Laboratory 5: State feedback and linearization of nonlinear models.

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Abstract--- Robotic manipulators are an essential part of the automation industry and can be controlled through different methods. In this document, the modeling of a robotic manipulator made up of a DC motor with real constants coupled to a bar through a flexible joint. The manipulator is controlled with feedback for the linearized system, which allows the angular speed of the arm to be controlled. And an observer is made to control the position of the arm.

Abstract--- Robotic manipulators are an essential part of the automation industry and can be controlled through different methods. In this document, the modeling of a robotic manipulator made up of a DC motor with real constants coupled to a rod by a flexible joint is performed. The manipulator is controlled with feedback for the linearized system, which allows controlling the angular velocity of the arm. And an observer is made to control the position of the arm.

Keywords--- Manipulator, Feedback, State, Observer.

General Objective--- Control a nonlinear system using feedback and state observers.

Specific objectives--

* Check the mathematical model of the guide using the Newton-Euler and Euler-Lagrange theory.

* Find the linear representation of a nonlinear system, considering the equilibrium and operating points.

* Realize a state feedback controller that ensures error in a stable state equal to zero for step, ramp and acceleration input.

* Design the state observer that allows the estimation of the states and the implementation of the controller.

1. INTRODUCTION

Theoretical framework

A modern control system can have many inputs and many outputs, and these are interrelated in a complex way. State-space methods for the analysis and synthesis of control systems are best suited to dealing with multi-input, multi-output systems that are required to be optimal in some sense.

This method is based on the description of the system in terms of *n* first-order difference or differential equations, which can be combined into a first-order difference or differential matrix equation. The use of matrix notation greatly simplifies the mathematical representation of systems of equations.

The methods in the state space allow you to include initial conditions within the design. This is a very important feature not considered in conventional design methods [1].

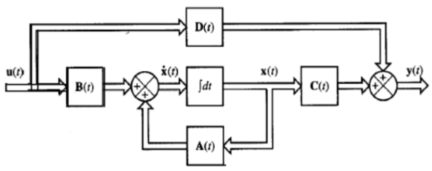
State: The state of a dynamical system is the smallest set of variables (called state variables) such that the knowledge of said variables at $t = t0$ together with the knowledge of the input for $t \geq t0$. They completely determine the behavior of the system for any time $t \geq t0$.

State variables: The state variables of a dynamic system are those that make up the smallest set of variables that determine the state of the dynamic system. If to fully describe the behavior of a dynamical system, at least n variables $x1$, $x2$, ⋯ , xn are required (so that once the input for $t \geq t0$ and the initial state at $t = t0$, the future state of the system is completely determined), then these n variables are considered a set of state variables.

State vector: If n state variables are needed to completely describe the behavior of a given system, then these n state variables can be considered as the n components of a vector $\vec{x}$. Such a vector is known as a state vector. A state vector is, therefore, a vector that uniquely determines the state $\vec{x}(t)$ of the system for any time $t \geq t0$, once the state at $t = t0$ is given and the input $u(t)$for $t > t0$.

State space: The n dimensional space whose coordinate axes are formed by the *eje* $x1$, *eje* $x2$, ⋯ , *eje* xn is known as the state space. Any state can be represented by a point within that state space [2].

Image **1** shows the basic diagram of the representation by system states and the equations.



$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

***Image 1**: Representation of states of a system.*

State feedback regulators are based on the formulation of the control law $u = -K\vec{x}$, where $\vec{x}$ are the states of the system (measurable and/or estimated as applicable), and K is the feedback gain. If zero steady state error is wanted in the feedback design, an additional feedback loop is made, where the output is directly compared to the desired reference (In(t)-Out(t)). This error must then be multiplied by a gain and integrated to be added to the aforementioned feedback terms.

State Observers: When not all states $\vec{x}$ (as is the common case), an observer can be constructed to estimate them, while only measuring the output $(t) = C\vec{x}(t)$. The scheme is shown in image 2.

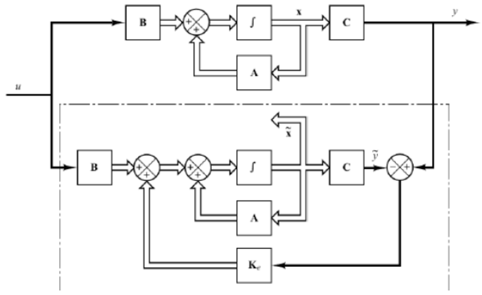


***Image 2** : Representation of states of a system.*

The observer is basically a copy of the plant; has the same input and almost always the same differential equation. An extra term compares the current output measurement with the estimated output $\hat{y}(t)$; causing the estimated states to approach the actual values of the states. The dynamics of the observer error is given by the poles of $(A - LC)$. [3]

2. MATERIALS

* Software: MATLAB ®

* White coat.

3. PROCEDURE

To carry out the practice, first assign values to the constants of the robotic manipulator. Then, for both the motor and the bar, there must be constants that are preferably obtained from practice, since then the application would not have inconsistent results and could be applied to a case which is going to be tested physically.

The schematic of the manipulator to be modeled can be found in **image 3**.

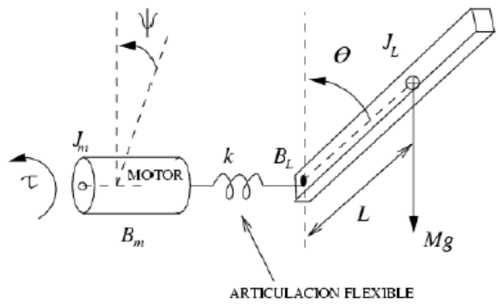


Image 3: *Flexible robotic manipulator model*

Motor:

For the DC motor that rotates the bar, the physical constants obtained from a real motor are used, during a practice in the engineering undergraduate laboratories of Carnegie Mellon University located in Pittsburg, USA. [4]

Based on the referenced constants, the moment of inertia of the motor is:

$$J_m = 3.2284x10^{-6}\ [kg \cdot m^2]$$

And the viscous friction of the motor:

$$B_m = 3.5077x10^{-6}\ [N \cdot m \cdot s]$$

Bar:

For the bar, it must be taken into account that it should not have a very large mass, and dimensions similar to that of a real-size arm.

The following values are chosen:

m = 1kg.

a = 0.05m

b = 1m

L = 0.5m

Being a and b the dimensions of the bar observed in **image 4**.

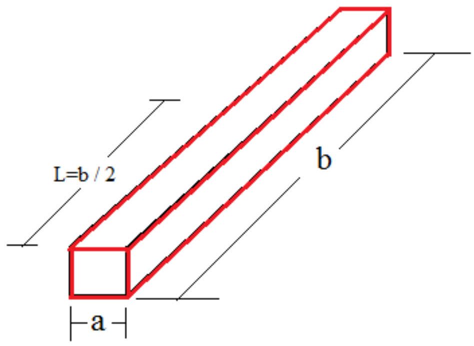


Image 4*: Robotic manipulator: model of the bar.*

And since the moment of inertia of the bar must be with respect to one end of it (which is where it will be coupled to the motor), the equation of table 10.2 of the book Physics for science and engineering [5] and that It is located in **image 5** of this document.

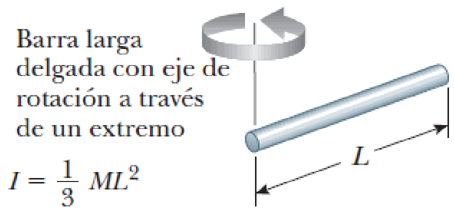


Image 5: *Moment of inertia of a bar*

Applying the named equation, the moment of inertia for the bar of this application is as follows.

$$J_L = \frac{1}{3}(1kg)(1m)^2$$

$$J_L = 0.33\ [kg \cdot m^2]\ \psi$$

Newton-Euler motor modeling:

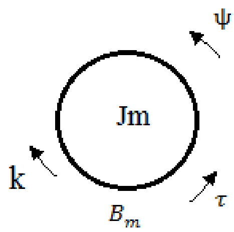


Image 6: *Motor diagram*

from **image 6** , the position of the motor axis is subtracted from the angular position of the arm due to the fact that they are coupled.

$$J_m\ddot{\psi} + B_m\dot{\psi} - k(\theta - \psi) = \tau \qquad (1)$$

Euler-Lagrange motor modeling:

The diagram of the motor present in image 6 is taken to perform the modeling by the Euler-Lagrange equation:

$$\frac{d}{dt}\left(\frac{dL}{d\dot{q}}\right) - \frac{dL}{dq} + \frac{dP}{d\dot{q}} = Qi$$

Where the variables have the following equivalence for this model.

$Lagrangian: L = K_e - V$

$Kinetic\ energy: K_e = \frac{1}{2}J_m\dot{\psi}^2$

$Potential\ energy: V = \frac{1}{2}K(\psi - \theta)^2$

$Dissipative\ energy: P = \frac{1}{2}B_m(\dot{\psi})^2$

$Qi = Forces\ generalized$

$q = Generalized\ Coordinate$

Where for this application the generalized coordinate will be ψ and the generalized force τ .

The Lagrangian is solved first.

$$L = \frac{1}{2}J_m\dot{\psi}^2 - \frac{1}{2}K(\psi - \theta)^2$$

And each one of the terms of the Lagrange equation are solved, to later join them, so that the development is more simplified.the Lagrangian first with respect to $\dot{\psi}$ and then with respect to time, resulting:

$$\frac{d}{dt}\left(\frac{dL}{d\dot{\psi}}\right) = \frac{d}{dt}(J_m\dot{\psi}) = J_m\ddot{\psi}$$

Now, derive the Lagrangian with respect to the angle.

$$\frac{dL}{d\psi} = - k(\psi - \theta)$$

And finally, the derivative of the dissipative energy with respect to the angular velocity.

$$\frac{dP}{d\dot{\psi}} = B_m\dot{\psi}$$

And finally, the terms in the general equation are replaced:

$$J_m\ddot{\psi} - (- k(\psi - \theta)) + B_m\dot{\psi} = \tau$$

Simplifying and rearranging, the equation is equal to the equation (1).

$$J_m\ddot{\psi} + B_m\dot{\psi} - k(\theta - \psi) = \tau$$

Newton-Euler bar modeling:

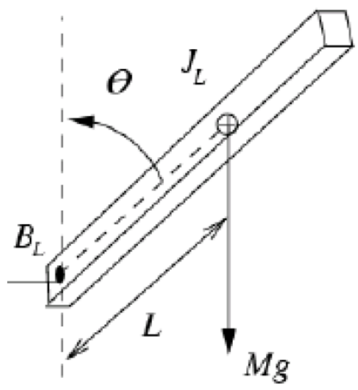


Image 7: *Variables and constants of the bar*

Where we have the mass M, of 1kg, the moment of inertia and the viscous friction of the bar. What are the constants that directly influence after modeling, to replace to obtain true data.

And the force diagram is taken from the diagram proposed for the bar in **image 7**:

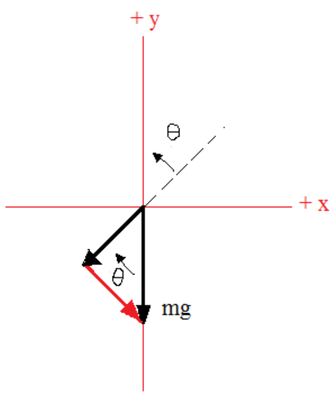


Image 8: *Variables and constants of the bar*

Image **8** shows the free-body diagram of the bar, and proceeds to obtain the equation (two).

$$-B_L\dot{\theta} - k(\theta - \psi) - mgLsin\theta = J_L\ddot{\theta}$$

Solving and setting (2) equal to 0, we get (3).

$$J_L\ddot{\theta} + B_L\dot{\theta} + K(\theta - \psi) + mgLsin\theta = 0$$

For when $\bar{\tau} = 0$

Balance points:

Engine:

From (1), if the derivatives are equal to zero, it remains as shown in 4).

$$-k(\bar{\theta} - \bar{\psi}) = \bar{\tau} \qquad (4)$$

Knowing

$$\bar{\theta} = \bar{\tau}$$

$$\bar{\psi} = 0$$

Bar:

From (3), the variations are also equal to zero, leaving the equation as follows:

$$k(\bar{\theta} - \bar{\psi}) + mgLsin\bar{\theta} = 0$$

$$mgLsin\bar{\theta} = 0$$

This occurs when the angles are small , less than 5 degrees, which is the case of the application.

$$\bar{\theta} = 0$$

Linear system:

Motor:

Starting from (1), $\ddot{\psi}$.

$$J_m\ddot{\psi} = -B_m\dot{\psi} + k(\theta - \psi) + \tau$$

Dividing the equation by J_m.

$$\ddot{\psi} = \frac{-B_m}{J_m}\cdot\dot{\psi} + \frac{k}{J_m}\cdot(\theta - \psi) + \frac{\tau}{J_m}$$

Their nominal values are subtracted, in order to simplify later.

$$\ddot{\psi} - \bar{\ddot{\psi}} = \frac{-B_m}{J_m}(\dot{\psi} - \bar{\dot{\psi}}) +$$

$$\frac{k}{J_m}(\theta - \bar{\theta}) - \frac{k}{J_m}(\psi - \bar{\psi}) + \frac{1}{J_m}(\tau - \bar{\tau})$$

Leaving in terms of variations.

$$\ddot{\psi}d = \frac{-B_m}{J_m}\cdot\bar{\dot{\psi}}d + \frac{k}{J_m}\cdot\theta d - \frac{k}{J_m}\cdot\psi d \quad (5)$$

Bar:

Clearing from (2) $\ddot{\theta}$, and passing the variables to variations, the equation is as follows.

$$\ddot{\theta}d = \frac{-B_L}{J_L}\dot{\theta}d - \left(\frac{k+mgL}{J_L}\right)\theta d + \frac{k}{J_L}\psi d \ (6)$$

Before passing to the representation in state space, changes of variable are made, because in the equation of state space there are no second derivatives, the changes are as follows:

For the angular position of the robotic manipulator bar.

$$\theta = \theta_1$$

$$\dot{\theta}_1 = \theta_2$$

$$\ddot{\theta}_1 = \dot{\theta}_2$$

And for the angular position of the motor:

$$\psi = \psi_1$$

$$\dot{\psi}_1 = \psi_2$$

$$\ddot{\psi}_1 = \dot{\psi}_2$$

With equations (5) and (6) matrix A is created.

Image **9** shows the state representation of the manipulator flexible with which the practice is developed.

$$\begin{bmatrix} \dot{\theta}_1 d \\ \dot{\theta}_2 d \\ \dot{\psi}_1 d \\ \dot{\psi}_2 d \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -(k+mgL)/J_L & -B/J_L & k/J_L & 0 \\ 0 & 0 & 0 & 1 \\ k/J_m & -k/J_m & -k/J_m & -B_m/J_m \end{bmatrix} \begin{bmatrix} \theta_1 d \\ \theta_2 d \\ \psi_1 d \\ \psi_2 d \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/J_m \end{bmatrix} \tau d$$

$$y(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 d \\ \theta_2 d \\ \psi_1 d \\ \psi_2 d \end{bmatrix}$$

Image 9: *Representation of system states.*

The variable k is assigned a value of 3*10^-4 N/m, which represents the coefficient of elasticity of the motor shaft.

Design criteria:

It is decided to implement a closed-loop settling time of 2 s with a damping coefficient of 0.9. With this and a desired margin of error of 1%, a natural frequency of 2.5 rad/s is given.

4. ANALYSIS OF RESULTS

Linear system

Step input:

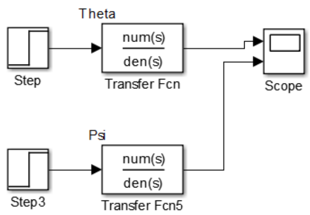


Image 10: *Open loop step input.*

Image **10** shows the block diagram that allows the open-loop analysis of the variables θ and ψ before a step input at t = 1 s.

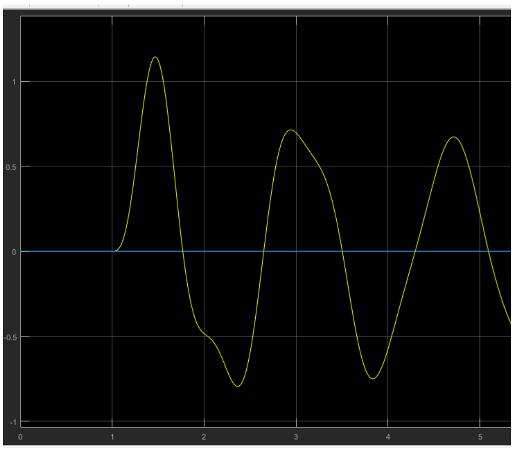


Image 11: *Variable* θ.

the **Image 11** open-loop response to the step input of the variable θ. As can be seen, the system exhibits oscillations, but its progressive decrease in amplitude indicates that it can be said to be stable when "t" tends to infinity.

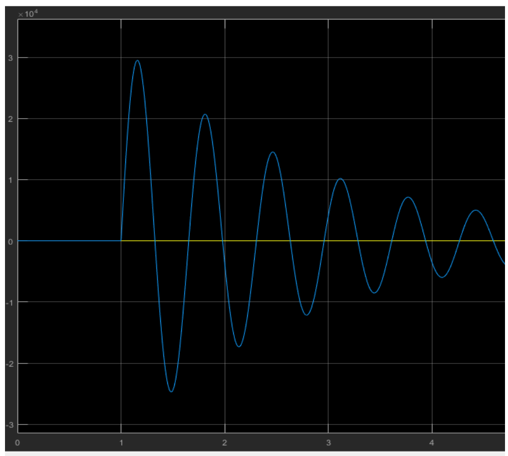


Image 12: *Variable* ψ

The **Image 12** open-loop response to the step input of the variable ψ. The same oscillations and progressive amplitude decrease are presented, it is observed that the system tends to stabilize when t tends to infinity, it can be seen that the scale on the vertical axis is x10^4, so it is said that the variable ψ takes much longer than the variable θ to be considered stable.

Observing the result obtained in **annex 1,** it is seen that at the moment of making a term-by-term association of the denominator of the transfer function with the desired polynomial, a system of 4 equations with 5 unknowns is generated, with this, it is decided leave the variable Ki in terms of the variable k2 and assign values to the latter such that the error is as close to zero as possible.

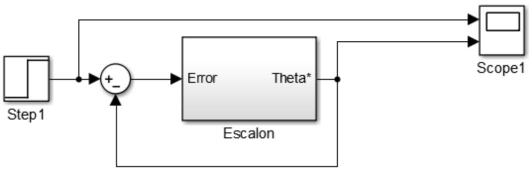


Image 13: *loop diagram*

- **Image 13** shows the system diagram for feedback step input, simplifying the plant and the control with Simulink tools.

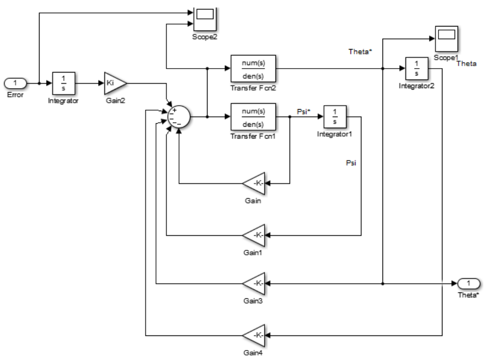
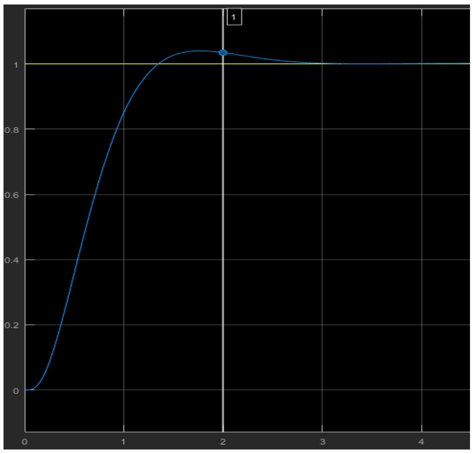


Image 14: *Feedback control: step*

Image **14** presents the particular diagram of the system with status feedback and the integral action.

The values of the variables obtained are shown in **image 16.**



	Time	Value
1	2.000	1.034e+00
2	7.500	1.002e+00

Image 15: *Scope: Controlled system*

At t = 2s the output presents an error of 3%, and when t tends to infinity, an error of 0.2% is presented, this due to the uncertainty presented by the values k2 and Ki.

```
%Entrada escalòn
K1e=5.46;
K2e=-36.32;
K3e=1.55e-4;
K4e=1.92e-3;
Ki=-22.43-K2e;
```

Image 16: *Feedback constants: step input.*

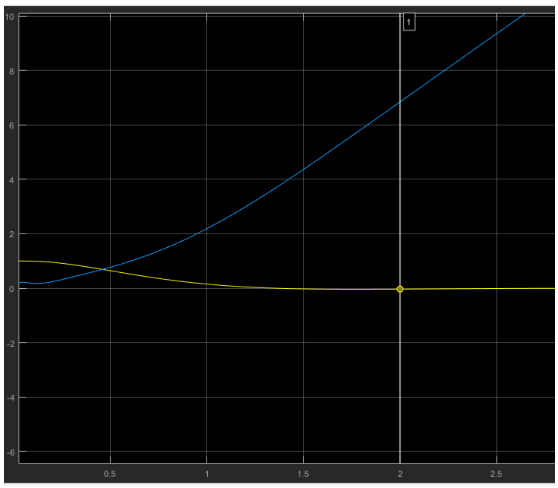


Image 17: *Error signal (Yellow) and control signal (Blue)*

As you can see, the error signal does not go to zero as fast, this is because the control signal is soft at the beginning, although it tends to be very slow. large as t approaches infinity.

Ramp input:

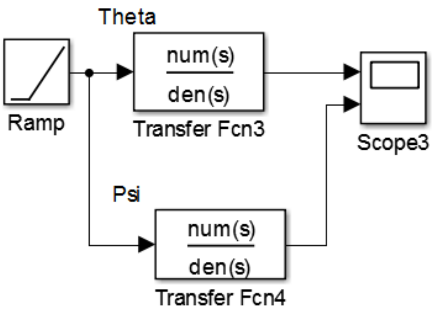


Image 18: *Open loop ramp input.*

It is performed both for the angular speed of the motor and for that of the bar.

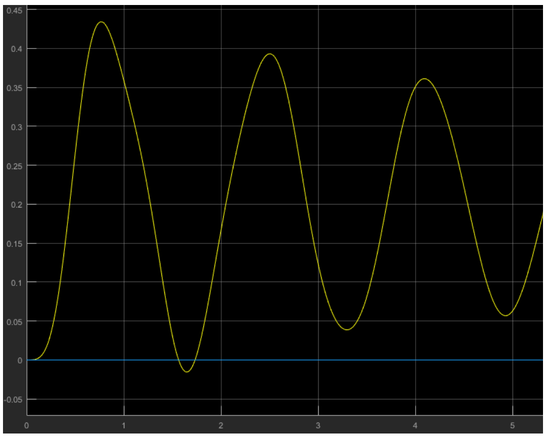


Image 19: Variable θ.

Image **19** shows the output of the angular velocity of the bar before a ramp input, in open loop, a low amplitude is observed, which reaches a little more than 0.4 u.

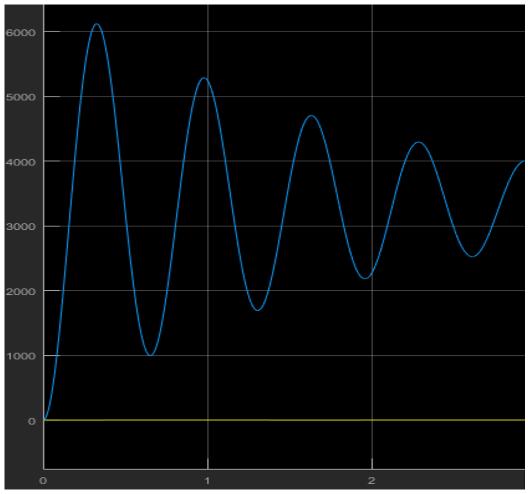


Image 20: Variable ψ

Image **20** shows the open-loop response to a ramp input of the variable ψ', and a very large amplitude is observed, in addition to oscillation in the system.

Carrying out the same procedure set out in **annex 1,** the following control system is obtained:

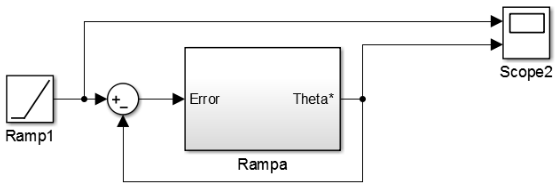


Image 21: *loop diagram*

- **Image 21** shows the diagram of the closed-loop system, which is internally given by what is shown in **image 22.**

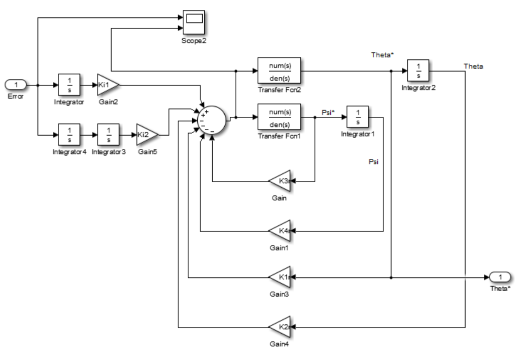
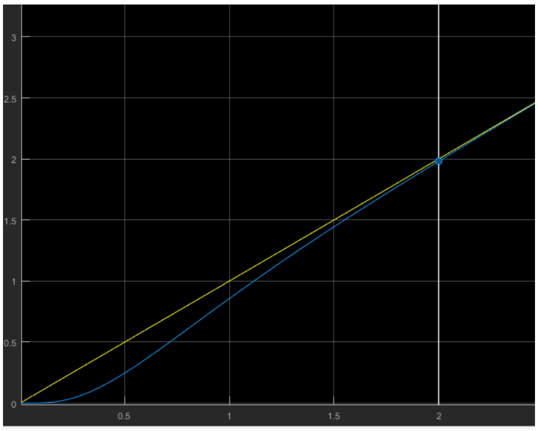


Image 22: *Feedback control: ramp.*

The answer obtained is the following:



	Time		Value
1	2.000		1.981e+00
2	10.000		1.000e+01
ΔT	8.000 s	ΔY	8.019e+00

It is observed that at t = 2s there is an error with respect to the input of at least 1% and that when t tends to infinity the output follows the input, so it is said that the constant k2 used is adequate in such a way that the design parameters are met.

The constants used in this feedback are presented in **image 24**, in the same way, the constant Ki1 remains in terms of the constant k2, so the system is expected to present deviations in the design parameters.

```
%Entrada Rampa
K1r=55.5;%Constante control theta
K2r=-96.32;%Constante control theta
K3r=2.28e-4;%Constante control Psi
K4r=5.5e-3; %Constante control Psi
Ki1=110.7-K2r;%Constante proporcional
Ki2=229.65;%Constante proporcional
```

Image 24: *Feedback constants: ramp input.*

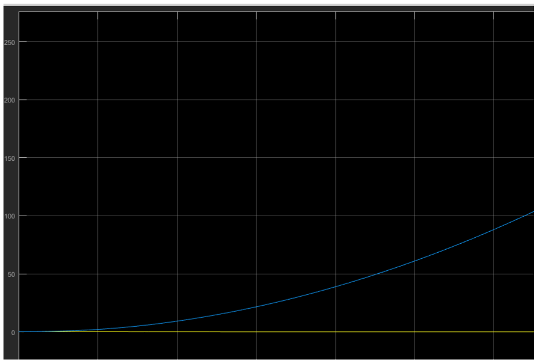
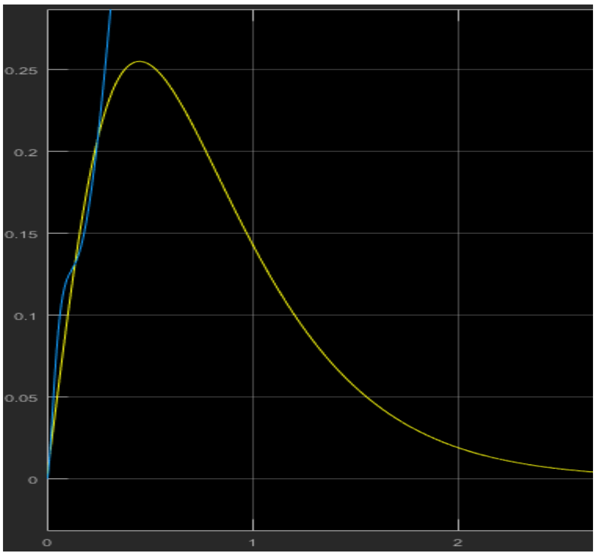


Image 25: *Scope of error and control signals.*

the **Image 25** error signals (Yellow) and the control signal (Blue), it can be seen that the error is practically zero, but if you zoom in, it appears that :



Acceleration input:

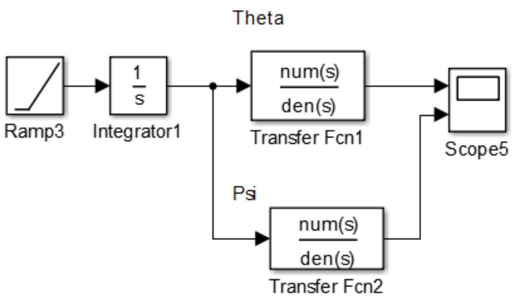


Image 26: Open loop acceleration input.

Open-loop analysis is performed for both Psi' and theta'.the **Image 26** diagram and before an integrated ramp input, which produces an acceleration.

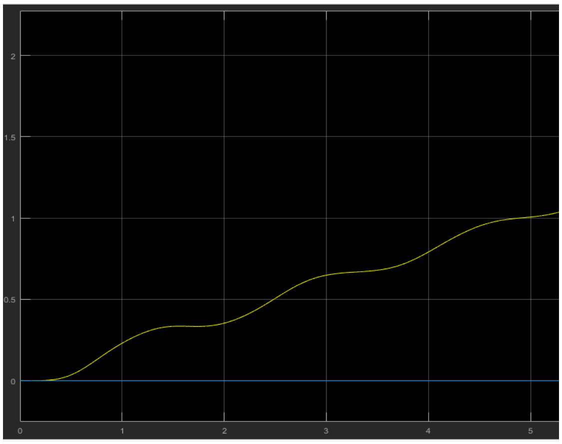


Image 27*: Variable* θ.

The response to acceleration input, of the variable theta*, is shown in **image 27**. It can be seen that the angular velocity of the arm has a low amplitude, despite facing an acceleration input.

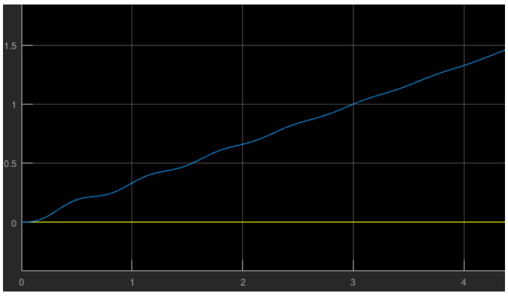


Image 28: *Variable* ψ

The open-loop response to an acceleration input for the variable ψ can be seen in **image 28**, the amplitude is much larger than the variable theta, since the vertical scale is at x10^4.

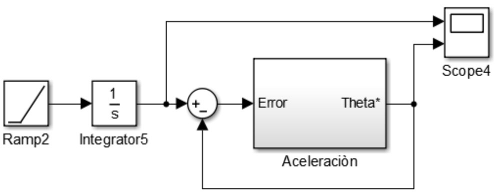


Image 29: *Closed loop diagram.*

In **image 29** , although a block diagram is observed, whose internal structure is observed in **image 30.** Its function is to carry out the corresponding control.

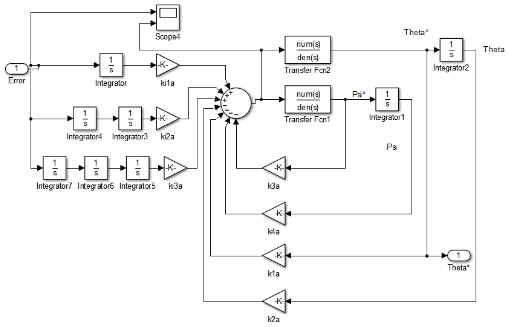


Image 30:*Feedback control: acceleration.*

The response obtained is shown in **image 31.**

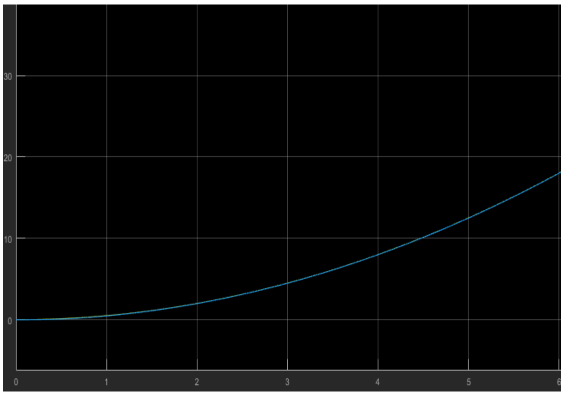


Image 31: *Scope4 - Input and controlled signal.*

As can be seen, both signals are practically the same, which would indicate a zero error and a very strong control action since the error practically becomes zero et = 0s.

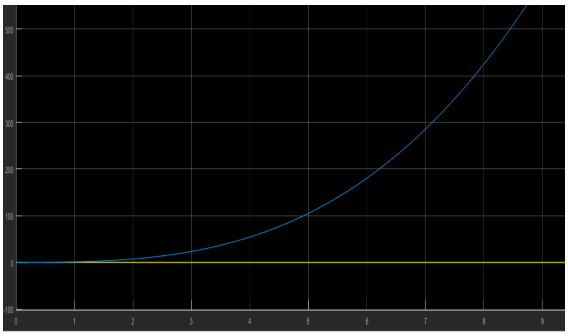


Image 32: *Error signal (Yellow), control signal (Blue).*

The constants obtained for this feedback can be found in **image 33.**

```
%Entrada Aceleraciòn
K1a=186.11;
K2a=-96.32;
K3a=3e-4;
K4a=10.7e-3;
Ki1a=1388-K2a;
Ki2a=4639;
Ki3a=5167;
```

Image 33:*Acceleration feedback constants.*

Observers:

Since the only measurable output is the angular position, the matrix C in the state space changes to be:

$$C = [1\ 0\ 0\ 0]$$

Using the matrix method (The matrix A' becomes AL*C, where L is the observer matrix, in this case it is 4x1, and the determinant of the matrix SI-A' is obtained) the observer constants are found, taking into account that it must be 10 times faster than the system, these are seen in **image 34**.

```
%Para el Observador
L1=494.8;
L2=71.2884e6;
L3=2.51e9;
L4=2.5e10;
```

Image 34: *Constants of the observer*

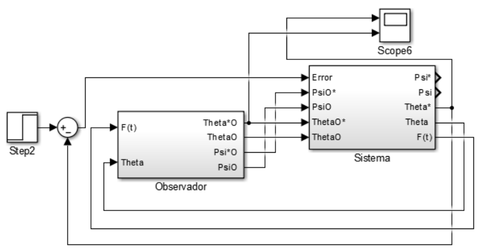


Image 35: *System with observer*

Two blocks are made to simplify the presentation of the diagrams in matlab, in one the observer is found and in the other the system is found. Each of the blocks is shown below .

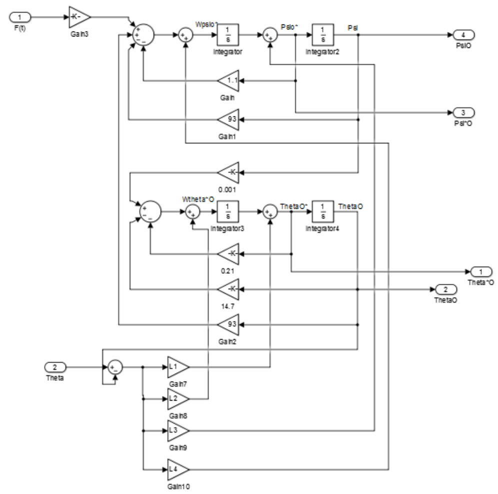


Image 36*: Observer block.*

The diagram presented in **image 36** is the content of the "Observer" block found in image 35. The observer is created from the same state equations of the system, the difference is that the variables are the required estimates, and to each equation a correction is added that depends on the values of the variables of the observer.

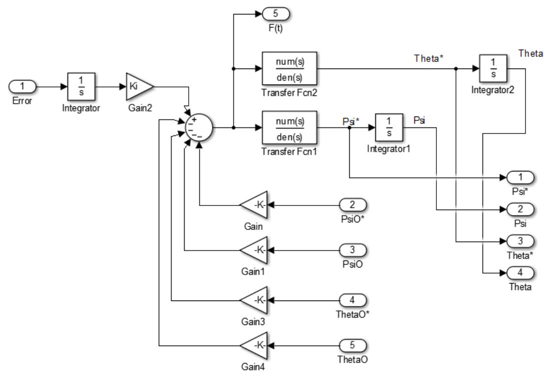


Image 37: *System block*

the **Image 37** content of the "system" block that is connected to the observer, therefore this diagram is the same as that found in **image 15** and retains its same constants, but in this case, each constant must be connected to the observer so that it fulfills its function. Although while Psi affects the system, the output being measured is Theta.

Implementing the system with observer given in **image 35** , the following response is obtained:

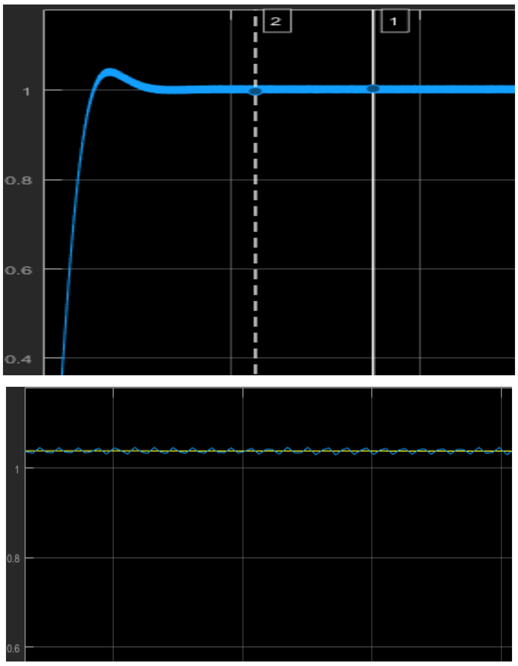


Image 38: *Response of the observer (Blue) with respect to that of the system (Yellow) for step input*

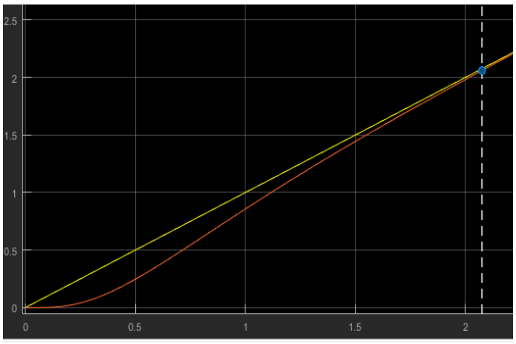


Image 39: *Response of the observer (Orange) with respect to the of the system (Blue) for ramp input*

The observer responds in the same way as the system, which indicates that the estimates are practically equal to the real values, and that the corrections to be made are zero.

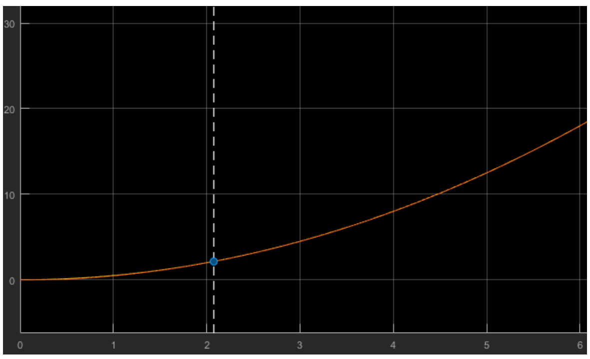


Image 40: *Response of the observer (Orange) with respect to that of the system (Blue) for acceleration input (Yellow)*

Image 40 shows the response of the linear system with an observer against an acceleration input, and the response of the The observer follows the response of the controlled system it controls to an acceleration input. That all the signals are one on top of the other indicates that the system controls the stable state with an error equal to 0 and that the observer fulfills its function correctly.

Nonlinear model

The diagram given for the nonlinear system is as follows:

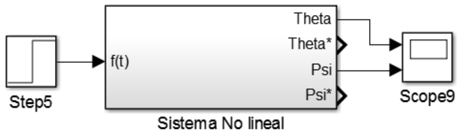


Image 41: *Diagram of the nonlinear model.*

Whose internal structure is observed in **image 42.**

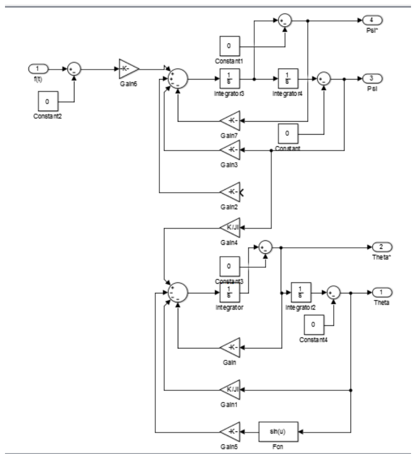


Image 42: *Internal structure of the non-linear model.*

The response of the non-linear system, before a step input (with the aim of comparing it against the linear model) is presented in **Annex 2.** As can be seen, it behaves practically in the same way, so its correct operation is verified.

Implementing the nonlinear model with the different controls designed, the following responses are obtained:

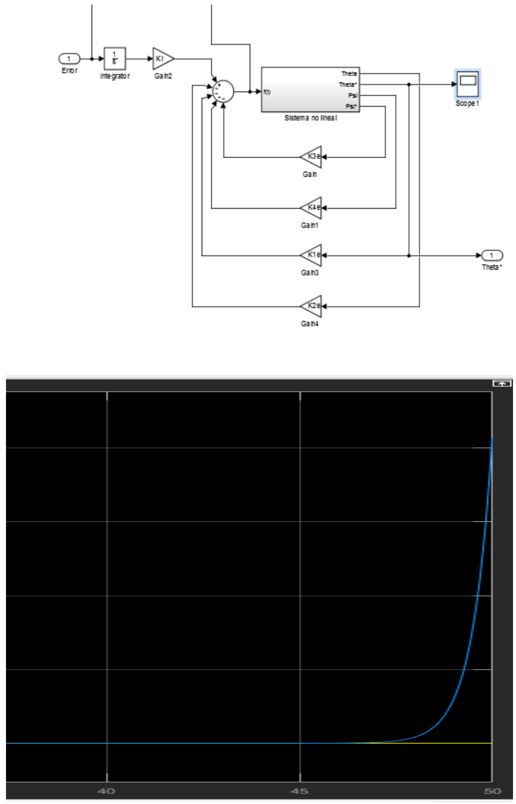


Image 43-44: *Block diagram and response of the nonlinear model with the control for a step input.*

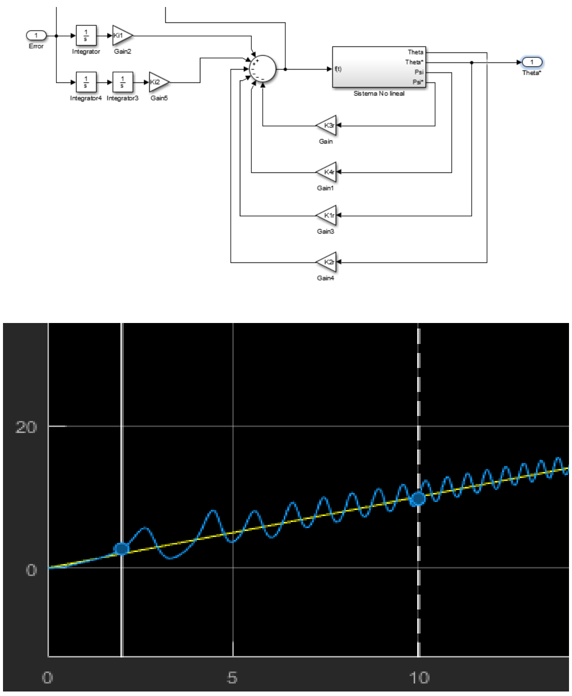


Image 45-46: *Block diagram and response of the nonlinear model with the control for a ramp input.*

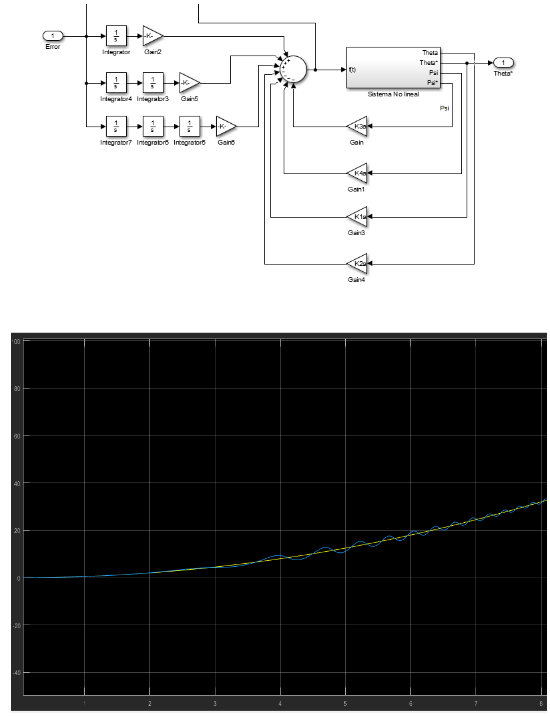


Image 47-48: *Block diagram and response of the nonlinear model with the control for an acceleration input.*

In **image 47-48** it is observed that for the acceleration input, the output of the nonlinear system tends to follow the input signal, but it presents oscillations from 3 seconds and these oscillations begin to attenuate until in a time of approximately 100s is fully controlled, with the steady state error equal to 0.

Now the observer is coupled to the nonlinear system and its response is observed.

As in **image 47-48,** the model given in **image 37 is followed,** coupling the non-linear model.

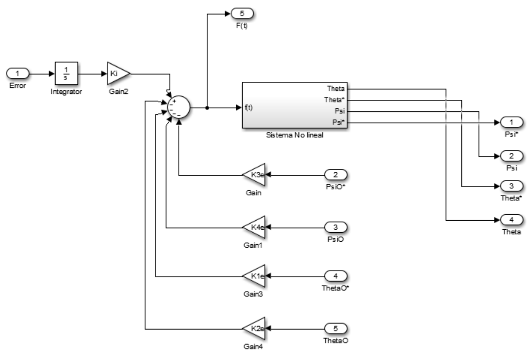


Image 49: *Block diagram of the nonlinear system with observer.*

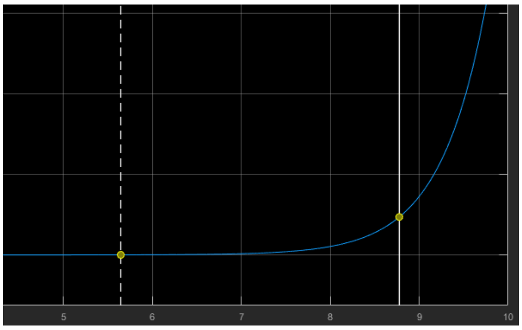


Image 50: *Response of the observer (Blue) and the system (Yellow) for a step input.*

For the case presented in **image 50,** the signals are seen to be one on top of the other, but only the observer's response (blue) and the cursors on the system's response (yellow) are observed. For this case, in a much longer time, the signal tends to a very large amplitude, although it is controlled at the beginning.

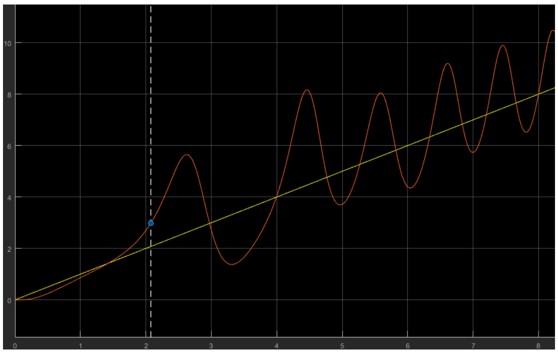


Image 51: *Response of the observer (Orange) and of the system (Blue) for a ramp input (Yellow)*

Image 51 shows the response of the observer (orange) and the response of the nonlinear system as they have an oscillatory movement around the ramp input (yellow). That this happens may be due to the only change that is made between the linear and non-linear system, which in our case where the equilibrium points are equal to zero and do not affect the change from the linear system to the non-linear system, which is adding the sine function to the controlled variable.

CONCLUSIONS

*It is verified that by modeling the system both by Newton-Euler and by Euler-Lagrange, the same equation proposed in the guide is reached.

* The linearization of the model allows the system to be analyzed at specific balance and operation points, which make the development easier when performing a linear control.

* The implementation and design of the observer constants were optimal so that it followed the output of the system and had a minimum error of almost zero.

* When implementing the nonlinear model in the system with an observer, there was an inconsistency. The problem lies in the term-by-term association of the denominator of the transfer function with the desired polynomial.

* The difference between the control of the linear system and the non-linear system is observed in the oscillations that appear in the analysis of the latter, when connected together with the observer. One of the factors that produces this change may be due to adding the function sen() to the state variable. Leaving the system non-linearized, since the equilibrium points for this case are zero and do not otherwise affect the non-linear response.

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[3] F. d. I. dl UNAM, "Control Tutorials with MATLAB," Regents of University of Michigan, August 18, 1997. [Online].

[4] << Modeling DC Motor Position, Control Tutorials >>, Developed by Professor Dawn Tilbury (Michigan University) and Professor Bill Messner (Carnegie Mellon University). National Instruments. [Online] Available at:

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ANNEX 1

This annex will show the procedure to obtain the coefficients of the back of states and of the integral action such that a steady state error equal to zero is presented to a step.

Note: The same procedure is followed for the other entries.

Procedure:

An input signal f (t) for the plant is defined, which is given by the status feedback, the integral action and the reference signal.

$$f(t) = (Ref - \theta') * \frac{Ki}{S} - k1 * \theta' - k2 * \theta - k3 * \psi' - k4 * \psi$$

Given the system equations for the variable Psi and Theta, it follows that:

$$\psi' = W\psi$$

$$W\psi' =- 1.1 * \psi' - 93 * \psi + 93 * \theta + 3.1e5 * f(t)$$

$$\theta' = W\theta$$

$$W\theta' =- 0.21 * \theta' - 14.7 * \theta + 0.001\psi$$

Using the Laplace transform , and algebraic management the following is obtained:

$$W\psi =- \theta * \left[\frac{3.1e5*k1*s-93+3.1e5*Ki+3.1e5*k2}{S}\right] - \psi\left[\frac{(1.1+3.1e5*k3)s+93+3.1e5*k4}{S}\right] + \frac{3.1e5*Ki*Ref}{S^2}$$

$$\psi =- \theta * \left[\frac{3.1e5*k1*S-93+3.1e5*Ki+3.1e5*k2}{S^2+(1.1+3.1e5*k3)S+93+3.1e5*k4}\right] + \frac{3.1e5*Ki*Ref}{S*[S^2+(1.1+3.1e5*k3)S+93+3.1e5*k4]}$$

$$W\theta = -\theta * \left[\frac{0.21s+14.7}{S}\right] - \psi\left[\frac{0.001}{S}\right]$$

$$s\theta - \theta[\frac{0.21s+14.7s}{]} - \theta[\frac{(-0.093+310ki+310k2)+310k1s}{s^3+(1.1+(3.1e5)k3)s^2+(93+(3.1e5)k4)s}] + \frac{310*ki*Ref}{s^4+(1.1+(3.1e5)k3)s^3+(93+(3.1e5)k4)s^2}$$

$$\frac{\theta}{Ref} = \frac{310*ki}{[s^5+(1.31+3.1e^5k3)s^4(107.931+3.1e^5k4+65.1e3*k3)s^3+(35.7+65.1e^3k4+4.5e6k3+310k1)s^2+(1367.1+4.5e6k4+310ki+310k2+0.093)s}$$

Here the transfer function of the system for the variable Theta was obtained; since it is required to control the angular velocity instead of the angular position of the arm, it is multiplied by a factor S on both sides of the equality:

$$\frac{\theta'}{Ref} = \frac{310ki}{s^4+(1.31+3.1e5k5)s^3+(107.7+3.1\,e5k4+65.1e3k3)s^2+(35.7+65.1e3k4+4.5e6k3+310k1)s+(1367.1+4.5e6k4+310ki+310k2+0.093)}$$

ANNEX 2