

17/2/14 DAVID WETTBRECHT 12300041 sheet 5 week 6

1(1) $\begin{pmatrix} 1 & 2 & 0 \\ 1 & -2 & -1 \end{pmatrix} \quad r_2 - r_1 \quad \begin{pmatrix} 1 & 2 & 0 \\ 0 & -4 & -1 \end{pmatrix} \quad \hat{A} = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & -\frac{1}{4} \end{pmatrix}$

$\frac{7.5}{6}$

$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & -\frac{1}{4} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 1 & 2 & 0 \\ 0 & -4 & -1 \end{pmatrix} \div -4 \quad \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & \frac{1}{4} \end{pmatrix}$

$x_1 + 2x_2 = 0$
 $x_2 - \frac{1}{4}x_3 = 0$

$x_1 = -\frac{1}{2}x_3$
 $x_2 = \frac{1}{4}x_3$

$\begin{pmatrix} -\frac{1}{2} \\ \frac{1}{4} \\ 1 \end{pmatrix} (t)$

$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}t \\ \frac{1}{4}t \\ t \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{4} \\ 1 \end{pmatrix} t = \text{basis for null space.}$

Column Space = $c_1, c_2 = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} \right\}$

row space = rank = 2 Row space = 2 dimension = 2
 $= \left\{ (1, 2, 0), (0, 1, -\frac{1}{4}) \right\}$
or $\left\{ (1, 2, 0), (1, -2, -1) \right\}$

$\frac{2.5}{3}$

Dim of column space = 2

$\begin{pmatrix} 1 & 1 \\ -2 & 2 \\ -1 & 0 \end{pmatrix} \quad r_2 + 2r_1 \quad \begin{pmatrix} 1 & 1 \\ 0 & 3 \\ -1 & 0 \end{pmatrix} \quad r_3 \div 3 \quad \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 0 \end{pmatrix} \quad r_3 + r_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$

rank = 2 column space = $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} \right\}$
dimension = 2

2

Q 1 ii. DAVID WEITBRECH 12300644

$$\begin{pmatrix} -1 & 2 \\ 3 & -6 \\ 4 & -8 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -2 \\ 3 & -6 \\ 4 & -8 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

rank = 1
 Dim = 1
 row space = $\{(1, -2)\}$ ✓

$$\begin{pmatrix} 2 & -6 & -8 \\ -1 & 3 & 4 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_2} \begin{pmatrix} -1 & 3 & 4 \\ 2 & -6 & -8 \end{pmatrix} \xrightarrow{r_2 + r_1} \begin{pmatrix} -1 & 3 & 4 \\ -1 & 3 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -3 & -4 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{rank} = 1 \quad \text{column space} = \left\{ \begin{pmatrix} 1 \\ -3 \\ -4 \end{pmatrix} \right\} \quad \text{dim} = 1$$

null space

$$\begin{pmatrix} -1 & 2 \\ 3 & -6 \\ 4 & -8 \end{pmatrix} \begin{matrix} x_1 \\ x_2 \end{matrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_1 - 2x_2 = 0 \quad x_1 = 2x_2 \quad x_2 = t$

$$\begin{pmatrix} t \\ \frac{1}{2}t \end{pmatrix} = t \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} = \text{null space}$$

2.5
3

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Q2 i. $\begin{pmatrix} 3 & -3 & -3 \\ -2 & 2 & 2 \end{pmatrix} \div 3$

$\begin{pmatrix} 1 & -1 & -1 \\ -2 & 2 & 2 \end{pmatrix}$

$\begin{pmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$

$r_2 + 2r_1$
rank = 1
nullity = 2

ii. $\begin{pmatrix} 4 & -4 & -4 \\ 1 & 1 & 1 \\ 3 & 0 & 0 \end{pmatrix} \quad r_1 \div 4$

$\begin{pmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 0 & 0 \end{pmatrix}$

$\begin{pmatrix} 1 & -1 & -1 \\ 0 & 2 & 2 \\ 0 & 3 & 3 \end{pmatrix}$

$\begin{pmatrix} 1 & -1 & -1 \\ 0 & 2 & 2 \\ 0 & 3 & 3 \end{pmatrix}$

$\begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 3 & 3 \end{pmatrix}$

$\begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

$\begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

rank = 3
nullity = 0