Math 1 2011 Exam Pap

Q5. A Independent if $\frac{df}{dy} = \frac{dg}{dx}$

$4y \stackrel{?}{=} 4y$ ✓ path independent

B $\frac{d\phi}{dx} = -3x^2 + x + 2y^2$ $\frac{d\phi}{dy} = -3y + 4xy$

integral

$\phi = -x^3 + \frac{x^2}{2} + 2xy^2 + C_1(y)$. where $C_1(y)$ depends on y.

diff wrt y

$4xy + \frac{dC_1}{dy} = -3y + 4xy$

$\frac{dC_1}{dy} = -3y$

$C_1 = \frac{-3y^2}{2}$

$\phi = -x^3 + \frac{x^2}{2} + 2xy^2 - \frac{3y^2}{2} + C$ where C is any constant

C $(\frac{1}{2}, 1)$ to $(-1, 0)$

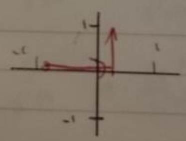
$\left[(-\tfrac{1}{2})^3 + \frac{(\frac{1}{2})^2}{2} + 2(\tfrac{1}{2})(1)^2 - \frac{3(1)^2}{2}\right] - \left[-(-1)^3 + \frac{(-1)^2}{2} + 2(0)(-\tfrac{1}{2}) - \frac{3(0)}{2}\right]$

$1 + \frac{1}{2} + 0 \quad 0$

$\left[-\frac{1}{2}\right] - [1.5]$

$= -2$

D



ii. $r(t) = (1-t)r_0 + t\,r_1$

$C_1: (-1,0)\ (\tfrac{1}{2},0) \quad (1-t)(-1,0) + t(\tfrac{1}{2},0) = (-1+\tfrac{3}{2}t, 0)$

$C_2: (\tfrac{1}{2},0)\ (\tfrac{1}{2},1) \quad (1-t)(\tfrac{1}{2},0) + t(\tfrac{1}{2},1) = (\tfrac{1}{2}, t)$

To Side $C_1: x = t,\ y = 0 \qquad -1 \le t \le \tfrac{1}{2} \qquad dy = 0$

$C_2: x = \tfrac{1}{2} \quad y = t \qquad 0 \le t \le 1 \qquad dx = 0$

$\int_{C_1} (-3x^2 + x + 2y^2)\,dx - (3y - 4xy)\,dy$

$= \int_{-1}^{1/2} (-3t^2 + t)\,dt \qquad = \left[-t^3 + \frac{t^2}{2}\right]_{-1}^{1/2} = -\tfrac{3}{2}$

$\int_{C_2} (-3x^2 + x + 2y^2)\,dx \qquad (3y - 4xy)\,dy$

$\int_0^1 -3t + 2t\,dt \qquad = \int_0^1 -t\,dt = -\frac{t^2}{2}\Big|_0^1 = -\frac{1}{2}$

$-\tfrac{3}{2} - \tfrac{1}{2} = -2.$

Q1 Jan 2011

Q1 $y'' + 4y = 8u(t-\pi) - 8u(t-3\pi)$ $y(0) = 2$ $y'(0) = 0$

$s^2 Y(s) - sy(0) - y'(0) + 4L(y) = \frac{8e^{-\pi s}}{s} - 8e^{-3\pi s}$

$s^2 Y - 2s + 4Y = \frac{8e^{-\pi s}}{s} - 8e^{-3\pi s}$

$Y(s^2+4) = \frac{8e^{-\pi s}}{s} - 8e^{-3\pi s} + 2s$

$Y = \frac{8e^{-\pi s}}{s(s^2+4)} - \frac{8e^{-3\pi s}}{s^2+4} + \frac{2s}{s^2+4}$

$\frac{2e^{-\pi s}}{s} - \frac{2se^{-\pi s}}{s^2+4} - 2\frac{4e^{-3\pi s}}{s^2+2^2} + \frac{2s}{s^2+2^2}$

$u(t-\pi) + u(t-3\pi)\sin(2t-6\pi) - 2u(t-3\pi)\sin(2t-6\pi) + 2\cos 2t$

Q2 $f(x,y,z) = \sqrt{y^2 - \sin(3x-2z)}$ $\quad P(2,-1,3)$

$$\frac{df}{dx} = \frac{1}{2}\frac{-\cos(3x-2z)(3)}{\sqrt{y^2-\sin(3x-2z)}}\Bigg|_{(2,-1,3)} \quad \frac{-3}{2}$$

$$\frac{df}{dy} = \frac{1}{2}\frac{2y}{\sqrt{y^2-\sin(3x-2z)}} = -1$$

$$\frac{df}{dz} = \frac{1}{2}\frac{-\cos(3x-2z)(-2)}{\sqrt{y^2-\sin(3x-2z)}} = 1$$

vector $= \nabla f \cdot \left(-\frac{3}{2}i,\ -1j,\ 1k\right)$

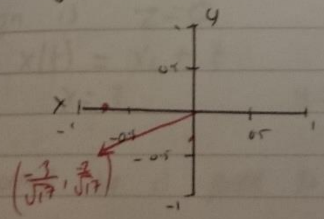
$$-\frac{3}{2}i - 1j + k$$

$$\|\nabla f\| = \sqrt{\left(\frac{-3}{2}\right)^2 + (-1)^2 + (1)^2} = \frac{\sqrt{17}}{2}$$

$$\Rightarrow u = \frac{-3/2}{\sqrt{17}/2},\ \frac{-1}{\sqrt{17}/2}$$

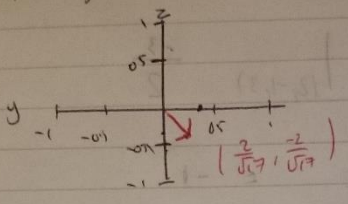
$$\left(-\frac{3\sqrt{17}}{17}i,\ -\frac{2}{\sqrt{17}}j,\ \frac{2}{\sqrt{17}}\right)$$

B For xy plane drop z coord



C Decrease most rapidly is $-u \Rightarrow \left(\frac{3}{\sqrt{17}}, \frac{2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}\right)$

D y-z plane, ignore x coord



C Rate of change in direction of $\pm u$ is equal to $\pm \|\nabla f\|$

$= \pm \frac{1}{2}\sqrt{17}$ $\quad \frac{1}{2}\sqrt{17}$ is rate of change

Q3 2011 MATHS 1.

3 a $z = \ln \sqrt[3]{\frac{2x^2-3y^3-3xy^2+21}{3}}$ $P(3,-2,0)$

rewrite $\frac{1}{3}\ln(2x^2-3y^3-3xy^2+21) - \ln(3)$

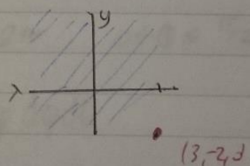
$\frac{df}{dx} = \frac{1}{3} \cdot \frac{4x - 3y^2}{(2x^2-3y^3-3xy^2+21)} \Big|_{(3,-2)}$ $\frac{0}{''} = 0$

$\frac{df}{dy} = \frac{1}{3} \frac{-9y^2 - 6xy}{(2x^2-3y^3-3xy^2+21)} \Big|_{(3,-2)}$ $-36 - 6(3)(-2) = 0$ $= 0.$

$F(3,-2) = 0.$

eqn $= 0 + f_x(x-x_0) + f_y(y-y_0)$
$z = 0 + 0(x-3) + 0(y+2)$
$z = 0$ = tangent plane

b $z = 0 \Rightarrow$ just the xy plane



c Eqn is $z = 0.$
$x(t) = x_0 + t$
$x = 3$ $y = -2,$ $z = 0$

d Normal line is parallel to the z-axis

2011 MATHS 1

Q6 a) Rectangular (x, y, z) Spherical $(\overset{x}{\rho\sin\phi\cos\theta}, \overset{y}{\rho\sin\phi\sin\theta}, \overset{z}{\rho\cos\phi})$

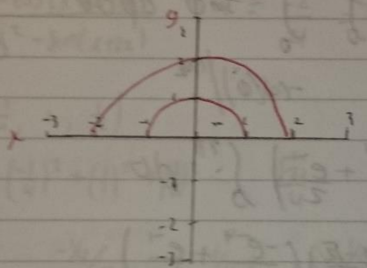
Jacobian $\rho^2 \sin\phi$

b i. $x^2+y^2+z^2=1$ is Sphere radius 1 centre $(0,0,0)$

ii. $x^2+y^2+z^2=4$ is Sphere radius 2 centre $(0,0,0)$

iii. $z=0$ is the xy plane

iv.



v. $1 \le \rho \le 2 \qquad 0 \le \theta \le 2\pi \qquad 0 \le \phi \le \pi/2$

$$\int_0^{2\pi}\int_0^{\pi/2}\int_1^2 \rho^2 \sin\phi \; d\rho\, d\phi\, d\theta$$

$$\left.\frac{\rho^3}{3}\right|_1^2$$

$$\frac{7}{3}\int_0^{2\pi}\int_0^{\pi/2} \sin\phi \; d\phi\, d\theta$$

$$+\frac{7}{3}\int_0^{2\pi} \left.\cos\phi\right|_0^{\pi/2} \; d\theta$$

$$+\frac{7}{3}(2\pi) = +\frac{14}{3}\pi$$

Vr $\delta(x,y,z) = \dfrac{e^{-(x^2+y^2+z^2)}}{\sqrt{x^2+y^2+z^2}}$ $\qquad x^2+y^2+z^2=\rho^2$

$$\int_0^{2\pi}\int_0^{\pi/4}\int_1^2 \frac{e^{-\rho^2}}{\rho}\rho^2\sin\phi$$

$$\iint\int_1^2 \rho e^{-\rho^2}\sin\phi \; d\rho\, d\phi\, d\theta$$

$$-\frac{e^{-\rho^2}}{2}\Big|_1^2$$

$$\left(\frac{-e^{-4}}{2}+\frac{e^{-1}}{2}\right)\int_0^{2\pi}\int_0^{\pi/4}\sin\phi\; d\phi\, d\theta$$

$$\left(\frac{e^{-4}}{2}+\frac{e^{-1}}{2}\right)\int_0^{2\pi} -\cos(\phi)\Big|_0^{\pi/4}$$

$$\left(\frac{2-\sqrt{2}}{2}\right)\left(\frac{-e^{-4}}{2}+\frac{e^{-1}}{2}\right)\int_0^{2\pi} d\theta$$

$$=(2\pi)\left(\cancel{\frac{2-\sqrt{2}}{2}}\right)\left(\frac{-e^{-4}}{2}+\frac{e^{-1}}{2}\right)$$

$$= (-e^{-4}+e^{-1})\pi$$