

MA1E02 Tutorial Sheet 4.

Week 6 2013

Questions Evaluate the following integrals:

1.

$$\int \frac{x^2 + 1}{(x-3)(x-2)^2} dx \quad (1)$$

2.

$$\int \frac{x^2 + 2x + 4}{x^2 + 2x + 5} dx \quad (2)$$

3.

$$\int \frac{10}{(x-1)(x^2+9)} dx \quad (3)$$

No log or log

2.  $\int \frac{x+4}{x^2+6x+5} dx$

$$\frac{x+4}{x^2+6x+5} = \frac{x+4}{(x+1)(x+5)}$$

$$\frac{A}{x+1} + \frac{B}{x+5}$$

$$\frac{Ax + 5A + Bx + B}{(x+1)(x+5)} = \frac{(A+B)x + (5A+B)}{(x+1)(x+5)} = \frac{x+4}{(x+1)(x+5)}$$

$$A+B=1$$

$$5A+B=4$$

$$A=1-B \Rightarrow 5(1-B)+B=4$$

$$5-5B+B=4$$

$$-4B=-1$$

$$4B=1$$

$$B=\frac{1}{4}$$

$$A=\frac{3}{4}$$

$$\frac{\frac{1}{4}}{x+1} + \frac{\frac{3}{4}}{x+5}$$

$$\int \frac{\frac{1}{4}}{x+1} + \int \frac{\frac{3}{4}}{x+5}$$

$$= \frac{1}{4} \ln|x+1| + \frac{3}{4} \ln|x+5| + C$$

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3  $\int \frac{10}{(x-1)(x^2+9)} dx$

$$\frac{10}{(x-1)(x+3)(x-3)}$$

$$\frac{A}{(x-1)} + \frac{Bx+C}{(x^2+9)}$$

$$\frac{Ax^2+9A + (Bx+C)(x-1)}{(x-1)(x^2+9)}$$

$$\frac{Ax^2+9A + Bx^2 - Bx + Cx - C}{(x-1)(x^2+9)}$$

$$(A+B)x^2 + (-B+C)x + (9A-C) = 10$$

$$A+B=0 \quad C-B=0 \quad 9A-C=10$$

$$B=-A \quad C=B \quad 9A-B=10$$

$$9A+A=10$$

$$A=1$$

$$B=-1$$

$$C=-1$$

$$\int \frac{1}{(x-1)} + \frac{-x-1}{(x^2+9)}$$

$$\int \frac{1}{x-1} + \int \frac{-x-1}{x^2+9^2} \leftarrow \begin{matrix} \tan^{-1} \text{ function} \\ \tan^{-1}(\frac{x}{a}) \end{matrix}$$

$$\ln|x-1| + \ln \sqrt{x^2+9} + C$$

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$$1 \int \frac{x^2+1}{(x-3)(x-2)^2} = \frac{A}{(x-3)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$$\frac{A(x-2)^2 + B(x-2) + C(x-3)}{(x-3)(x-2)^2}$$

$$A(x^2-4x+4) + Bx-2B + Cx-3C$$

$$Ax^2 - 4Ax + 4A + Bx - 2B + Cx - 3C$$

$$\frac{(A)x^2 + (-4A+B+C)x + (4A-2B-3C)}{(x-3)(x-2)^2} = \frac{x^2+1}{(x-3)(x-2)^2}$$

$$(A=1)$$

$$(-4A+B+C)=0$$

$$-4+B+C=0$$

$$B+C=4$$

$$C=4-B$$

$$(4A-2B-3C)=1$$

$$4-2B-3C=1$$

$$-2B-3C=-3$$

$$\Rightarrow -2B-3(4-B)=-3$$

$$-2B-12+3B=-3$$

$$(B=9)$$

$$C=4-B$$

$$C=4-9$$

$$(C=-5)$$

$$= \frac{10}{(x-3)} - \frac{9}{(x-2)} - \frac{5}{(x-2)^2}$$

$$\int \frac{10}{(x-3)} dx - \int \frac{9}{(x-2)} dx - \int \frac{5}{(x-2)^2} dx$$

$$= 10 \ln|x-3| - 9 \ln|x-2| + \frac{5}{x-2} + C$$

absolute value

No log of neg numbers

$$u=x-2$$

$$\frac{du}{dx}=1$$

$$du=dx$$

$$5 \int u^{-2} du$$

$$5 \cdot \frac{1}{-1} u^{-1}$$

$$= -\frac{5}{x-2}$$