Laboratory 2: Systems of first, second order and PID.

Galvis, David. Lopez, Daniel.

{u1802584,u1802530}@unimilitar.edu.co

Nueva Granada Military University

Summary---

This document presents the necessary procedures for the design and development of first and second order electronic systems implementing the filter theory; the analysis of the temporary responses to a step input is shown, and the concept and implementation of a PID-type controller is presented.

Abstract---

In this document, the steps needed for the design and development of first and second order electronic systems are presented, using filters theory; the analysis of its temporal response due to a step signal input is shown, and the concept and implementation of a PID controller is presented, for error minimization, stabilization time improving and overvalue handling.

Keywords--- *PID, First order, Second order, Transfer function*

General Objective--- Review first and second order responses, and from there, project the desired responses in PID controls.

Specific objectives--

*Study the response of first-order systems to steps.

*Study the responses of second-order systems before the step.

*Study PID regulators.

INTRODUCTION

In the study of the behavior of a system, the first and second order temporal response stands out, since several design techniques are based on being able to achieve behavior in time similar to these; It is not enough to vary a regulator until the desired response is obtained, existing design theories must be taken into account.

1. THEORETICAL FRAMEWORK

First Order System:

This system is characterized by the relationship between the response time and a time constant that depends on the elements that make up the system.

$$H(s) = \frac{X(s)}{F(s)} = \frac{k}{\tau s+1}$$

Image 0. First-order transfer function

Second-order system:

As in first-order systems, the behavior is determined by the components that compose it; several types of response can be obtained.

$$H(s) = \frac{X(s)}{F(s)} = \frac{kw_n^2}{s^2+2\xi w_n s+w_n^2}$$

Image 1. Second order transfer function

Differential Integer System

This type of system is characterized in that the output is equal to the integral or the derivative of the input, with gain depending on the elements that compose it; An example of this is the op amp in integrator or shunt mode.

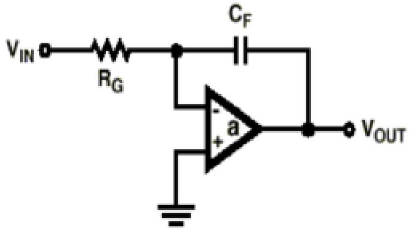


Image 2. Operational amplifier as integrator

Feedback System

This system is characterized by taking the current value of the output, comparing it with a reference value to which it is desired to arrive, and the difference between the input and the output, also known as error, is used again in the system to get a better response.

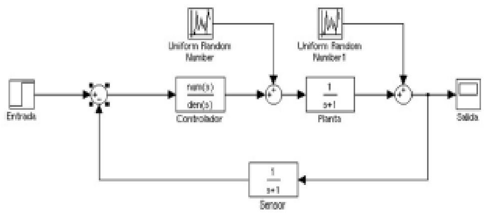


Image 3. Example of a schematic of a feedback system.

PID regulator

It is a type of control in which, through proportional, integral or derivative constants (and their combinations), applied to the error, they determine its reaction form. To obtain these constants, the analytical method can be carried out, which consists of formulating the desired system and comparing it with the model in terms of the constants and relating terms; or through tuning methods, such as Ziegler-Nichols, which experimentally determined the coefficients for each type of PID controller from certain parameters.

2. MATERIALS

Measurement:

Oscilloscope, Multimeter

Software:

MATLAB

Material:

Breadboard, resistors, capacitors, operational amplifiers, wire, connecting cables.

Security elements:

White

3. coat PROCEDURE

What is the tuning technique by Ziegler and Nichols? For each method make an example. Show an example.

The process of selecting controller parameters that meet given performance specifications is known as controller tuning. Ziegler and Nichols suggested rules for tuning PID controllers (this means giving values to Kp, Ti, and Td) based on the experimental step responses or on the value of Kp that produces marginal stability when only proportional control action is used.

First method: If the plant does not contain complex conjugate integrators or dominant poles, the unit step response curve can be S-shaped, for which it is valid. Such step response curves can be generated experimentally or from a dynamic simulation of the plant.

The S-shaped curve is characterized by 2 parameters: the lag time L and the time constants T. The lag time and the time constant are determined by drawing a tangent line at the inflection point of the S-shaped curve. S and determining the intersections of this tangent with the time axis and with the line c(t)=k. [1]

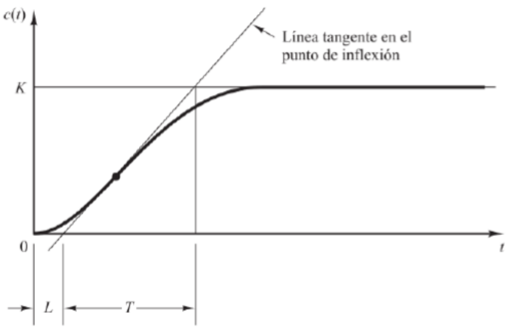


Figure 1: First Ziegler and Nichols method

.After obtaining the values of L and T from the graph of the response to a step input,

Tipo de controlador	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

Table 1: Constants, 1st Ziegler and Nichols method can be found.

Second method: First, set Ti=∞ and Td=0. Using only proportional control action, Kp is increased from 0 to a Kcr value where the output first exhibits sustained oscillations. Therefore, the critical gain Kcr and the corresponding period Pcr are determined experimentally. If the output does not exhibit sustained oscillations for whatever value Kp might take, this method is not applied. Ziegler-Nichols suggested that the values of the parameters Kp, Ti and Td were established according to the table: [2]

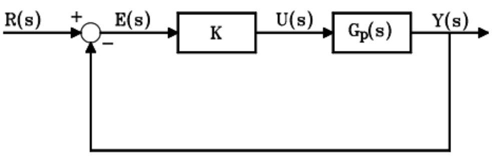


Figure 2: Second method, using closed loop

The constants for this second method are found in the information in table 2 :

Tipo de controlador	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Table 2: Constants of the second Ziegler-Nichols method.

An example of the second Ziegler-Nichols tuning method is shown below:

Having the system represented in figure 3, the second method of the tuning rule is applied to find the respective values of Kp, Ti and Td.

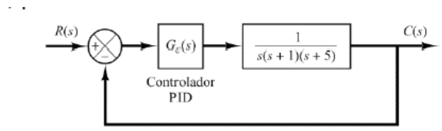


Figure 3: Feedback system, to perform the second Ziegler-Nichols method.

Closed loop transfer function as follows.

$$\frac{C(s)}{R(s)} = \frac{K_p}{s(s+1)(s+5)+K_p}$$

The value of Kp that makes the system marginally stable for a sustained oscillation to occur is obtained by the Routh stability criterion.

$$s^3 + 6s^2 + 5s + K_p = 0$$

Performing the Routh matrix, it is as follows:

$$\begin{array}{lcc} s^3 & 1 & 5 \\ s^2 & 6 & K_p \\ s^1 & \frac{30-K_p}{6} & \\ s^0 & K_p & \end{array}$$

In the Routh criterion, the critical value occurs when a variable in the matrix is zero, and instability occurs when there is a change in sign from one row to another, therefore the critical value Kcr is 30.

$$s^3 + 6s^2 + 5s + 30 = 0$$

Replacing the gain kp which is kcr=30. In the previous equation, which is the characteristic equation, the substitution of s=jw is made.

$$(j\omega)^3 + 6(j\omega)^2 + 5(j\omega) + 30 = 0$$

Then by clearing ω

$$\omega^2 = 5 \text{ o } \omega = \sqrt{5}.$$

$$P_{cr} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{5}} = 2.8099$$

Then, using table 2 for a PID controller, the Kcr and Pcr values already obtained are replaced.

$$K_p = 0.6K_{cr} = 18$$

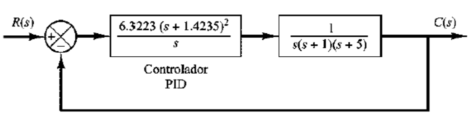
$$T_i = 0.5P_{cr} = 1.405$$

$$T_d = 0.125P_{cr} = 0.35124$$

Therefore, the transfer function of the controller is

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) = 18\left(1 + \frac{1}{1.405s} + 0.35124s\right)$$

$$\frac{C(s)}{R(s)} = \frac{6.3223s^2 + 18s + 12.811}{s^4 + 6s^3 + 11.3223s^2 + 18s + 12.811}$$



What are self-tuning, self-tuning control systems? Use matlab and present an example.

The auto-tuning control system has the property that the controller automatically adjusts itself to a desirable configuration of the feedback system. The basic scheme of an auto-tuning controller is shown in Figure 3. Through some design method, the controller must be adjusted to meet the requirements of the closed-loop system.[3]

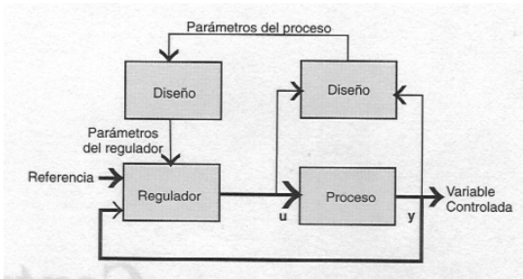


Figure 4: Block diagram of a Self-tuning regulator.

How are the Matlab commands "place" and "acker" used? Illustrate with examples.

The 'place' command is defined as the assignment of poles in closed loop by feedback status; can be used in such a way as to generate a feedback state matrix with certain conditions relative to the eigenvalues of its parameter relation.

The 'acker' function is defined as the pole positioning gain selection using the Ackerman formula.

4. ANALYSIS OF RESULTS

Proportional, integral, derivative circuits were designed and simulated at a cut-off frequency and a specific working frequency, which in practice and physical assembly each of the controls did not work at the previously designed frequency. This error had to be carried out by creating a voltage follower, which couples the impedances and reduces the error generated by the use of the operational amplifier.

Also, to have a proportional control of the integrative and derivative controllers, a proportional operational amplifier is implemented before the control in order to add a gain to it.

5. CONCLUSIONS

All responses from each of the PID controllers worked in the simulation software. However, in the physical montage, only the integrative action was the one that did not fulfill its original function.

The impedance coupling between amplifiers was not entirely correct, so the system did not respond as expected.

The offset adjustment makes it possible to have a good development of the plant and adjust to the desired response. But when the offset affects the signal, it is not optimal to implement.

The components that make up the circuit determine the gains and the behavior of the system, as these have certain tolerances, an error in the measurements with respect to the simulations is always expected.

Although the mathematical guidelines for the design of a PID were followed, the implementation was not completely complete, this is reflected in the non-operation of the controller.

6. REFERENCES

[1] Experimental tuning of controllers [online], Department of automatic and systems engineering, Spain: Polytechnic University of Cartagena UPCT, pp. 123 Available at: http://repositorio.upct.es/bitstream/handle/10317/174/Cap%EDtulo;jsessionid=46310D2741AF1C77389186664869C64A?sequence=13

[2] Experimental tuning of controllers [online], Department of systems engineering and automatic, Spain: Polytechnic University of Cartagena UPCT, pp. 124 Available at: http://repositorio.upct.es/bitstream/handle/10317/174/Cap%EDtulo;jsessionid=

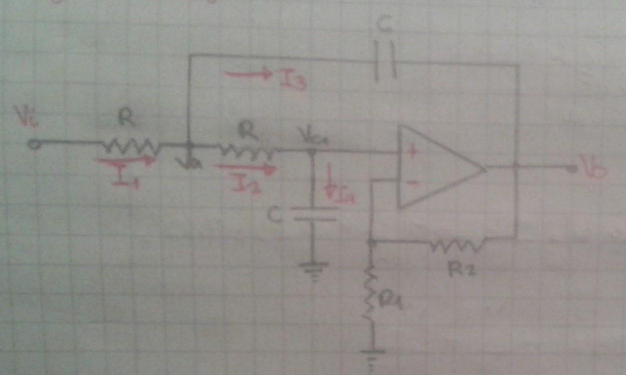
46310D2741AF1C77389186664869C64A?sequence=13

[3] E. Franco, JJ Martínez, H. Calero, Self-Controller Tuning: Applied to process control with highly variable dynamics [online], research article, Department of Engineering, Universidad del Valle, Cali, 1998. Available at: https://www.google.com.co/url?sa=t&rct = j & q = & ESRC = s & source = web & cd = 1 & cad = rja & uact = 8 & ved = 0ahUKEwiv247EprTSAhXGRiYKHfBtBQAQFggYMAA & url = http% 3A% 2F% 2Fbibliotecadigital.univalle.edu.co% 2Fbitstream% 2F10893% 2F1498% 2F1% 2Finycompe_v1_n2_a5.pdf & usg = AFQjCNEWVKsZzdhkZuHN80oC_QniNhOjbA & bvm = bv.148441817, d .eWE

[4]

ANNEX 1

Segundo orden



$$I_1 = I_2 + I_3$$

$$\frac{V_i - V_a}{R} = \frac{V_a - V_{c1}}{R} + C\frac{d(V_a - V_o)}{dt}$$

$$\frac{V_i - V_a}{R} = \frac{V_a - V_{c1}}{R} + C\frac{dV_a}{dt} - C\frac{dV_o}{dt}$$

$$I_2 = I_4$$

$$\frac{V_a - V_{c1}}{R} = C\frac{dV_{c1}}{dt}$$

$$V_a = RC\frac{dV_{c1}}{dt} + V_{c1}$$

$$V_{a(s)} = RCs\,V_{c1(s)} + V_{c1(s)}$$
$$= V_{c1(s)}(RCs+1)$$

$$V_{c1(s)} = \frac{1}{k}V_{o(s)}$$

$$\frac{V_{i(s)}}{R} = V_{a(s)}\left(\frac{2}{R} + Cs\right) - \frac{V_{c1(s)}}{R} - Cs\,V_{o(s)}$$

$$\frac{V_{i(s)}}{R} = V_{c1(s)}\left[(RCs+1)\left(\frac{2}{R} + Cs\right) - \frac{1}{R}\right] - Cs\,V_{o(s)}$$

$$\frac{V_{i(s)}}{R} = V_{c1(s)}\left[2Cs + RC^2s^2 + \frac{2}{R} + Cs - \frac{1}{R}\right] - Cs\,V_{o(s)}$$

$$\frac{V_{i(s)}}{R} = V_{c1(s)}\left[RC^2s^2 + 3Cs + \frac{1}{R}\right] - Cs\,V_{o(s)}$$

$$V_{i(s)} = V_{c1(s)}(R^2C^2s^2 + 3RCs + 1) - RCs\,V_{o(s)}$$

$$V_{i(s)} = V_{o(s)}\left(\frac{R^2C^2s^2 + 3RCs + 1}{k} - RCs\right)$$

$$\frac{V_{o(s)}}{V_{i(s)}} = \frac{k}{R^2C^2s^2 + (3-k)RCs + 1}$$

ANNEX 2.

$R = 100\,k\Omega$

$\cancel{s^2} + 2\xi\omega_n s + \omega_n^2 = s^2 + s + 1$

$\omega_n = 1 \rightarrow \xi = 0.5 \rightarrow t_r|_{LA} = 9\,s \rightarrow t_r|_{LC} = 9 \cdot 0.85 = 7.65\,s$

$\frac{4.5}{\xi\omega_n} = 7.65 \rightarrow \xi\omega_n = 0.588$

$0.25 = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$

$\cancel{-}1.386 = \frac{\cancel{-}\pi\xi}{\sqrt{1-\xi^2}}$

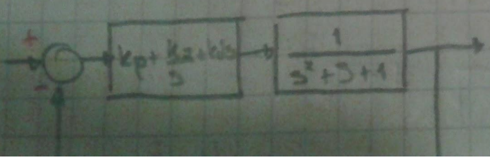
$2 = \frac{\pi^2\xi^2}{1-\xi^2} \rightarrow 2 - 2\xi^2 = \pi^2\xi^2$

$2 - 2\xi^2 = 9.9\xi^2$

$2 = 11.9\xi^2$

$\xi^2 = 0.168$

$\xi = 0.409 \rightarrow \omega_n = 1.438$



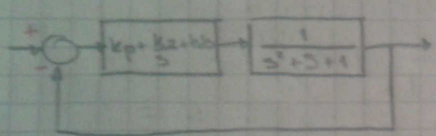
$2 = \frac{\pi^2\xi^2}{1-\xi^2} \rightarrow 2 - 2\xi^2 = \pi^2\xi^2$

$2 - 2\xi^2 = 9.9\xi^2$

$2 = 11.9\xi^2$

$\xi^2 = 0.168$

$\xi = 0.409 \rightarrow \omega_n = 1.438$



$$G(s) = \frac{\frac{k_p + k_I/s + k_d s}{s^2+s+1}}{1 + \frac{k_p + k_I/s + k_d s}{s^2+s+1}} = \frac{k_p + k_I/s + k_d s}{s^2+s+1+k_p+k_I/s+k_d s} \cdot \frac{s}{s} = \frac{k_d s^2 + k_p s + k_I}{s^3 + s^2(k_d+1) + (1+k_p)s + k_I}$$

→ Polinomio requerido

$s^2 + 2(0.409)(1.438)s + 2.0678 = s^2 + 0.588s + 2.0678$

$s = \frac{-0.588 \pm \sqrt{0.346 - 8.2712}}{2}$

* Polo adicional → $(s+3)$

$(s^2 + 0.588s + 2.0678)(s+3) = s^3 + s^2(k_d+1) + (1+k_p)s + k_I$

$\cancel{s^3} + 3.588s^2 + 3.8318s + 6.2034 = \cancel{s^3} + s^2(k_d+1) + (1+k_p)s + k_I$

$k_d + 1 = 3.588 \quad 1 + k_p = 3.8318 \quad k_I = 6.2034$

$k_d = 2.588 \quad k_p = 2.8318$