2013 Maths 2 exam paper

Q1 i. Parametric eqⁿ of line spanned by $(-1, 2, 4, 0)$

$t\vec{v} =$ $x = -t$
$y = 2t$
$z = 4t$
$w = 0$

ii. Eqⁿ for plane spanned by $(-1, 2, 3), (1, 1, 0)$

Eqⁿ for plane $Ax + By + Cz = 0$

$\begin{cases} -A + 2B + 3C = 0 \\ A + B + 0C = 0 \end{cases}$ ~~$3B = 0 \Rightarrow A = 0$~~

$3B + 3C = 0 \quad B = -C \quad A = C$

$\Rightarrow \quad Cx - Cy + Cz = 0$

$x - y + z = 0$ final answer

Q2 Rank and nullity

$\begin{pmatrix} 6 & -2 & -4 \\ -3 & 1 & 2 \end{pmatrix}$ $\begin{pmatrix} 6 & -2 & -4 \\ -3 & 1 & 2 \\ 1 & 0 & -1 \end{pmatrix}$

Rank = # of independent rows and columns

$\begin{pmatrix} 1 & 0 & -1 \\ 6 & -2 & -4 \\ -3 & 1 & 2 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 & -1 \\ 6 & -2 & -4 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow$ Rank $= 2$

Nullity = # of columns - rank $3 - 2 = 1$

Q3 Basis and dimension for row, column, null space

$\begin{pmatrix} 0 & 2 & -4 & 2 \\ 0 & -3 & 6 & -3 \end{pmatrix}$ $\begin{pmatrix} 0 & 1 \\ 0 & 2 \\ -2 & 1 \end{pmatrix}$

$\rightarrow \begin{pmatrix} 0 & 2 & -4 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ row space basis: $(0, 1, -2, 1)$

Dimension $= 1$ = row space

2

column space take leading 1 colum and take from original vol

column base = (b) colum = $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$

Null Space $Ax = 0$.

$$\begin{pmatrix} 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = 0$$

column-rank $4-3$ = dimension is 3.

$x_2 - 2x_3 + x_4 = 0$

x_1 $\quad x_2 = 2x_3 - x_4$

$x_1 = s$, $x_3 = w$ $x_4 = t$.

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{matrix} s \\ 2w \\ w \\ t \end{matrix} = s\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + w\begin{pmatrix} 0 \\ 2 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

basis $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$

Q4 Least Squared approx solution

$x = -1$

$y = 1$

$x + 4y = 0$

$\bar{A}x = b$

$$\begin{array}{ccc} \bar{A} & x & \bar{b} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 4 & 0 \end{pmatrix} & \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = & \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \end{array}$$

$A^T A x = A^T b$. $A^T = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{pmatrix}$

Solve 4

Q5. Eigen values and Eigenvectors

$$\begin{pmatrix} -3 & 0 & 0 \\ 1 & 2 & 3 \\ 2 & -1 & -2 \end{pmatrix}$$

Det $(A - \lambda I) = 0$

$$\det \begin{pmatrix} -3-\lambda & 0 & 0 \\ 1 & 2-\lambda & 3 \\ 2 & -1 & -2-\lambda \end{pmatrix}$$

$(-3-\lambda)[(2-\lambda)(-2-\lambda) - (3)(-1)] + 0[\cdots] + 0[\cdots] = 0$

$3-\lambda\ [4 + \lambda^2 + 3] = 0$

$3-\lambda\ [-1 - \lambda^2] = 0$

$-(3-\lambda)(\lambda-1)(\lambda+1) = 0$ $\quad \lambda = -3, 1, -1$

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Q5) cont.

Eigen vector $Av = \lambda v$

$(A - \lambda I)v = 0$ $A\lambda$:

$$\begin{pmatrix} 3-\lambda & 0 & 0 \\ 1 & 2-\lambda & 0 \\ 2 & -1 & -2-\lambda \end{pmatrix}\begin{pmatrix} \\ \\ \end{pmatrix} = 0$$

$\lambda = -3 \Rightarrow$

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 5 & 3 \\ 2 & -1 & 1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = 0.$$

reduce $\begin{pmatrix} 1 & 5 & 3 \\ 0 & -11 & -5 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 3+\frac{25}{11} \\ 0 & 11 & -5 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = 0$

$\Rightarrow \begin{pmatrix} 1 & 0 & \frac{54}{11} \\ 0 & -1 & -5 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = 0.$ $\quad v_1 = \frac{-54}{11}v_3$, $v_2 = \frac{-5}{11}v_3$ $\Rightarrow \begin{pmatrix} -54/11 \\ -5/11 \\ 1 \end{pmatrix}$ eigen vector for $\lambda = 3$

zero is never eigen vector

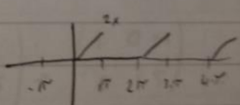
repeat for $\lambda = -1$ and $\lambda = 1$

Q5 ii) P, D $\quad P^{-1}AP = D$

$$P = \left(\begin{pmatrix} v_1 \\ \\ \end{pmatrix}\begin{pmatrix} v_2 \\ \\ \end{pmatrix}\begin{pmatrix} v_3 \\ \\ \end{pmatrix}\right) \qquad D = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

column of eigen vector.

Q6 $f(x) = \begin{cases} 0 & -\pi \le x < 0 \\ 2x & 0 \le x \le \pi \end{cases}$



Even function

$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$

$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx \quad \frac{1}{2\pi}\int_{-\pi}^{0} 0 \quad \int_{0}^{\pi} 2x\,dx = \frac{x^2}{2\pi}\Big|_0^{\pi} = \pi/2$

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$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$$

$$\frac{1}{\pi}\int_{-\pi}^{0} 0\cos(nx)\,dx + \int_{0}^{\pi} 2x\cos nx\,dx$$

0

By parts

$\frac{2}{\pi}$ $u=x$ $du=dx$ $dv=\cos(nx)dx$ $v=\frac{\sin(nx)}{n}$

$\int u\,dv$ $uv - \int v\,du$

$$= \frac{2}{\pi}\left[x\frac{\sin(nx)}{n}\Big|_0^{\pi}\right] - \int_0^{\pi}\frac{\sin(nx)}{n}\,dx$$

$$0 \quad + \frac{2\cos(nx)}{\pi n^2}\Big|_0^{\pi} = \frac{2}{\pi n^2}\left[(-1)^n - 1\right]$$

Q1 2012

Write matrix for: ~~rotation~~ reflecion about ~~y~~ x-axis, orthogonal Projection onto y-axis, rotate $(-2,-1)$

About x-axis $= \begin{pmatrix}1&0\\0&-1\end{pmatrix}$ $\rightarrow (-2,1)$

projection $\begin{pmatrix}0&0\\0&1\end{pmatrix}$ onto y-axis $(0,-1)$

rotation: $\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$ } nearly not right sure! $\begin{pmatrix}2\cos\theta - \sin\theta \\ -2\sin\theta \quad \cos\theta\end{pmatrix}$

- Gram Schmidt
- linear independent - check! basis spanning of set of vectors
- Subset of vectors that span the vector set.

$(1,0,1)$ $(-1,0,-1)$ $(2,0,2)$ ← Dependent

2013 MATHS 4 Exam Papers

Q1 i Parametric eqn $\vec{v}$ $(1,-2,4,0)$

$t\vec{v} = t\begin{pmatrix}1,-2,4,0\end{pmatrix}$

$x = t$
$y = -2t$
$z = 4t$
$w = 0$

ii Eqn in form of $Ax + By + Cz = 0$

$-A + 2B + 3C = 0 \rightarrow 3B + 3C = 0$
$A + B = 0 \Rightarrow A = -B$ $-3A = -3C$
$C = A$

$Ax + By + Cz = 0$
$Cx - Cy + Cz = 0$
$x - y + z = 0$

2A. Rank and nullity

$\begin{pmatrix} 6 & -2 & -4 \\ -3 & 1 & 2 \end{pmatrix}$ $\begin{pmatrix} 3 & -1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$ Rank = # independent rows (rows with values) / leading 1's)

$\begin{pmatrix} 1 & -1/3 & -2/3 \\ 0 & 0 & 0 \end{pmatrix}$

Rank (A) = 1
Nullity (A) = # columns - Rank = 3-2 = 2

B $\begin{pmatrix} 6 & -2 & -4 \\ -3 & 1 & 2 \\ 1 & 0 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & -1 \\ 3 & -1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & -5 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{pmatrix}$

Rank (A) = 2
Nullity = # columns - Rank = 3-2 = 1

3.A. Basis and dimension for row, column and null space

$$\begin{pmatrix} 0 & 2 & -4 & 2 \\ 0 & -3 & 6 & -3 \end{pmatrix} \begin{matrix} \div 2 \\ \div 3 \end{matrix} \quad \cancel{\begin{pmatrix} 0 & 1 & -2 & 1 \\ 0 & -1 & 3 & -1 \end{pmatrix} r_2 + r_1 \begin{pmatrix} 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}}$$

$$\begin{pmatrix} 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad \text{row space}$$

Rank (A) = dim Row (A) = dim col (A) = 1
Nullity (A) = 4-3 = 3

Basis for row space row (A) = Span$\{(0,1,-2,1)\}$
Basis for column space col (A) = Span$\left\{\begin{pmatrix} 2 \\ -3 \end{pmatrix}\right\}$

Basis for null space $A\bar{x} = 0$

$$\begin{pmatrix} 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{Dimension} = 3.$$

$$\begin{pmatrix} 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = 0 \qquad y - 2z + w = 0 \qquad x_1 = s_1$$

$$y = 2z - w \qquad x_3 = s_3$$

$$x_4 = s_4$$

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} s_1 \\ 2s_3 - s_4 \\ s_3 \\ s_4 \end{pmatrix} = s_1\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + s_3\begin{pmatrix} 0 \\ 2 \\ 1 \\ 0 \end{pmatrix} + s_4\begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

Null(A) = Span $\{(1,0,0,0), (0,2,1,0), (0,-1,0,1)\}$

B Basis and dimension for row column and null space

$$\cancel{\begin{pmatrix} 0 & 1 \\ 0 & -2 \\ -2 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -2 & 1 \\ 0 & 0 & -2 \end{pmatrix} r_2 \times -\tfrac{1}{2} r_1 \begin{pmatrix} 1 & -2 & 1 \\ 0 & 0 & 1 \end{pmatrix}} \quad \begin{pmatrix} 0 & 1 \\ 0 & -2 \\ -2 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} -2 & 1 \\ 0 & 1 \\ 0 & -2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -\tfrac{1}{2} \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

Rank (A) = dim (Row (A)) = dim col(A) = 2
Nullity (A) = ~~3~~ 2-2 = 0

Basis for row space = row(A) = Span $\{$ ~~(1,-2,1), (0,0,1)~~ $\}$ $(1\ -\tfrac{1}{2}), (0,1)$
Basis for column space col(A) = Span $\{$ [illegible] $\}$ $= \left\{\begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}\right\}$

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Basis for null space

$A\bar{x}=0$ $\begin{pmatrix}1 & -\frac{1}{2}\\ 0 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$ $x-\frac{1}{2}y=0$ $x=0$

$y=0$ $y=0$

$\Rightarrow x=0$

Null Space $=\text{Span}\left\{\begin{pmatrix}0\\ 0\end{pmatrix}\right\}$

4 $Ax=b$

$\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & 4 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\begin{pmatrix}-1\\ 1\\ 0\end{pmatrix}$ $A^TAx=A^Tb$

$A\bar{x}$

$\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & 4 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ \cancel{x_3}\end{pmatrix}\begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 4\\ \cancel{0} & \cancel{0} & \cancel{0}\end{pmatrix}=\begin{pmatrix}-1\\ 1\\ 0\end{pmatrix}\begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 4\\ 0 & 0 & 0\end{pmatrix}$

$\begin{pmatrix}1 & 0\\ 0 & 1\\ 1 & 4\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}\begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 4\end{pmatrix}=\begin{pmatrix}-1\\ 1\\ 0\end{pmatrix}\begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 4\end{pmatrix}$

$\begin{pmatrix}2 & 4\\ 4 & 17\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}-1\\ 1\end{pmatrix}$

$2x+4y=-1$ $\times 2$ $-4x-8y=2$

$4x+17y=1$ $4x+17y=1$

$9y=3$ $y=\frac{1}{3}$ $\Rightarrow 2x+\frac{4}{3}=-1$

$2x=\frac{-7}{3}$

$x=\frac{-7}{6}$

$(x,y)=\left(\frac{-7}{6},\frac{1}{3}\right)$

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5. $A = \begin{pmatrix} -3 & 0 & 0 \\ 1 & 2 & 3 \\ 2 & -1 & -2 \end{pmatrix}$ Det $(A - \lambda I) = 0$

$\det\left[\begin{pmatrix} -3 & 0 & 0 \\ 1 & 2 & 3 \\ 2 & -1 & -2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}\right] = 0$

Det. $\begin{pmatrix} -3-\lambda & 0 & 0 \\ 1 & 2-\lambda & 3 \\ 2 & -2 & -2-\lambda \end{pmatrix}$

$+(-3-\lambda)\big((2-\lambda)(-2-\lambda) - (0)(-2)\big) - 0(\sim) + 0(\sim)$

$(-3-\lambda)\,(-4 - 2\lambda + 2\lambda + \lambda^2)$

$(-3-\lambda)\,(\lambda^2 - 4)$

$-(-3-\lambda)(\lambda+1)(\lambda-1) = 0.$

$\lambda = -3$ or $\lambda = +1$ $\lambda = -1.$

Eigen vec $Av = \lambda v.$

$\left[\begin{pmatrix} -3 & 0 & 0 \\ 1 & 2 & 3 \\ 2 & -1 & -2 \end{pmatrix} - \begin{pmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix}\right]\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$

$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 5 & 3 \\ 2 & -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$ $x + 5y + 3z = 0$ $x + 5z - 10x + 3z = 0$

$2x - y - z = 0.$ $-9x + 8z = 0.$

$\Rightarrow y = z - 2x$

$r_3 - 2r_1 =$ $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 5 & 3 \\ 0 & -11 & -5 \end{pmatrix}$ $x + 5y + 3z = 0$

$-11y - 5z = 0$ $11y = -5z$ $\boxed{y = \frac{-5}{11}z}$

$x \; \frac{-25}{11}z + 3z = 0$

$x + \frac{8}{11}z = 0$ $\boxed{x = \frac{-8}{11}z}$

$z = -11,$ $x = 8$ $y = 5$

$v_1 = (8, 5, -11)$

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Q5 cont

$\lambda = 1$

$$\left[\begin{pmatrix} -3 & 0 & 0 \\ 1 & 2 & 3 \\ 2 & -1 & -2 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

$$\begin{pmatrix} -4 & 0 & 0 \\ 1 & 1 & 3 \\ 2 & -1 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$-4x = 0$ $\quad x = 0$

$x + y + 3z = 0 \Rightarrow y + 3z = 0 \quad y = -3z$

$2x - y - 3z = 0 \quad -y - 3z = 0 \quad y = 0 \quad z = 0$

~~$\bar{v}_2 \quad (0,0,0)$~~

$\bar{v}_3$ for $\lambda = -1 = (0,0,0)$ as well

$r_1 \div 4$

$r_2 + r_3$

$$\begin{pmatrix} -1 & 0 & 0 \\ 3 & 0 & 0 \\ 2 & -1 & -3 \end{pmatrix} \xrightarrow[r_3 - 2r_1]{r_2 + 3r_1} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$x = 0$

$-y - 3z = 0$

$y = -3z \quad x = 0$

$$\Rightarrow \bar{v}_3 = \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix}$$

2

$\lambda = -1$

$$\left[\begin{pmatrix} -3 & 0 & 0 \\ 1 & 2 & 3 \\ 2 & -1 & -2 \end{pmatrix} - \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}\right]\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 0 & 0 \\ 1 & 3 & 3 \\ 2 & -1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$r_1 + (r_1 + r_3)$ $\begin{pmatrix} 0 & -1 & -1 \\ 1 & 3 & 3 \\ 2 & -1 & -1 \end{pmatrix}$ $r_3 - 2r_2$ $\begin{pmatrix} 0 & -1 & -1 \\ 1 & 3 & 3 \\ 0 & -7 & 7 \end{pmatrix}$ $r_3 - 7r_1$

$$\begin{pmatrix} 0 & -1 & -1 \\ 1 & 3 & 3 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 3 & 3 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{r_1 + 3r_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x = 0 \quad x = 0$

$-y - z = 0 \quad y = -z$

$$\bar{v}_2 = \begin{pmatrix} 0 \\ +1 \\ -1 \end{pmatrix}$$

B $D = P^{-1}AP$

$$P = \begin{pmatrix} 8 & 0 & 0 \\ 5 & 1 & 3 \\ -11 & -1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = D$$

Q6 $f(x) = \begin{cases} 0 & -\pi \le x < 0 \\ 2x & 0 \le x \le \pi. \end{cases}$

Even funcion. $f(x) = a_0 + \Sigma (a_n \cos(nx) + b_n \sin(nx))$

$a_0 = \frac{1}{2\pi} \int f(x)dx = \frac{1}{2\pi} \int_{-\pi}^{0} 0\,dx \int_0^{\pi} 2x\,dx = x^2 \Big|_0^{\pi} = \frac{\pi^2}{2\pi} = \frac{\pi}{2}$

$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\cos(nx)\,dx = \frac{1}{\pi} \int_{-\pi}^{0} 0\,dx + \int_0^{\pi} 2x\cos(nx)\,dx.$

By parts $u = x$ $du = dx$ $dv = \cos(nx)\,dx$ $v = \sin(nx)$

$= \frac{2}{\pi} \left[x\frac{\sin nx}{n} \Big|_0^{\pi} \right] - \int_0^{\pi} \sin\frac{(nx)}{n}\,dx$

0

$= +\frac{2}{\pi}\frac{\cos(nx)}{n^2} \Big|_0^{\pi} = \frac{2}{\pi n^2} [(-1)^n - 1]$

b $f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \left[\left(\frac{(-1)^n - 2}{\pi n^2} \right) \cos(nx) \right]$