Control of Furuta pendulum, final project.

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Abstract--- In this laboratory practice it was required to design a servosystem in order to control the position of the upward vertical bar of the Furuta pendulum, when being This is a non-linear system, the model must be found in order to perform a dynamic control over the entire plant. The techniques learned in the theoretical class are used to develop each of the proposed objectives.

Abstract--- In this laboratory practice it was required to design a servo system in order to control the position of the vertical bar upwards of the Furuta pendulum, since this is a non-linear system, the model must be found in order to perform a dynamic control over Whole plant. The techniques learned in the theoretical class are used to develop each of the proposed objectives.

Keywords--- Servosystem, Observer, Nonlinear Control, Point of operation.

General Objective--- Design and implement a plant whose variable to be controlled is indirectly affected by an actuator.

Specific objectives--

* Develop a data acquisition interface for the plant.

* Verify the plant model.

* Get the model error.

* Design an application to check the behavior of the plant in closed loop.

* Calculate and transmit the error signal in a closed loop without a controller.

* Graph the behavior of the plant in a closed loop.

* Calculate the operating range of the system.

* Design and implement a digital controller, which allows achieving the three responses to the system output (underdamped, critically damped)

* Graph the behavior of the controlled plant.

* Describe control operation considerations.

INTRODUCTION

The Furuta Pendulum must be built, it must sense the angular positions of each bar, in addition to having a DC motor as an actuator.

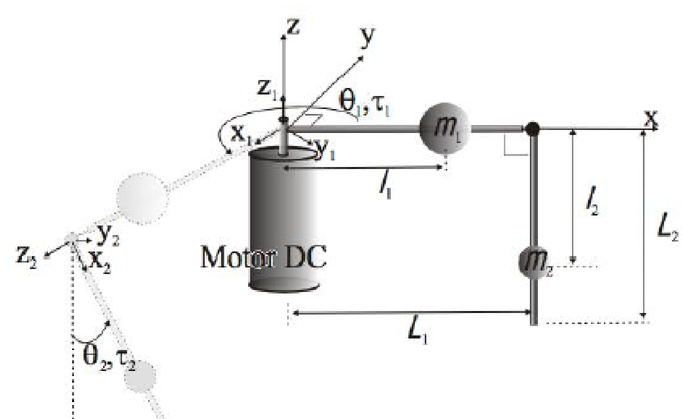

***Figure 1**: Furuta's pendulum*

1. MATERIALS

* Software: MATLAB ®

* White coat.

* DC geared motor, potentiometer, MDF support.

2. PROCEDURE

To model something similar to the direct kinematics of a manipulator robot is performed, in this it is sought to find the corresponding position vectors for the end of the arm and for the end of the pendulum. Finding those vectors are derived to find their equivalent velocity vectors. This in order to find the magnitude of that velocity vector.[1]

Having the magnitude of the speed, we proceed to make an analysis of kinetic and potential energy for the motor, for the arm and for the pendulum. Having the energy representations as a function of the state variables, which are the angles and the angular velocities of the arm and the pendulum, the Euler-Lagrange method is carried out to obtain the system model. The resulting model is the following:

$$\tau = \left(\frac{ke}{R}\right)V - \left(\frac{ke^2}{R}\right)\dot{\phi}$$

$$\dot{x1} = x2$$

$$\dot{x2} = \frac{B\Upsilon(sen(\theta))^2-1)sen(\theta)\dot{\phi}^2 - 2B^2cos(\theta)sen(\theta)\dot{\phi}\dot{\theta} + B\Upsilon sen(\theta)\dot{\theta}^2 - \Upsilon\delta cos(\theta)sen(\theta) + B\tau}{B\Upsilon - \Upsilon^2 + (B^2+\Upsilon^2)sen(\theta)^2}$$

$$\dot{x3} = x4$$

$$\dot{x4} = \frac{B(\alpha + Bsen(\theta)^2)cos(\theta)sen(\theta)\dot{\phi}^2 + 2B\Upsilon(1-sen(\theta))^2)sen(\theta)\dot{\phi}\dot{\theta} - \Upsilon^2cos(\theta)sen(\theta)\dot{\theta}^2 + \delta(\alpha+Bsen(\theta)^2)sen(\theta) - \Upsilon c}{B\Upsilon - \Upsilon^2 + (B^2+\Upsilon^2)sen(\theta)^2}$$

$$\alpha = J + \left(\frac{1}{3}ma + mp\right)ra^2$$

$$\Upsilon = \frac{1}{2}(mp)(ra)rp$$

$$B = \frac{1}{3}(mp)rp^2$$

$$\delta = \frac{1}{2}mp(g)(rp)$$

Where x1 is the angle of the arm, x2 the angular velocity of the arm, x3 the angle of the pendulum and x4 the angular velocity of the pendulum.

The following values are used for the constants:

```
ma =0.07;
mp =0.01;
J = 8e-7;
ra = 0.08;
rp = 0.2;
g = 9.81;
R = 12;
ke = 27e-3;
```

Replacing these values proceeds to linearize, for that the operating points of the system are found equaling the derivatives of the state variables to zero:

$$0 = x2$$

$$0 = x4$$

$$0 = cos(\theta)sen(\theta)$$

$$0 = (\alpha + Bsen(\theta)^2)\delta sen(\theta)$$

$$\theta = k\pi$$

It is obtained that the angle of the pendulum must be a multiple of pi. Since we want the system to stabilize at zero, it is taken as zero.

Having the operating points, the Jacobian is carried out, which remains:

$$\begin{bmatrix} \hat{\dot{x1}} \\ \hat{\dot{x2}} \\ \hat{\dot{x3}} \\ \hat{\dot{x4}} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{\delta\Upsilon}{\alpha B - \Upsilon^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{\alpha\delta}{\alpha B - \Upsilon^2} & 0 \end{bmatrix} \begin{bmatrix} \hat{x1} \\ \hat{x2} \\ \hat{x3} \\ \hat{x4} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{B}{\alpha B - \Upsilon^2} \\ 0 \\ -\frac{\Upsilon}{\alpha B - \Upsilon^2} \end{bmatrix} \tau$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x1} \\ \hat{x2} \\ \hat{x3} \\ \hat{x4} \end{bmatrix}$$

The output is set to the angle of the arm, since if said angular position is controlled indirectly, the other state variables are set to zero. Replacing values:

```
A =

     0    1.0000         0         0
     0         0  -35.4294         0
     0         0         0    1.0000
     0         0   94.8326         0


B =

   1.0e+03 *

        0
   6.0193
        0
  -3.6116
```

3. ANALYSIS OF RESULTS

Simulation in Matlab of the static system:

Being the 4th order system and designing for a desired settling time of 1s. In discrete time, the vector K of the servo system remains as:

```
K =
   -0.0257   -0.0075   -0.1683   -0.0206
```

Y the integral constant.

```
ki =
   -0.0017
```

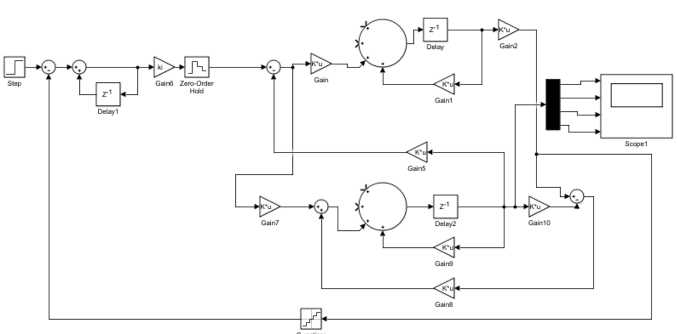

***Figure 2**: Simulink diagram of the static system*

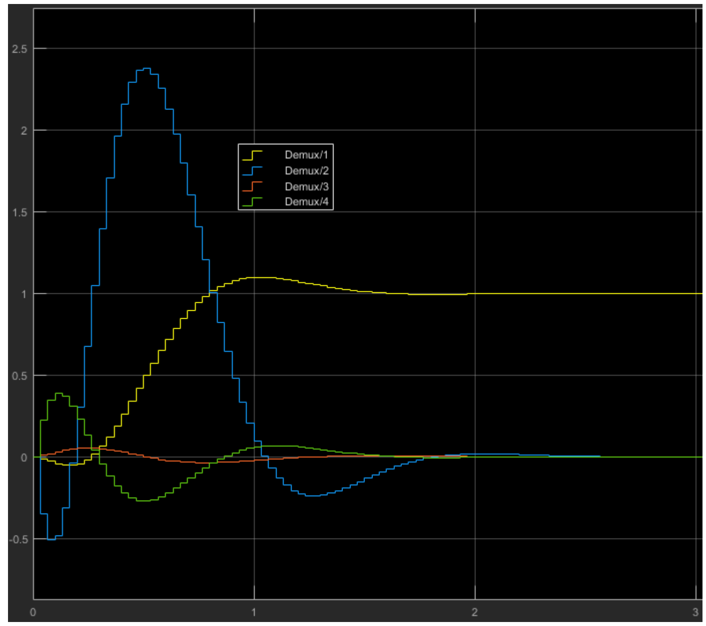

***Figure 3**: Scope of the static system where the yellow signal (yellow) is the controlled variable (angular position of the motor)*

Being the vector of state variables:

$$[\varphi \;\; \dot{\varphi} \;\; \theta \;\; \dot{\theta}]$$

where φ is the angular position of the motor and θ is the angular position of the second bar of the pendulum. And when observing the scope, it can be seen that the first state variable is controlled, which is the position of the motor (yellow signal) and stabilizes at the reference while the others are taken to zero.

4. CONCLUSIONS

*When performing the static servosystem, the desired underdamped response was obtained for the designed settling time, this represents that the linearization of the system was performed correctly.

*A state variable can be indirectly controlled only in the event that said variable must be stabilized at zero, due to the most important property of control by retrostates.

* The angular position of the pendulum is not observable, so it was decided to control the angular position of the arm to bring the angle of the pendulum to zero.

5. REFERENCES

[1] Global control of the Furuta Pendulum using artificial neural networks and feedback of state variables. Available at: *http://www.scielo.org.co/scielo.php?script=sci_arttext&pid=S0123-77992013000100005*