120 [6]	120 [15] −120=0	30 [10]	150
	$u_3=0$ G	[7]	[6]	100 [10]	100
		180	140	120.	

$U_1+V_1 = C_{11} = 9 = 0+V \quad V_1=9$

$U_1+V_2 = 11 \Rightarrow 0+V_2=11 \quad V_2=11$

$U_2+V_2 = 15 \Rightarrow U_2+11=15 \quad U_2=4$

$U_2+V_3 = 10 \Rightarrow 4+V_3=10 \quad V_3=6$

$U_3+V_3 = 6 \quad U_3+6=6 \quad U_3=0$

C_{13} (CI) $= 8-0-6=2$

C_{21} (CI) $= \cancel{4+9}\ 6-4-9 = -7$ $\boxed{\text{do first}}$

C_{31} (CI) $= 7-0-9 = -2$

C_{32} (CI) $= 6-0-11 = -5$

	W ($v_1=9$)	S ($v_2=11$)	A ($v_3=13$)	D demand
D ($u_1=0$)	60 [9] −30	140 [11]	+30 [8]	200
C ($u_2=-3$)	120 [6] +30	[15]	30−30 [10]	150
G ($u_3=-7$)	[7]	[6]	[10]	100
	180	140	130	

$V_1=9 \quad U_2+V_1=6 \quad U_2+9 \quad U_2=-3$

$V_2=11, \quad V_2+V_3=10 \Rightarrow -3+V_3 \quad V_3=13$

$U_3+V_3=6 \Rightarrow U_3+13 \Rightarrow U_3=-7$

$C_{13}\ (C1) = 8 - 0 - 13 = -5$ oo!

$C_{22}\ (C1) = 15 - (-3) - 11 = 7$

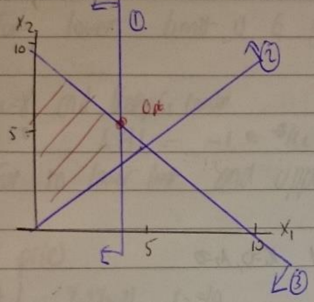
$C_{31}\ (C1) = 7 - \cancel{6} - 9 = 15$

$C_{32}\ (C1) = 6 - -7 - 11 = 2$

$TC = 30 \times 9 + 30 \times 8 + 150 \times 6 + 140 \times 11 + 100 \times 6$

Assignment

Q1. - constraint 1, give $x_1=4, x_2=0$
- Constraint 2 gives $x_1 \leq x_2$ and $x_2 \geq x_1$
- Constraint 3 gives $x_1=0, x_2=10$, and $x_1=10, x_2=0$
- All less than constraints but be careful



Value at optimum = $x_1=4$, $x_2=6$ $z=14$

Objective function coef.

Slope of ob func $z=2x_1+x_2$

$\Rightarrow x^2 = -2x_1 + z$ giving slope as -2

- Slope of constraint 1 = 0.
- Slope of constraint 3: $x_1+x_2 \leq 10$, $x_2 = x_1 - 10$, slope $= 1$
- Slope of O.F. is between $0 \leq -c_1/c_2 < -1$

- let $c_2 = 1$ and keep c_1 constant
- LHS = 0 RHS = $-c_1 \leq -1 \rightarrow 2$

Shadow price

let rhs of constraint 1 = 5

$x_1 = 5$

$x_1 + x_2 = 10$

Solving gives $x_1 = 5$ $x_2 = 5$ $z = 15$ therefore shadow price is 1.

Constraint 2 is nonbinding, shadow price = 0.

2.
Let rhs of constraint 3 = 11

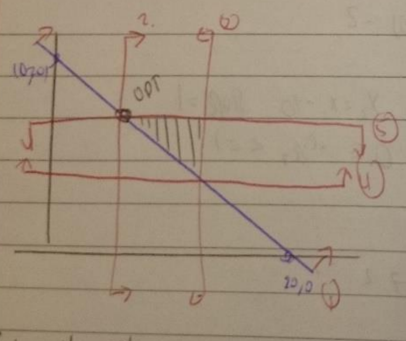
- $x_2 = 4$
- $x_1 + x_2 = 1$

- Solving gives $x_1 = 4$ and $x_2 = 7$ $z = 15$ therefore shadow price = 1.

		Shadow price
Constraint	1	1
Constraint	2	0
Constraint	3	1

Q2 Minimise problem

- Constraint one gives $x_1 = 20$ $x_2 = 0$ and $x_1 = 0, x_2 = 20$
- C2 gives $x_1 = 5$
- C3 gives $x_1 = 12$
- C4 gives $x_2 = 6$
- C5 gives $x_2 = 10$
- constraint 1,2 and 4 ≥ 0 · 3,5 ≤ 0



Value at optimal $x_1 = 10, x_2 = 10$ $z = 140$

Objective function coefficient

- Slope of obj function $z = 8x_1 + 6x_2$
  $\Rightarrow x_2 \; -8/6\, x_1 \; -z/6$ slope $= -4/3$
- Slope constraint 1 = 1
- Slope C5 : $x_2 = 10$, slope = 0 slope then $-1 \leq -\frac{c_1}{c_2} \leq 0$

3 MangSci Assignment one solution

- Slope between $-1 \leq \frac{-c_1}{c_2} \leq 0$
- let $c_2 = 6$ and keep c_1 constant

- RHS = 0 LHS = $-1 \leq -c_1/6 \Rightarrow c_1 \geq 6$
Therefore lower limit is 6 and no upper limit.

- let $c_1 = 8$ and keep c_2 const.
- RHS = 0 LHS = $-1 \leq -8/c_2 \Rightarrow c_2 \leq 8$
- Therefore no lower limit and upper limit is 8.

Shadow prices
- Constraint 1 $x_1 + x_2 = 21$ $x_2 = 10$
Solving gives $x_1 = 11$ and $x_2 = 10$ $z = 148$ Shadow price = -8. = 8 because min problem

Constraint 2,3,4 non binding.

- Constraint 5 $x_1 + x_2 = 20$ $x_2 = 1$
- Solving gives $x_1 = 9$ and $x_2 = 11$ $z = 138$ and shadow price = 2.

RHR
C1 can increase until it meets point of intersection of constraint 3 and 5 until, which is when $x_1 = 12$ $x_2 = 10$, putting these in constraint 1 gives 22, this is upper limit
- C1 can decrease until it meets the point of intersection of constraints 2 and 5 which is when $x_1 = 5$ and $x_2 = 10$, sub in equal 15 meaning lower limit is 15.

$15 \leq c_1 \leq 22$

EOQ HER NOTES

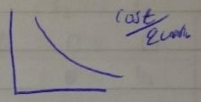
Costs Associated with Stock

- Ordering costs, Labour, overheads (telephone post), transport
- holding cost, → capital tied up, storage, wages of staff in warehouse, deterioration etc
- Shortage costs – loss of customer goodwill, emergency orders

Ordering Costs

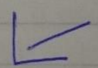
- Increase as number of orders increase
- Smaller Q ordered → more orders made → cost increases
- Order cost not linear as the order costs are proportional to the reciprocal of the order size

order cost



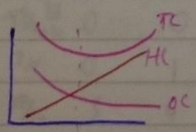
Holding costs

- Vary according to amount ordered
- If large amounts are ordered infrequently, holding costs high
- Assume linear relationship between order q and holding cost



Minimum total costs

Minimum cost will be when the ordering and holding costs are equal. This is basis of EOQ model



carrying cost $C_c \frac{Q}{2} = C_o \frac{D}{Q}$

$C_c Q = 2C_o \frac{D}{Q}$

$C_c Q^2 = 2C_o D$

carrying order cost $\frac{D}{Q}$

$Q^2 = \frac{2C_oD}{C_c}$

$Q = \sqrt{\frac{2C_oD}{C}}$ or $C_c = K_c \times P$ / % × price

Assumptions

- Demand constant rate
- Demand known - certain
- entire order Q received at once
- new deliveries arrive exactly at stock = 0
- lead time zero (place and receive order at same time)
- per unit holding cost and ordering cost are constant