Theoretical Analysis of Mean Effective Gain of Mobile Terminal Antennas in Ricean Channels

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***Abstract*—The Mean Effective Gain (MEG) for Rice distributed signals impinging at the terminal receiver antenna is presented. Numerical results are presented for three different angular probability distributions of impinging waves. Finally, it is shown that contrary to the Rayleigh channels in Ricean channels there is no inclination angle at which the MEG of the half-wave dipole antenna remains constant regardless the cross-polarization ratio (XPR) of the channel.**

Keywords—Mean Effective Gain; Ricean channels

I. INTRODUCTION

The performance analysis of mobile terminal antennas has become a hot research topic and a plethora of different methods has been devised. Presently, most manufacturers and operators have developed a method of their own as well as other companies in the antenna business, but still none has succeeded to become a standard. Currently, activities towards such a common standard for terminal antenna performance characterization are taking place in the international standardization body 3GPP.

Among the existing methods, it is possible to discern two different approaches. The first one proposes absolute figures of merit to be measured, for instance the total radiated power (TRP). In the second one, the performance of the device under test is compared to a well-known reference. A method that belongs to the latest approach and assesses the Mean Effective Gain (MEG) of mobile antennas will be considered in the next sections. This method was proposed by Anderssen and Hansen [1], and then developed by Taga [2], which gave the final formulation of the MEG as it is known today.

The MEG considered in [1&2], assumes Rayleigh fading channels, where neither line of sight nor strong contributions to the total scattered field is present. The effects of de-polarization of the deterministic wave component in Ricean channels have barely been studied; at least no results on this topic are available in the published scientific-technical literature. The analysis presented below is aimed at understanding the influence of the depolarization effects on the terminal antenna performance in Ricean channels.

II. AVERAGE RECEIVED POWER AND MEG FOR RICE DISTRIBUTED SIGNALS

The mean effective gain (MEG) G_e [2], is defined as the ratio of the average power P_{rec} received at the mobile antenna and the sum of the average power of the vertically and horizontally polarized waves received by isotropic antennas:

$$G_e = P_{rec} / (P_V + P_H) \tag{1}$$

Equation (1) is a general formulation of the MEG, though until now the underlying parameters have been obtained assuming Rayleigh statistics for envelope of the received signal. A closed form expression for P_{rec} is presented in [3]. Below a more general expression for MEG that accounts for the more general Rice distributed channel envelopes is derived.

Now, assume a mobile scenario in a spherical co-ordinate system. The mobile is placed at the origin and is moving in the *x-y* plane along the positive *y*-axis with velocity **V**. The antenna field pattern of the mobile terminal is,

$$\mathbf{E_a} = E_\theta(\Omega)\mathbf{e}_\theta + E_\phi(\Omega)\mathbf{e}_\phi \tag{2}$$

where $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$ are the unitary vectors associated with a specific direction $\Omega = (\theta, \phi)$ on the spherical co-ordinate system, and E_θ, E_ϕ are the complex envelopes of the far-field electric pattern. Similarly, the incident field is,

$$\mathbf{A} = A_\theta(\Omega)\mathbf{e}_\theta + A_\phi(\Omega)\mathbf{e}_\phi \tag{3}$$

where A_θ, A_ϕ are the complex envelopes of the incident electric field.

In general, in a multipath environment, the incident field can be described by random variable. When the incident field besides the random field component comprises a direct or deterministic field component, the resulting signal envelope statistics may be described by the Rice distribution. In this case, the pattern of the incident field is given by two terms, the first one denoting the random scattered field $\mathbf{A}^0$ and the second one represents the deterministic component $\mathbf{B}$, coming from any arbitrary direction $\boldsymbol{\Omega_0}$,

$$\mathbf{A} = \mathbf{A^0(\Omega)} + \mathbf{B(\Omega - \Omega_0)} \tag{4}$$

In this case the following assumptions may be done:

1. The phase angles of the vertically polarized waves are independent for different directions of arrival Ω and Ω', except for the direction denoted by Ω_0,

$$\begin{aligned}\left\langle A_\theta(\Omega) A_\theta^*(\Omega')\right\rangle = \left\langle A_\theta^0(\Omega) A_\theta^{0*}(\Omega)\right\rangle \delta(\Omega-\Omega') + \\ + B_\theta B_\theta^* \delta(\Omega-\Omega_0)\delta(\Omega'-\Omega_0)\end{aligned} \tag{5}$$

where A_ϑ^0 and B_ϑ, are the complex envelopes of the random, respective the deterministic components of the incident electric field.

2. The phase angles of the vertically and horizontally polarized waves are independent for different directions of arrival Ω and Ω' except for direction denoted by Ω_0,

$$\left\langle A_\theta(\Omega) A_\phi^*(\Omega')\right\rangle = B_\theta B_\phi^* \delta(\Omega-\Omega_0)\delta(\Omega'-\Omega_0) \tag{6}$$

where the asterisk denotes the complex conjugate.

As stated in [3] the complex envelope of the received signal may be computed as follows:

$$V(t) = c\oint \mathbf{E_a}(\Omega)\mathbf{A}(\Omega) e^{-j\beta V e_r(\Omega) t} d\Omega \tag{7}$$

where c is a complex proportionality constant, $\mathbf{E_a}(\Omega)$ is the electric field pattern of the receiving antenna (2), $\mathbf{A}(\Omega)$ is the electric field pattern of the impinging plane wave (3) and the exponent is the Doppler shift due to the movement of the mobile antenna. The integration is performed over a sphere of unit radius.

The autocorrelation function of this stochastic complex variable is then computed as,

$$R_V(\tau) = \left\langle V(t) V^*(t+\tau)\right\rangle \tag{8}$$

Substituting (7) in (8) the following expression is obtained,

$$R_V(\tau) = cc^* \oiint \left\langle \{\mathbf{E_a}(\Omega)\mathbf{A}(\Omega)\}\{\mathbf{E_a}^*(\Omega')\mathbf{A}^*(\Omega')\}\right\rangle e^{-j\beta V(e_r(\Omega)-e_r(\Omega'))t + j\beta V e(\Omega)\tau} d\Omega d\Omega' \tag{9}$$

Further, making use of equations (2) to (4) together with conditions given by (5) and (6) the autocorrelation is expressed as follows,

$$\begin{aligned} R_V(\tau) = cc^* \oiint \left(|E_\theta(\Omega)|^2 \left\langle |A_\theta(\Omega)|^2 \right\rangle + |E_\phi(\Omega)|^2 \left\langle |A_\phi(\Omega)|^2 \right\rangle \right) e^{j\beta V e(\Omega)\tau} d\Omega + \\ + cc^* \{ |E_\theta(\Omega_0)|^2 |B_\theta(\Omega_0)|^2 + |E_\phi(\Omega_0)|^2 |B_\phi(\Omega_0)|^2 + \\ + E_\theta^*(\Omega_0) E_\phi(\Omega_0) B_\theta^*(\Omega_0) B_\phi(\Omega_0) + \\ + E_\theta(\Omega_0) E_\phi^*(\Omega_0) B_\theta(\Omega_0) B_\phi^*(\Omega_0) \} e^{j\beta V e(\Omega_0)\tau} \end{aligned} \tag{10}$$

The angular power distribution is then obtained averaging the received power over time, which for the vertically polarized component will be [3],

$$\left\langle |A_\theta(\Omega)|^2 \right\rangle = 2 P_V P_\theta(\Omega) \tag{11}$$

An analogue expression is obtained for the horizontal polarization.

Further, considering that both the receiving antenna field and the impinging wave are linearly polarized the phase difference of the two orthogonal field components is an integer multiple of π for both the electric field of the receiving antenna and the electric field of the incoming "deterministic wave". Finally, taking into account that the antenna pattern is proportional to the square of the electric field the autocorrelation function of the received signal by a mobile antenna in Ricean fields is obtained,

$$\begin{aligned} R_V(\tau) = C \oiint \left(P_V G_\theta(\Omega) P_\theta(\Omega) + P_H G_\phi(\Omega) P_\phi(\Omega) \right) e^{j\beta V e(\Omega)\tau} d\Omega + \\ + C\left(\sqrt{G_\theta(\Omega_0)} |B_\theta(\Omega_0)| + \sqrt{G_\phi(\Omega_0)} |B_\phi(\Omega_0)| \right)^2 e^{j\beta V e(\Omega_0)\tau} \end{aligned} \tag{12}$$

The received average power is given by $P_{rec} = \left\langle V(t) V^*(t) \right\rangle / 2$, which finally results in,

$$\begin{aligned} P_{rec} = \oiint \left(P_V G_\theta(\Omega) P_\theta(\Omega) + P_H G_\phi(\Omega) P_\phi(\Omega) \right) d\Omega + \\ + \left(\sqrt{P_V K_V G_\theta(\Omega_0)} + \sqrt{P_H K_H G_\phi(\Omega_0)} \right)^2 \end{aligned} \tag{13}$$

where K_V and K_H are the Rice K-factors of the vertical and the horizontal polarization components respectively and are defined as follows,

$$K_V = B_\theta B_\theta^* / P_V \tag{14}$$

$$K_H = B_\phi B_\phi^* / P_H \quad (15)$$

P_V and P_H are the average incident power of the vertically and horizontally polarized of the random component assuming isotropic antennas. The numerators of equations given by (14) and (15) are the power of the deterministic component of the vertical and horizontal polarizations respectively.

Finally, the amount of depolarization experienced by the transmitted signals is given by the cross-polarization ratio (XPR). The XPR is defined as the ratio of the received average power of the vertically polarized component to the average of the received horizontal component,

$$XPR = \frac{P_V + B_\theta B_\theta^*}{P_H + B_\phi B_\phi^*} = \frac{P_V}{P_H}\left(\frac{1 + B_\theta B_\theta^* / P_V}{1 + B_\phi B_\phi^* / P_H}\right) \quad (16)$$

Then making use of the K factors given above (14, 15) the following expression for the XPR in Ricean channels is obtained,

$$XPR_{Rice} = XPR_{Rayleigh}\left(\frac{1+K_V}{1+K_H}\right) \quad (17)$$

where $XPR_{Rayleigh}$ is the corresponding cross-polarization power ratio in Rayleigh channels.

$$XPR_{Rayleigh} = P_V / P_H \quad (18)$$

It is worthwhile to notice that the XPR_{Rice} given by equation (17) is valid as long as K_V and K_H are finite.

Then, making use of equations (1), (13), (14), (15), (17) and (18) and after some algebraic manipulations the final expression for the MEG in Ricean channels is obtained:

$$G_e = \frac{\oint\left(\frac{XPR_{Rice} G_\theta P_\theta}{1+K_V} + \frac{G_\phi P_\phi}{1+K_H}\right) d\Omega}{1 + XPR_{Rice}} + \frac{\left(\sqrt{\frac{K_V XPR_{Rice} G_\theta(\Omega_0)}{1+K_V}} + \sqrt{\frac{K_H G_\phi(\Omega_0)}{1+K_H}}\right)^2}{1 + XPR_{Rice}} \quad (19)$$

A closer analysis of equation (19) reveals, that the MEG in Ricean environments does strongly depend on the K-parameters of both orthogonal polarizations. Some limit cases may be elucidated from this general equation. Indeed, if the corresponding Ricean K-parameters for both polarizations are null, i.e. pure Rayleigh fading, then equation (19) will be identical to the equation by Taga [2].

$$G_e = \frac{\oint (XPR_{Rayleigh} G_\theta P_\theta + G_\phi P_\phi) d\Omega}{1 + XPR_{Rayleigh}} \quad (20)$$

In the other limit case when the fading stochastic component is absent and both K_V and K_H go to infinity, the equation (19) becomes:

$$G_e = \frac{\left(\sqrt{XPR_{Rice} G_\theta(\Omega_0)} + \sqrt{G_\phi(\Omega_0)}\right)^2}{1 + XPR_{Rice}} \quad (21)$$

The cross-polarization is then given by:

$$XPR_{Rice} = B_\theta B_\theta^* / B_\phi B_\phi^* \quad (22)$$

In this case, the MEG has a form similar to the Friis transmission formula and the Radar Range equation [4] if multiplied by the total incident power. Namely, consider equation (21), and denote the power of the vertically and horizontally polarized components, by $S_\theta = B^*_\theta B_\theta$ and $S_\phi = B^*_\phi B_\phi$ respectively. Thus, making use of the cross-polarization ratio and the definition of the scalar product we obtain the following expression for the MEG,

$$G_e = \frac{\left(\sqrt{S_\theta G_\theta(\Omega_0)} + \sqrt{S_\phi G_\phi(\Omega_0)}\right)^2}{S_0 + S_\phi} = \frac{G_r(\theta_0,\phi_0) S(\theta_0,\phi_0) \cos^2(\vec{p}_r \vec{p}_s)}{S(\theta_0,\phi_0)} \quad (23)$$

where $G_r(\theta,\phi)$ is the gain of the receiving antenna and $S(\theta,\phi)$ is the total received power at the antenna front end. The vectors $\mathbf{p}_r$ and $\mathbf{p}_s$ are polarization vectors of the receiving antenna and the incident field (direct or scattered field) respectively. Further, it is well known that the incident power is directly proportional to the antenna gain of the transmitting antenna, which leads either to the Friis formula or the Radar Range Equation in the numerator of (13) depending on the specific propagation scenario.

$$G_e = \frac{\kappa G_r(\theta_0,\phi_0) G_t(\theta_0,\phi_0) \cos^2(\vec{p}_r \vec{p}_s)}{S(\theta_0,\phi_0)} = G_r(\theta_0,\phi_0)\cos^2(\vec{p}_r \vec{p}_s) \quad (24)$$

The parameter κ above depends on the specific propagation scenario [4]. Then, as expected, MEG is proportional to the antenna gain in the direction of observation times the polarization mismatch.

Now, in order to perform terminal antenna efficiency measurements in laboratory conditions using MEG as the figure of merit [6] it is interesting to find out if there is any position at which the performance of the reference antenna will be independent of the channel characteristics. In [2] it is shown that if the half-wave dipole antenna is inclined about 55° from the vertical the MEG is –3 dBi regardless the XPR and the angular power distribution of the impinging waves in Rayleigh channels. This can be obtained if the derivative of MEG with respect to the XPR is zero:

$$\frac{\partial G_e}{\partial XPR} = 0 \tag{25}$$

Substituting (20) in (25) and performing the corresponding algebraic operations it is obtained that,

$$\frac{\partial G_e}{\partial XPR} = \frac{\oint (G_\theta P_\theta - G_\phi P_\phi) d\Omega}{\left(1 + XPR_{Rayleigh}\right)^2} = 0 \tag{26}$$

Hence, the necessary and sufficient condition is that the total radiated power with theta polarization equals the total radiated power of the phi polarization.

In a similar way it may be shown that the MEG in Ricean channels (19) will in general always dependent on the XPR, which in lack of space will not be presented here.

III. Probability Distribution Models of Incident Waves

As is clear from the section above the distribution of waves incident onto the mobile terminal as well as the XPR are included in the MEG. However, the knowledge of such a distribution is critical not only to the theoretical study of the MEG but also to the development of accurate antenna performance measurement methods.

Below, three theoretical distributions will be considered. The first one describes a uniform distribution in space for both polarizations and both azimuth and elevation angles:

$$P_\theta(\theta,\phi) = P_\phi(\theta,\phi) = \begin{cases} 1/4\pi & 0 \le \theta \le \pi \quad 0 \le \phi \le 2\pi \\ 0 & \mathit{othervise} \end{cases} \tag{27}$$

This distribution has been largely used for indoor radio channel modeling. In this case, the average angle of arrival is likely to be equally probable in most directions if omni-directional antennas are used.

The second distribution is the one proposed by Taga [2] and models outdoor environments. Taga assumed a Gaussian model for the elevation angle dependence of the received power for both the vertical and horizontal polarizations. Further, it was assumed a uniform distribution in azimuth, which leads to the following equation,

$$P_\theta(\theta,\phi) = \begin{cases} A_\theta e^{-\frac{\left\{\theta - \left[\pi/2 - m_V\right]\right\}^2}{2\sigma_V^2}} & 0 \le \theta \le \pi \quad 0 \le \phi \le 2\pi \\ 0 & \mathit{otherwise} \end{cases} \tag{28}$$

A similar distribution is assumed for the horizontal polarization with corresponding parameters m_H, A_ϕ and σ_ϕ. This distribution has been obtained fitting measurement data, where both the receiver and the transmitter were placed outdoors.

It should be noted that this model was obtained by fitting the Gaussian function to just a few points. Furthermore, the shape of the dependence is Gaussian but this doesn't mean that the distribution of the angles is Gaussian. It will rather be a truncated Gaussian distribution.

The third and final distribution considers the outdoor-to-indoor propagation scenario, which may be regarded as the worst case for mobile communications in terms of transmitted signal attenuation. In [5] an elliptical angular power distribution function for the azimuth angle has been proposed based on measurements, while the corresponding distribution for the elevation angles is modeled by the Gaussian function:

$$P_\vartheta(\theta,\phi) = \begin{cases} A_\vartheta e^{-\frac{\left\{\theta - \left[\pi/2 - m_V\right]\right\}^2}{2\sigma_V^2} + \gamma(\phi)} & 0 \le \theta \le \pi \quad 0 \le \phi \le 2\pi \\ 0 & \mathit{otherwise} \end{cases} \tag{29}$$

$$\gamma(\phi) = a_{\theta 1}\sin^2(\phi) + b_{\theta 0}\cos(\phi) + b_{\theta 1}\cos^2(\phi) \tag{30}$$

The parameters $a_{\theta 1}$, $b_{\theta 1}$ and $b_{\theta 0}$ define the distribution variables. A similar distribution is assumed for the horizontal polarization with corresponding parameters $a_{\phi 1}$, $b_{\phi 1}$ and $b_{\phi 0}$ and A_ϕ. In the next section numerical values for the parameters of the enumerated distribution are given.

IV. MEG of Inclined Half-Wave Dipole Antennas, Numerical Results

In this section, some numerical results for the MEG of an inclined half-wave dipole antenna are presented. The MEG has been obtained for the three different models for the distribution of the angle of arrival presented in section III. The MEG is presented as a function of the inclination angle for different cross-polarization parameters and Rice K factors for both the vertical and horizontal polarizations.

For the sake of simplicity the Rice K factor of the vertically and horizontally polarized incoming waves are assumed equal. Four different values of K_V=K_H=0, 1 and ∞ have been considered. It should be noted that K=0 corresponds to the Rayleigh distribution. Further, following the same approach of Taga [2], seven cross-polarization parameters were considered, XPR=50, 6, 3, 0, -2, -6, -9 dB. Finally, three direction of arrivals for the deterministic component were considered, (ϕ,θ)=(0°, 90°) and (90°, 90°). It has also been assumed that the inclination angle lies on the x-z plane.

Specific values for the considered distributions are given in table I below.

TABLE I. PARAMETERS OF ANGULAR POWER DISTRIBUTIONS

Outdoor Model	Outdoor-to-indoor Model
$m_V=m_H=0°$ $\sigma_V=\sigma_H=30°$	$m_V=m_H=0°$ $\sigma_V=\sigma_H=30°$ $a_{\theta 1}=0.71$, $a_{\phi 1}=0.98$ $b_{\theta 1}=2.12$, $b_{\phi 1}=1.18$ $b_{\theta 0}=0.70$, $b_{\phi 0}=0.46$

Figure 1 shows the MEG dependence on the antenna inclination angle for three different power angle distributions. They model the average power distributions of the waves impinging at the terminal antenna. It should be noted that the MEG obtained below are not in any case an average MEG, they depend on the orientation of the mobile antenna and are directional, [5].

Only three directions of the deterministic component have been chosen to illustrate the impact of the direction of arrival of the Rice component of the field at the receiving antenna. Results for the three considered models have been shown on the same plot. The dashed line is the indoor model (27), the solid line represents the Taga model [2], which is denoted as outdoor model (28), and finally the dotted line corresponds to the outdoor-to-indoor model (29&30), [5].

The plots in Fig. 1 reveals that for the considered half-wave dipole antenna, the MEG is quite insensitive to the angular power distribution on the horizontal plane, that is, the azimuth. Only a minor divergence is seen regarding the angle of constant MEG, which is independent of the inclination angle (observe that only one antenna orientation have been considered for the outdoor-to-indoor model). Some dB difference is obtained for the studied distributions. The largest difference is observed when the Ricean K factor is different from null and different angle of arrivals are observed for the incident deterministic component. As can be seen from equation (19) the larger the K factor the less the impact of the Rayleigh component on the MEG, which is natural to expect.

As is clear from Fig. 1, the MEG of a half-wave dipole in Ricean channels with K factors around one and below are quite sensitive to the direction of arrival of the deterministic component. And in general there is no inclination angle that gives the same MEG regardless the XPR. For some directions there will still be such an angle, however for other directions it will completely disappear.

V. SUMMARY

A generalization of the Mean Effective Gain for Rice distributed signals impinging at the receiver antenna has been done. This formula introduces a new degree of freedom in the analysis of the MEG for such cases where the distribution of the incoming waves is close to the Rayleigh distribution, that is Rice distributed with K parameters of the order of one and less. Further it is shown that in order to the MEG of an antenna in Rayleigh channels be independent of the XPR, the total radiated power with theta polarization must equal the total radiated power of the phi polarization. It is also shown that for the half-wave dipole antennas in Rice channels there is no such inclination angle at which the MEG will be independent of the XPR.

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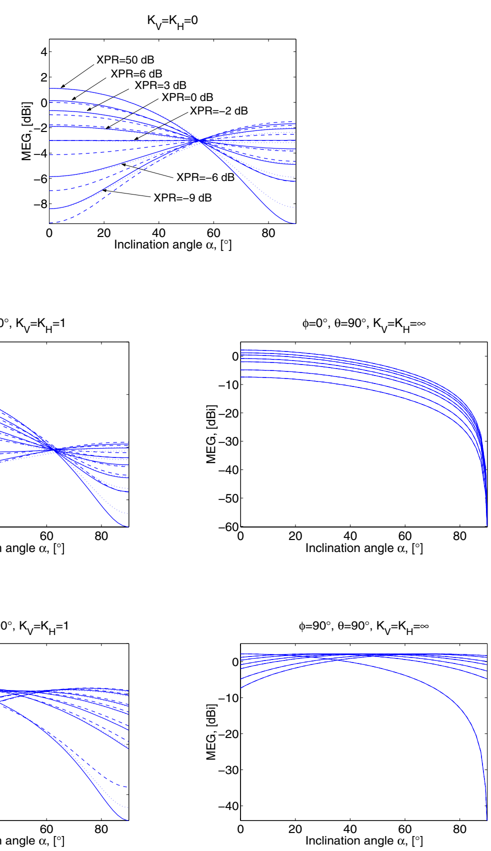

Figure 1. Mean Effective Gain versus inclinaton angle for different values of XPR and Ricean K-factors

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