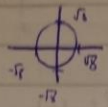
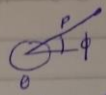
Maths 1 - Triple Integration

2013 Q4 A $z = \sqrt{x^2+y^2}$ $\qquad x^2+y^2+z^2=8$



b Spherical coords (p, θ, ϕ)



$x = p\sin\phi\cos\theta \quad y = p\sin\phi\sin\theta \quad z = p\cos\phi \quad$ Jacobian $p^2\sin\phi$

$0 \le p \le \sqrt{8} \qquad 0 \le \theta \le 2\pi$

$z = \qquad z = \sqrt{x^2+y^2}$

$p\cos\phi = \sqrt{p^2\sin^2\phi\cos^2\theta + p^2\sin^2\phi\sin^2\theta}$

$p\cos\phi = \sqrt{p^2\sin^2\phi}$

$p\cos\phi = p\sin\phi$

$\cos\phi = \sin\phi$

$\cos\phi = \sin\phi$ or $\phi = 45^\circ \quad \therefore \quad 0 \le \phi = \pi/4$

$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\sqrt{8}} p^2\sin\phi \; dp\, d\phi\, d\theta$$

$$\frac{p^3}{3}\Big|_0^{\sqrt{8}} \sin\phi$$

$$\frac{16\sqrt{2}}{3}\int_0^{2\pi}\int_0^{\pi/4} \sin\phi \; d\phi\, d\theta$$

$$\frac{16\sqrt{2}}{3}\int_0^{2\pi} -\cos\phi\Big|_0^{\pi/4} \; d\theta$$

$$\frac{16\sqrt{2}}{3} \; \frac{2\sqrt{2}}{2} \int_0^{2\pi} d\theta$$

$$\frac{-16+16\sqrt{2}}{3}(2\pi) = \frac{-32+32\sqrt{2}}{3} = \text{volume}$$

c Mass: $\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\sqrt{8}} p\cos\phi \, p^2\sin\phi \; dp\, d\phi\, d\theta \qquad u = \cos\phi \quad du = -\sin\phi$

$$\frac{p^4}{4}\Big|_0^{\sqrt{8}} \int_0^{2\pi}\int_0^{\pi/4} \cos\phi\sin\phi \; d\phi\, d\theta$$

$16 \qquad \int_0^{\pi/4} -u \, du$

$$\int_0^1 \frac{+u^2}{2}\Big|_0^{\pi/4}$$

$$-\cos 2\phi$$

$16(2\pi)\left(-\frac{1}{2}+1\right) = 16\pi = \text{mass.}$

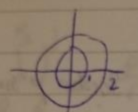
2012 Q6 Rectangular coord x, y, z

cylindrical $x = \rho\cos\theta$ $y = \rho\sin\theta$ z function of ρ

Surface $x^2+y^2=1$ = circle radius 1, centre (0,0)

Surface $x^2+y^2=4$ = circle rad 2, centre (0,0)

$z=0$ is plane $z=3$ is plane



V. $x = \rho\cos\theta$ $y = \rho\sin\theta$ $z = z$

$$\int_0^3\int_1^2\int_0^{2\pi} \rho \; d\rho\, d\theta\, dz$$

$$\frac{\rho^2}{2}\Big|_1^2$$

$$\frac{3}{2}\int_0^3\int_0^{2\pi}$$

$$\frac{3}{2}(3)(2\pi) = 9\pi = \text{vol}$$

v. $\delta(x,y,z) = e^{-(x^2+y^2+z)}$ $e^{-(r^2+z)}$

$$\int_0^3\int_1^2\int_0^{2\pi} re^{-(r^2+z)}\, d\theta\, d\rho\, dz$$

$$2\pi \int_0^3\int_1^2 re^{-(r^2+z)}$$

$$\int_0^3\int_1^2 r(e^{-r^2}+e^{z})$$

$$\int_0^3 e^{-z} \quad \int_1^2 re^{-r^2}$$

$u = -r^2$ $du = -2r\,dr$

$$-e^{-z}\Big|_0^3 \quad \frac{1}{2}\int_1^2 u\; du$$

$$(-e^{-3}+1)(2\pi)\frac{1}{-2}\cdot\frac{u^2}{2}\Big|_1^2$$

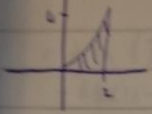
$$\frac{1}{-2}e^{u}$$

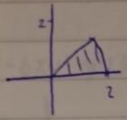
$$(-e^{-3}+1)(2\pi)(e^{-4}-e^{-1})$$

$$-\pi(1-e^{-3})(e^{-4}-e^{-1})$$

Maths 3 Double Integration

2013 Q3





b $\int_{\sqrt{y}}^{2} \int_{0}^{4} (x^3+1)^{\frac{1}{2}}\, dy\, dx$

$y(x^3+1)^{\frac{1}{2}} \Big|_0^4$

$4 \int_{\sqrt{y}}^{2} (x^3+1)^{\frac{1}{2}}\, dx$

$= 4 \left[\frac{3x^2}{2\sqrt{x^3+1}} \right] \Big|_{\sqrt{y}}^{2}$

$= 4 \left(\frac{6}{\sqrt{9}} - \frac{3y}{2\sqrt{y\sqrt{y}+1}} \right. \qquad = 8 - \frac{6y}{\sqrt{y \cdot \sqrt{y}+1}}$

c $x = r\cos\theta \qquad y = r\sin\theta \qquad 0 \le \theta \le \pi/4 \quad 0 \le r \le 2$

$r^2\cos^2\theta \, (r^2\cos^2\theta + r^2\sin^2\theta)$

$r^2\cos^2\theta \, r^2 \, r \cdot d\theta\, dr$

$\int_0^2 \int_0^{\pi/4} r^5 \cos^2\theta \, d\theta\, dr$

$= \int_0^2 r^5 \int_0^{\pi/4} \cos^2\theta \, d\theta\, dr$

$\left[\frac{r^6}{6}\right]\Big|_0^2 \quad \left[\frac{1}{2} \left(x + \frac{1}{2}\sin 2x \right) \right]_0^{\pi/4}$

$32/3 \quad \left[\frac{1}{2} \left(\pi/4 + \frac{1}{2}\sin \pi/2 \right) - 0 \right]$

$= 32/3 \, (\pi/8 + 1/4) \quad = \quad 4\pi/3 + 8/3$

2012 Q3 Maths 3

2012 Q3

$$z = \ln \frac{\sqrt[3]{3\cos(2x-y)+6x^2-6xy^2-y^3+31}}{2}$$

$$= \frac{1}{3}\ln\left(3\cos(2x-y)+6x^2-6xy^2-y^3+31\right) - \ln(2)$$

$$\frac{df}{dx} = \frac{-6\sin(2x-y)+12x-6y^2}{(3\cos(2x-y)+6x^2-6xy^2-y^3+31)}\Bigg|_{(1,2)} \quad \frac{-12}{24} \quad = \cancel{\tfrac{12}{4}} \; \cancel{-\tfrac{3}{4}} \; -\frac{1}{2}$$

$$\frac{df}{dy} = \frac{3\sin(2x-y)-12xy-3y^2}{(3\cos(2x-y)+6x^2-6xy^2-y^3+31)}\Bigg|_{(1,2)} \quad \frac{-36}{8} \quad = \cancel{\tfrac{12}{2}} \; -\frac{3}{2}$$

$$z = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

$$z = \tfrac{1}{3}\ln(8) - \ln(2) + \frac{-1}{2}(x-x_0) \quad -\tfrac{3}{2}(y-y_0)$$

$$z = 0 \quad -\frac{1}{2}(x-1) \quad -\tfrac{3}{2}(y-2)$$

$$z = -\frac{x}{2} + \frac{1}{2} \quad -\frac{3}{2}y + 3$$

$$z + \frac{x}{2} + \frac{3}{2}y = 3.5$$

$$2z + x + 3y = 7 \quad = \text{eq}^n \text{ of tangent plane}$$

b Points of intersection

~~x-axis $x=0$. $2z+3y=8$ $(z=2\ y=4)$~~

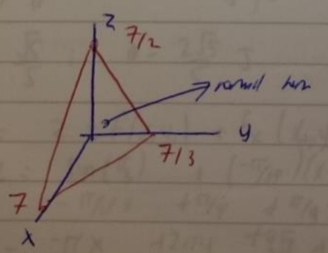
~~y-axis~~ $y=0$.

(i) x-axis $y=z=0$ $x=7$ $(7,0,0)$

(ii) y-axis $x=z=0$ $y=7/3$ $(0, 7/3, 0)$

(iii) z-axis $y=x=0$ $z=7/2$ $(0,0,7/2)$

c



d normal to $n = (-f_x(x_0, y_0), -f_y(x_0, y_0), 1)$

$n = \left(\frac{1}{2}, \frac{3}{2}, 1\right)$

$r_t = r_0 + t(r_1)$

$(1, 2, 0) + t(1/2, 3/2, 1)$

$x = 1 + 1/2 t \quad y = 2 + 3/2 t \quad z = 0 + t$

$(1, 2, 0) + t(1, 3, 2)$

$x = 1 + t \quad y = 2 + 3t \quad z = 2t$ = parametric eqn