Math Paper 1 2010

1 a $\dfrac{(x-1)(x+1)}{(x+1)(x+2)} \quad \dfrac{x-1}{x+2} \quad x \to -1 \quad \dfrac{-1-1}{-1+2} \quad \dfrac{-2}{1} = -2$

b $f(x) = \begin{cases} \dfrac{\tan kx}{x} & \text{for } \dfrac{-\pi}{2k} < x < 0 \\ \sqrt{\dfrac{x+k^2}{2}} - k & \text{for } x \geq 0 \end{cases}$

$\dfrac{\tan kx}{x} = \lim_{x \to 0} \dfrac{\sin kx}{x} \cdot \dfrac{1}{\cos kx} \qquad \dfrac{k}{k}$

$k \dfrac{\sin kx}{kx} \cdot \dfrac{1}{\cos kx}$

$k(1) \cdot k$

$= k^2$ for $\dfrac{-\pi}{2k} < x < 0$

need $\lim_{x \to 0} f(x) = f(0)$

$f(0) = \dfrac{\sqrt{x+k^2} - k}{2} = k^2$

$\sqrt{x+k^2} - k = 2k^2$

$\sqrt{x+k^2} = 2k^2 + k$

$x + k^2 = 4k^4 + 4k^3 + k^2 - k^2 - x$

$0 \quad 4k^4 + 4k^3 - x = 0$

2 ai $\frac{d}{dy} \sinh(\ln y)$ $\sinh(\log y) = \frac{y^2-1}{2y}$

$\frac{e^{\ln y} - e^{-\ln y}}{e^{(\ln y)} 2}$

$\frac{dy}{dx}$ of $\left(\frac{y^2-1}{2y}\right) = \frac{1}{2}\left(\frac{1}{y^2}+1\right)$

aii $\frac{d}{dx} \tan^{-1}(x^2-1)^{1/2}$ $\frac{1}{((x^2+1)^{1/2})^2+1} \cdot \frac{1}{2}(x^2-1)^{-3/2} \cdot 2x$

$\frac{2x}{x^2+2}(x^2+1)^{-3/2}$

aiii $\frac{d}{dx} \frac{\sqrt{x}e^{2x}}{x^2+1}$ $\frac{u}{v}$ $\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$\frac{(x^2+1)\left(\sqrt{x}\,2e^{2x} + \frac{1e^{2x}}{2}(x)^{-1/2}\right) - \sqrt{x}e^{2x}(2x)}{(x^2+1)^2}$$

$= \frac{e^{2x}\sqrt{x}}{x^2+1}$

d $f(x) = (x^2-16)^{1/3}$

$\frac{1}{3}(x^2-16)(2x) = 0$

$\frac{1}{3}(x-4)^{-2/3}(x+4)(2x) = 0$

$x=4$ $x=-4$ $y=0$

min at $x=4$ $y=0$

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3 a $\int \sqrt{x-2}\,(x^2+x+1)\,dx$ $u = \sqrt{x-2}$ $\frac{du}{dx} = \frac{1}{2}(x-2)^{-1/2}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x-2}}$

$du = \frac{dx}{2\sqrt{x-2}}$

$\cancel{dv = (x^2+x+1)dx}$

$\cancel{v = \int dv = \int x^2+x+1\,dx}$

$\cancel{= \frac{x^3}{3} + \frac{x^2}{2} + x}$

$\int u\,dv = uv - \int v\,du$

$\left(\frac{x^3}{3}+\frac{x^2}{2}+x\right)(\sqrt{x-2}) - \int \frac{x^3}{3}+\frac{x^2}{2}+x \,\frac{dx}{2\sqrt{x-2}}$

$u = \sqrt{x-2}$ $du = \frac{1}{2\sqrt{x-2}}\,dx$

$\int \frac{\sqrt{x-2}\,(x^2+x+1)\,dx}{2\sqrt{x-2}}$

$= 2\int u^2\,(u^2 + (u^2+2)^2 + 3)\,du$

$= 2\int u^6 + 5u^4 + 7u^2$

$= \frac{2u^7}{7} + 2u^5 + \frac{14u^3}{3}$

$u = \sqrt{x-2}$ $2(x-2)^{5/2} + \frac{14}{3}(x-2)^{3/2} + \frac{2}{7}(x-2)^{7/2} + C$

b $\int \frac{\ln x}{x}$ $u = \ln x$ $\frac{du}{dx} = \frac{1}{x}$

$dx = \frac{1}{x}\,du$

$\int \frac{u}{x}\Big/\frac{1}{x}$ $\int \frac{u}{\cancel{x}}\Big/\frac{1}{x}\,du$

$\int \frac{u}{\cancel{x}}\cancel{x}\,du$

$\int u\,du$

$= \frac{u^2}{2} = \frac{(\ln x)^2}{2} + C$

c. $\int_0^1 \frac{x+3}{x^2+6x+25}$ $u = x^2+6x+25$

$du = 2x+6\,dx$

$= \int \frac{x+3}{u\ \ 2x+6}\,du$

$= \frac{1}{2}\int \frac{1}{u}$

$\frac{1}{2}\left(\ln(u)\right) = \frac{\ln(u)}{2} = \frac{\ln(x^2+6x+25)}{2}$

$\frac{\ln(1^2+6+25)}{2} - \ln\left(\frac{0+0+25}{2}\right)$

$1.7328 - 1.6094 = - 0.1234$

4 length $= \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ $\frac{dy}{dx}$ of $\frac{1}{4}y^2 - \frac{1}{2}\ln y$

$\int_1^2 \sqrt{1+\left(\frac{y}{2}-\frac{1}{y}\right)^2}$ $\frac{y}{2}\left(5 - \frac{1}{2}\frac{1}{y}\right.$

$\int_1^2 \sqrt{1+\frac{y^2}{4}+y+\frac{1}{y^2}}\,dy$ $\frac{y}{2} - \frac{1}{y}$

$u = 1+\frac{y^2}{4}+y+\frac{1}{y^2}$ $\frac{-y}{2y}$

$du = \frac{y}{2}$

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