1

Mang Sci 2 2013 PAPER

4 A Pseudo random number.
Exhibits Statistical randomness
Generated through a complex algorithm using a seed value

Desirable properties
Uncorrelated (independent) Sequence → sequence of random numbers should be serially uncorrelated (independent)

Long Period → Generator should be of long period, ideally generator should not repeat, practically the repetition should occur only after the generation of a very large set of Random Numbers

Uniformity → sequence should be uniform and unbiased

Efficiency → Generator should be efficient and easily calculated to save costs

B Linear Congruential generator m
multiplier = 13, increment = 37 modulus = 50

$[X(13) + 37] \bmod 50$

$X_i = (13X_{i-1} + 37) \bmod 50.$

C $X_0 = 27.$

$X_1 = (13(27) + 37) \text{ Mod } 50$
$388 \text{ Mod } 50 = 38$ $R_1 = 38/50 = 0.76$

$X_2 = (38(13)+37) \text{ Mod } 50 = 31$ $R_2 = 31/50 = 0.62$

$X_3 = (31(13) + 37) \text{ Mod } 50 = 40$ $R_2 = 40/50 = 0.8$

D $F(R=r)=r$? $= P(R \le r)$ $\quad R = F(x)$

$P(F(x) \le r)$

$P(F^{-1}(F(x)) \le F^{-1}(r))$

$P(X \le F^{-1}(r))$ this is cdf

this is $F(F^{-1}(r)) = r$

4E. Inverse Transform

$F(X=x) = 1 - e^{-\lambda x}$

$y = 1 - e^{-\lambda x}$

$y - 1 = -e^{-\lambda x}$

$1 - y = e^{-\lambda y}$

$\ln(1-y) = -\lambda y$

$y = -\frac{\ln(1-y)}{\lambda}$ $\quad$ 1-r is also uniform on (0,1)

$= \frac{-\ln(r)}{\lambda}$ $\quad \lambda = 2 \Rightarrow \frac{-\ln(r)}{2}$

$\frac{\log(r)}{2}$ $= X_1 = 0.137$

$X_2 = 0.239$

$X_3 = 0.111$

3.

May SCI 2 Exam Paper 2013

Q5. A. Benefits of Simulation
- Future prediction of performance especially before implementation
- Analytic solution may be infeasible, need simulation
- Cheaper in certain situations - sensitivity analysis
- To identify bottle necks - problem areas in system

B. Definitions:

Discrete: Things occur in discrete points in time, not a constant occurance of things happening like river flowing

Dynamic: The system is evolving/changing over time

Stochastic: Completely random, uncertainty in system, to what events will occur.

C. World Views on Simulation

Event Scheduling → concentrate on events and their effect on system states. Update time to next event

Process Interaction → Concentrated on process → a process is a time sequenced list of events, activities and delays that define the life cycle of one entity as it moves through system.
- Usually many process are active simultaneously in model, and interaction can be complex - (follow the clump model

Activity Scanning → Concentrated on activities of a model and the conditions that allow an activity to begin
- At each clock advance, the conditions for each activity are checked, and if conditions are true, then the corresponding activity begins

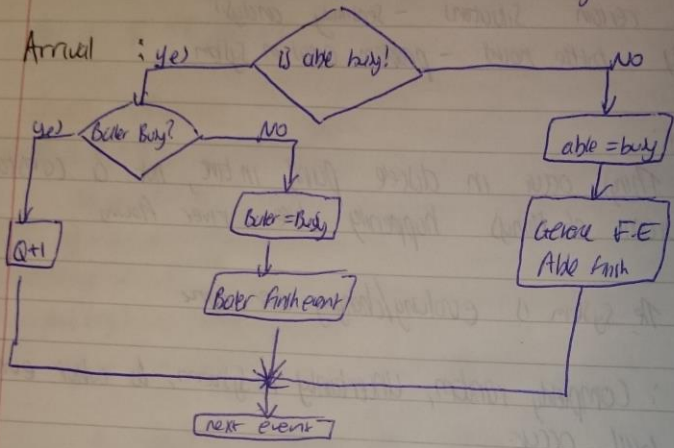
4

5D Able-Baker Call Center example

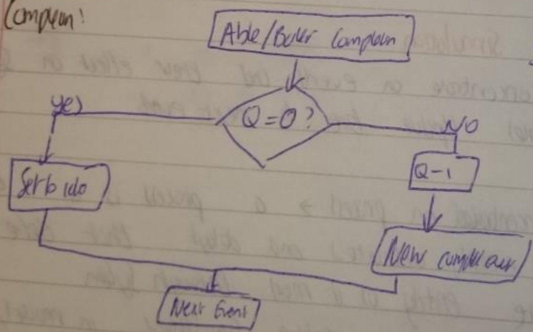
System State variables: Able busy? Baker busy? Queue size

Events: Arrival, Completion by Able, Completion by Baker.

Arrival:



Completion:



Same for Able and Baker

E Definitions

Calling Population - Amount of people that can enter into Q.
Infinite OR Finite

Capacity - Number of entities within queue and service in given time
Infinite or finite ∞/10.

5

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QE. E.. Arrival Process → Distribution to which entities arrive at Q.
poisson or normal non-homogeneous

Queue Behaviour - What customers can do in Q.
leave Q, wait for service

Queue Discipline - Rule specifying who in que is served next
FIFO, LIFO, priority

Service Time - Time taken to serve
Random, exponential

Service Mechanism - Ordering in how servers are controlled
Parallel or single server?

7

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DUMP TRUCK EXAMPLE.

Q6 A STATE VARIABLES
Trucks in loading Q
Trucks weighing
Trucks in weighing Q
Trucks loading

B System Events
Arrival of loading Q
End loading
End weighing

C Simulation
Time = 0 SV = (2,0,2,0)
R_1 = 0.91 Loading to 10 time 10
R_2 = 0.75 Loading at t+6 time 6

Time = 10 SV = (2, 1, 0, 1)
R_3 0.94 loading end at t+10 t = 20
R_4 0.09 loading end at t+5 t = 15
R_5 0.61 end weight t+7 t = 17

t = 15 SV = (1, 1, 0, 2)

T = 17 SV = (1,1,0,1)
R_6 = 0.86 weighing t+7 t = 24
R_7 = 0.55 travel t+45 t = 62

T = 20 SV = (0,1,0,2)

t = 24 SV (0,1,0,1)
R_8 = 0.2 -> end weigh t+5 t = 24
R_9 = 0.04 -> travel for t+21 t=54

8.

$t = 29$ $SV = (0, 1, 0, 0)$

$R_4 = 0.76$ end weigh $t+7$ $t = 36$

Cut of Reward rule

6.1) Acceptance or rejection

1. Generate y from $g(x)$
2. Generate r from $u(0,1)$
3. If $r \leq \frac{f(y)}{c\,g(y)}$ accept y otherwise go 6.1

$c = \sup_{x} \left(\frac{f(x)}{g(x)} \right)$

26/3/14 Section B – Introduction to Simulation

Last year's paper

4 a. Pseudo random numbers

Deterministic process, mimics randomness

- Independence, uniform, high period, easily generated / calculated

b linear congruential

$(x(13) + 37) \text{ Mod } 50$

$X_i = (13X_{i-1} + 37) \text{ Mod } 50$

c. $X_0 = 27$ $\quad X_1 = (13(27) + 37) \text{ Mod } 50$

$(351 + 37) \text{ Mod } 50$

$388 \text{ MOD } 50 = 38$

$X_2 = (38(13) + 37) \text{ Mod } 50 = 31$

$X_3 = (31(13) + 37) \text{ Mod } 50 = 40$

Divide by 50 to get $[0,1]$

$R_1 = 0.76$

$R_2 = 0.62$

$R_3 = 0.8$

d. $F(R=r) = P(R \leq r) \qquad R = F(x)$

$= P(F(x) \leq r)$

$P(F^{-1}(F(x)) \leq F^{-1}(r))$

$P(X \leq F^{-1}(r))$ this is cdf

That i) $= F(F^{-1}(r)) = r$

2 Man Sci Exam Paper

4e. $F(X=x) = 1 - e^{-\lambda x}$

$r - 1 = -e^{-\lambda x}$

$1 - r = e^{-\lambda x}$

$\ln(1-r) = \lambda x$

$\frac{\cancel{\ln}(1-r)}{\lambda} = x$ $1-r$ is also unifr on $(0,1)$

$= \frac{-\log(r)}{\lambda}$ $\lambda = 2 \Rightarrow \frac{-\log(r)}{2}$

$\frac{\log(r)}{2}$ $\frac{\log(0.76)}{2} = x_1 = 0.137$

$x_2 = 0.239$

$x_3 = 0.111$

Q5 A - Future prediction of preformance esp before implementation.
- Analytical Solution may be infeasible, need Simulation
- Cheaper in certain situations - Sensitivity analysis.
- To identify bottle necks - problem areas in System

B Discrete: Things occur in discrete points in time, not a constant occurance of things happening not like river flowing into lake

Dynamic: System is evolving/changing over time

Stochastic: random, uncertainty in system to what events will occur.

C. Event Scheduling - at each event, update to system variables collects stats and fast forward
- focus on event point generate future events

3

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4c.

Process interaction - track each individual entity within system and track its progress through simulation

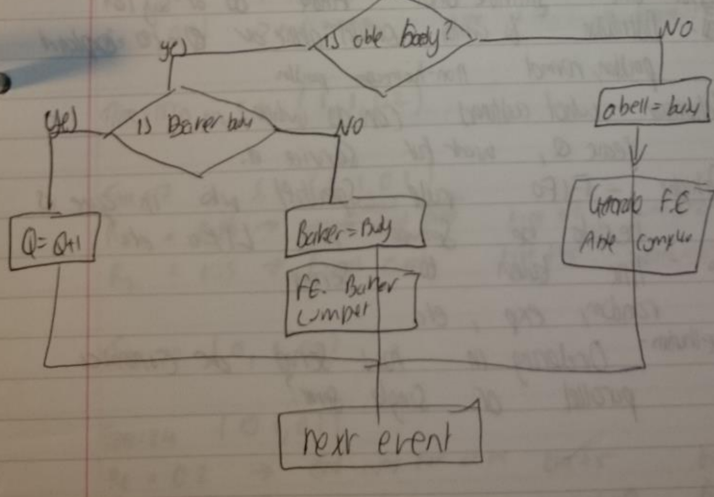
Activity scanning - like event scheduling check at each time point if something is happen at time t - anything need to do? No → time t+1 anyth ... ? yes → do it.

b. System state variables: Able idle or not
Baker idle or not
Size Queue

events: Arrival
Completion of service Able
Completion of service by Baker

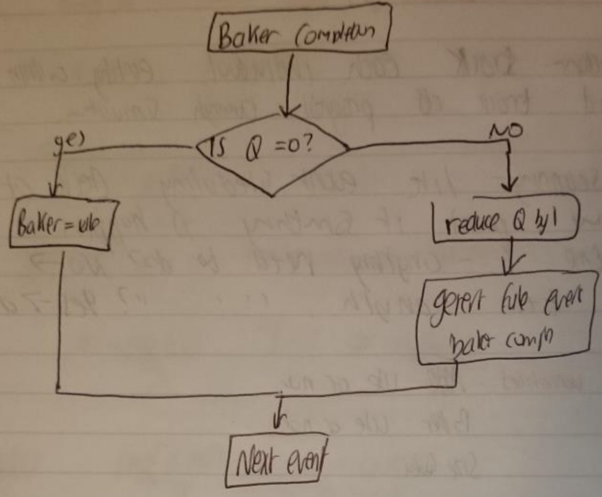
Flow chart

Arrival event.



4

4d



Same chart for Able Completion

4e Calling population - amount of people that can enter into que
infinite or finite
Capacity - Number of entities within queue and service in given time. Infinite and Finite ∞ or say 10
Arrival Process - Distribution to which entities arrive at Q - to explained
poisson, normal non-homogen poisson
Queue Behaviour - what customers can do while in Q
leave Q, wait for service etc.
Queue Discipline - FIFO rule specifies who in que is next to be served rand LIFO etc
Service Time - Time taken to service
random, exp, etc
Service mechanism - Ordering in how served are connected parallel or single server?

2

Time = 24 $\quad$ s.v = (0, 1, 0, 0)

R_{10} = 0.76 → end marry $\quad$ time + 7 $\quad$ time = 36

→ travel time $\quad$ out of random number

6Q 1 Generate y from g(x)

2 Generate r from u(0,1)

3 If $r \leq \frac{f(y)}{c\,g(y)}$ accept y, otherwise go to 1

$c = \sup_x \left(\frac{f(x)}{g(x)}\right)$