Laboratory 3: Position and speed control of a DC motor.

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Abstract--- In this practice, the position and speed variables of a DC motor were controlled. Carrying out a PID control for the motor and that meets the control parameters (ess= 0, ζ = 0.7 and ts=85% of the establishment time in open loop) required before a step, ramp and acceleration input.

Abstract--- In this practice is presented the control of position and speed variables of a DC Motor. Calculating a PID for the motor and making it satisfy the control parameters (ess= 0, ζ = 0.7 and ts=85% of the setting time in open loop)

Keywords--- *Control, Position, Angular, Response.*

General Objective--- Control the position and speed of a DC motor for step, ramp and acceleration type inputs, considering saturation effects of a real system.

Specific objectives--

* Find the mathematical model for a DC motor and compare its response with a real system.

* Find the constants that allow controlling the position and speed of a DC motor considering step, ramp and acceleration type inputs.

* Verify the constants obtained in the simulation files of the real system, in such a way that the effects of saturation and choice of adequate parameters can be seen when designing the controllers.

1. INTRODUCTION

We have the block diagram of the position control of a DC motor, using the SRV02 module:

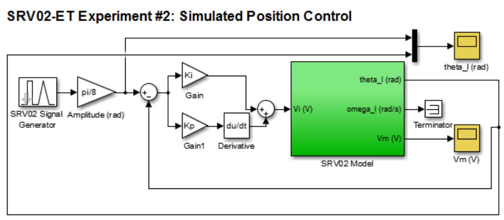


Figure 1: SRV02 position control.

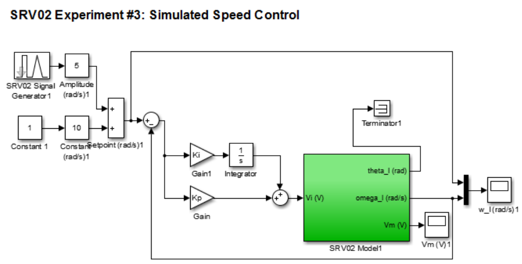


Figure 2: Speed control SRV02

2. THEORETICAL FRAMEWORK

The DC motor is one of the most widely used motor sources in the industry, it is basically a torque transducer that converts electrical energy into mechanical energy. The torque developed on the motor shaft is directly proportional to the flux in the field and the current in the armature. [1]

DC motor modeling.

To represent the mathematical model of a DC motor, with transmission of movement through gears and load, the scheme shown in Figure 1 can be used.

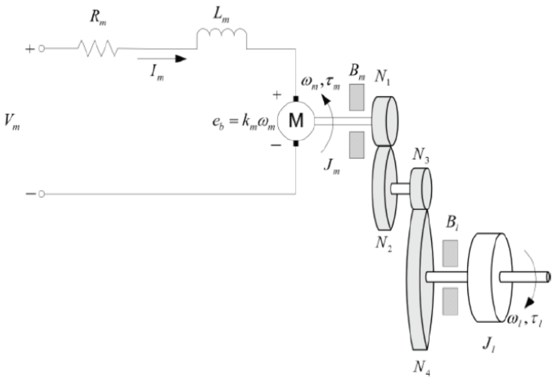


Figure 3: Diagram of DC motor with transmission. [2]

Considering this scheme, the system must be analyzed from the electrical and mechanical parts in order to arrive at the function that relates the load speed to the motor voltage.

Electrical equations:

The voltage produced by the counter electromotive force e_b is:

$$e_b(t) = k_m \omega_m(t) \qquad Eq. 1$$

Using Kirchhoff's voltage law for the circuit mesh.

$$V_m(t) - R_m I_m(t) - L_m \frac{dI_m(t)}{dt} - k_m \omega_m(t) = 0$$

Because the motor inductance L_m is less than the resistance, this inductance can be ignored:

$$V_m(t) - R_m I_m(t) - k_m \omega_m(t) = 0 \; Eq. 2$$

Where k_m is the back emf constant and ω_m is the motor shaft speed.

$$I_m(t) = \frac{V_m(t) - K_m \omega_m(t)}{R_m} \qquad Eq. 3$$

Mechanical equations:

For the analysis of the mechanical equations, the motor and the coupling to the gears are taken into account.

Equation 4 relates the moment of inertia J to the angular acceleration of the system α , τ which is the sum of the torques applied to the body.

$$J\alpha = \tau \qquad Eq. 4$$

Then, modeling the mechanical system, the friction acting on the load B_l, and being a sum of Torques, this friction is multiplied by the angular velocity of the load ω_l.

$$J_l \frac{d\omega_m(t)}{dt} + B_l \omega_l(t) = \tau_1(t) \; Eq. 5$$

where J_l is the moment of inertia of the load and τ_l is the total torque applied to the load. The motor shaft equation is:

$$J_m \frac{d\omega_m(t)}{dt} + B_l \omega_l(t) + \tau_{m1}(t) = \tau_m(t)$$

where J_m is the moment of inertia of the motor shaft and τ_{m1} is the torque resultant acting on the motor shaft from the load torque. The torque equation on the load shaft from a torque applied to the motor can be written as follows:

$$\tau_1(t) = \eta_g k_g \tau_{m1}(t)$$

where k_g is the radius of the gear and η_g is the efficiency of the gearbox . The planetary gearbox that is directly mounted on the mechanism motor is represented by the gears N1 and N2 represented in the following equation:

$$k_{gi} = \frac{N_1}{N_2}$$

This equation comprises the internal part of the gearbox, the gear of the motor N_3 and load gear N_4 , comprise the external part of the gearbox which has the following equation:

$$k_{ge} = \frac{N_4}{N_3}$$

The radius of the mechanism gear is given by:

$$k_g = k_{gi} k_{ge}$$

Then, the torque seen from the motor shaft to the gears can be expressed as:

$$\tau_{m1}(t) = \frac{\tau_1(t)}{\eta_g k_g}$$

Intuitively, the motor shaft must turn k_g times for the output shaft to turn one turn .

Simplifying and substituting the mechanical equations, we finally arrive at:

$$J_{eq}\frac{dw_l(t)}{dt} + Beq\,\omega_l(t) = \eta_g k_g \tau_m(t)$$

3. MATERIALS

* Software: MATLAB ®

* White coat

4. PROCEDURE

From the transfer function determined in the theoretical framework, the corresponding values are assigned according to Table 1.

The procedure is developed in more detail in the annex at the end of this document.

Symbol	Value
η_g	0.9
η_m	0.69
R_m	2.6 Ω
k_g	70
k_t	7.68x Nm/A
k_m	7.68x Nm/A
B_{eq}	0.015 Nm/rad/s
J_{eq}	0.00213 kg

Table 1. System values.

As a result, it is obtained that:

```
tf1 =

     1.528
  -------------
  0.02535 s + 1
```

Image 1. Transfer function (Angular Velocity)

Using the *tf2ss* , it is possible to obtain the representation in state variables of the system.

```
Ng=0.9;
Nm=0.69;
Rm=2.6;
Kg=70;
Kt=7.68*(10^-3);
Km=7.68*(10^-3);
Beq=0.015;
Jeq=0.00213;

Am=(Ng*Kg*Nm*Kt)/Rm;
Beqf= Beq+ (Ng*(Kg^2)*Nm*Kt*Km)/Rm;
K=Am/Beqf;
t=Jeq/Beqf;
num=K;
den=[t 1];

tf1=tf(num,den);
[A,B,C,D]=tf2ss(num,den);
```

Image 2. MATLAB code to obtain the state space.

Where A is the matrix that represents the dynamics of the system, B is the matrix that relates the inputs to the system, C is the output matrix and D is the matrix that relates the inputs to the output, this is generally null.

```
>> MotorDC
>> A
A =
  -39.4506
>> B
B =
     1
>> C
C =
   60.2834
>> D
D =
     0
```

Image 3. State arrays.

Now it is requested to carry out a PID control for the speed and angular position of the system, taking into account that an error in stable state () equal to 0 is required before step, ramp and acceleration inputs; a damping coefficient (ξ) equal to 0.7 and a settling time equal to 85% of the settling time in open loop.

From the transfer function shown in image 1, the time constant (t) of the system is determined, this being the term that accompanies the *s*. Therefore:

$$\tau = 0.02535\, s$$

For a first order system, it is said that in 5 time constants the system is already stable, so the open-loop settling time will be equal to 5t.

$$ts_{LA} = 5 * \tau = 5 * (0.02535) = 0.12675\, s$$

$$ts_{LC} = 0.85 * ts_{LA} = 0.85 * (0.12675) = 0$$

With the settling time already defined, and a damping coefficient set, find the frequency desired nature.

$$ts = \frac{4.5}{\xi * \omega_n} \rightarrow \omega_n = \frac{4.5}{\xi * ts} = 59.68$$

For the angular velocity of the system:

- **Step input**:

The open-loop system is type 0, so by theory it is known that it presents an error in stable state of:

$$\frac{1}{1 + K}$$

Where K is the error constant.

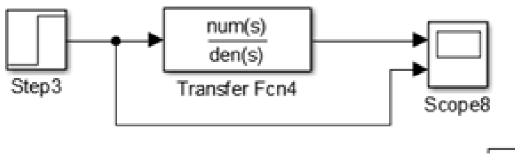


Image 4. Block diagram of the system

To reduce the error, an integral action is implemented in the control, this will make the system become second order, so it is decided to implement a PI control, so the system in loop closed presents the following transfer function:

$$H(s) = \frac{1.528(K_p s + K_I)}{0.0253s^2 + (1.528K_p + 1)s + 1.528K_I}$$

With the damping coefficient and the natural frequency of the system, the desired polynomial is generated:

$$s^2 + 2\xi\omega_n s + {\omega_n}^2 = s^2 + 83.56s + 3561.7$$

With this, terms of the transfer function of the system are associated with those of the desired polynomial, thus obtaining the values of the control constants.

$$\frac{1.528K_p + 1}{0.02535} = 83.56$$

$$K_p = 0.732$$

$$\frac{1.528K_I}{0.02535} = 3561.7$$

$$K_I = 59.1$$

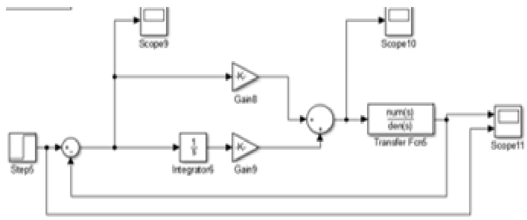


Image 5. Block diagram of the closed loop system.

- **Ramp input**

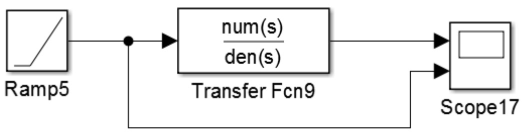


Image 6. Block diagram of the open loop system

The system needs to be type 2 so that it presents an error in stable state of 0 before a ramp input, so a double integral action is implemented, with this, the system becomes third order, so a PI^2, with this the transfer function is equal to:

$$H(s) = \frac{1.528(Kps^2 + K_{I1}s + K_{I2})}{[0.02535s^2 + s]s + 1.528(Kps^2 + K_{I1}s + K_{I2})}$$

From the roots of the desired polynomial, an additional pole is determined that can be considered negligible, and an alternative polynomial is generated, with this it is related term by term and the values of the constants are found.

$$K_p = 7.66$$

$$K_{I1} = 638.2$$

$$K_{I2} = 37719.17$$

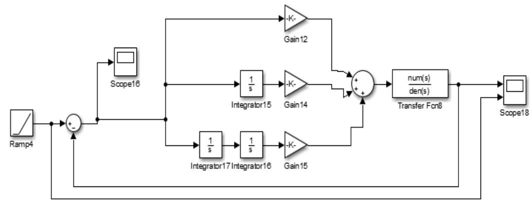


Image 7. Block diagram of the closed loop system.

- **Acceleration input:**

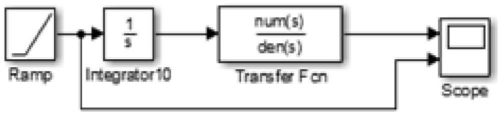


Image 8. Block diagram of the open loop system.

For the system to present an error in stable state before an acceleration input, a triple integral action is implemented, with this the system becomes fourth order, so a PI^3.

$$H(s) = \frac{1.528(K_p s^3 + K_{I1}s^2 + K_{I2}s + K_{I3})}{0.0253s^4 + (1.528K_p + 1)s^3 + 1.528K_{I1}s^2 + 1.528K_{I2}s + 1.5281.528K_{I3}}$$

With the roots of the desired polynomial, the value of the additional poles that must be added to it is determined, such that its effect is negligible; from this a new desired fourth degree polynomial is generated.

$$s^4 + 903.56s^3 + 240180.9s^2 + 16967030s + 598721770$$

Associating terms with those of the transfer function, it follows that:

$$K_p = 14.336$$

$$K_{I1} = 3984.7$$

$$K_{I2} = 281488.36$$

$$K_{I3} = 9932982$$

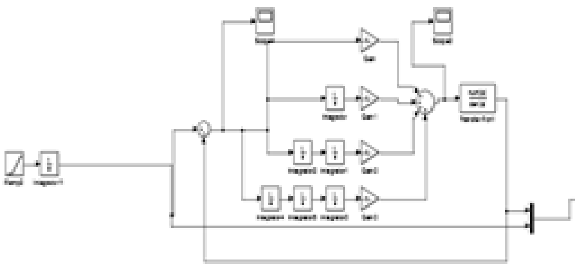


Image 9. Block diagram of the closed loop system.

For the angular position of the system:

Being the angular position the integral of the angular velocity, the angular position transfer function is:

$$H(s) = \frac{1.528}{[0.02535s+1]s}$$

- **Step input:**

The system is type 1, of second order, and by theory it is known that the error in a type 1 system is 0 before a step input, this can be checked using the final value theorem.

$$ess = \lim_{s \to 0} sE(s)Ref(s)$$

Where Ref(s) is the input, in this case, a step (1/s), and E(s) is the error signal, defined as:

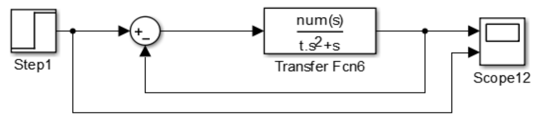
$$E(s) = \frac{1}{1+CP}$$

Being C, the control signal, but in this case it is equal to 1, so:

$$E(s) = \frac{[0.02535s+1]s}{[0.02535s+1]s+1.528 \; Going\ back}$$

to the theorem of final value, it is given that:

$$ess = \lim_{s \to 0} s * \frac{[0.02535s+1]s}{[0.02535s+1]s+1.528} * \frac{1}{s} = 0$$



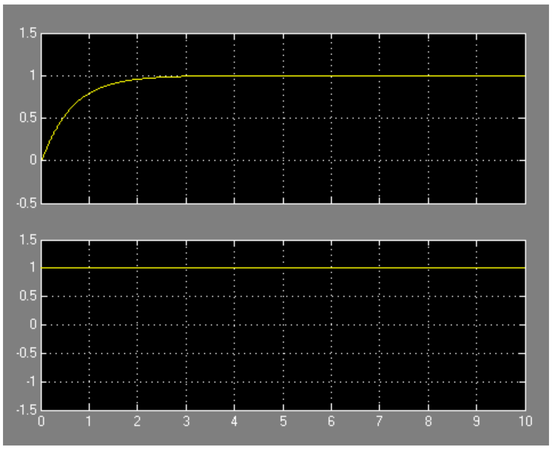


Image 10. System response in closed loop.

In image 10 it is observed in the graph of the upper part that the response of the system in closed loop has a stable state error of zero, the graph of the lower part represents the step type input.

Already with an error in stable state equal to 0, a PD control is decided to ensure the settling time; in closed loop a transfer function is given:

$$H(s) = \frac{1.528(Kp + K_d s)}{[0.02535s + 1]s + 1.528(Kp + K_d s)}$$

Carrying out the same previous procedure, comparing terms with those of the desired polynomial, the values of the constants are found.

$$K_p = 59.1$$

$$K_d = 0.73$$

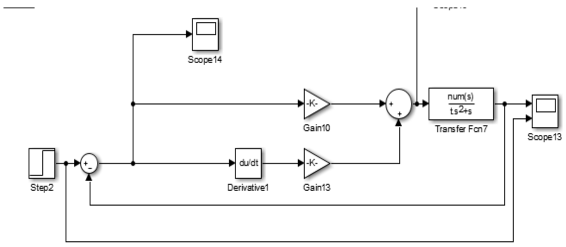


Image 11. Block diagram of the closed loop system.

- **Ramp input**

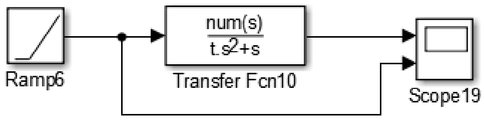


Image 12. Block diagram of the open loop system.

For a reduction of the error, an integral action is implemented, such that the system becomes type 2, of third order, with this, a PID control is designed and its transfer function in closed loop is equivalent to:

$$H(s) = \frac{1.528(Kds^2 + K_p s + K_I)}{[0.02535s^2 + s]s + 1.528(Kds^2 + K_p s + K_I)}$$

With this, it is obtained that :

$$K_p = 638.2$$

$$K_I = 37719.17$$

$$K_d = 7.66$$

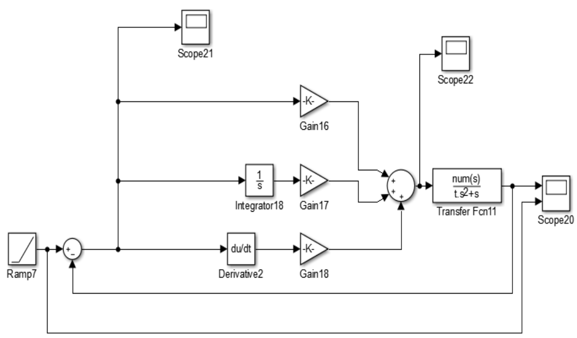


Image 13. Block diagram of the closed loop system.

- **Accelerating entry.**

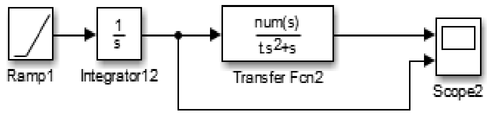


Image 14. Block diagram of the open loop system.

To reduce the error before an acceleration input, a double integral action is used, this makes the system become fourth order, so a PI^2D, with this, the transfer function in closed loop remains as follows:

$$H(s) = \frac{1.528(Kds^3 + Kps^2 + K_{I1}s + K_{I2})}{[0.02535s^3 + s^2]s + 1.528(Kds^3 + Kps^2 + K_{I1}s + K_{I2})}$$

Following the same procedure, it follows that:

$$K_p = 3984.7$$

$$K_{I1} = 281488.4$$

$$K_{I2} = 9932982$$

$$K_d = 14.33$$

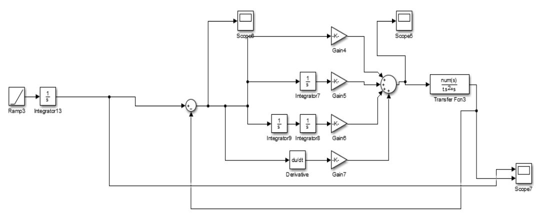


Image 15. Block diagram of the closed-loop system.

5. ANALYSIS OF RESULTS

speed control

Step

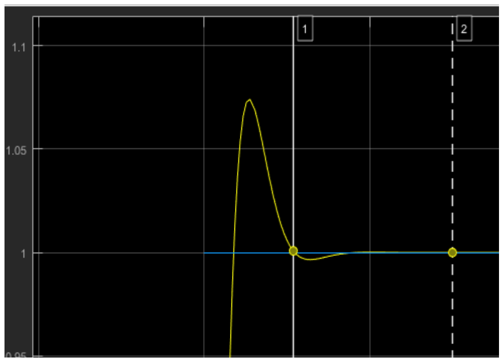
In open loop, the system presents the following behavior:

▼ Measurements

	Time		Value
1	0.127		1.518e+00
2	0.600		1.528e+00
ΔT	473.045 ms	ΔY	1.032e-02

The settling time agrees with that presented in the procedure since in stable state the system presents a value of 1.528 u, and in the value of 5τ has a value of 1.518, this is equivalent to 99.34% of the final value, so it can be said that at that moment the system is already stable.

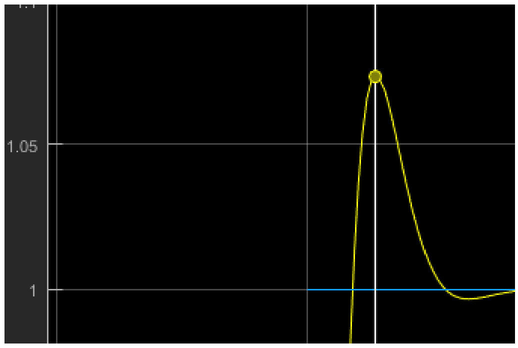
Implementing the designed controller (see **image 5**), the system presents the following response:



▼ Measurements

	Time		Value
1	0.108		1.001e+00
2	0.300		1.000e+00
ΔT	192.300 ms	ΔY	7.855e-04

The system is said to be stable at the established time, which is equivalent to 85% of the establishment time in open loop, it can even be said that the system is stable. stabilizes before due to the managed scale.

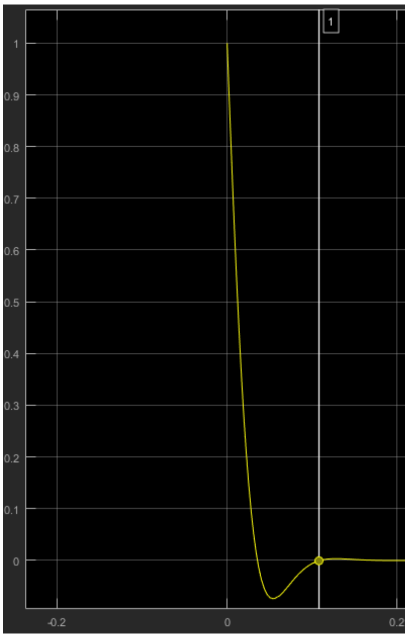


▼ Measurements

	Time		Value
1	0.054		1.073e+00
2	0.300		1.000e+00
ΔT	246.312 ms	ΔY	7.348e-02

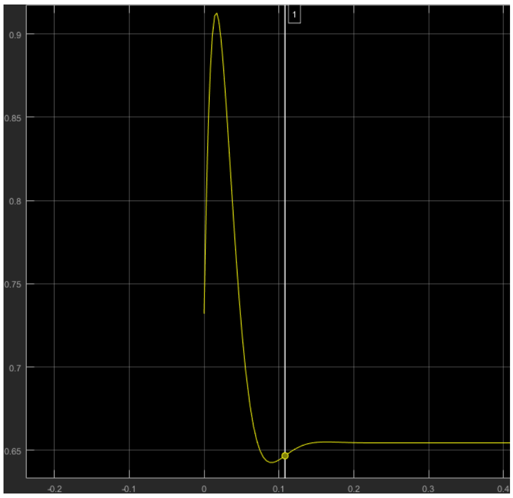
The peak that the system presents is equivalent to 7.3% of the value in stable state, theoretically, with a damping coefficient of 0.7, it should be approximately 4.6%, which indicates that in the closed-loop system there is a coefficient of damping approximately equal to 0.41, which indicates that the established parameters are not met, this may be due to the approximations made in the calculations.

Now we can see how the control is behaving with respect to the error signal.



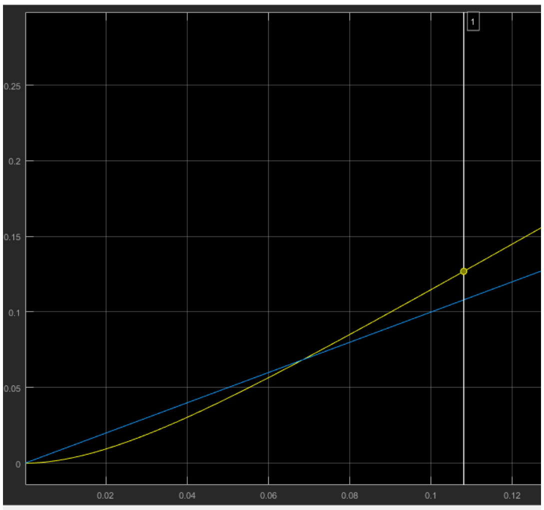
▼ Measurements			
	Time		Value
1	0.108		-6.442e-04
2	1.227		0.000e+00
ΔT	1.119 s	ΔY	6.442e-04

The error signal is shown to be practically null in the established establishment time, which is what is sought with the implementation of the control in the system.



Ramp:

In open loop, the system presents the following response:

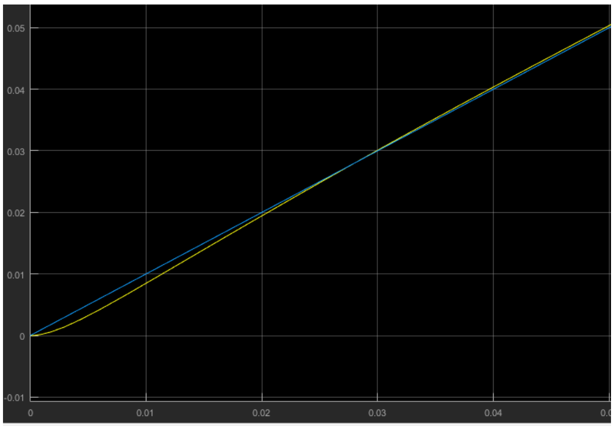


▼ Measurements		
	Time	Value
1	0.108	1.268e-01
2	1.227	1.836e+00

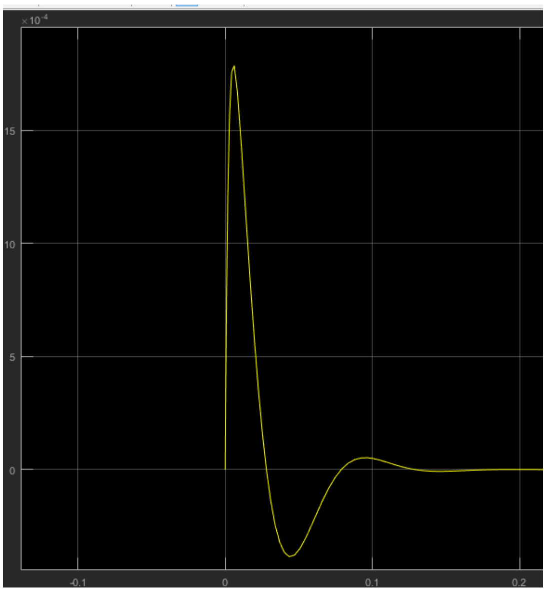
Since the system is type 0, by theory it is known that in closed loop, time tends to be infinite, the error also tends to be infinite, because the difference between slopes of both lines is not zero.

Implementing the designed control (see **image**

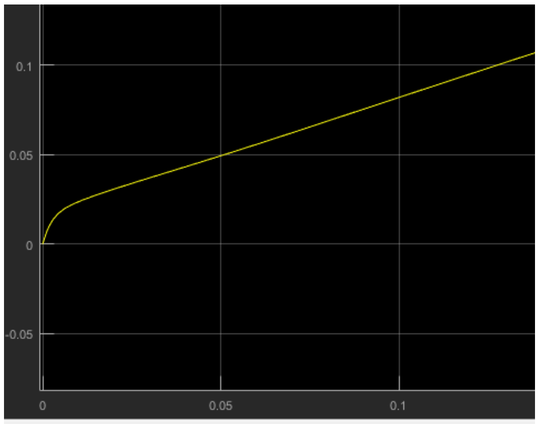
7) it is obtained that the system responds in the following way:



Although there is an error in stable state equal to 0, the system stabilizes in a much shorter time than established, for this reason the signals are observed error and control.



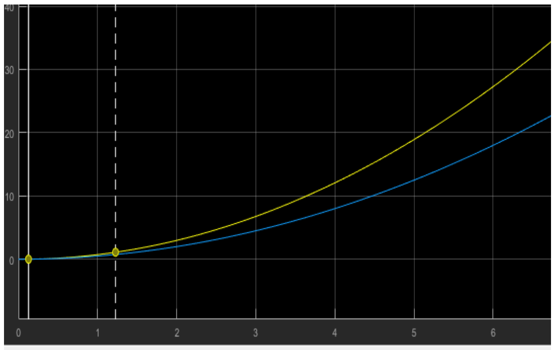
As can be seen, the error signal that is handled is very small, so it is assumed that the control performed is very strong, since the error is practically zero (Observe the scale of the axis).



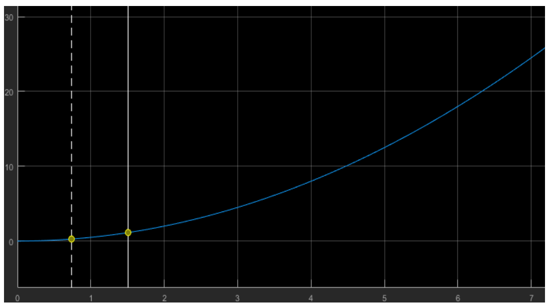
The control shows a very strong integral action, which is why the error decreases in such a way; Although this type of error reduction is desirable, it requires a greater control action. This is involved in a higher energy expenditure of the components of the control system.

Acceleration:

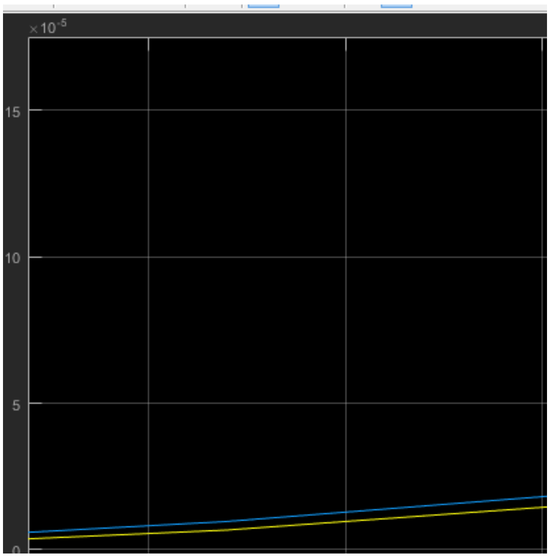
In open loop the following behavior



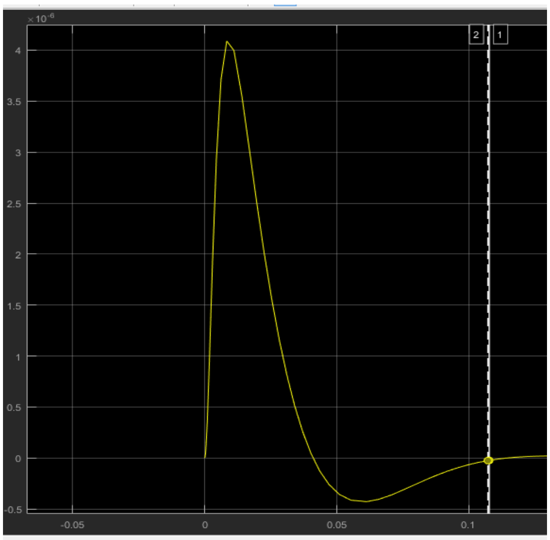
is presented: Indeed an infinite error is presented, to reduce it, the designed control is implemented (see **image 9**).



It seems that the output is exactly at the input, if the scale is reduced, the transient it presents is observed.

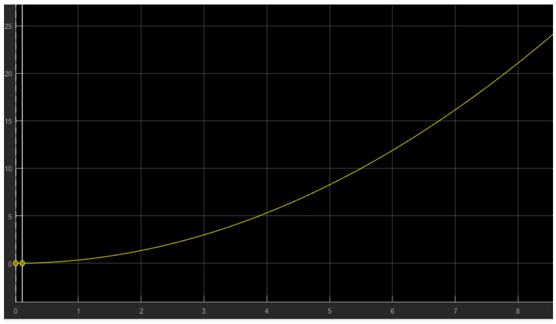


The difference that appears from t=0 is minimal, this would imply, as observed in the previous item, that the error signal is practically 0, and that the control signal is implementing very strong actions.



	Time	Value
1	0.108	-2.404e-08
2	0.107	-2.610e-08
ΔT	516.191 µs	ΔY 2.055e-09

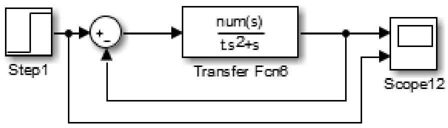
Indeed, the error is very small, and practically 0 is reached in the desired establishment time, although it can be said that from t=0 the error is null.



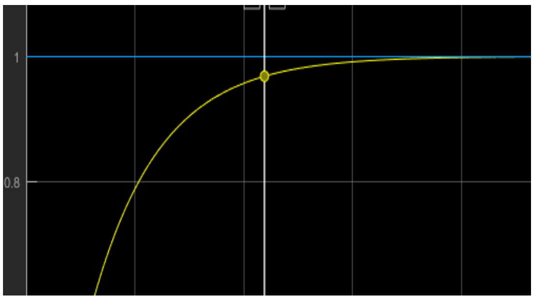
(...)

position control

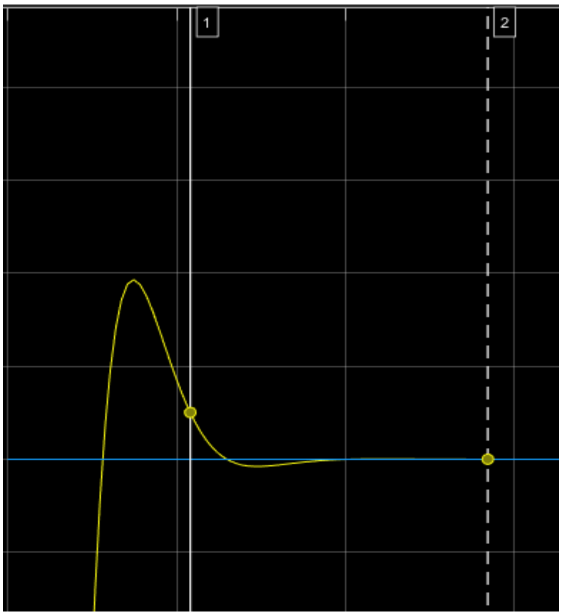
Step



Because it is type 1, the system already presents an error of 0 in stable state before a step input in closed loop.



For this reason, what is sought with the designed control is to ensure a settling time. Implementing the control seen in **image 11**, it follows that:



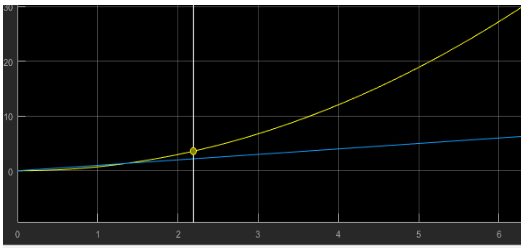
▼ Measurements

	Time	Value
1	0.108	1.010e+00
2	0.284	1.000e+00

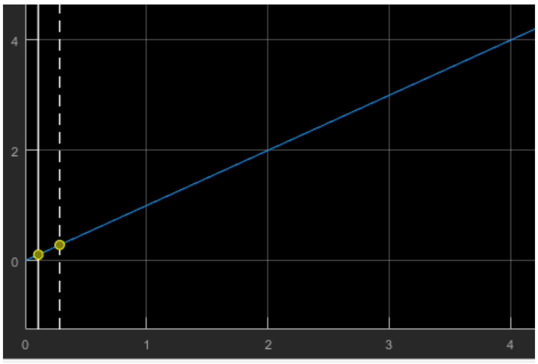
For the desired settling time of 0.1077 s, exactly 1% error is obtained, which is what it was designed for.

Ramp:

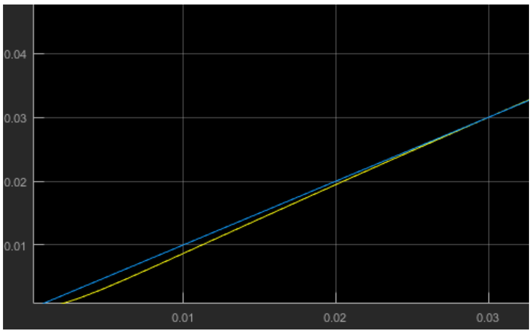
In open loop, the system presents the following response:



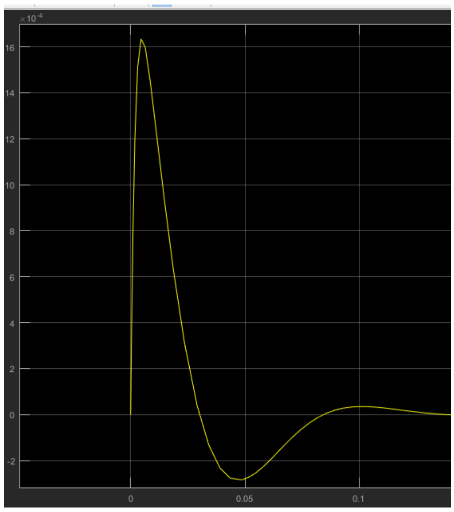
Being a type 1 system, the error in closed loop is constant, so the designed control is implemented (see **image 13**)



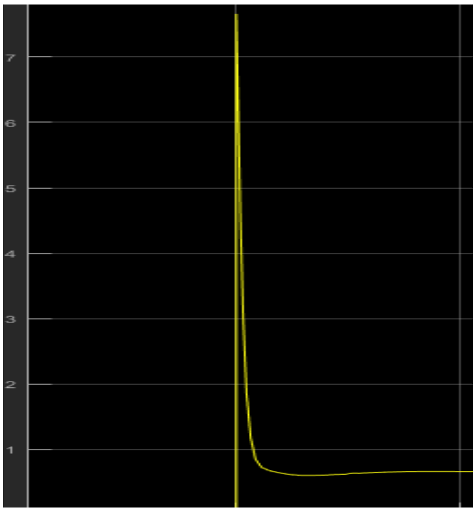
. Again, it is observed that the input and output are practically equal, but at a t very close to 0 it is observed that:



Following the previous reasoning, it can be said that the error signal can be assumed equal to 0.



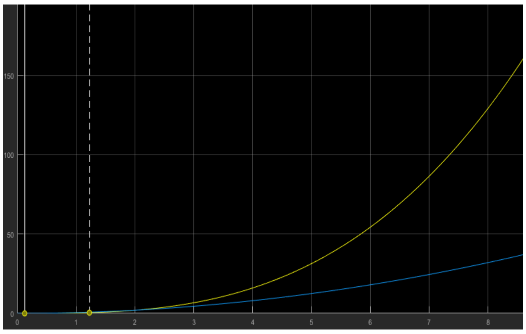
Indeed, it is seen that the error is reduced very quickly by which is expected a very strong control signal.



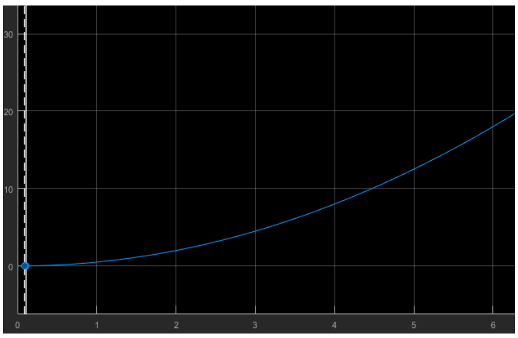
(...)

Acceleration:

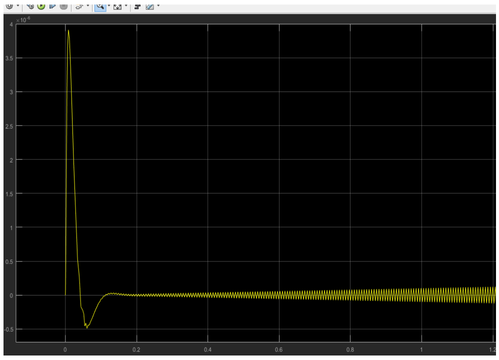
In open loop, the system responds as follows:



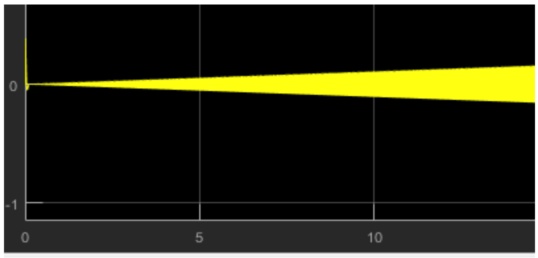
Implementing the control seen in **image 15,** the response to an acceleration input is:



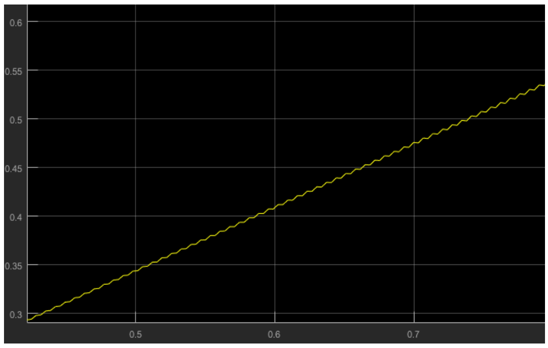
It is said that the output is equal to the input, so the system is responding very fast, decrementing the error almost immediately.



The error signal, although it is practically 0, tends to obtain an oscillatory behavior, it can occur due to the implementation of a triple integral action that makes the error obtain such behavior.



Given this behavior in the error, the control signal is expected to behave very similarly.



Indeed, the signal shows an oscillatory behavior, this shows that the control signal "adjusts" to the error signal, which is what is sought.

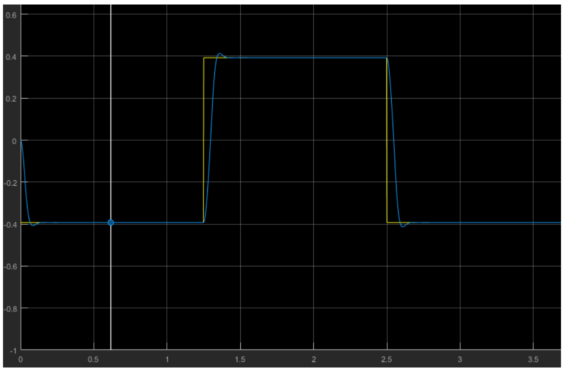
Simulation of the SRV02

Using the material attached to the laboratory guide, the process of the motor connected to the SRV02 driver is simulated, as shown in figure 1 at the beginning of the document.

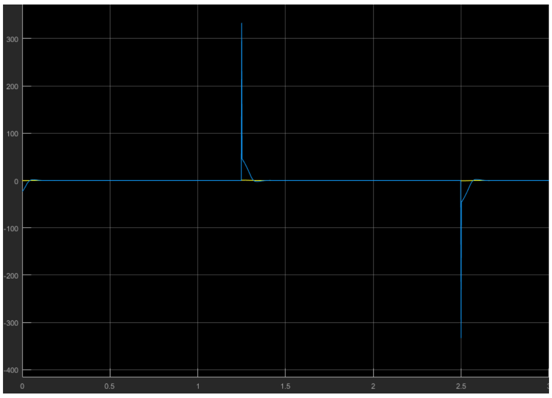
Position control:

step input

For a step reference signal, implementing the design made, the system responds as follows.

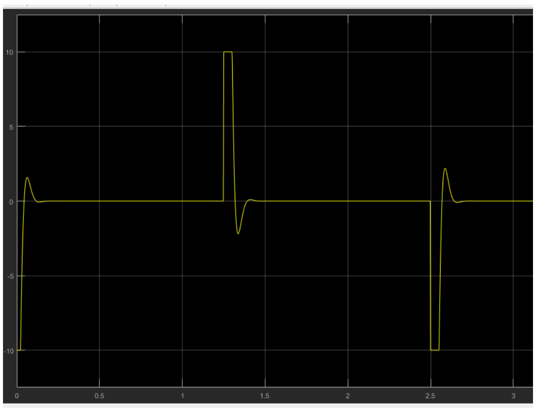


The error signal and the control signal are shown as follows:



It is seen that the error is minimal, but for the same reason, the control signal presents very strong actions, this is reflected in the power necessary for it to be activated. I managed to comply.

The voltage shown is given by:

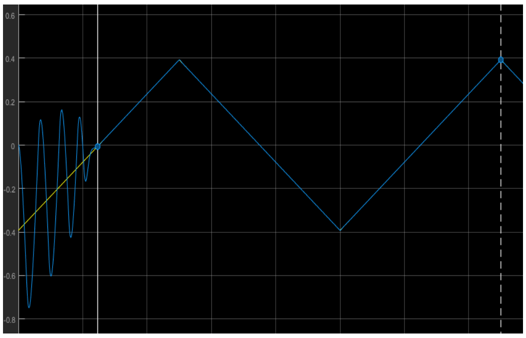


The voltage provided by the motor driver tends to the values of the control signal, but internally there are saturators, so the signal is clipped; trying to implement this control would result in motor damage, so there are several parameters to correct for this.

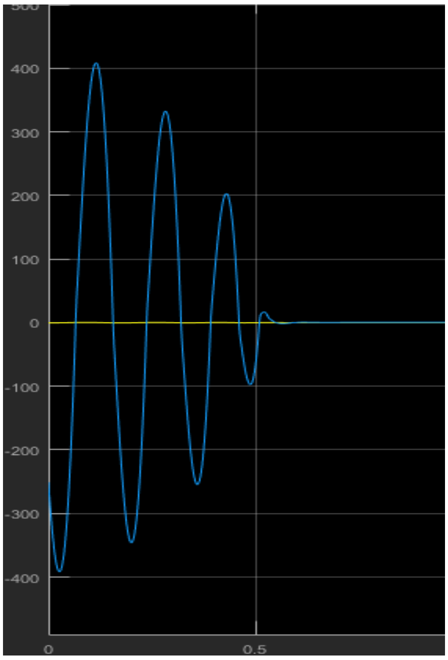
You can decide to increase the damping coefficient, as well as the settling time, with this lower coefficients would be expected in the desired polynomial, which would result in lower value control constants.

Ramp entrance.

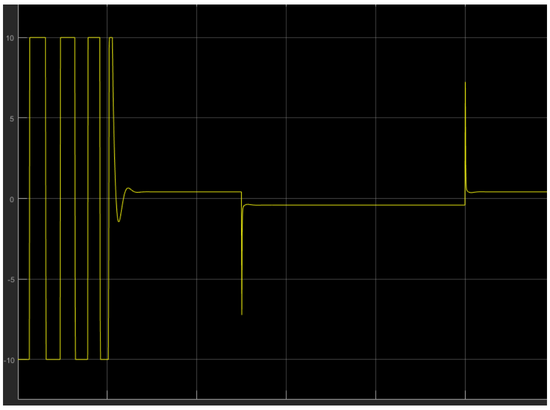
Implementing the designed control, the system responds in the following way:



With this a very strong control action is expected, seeing this reflected in a very large value of required voltage, so the damage or saturation has to be given of the elements.



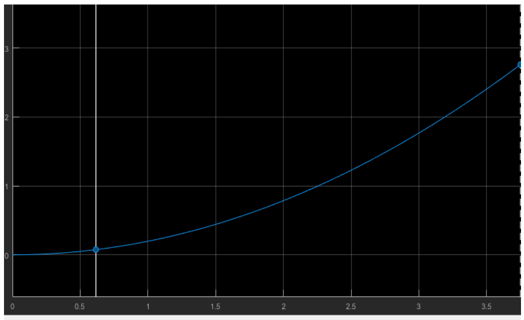
Although the error signal is practically zero, voltage peaks of up to 400 v are observed in the control, and in a 5 v motor it would imply the total damage of the element, for which the previously stipulated is confirmed; the voltage signal, making use of the driver saturator, is observed as follows:



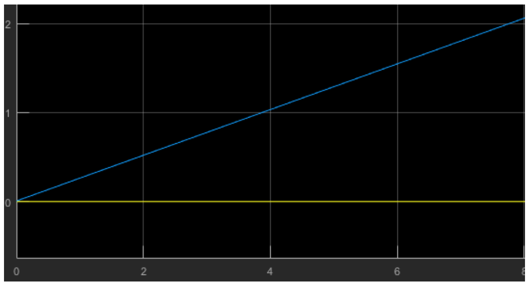
With this, the control must be redesigned, varying and determining parameters such as the settling time and the control coefficients.

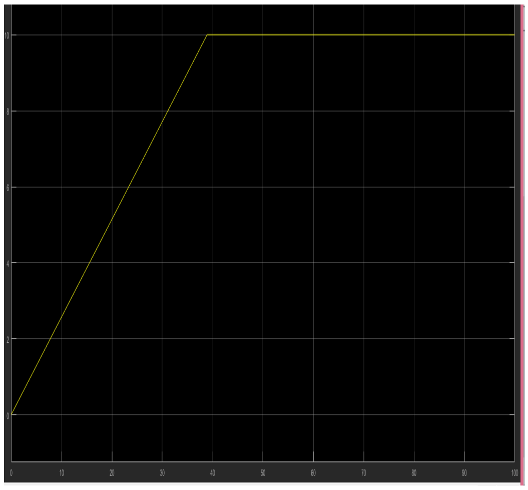
Acceleration input:

With the designed control, the system presents the following response:



The output is practically the same input, so a null error signal and a very strong control signal are expected, and with this, a voltage signal saturated.





Speed control

Step input

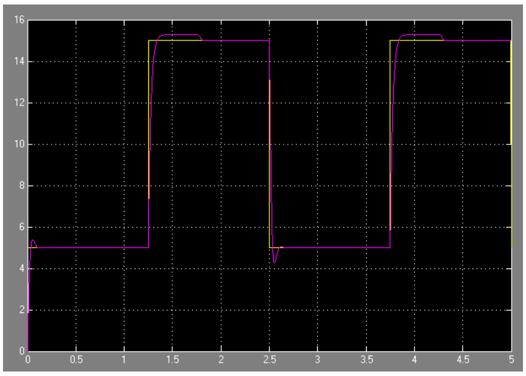


Image . Angular velocity output ω = rad/s

In the image, the purple signal represents the angular velocity of the motor, and the yellow signal is the input (step).

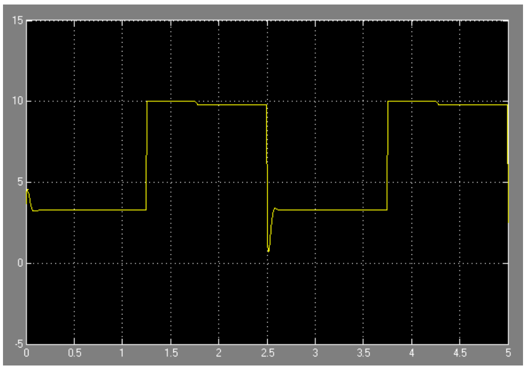
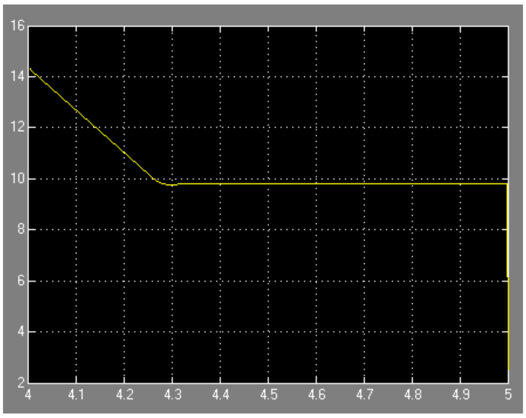
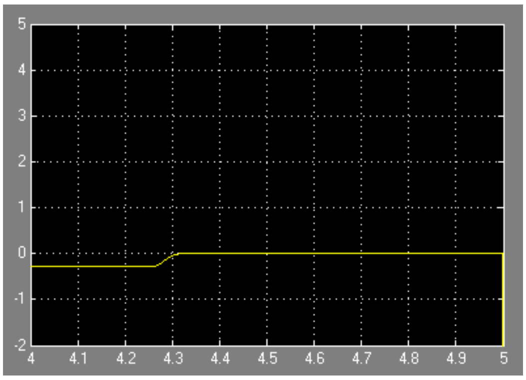


Image . Voltage output for speed control

The image shows the voltage of the motor voltage signal, which reaches a maximum of 10v



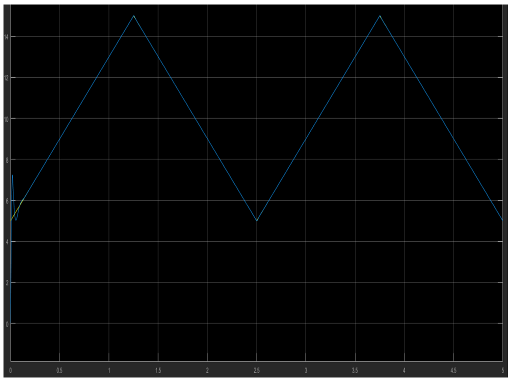
. The control signal is found in the previous image, in which there is a peak of 14.33v.

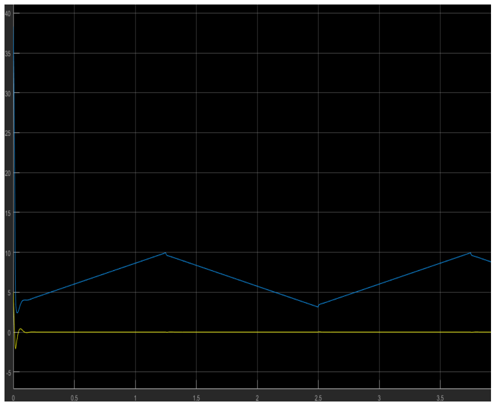


System error signal before step input, for speed control.

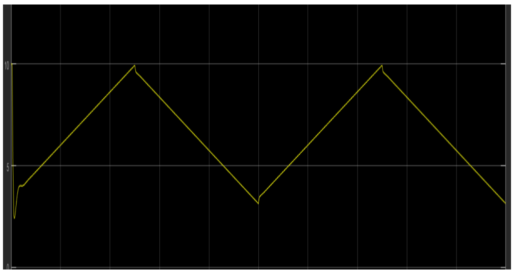
Ramp input:

Implementing the designed control, the system shows a response equal to:



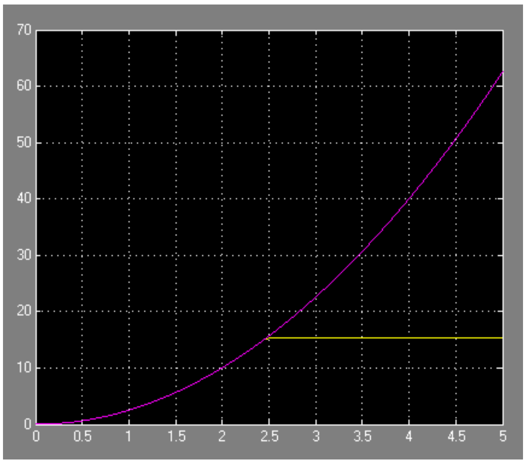


The error is practically 0, but the control signal has a peak of almost 40 v, although it is not as excessive as those seen in the position control, the same means a damage to the elements, the voltage presented after the saturator is:

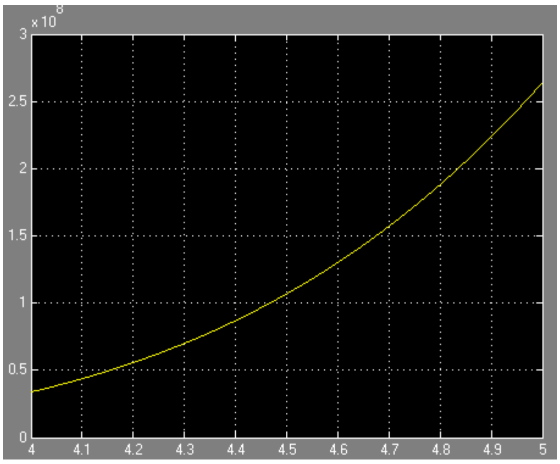


Acceleration input:

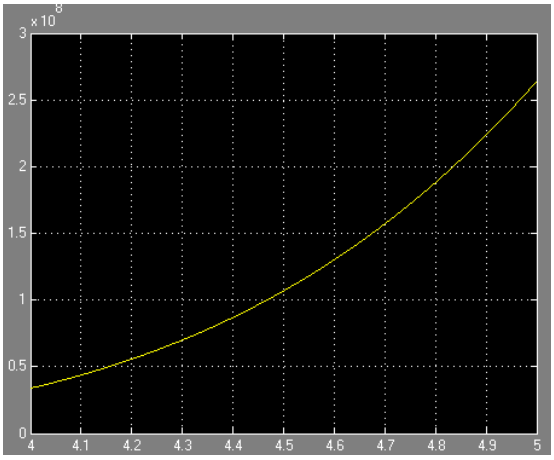
The motor output signal is found in the following image, where the purple signal is the input signal and the yellow signal is the of the output (angular velocity) It is observed that it stabilizes at 15V at approximately 2.5s.



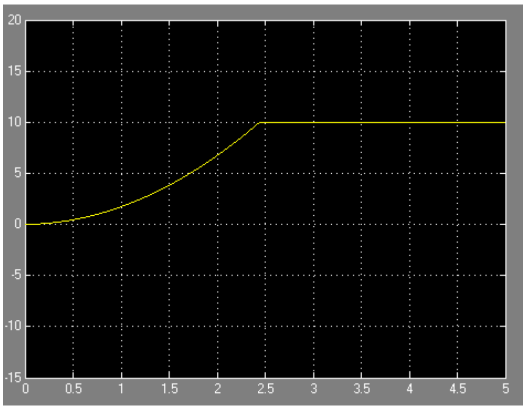
The system control signal to this acceleration input is presented in the following image.



The error signal is observed below:



The system voltage output is found in the following graph:



6. CONCLUSIONS

- The approximations that are made in the calculations, at the moment of finding the control constants, can result in the response presenting variations in the specifications for which the design was made
- For a small settling time in a first-order system, where the time constant is also small, and a damping coefficient between 0 and 1 is required such that the system is underdamped, would expect a natural frequency of the system with very large values compared to ξ, this generates large coefficients in the desired polynomial, generating large values in the control constants.
- For control systems (...)(Reference to a PI, PID or PD), it is expected that the

control actions are not very strong, and that the error is not almost immediately 0.

- For speed and acceleration inputs, in systems of type 0 or 1 in certain cases, implement comprehensive actions, especially those of such magnitude as those obtained, present very strong control actions.
- The final value theorem applies to closed-loop systems,

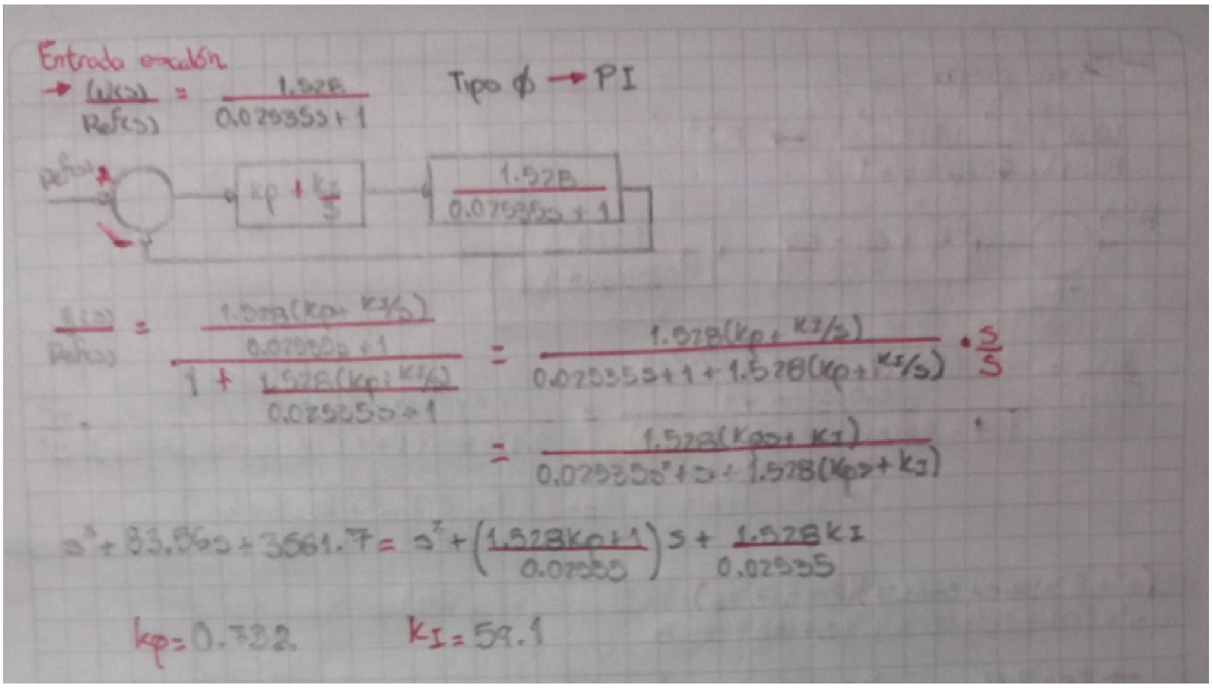
7. REFERENCES

[1] B. Kuo, Automatic control systems, Prentice Hall, 1996.

[2] Q. INC, Instructor Workbook: SRV02 Base Unit Experiment for Matlab, 2011.

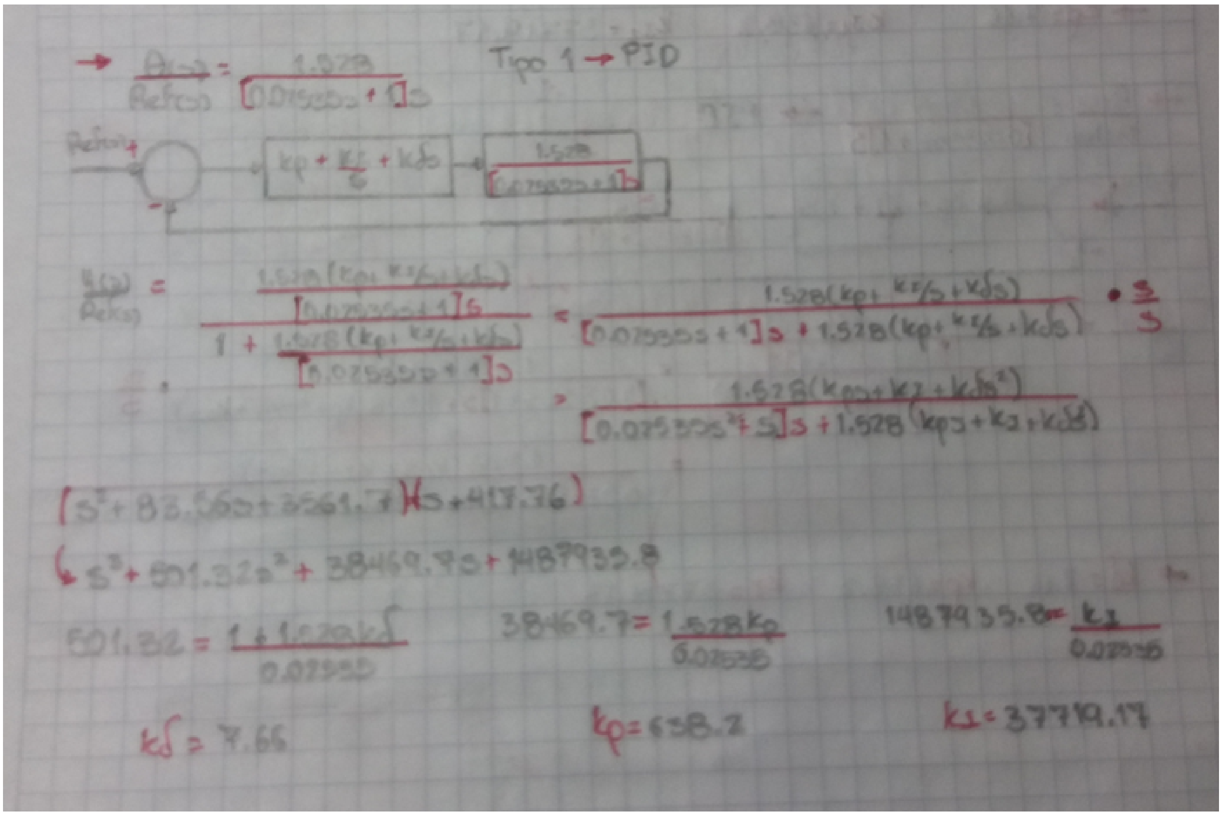
ANNEXES

Annex 1



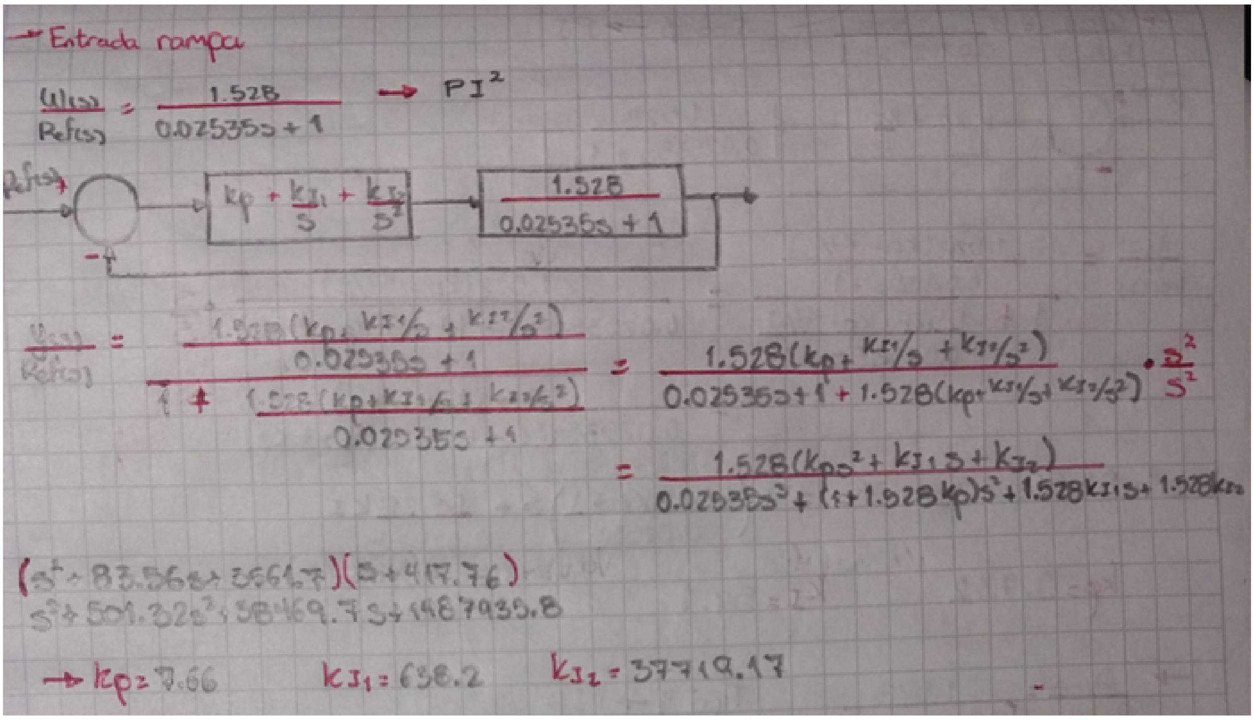
In **annex 1**, there is the procedure to obtain the values of the constants of the controller, for the case of the step-type input signal **for speed control.** The speed control transfer function is type 0, so a PI controller is implemented. Then the constants kp and ki of the controller are found. These are: Kp=0.732 and Ki=59.1.

ANNEX 2



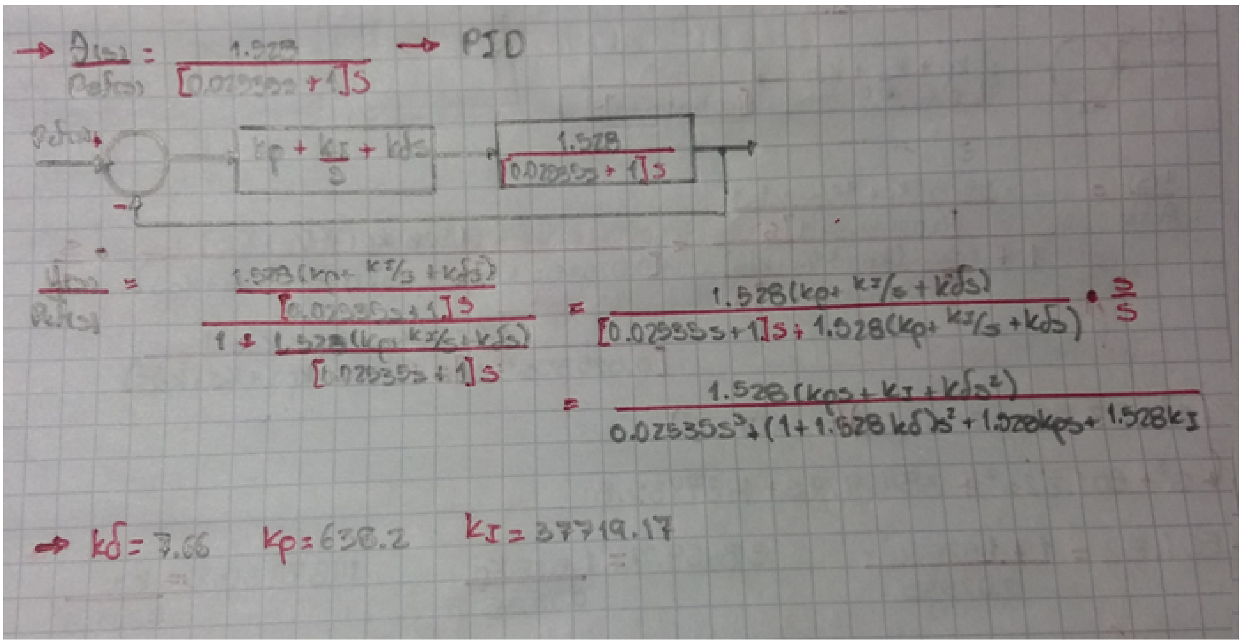
In **annex 2** the procedure is developed to obtain the values of the constants of the respective controller, knowing that the transfer function for **position control before a step-type input** is type 1, a PID controller is implemented and they are found. their respective constants Kp=638.2 , Ki=37719.17 and Kp=7.66.

ANNEX 3



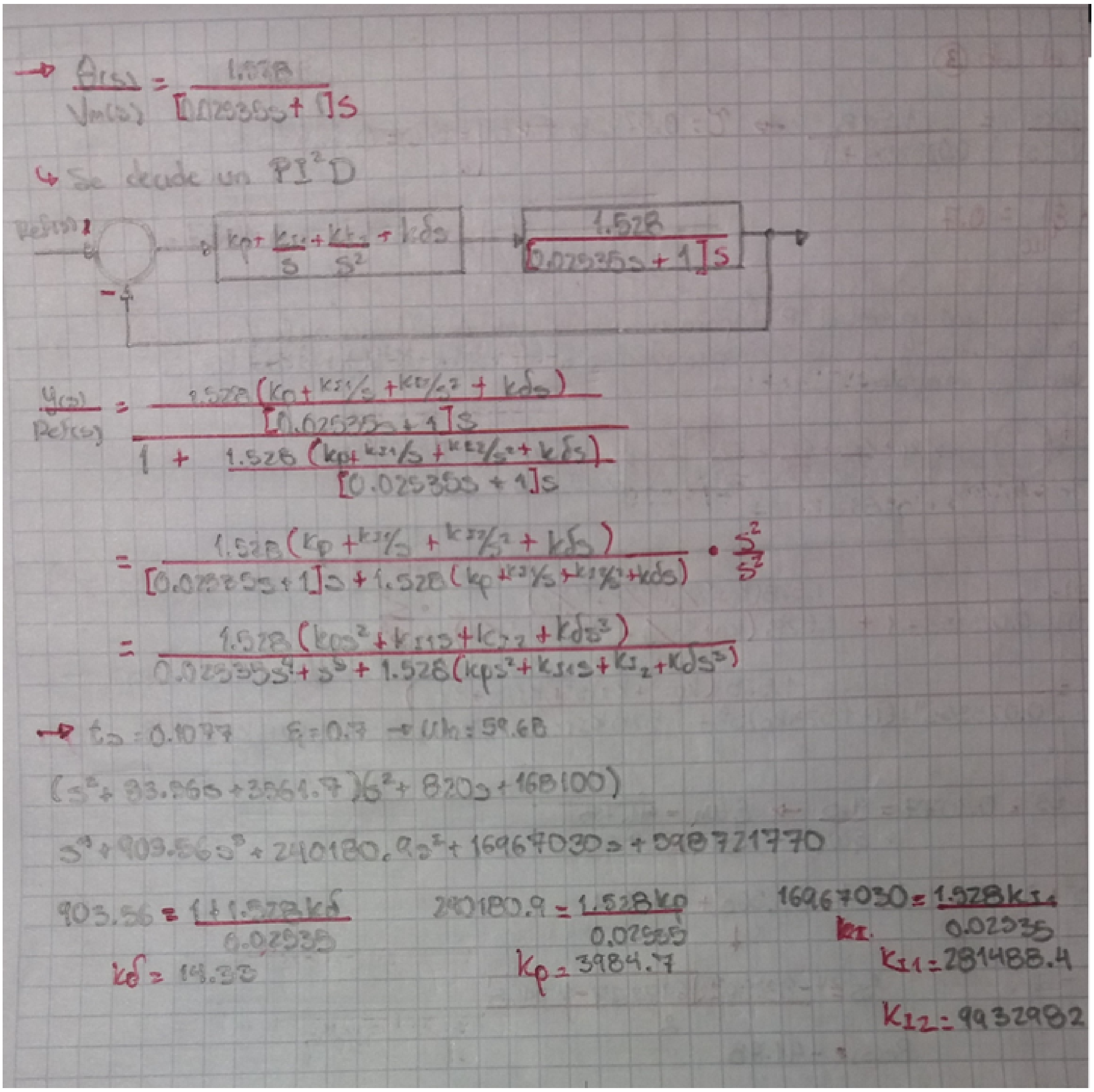
In **annex 3** the procedure to obtain the values of the constants of the respective controller is developed, knowing that the transfer function for the **speed control before a ramp-type input** is type 1, a PI^2 and find their respective control constants Kp=7.66 , Ki=638.2 and Ki_2=37719.17.

ANNEX 4



In **annex 4** the procedure is developed to obtain the values of the constants of the respective controller, knowing that the transfer function for **position control before a ramp-type input** is type 1, a PID controller is implemented and they are found. their respective constants Kp=638.2 , Ki=37719.17 and Kd=7.66.

ANNEX 5



$$\rightarrow \frac{\theta(s)}{V_m(s)} = \frac{1.528}{[0.02535s+1]s}$$

↳ Se decide un PI^2D

Ref(s)

$$k_p + \frac{k_{i1}}{s} + \frac{k_{i2}}{s^2} + k_d s \qquad \frac{1.528}{[0.02535s+1]s}$$

$$\frac{Y(s)}{Ref(s)} = \frac{\dfrac{1.528\left(k_p + k_{i1}/s + k_{i2}/s^2 + k_d s\right)}{[0.02535s+1]s}}{1 + \dfrac{1.528\left(k_p + k_{i1}/s + k_{i2}/s^2 + k_d s\right)}{[0.02535s+1]s}}$$

$$= \frac{1.528\left(k_p + k_{i1}/s + k_{i2}/s^2 + k_d s\right)}{[0.02535s+1]s + 1.528\left(k_p + k_{i1}/s + k_{i2}/s^2 + k_d s\right)} \cdot \frac{s^2}{s^2}$$

$$= \frac{1.528\left(k_p s^2 + k_{i1} s + k_{i2} + k_d s^3\right)}{0.02535s^4 + s^3 + 1.528\left(k_p s^2 + k_{i1} s + k_{i2} + k_d s^3\right)}$$

$\rightarrow t_s = 0.1077 \quad \xi = 0.7 \rightarrow \omega_n = 59.68$

$$(s^2 + 83.56s + 3561.7)(s^2 + 820s + 168100)$$

$$s^4 + 903.56s^3 + 240180.9s^2 + 16967030s + 598721770$$

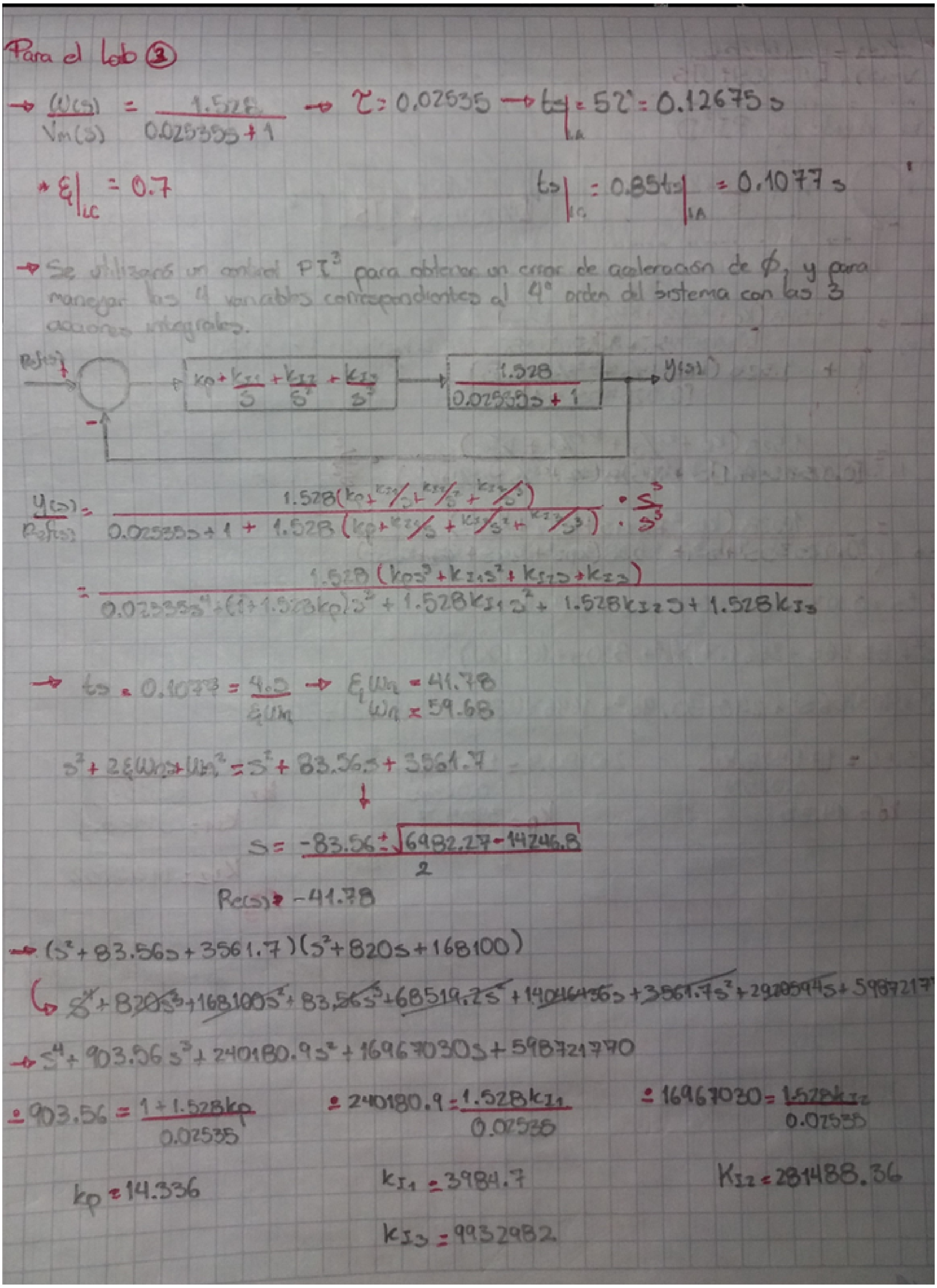
$$903.56 = \frac{1 + 1.528k_d}{0.02535} \qquad 240180.9 = \frac{1.528k_p}{0.02535} \qquad 16967030 = \frac{1.528k_{i1}}{0.02535}$$

$k_d = 14.33 \qquad k_p = 3984.7 \qquad k_{i1} = 281488.4$

$k_{i2} = 9932982$

In **appendix 5** the procedure to obtain the values of the constants of the respective controller is developed, knowing that the transfer function for **position control before an acceleration type input** controller is implemented PI^2D and find their respective control constants Kp=3984.7 , Ki=281488.4 and Ki_2=9932982 Y Kd=14.33..

ANNEX 6



Para el lab ③

→ $\frac{\omega(s)}{V_m(s)} = \frac{1.528}{0.02535s+1}$ → $\tau = 0.02535$ → $t_s|_{LA} = 5\tau = 0.12675\ s$

* $\xi|_{LC} = 0.7$ $t_s|_{LC} = 0.85 t_s|_{LA} = 0.1077\ s$

→ Se utilizará un control PI^3 para obtener un error de aceleración de ϕ, y para manejar las 4 variables correspondientes al 4° orden del sistema con las 3 acciones integrales.

$$\frac{y(s)}{Ref(s)} = \frac{1.528\left(k_p + \frac{k_{I1}}{s} + \frac{k_{I2}}{s^2} + \frac{k_{I3}}{s^3}\right)}{0.02535s + 1 + 1.528\left(k_p + \frac{k_{I1}}{s} + \frac{k_{I2}}{s^2} + \frac{k_{I3}}{s^3}\right)} \cdot \frac{s^3}{s^3}$$

$$= \frac{1.528\left(k_p s^3 + k_{I1}s^2 + k_{I2}s + k_{I3}\right)}{0.02535s^4 + (1+1.528k_p)s^3 + 1.528k_{I1}s^2 + 1.528k_{I2}s + 1.528k_{I3}}$$

→ $t_s = 0.1077 = \frac{4.5}{\xi\omega_n}$ → $\xi\omega_n = 41.78$, $\omega_n = 59.68$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + 83.56s + 3561.7$$

$$s = \frac{-83.56 \pm \sqrt{6982.27 - 14246.8}}{2}$$

$Re(s) = -41.78$

→ $(s^2 + 83.56s + 3561.7)(s^2 + 820s + 168100)$

$\hookrightarrow \cancel{s^4} + 820s^3 + 168100s^2 + 83.56s^3 + 68519.2s^2 + 14046436s + 3561.7s^2 + 2920594s + 5987217$

→ $s^4 + 903.56s^3 + 240180.9s^2 + 16967030s + 598721770$

$903.56 = \frac{1 + 1.528k_p}{0.02535}$ $240180.9 = \frac{1.528k_{I1}}{0.02535}$ $16967030 = \frac{1.528k_{I2}}{0.02535}$

$k_p = 14.336$ $k_{I1} = 3984.7$ $K_{I2} = 281488.36$

$k_{I3} = 9932982$

In **annex 6** the procedure is developed to obtain the values of the constants of the respective controller, knowing that the transfer function for the **speed control before an acceleration type input** controller is implemented PI^{3} and they are found their respective control constants Kp=14.336 , Ki_1=3984.7 and Ki_2=281488.36 Y Ki_3 =9932982.