1. Linear Programming Sensitivity Analysis

Sensitivity analysis is the study of how the changes in the coefficients of an optimization model affect the optimal solution

1. How will a change in a coefficient in the objective function affect the optimal solution
2. How will a change in the right hand side value for a constraint affect the optimal solution

Example Par inc problem: Max $10S + 10D$

ST: $\frac{7}{10}S + 1D \leq 630$ cutting + dyeing

$\frac{1}{2}S + \frac{5}{6}D \leq 600$ sewing

$1S + \frac{2}{3}D \leq 708$ finishing

$\frac{1}{10}S + \frac{1}{4}D \leq 135$ inspection + packaging $S, D \geq 0$

Optimal Solution is $S = 540$ standard bags and $D = 252$ Delux bags

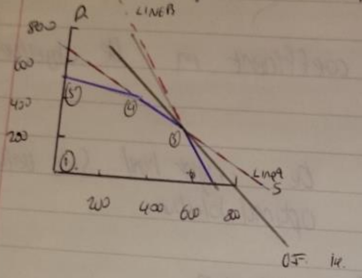
Objective Function Coefficients

- Graphical Sensitivity analysis
- The range of optimality for each objective function coefficient provides the range of values over which the current solution will remain optimal
- See figure. As long as slope of O.F. line is between the slope of line A and the slope of line B, extreme point 3 with $S = 540$ and $D = 252$ will be optimal
- Changing an objective function coef of S or D will cause the slope of the objective function line to change

- Rotating O.F line counterclockwise increases the slope
- When the O.F rotates counterclockwise enough to coincide with line A we obtain alternative optimal solutions between 3 and 4

- Any further anticlockwise rotation will cause ③ to be nonoptimal
- Slope of line A provides upper limit

- Clockwise ⇒ Slope of B is lower limit for slope of O.F.



- Extreme point will be optimal as long as
  Slope of line B ≤ Slope of O.F ≤ Slope of line A
- Eqn of line A $\frac{7}{10}S + 1D = 630$ $D = -\frac{7}{10}S + 630$
  Slope A $= -\frac{7}{10}$ intercept $= 630$

- B : $1S + \frac{2}{3}D = 708$ Slope $= -\frac{3}{2}$ intercept $= 1062$

$$-\frac{3}{2} \leq \text{Slope of O.F} \leq -\frac{7}{10}$$

- O.F line can be written as $P = C_S S + C_D D$ (← profit S, ← profit D)

$$C_D D = -C_S S + P$$

$$\Rightarrow D = \frac{-C_S}{C_D} S + \frac{P}{C_D}$$

$$\Rightarrow \quad -\frac{3}{2} \leq -\frac{C_S}{C_D} \leq -\frac{7}{10}$$

To compute range of optimality for the standard bag contribution, we hold the profit contribution for delux bag fixed at its initial value of $C_D = 9$.

$$\Rightarrow \quad \frac{-3}{2} \leq \frac{-C_S}{9} \leq -\frac{7}{10}$$

3.

From left hand side inequality:

$-3/2 \le -C_s/9$ or $3/2 \geq C_s/9$

Thus $27/2 \geq C_s$ or $C_s \le 27/2 = 13.5$

For right hand side we have

$\frac{-C_s}{9} \le -7/10$ or $\frac{C_s}{9} \geq \frac{7}{10}$

Thus $C_s \geq 63/10$ or $C_s \geq 6.3$

Combining calculated limits we know have C_s which provides us with the range of optimality for the standard bag contribution

$6.3 \le C_s \le 13.5$

- Everything else unchanged, profit for standard bag can range from £6.3 to £13.5 and solution will remain optimal
- Profit contribution will change
- For delux bag with $C_s = 10 \Rightarrow 6.67 \le C_D \le 14.29$
- In case of vertical line, we will have one limit and the other will be ∞ eg $13.5 \le C_s < \infty$

Right hand side:

The change in the value of the optimal solution per unit increase in the right hand side of the constraint is called the dual value

Example, add 10 extra units to constraint and find new optimal solution. Find profit for solution and difference between old and new profit

difference $\div$ 10 is then the dual value