Travelling Salesman

Travelling Salesman visits all the nodes in a network, known as Hamilton's algorithm

Chinese postman travels along all the arcs - Euler's algorithm

Hamiltonian Algorithm

	A	B	C	D
A	-	4	6	9
B	4	-	9	3
C	6	9	-	7
D	9	3	7	-

Solution Methods

- Linear programming will give an optimal solution
- Enumeration will also give an optimal solution but there are $(N-1)!$ solutions, so for 6 city problem there are 120 options.

- Heuristics - rules of thumb or good solutions that are not necessarily an optimum solution but could be

Heuristics

- Nearest neighbour
- Nearest insertion

Linear programming

Formulate as LP

$$Min\ z = 4x_{AB} + 6x_{AC} + 9x_{AD} + 4x_{BA} + 9x_{BC} + 3x_{BD}$$
$$+ 6x_{CA} + 9x_{CB} + 7x_{CD} + 9x_{DA} + 3x_{DB} + 7x_{DC}$$

Entering Constraints

$x_{BA} + x_{CA} + x_{DA} = 1$ going into A

$x_{AB} + x_{CB} + x_{DB} = 1$

$x_{AC} + x_{BC} + x_{DC} = 1$

$x_{AD} + x_{BD} + x_{CD} = 1$

Leaving Constraints

$x_{AB} + x_{AC} + x_{AD} = 1$ leaving A.

$x_{BA} + x_{BC} + x_{BD} = 1$ "

$x_{CA} + x_{CB} + x_{CD} = 1$ "

$x_{DA} + x_{DB} + x_{DC} = 1$ "

All variables $\{0,1\}$

Solution (1)

$x_{AC} = 1$, $x_{BD} = 1$ $x_{CA} = 1$ $x_{DB} = 1$

Total distance = 18

Two tours A – C – A and B – D – B

Solution (2)

Additional constraints required.

Eliminate A – C – A by adding:

$x_{AC} + x_{CA} \leq 1$

$x_{AC} = x_{CD} = x_{DB} = x_{BA} = 1$

New distance = 20

Nearest Neighbour Heuristic

	A	B	C	D	E	F
A	–	13	12	18	7	14
B	13	–	21	26	15	25
C	12	21	–	11	6	4
D	18	26	11	–	12	14
E	7	15	6	12	–	9
F	14	25	4	14	9	–

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$A \to E \to C \to F \to D \to B \to A = 70$

	A	B	C	D	E	F
A	–	13	12	18	7	14
B	13	–	21	26	15	25
C	12	21	–	11	6	4
D	18	26	11	–	12	14
E	7	15	6	12	–	9
F	14	25	4	14	9	–

$13 + 7 + 6 + 4 + 14 + 26 = 70$

Other Solutions

A E C F D B A = 70
B A E C F D B = 70
C F E A B D C = 70
D C F E A B D = 70
E C F A B D E = 75
F C E A B D F = 70

Multiple Solution (not necessarily optimum)

Nearest Insertion

	A	B	C	D
A	–	4	6	9
B	4	–	9	3
C	6	9	–	7
D	9	3	7	–

$A \to A$ leaves B, C and D to be included

Min {AB, AC, AD} Min (4, 6, 9)
= 4 = AB
A–B–A, C and D not included

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(4) An A - B - A
Min {AC, BC, BD, AD}
Min {6, 9, 3, 9}
= 3
= BD

A - D - B A = 9 + 3 + 4 = 16 OR
A - B - D - A = 4 + 3 + 9 = 16 Choose Either

(5) A - D - B - A C to be included
Min {AC, DC, BC}
A - C - D - B - A = 6 + 7 + 3 + 4 = 20 OR
A - D - C - B - A = 9 + 7 + 9 + 4 = 29 OR
A - D - B - C - A = 9 + 3 + 9 + 6 = 27

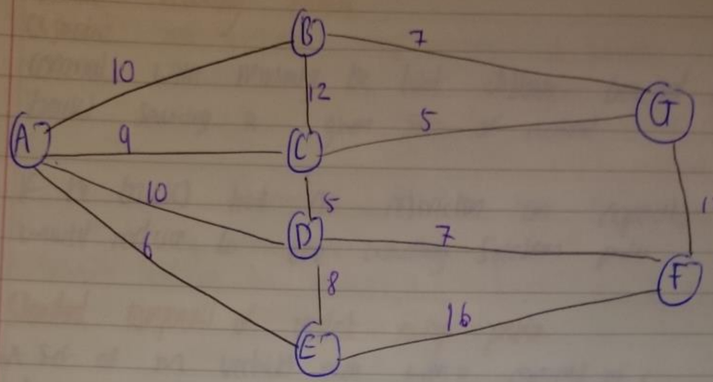
Best is A - C - D - B - A = 6 + 7 + 3 + 4 = 20

- Above was for a symmetrical problem
- If problem is not symmetrical must look at total length to be added
- e.g. if we have A - B - A, when adding C look at length of A - C - B and compare with B - C - A

Chinese Postman / Euler Algorithm

Solution method
- If there are NO arcs of ODD degree then the solution is just the sum of all the arcs
- Identify the nodes of odd degree
- Pair them in the shortest way (see all NCr pairs possible)
- Add extra arcs to network
- Calculate length of tour

5.



Solution (1)

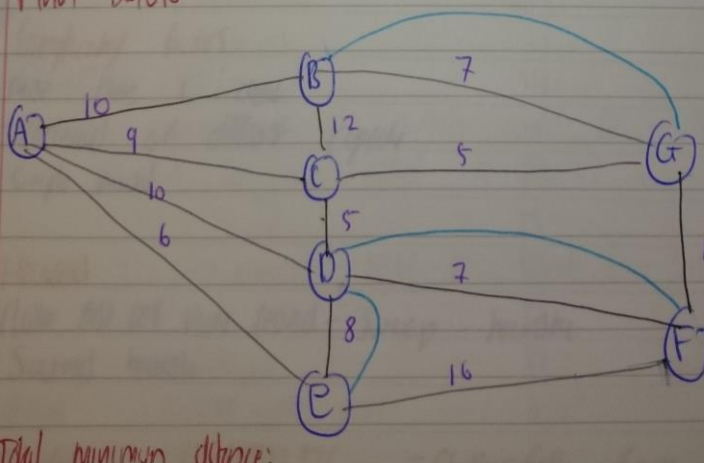
Total length of arcs = 108

Node	A	B	C	D	E	F	G.
Degree	4	3	4	4	3	3	3

Nodes of odd degree B E F G.

These must be paired

Final Solution



Total minimum distance:

Original dist + extra added arcs.

108 + 7 + 7 + 8

108 + 22 = 130