Laboratory 6: Frequency compensation, analog control position of a mass.

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Abstract--- In this laboratory practice a compensator is designed in order to control the position of a mass before certain control design parameters, the mass is located in a hoist system in which the force is exerted by a motor. The compensators allow the control of a system through the analysis of frequency, gain margin, phase, assignment of poles and zeros, totally different from PID where the control is carried out starting from time.

Abstract--- In this laboratory practice a compensator is designed in order to control the position of a mass before certain parameters of control design, the mass is in a system hoist which the force exerts on a motor. The compensators allow the control of a system through the analysis in frequency, gain margin, phase, poles and zeros assignment, totally different to the PID where the control is made starting from the time.

Keywords--- Compensator, delay, gain margin, phase margin, root locus

General Objective--- Implement a position control of a mass, based on compensator techniques.

Specific objectives--

* Plot the Bode plot of the transfer functions.

* Identify the gains and phases that a system must have to meet the desired control design.

* Design regulators from the frequency domain - Compensators.

1. INTRODUCTION

There are several methods to control a system, one of them is based on the frequency response of the system and phase or gain compensation, to meet the design requirements (advance and/or delay networks + combination of them). This is done by analyzing the gain margin and phase of the system by implementing certain parameters that will be explained later.

Theoretical framework:

Both the phase margin and the gain margin are obtained from the Bode diagrams of the system, where the phase margin is defined as the missing angle so that at the frequency where a gain of 0 dB occurs, a phase occurs. of -180°, and the gain margin is the missing gain so that at the frequency where a phase of -180° occurs, 0 dB is given. The methods used in the design of compensators are based on adding phase or gain to meet the desired objectives. Generally, for the determination of what must be compensated, it is necessary to meet position, velocity or acceleration error, and/or temporal response, reflected through the bandwidth and resonance peak of the controlled system, which, in turn , can be related to the phase margin.

One way to write the compensator equation is $Hc = K\ Ts+1\ \alpha Ts+1$, where K is the gain or attenuation provided by the compensator, T determines the position of the pole or zero (relates the cutoff frequency) and α is a factor that multiplies the coefficient that accompanies the term s, in charge to set the compensator in lagging ($0 < \alpha < 1$) or leading ($\alpha > 1$) settings.

For the design, the literature presents various ways of dealing with the procedure. See [1][2][3][4].

2. MATERIALS

* Software: MATLAB ®

* White coat.

* Plant (DC Motor, Support, pulleys, rope, mass, linear potentiometer).

* Compensator circuit (resistors, breadboard and jumpers, capacitors and trimmers, operational amplifiers).

3. PROCEDURE

Solution of the **questionnaire**:

* How is the design of regulators by compensators (networks of delay, advance and combination of them)? Illustrate with examples.

*It must be borne in mind that the compensator modifies only the transient response, not the stable state error (ess) and if you want to modify this, you must make sure before carrying out the next step by implementing integrators to the plant, and this set works..

*Determine the gain K that ensures the position, velocity or acceleration error.

* Integrating the calculated gain to the transfer function of the system, the Bode diagram is drawn and the phase and gain margin is determined, so that what is missing to meet the margins requested by the design is identified.

Lead: calculate the necessary phase angle ϕ *missing* $= \phi$ *desired* $- \phi$ *found in graph*.

Backlog: Evaluate the missing angle ϕ *missing* $= -180 + \phi$ *found in graph*, and at the frequency where it happens - ω *to be found* , the magnitude $|H(j\omega)|$.

*Now to see where the pole or zero of the compensator will be located:

Lead: determine the attenuation factor α ($K = Kc\alpha$), from $\sin(\phi$ *missing* $) = 1-\alpha\ /\ 1+\alpha$. Then find the frequency (ω), where the profit graph has the value $|H(j\omega)| = -20log\ 1\ \sqrt{\alpha}$. The frequency found is $\omega\ = 1\ /\ T\sqrt{\alpha}$.

Lag: determine the attenuation factor α ($K = Kc\alpha$), from $|H(j\omega)| = 20\log(\alpha)$. Now, find the value of T, such that, $\omega\ 10 \leq \omega cero \leq \omega\ 2$, and $\omega\ pole = \omega\ zero\ \alpha$.

*The gain of the compensator Kc is adjusted *($s+1/T\ /\ s+\ 1/\alpha T$), and it is verified if what is required by the design is fulfilled. If it is not fulfilled, the calculations must be carried out from point 2 again, adjusting the coefficients that are created until the design objectives are met.

EJEMPLO DEL DISEÑO DE UN COMPENSADOR EN ADELANTO.

Diseñe un compensador para que el sistema con

$$GH(s) = \frac{4}{s(0.5 \cdot s + 1)(0.02 \cdot s + 1)}$$

cumpla con las siguientes especificaciones:

.- $K_v \geq 40\ seg^{-1}$,

.- $MF \geq 45°$,

.- MG tal que $W \geq 15$ rad/seg. (W es la frecuencia que corresponde a una magnitud de -3 db en el diagrama de Bode de lazo abierto).

PROCEDIMIENTO DE RESOLUCION:

a) Determine la ganancia necesaria para cumplir el requisito de coeficiente de error.

Kc'=10 para que $K_v=40 \Rightarrow$

$$K_c' \cdot GH(s) = \frac{40}{s(0.5 \cdot s + 1)(0.02 \cdot s + 1)}$$

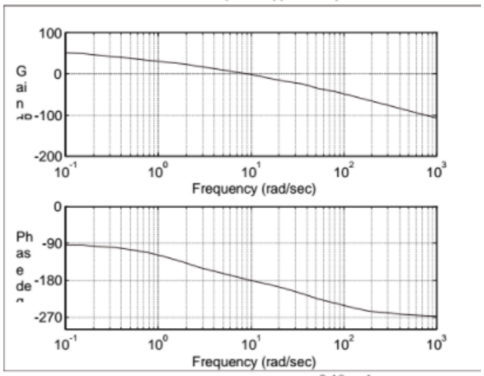
b) Trace el diagrama de Bode del sistema de lazo abierto, con la ganancia hallada en (a), y encuentre los actuales MF, MG, y W.

$$(MF \approx 5°;\ W \approx 10 rad/seg.)$$

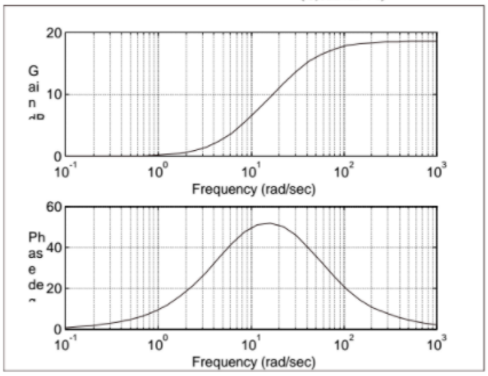
c) Determine el adelanto de fase necesario que debe ser agregado al sistema:

$$\phi = (45°-5°) + 12° = 52°$$

Sistema sin Compensar con K=40 $G(s)=\dfrac{40}{s(0.5s+1)(0.02s+1)}$



Bode del compensador en adelanto diseñado: $G(s)=\dfrac{0.19s+1}{(0{,}02242s+1)}$



d) Halle α, sabiendo que: $\operatorname{sen}\phi=\dfrac{1-\alpha}{1+\alpha}\Rightarrow\alpha=0.118$

e) Determine la frecuencia a la cual el sistema no compensado tiene una magnitud igual a $-20\log\left(\dfrac{1}{\sqrt{\alpha}}\right)=10\log\alpha=-9.28\,db$; esta frecuencia es W=15rad/seg. Fije esta W, que es la frecuencia media W_m del compensador, como la nueva frecuencia de transición de la ganancia, (en esta frecuencia se produce precisamente el ϕ_m que aporta el compensador).

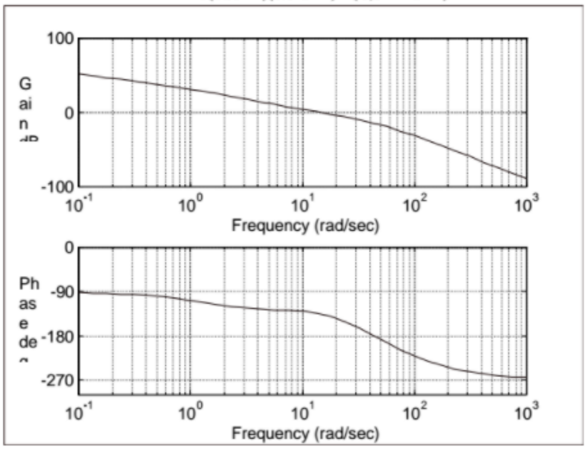
f) Determine el cero y el polo de la red de adelanto como se muestra ya que:

$$W_m=-15=\left(\frac{1}{\sqrt{\alpha}\cdot T}\right)\Rightarrow T=0.19\Rightarrow\frac{1}{T}=5.15\,rad/seg\quad y\quad\frac{1}{\alpha\cdot T}=43.66\,rad/seg$$

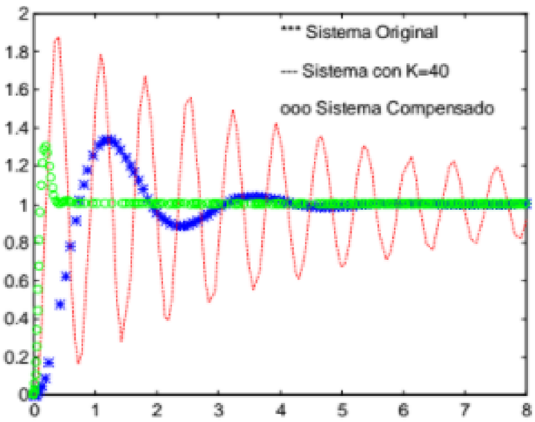
g) Determine la "ganancia" de la red de adelanto: $K'_c=K_c/\alpha=10/0.118=84{,}75$.

h) Compruebe en el diagrama de Bode del sistema compensado, que se cumplan las especificaciones requeridas (MF ≈ 45º; W ≈ 20rad/seg).

Sistema Compensado $G(s)=\dfrac{40}{s(0.5s+1)(0.02s+1)}*\dfrac{0.19s+1}{(0{,}02242s+1)}$



Compensador en adelanto
Respuesta transitoria ante una entrada escalón unitaria



<u>EJEMPLO DEL DISEÑO DE UN COMPENSADOR EN ATRASO.</u>

Un sistema tiene la siguiente función de transferencia de lazo abierto:

$$GH(s)=\frac{2}{(1+0.25\cdot s)(1+0.1\cdot s)s}$$

Diseñar un compensador en atraso tal que el sistema tenga:

.- $K_v\geq 4\ seg^{-1}$.

.- MF ≥ 40º

.- MG ≥ 12db.

PROCEDIMIENTO DE RESOLUCION:

a) Determinar la ganancia de lazo abierto tal que se satisfaga el requerimiento de coeficiente de error:

$$K_v\ (\text{actual}) = 2 \Rightarrow K=2K_v=4 \Rightarrow K_c \cdot GH(s) = \frac{4}{(1+0.25\cdot s)(1+0.1\cdot s)s}$$

b) Con la ganancia así determinada, trazar el diagrama de Bode (de lazo abierto) del sistema no compensado, y determinar MF y MG. Si son insuficientes, proceder con los otros pasos.
(Condiciones actuales del sistema: MG ≈ 7db ; MF ≈ 24º)

c) Hallar el valor de la frecuencia que corresponde a un margen de fase igual al margen de fase requerido más un agregado entre 5 y 12º (que se añaden para compensar el retardo de fase que produce la red de atraso). Se elige esta frecuencia como la nueva frecuencia de transición de la ganancia: W_T'
$\Rightarrow W_T$' (corresponde a MF= 40º+5º=45º)=2.4rad/seg.

d) Se elige 1/T una década por debajo de W_T' $\Rightarrow$ 1/T=0.24rad/seg. (para asegurar T suficientemente grande y que la curva de fases no se vea afectada en la zona de transición de la ganancia).

e) Determinar la atenuación necesaria para reducir la curva de amplitud a 0 db en W_T', atenuación que corresponde a -20 log α.
En el diagrama de bode: W_T' corresponde a 4.2db $\Rightarrow$ -20 log α=-4.2db.

$$\Rightarrow \alpha = 1.62 \Rightarrow 1/\alpha\cdot T = 0.15\text{rad/seg.}$$

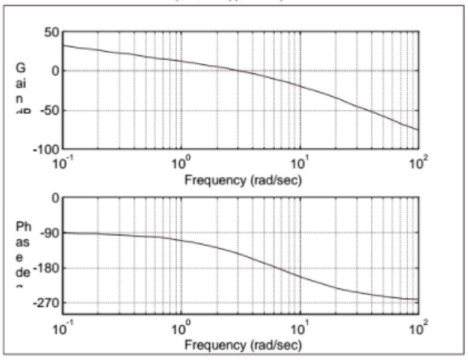
(Este último punto: el polo de la red de atraso, también puede determinarse como el punto de corte en la recta de 0 db, de una línea recta de pendiente - 20db/dec que parte del punto correspondiente al cero de la red, en la atenuación deseada).

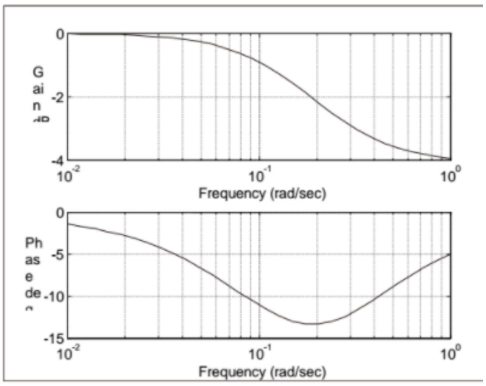
f) Finalmente, se obtiene el diagrama de Bode del sistema compensado, y se comprueban las especificaciones.
(MF ≈ 43º; MG ≈ 13db).

Función de transferencia del sistema compensado:

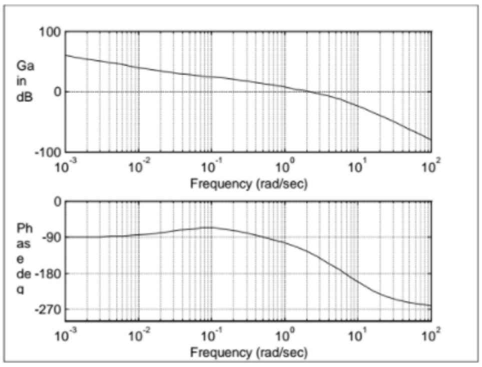
$$G_c \cdot GH(s) = \frac{1}{1.62}\,\frac{s+0.24}{s+0.15}\,\frac{4}{(1+0.25\cdot s)(1+0.1\cdot s)s}$$

Sistema sin Compensar : $G(s) = \dfrac{4}{s(0.25s+1)(0.1s+1)}$



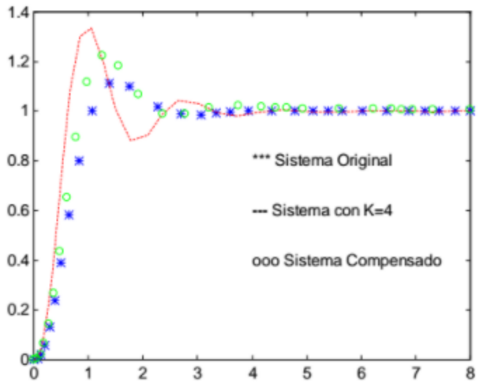


Sistema Compensado: $G(s) = \dfrac{4}{s(0.25s+1)(0.1s+1)} * \dfrac{1}{1.62} * \dfrac{s+0.24}{(s+0.15)}$



Compensador en atraso

Respuesta transitoria ante una entrada escalón unitaria



EJEMPLO DEL DISEÑO DE UN COMPENSADOR EN ADELANTO-ATRASO.

Un sistema tiene la siguiente función de transferencia:

$$G(s) = \frac{K}{s(1+0.25\cdot s)(1+0.1\cdot s)}$$

Se quiere que el sistema de lazo cerrado con retroalimentación unitaria tenga:

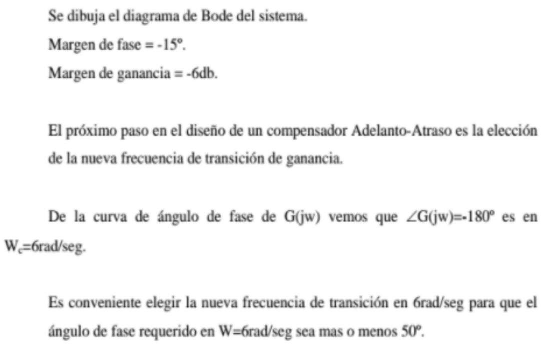
1.- Coeficiente de error de velocidad: K_v = 15.

2.- MF ≥ 50º.

3.- MG ≥ 10 db.

Del requisito de coeficiente de velocidad

$$K_v = \lim_{s\to 0} s\cdot G(s) = \lim_{s\to 0} \cancel{s}\frac{K}{\cancel{s}(1+0.25\cdot s)(1+0.1\cdot s)} = 15$$



Se dibuja el diagrama de Bode del sistema.

Margen de fase = -15°.

Margen de ganancia = -6db.

El próximo paso en el diseño de un compensador Adelanto-Atraso es la elección de la nueva frecuencia de transición de ganancia.

De la curva de ángulo de fase de G(jw) vemos que ∠G(jw)=-180° es en W_c=6rad/seg.

Es conveniente elegir la nueva frecuencia de transición en 6rad/seg para que el ángulo de fase requerido en W=6rad/seg sea mas o menos 50°.

* How can the gain Mg and phase margin $M\phi$ to the time coefficients (ts and ξ) typical of a response of a second-order system?

Phase Margin: We know that the greater the damping coefficient, the greater the phase margin (MF ∝ ξ). On the other hand, we know that the maximum overshoot is inversely related to the damping ratio, so MF ∝ 1/ Mp .

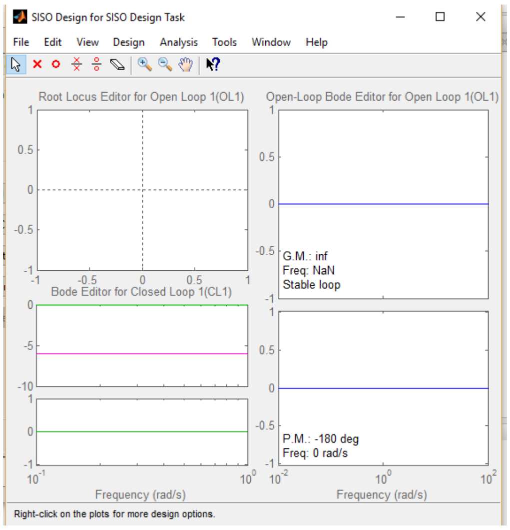
Gain Margin: If we increase the Gain Margin, the system will be slower, since we are reducing the gain of the closed loop system. Therefore the rise time is directly proportional to the Gain Margin (MG ∝ tr).

*What are the sisotool, rltool, ident tools used for? Illustrate with examples:

sisotool:

This tool allows you to design controllers for feedback systems already modeled in Matlab or Simulink, with single input and single output, as its acronym in English SISO which stands for Single-Input, Single-Output.

Controllers can be designed using the Bode plot, root locus, and Nichols graph method to add or remove poles, zeros, and gains. As well as automated PID, control analysis in both the time domain and frequency domain.



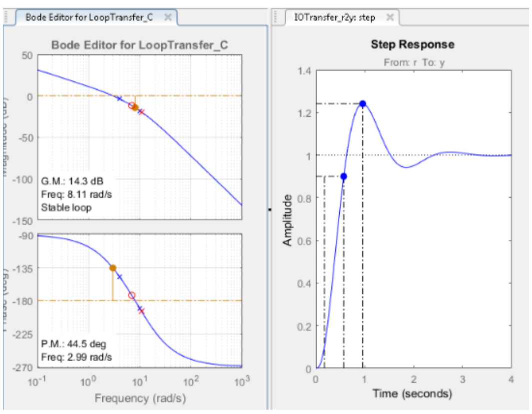
An example of the use of this tool is for a transfer function (as in this case, that of a DC motor).

$$G = \frac{1.5}{s^2 + 14s + 40.02}$$

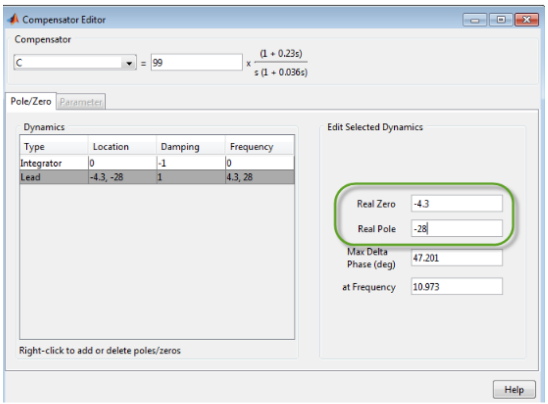
This is the transfer function of the plant and it is desired to carry out a control that meets the following requirements.

- Rise time of less than 0.5 seconds
- Steady-state error of less than 5%
- Overshoot of less than 10%
- Gain margin greater than 20 dB
- Phase margin greater than 40 degrees

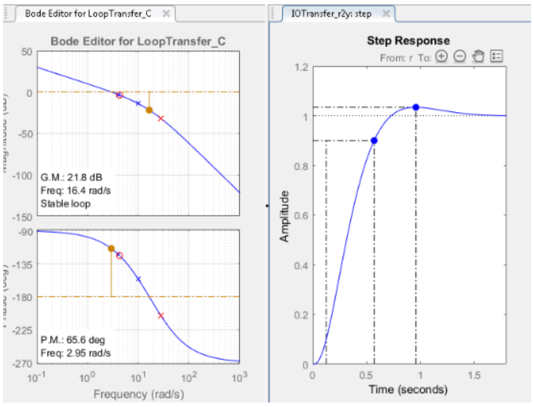
Having the response of the system in frequency and then adding the parameters, the respective adjustment of magnitude and phase is made.



The software then indicates where the pole and zero should be located so that the design parameters are met. This being for the case where you want to control by phase advance or delay compensators.

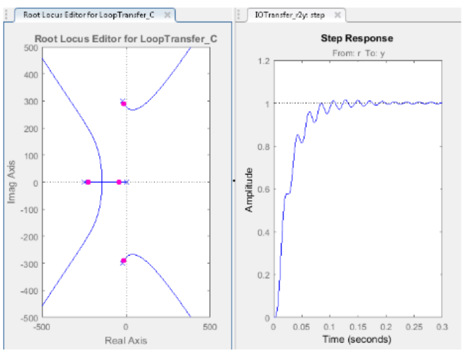


Until finally, adding the compensator, the expected control is reached.

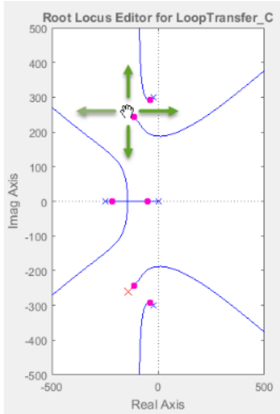


rltool:

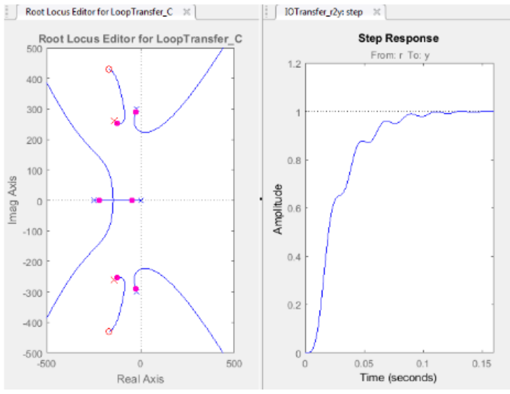
This tool allows the design of root locus control, a basic control technique where the gain of a compensator is designed, and the location of the poles and zeros starting from the root locus diagram.



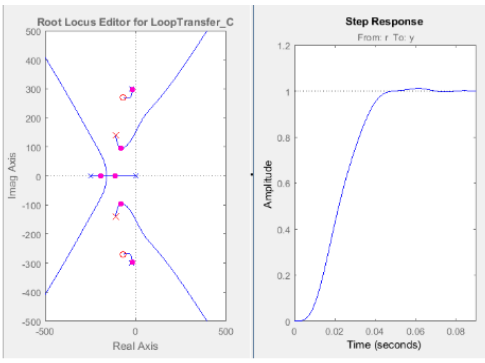
The diagram first shows the original, uncontrolled system response, with their respective poles and zeros located at the root loci.



And that also allows moving in the plane, and observing the change that this action makes in the response of the system.



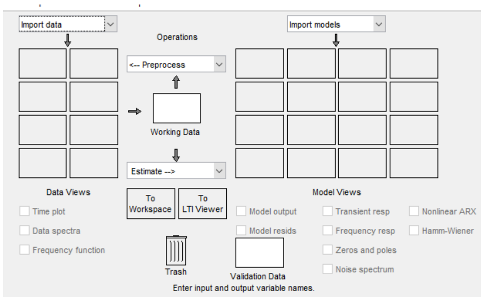
And to be able to go from a very oscillatory response to a common response of a controlled system that meets the parameters established in the design.

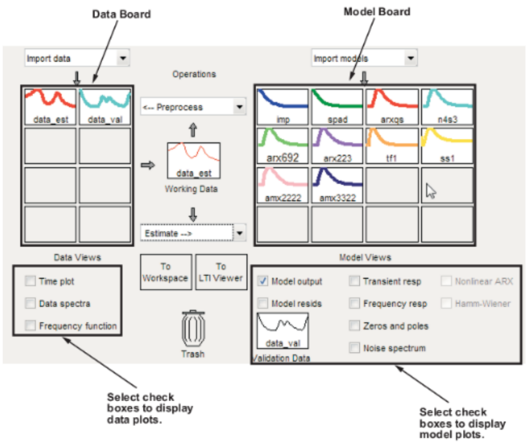


ident:

Matlab's ident tool allows obtaining

information and identifying different real systems through its response, with the system's response it is possible to know the parameters that characterize it and even its transfer function.



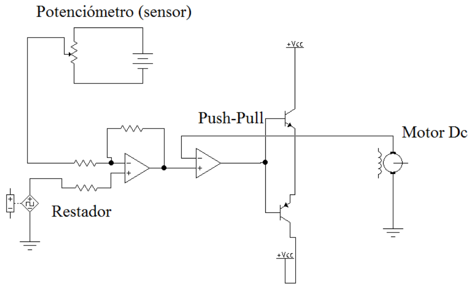


For example, the transfer function of a system can be found through the response obtained from the oscilloscope, for example for the

case in which we have the values in excel of time and level of an oscilloscope signal, which when entered in the ident you can get its transfer function, order and type, and other parameters such as rise time, Wn , settling time among others.

Practice

We then have the graph of the plant feedback with the sensor (potentiometer):

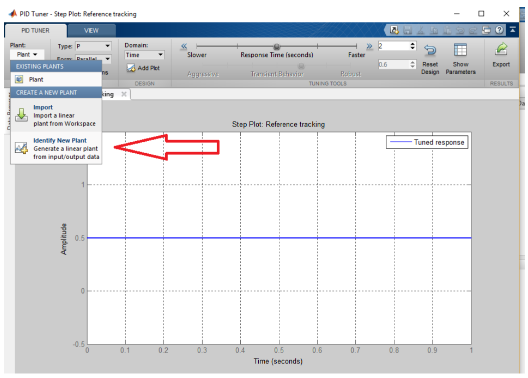


Circuit of the plant with feedback

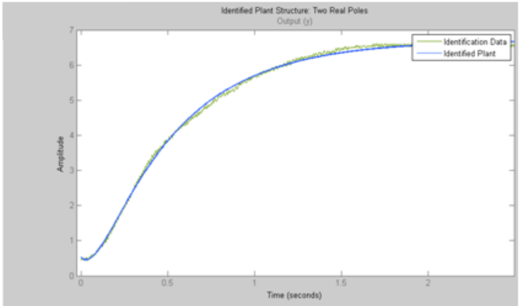
Response in closed loop of the system

We proceed to copy the values shown in the oscilloscope to a usb, and then in Microsoft Excel, make the respective adjustments because the oscilloscope only has 1 output column where the values of the 2 channels are separated with a comma. They must be separated for analysis in Matlab.

Then the approximation of the graph is made using the "identify new plant" option of the "pidtool" tool of the matlab software.



Having the following response, which resembles the one presented in the oscilloscope.



And then the equation of the transfer function of the system in closed loop is obtained:

$$\frac{0.9277}{s^2+13.15s+23.15}$$

Equation 1

But this transfer function is that of the plant fed back with the sensor (potentiometer), therefore it must undo that feedback, and is presented later in equation 2.

The following design criteria are used:

*Settling time 85% of the open loop.

*25% overshoot.

*Error in steady state ess=0.

With these parameters we can know the desired pole, with which the compensator will be designed. The pole must be second order in order to control the maximum overshoot and settling time.

So first the damping coefficient must be known ξ, then how it should be 25% and using the following equation.

$$M_p = e^{-\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)\pi}$$

Where Mp is 0.25 so that it is 25%, and ξfor the remaining value is:

$$\xi = 0.404$$

Having the damping coefficient, the settling time is now found, which is 85% and are 1.6745s and using the 5% criterion.

$$5\% \Rightarrow \frac{3}{\xi Wn} = 1.67s$$

And then, Wn, the natural frequency has a value of 4.446rad/s.

Then we already have the desired polynomial, which is:

$$s^2 + (2 * 0.404 * 4.446)s + 4.446^2$$

And factoring, the desired pole is as follows:

$$s = -1.795 + 4.06i$$

With which it must be replaced in the plant transfer function.

Characteristics of the transfer function of the plant:

```
        0.9277
---------------------
s^2 + 13.16 s + 22.22
```

Equation 2

From equation 2, the denominator can be simplified by factoring the terms, being as follows:

$$\frac{0.9277}{(s+11.5)(s+2)}$$

And now replacing the pole M1M2

$$\frac{\_}{\_\angle 110.57°}$$

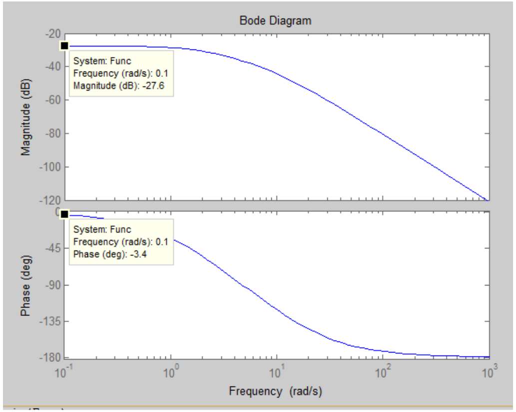
$$\frac{0.9277\angle -110.57°}{M1M2}$$

$$\varphi = -180 - (-110.57) = -70°$$

Since the angle was -70°, a delay compensator must be added.

4. ANALYSIS OF RESULTS

First, an analysis of the Bode plot of the transfer function of the plant is carried out.



It is observed from the previous Bode plot that the system is unstable because the profit margin never becomes positive, and it is observed in the graph of the gain that it never goes through zero.

```
>> [mag,phase,w] = bode (Func,4)

mag =

    0.0175


phase =

  -82.2436


w =

     4
```

To make the system stable, first add the missing profit, which can be found with the following equation.

$$20 * log(mag) = dB$$

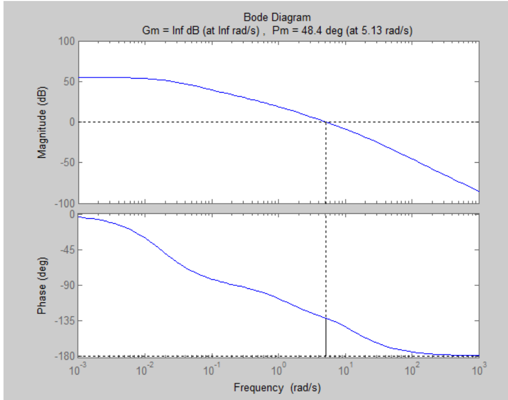
So, since mag is 0.0175, it is plugged into the above equation.

$$20 * log(0.0175) =- 35dB$$

It means that 35dB must be added for the system to be stable, it can be added by means of a proportional gain, this gain is found as follows:

$$1/mag = 1/0.0175 = 57.14$$

The gain proportional that is added will be 57.14, and now the Bode diagram of the system with only this gain is:



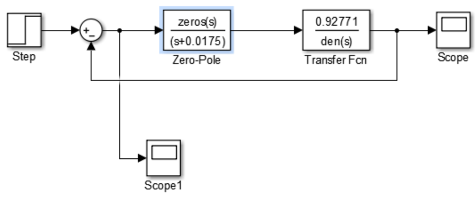
And it is observed in the Bode diagram that the system is stable.

Now comes the assignment of the pole and zero.

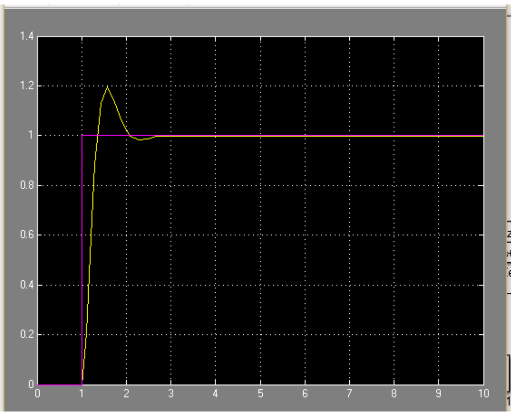
When the compensation is in delay, first a value can be assigned to the zero that is in the region of the locus of the roots, in this case -4.06 is assigned as zero of

The compensator equation is:

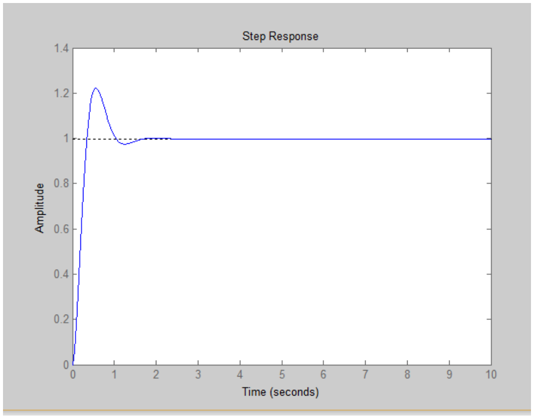
$$C(s) = 57.14 * \frac{s + 4.06}{s + 0.0175}$$

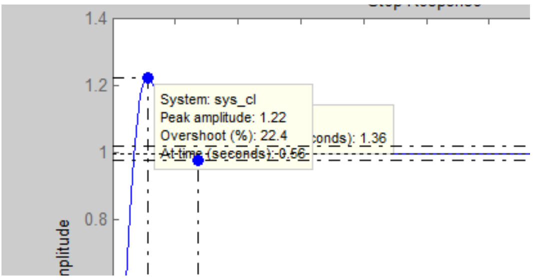


Block diagram of the system.

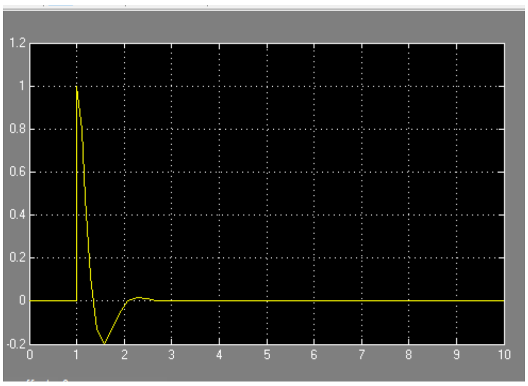


Where the output signal is yellow, and the input is magenta.



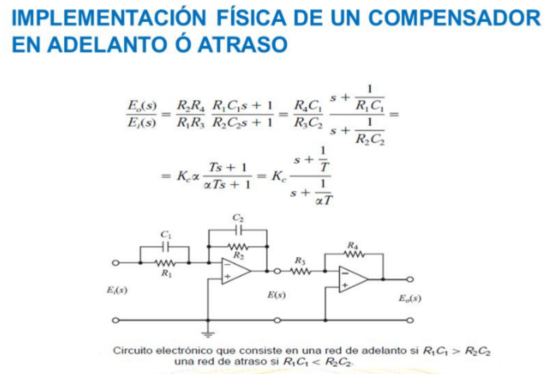


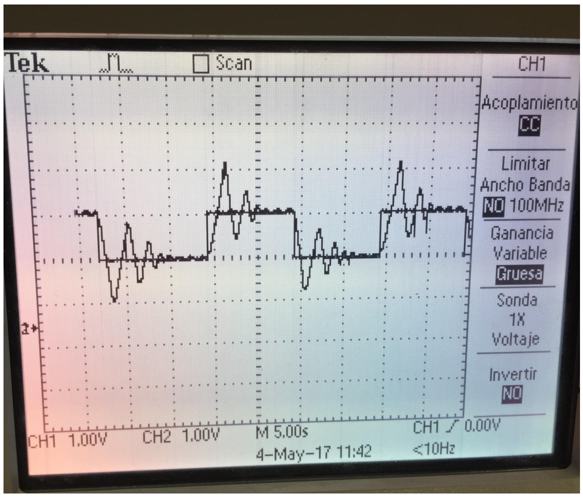
Where an overshoot close to the 25% requested in practice



is observed AND the system control signal, where it is observed that it reaches the maximum amplitude of the step.

The electronic circuit of a compensator is observed in the following image with a brief description that presents it.





The previous image shows the response of the system to a square input with an amplitude of 1Vpp, an offset of 2Vdc at a frequency of 50mHz. The establishment time of 85% is ensured and also the overshoot of less than 25%.

Remark:

The compensated system complies with the settling time and overshoot parameters, with respect to the error in stable state, when using a very large compensator gain, this error tends to be approximately zero, but not equal to zero, this occurs, because there is a guideline in the procedure that indicates that before calculating any variable, an error in stable state equal to 0 is ensured by implementing integrators to the plant, and from this set is that the Bode diagrams are analyzed and calculations.

5. CONCLUSIONS

*The regulation by compensators, although a little more extensive in its calculations, presents a greater simplicity in its physical implementation.

*Although a near-zero steady-state error can be ensured by using a very large compensator gain (Possible Instability), it is better to ensure the steady-state error before performing the compensator calculation from the use of pure integrators.

6. REFERENCES

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