Disturbances on control, and review of control in state space.

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Abstract--- In this document we will study the behavior of a mass-spring system with disturbance, modeling it through a representation in state space in order to be able to take it to its correct form, controllable canonical and observable canonical. By obtaining these representations, a controller can be designed that obeys the control law using three different methods to calculate the controller constants in a vectorial way. Likewise, a state observer can be calculated by means of three methods, through which the behavior of any state variable that is occasionally difficult to physically measure can be shown. The behavior of the aforementioned will be shown, but adding an integration constant, which will turn it into a servo system.

Abstract--- In this document the behavior of a spring mass system with perturbation modeling will be studied by means of a representation in space of states so as to be able to take it to its controllable canonical and observable canonical form. By obtaining such representations a controller can be designed that obeys the control law by means of three different methods to calculate in a vector way the constants of the controller. Likewise, a state observer can be calculated by means of three methods, by means of which the behavior of any state variable that is occasionally difficult to measure physically can be shown. It will show the behavior of the above mentioned but adding an integration constant, which will turn it into a servosystem.

Keywords--- State space, control law, observer, servosystem.

General Objective--- Reinforce the acquired concepts of control in state space.

Specific objectives--

* Model a system by means of representation in state variables under the controllable canonical and observable canonical forms.

* Simulate the system in state space using MATLAB.

* Control the system using the control law u= - kx.

* Design state watchers.

* Design servo systems.

1. INTRODUCTION

A modern control system can have many inputs and many outputs, and these are interrelated in complex ways. State-space methods for the analysis and synthesis of control systems are best suited to dealing with multi-input, multi-output systems that are required to be optimal in some sense.

This method is based on the description of the system in terms of n first-order difference or differential equations, which can be combined into a first-order difference or differential matrix equation. The use of matrix notation greatly simplifies the mathematical representation of systems of equations.

The methods in the state space allow you to include initial conditions within the design. This is a very important feature not considered in conventional design methods [1].

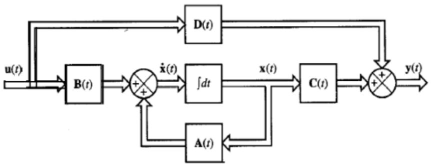
State: The state of a dynamical system is the smallest set of variables (called state variables) such that the knowledge of said variables at $t = t0$ together with the knowledge of the input for $t \geq t0$. They completely determine the behavior of the system for any time $t \geq t0$.

State variables: The state variables of a dynamic system are those that make up the smallest set of variables that determine the state of the dynamic system. If to fully describe the behavior of a dynamical system, at least n variables $x1$, $x2$, $\cdots$, xn are required (so that once the input for $t \geq t0$ and the initial state at $t = t0$, the future state of the system is completely determined), then these n variables are considered a set of state variables.

State vector: If n state variables are needed to completely describe the behavior of a given system, then these n state variables can be considered as the n components of a vector $\vec{x}$. Such a vector is known as a state vector. A state vector is, therefore, a vector that uniquely determines the state $\vec{x}(t)$ of the system for any time $t \geq t0$, once the state at $t = t0$ is given and the input $u(t)$for $t > t0$.

State space: The n dimensional space whose coordinate axes are formed by the $eje\ x1$, $eje\ x2$, $\cdots$, $eje\ xn$ is known as the state space. Any state can be represented by a point within that state space [2].

Image 1 shows the basic diagram of the representation by system states and the equations.



$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

Figure 1: Representation of states of a system.

State feedback regulators are based on the formulation of the control law $u = -K\vec{x}$, where $\vec{x}$ are the states of the system (measurable and/or estimated as applicable), and K is the feedback gain. If zero steady state error is wanted in the feedback design, an additional feedback loop is made, where the output is directly compared to the desired reference (In(t)-Out(t)). This error must then be multiplied by a gain and integrated to be added to the aforementioned feedback terms.

State Observers: When not all states $\vec{x}$ (as is the common case), an observer can be constructed to estimate them, while only measuring the output $(t) = C\vec{x}(t)$. The scheme is shown in image 2.

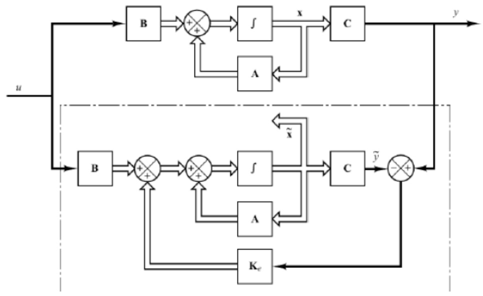


Figure 2: Representation of states of a system with observer.

The observer is basically a copy of the plant; has the same input and almost always the same differential equation. An extra term compares the current output measurement with the estimated output $\hat{y}(t)$; causing the estimated states to approach the actual values of the states. The dynamics of the observer error is given by the poles of $(A - LC)$. [3]

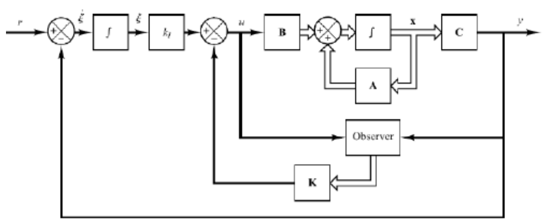


Figure 3: Representation of a servosystem

2. MATERIALS

* Software: MATLAB ®

* White coat.

3. PROCEDURE

For the following system, the model must be made in state space, taking into account the disturbance as a sinusoidal displacement.

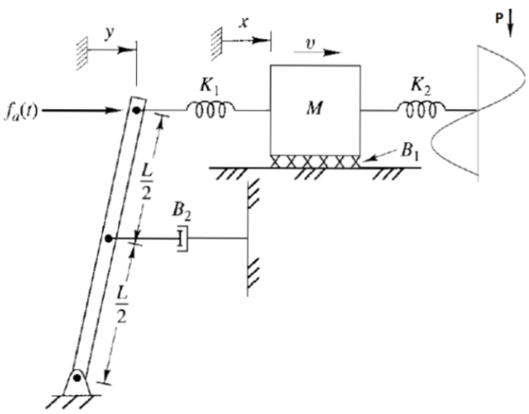


Figure 4: Spring mass system.

The modeling was carried out representing the free body diagrams for the mass and for the bar which is anchored to the ground, which indicates its pivot point. For the sum of torques, an angle formed by the bar and the vertical is taken into account, but since all the terms of the resulting equation contain terms of said angle and also of the length of the bar, then they will be eliminated.

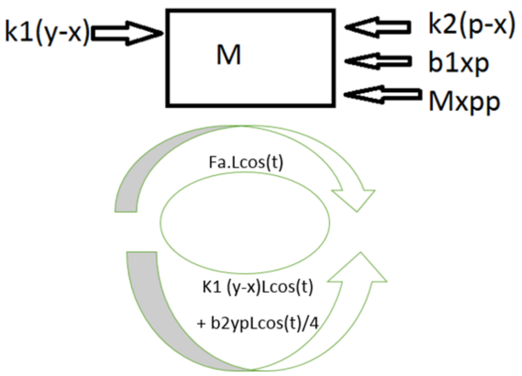


Figure 5: Free body diagrams to add forces and torques.

$$\frac{b2\,\dot{y}\,L\cos\theta}{4}+k1(y-x)L\cos\theta-FaL\cos\theta=0$$

$$\dot{y}=Fa\left(\frac{4}{b2}\right)+x\left(\frac{4k1}{b2}\right)-y\left(\frac{4k1}{b2}\right) \qquad (1)$$

$$v=\dot{x} \qquad (2)$$

$$M\dot{v}+b1v+k2(p-x)-k1(y-x)=0$$

$$\dot{v}=-v\left(\frac{b1}{M}\right)-x\left(\frac{k1-k2}{M}\right)+y\left(\frac{k1}{M}\right)-p\left(\frac{k2}{M}\right) \qquad (3)$$

After modeling, the system matrices are as follows. Where E is the disturbance matrix.

```
A =
[              0,      1,          0]
[ -(k1 + k2)/m, -b1/m,         k1/m]
[      (4*k1)/b2,      0, -(4*k1)/b2]
B =
    0
    0
  4/b2
C =
     1     0     0
D =
     0
E =
     0
  -k2/m
     0
```

Figure 6: System matrices in terms of variables.

And then substituting the values of the constants:

```
k1 = 0.6; % N/m 5
k2 = 0.8; % N/m 10
b1 = 0.2; % N*s/m 20
b2 = 0.5; % N*s/m 10
m = 0.5; % kg
```

Figure 7: System constants.

These values are taken randomly but taking into account a real situation.

```
A =
         0    1.0000         0
   -2.8000   -0.4000    1.2000
    4.8000         0   -4.8000
B =
     0
     0
     8
C =
     1     0     0
D =
     0
E =
         0
   -1.6000
         0
```

Figure 8: System arrays with values.

Then, to find the matrices of the canonical controllable and canonical observable forms, one must first find the matrix W, given by the general form:

$$W = \begin{bmatrix} a_{n-1} & a_{n-2} & a_1 & 1 \\ a_{n-2} & \dots & 1 & 0 \\ \dots & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

```
W =
    4.7200    5.2000    1.0000
    5.2000    1.0000         0
    1.0000         0         0
```

Figure 9: Matrix W with the values replaced.

For the FCC, the controllability matrix must be found.

```
MatCont = [ B A*B (A^2)*B] ;
```

And the following procedure is carried out, where the FCCs are defined for all the matrices.

```
T = MatCont*W;
Tinv = inv(T);

A2FCC = Tinv*A*T;
B2FCC = Tinv*B;
C2FCC = C*T;
E2FCC = Tinv*E;
```

Then, to represent the observable canonical form, the observability matrix must be found and the following procedure must be carried out.

```
MatObs = [ C ; C*A ; C*(A^2)];

T2 = inv(W*MatObs);
T2inv = inv(T2);

A2FCO = T2inv*A*T2;
B2FCO = T2inv*B;
C2FCO = C*T2;
E2FCO = T2inv*E;
```

REGULATORS

A state feedback regulator is designed using the three design methods:

Methods

- Weight matrix compensation.
- Equalization of polynomials.
- Ackermann's method.

To carry out the state space control, we start from now on having a characteristic polynomial (obtained from the plant) and a desired polynomial (obtained from the design criteria)

. The characteristic polynomial is the following.

$$s^3 + 5.2s^2 + 4.72s + 7.68$$

Obtained by performing the determinant of (SI -A).

The control objectives

: Settling time: 8s

Damping coefficient zita ξ: 0.7

Desired polynomial:

$$s^3 + 6s^2 + 5.51s + 2.55$$

This polynomial adds the following poles to the system:

```
                        -5
  - (51^(1/2)*1i)/14 - 1/2
    (51^(1/2)*1i)/14 - 1/2
```

Regulator: method 1

This method 1 is through compensation by weight matrix.

To find the constant K a using method 1, the following formula is used:

$$\mathbf{K} = \left[\alpha_3 - a_3 \;\vdots\; \alpha_2 - a_2 \;\vdots\; \alpha_1 - a_1\right]\mathbf{T}^{-1}$$

Where the values of α are constants of the equation of the desired polynomial, which in our case will be:

```
alf1 = 6 ; alf2 = 5.5 ; alf3 = 2.55;
```

And the values of 'a' will be those of the characteristic polynomial.

```
a1 = 5.2; a2 = 4.72; a3 = 7.68;
```

And in addition to the inverse of the matrix T where this is:

T=M*W

M = Controllability matrix

W = Weight matrix

Regulator: method 2

For the second method, equalization of polynomials, the determinant of the expression must be found:

$$|s\mathbf{I} - \mathbf{A} + \mathbf{BK}|$$

Where K is the vector:

K = [k1met2 k2met2 k3met2]

```
syms k1met2 k2met2 k3met2
Kmet2 = [ k1met2 k2met2 k3met2];
SIABK = det((MatDeEses - A) + (B*Kmet2));
SIABK = vpa(SIABK)
k3met2 = (alf1-5.2)/8;
k2met2 = (alf2-3.2*k3met2-4.72)/9.6;
k1met2 = (alf3-7.68-22.4*k3met2)/9.6;
```

Being the determinant:

*6*k1met2 + 22.4*k3met2 + 4.72*s + 9.6*k2met2*s + 3.2*k3met2*s + 8.0*k3met2*s^2 + 5.2 *s^2 + s^3 + 7.68*

which equates to the desired polynomial.

And the result of this determinant must be equal to the desired polynomial, by doing this it is possible to find the values of K, which must **BE EQUAL BY THE 3 METHODS.**

Regulator: method 3

Ackerman's method, where the state space feedback constant is obtained as follows.

$$\mathbf{K} = [0 \quad 0 \quad 1]\left[\mathbf{B} \;\vdots\; \mathbf{AB} \;\vdots\; \mathbf{A}^2\mathbf{B}\right]^{-1}\phi(\mathbf{A})$$

Being ΦA:

$$\alpha_3\mathbf{I} + \alpha_2\mathbf{A} + \alpha_1\mathbf{A}^2 + \mathbf{A}^3 = \phi(\mathbf{A})$$

Then performing this procedure in MATLAB.

```
FiA = (A)^3 + alf1*((A)^2) + alf2*A + alf3*MatIdnt;

BABA2B = [B A*B (A^2)*B];
InvBABA = inv(BABA2B);

Kmet3 = [ 0 0 1]*(InvBABA)*FiA
```

OBSERVERS

For each observer method, the observer constant ke must be found.

Observer: method 1

have the following expressions:

$$\mathbf{Q} = (\mathbf{WN}^*)^{-1}$$

$$\mathbf{N} = [\mathbf{C}^* \,\vdots\, \mathbf{A}^*\mathbf{C}^* \,\vdots\, \cdots \,\vdots\, (\mathbf{A}^*)^{n-1}\mathbf{C}^*]$$

Where C* and A* are the transposed matrices of the C and A matrices of the system, respectively.

$$\mathbf{W} = \begin{bmatrix} a_{n-1} & a_{n-2} & \cdots & a_1 & 1 \\ a_{n-2} & a_{n-3} & \cdots & 1 & 0 \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ a_1 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

To find the constant Ke of the observer using method 1, the following expression is solved:

$$\mathbf{K}_e = (\mathbf{WN}^*)^{-1}\begin{bmatrix} \alpha_2 - a_2 \\ \alpha_1 - a_1 \end{bmatrix}$$

For the case of a desired second-order polynomial

Then, for the case of a system where the polynomial is of the third degree, there will be an alpha 3 and an a3.

```
Atrans = A';
Ctrans = C';
N = [Ctrans Atrans*Ctrans (Atrans^2)*Ctrans];
Ntrans = N';
WN = W*Ntrans;
Q = inv(WN);
Ke = Q*[alf3 - a3 ; alf2 - a2 ; alf1 - a1]
```

Observer: method 2

To find the constant vector Ke using method 2, it is assumed that the determinant of the following expression is equal to 0

$$|s\mathbf{I} - \mathbf{A} + \mathbf{K}_e\mathbf{C}| = 0$$

Where Ke = [ke1 ; ke2 ; ke3]

Where the result is a polynomial of degree 3 in terms of Ke, said polynomial is the following.

*1.92*ke1 + 4.8*ke2 + 1.2*ke3 + 4.72*s + 5.2*ke1*s + ke2*s + ke1*s^2 + 5.2*s^2 + s^3 + 7.68*

which must be set equal to the polynomial desired stated above.

Carrying out this procedure in MATLAB where the constants of the vector Ke are already cleared is:

```
syms ke1 ke2 ke3
Kemet2 = [ ke1 ; ke2 ; ke3];

DetObsMet2 = det(MatDeEses - A + Kemet2*C);
DetObsMet2 = vpa(DetObsMet2)

ke1 = alf1-5.2;
ke2 = alf2-4.72-ke1*5.2;
ke3 = (alf3-4.8*ke2-1.92*ke1-7.68)/1.2;
Kemet2 = [ ke1 ; ke2 ; ke3]
```

Observer: method 3

The equation to find the constant Ke using the Ackerman method is the following.

$$\mathbf{K}_e = \mathbf{K}^* = \phi(\mathbf{A}^*)^* \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{CA}^{n-2} \\ \mathbf{CA}^{n-1} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix} = \phi(\mathbf{A}) \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{CA}^{n-2} \\ \mathbf{CA}^{n-1} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix}$$

Where Fi is A.

$$\alpha_3\mathbf{I} + \alpha_2\mathbf{A} + \alpha_1\mathbf{A}^2 + \mathbf{A}^3 = \phi(\mathbf{A})$$

Performing these operations in MATLAB.

```
ObsFiA = A^3 + alf1*A^2 + alf2*A + alf3*MatIdnt;

CCA = [ C ; C*A ; C*(A^2)];

invCCA = inv(CCA);

Kemet3 = ObsFiA*invCCA*[ 0; 0; 1]
```

Servosystems

For the servosystems, a state space was built, represented by packed matrices, which are constructed as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & o \end{bmatrix} \begin{bmatrix} x \\ \xi \end{bmatrix} + \begin{bmatrix} B \\ o \end{bmatrix}$$

$$y = \begin{bmatrix} C & o \end{bmatrix} \begin{bmatrix} x \\ \xi \end{bmatrix}$$

In this way, the packed matrices A, B, and C are as follows:

```
Aempq =

         0    1.0000         0         0
   -2.8000   -0.4000    1.2000         0
    4.8000         0   -4.8000         0
   -1.0000         0         0         0


Bempq =

     0
     0
     8
     0


Cempq =

     1     0     0     0
```

After having these matrices, the matrix of controllability, the matrix M and the matrix T. This is because the matrices changed and for the first method the matrix T is used to calculate the constants of the controller vector.

```
MattCont =

         0         0    9.6000  -49.9200
         0    9.6000  -49.9200  214.2720
    8.0000  -38.4000  184.3200 -838.6560
         0         0         0   -9.6000
```

The characteristic polynomial of this new representation is obtained with the determinant of sI-Aempq which is also used to calculate the matrix M and subsequently the matrix T.

```
eqnMM =

s^4 + 5.2*s^3 + 4.72*s^2 + 7.68*s


MM =

    7.6800    4.7200    5.2000    1.0000
    4.7200    5.2000    1.0000         0
    5.2000    1.0000         0         0
    1.0000         0         0         0


TTT =

         0    9.6000         0         0
         0         0    9.6000         0
         0   22.4000    3.2000    8.0000
   -9.6000         0         0         0
```

must be taken into account that the vector K that is going to be obtained is the following:

$$[k1 \;\; k2 \;\; k3 \;\; -ki]$$

Then this last value of the vector must be negative to obtain the real value of the integration constant ki, and with the others group them in a new vector K.

Method 1

Using the weight matrix compensation method, the vector K of the following form:

$$[\alpha 4 - a4 \quad \alpha 3 - a3 \quad \alpha 2 - a2 \quad \alpha 1 - a1]T^{-1}$$

the a values are obtained from the characteristic polynomial, which are the following:

aa1 = 5.2; aa2 = 4.72; aa3 = 7.68; aa4=0

the values of αare obtained from the desired polynomial, which in this case will change since now it will be of cautious order and therefore two non-dominant poles must be added.

s^4 + 10.7*s^3 + 32.25*s^2 + 20.0*s + 6.25

alff1 = 10.7 ; alff2 = 32.25 ; alff3 = 20 ; alff4= 6.25

Method 2

For the polynomial equalization method, the determinant is found

$$|s\mathbf{I} - \mathbf{A} + \mathbf{BK}|$$

Where K=[k1 k2 k3 k4]

Taking into account that k4=-ki the resulting equation is equal to the desired polynomial to clear the values of the constants of the vector

```
syms k1sermet2 k2sermet2 k3sermet2 kisermet2
KKsermet2 = [ k1sermet2 k2sermet2 k3sermet2 kisermet2]

SIABKempq = vpa(det((SIIII-Aempq) + (Bempq*KKsermet2)))

k3sermet2 = (alff1-5.2)/8;
k2sermet2 = (alff2-3.2*k3sermet2-4.72)/9.6;
k1sermet2 = (alff3-7.68-22.4*k3sermet2)/9.6;
kisermet2 = (-alff4)/9.6
KKsermet2 = [ k1sermet2 k2sermet2 k3sermet2 kisermet2]
kimet2=-KKsermet2(4)
Ksermet2=[k1sermet2 k2sermet2 k3sermet2]
```

The result of that determinant is:

7.68*s - 9.6*kisermet2 + 9.6*k1sermet2*s + 22.4*k3sermet2*s + 9.6*k2sermet2*s^2 + 3.2*k3sermet2*s^2 + 8.0*k3sermet2*s^3 + 4.72*s^2 + 5.2*s^3 + s^4

Method 3

Using the ackerman method, the vector K is obtained as follows:

$$\alpha_4 I + \alpha_3 A_{empq} + \alpha_2 A_{empq}{}^2 + \alpha_1 A_{empq}{}^3 = \Phi(A_{empq})$$
$$K = [0 \ \ 0 \ \ 0 \ \ 1] T^{-1} [B \ \ AB \ \ A^2B \ \ A^3B] \ \ \Phi(A_{empq})$$

Then the following procedure was carried out in MATLAB.

```
FiiA = (Aempq)^4 + alff1*((Aempq)^3) + alff2*((Aempq)^2) + alff3*Aempq + alff4*MattIdnt;
InvvBABA = inv(MattCont);
KKsermet3 = [ 0 0 0 1]*(InvvBABA)*FiiA
k1sermet3=KKsermet3(1);
k2sermet3=KKsermet3(2);
k3sermet3=KKsermet3(3);
kimet3=-KKsermet3(4)
Ksermet3=[k1sermet3 k2sermet3 k3sermet3]
```

4. ANALYSIS RESULTS

FCC

The matrices of the FCC form are as follows.

```
A2FCC =
         0    1.0000         0
   -0.0000    0.0000    1.0000
   -7.6800   -4.7200   -5.2000
B2FCC =
     0
     0
     1
C2FCC =
    9.6000         0         0
E2FCC =
         0
   -0.1667
    0.0667
```

Making the diagram in simulink in the controllable canonical form and simulating, the following response is obtained.

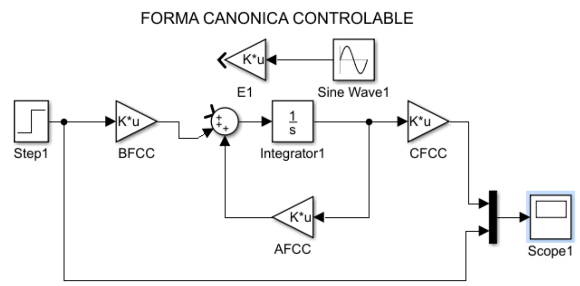


Figure 10: FCC state space.

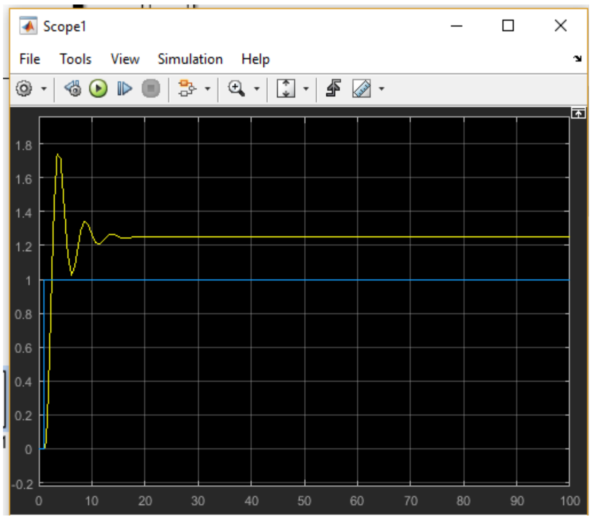


Figure 11: Open loop response of the FCC representation.

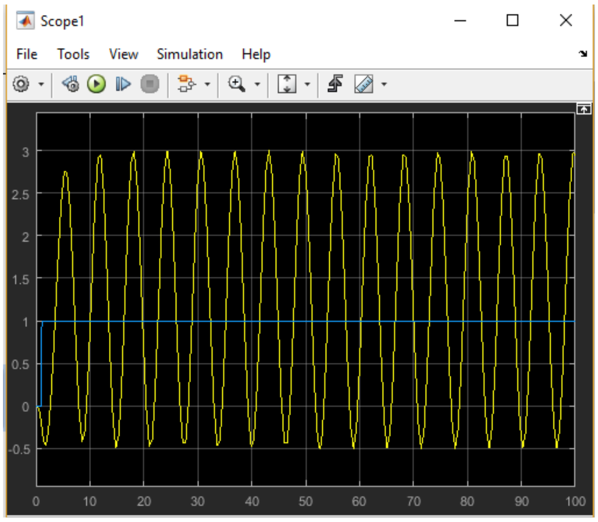


Figure 12: Response with disturbance of the FCC representation.

FCO

```
A2FCO =
     0.0000    -0.0000    -7.6800
     1.0000     0.0000    -4.7200
    -0.0000     1.0000    -5.2000
B2FCO =
     9.6000
     0.0000
     0.0000
C2FCO =
    -0.0000     0.0000     1.0000
E2FCO =
    -7.6800
    -1.6000
    -0.0000
```

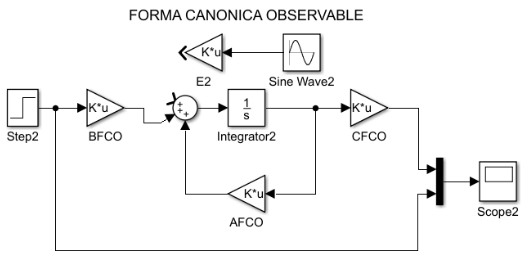


Figure 13: FCO state space

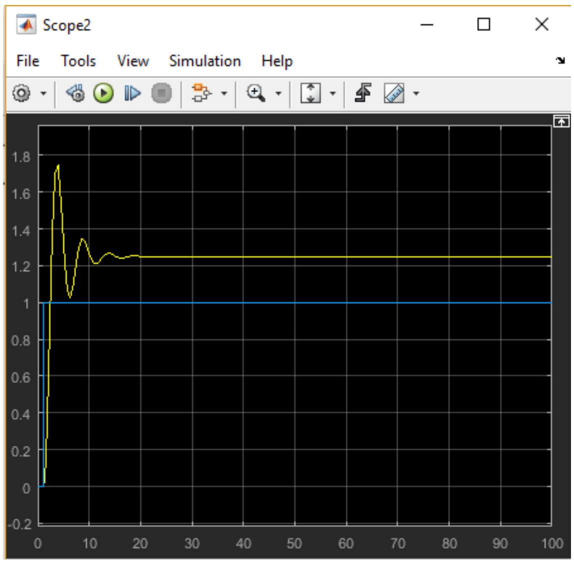


Figure 14: Open-loop response of the FCO representation

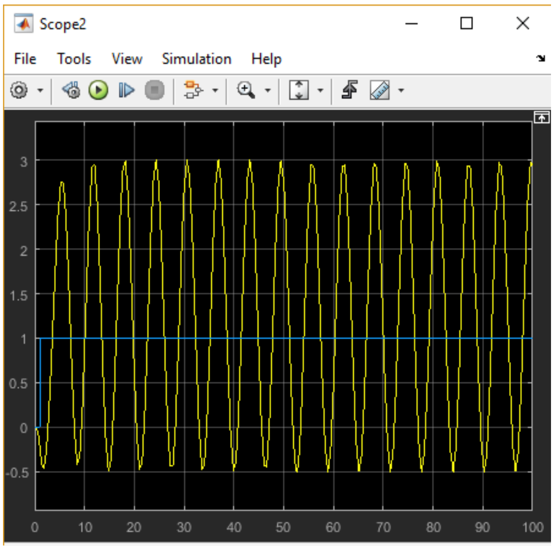


Figure 15: Perturbation response of the FCO representation.

In both cases FCC and FCO the same answer is obtained.

Regulator: method 1

Using the equation for method 1

```
Kmet1 = [(alf3-a3) (alf2-a2) ((alf1-a1))]*Tinv
```

```
Kmet1 =
    -0.7677     0.0479     0.1000
```

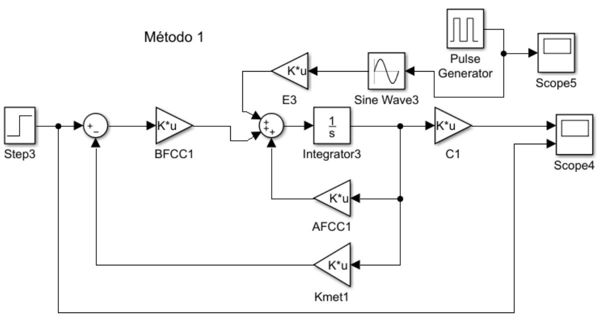


Figure 16: State space with feedback gain K method 1.

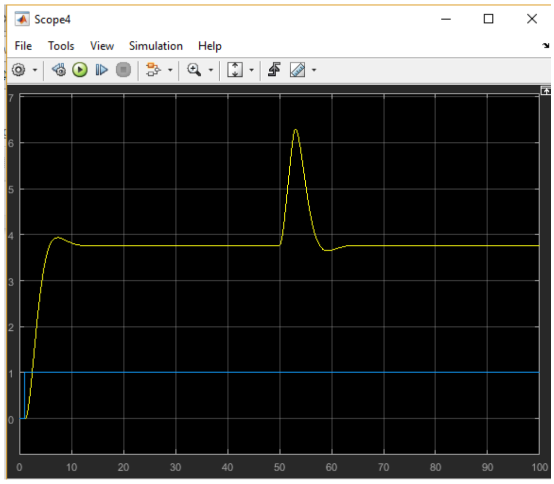


Figure 17: Scope response, method 1.

The underdamped response is observed stabilizing at 8 seconds and the signal controlling just after adding a sinusoidal disturbance a when t = 50s.

Regulator: method 2

Having then the K obtained previously, the simulink is entered.

```
Kmet2 = [ k1met2 k2met2 k3met2]
```

```
Kmet2 =
    -0.7677     0.0479     0.1000
```

Then the vector kmet2 is entered as gain in the diagram made in simulink.

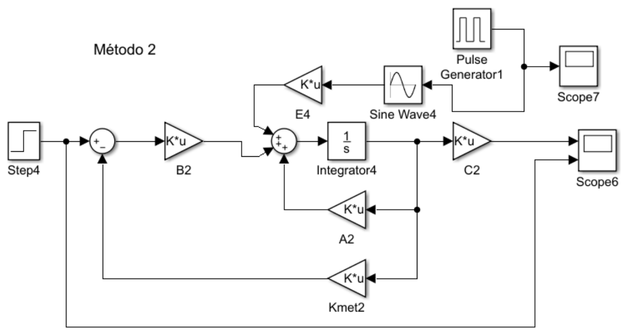


Figure 18: State space with feedback gain K method 2.

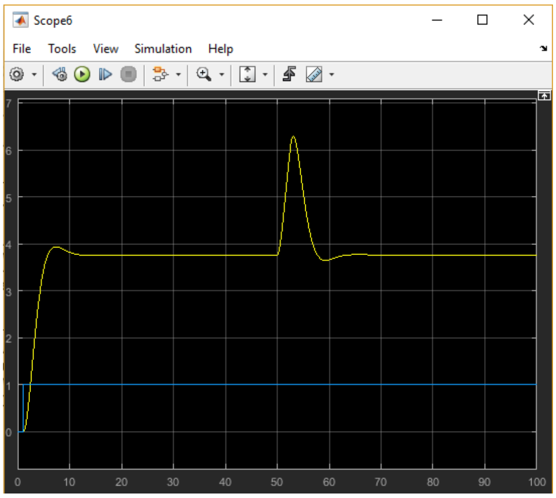


Figure 19: Scope response, method 2.

Regulator: method 3

The K is obtained using the Ackerman method:

```
Kmet3 =
   -0.7677    0.0479    0.1000
```

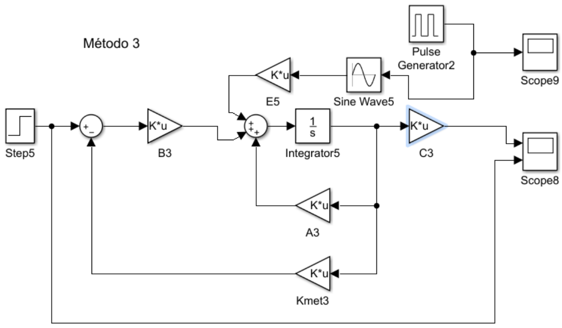


Figure 20: State space with feedback gain K method 3.

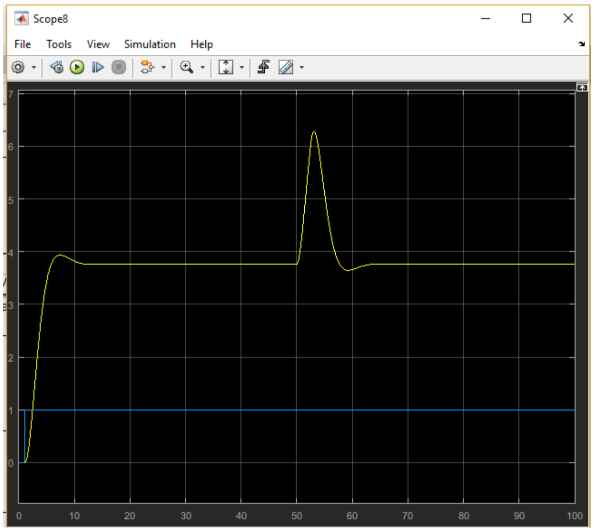


Figure 21: Scope response, method 3.

Where the same output is observed with respect to the other two previous methods, this means that the procedure was carried out correctly.

Observer method 1

The observer constant will be

```
Ke =
     0.8000
    -3.3800
     7.9650
```

Y will be the one implemented in the simulink diagram.

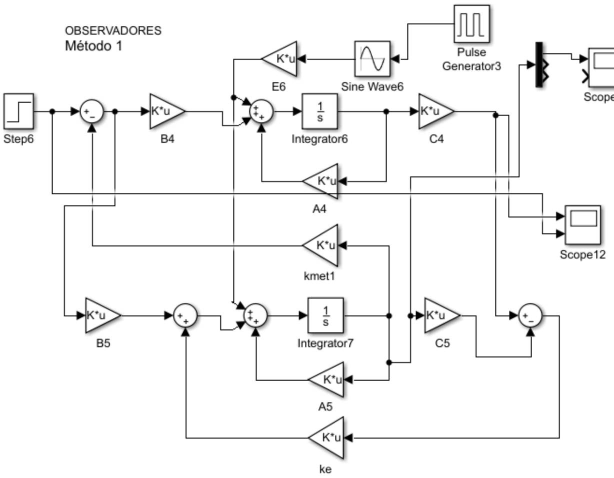


Figure 22: State space with observer, method 1.

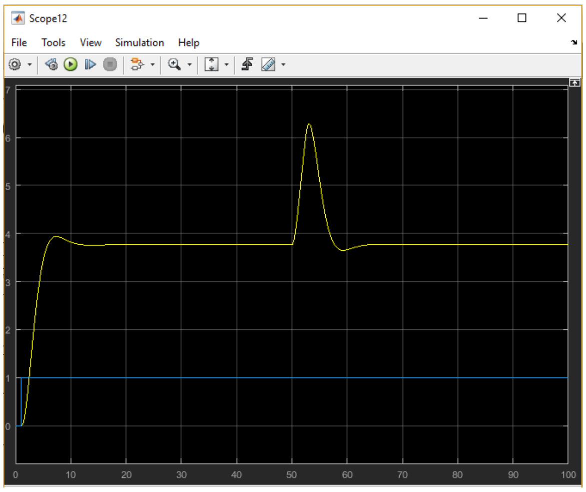


Figure 23: Scope response, observer method 1

Using the Ke of the observer, the same response as in the previous cases is obtained.

Observer method 2

Obtaining the vector ke by method 2, it is then implemented in the simulink.

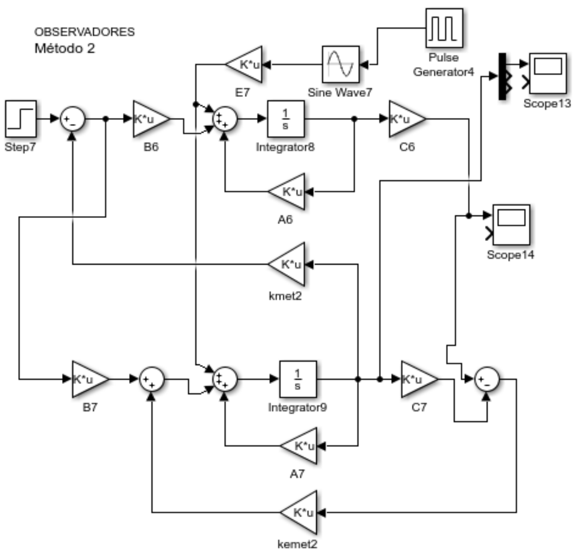


Figure 24: State space with observer, method 2

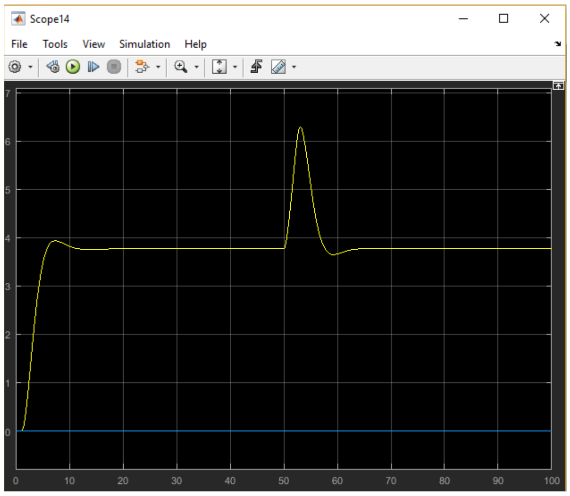


Figure 25: Scope response, observer method 2

Where again it is observed that the response is equal to both the observer of method 1 and the regulator.

Observer method 3

The constant Kemet3 is the one obtained by the Ackerman method for observers, where the result is equal to the previous ones.

```
Kemet3 =
    0.8000
   -3.3800
    7.9650
```

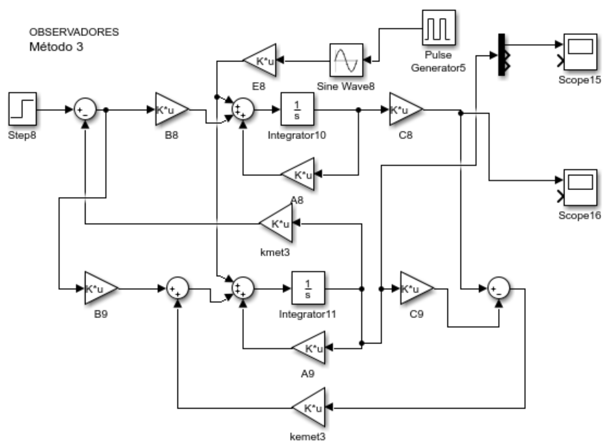


Figure 26: State space with observer, method 3

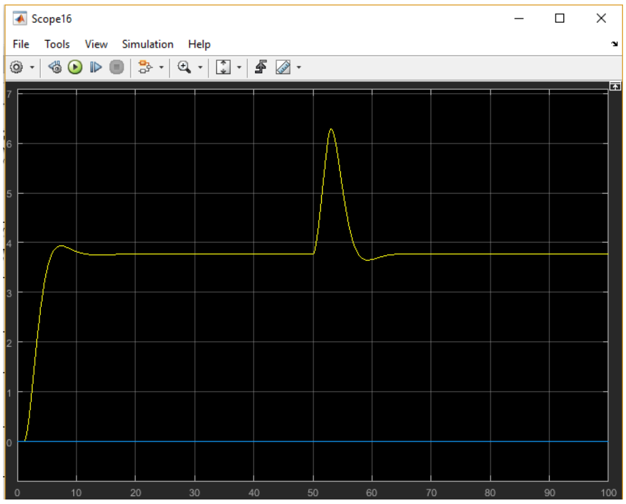


Figure 27: Scope response, observer method 3

And the same response is obtained, which indicates that the procedure was carried out correctly, this time by the third method.

Method 1 servo system

The constants obtained by the weight matrix compensation method for the controller, and the constant ki obtained are

```
KKsermet1 =

   -0.3208    2.6385    0.6875   -0.6510


kimet1 =

    0.6510


Ksermet1 =

   -0.3208    2.6385    0.6875
```

These are the control constants K and the integration constant ki that are implemented in simulink

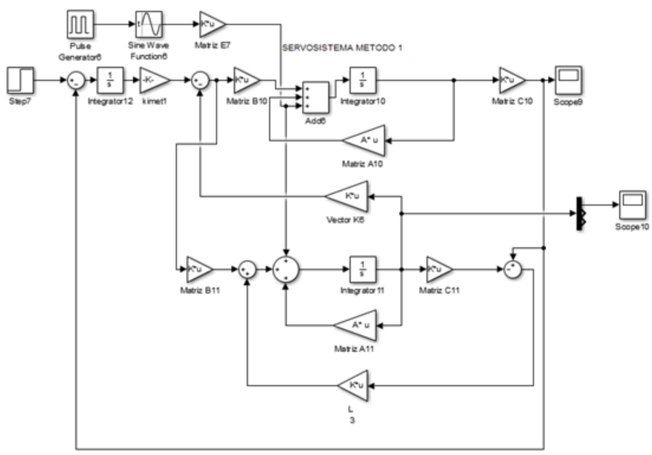


Figure 28: Block diagram of the servosystem found by method 1.

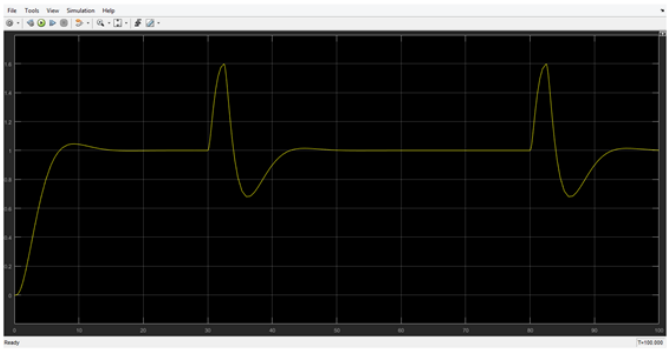


Figure 29: Servosystem response with method 1

Servosystem method 2

The constants obtained by the polynomial equalization method are

```
kimet2 =

    0.6510


Ksermet2 =

   -0.3208    2.6385    0.6875
```

These are the control constants K and the integration constant ki that are implemented in simulink

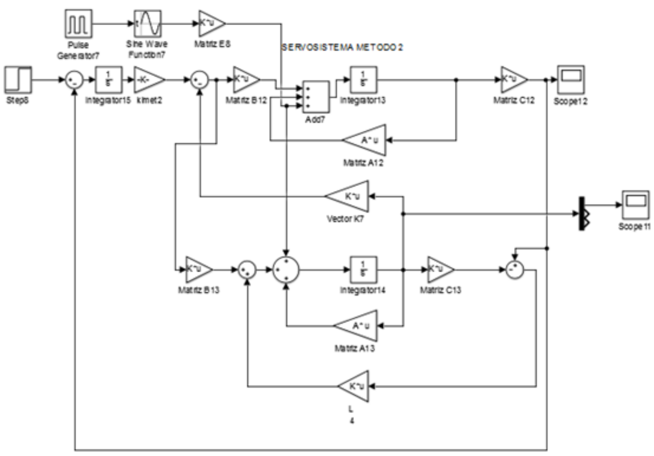


Figure 30: Block diagram of the servo system found by method 2.

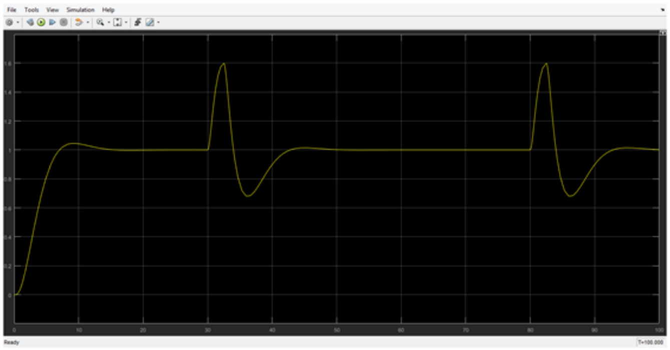


Figure 31: Response of the servo system with method 2

Servo system method 3

The constants obtained by the Ackerman method are

```
KKsermet3 =

   -0.3208    2.6385    0.6875   -0.6510


kimet3 =

    0.6510


Ksermet3 =

   -0.3208    2.6385    0.6875
```

These are the control constants K and the integration constant ki that are implemented in simulink

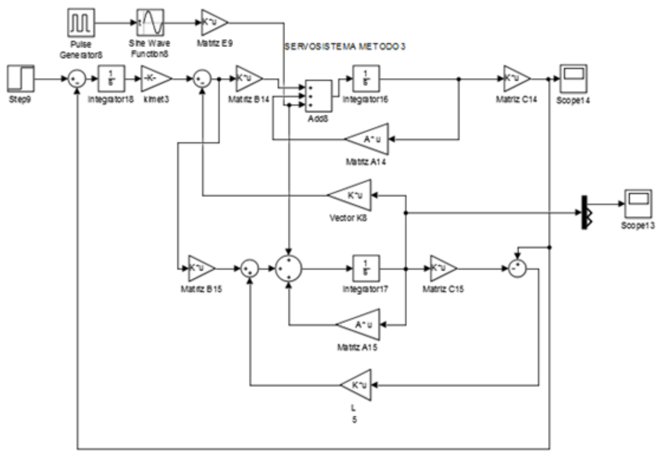


Figure 32: Block diagram of the servosystem found by method 3.

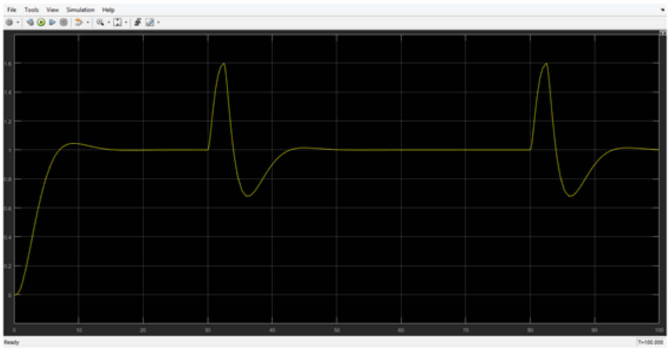


Figure 33: Servosystem response ma with method 3

5. CONCLUSIONS

When making the regulators by the different methods, the result was the same, and these cases are fulfilled for systems of any order.

With an observer, any state variable can be measured and it is observed that the measurement of the specific variable to be controlled is the same with the observer or with the controller alone.

Using a servo system greatly improves the behavior of the controlled system, thus achieving the control objectives.

The servo system fulfills the great function of following a reference, which makes it the most used system in the industry due to this characteristic, in addition to the fact that its development is very simple.

6. REFERENCES

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