Laboratory 1: Modelling of mechatronic systems.

Galvis, David., Lopez, Daniel

{u1802584,u1802530}@unimilitar.edu.co

Universidad Militar Nueva Granada

Resumen--- En este informe se desarrollan distintos sistemas mecatrónicos, los cuales se solucionarán por el método de Newton-Euler y el método Euler-Lagrange. Se simula su respuesta dinámica en el software de MATLAB y analiza el resultado.

Abstract--- In this paper various mechatronics systems are developed and analyzed, through Newton-Euler or Euler-Lagrange method. Its dynamic response is simulated using MATLAB for further analyzing.

Key words --- System, state variable, dynamic response.

General objective --

Strenghten knowledge related to the modeling of mechatronic systems and their various representations, after the use of Newton-Euler theory.

Specific objectives--

*Model mechatronic systems using Newton-Euler and Euler-Lagrange theory.

*Find the various representations of mechatronic systems.

*Find the answer of the dynamics of mechatronic systems and observe their behavior by varying the parameters of the model that represents them.

*Use analogies to simplify the finding of mechatronic system models.

1. INTRODUCTION

In this document is the development of 9 mechatronic systems proposed in order to analyze each of the systems and find their main characteristics of the dynamic response of the system. In addition, questions are solved that help a better understanding of what a linear system is about, its analogies and what composes it.

2. THEORETICAL FRAMEWORK

The modeling or mathematical representation of a dynamical system allows the understanding and characterization of their behavior.

The systems can be classified into mechanical, electrical, hydraulic, thermal among others, or the interaction of them. Within the modeling, you have mathematical tools that help the study of the behavior of its dynamics.

Differential equations, transfer functions, equations of state and energies, allow to find such mathematical models or also known in the control literature as simulation models. Within the theory used for this work, there is the modeling by Newton-Euler (Defined in the static and dynamic physical laws) and Euler-Lagrange (Defined by the laws of conservation of energy)

3. MATERIALS

Software: MATLAB 2014

Security elements: Lab coat

4. PROCESS

What kind of analogies are there??

There are two types of analogies, between mechanical and electrical systems.

Mechanic system	Variable	Electric system	Variable
Force (Couple)	F T	Voltage	V
Mass	M	Inductance	L
Viscous friction	Fv , D	Resistance	R
Elastic constant	K	Capacitance	C
Angular displacement	X , θ	Electric charge	q
Angular speed	V , ω	Current	i

Table 1: Analogy between systems mechanic-electric

Thermal system	Variable	Electric system	Variable
Heat transfer	q	Current	I
Temperature difference	Δθ	Voltage difference	ΔV
Thermal resistance	Rt	Resistance	R
Thermal inertia	Ct	Capacitance	C

Table 2: Analogy between systems thermal-electric

What elements are needed to model the different types of systems (strength, mass, height, etc.)? What are their units?

a) Translation mechanical system:

Mass [kg], Elastic constants [N/m], Damping coefficients [(N*s)/m^2], friction coefficients [N/m], lever distances[m], external forces[N].

b) Rotational mechanical system:

Inertia[kg*m^2], elastic constans [N/m], damping coeficientes [(N*s)/m^2], friction coefficients [N/m], gear ratio, external torque[N.m].

c) Electric system:

Resistance[Ω], capacitance[F], inductance[H].

d) Liquid level systems:

Capacitance [m3], resistance [R], height[m]

e) Thermal systems:

Thermal capacitance [c*m], thermal resistance [$m^2 \cdot K \cdot W^{-1}$], external temperatures[K].

*** What properties must a linear system meet?**

A linear system must comply with two fundamental parameters, that of homogeneity and that of superposition; homogeneity is described as follows:

Given:

$$f(x) = y$$

Then:

$$f(kx) = ky$$

What involves a proportionality or homogeneity between output and input.

The superposition indicates that given:

$$f(x) = y\,; f(w) = v$$

Then:

$$f(kx + kw) = ky + kv$$

This indicates that when two causative factors (generating effects) are presented in the system, the output can be represented as the sum of the individual effects; see that the homogeneity parameter is already included.

you must comply that the output of the system is an expression that depends linearly on the input.

*** How can the linearity of a system be determined?**

The output must have a linear relationship with the input, so that the linearity of the system is met.

The linearity of a system can be determined by performing the mathematical verification of both previously established parameters.

***How does a state variable defined?**

It is the variable of the system that changes during the process in which the model comes into operation or responds to an input.

*** What dimension should each of the state space matrix have if it has q inputs, n states, and p outputs?**

Matrix A should have dimention **nxn**.
Matrix B should have dimention **nxq**.
Matrix C should have dimention **1xp**.

***Solve the systems presented from Figure 4 to 12 proposed by Newton-Euler.**

See Annex 1

* **Systems presented in Figure 6,7 and 12 to propose Euler-Lagrange the mathematical models that represent the dynamics of the systems.**

See Annex 1

5. ANALYSIS OF RESULTS

For all systems, a unitary step was configured as an input. SEE SOLUTIONS IN ANNEX 1

The results presented by MATLAB software are contained in Annex 2.

a) Figure 4:

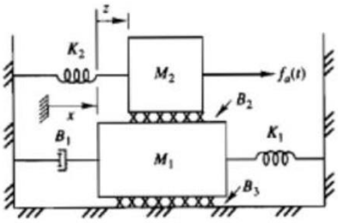


Figura 4: Sistema mecánico traslacional

According to the values established for the system, it responded with an under-damped response.

b) Figure 5:

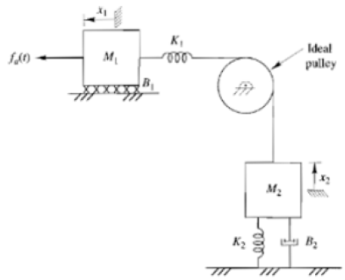


Figura 5: Sistema mecánico con polea

According to the values established for the system, it is unstable, which implies that, in the characteristic polynomial, there is a root located in the right half-plane.

c) Figure 6:

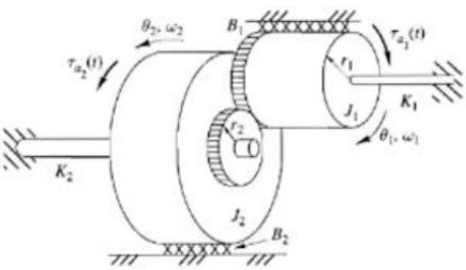


Figura 6: Sistema mecánico rotacional

According to the values established for the system, it responded with a sub-damped behavior, unlike that described in ***Figure 4***, it performs fewer oscillations before stabilizing, which indicates the presence of a zero closer to the horizontal axis.

d) Figure 7:

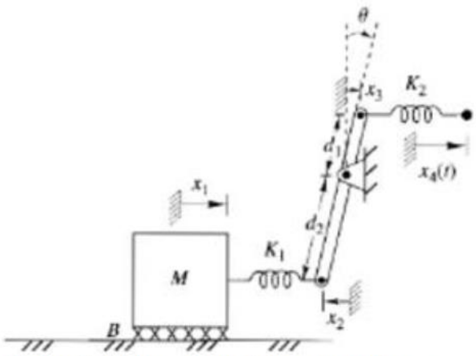


Figura 7: Sistema mecánico con palanca

According to the values established for the system, it responded with an over-damped behavior, which implies a longer response time compared to a critically damped system.

e) Figure 8:

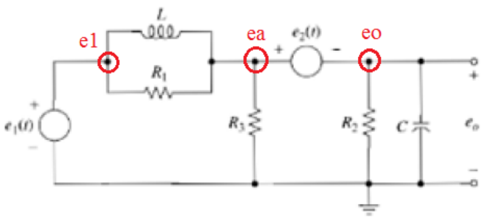


Figura 8: Sistema eléctrico

According to the values established for the system, it responded with a sub-dampened behavior, its difference to the others is that a damping coefficient is presented such that the response is almost immediate.

f) Figure 9:

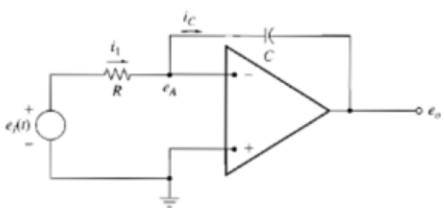
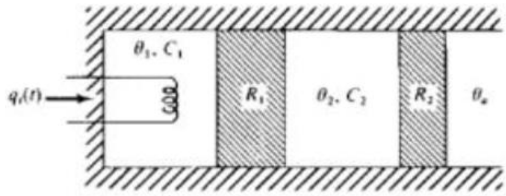


Figura 9 : Sistema eléctrico con operacionales

According to the values established for the system, it demonstrates a behavior typical of a pure investor integrator.

g) Figure 10:



According to the values established for the system, it responded with an unstable behavior, unlike the one presented in Figure 5, it increases more slowly.

h) Figure 11:

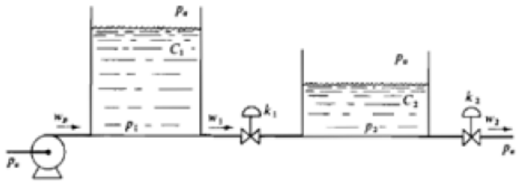


Figura 11: Sistema hidráulico

According to the values established for the system, it responded with an under-dampened behavior; according to the scale presented in the simulation it is shown that the output is not very significant with respect to the input, so it could be said that the system did not vary.

6. CONCLUSIONS

Through the physical analysis of certain systems, a representation of this system was reached through the state space and the transfer function of each model.

There are different types of analysis that can be performed on a system to reach its respective model, in this case being through Newton-Euler or Euler-Lagrange.

Depending on the values given to the elements that make up the system, it responds in different ways.

7. REFERENCES

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[2] K. Ogata, "Dinámica de sistemas", 1era ed,vol 1, Ed Prentice hall, 1987,pp.596 "Ecuaciones de Lagrange".

ANEXO 1

FIGURE 4: TRANSLATIONAL MECHANICAL SYSTEM

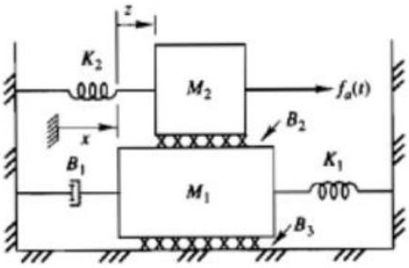
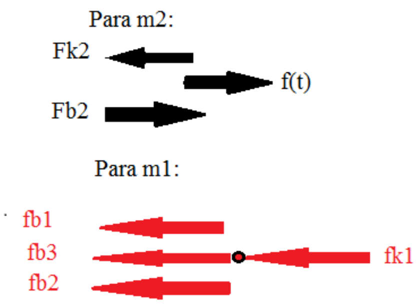


Figura 4: Sistema mecánico traslacional



a) For m2

$$f(t) + B_2(\dot{x}_1 - \dot{x}_2) - k_2x_2 = m_2\ddot{x}_2$$

$$\ddot{x}_2 + \frac{B_2}{m_2}\dot{x}_2 + \frac{k_2}{m_2}x_2 = \frac{B_2}{m_2}\dot{x}_1 + \frac{1}{m_2}f(t) \qquad Ec.1$$

For m1

$$-B_1\dot{x}_1 - B_2(\dot{x}_1 - \dot{x}_2) - B_3\dot{x}_1 - k_1x_1 = m_1\ddot{x}_1$$

$$\ddot{x}_1 + \frac{(B_1 + B_2 + B_3)}{m_1}\dot{x}_1 + \frac{k_1}{m_1}x_1 = \frac{B_2}{m_1}\dot{x}_2 \qquad Ec.2$$

b) $\dot{x}_1 = x_3$

$\dot{x}_2 = x_4$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ \frac{-k_1}{m_1} & 0 & \frac{(B_1 + B_2 + B_3)}{m_1} & \frac{B_2}{m_1}\\ 0 & \frac{-k_2}{m_2} & \frac{B_1}{m_2} & \frac{-B_2}{m_2}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ \frac{1}{m_2}\end{bmatrix}[f(t)]$$

c) Initial conditions = 0

$$: s^2X_2(s) + \frac{B_2}{m_2}sX_2(s) + \frac{k_2}{m_2}X_2(s) = \frac{B_2}{m_2}sX_1(s) + \frac{1}{m_2}F(s)$$

$$: s^2X_1(s) + \frac{(B_1 + B_2 + B_3\ )}{m_1}sX_1(s) + \frac{k_1}{m_1}X_1(s) = \frac{B_2}{m_1}sX_2(s)$$

$$: X_1(s)\left(s^2 + \frac{(B_1 + B_2 + B_3\ )}{m_1}s + \frac{k_1}{m_1}\ \right) = \frac{B_2}{m_1}sX_2(s)$$

$$: X_1(s) = \frac{B_2 s}{m_1 s^2 + (B_1 + B_2 + B_3\ )s + k_1}X_2(s)$$

$$: \ X_2(s)\left(s^2 - \frac{{B_2}^2 s^2}{m_1 m_2 s^2 + m_2(B_1 + B_2 + B_3\ )s + \ k_1 m_2}\ + \frac{B_2}{m_2}s + \frac{k_2}{m_2}\right) = \frac{1}{m_2}F(s)$$

$$: \ \ X_2(s)\left(s^2 m_2 + B_2 s + k_2 - \frac{{B_2}^2 s^2}{m_1 s^2 + (B_1 + B_2 + B_3\ )s + k_1}\right) = F(s)$$

$$: \ \frac{X_2(s)}{F(s)} = \frac{m_1 s^2 + (B_1 + B_2 + B_3\ )s + k_1}{m_1 m_2 s^4 + (m_2(B_1 + B_2 + B_3) + B_2 m_1)s^3 + (k_1 m_2 + B_2(B_1 + B_3) + k_2 m_1)s^2 + (B_2 k_1 + k_2(B_1 + B_2 + B_3\ ))s}$$

The denominator of the previous equation will be named as P(s) then the final equation is:

$$: \ \frac{X_1(s)}{F(s)} = \frac{B_2 s}{P(s)} \qquad Ec.3$$

FIGURE 5: MECHANICAL SYSTEM WITH PULLEY

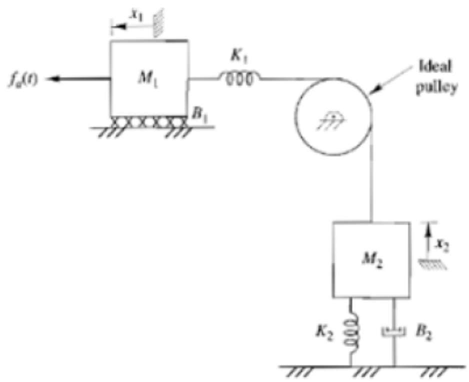
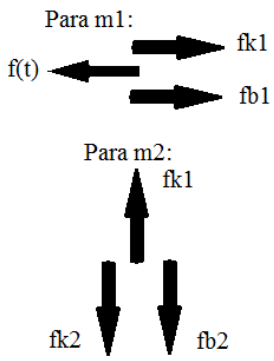


Figura 5: Sistema mecánico con polea



a) For m_1 :

$$f(t) - k_1(x_1 - x_2) - B_1\dot{x_1} = m_1\ddot{x_1}$$

$$\ddot{x_1} + \frac{B_1}{m_1}\dot{x_1} + \frac{k_1}{m_1}x_1 = \frac{k_1}{m_1}x_2 + \frac{1}{m_1}f(t) \quad Ec. 4$$

For m_2 :

$$k_1(x_1 - x_2) - k_2x_2 - B_2\dot{x_2} = m_2\ddot{x_2}$$

$$\ddot{x_2} + \frac{B_2}{m_2}\dot{x_2} + \frac{(k_1 + k_2)}{m_2}x_2 = \frac{k_1}{m_2}x_1 \quad Ec. 5$$

b)

$\dot{x_1} = x_3$

$\dot{x_2} = x_4$

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\\ \dot{x_4}\end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ \frac{-k_1}{m_1} & \frac{k_1}{m_1} & \frac{-B_1}{m_1} & 0\\ \frac{k_1}{m_2} & \frac{-(k_1 + k_2)}{m_2} & 0 & \frac{-B_2}{m_2}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{1}{m_1}\\ 0\end{bmatrix}[f(t)]$$

c) Initial conditions = 0

$$: \ s^2X_1(s) + \frac{B_1}{m_1}sX_1(s) + \frac{k_1}{m_1}X_1(s) = \frac{k_1}{m_1}X_2(s) + \frac{1}{m_1}F(s)$$

$$: \ s^2X_2(s) + \frac{B_2}{m_2}sX_2(s) + \frac{(k_1 + k_2)}{m_2}X_2(s) = \frac{k_1}{m_2}X_1(s)$$

$$: \ X_2(s)\left(s^2 + \frac{B_2}{m_2}s + \frac{(k_1 + k_2)}{m_2}\right) = \frac{k_1}{m_2}X_1(s)$$

: $X_2(s) = \dfrac{k_1}{m_2 s^2 + B_2 s + k_1 + k_2} X_1(s)$

: $X_1(s)\left(s^2 + \dfrac{B_1}{m_1}s - \dfrac{{k_1}^2}{m_2 s^2 + B_2 s + k_1 + k_2} + \dfrac{k_1}{m_1}\right) = \dfrac{1}{m_1}F(s)$

: $\dfrac{X_1(s)}{F(s)}$

$$= \frac{m_2 s^2 + B_2 s + k_1 + k_2}{m_1 m_2 s^4 + (m_1 B_2 + B_1 m_2)s^3 + (m_1 k_1 + m_1 k_2 + B_1 B_2 + k_1 m_2)s^2 + (B_1 k_1 + B_1 k_2 + B_2 k_1)s + {k_1}^2 - m_1 {k_1}^2 + k_1 k_2}$$

The denominator of the previous equation will be named as $P(s)$, then:

: $\dfrac{X_2(s)}{F(s)} = \dfrac{k_1}{P(s)}$ $\quad Ec. 6$

FIGURE 6: ROTATIONAL MECHANICAL SYSTEM

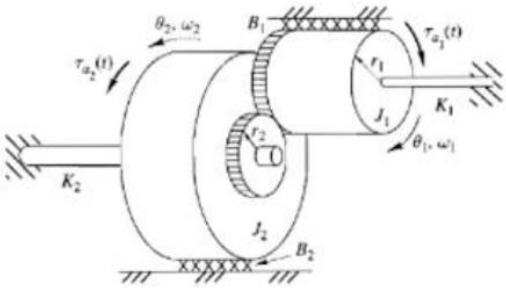
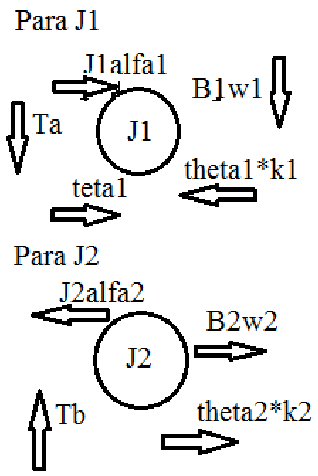


Figura 6: Sistema mecánico rotacional



a) For J_1 :

$$\tau_a - B_1\dot{\theta}_1 - \theta_1 k_1 = J_1\ddot{\theta}_1$$

$$\ddot{\theta}_1 + \frac{B_1}{J_1}\dot{\theta}_1 + \frac{k_1}{J_1}\theta_1 = \tau_a(t) \qquad Ec. 7$$

For J_2 :

$$\ddot{\theta}_2 + \frac{B_2}{J_2}\,\dot{\theta}_2 + \frac{k_2}{J_2}\theta_2 = \frac{r_2}{r_1}\,\tau_a(t) \qquad Ec. 8$$

b)

$\dot{\theta}_1 = \theta_3$

$\dot{\theta}_2 = \theta_4$

$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\\ \dot{\theta}_4\end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ \dfrac{-k_1}{J_1} & 0 & \dfrac{-B_1}{J_1} & 0\\ 0 & \dfrac{-k_2}{J_2} & 0 & \dfrac{-B_2}{J_2}\end{bmatrix}\begin{bmatrix}\theta_1\\ \theta_2\\ \theta_3\\ \theta_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\\ \dfrac{r_2}{r_1}\end{bmatrix}[\tau_a(t)]$$

c) Initial conditions = 0

$$:\ s^2\theta_1(s) + \frac{B_1}{J_1}\,s\theta_1(s) + \frac{k_1}{J_1}\,\theta_1(s) = \tau_a(s)$$

$$:\ s^2\theta_2(s) + \frac{B_2}{J_2}s\theta_2(s) + \frac{k_2}{J_2}\,\theta_2(s) = \frac{r_2}{r_1}\tau_a(s)$$

$$:\ \theta_2(s)\left(s^2 + \frac{B_2}{J_2}s + \frac{k_2}{J_2}\right) = \frac{r_2}{r_1}\tau_a(s)$$

$$:\ \frac{\theta_2(s)}{\tau_a(s)} = \frac{J_2}{J_2 s^2 + B_2 s + k_2}\,\frac{r_2}{r_1}$$

$$:\ \theta_1(s)\left(s^2 + \frac{B_1}{J_1}s + \frac{k_1}{J_1}\right) = \tau_a(s)$$

$$:\ \frac{\theta_1(s)}{\tau_a(s)} = \frac{J_2}{J_1 s^2 + B_1 s + k_1} \qquad Ec. 9$$

Using Euler-Lagrange

$L = k_e - U$

$k_e = \frac{1}{2}J_1\dot{\theta}_1{}^2 += \frac{1}{2}J_2\dot{\theta}_2{}^2$

$U = \dfrac{1}{2}k_1\theta_1{}^2 + \dfrac{1}{2}k_2\theta_2{}^2$

$L = \frac{1}{2}J_1\dot{\theta}_1{}^2 += \frac{1}{2}J_2\dot{\theta}_2{}^2 - \frac{1}{2}k_1\theta_1{}^2 - \frac{1}{2}k_2\theta_2{}^2$

FIGURE 7: MECHANICAL SYSTEM WITH LEVER

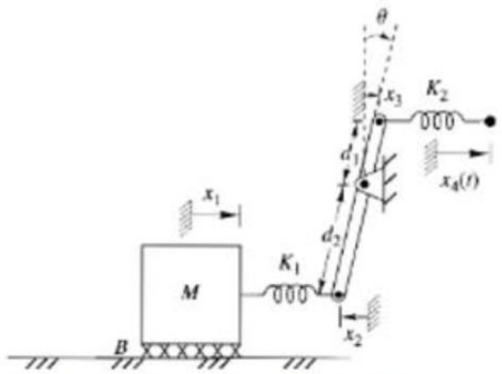
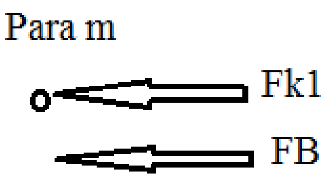


Figura 7: Sistema mecánico con palanca



a) For m:

$$-k_1\,(x_1 + x_2) - B\dot{x_1} = m\ddot{x_1}$$

$$\ddot{x_1} + \frac{B}{m}\,\dot{x_1} + \frac{k_1}{m}x_1 = \frac{-k_1}{m}x_2 \qquad Ec.10$$

For lever:

$$x_3 = \frac{d_1}{d_2}x_2$$

$$d_1 f_{k_2} = d_2 f_{k_1}$$

$$d_1 k_2 (x_4 - x_3) = d_2 k_1 (x_1 + x_2)$$

$$d_1 k_2 \left(x_4 - x_3 = \frac{d_1}{d_2}x_2\right) = d_2 k_1 (x_1 + x_2)$$

$$d_1 k_2 x_4 - \; d_2 k_1 x_1 = \left(\frac{{d_1}^2}{d_2}k_2 + \frac{{d_2}^2}{d_2}k_1\right)x_2 \;\rightarrow\; x_2 = \frac{d_1 d_2 k_2 x_4 - {d_2}^2 k_1 x_4}{{d_1}^2 k_2 + {d_2}^2 k_1}$$

$$m\ddot{x_1} + B\dot{x_1} + \frac{k_1 k_2 {d_2}^2}{{d_1}^2 k_2 + {d_2}^2 k_1}x_1 = -\frac{k_1 k_2 d_1 d_2}{{d_1}^2 k_2 + {d_2}^2 k_1}x_4$$

$$\dot{x_1} = w_1 = \begin{bmatrix} \dot{x_1} \\ \dot{w_1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \dfrac{k_1 k_2 {d_2}^2}{m({d_1}^2 k_2 + {d_2}^2 k_1)} & -\dfrac{B}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ w_1 \end{bmatrix} + \begin{bmatrix} 0 \\ -\dfrac{k_1 k_2 d_1 d_2}{m({d_1}^2 k_2 + {d_2}^2 k_1)} \end{bmatrix} [x_4(t)]$$

Initial conditions = 0.

$$: \quad ms^2 X_1(s) + BsX_1(s) + \frac{k_1 k_2 {d_2}^2}{({d_1}^2 k_2 + {d_2}^2 k_1)} X_1(s) = -\frac{k_1 k_2 d_1 d_2}{{d_1}^2 k_2 + {d_2}^2 k_1} X_4(s)$$

$$: \quad X_1(s)\left(ms^2 + Bs + \frac{k_1 k_2 {d_2}^2}{({d_1}^2 k_2 + {d_2}^2 k_1)}\right) = -\frac{k_1 k_2 d_1 d_2}{{d_1}^2 k_2 + {d_2}^2 k_1} X_4(s)$$

$$: \quad \frac{X_1(s)}{X_4(s)} = \frac{k_1 k_2 d_1 d_2}{s^2(m{d_1}^2 k_2 + {d_2}^2 k_1) + s(B{d_1}^2 k_2 + B{d_2}^2 k_1) + k_1 k_2 {d_2}^2} \qquad Ec. 11$$

FIGURE 8: ELECTRIC SYSTEM

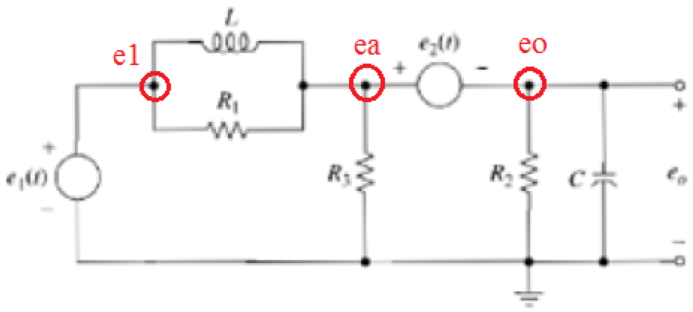


Figura 8: Sistema eléctrico

Input: $e_1(t)$

State variables: i_L , e_C

Output: e_O

Equations and initial relations to simplify the replacement of variables:

$$e_1(t) - e_A = e_L(t)$$

$$e_1(t) - e_A = L\dot{i_L}$$

$$e_1(t) - L\dot{i_L} = e_A$$

$$e_A - e_O = e_2(t)$$

$$e_1(t) - L\dot{i_L} - e_O = e_2(t)$$

$$e_1(t) - e_2(t) - e_O = L\dot{i_L}$$

$$e_O = e_C$$

e_A is the node above the resistance R3.

Node $e_1(t)$

$$e_1(t) = i_L + i_{R1}$$

$$e_1(t) = i_L + \frac{e_1(t) - e_A}{R_1}$$

$$i_{e_1(t)} + i_{e_2(t)} = i_{R3}$$

$$i_L + L\frac{\dot{i_L}}{R_1} + i_{e_2(t)} = \frac{e_1(t) - L\dot{i_L}}{R_3} \qquad Ec.12$$

$$i_{e_2(t)} = i_{R2} + i_C = i_{R3} - i_{e_1(t)}$$

$$\frac{e_O}{R_2} + C\dot{e_O} = i_{e_2(t)}$$

$$\frac{e_O}{R_2} + C\dot{e_O} = \frac{e_1(t) - L\dot{i_L}}{R_3} - i_L + \frac{e_1(t) - e_A}{R_1}$$

$$\frac{e_O}{R_2} + C\dot{e_O} = \frac{e_1(t) - (e_1(t) - e_2(t) - e_O)}{R_3} - i_L + \frac{e_1(t) - (e_1(t) - (e_1(t) - e_2(t) - e_O))}{R_1}$$

$$\frac{e_O}{R_2} + C\dot{e_O} = \frac{e_O}{R_3} + \frac{e_2(t)}{R_3} - i_L + \frac{e_1(t)}{R_1} - \frac{e_O}{R_1} - \frac{e_2(t)}{R_1} \qquad Ec.13$$

$$\dot{e_O} = \frac{1}{C}\left(e_C\left(\frac{1}{R_3} - \frac{1}{R_2} - \frac{1}{R_1}\right) + \frac{e_1(t)}{R_1} - i_L + e_2(t)\left(\frac{1}{R_3} - \frac{1}{R_1}\right)\right) \qquad Ec.14$$

Clearing $C\dot{e_O}$ from Ec.13 and replacing it in Ec.12:

$$i_L + L\frac{\dot{i_L}}{R_1} + \frac{e_O}{R_2} + \frac{e_O}{R_3} + \frac{e_2(t)}{R_3} - i_L + \frac{e_1(t)}{R_1} - \frac{e_O}{R_1} - \frac{e_2(t)}{R_1} - \frac{e_O}{R_2} = \frac{e_1(t) - L\dot{i_L}}{R_3}$$

Clearing $\dot{i_L}$:

$$\dot{i_L} = \frac{1}{L}\left(\frac{R_1 R_3}{R_1 + R_3}\left(e_1(t)\left(\frac{1}{R_3} - \frac{1}{R_1}\right) + e_2(t)\left(\frac{1}{R_1} - \frac{1}{R_3}\right) + e_O\left(\frac{1}{R_1} - \frac{1}{R_3}\right)\right)\right) \qquad Ec.15$$

$$\begin{bmatrix}\dot{e_C}\\ \dot{i_L}\end{bmatrix} = \begin{bmatrix}\frac{1}{C}\left(\frac{1}{R_3} - \frac{1}{R_2} - \frac{1}{R_1}\right) & -1\\ \frac{R_1 R_3}{R_1 + R_3}\cdot\frac{1}{L}\left(\frac{1}{R_1} - \frac{1}{R_3}\right) & 0\end{bmatrix}\begin{bmatrix}e_C\\ i_L\end{bmatrix} + \begin{bmatrix}\frac{1}{C}\\ \frac{R_1 R_3}{R_1 + R_3}\cdot\frac{1}{L}\left(\frac{1}{R_3} - \frac{1}{R_1}\right)\end{bmatrix}[e_1(t)]$$

$$y = e_O = [1 \quad 0]\begin{bmatrix}e_C\\ i_L\end{bmatrix}$$

FIGURE 9: ELECTRIC SYSTEM WITH OPERATIONAL AMPLIFIERS

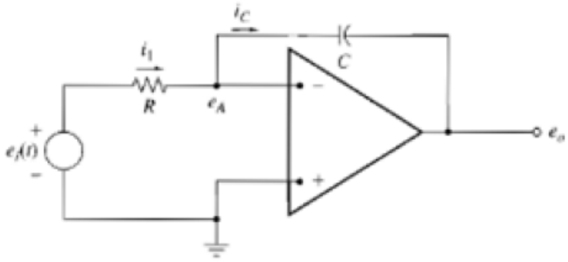


Figura 9 : Sistema eléctrico con operacionales

Input: $e_1(t)$

State variables: e_C

Output: e_O

Solution:

$$e_A = 0$$

$$i_1 = \frac{e_1(t) - e_A}{R}$$

$$i_C = C\dot{e_C}$$

$$e_O - e_A = e_C$$

We take current i_C in the opposite direction shown in figure 9

$$e_O = e_C$$

$$i_1 + i_C = 0$$

$$\frac{e_1(t)}{R} = -C\dot{e_O}$$

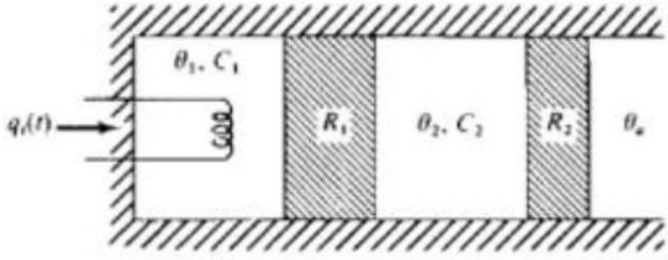
$$-\frac{e_1(t)}{RC} = \dot{e_O}$$

$$e_1(s) = -s[e_O(s)RC]$$

$$\frac{e_O(s)}{e_1(s)} = -\frac{1}{RCs}$$

$$[\dot{e_C}] = [\ 0\ ][e_C] + \left[-\frac{1}{RC}\right] e_1(t)$$

FIGURE 10: THERMAL SYSTEM



Input: $\widehat{Q\iota}$

State variables: $\widehat{\dot{\theta_1}}$ y $\widehat{\dot{\theta_2}}$

Output: $\widehat{\theta_2}$

$$\bar{q_\iota}(t) = \frac{1}{R_1}(\overline{\theta_1} - \overline{\theta_2}) \qquad Ec. 16$$

$$q_{out2} = \frac{1}{R_2}(\theta_2 - \theta_a) \qquad Ec. 17$$

$$\frac{1}{R_1}(\overline{\theta_1} - \overline{\theta_2}) = \frac{1}{R_2}(\overline{\theta_2} - \theta_a) \qquad Ec. 18$$

$$\dot{\theta_1} = \frac{1}{C_1}[q_{i(t)} - \bar{q_i}(t)]$$

$$\dot{\theta_1} = \frac{1}{C_1}\left[q_{i(t)} - \frac{1}{R_1}(\theta_1 - \theta_2)\right] \qquad Ec. 19$$

$$\dot{\theta_2} = \frac{1}{C_2}\left[\frac{1}{R_1}(\theta_1 - \theta_2) - \frac{1}{R_2}(\theta_2 - \theta_a)\right] \qquad Ec. 20$$

$$\overline{\theta_1} = R_1\bar{q_\iota} + \overline{\theta_2}$$

Replace in Ec.18

$$\bar{q_\iota} = \frac{1}{R_2}\overline{\theta_2} - \frac{1}{R_2}\theta_a$$

Clearing of $\overline{\theta_2}$

$$\overline{\theta_2} = \bar{q_\iota}R_2 + \theta_a$$

Replace $\overline{\theta_2}$ in Ec.16 and clearance of $\overline{\theta_1}$

$$\overline{\theta_1} = \bar{q_\iota}(R_1 + R_2) + \theta_a$$

Move Ec.19 as an incremental variable

$$\dot{\widehat{\theta_1}} = \frac{1}{C_1}\overline{q_\iota} + \frac{1}{C_1}\widehat{q_\iota} + \frac{1}{C_1R_1}\left(-\overline{\theta_1} - \widehat{\theta_1} + \overline{\theta_2} + \widehat{\theta_2}\right)$$

Replacing $\overline{\theta_1}$ and $\overline{\theta_2}$ in the previous equation

$$\dot{\widehat{\theta_1}} = \frac{1}{C_1}\overline{q_\iota} - \frac{1}{C_1R_1}\widehat{\theta_1} + \frac{1}{C_1R_1}\widehat{\theta_2} \qquad Ec.21$$

Moving $\dot{\theta}_2$ to incremental variables, with the same process to obtain $\dot{\widehat{\theta_1}}$, we got:

$$\dot{\widehat{\theta_2}} = \frac{1}{C_2R_1}\widehat{\theta_1} - \widehat{\theta_2}\left(\frac{1}{C_2R_1} - \frac{1}{C_2R_2}\right) \qquad Ec.22$$

Using LaPlace Transformation from Ec.21 so then use the Transformation in Ec.22 and obtain a relation with the structure $\frac{Output}{Input}$. Then

$$\widehat{\theta_1}(s) = \frac{R_1\widehat{Q\iota}(s) + \widehat{\theta_2}(s)}{C_1R_1s + 1}$$

$$\widehat{\theta_2}(s)\left(s + \frac{1}{C_2R_1} - \frac{1}{C_2R_2}\right) = \frac{1}{C_2R_1}\widehat{\theta_1}(s)$$

Replacement of $\widehat{\theta_1}(s)$ in the previous equation so we can obtain the transfer function:

$$\frac{\widehat{\theta_2}(s)}{\widehat{Q\iota}(s)} = \frac{\frac{1}{R_1C_1C_2}}{s^2 + s\left(\frac{1}{C_1R_1} + \frac{1}{C_2R_1} + \frac{1}{C_2R_2}\right) + \frac{1}{C_1C_2R_1R_2}}$$

$$\begin{bmatrix} \dot{\widehat{\theta_2}} \\ \dot{\widehat{\theta_1}} \end{bmatrix} = \begin{bmatrix} \frac{1}{C_2R_1} - \frac{1}{C_2R_2} & \frac{1}{C_2R_1} \\ -\frac{1}{C_1R_1} & \frac{1}{C_1R_1} \end{bmatrix} \begin{bmatrix} \widehat{\theta_2} \\ \widehat{\theta_1} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C_1} \end{bmatrix} \widehat{Q\iota}$$

$$y = \widehat{\theta_2} = [1 \quad 0]\begin{bmatrix} \widehat{\theta_2} \\ \widehat{\theta_1} \end{bmatrix}$$

FIGURE 11: HYDRAULIC SYSTEM

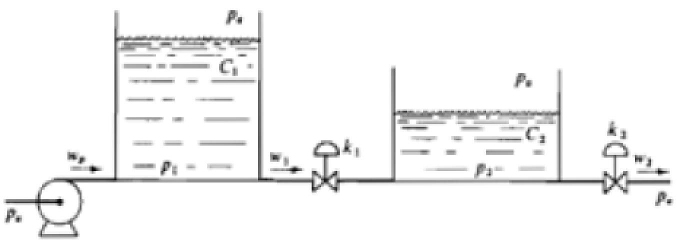


Figura 11: Sistema hidráulico

Input: $\widehat{w_\iota}$

State variables: $\dot{\widehat{P_1}}$ y $\dot{\widehat{P_2}}$

Output: $\widehat{P_2}$

$$P_1 - P_2 = \Delta P$$

$$w_i = K\sqrt{\Delta P}$$

$$\Delta P = K' w_i$$

$$\Delta P = K'K\sqrt{\Delta P}$$

$$\Delta P = (K'K)^2$$

$$\overline{w_i} = K'K^2$$

$$\dot{P_1} = \frac{1}{C_1}(w_i - w_1)$$

$$\dot{P_1} = \frac{1}{C_1}(w_i - k_1\sqrt{P_2 - P_a})$$

$$\dot{P_2} = \frac{1}{C_2}(w_1 - w_2)$$

$$\dot{P_2} = \frac{1}{C_2}(k_1\sqrt{P_2 - P_a} - k_2\sqrt{P_2 - P_a})$$

Convert $\dot{P_1}$ and $\dot{P_2}$ to incremental variables.

$$\widehat{\dot{P_1}} = \frac{1}{C_1}\left(\overline{w_i} - k_1\sqrt{\overline{\Delta P}}\right) + \frac{1}{C_1}\left(\widehat{w_i} - \left(\frac{k_1}{2\sqrt{\overline{\Delta P}}}\right)\widehat{P_1}\right)$$

$$\widehat{\dot{P_1}} = \frac{1}{C_1}\widehat{w_i} - \frac{k_1}{2C_1}\widehat{P_1}$$

$$\widehat{\dot{P_2}} = \frac{1}{C_2}\left(\overline{w_i} - k_2\sqrt{\overline{P_2} - P_a}\right) + \frac{1}{C_2}\left(\widehat{w_i} - \left(\frac{k_2}{2\sqrt{\overline{P_2} - P_a}}\right)\widehat{P_2}\right)$$

$$\widehat{\dot{P_2}} = \frac{1}{C_2}\widehat{w_i} - \frac{k_2}{2C_2}\widehat{P_2}$$

$$\begin{bmatrix}\widehat{\dot{P_1}}\\ \widehat{\dot{P_2}}\end{bmatrix} = \begin{bmatrix}\frac{-k_1}{2C_1} & 0\\ 0 & \frac{-k_2}{2C_2}\end{bmatrix} = \begin{bmatrix}\widehat{P_1}\\ \widehat{P_2}\end{bmatrix} + \begin{bmatrix}\frac{1}{C_1}\\ \frac{1}{C_2}\end{bmatrix}[\widehat{w_i}]$$

$$y = \widehat{P_2} = [0 \quad 1]\,[\begin{matrix}\widehat{P_1}\\ \widehat{P_2}\end{matrix}]$$

FIGURE 12: DOUBLE PENDULUM

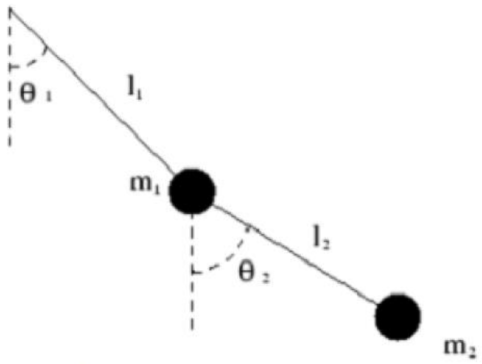


Figura 12: Péndulo doble

Position of pendulum:

$$x = l_1 \sin\theta_1 + l_2 \sin\theta_2$$

$$y = l_1 \cos\theta_1 + l_2 \cos\theta_2$$

Speed:

$$\dot{x} = l_1 \cos\theta_1 \dot{\theta}_1 + l_2 \cos\theta_2 \dot{\theta}_2 \qquad Ec.23$$

$$\dot{y} = -l_1 \sin\theta_1 \dot{\theta_1} - l_2 \sin\theta_2 \dot{\theta}_2 \qquad Ec.24$$

$$\dot{v_2} = \dot{x}^2 + \dot{y}^2 \qquad Ec.25$$

$$v_1 = l_1 \dot{\theta}_1$$

Replacing Ec.24 on Ec.25

$$v_2 = (l_1 \cos\theta_1 \dot{\theta}_1 + l_2 \cos\theta_2 \dot{\theta}_2)^2 + (-l_1 \sin\theta_1 \dot{\theta_1} - l_2 \sin\theta_2 \dot{\theta}_2)^2 \qquad Ec.26$$

The speed of pendulum 1 (m1) depends only from distance l1.

$$K_e = {}^1\!/_2\, m_1 v_1 + {}^1\!/_2\, m_2 v_2 \qquad Ec.27$$

Replacing Ec.26 in Ec.27.

So the kinetic energy of the system is:

$$K_e = {}^1\!/_2\, m_1 (l_1 \dot{\theta}_1)^2 + {}^1\!/_2\, m_2 \left((l_1 \cos\theta_1 \dot{\theta}_1 + l_2 \cos\theta_2 \dot{\theta}_2)^2 + (-l_1 \sin\theta_1 \dot{\theta_1} - l_2 \sin\theta_2 \dot{\theta}_2)^2 \right) \qquad Ec.28$$

Now, to calculate potential energy U:

$$U = m_1 g l_1 (1 - \cos\theta_1) + m_2 g [l_1 (1 - \cos\theta_1) + l_2 (1 - \cos\theta_2)] \qquad Ec.29$$

Then, to obtain the Lagrangian $L = K_e - U$

$$L = {}^1\!/_2\, m_1(l_1\dot{\theta}_1)^2 + {}^1\!/_2\, m_2\left((l_1\cos\theta_1\,\dot{\theta}_1 + l_2\cos\theta_2\,\dot{\theta}_2)^2 + (-l_1\sin\theta_1\dot{\theta}_1 - l_2\sin\theta_2\,\dot{\theta}_2)^2\right) - m_1 g l_1(1-\cos\theta_1) + m_2 g[l_1(1-\cos\theta_1) + l_2(1-\cos\theta_2)] \qquad Ec.30$$

Lagrange equations:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0 \qquad Ec.31$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = 0 \qquad Ec.32$$

Solving the derivates of the system to obtain state variables:

$$\frac{\partial L}{\partial \dot{\theta}_1} = (m_1 + m_2){l_1}^2\dot{\theta}_1 + m_2 l_1 l_2\dot{\theta}_2\cos(\theta_2 - \theta_1) \qquad Ec.33$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = (m_1 + m_2){l_1}^2\dot{\theta}_1 + m_2 l_1 l_2\dot{\theta}_2\cos(\theta_2 - \theta_1) \qquad Ec.34$$

$$\frac{\partial L}{\partial \dot{\theta}_2} = m_2{l_2}^2\dot{\theta}_2 + m_2 l_1 l_2\dot{\theta}_1\cos(\theta_2 - \theta_1) \qquad Ec.35$$

$$\frac{\partial L}{\partial \theta_2} = -m_2 l_1 l_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_2 - \theta_1) - m_2 g l_2\sin(\theta_2) \qquad Ec.36$$

Replacing the derivates on Lagrange equations, the following equations are obtained.

$$(m_1 + m_2){l_1}^2\ddot{\theta}_1 + m_2 l_1 l_2\left[\ddot{\theta}_2\cos(\theta_2 - \theta_1) - \dot{\theta_2}^2\sin(\theta_2 - \theta_1)\right] + (m_1 + m_2)g\sin(\theta_1) = 0 \qquad Ec.37$$

$$l_2\ddot{\theta}_2 + l_1[\ddot{\theta}_1\cos(\theta_2 - \theta_1) + \dot{\theta_1}^2\,\mathrm{sen}(\theta_2 - \theta_1)] + g\,\mathrm{sen}\,(\theta_2) = 0$$

$$\ddot{\theta}_1 + \left(\frac{m_2}{m_1+m_2}\right)\left(\frac{l_2}{l_1}\right)[\ddot{\theta}_2\cos(\theta_2 - \theta_1) - \dot{\theta_2}^2\,\mathrm{sen}(\theta_2 - \theta_1)] + \frac{g}{l_1}\mathrm{sen}(\theta_2) = 0 \qquad Ec.38$$

$$\ddot{\theta}_2 + \left(\frac{l_1}{l_2}\right)\left[\ddot{\theta}_1\cos(\theta_2 - \theta_1) - \dot{\theta_1}^2\,\mathrm{sen}(\theta_2 - \theta_1)\right] + \frac{g}{l_2}\mathrm{sen}(\theta_2) = 0 \qquad Ec.39$$

ANNEX 2

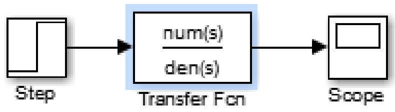
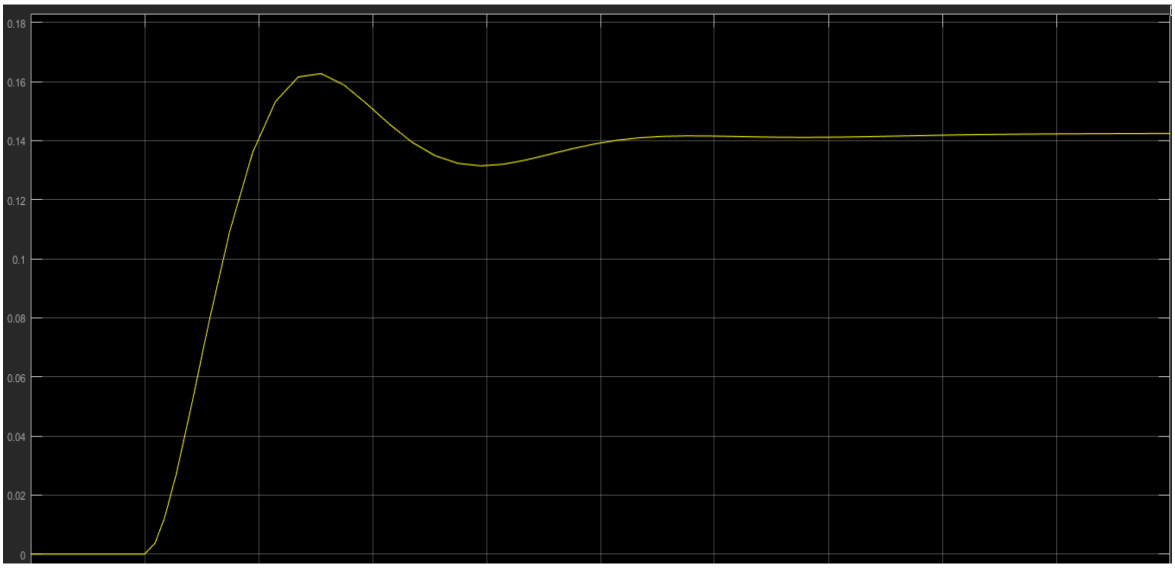


Figure 0. Diagram on Simulink

For all models, the diagram shown in Image 0 was implemented in Simulink, where a Transfer Fcn block or transfer function is used where later in each file .m, values are assigned to the numerator and denominator, which determines the behavior of the system; a Step block as input and a Scope block to display the system output

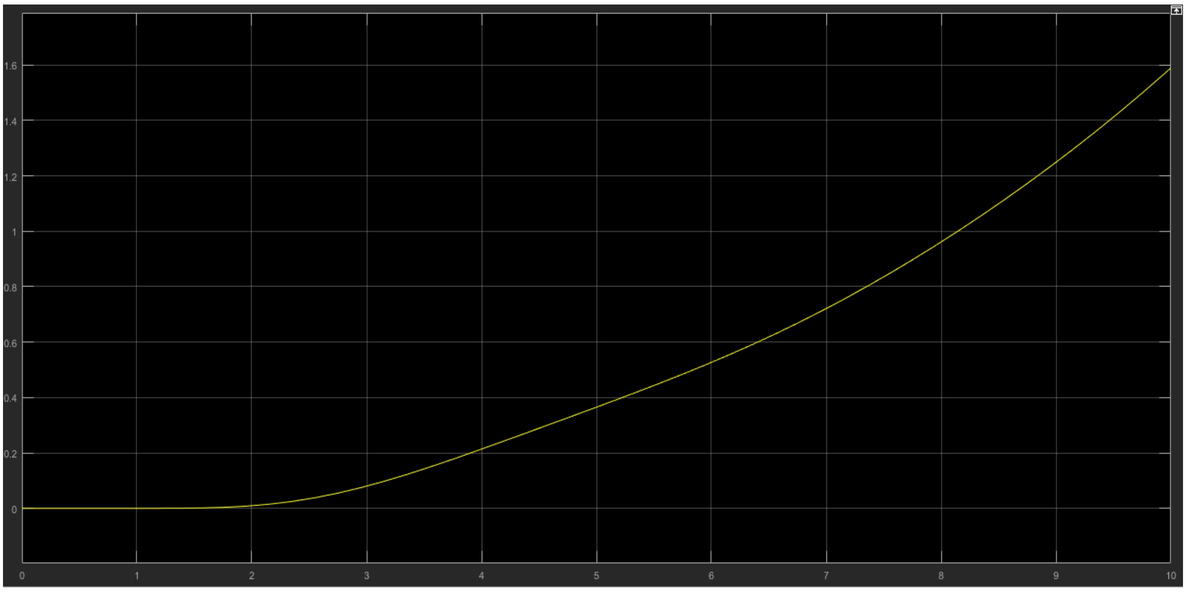
a) Figure 4:

```
m1=3;
m2=1;
k1=5;
k2=7;
b1=2;
b2=4;
b3=6;
num=[m1 (b1+b2+b3) k1];
den=[(m1*m2) (m2*(b1+b2+b3)+b2*m1) (k1*m2+b2*(b1+b3)+k2*m1) (b2*k1+k2*(b1+b2+b3)) k1*k2];
```



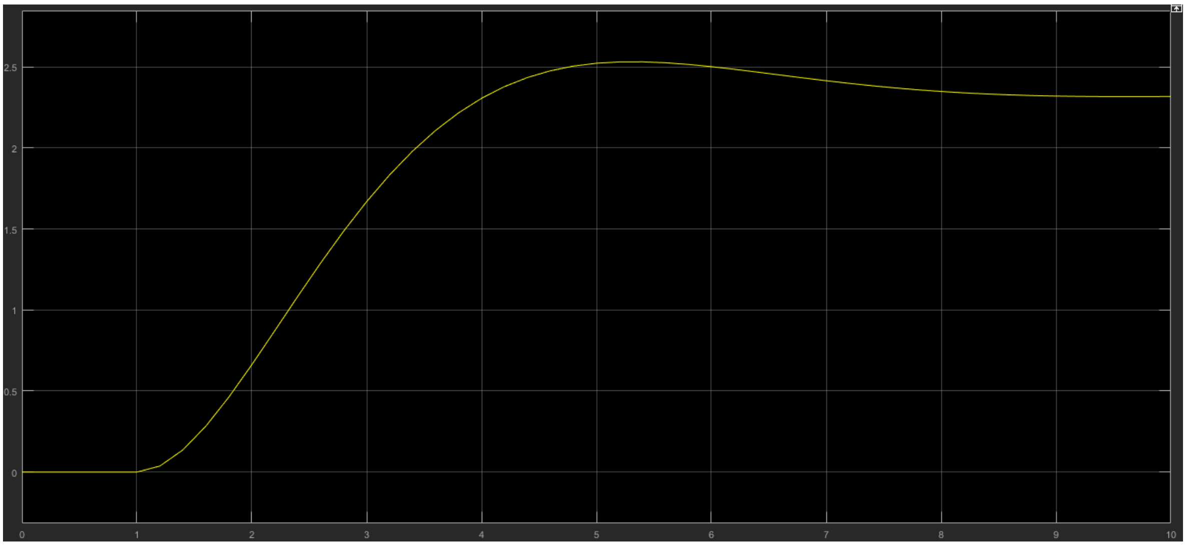
b) Figure 5:

```
m1=2;
m2=4;
k1=3;
k2=1;
b1=1;
b2=8;
num=k1;
den=[(m1*m2) (m1*b2+m2*b1) (m1*k1+m1*k2+b1*b2+k1*m2) (b1*k1+b1*k2+b2*k1) ((k1^2)-m1*(k1^2)+k1*k2)];
```



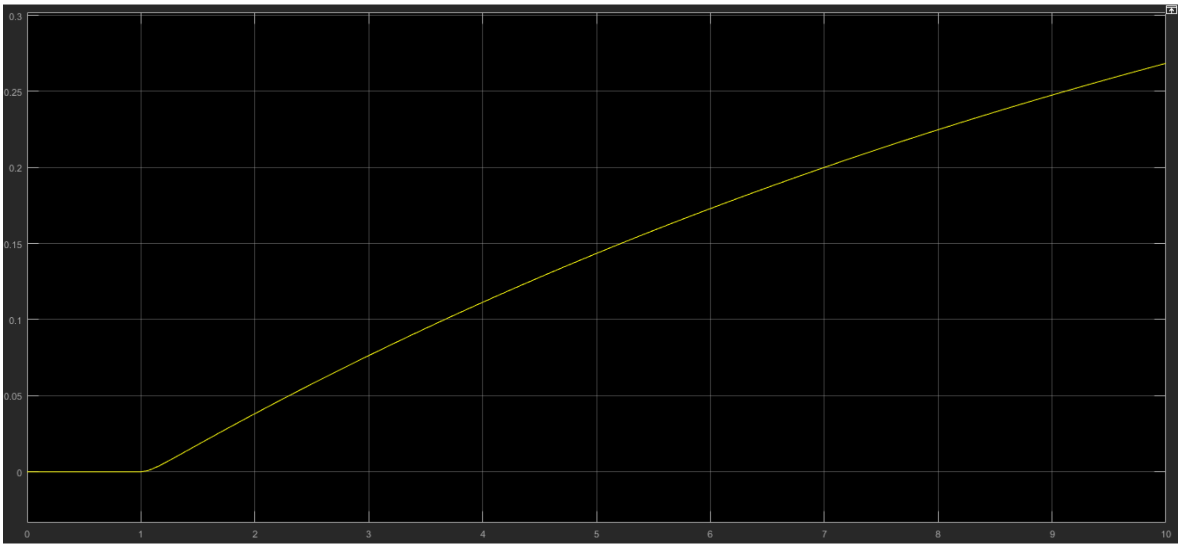
c) Figure 6:

```
j1=2;
j2=7;
k1=5;
k2=6;
b1=2;
b2=8;
r2=10;
r1=5;
num=[j2*r2];
den=[(j2*r1) (b2*r1) (k2*r1)];
```



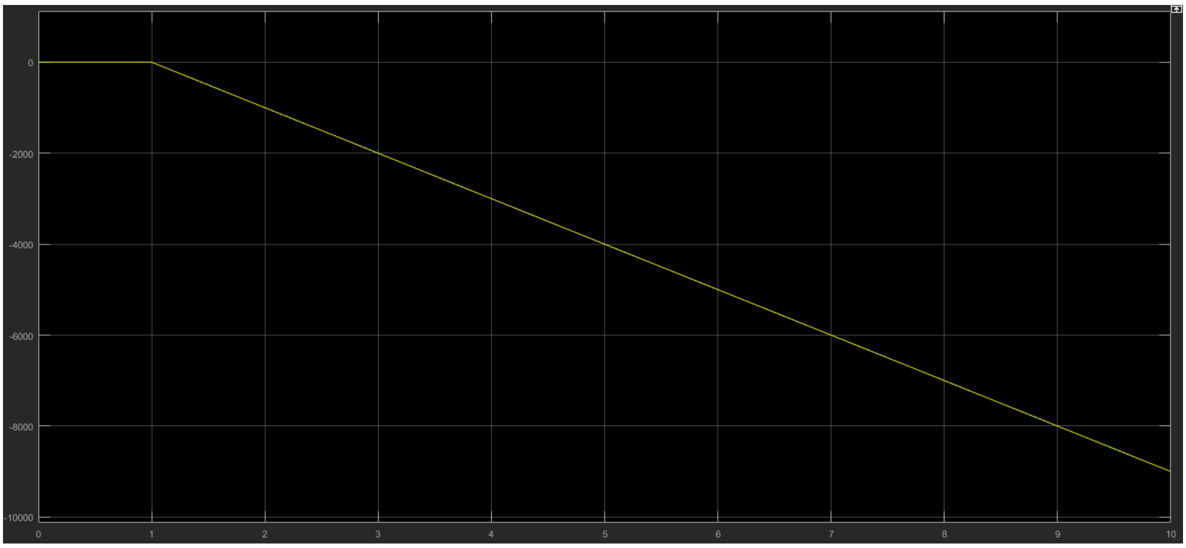
d) Figure 7:

```
m=5;
k1=3;
k2=2;
b=20;
d2=10;
d1=5;
num=[k1*k2*d1*d2];
den=[(m*(d1^2)*k2+(d2^2)*k1) (b*((d1^2)*k2+(d2^2)*k1)) (k1*k2*(d2^2))];
```



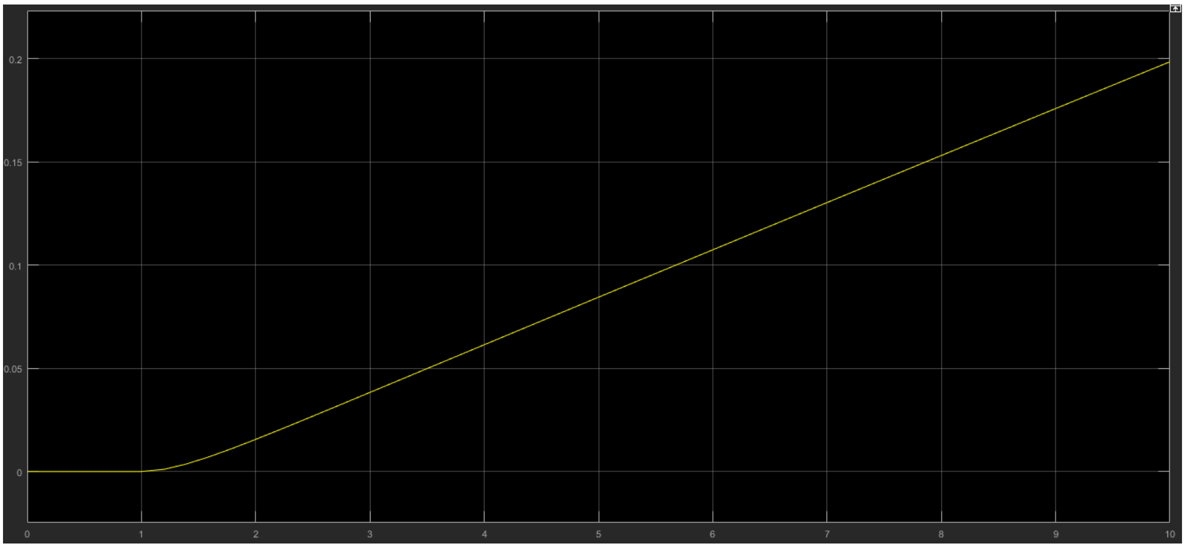
e) Figure 9:

```
R=1000;
C=10^(-6);
num=-1;
den=[R*C 0];
```



f) Figure 10:

```
r1=1;
r2=7;
c1=5;
c2=3;
num= [1/(r1*c1*c2)];
den=[1 ((1/c1*r1)+(1/c2*r1)+(1/c2*r2)) (1/(c1*c2*r1*r2))];
```



g) Figure 11

```
k1=7;
k2=10;
c1=4;
c2=9;
A=[(-k1/2*c1) 0; 0 (-k2/2*c2)];
B=[1/c1;1/c2];
C=[0 1];
D=0;
[num,den]=ss2tf(A,B,C,D,1);
```

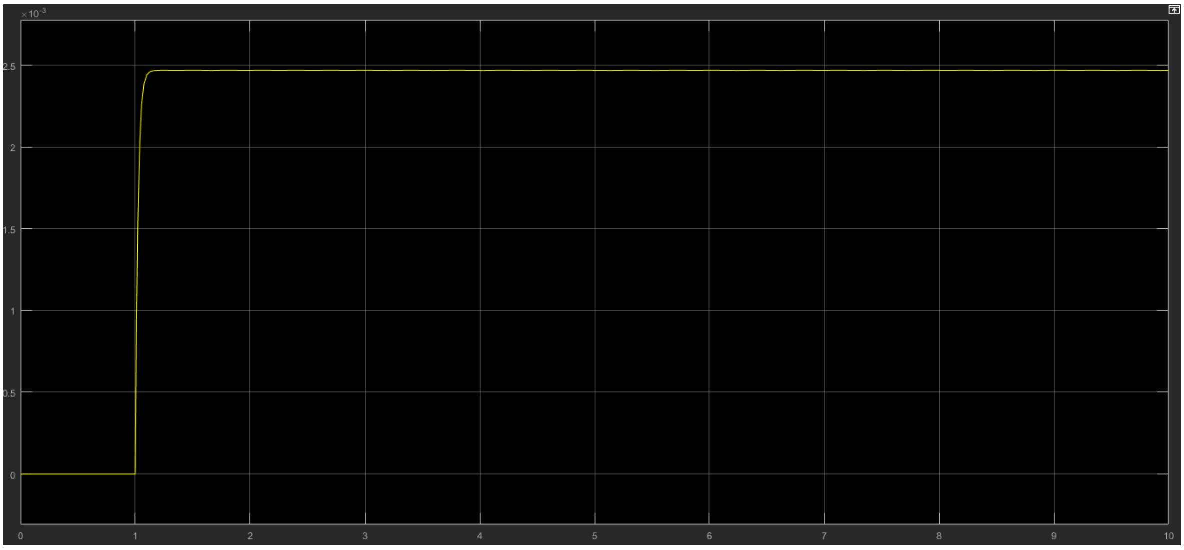


Figure 8:

```
r1=520;
r2=1000;
r3=1200;
c=10^(-6);
L=.1;
A=[(1/r3-1/r2-1/r1)/c -1; ((r1*r3)/r1+r3)*((1/r1-1/r3)/L) 0];
B=[1/c;((r1*r3)/r1+r3)*((1/r3-1/r1)/L)];
C=[1 0];
D=0;
[num,den]=ss2tf(A,B,C,D,1);
```