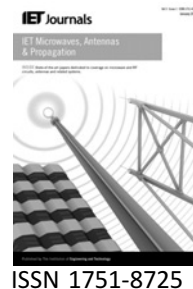


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# Mean effective gain of antennas in a wireless channel

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**Abstract:** The mean effective gain (MEG) is one of the most important parameters for the characterisation of antennas in wireless channels. An analysis of some fundamental properties of the MEG is provided and corresponding physical interpretations are given. Three points are analysed in detail: (i) closed-form expressions for MEG in a mixed environment with both stochastic and deterministic components are provided, showing that the MEG can be written as a sum of gains for the deterministic and stochastic components, (ii) it is shown that under some assumptions, the propagation channel and the antenna are equivalent in the sense that the impact of the channel cross-polarisation ratio (XPR) and the antenna effective cross-polar discrimination on the MEG are symmetrical, (iii) based on the fact that MEG depends on random variables, such as the XPR and antenna rotations because of user's movements, the average, the minimum and maximum MEG of antennas are defined, respectively. Finally, the maximum effective gain of antennas is derived and shown that it is bounded by  $4\pi\eta_{\text{rad}}$ , where  $\eta_{\text{rad}}$  is the radiation efficiency of the antenna.

## 1 Introduction

Mobile terminals are vital elements of wireless networks and have a significant impact on the overall system performance. The efficiency of the mobile terminal including the antenna has a strong impact on the link quality in both the downlink and uplink channels. In particular, the antenna gain directly enters the link budget, and thus (co-) determines the coverage and/or data rate that can be achieved. In wireless communication systems with a 'single-path' between the receiver and the transmitter, or generally, in systems with a strong line-of-sight (LOS) component, such as point-to-point links, the impact of the antennas on the link quality is fully quantified by the Friis equation [1]. This equation accounts for antenna directivity, radiation efficiency and polarisation mismatch in the LOS direction. On the other hand, in wireless systems where none-line-of-sight (NLOS) communications are predominant, that is, in multipath channels with no dominant component, a full characterisation of the impact on the link budget is obtained by the partial antenna gain patterns for orthogonal polarisations combined with the directional and polarisation properties of the propagation channel. However, such a full

functional characterisation of antenna and channel is too complicated for most practical purposes. It is thus desirable to use a single parameter that describes antenna, channel and their interaction.

The mean effective gain (MEG), which is a single parameter describing the impact of the antenna on the link budget, has emerged as the way of characterising the communication performance of handsets including the antennas in real propagation environments. Currently, mainly due to practical reasons, the total radiated power (TRP) isotropically radiated by the mobile terminal is used in the link budget calculations, together with an attenuation factor accounting for the losses in the user's body. Even if the TRP is an excellent parameter for the evaluation of the power radiated in free space, it is not a proper measure for the characterisation of the communication link quality. TRP does not account for the full interaction between the antennas and the channels, for example, the joint effects of polarisation and directivity mismatch. MEG, on the other hand, is the average received power that in the Rayleigh fading environment completely defines the first-order statistics of the signal

envelope of the small-scale fading. Moreover, MEG is a measure of how a deterministic device, the antenna, performs in the stochastic channel. Finally, MEG is the natural extension of the communication link quality concept introduced by Wheeler for single-path channels, [2], to the more general case of multipath channels. Wheeler defined the communication link quality as the ratio of the received power to the transmitted power and made use of the Friis equation.

The concept of MEG was introduced by Taga [3], who defined it as the average power received by the antenna under test in the propagation channel of interest to the sum of the average powers that would have been received in that same environment by two isotropic antennas, vertically and horizontally polarised, respectively. A more general definition and practical definition of MEG is defined relative a realistic reference antenna such as a half-wavelength dipole. In his paper, Taga gave a closed-form equation for the uncorrelated scattering case based on the Jakes' signal autocorrelation model given in [4]. Work since then has concentrated on evaluating the MEG of antennas in different Rayleigh fading environments, different antenna designs as well as different commercial handsets have been evaluated using channel models describing the spatial and polarisation response of the channel to the transmitted electromagnetic waves. In many occasions, the variability of the MEG because of the user's body, mainly the head and/or hand, has also been evaluated from measurements of handsets. Some examples can be found in [5–21]. A summary of research results and further references can be found in [22], [23].

Despite the large number of investigations of the MEG, there are still several important topics that have not been addressed yet; the current paper aims to close those gaps.

1. First of all, we investigate the MEG in Rician fading channels, that is, channels that contain an LOS component as well as random fields (all previous papers considered Rayleigh-fading random fields only). Rician channels are gaining more and more importance, for example, in communication between personal digital assistant (PDA) like devices to wireless local area network access points (WiFi), picocell base stations (3GPP) or relays (Wimax). We provide closed-form equations for the MEG in such channels that clearly show the influence of the different field components.

2. Next, we provide a physical interpretation of the factors influencing the MEG and analyse how it can be improved. As discussed above, the MEG accounts for the influence of both the antenna (as given by the polarised antenna patterns) and the channel (described by the directional spreading and depolarisation in the channel). We show that the MEG is determined by how well the polarisation characteristics of the antenna and the channel are matched to each other, and similarly for the directional characteristics; as a matter of fact, channel and antennas

show duality in their impact on the MEG. This gives an understanding of how a low MEG is generated, and how it can be improved.

3. While the MEG has always been treated in the literature as a single, fixed, number, this is only valid for a certain, fixed orientation of the antenna with respect to the direction of the multipath components in the propagation channel. In many practical situations, the orientation of the handset (and thus the antenna) cannot be reliably predicted. Moreover, channel properties such as the cross-polarisation ratio (XPR) and the angular spread are stochastic variables by nature. Thus, the MEG becomes a function of the angular spread, the XPR, the orientation angle relative to the incoming wave field and so on. We investigate the properties of this function, including its mean, minimum and maximum. The results enable a more realistic link budget that includes margins and outage probabilities. Finally, we provide closed-form expressions for the optimal link gain, which could be achieved by proper knowledge of the channel assuming that losses because of matching are minimised.

The remainder of the paper is organised as follows. Section 2 provides the derivation of the generalised MEG that incorporates the LOS component of the received field. Here, we also analyse some special or limiting cases of the new MEG equation. Section 3 investigates the channel-antenna duality, and shows how the matching between the channel and antenna characteristics influences the MEG. In Section 4, we analyse the maximum and the minimum MEG, more specifically, we address how the MEG changes as a function of the antenna orientation, but also as a function of other parameters. Finally, a summary with conclusions is given in Section 5.

## 2 MEG in Ricean channels

For a receive antenna, the MEG is defined as the ratio of signal power available at the antenna, that is, the power spectral density (PSD) of an underlying wide-sense stationary stochastic process, and the PSD of a reference signal. The reference signal is usually measured by a reference antenna with well-defined performance characteristics. In the definition by Taga [3], the reference is the mean power that would be measured by idealised isotropic antennas, which is equivalent to the actual average power of the incoming field. The field is assumed to be random, more specifically, it is assumed to be the superposition of a large number of multipath components similar in amplitude but with different, uniformly distributed random phases. The received power is computed as the ensemble average of the signal power induced at the antenna by this random field. The ensemble is created by different superpositions of the multipath components, as, for example, measured at different locations within a small-scale fading area. Assuming the underlying process to be ergodic, the ensemble average can be replaced by a temporal or spatial average. It is

furthermore assumed that the first-order statistics of the real and imaginary parts of the fields are independent and identically distributed zero-mean Gaussian variables; so that the envelope of this random process is then distributed according to the Rayleigh probability density. Under this assumption, uncorrelated fading of orthogonally polarised components follows. In Rayleigh channels, the MEG completely characterises the fading statistics of the signal envelope, since MEG is identical with the only parameter of the Rayleigh distribution, which is the average power.

Even though the Rayleigh-fading case is the most common in practice, the more general assumption of a mixture of unpolarised stochastic and polarised deterministic components is still of great interest [24, 25]. Therefore, we will investigate the MEG of an antenna in different types of fields depending on whether the field is purely stochastic or it also contains a deterministic component.

## 2.1 Derivation of MEG in uncorrelated random field with one deterministic component

The MEG is the ratio of the average power received by the antenna under test,  $P_r$ , to the average power received by a reference antenna in the same environment,  $P_{ref}$  [3]

$$G_e = \frac{P_r}{P_{ref}} \quad (1)$$

The average received power is given by the average of the squared magnitude of the open circuit voltage at the antenna port

$$P_r \propto \frac{1}{2} \langle V_{oc}(t) V_{oc}^*(t) \rangle \quad (2)$$

where  $\langle \cdot \rangle$  indicates averaging over ensemble or space as discussed above and  $(\cdot)^*$  denotes complex conjugate.

The time-dependent complex signal,  $V_{oc}(t)$  is given by the open-circuit voltage induced at the local port of the antenna [4, 25]

$$V_{oc}(t) = \int \mathbf{F}_r(\Omega) \cdot \mathbf{E}_i(\Omega) e^{-i(2\pi/\lambda)u \cdot \mathbf{e}_r(\Omega)t} d\Omega \quad (3)$$

where  $\mathbf{F}_r(\Omega)$  is the electric far field amplitude of the antenna (bold face variables denote vector magnitudes),  $\mathbf{E}_i(\Omega)$  is the electric field amplitude of the plane wave incident from the direction encompassed by the solid angle  $\Omega$ , that is,  $\Omega$  defines the angle of arrivals (AoA) that are given in spherical co-ordinates,  $u$  denotes the absolute value of the mobile velocity,  $\mathbf{e}_r(\Omega)$  is the projection of the mobile velocity on the direction of observation and  $t$  denotes time. The integral is calculated over the sphere of unit radius.

In multipath environments, the incident field is usually described by a random variable that emulates the stochastic behaviour of the received signal. The incident field has, in the general case, a direct or deterministic field component besides the random field component. In the presence of several strong specular reflections, several deterministic field components might exist. The resulting first-order statistics of the signal envelope are usually described by the Rice distribution.

In order to describe the Ricean fading, we make a generalisation of correlation properties of the received field in [4]

$$\begin{aligned} \langle E_{i\alpha} E_{i\beta}^* \rangle &= E_{0\alpha} E_{0\beta}^* \delta(\Omega - \Omega_0) \delta(\Omega' - \Omega_0) \\ &+ \langle |E_\alpha|^2 \rangle \delta(\Omega - \Omega') \delta_{\alpha\beta} \end{aligned} \quad (4)$$

where  $E_{i\alpha}$  and  $E_{i\beta}$  are the complex amplitudes of the random incident electric field in  $\alpha$  and  $\beta$  polarisations respectively,  $\delta_{\alpha\beta}$  denotes the Kronecker-delta function, and  $\delta(\cdot)$  denotes the Dirac-delta. It is important to note that a propagation channel or incident field can be characterised independently of the antenna [26], although, of course, the reception and detection of the field is done by means of antennas with specific characteristics.

Equation (4) states that: (1) the phases of the co-polarised waves are independent in different direction of arrivals (DoAs)  $\Omega$  and  $\Omega'$  and (2) the phases of the cross-polarised waves are also independent in different DoAs  $\Omega$  and  $\Omega'$  but correlated in a fixed direction  $\Omega_0$ .

The autocorrelation function of this stochastic and, in general, ergodic complex variable is then computed as

$$R_{V_{oc}}(\Delta t) = \frac{1}{2} \langle V_{oc}(t) V_{oc}^*(t + \Delta t) \rangle_t \quad (5)$$

Substituting (3) in (5) and making use of the conditions given in (4), the autocorrelation becomes

$$\begin{aligned} R_{V_{oc}}(\Delta t) &= \frac{1}{2} \int (|F_{r\theta}(\Omega)|^2 \langle |E_\theta(\Omega)|^2 \rangle \\ &+ |F_{r\phi}(\Omega)|^2 \langle |E_\phi(\Omega)|^2 \rangle) e^{i(2\pi/\lambda)u \cdot \mathbf{e}_r(\Omega)\Delta t} d\Omega \\ &+ (|F_{r\theta}(\Omega_0)|^2 |E_{0\theta}(\Omega_0)|^2 + |F_{r\phi}(\Omega_0)|^2 |E_{0\phi}(\Omega_0)|^2 \\ &+ 2\Re\{F_{r\theta}(\Omega_0)F_{r\phi}^*(\Omega_0) \\ &\times E_{0\theta}(\Omega_0)E_{0\phi}^*(\Omega_0)\}) e^{i(2\pi/\lambda)u \cdot \mathbf{e}_r(\Omega_0)\Delta t} \end{aligned} \quad (6)$$

where  $\Re(\cdot)$  denotes the real part of the complex variable. The power angular distribution is then obtained by averaging the received power over the small-scale fading

$$\begin{aligned} \langle |E_\theta(\Omega)|^2 \rangle &\propto 2P_\theta p_\theta(\Omega) \\ \langle |E_\phi(\Omega)|^2 \rangle &\propto 2P_\phi p_\phi(\Omega) \end{aligned} \quad (7)$$

where  $p_\theta(\Omega)$  and  $p_\phi(\Omega)$  denote the weighted power angular spectrum (PAS) (also known as the weighted probability density function of the AoA) of the stochastic components in the  $\theta$ -polarisation and  $\phi$ -polarisations, respectively, where  $\theta$  and  $\phi$  are the elevation and azimuth angle in a spherical coordinate system, respectively. According to the definition of probability density function,  $p_\theta(\Omega)$  and  $p_\phi(\Omega)$  are normalised as

$$\int p_\theta(\Omega) d\Omega = \int p_\phi(\Omega) d\Omega = 1 \quad (8)$$

The available powers of the stochastic components in the  $\theta$ -polarisation and  $\phi$ -polarisation are denoted by  $P_\theta$  and  $P_\phi$ , respectively. It should be noted that  $P_\theta$  and  $P_\phi$  are usually referred to as the powers in the vertical and horizontal polarisations, respectively. However, this is somewhat misleading if the field is purely vertically or horizontally polarised and the propagation occurs only in the horizontal plane (or, more generally, in the same plane). In this case, it is only correct to assume that the field is either horizontally or vertically polarised.

Finally, we can proceed to calculate the received average power and therefore the MEG. Taking into account that the antenna pattern is proportional to the squared magnitude of the electric field and (7)–(8), the autocorrelation function of the signal received by an antenna in a mixed stochastic and deterministic field can be computed as

$$\begin{aligned} R_{V_{oc}}(\Delta t) &= \int (P_\theta G_\theta(\Omega) p_\theta(\Omega) + P_\phi G_\phi(\Omega) p_\phi(\Omega)) e^{i(2\pi/\lambda)u \cdot e_r(\Omega) \Delta t} d\Omega \\ &+ \left( \sqrt{P_{0\theta} G_\theta(\Omega_0)} + \sqrt{P_{0\phi} G_\phi(\Omega_0)} \right)^2 e^{i(2\pi/\lambda)u \cdot e_r(\Omega_0) \Delta t} \end{aligned} \quad (9)$$

In (9),  $P_{0\theta} = (1/2)|E_{0\theta}(\Omega_0)|^2$  and  $P_{0\phi} = (1/2)|E_{0\phi}(\Omega_0)|^2$  denote the powers of the deterministic field in  $\theta$  and  $\phi$  polarizations, respectively. We associate the vertical and horizontal polarisations to the  $\theta$  and  $\phi$  polarisations, respectively. Hence, the received average power is obtained from (9) using the relationship  $P_r = R_{V_{oc}}(0) = \langle V_{oc}(t) V_{oc}^*(t) \rangle_t / 2$

$$\begin{aligned} P_r &= \int P_\theta G_\theta(\Omega) p_\theta(\Omega) + P_\phi G_\phi(\Omega) p_\phi(\Omega) d\Omega \\ &+ \left( \sqrt{P_\theta K_\theta G_\theta(\Omega_0)} + \sqrt{P_\phi K_\phi G_\phi(\Omega_0)} \right)^2 \end{aligned} \quad (10)$$

where  $K_\theta$  and  $K_\phi$  are the Ricean  $K$ -factors of the vertical and horizontal polarisation components, respectively, defined as

$$K_\theta = \frac{P_{0\theta}}{P_\theta}, \quad K_\phi = \frac{P_{0\phi}}{P_\phi} \quad (11)$$

The antenna gains are normalised with respect to the radiation efficiency,  $\eta_{rad}$ , as

$$\int G_\theta(\Omega) + G_\phi(\Omega) d\Omega = 4\pi\eta_{rad} \quad (12)$$

where the radiation efficiency is defined as the ratio (TRP),  $P_{rad}$ , to the input power at the antenna port,  $P_{in}$  [27]

$$\eta_{rad} = \frac{P_{rad}}{P_{in}} \quad (13)$$

and TRP is given by

$$P_{rad} = P_{in} \int \frac{G_\theta(\Omega) + G_\phi(\Omega)}{4\pi} d\Omega \quad (14)$$

The amount of polarisation power imbalance of the RF electromagnetic field is given by the XPR,  $\chi$ . The XPR is defined as the ratio of the average received power of the vertically polarised component to the average power received in the horizontal component. From (11), the XPR in Ricean channels can be computed as

$$\chi = \frac{P_{0\theta} + P_\theta}{P_{0\phi} + P_\phi} = \chi_{unpol} \frac{1 + K_\theta}{1 + K_\phi} \quad (15)$$

where  $\chi_{unpol}$  is the corresponding XPR of the stochastic (unpolarised) components. The XPR in the LOS scenario given by (15) is valid as long as  $K_\theta$  and  $K_\phi$  are finite.

In our case, the reference power is the total available power that stems from the random field and the deterministic components received by isotropic antennas, that is

$$P_{ref} = P_\theta + P_{0\theta} + P_\phi + P_{0\phi} \quad (16)$$

It is worthwhile to note that the isotropic antenna is an ideal antenna that cannot be constructed in practice. Usually, a calibrated dipole antenna is used as reference both for anechoic chamber measurements, [28], as well as MEG measurement.

By (1) and (10)–(16), we can after some algebraic manipulations obtain an expression for the MEG in Ricean channels:

**Proposition 1:** In a multipath environment characterised by a mixed field with both uncorrelated random, unpolarised, component and one deterministic, polarised, component, the MEG of an antenna is given by

$$\begin{aligned} G_e &= \frac{1}{1 + \chi} \int \frac{\chi G_\theta(\Omega) p_\theta(\Omega)}{1 + K_\theta} + \frac{G_\phi(\Omega) p_\phi(\Omega)}{1 + K_\phi} d\Omega \\ &+ \frac{1}{1 + \chi} \left( \sqrt{\frac{\chi K_\theta G_\theta(\Omega_0)}{1 + K_\theta}} + \sqrt{\frac{K_\phi G_\phi(\Omega_0)}{1 + K_\phi}} \right)^2 \\ &= G_e^{NLOS} + G_e^{LOS} \end{aligned} \quad (17)$$



In (17), the MEG is basically the sum of the MEGs due to the NLOS (unpolarised) component and the LOS (polarised) component of the incident field. Note that even though it is convenient to express the MEG as a sum of gains of the NLOS and LOS components, it should not be assumed that it actually is the sum of two independent parameters. Indeed, the total available power acts as a common reference. However, it is straightforward to show that when  $K_\theta$  and  $K_\phi$  both are zero, the MEG is completely defined by the stochastic, unpolarised NLOS component; on the other hand, when  $K_\theta$  and  $K_\phi$  both tend to infinity the MEG is completely defined by the deterministic, polarised LOS component.

Moreover, just like in the Rayleigh-fading case, the MEG is the same as the average received power. However, since the Rician probability density is a function of two parameters, besides the average power, the  $K$ -factor must be defined in order to fully characterise the signal envelope statistics.

## 2.2 MEG of an antenna in correlated deterministic field

As a sanity check, we look at the limit case when no scattered field components are present, that is,  $K_\theta \rightarrow \infty$  and  $K_\phi \rightarrow \infty$ . The MEG is given by

$$G_e = \frac{1}{1 + \chi} \left( \sqrt{\chi G_\theta(\Omega_0)} + \sqrt{G_\phi(\Omega_0)} \right)^2 \quad (18)$$

where  $\chi = P_{0\theta}/P_{0\phi} = |E_\theta|^2/|E_\phi|^2$  is the XPR of the LOS component. Further, the MEG equation can be then rewritten as

$$\begin{aligned} G_e &= \frac{\left( |E_\theta| \sqrt{G_\theta(\Omega_0)} + |E_\phi| \sqrt{G_\phi(\Omega_0)} \right)^2}{|E_\theta|^2 + |E_\phi|^2} \\ &= \frac{(|E_\theta|^2 + |E_\phi|^2)(G_\theta(\Omega_0) + G_\phi(\Omega_0))}{|E_\theta|^2 + |E_\phi|^2} \cos^2(\hat{\mathbf{p}}_r, \hat{\mathbf{p}}_t) \\ &= G(\Omega_0) \cos^2(\hat{\mathbf{p}}_r, \hat{\mathbf{p}}_t), \end{aligned} \quad (19)$$

where the unit vectors  $\hat{\mathbf{p}}_r$  and  $\hat{\mathbf{p}}_t$  are the polarisation vectors of the receiving and transmitting antennas respectively. Equation (19) states that MEG in an LOS scenario with no random field component is basically the gain of the receiving antenna (or, because of reciprocity, the transmitting one) in the direction of the LOS, times the polarisation matching efficiency. This equation can also be directly obtained from the Friis equation [1].

## 3 Physical interpretation of the MEG in Rayleigh fading

We now turn to the physical interpretation of the MEG in a Rayleigh-fading environment, that is, in the absence of an LOS component. We will focus on the polarisation

properties and show that the overall MEG depends on the polarisation discrimination of both channel and antenna. Here, it is worthwhile to remember that ‘intermixing’ of orthogonal polarisations can occur due to two reasons: (i) the channel can change the polarisation of an electromagnetic field, whereas LOS preserves the polarisation, each reflection process leads to a depolarisation of the waves and (ii) an antenna does not perfectly distinguish between orthogonal polarisations. We will show in the following that both the antenna and channel polarisation discrimination have an impact on the MEG, and that the two phenomena are duals of each other.

In a Rayleigh-fading environment, MEG is [3]

$$G_e = \int \frac{\chi}{\chi + 1} G_\theta(\Omega) p_\theta(\Omega) + \frac{1}{\chi + 1} G_\phi(\Omega) p_\phi(\Omega) d\Omega \quad (20)$$

This result also follows from Section 2 with  $K_\theta = K_\phi = 0$ . (The power of the total field as it would be measured by two ideal isotropic antennas is given by  $P_{\text{ref}} = P_\theta + P_\phi$ .)

Let us introduce the mean partial gains [14, 15], in the  $\theta$ -polarisation,  $\gamma_\theta$  and the  $\phi$ -polarisation,  $\gamma_\phi$ , respectively

$$\gamma_\theta = \int G_\theta(\Omega) p_\theta(\Omega) d\Omega, \quad \gamma_\phi = \int G_\phi(\Omega) p_\phi(\Omega) d\Omega \quad (21)$$

Further, we introduce the effective cross-polar discrimination (effective XPD) of the antenna,  $\kappa$

$$\kappa = \frac{\gamma_\theta}{\gamma_\phi} = \frac{\int G_\theta(\Omega) p_\theta(\Omega) d\Omega}{\int G_\phi(\Omega) p_\phi(\Omega) d\Omega} \quad (22)$$

and the total average gain  $\gamma_t$ , that is, the sum of partial gains of the antenna

$$\gamma_t = \gamma_\theta + \gamma_\phi \quad (23)$$

The interpretation of the mean partial gain is straightforward, it quantifies the actual mean gain for each polarisation in a multipath environment. Hence, the effective XPD is a measure of the polarisation imbalance of the antenna weighted by the channel in a multipath environment. It should be observed that the effective XPD substantially differs from the antenna XPD in the same way as MEG differs from antenna gain. Antenna XPD is evaluated at the maximum gain direction of the antenna as the ratio of the partial gains in the  $E$ - and  $H$ -planes.

By combining (20)–(23), we straightforwardly arrive at the following proposition.

**Proposition 2:** In a multipath environment characterised by uncorrelated random electromagnetic fields only, the MEG of an antenna is a symmetric function of the antenna effective XPD,  $\kappa \geq 0$ , and the channel XPR,  $\chi \geq 0$  and directly proportional to the total average gain  $\gamma_t$  of the antenna given by

$$G_e = \gamma_t \frac{\chi\kappa + 1}{(\chi + 1)(\kappa + 1)} \quad (24)$$

The physical interpretation of this proposition is that in multipath environments, MEG will evaluate any change in channel XPR in the same way as it evaluates any change in antenna effective XPD provided that the total average channel gain is kept constant. In this sense, the antenna and the channel are equivalent. Hence, (24) is a result of the ‘antenna-channel duality’.

**Proposition 3:** In a multipath environment characterised by uncorrelated random electromagnetic fields only, the MEG of an antenna is upper bounded by the largest of the partial gains of the antenna, that is

$$G_e \leq \max\{\gamma_\theta, \gamma_\phi\} \quad (25)$$

Equality  $G_e = \gamma_\phi$  holds iff  $\chi + \kappa = 0$  or  $G_e = \gamma_\theta$  iff  $1/\chi + 1/\kappa = 0$ , where  $\kappa \geq 0$  is the antenna effective XPD in the isotropic environment and  $\chi \geq 0$  is the channel XPR.

Rearranging (24) with  $\kappa \geq 0$  and  $\chi \geq 0$ , we see that

$$G_e = \gamma_t \frac{1}{1 + 1/(1/(1/\chi + 1/\kappa) + 1/(\chi + \kappa))} \leq \gamma_t$$

which straightforwardly results in (25).

The physical interpretation is that ‘perfect’ polarisation matching in multipath environments is only possible for purely polarised channels and antennas, that is, when both are vertically polarised ( $(1/\chi) + (1/\kappa) = 0$ ) or horizontally polarised ( $\chi + \kappa = 0$ ). In any other cases, there will be a polarisation mismatch loss quantified by the term  $0 \leq (\chi\kappa + 1)/(\chi + 1)(\kappa + 1) \leq 1$ . A closer inspection of this term reveals that it is an effective polarisation mismatch loss coefficient similar to that found for the deterministic case (19).

**Proposition 4:** In a multipath environment characterised by uncorrelated random electromagnetic fields only, the MEG of an antenna equals exactly half the total average gain of the antenna, that is

$$G_e = \frac{1}{2} \gamma_t \quad (26)$$

if either (i)  $\kappa = 1(\gamma_t = 2\gamma_\theta = 2\gamma_\phi)$  for all  $\chi \geq 0$  or (ii) if  $\chi = 1$  for all  $\kappa \geq 0$  and  $\gamma_t = \gamma_\theta + \gamma_\phi$ , where  $\kappa$  is the antenna effective XPD and  $\chi$  the channel XPR.

Equation (26) follows from Proposition 2.

The physical interpretation here is that if the antenna has completely balanced polarisations, the polarisation mismatch loss in multipath environments is on average always 1/2 (of the total average gain of the antenna) independently of the polarisation power balance of the incoming waves, since the antenna cannot sense the actual polarisation state. Similarly, if the channel is power balanced in the two orthogonal polarisations, the antenna has a power loss of 1/2 relative to the power of two isotropic antennas sensing the channel.

The antenna gain pattern is by definition the product of the radiation efficiency of the antenna,  $\eta_{\text{rad}}$ , and the antenna directivity pattern

$$G_\theta(\Omega) = \eta_{\text{rad}} D_\theta(\Omega), \quad G_\phi(\Omega) = \eta_{\text{rad}} D_\phi(\Omega) \quad (27)$$

In this case, the MEG can be expressed as the product of the radiation efficiency and the mean effective directivity (MED),  $D_e$  (see, e.g. [29] for further reference)

$$G_e = \eta_{\text{rad}} D_e \quad (28)$$

The MED is introduced in order to further discern between the different factors that might impact on the communication link quality. In this case, the radiation efficiency and the directivity properties of the antenna at two orthogonal polarisations are separately assessed. In practice, the radiation efficiency and the directivity of a radiating system, like for instance a mobile terminal, are interrelated in a very complex way. Obviously, for hundred percent efficient antennas, the MED is identical with the MEG.

### 3.1 $\lambda/2$ -dipole in the isotropic environment

The omnidirectional radiation pattern and high efficiency of the half-wavelength dipole antenna have made it attractive as a reference for studying the performance of handset antennas [28]. This simple, yet versatile antenna has also been used in numerous wireless communication devices, such as cellular handsets. In contrast to the isotropic antenna, the half-wavelength dipole antenna is a realistic antenna that can actually be constructed.

We define the antenna power patterns for the vertical polarisation,  $G_\theta(\theta, \phi)$ , the horizontal polarisation,  $G_\phi(\theta, \phi)$  and their sum, that is, the total gain  $G(\theta, \phi)$ . It is further assumed that the antenna is hundred percent efficient,  $\eta_{\text{rad}} = 1$ . The polarisation sensitivity changes with tilting of the antenna. Hence, the effective XPD of the two orthogonal polarisations is a function of the antenna inclination with respect to the vertical axis.

Usually, different models of the propagation channel are used in order to statistically account for the impact of the distribution of the AoA (or AoD if the uplink is

considered) at the mobile antenna position. The simplest, yet useful, model is the isotropic model (or 3D-uniform model). The isotropic model describes, as the name indicates, a scenario in which the AoAs (or angle of departure (AoDs)) are equally probable in all directions

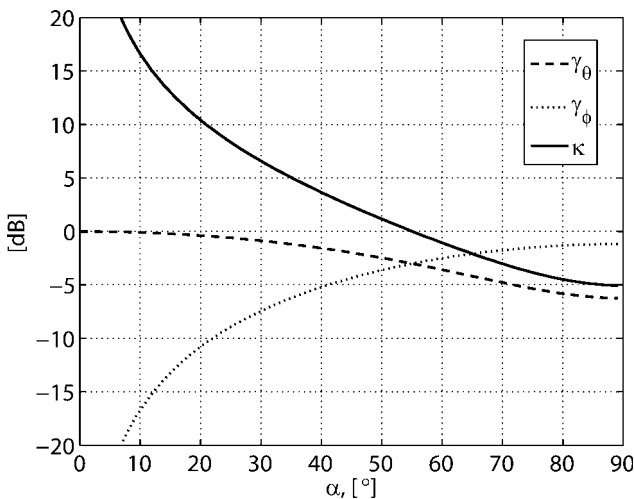
$$p_{\theta}(\theta, \phi) = p_{\phi}(\theta, \phi) = 1/4\pi \quad (29)$$

It is straightforward to show, from (21)–(24) and (29), that, in this case, the MEG is then given by

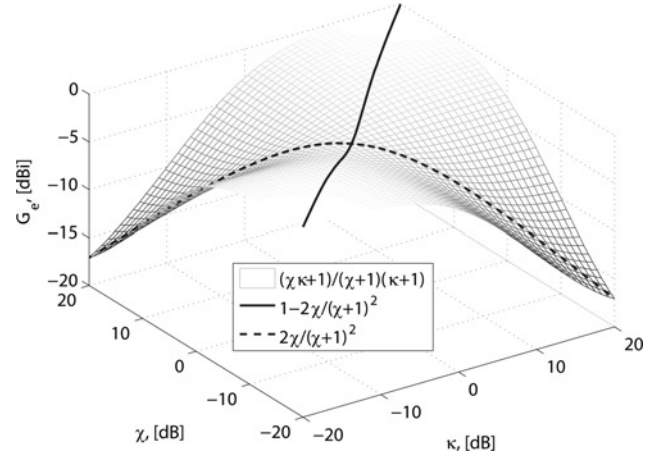
$$G_{ei} = \eta_{rad} \frac{\chi\kappa + 1}{(\chi + 1)(\kappa + 1)} \quad (30)$$

The physical meaning of (30) is again the ‘antenna-channel duality’, which was stated in Proposition 2.

In general, the effective XPD,  $\kappa$  is a function of the antenna orientation in space, that is, a tilted antenna will sense the vertical and horizontal polarisations differently depending on the tilting angle with respect to the coordinate system. For the hundred percent efficient  $\lambda/2$ -dipole in an isotropic environment, this dependence is plotted in Fig. 1. As can be seen from this figure, the effective XPD goes to infinity for a vertical dipole since no sensing is possible in the horizontal polarisation. The effective XPD further decreases monotonically with the tilt angle and changes sign at  $55^\circ$  (since the effective XPD is given in decibel in the plot). At this angle, the average partial gains in the two orthogonal polarisation are equal, that is, the effective XPD in decibel equals zero. The MEG in this case, as plotted in Fig. 2, will be constant and equal to  $-3$  dBi for all XPRs,  $\chi$ , of the channel. Another clear observation from Fig. 2 is that the MEG is always less than or equal to 0 dBi in the isotropic environment for all effective XPD and all XPR. Equality is achieved only in the limit, when both the channel and the



**Figure 1** Average partial gains and average XPD against antenna tilt angle



**Figure 2** MEG against antenna tilt angle for different values of channel XPR

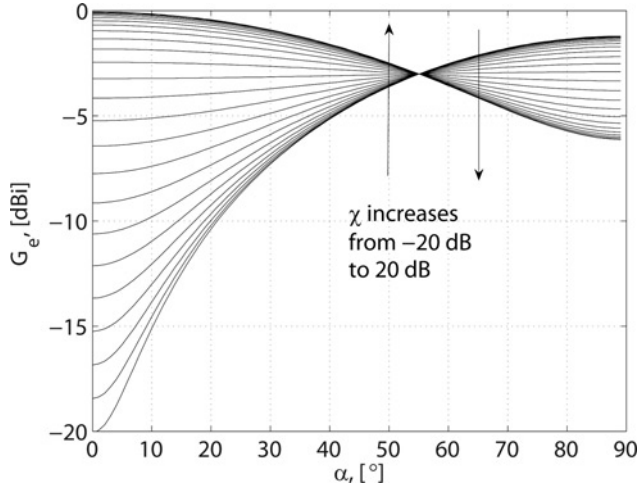
antenna are vertically polarised or when both are horizontally polarised.

Proposition 3 takes the form  $G_{ei} \leq \eta_{rad}$ , where equality is achieved if  $\chi + \kappa = 0$  or if  $1/\chi + 1/\kappa = 0$ , with the physical interpretation given above.

Proposition 4 in this case means that if either the channel or the antenna has completely balanced polarisations, the polarisation mismatch loss in multipath environments is always 1/2 independently of polarisation power balance of the other parameter, since it cannot sense the actual polarisation state. Therefore  $G_{ei} = \eta_{rad}/2$  if either  $\kappa = 1$  for all  $\chi \geq 0$  or if  $\chi = 1$  for all  $\kappa \geq 0$ , where  $\kappa$  is the antenna effective XPD and  $\chi$  is the channel XPR.

## 4 MEG variability

MEG is, as discussed, a measure of antenna performance in the channel fading, where the channel statistics and the antenna orientation are assumed to be stationary. This assumption means that the channel XPR, the AoA in both orthogonal polarisations, as well as the orientation of the antenna remain constant relative to the environment during that period of time or positions in space along the mobile path. However, in practice, this situation will seldom be observed due to the fact that the orientation of the user with respect to the incident field can change, in other words, the user can turn. Furthermore, the XPR observed in the channel can change as the mobile station moves over distances of several metres. This is clearly of paramount importance to a wireless network service provider since the performance variability will impact network dimensioning in terms of both coverage and capacity. We are therefore interested in evaluating the anticipated variability span of the MEG (Fig. 3).



**Figure 3** MEG as a function of XPR and XPD

#### 4.1 Average MEG

First, we evaluate the average (over the distribution of the antenna orientation) MEG conditioned on the channel XPR and the PAS of the AoA in both the  $\theta$ - and  $\phi$ -polarisations. Hence, we are only interested in the variations resulting from different antenna orientations in space. Models that provide the probability of usage at different tilt angles are, for example, given in [22]. However, in order to exemplify our point, we now assume that the orientation (tilt and rotation) of the antenna is uniformly distributed on the unit sphere, that is, all tilts and rotations are equiprobable. Hence, the average MEG conditioned on the channel XPR and the PAS of the AoA is given by

$$\begin{aligned} E_{\Omega'}\{G_e|\chi, p_\theta, p_\phi\} &= \int \frac{G_e(\theta', \phi')}{4\pi} d\Omega' \\ &= \frac{1}{4\pi} \int \left( \frac{\chi}{\chi+1} G_\theta(\theta, \theta', \phi, \phi') p_\theta(\theta, \phi) \right. \\ &\quad \left. + \frac{1}{\chi+1} G_\phi(\theta, \theta', \phi, \phi') p_\phi(\theta, \phi) \right) d\Omega d\Omega' \\ &= \frac{1}{4\pi} \int \frac{\chi}{\chi+1} \gamma_\theta(\theta', \phi') + \frac{1}{\chi+1} \gamma_\phi(\theta', \phi') d\Omega' \\ &= \gamma_{ta} \frac{\chi \kappa_a + 1}{(\chi+1)(\kappa_a + 1)} \end{aligned} \quad (31)$$

where

$$\kappa_a = \frac{\int \gamma_\theta(\theta', \phi') d\Omega'}{\int \gamma_\phi(\theta', \phi') d\Omega'} \quad (32)$$

and

$$\gamma_{ta} = \frac{1}{4\pi} \int \gamma_\theta(\theta', \phi') + \gamma_\phi(\theta', \phi') d\Omega' \quad (33)$$

If the channel is isotropic, then

$$\begin{aligned} \langle G_e \rangle &= \frac{1}{4\pi} \int \int \frac{\chi}{\chi+1} G_\theta(\theta, \theta', \phi, \phi') \\ &\quad + \frac{1}{\chi+1} G_\phi(\theta, \theta', \phi, \phi') d\Omega d\Omega' \\ &= \frac{\eta_{rad}}{2} \end{aligned} \quad (34)$$

where we have used the following identity

$$\begin{aligned} \frac{1}{4\pi} \int \int G_\theta(\theta, \theta', \phi, \phi') d\Omega d\Omega' \\ = \frac{1}{4\pi} \int \int G_\phi(\theta, \theta', \phi, \phi') d\Omega d\Omega' = \frac{\eta_{rad}}{2} \end{aligned} \quad (35)$$

The above computations show the following proposition

**Proposition 5:** In a multipath environment characterised by uncorrelated random electromagnetic fields only, the MEG of an antenna equals exactly half the radiation efficiency, when the AoAs are isotropically distributed and the probability that the antenna would be oriented at some angle relative to a reference coordinate system is also uniform on the unit sphere.

We now turn to averaging over the channel XPR distribution. It has been established by measurements that the channel XPR often can be modelled as a lognormal variable, [22], with probability density function

$$p_\chi(\chi) = \frac{1}{\chi \sigma_\chi \sqrt{2\pi}} e^{-(\ln(\chi) - \mu_\chi)^2 / 2\sigma_\chi^2} \quad (36)$$

Hence, the average MEG conditioned on the antenna orientation and probability density functions of the AoA  $p_\theta$  and  $p_\phi$ , is

$$\begin{aligned} E_\chi\{G_e|\Omega', p_\theta, p_\phi\} &= \langle G_e \rangle = \int p_\chi(\chi) G_e(\theta', \phi') d\chi \\ &= \int p_\chi(\chi) \left( \frac{\chi}{\chi+1} \gamma_\theta + \frac{1}{\chi+1} \gamma_\phi \right) d\chi \\ &= \gamma_\theta + (\gamma_\phi - \gamma_\theta) \int \frac{1}{\chi+1} p_\chi(\chi) d\chi \\ &= \gamma_\theta + (\gamma_\phi - \gamma_\theta) \sum_{n=1}^{\infty} (-1)^{n+1} e^{-2n\mu_\chi} \mu_\chi^n \end{aligned} \quad (37)$$

where  $\mu_\chi^n = e^{n\mu_\chi + n^2\sigma_\chi^2/2}$  is the  $n$ th moment of  $\chi$ . The computation of the integral  $\int (1/(\chi+1)) p_\chi(\chi) d\chi$  is given in the appendix.



## 4.2 Minimum, maximum, infimum and supremum MEG

In this section, we define the maximum and minimum (over different orientations in space) of the MEG of an antenna. If the antenna is tilted at an angle  $\theta'$  from the  $z$ -axis (vertical) and then rotated an angle  $\phi'$  from the  $x$ -axis, the shape of the antenna gain pattern will remain the same; however, the shape of the partial gains will change since the polarisation state of the antenna will change as the antenna is tilted, [30]. The MEG of the rotated antenna in an uncorrelated field is a function of the rotation angles  $\theta'$  and  $\phi'$

$$G_c(\theta', \phi') = \int \left( \frac{\chi}{\chi+1} G_\theta(\theta, \theta', \phi, \phi') p_\theta(\theta, \phi) + \frac{1}{\chi+1} G_\phi(\theta, \theta', \phi, \phi') p_\phi(\theta, \phi) \right) d\Omega \quad (38)$$

The maximum MEG,  $G_{eM}$ , is calculated as

$$G_{eM} = \frac{\chi \gamma_{\theta M} + \gamma_{\phi M}}{\chi + 1} \quad (39)$$

where the maximum partial gains  $\gamma_{\theta M}$  and  $\gamma_{\phi M}$  are defined as

$$\begin{aligned} \gamma_{\theta M} &= \int G_\theta(\theta, \theta_M, \phi, \phi_M) p_\theta(\theta, \phi) d\Omega \\ \gamma_{\phi M} &= \int G_\phi(\theta, \theta_M, \phi, \phi_M) p_\phi(\theta, \phi) d\Omega \end{aligned} \quad (40)$$

where

$$[\theta_M \quad \phi_M] = \arg_{\theta', \phi'} \max G_c(\theta', \phi') \quad (41)$$

In other words,  $\theta_M$  and  $\phi_M$  are the angles that maximise the MEG of the rotated antennas, as given by (38).

The minimum MEG,  $G_{em}$  is defined in a similar way as

$$G_{em} = \frac{\chi \gamma_{\theta m} + \gamma_{\phi m}}{\chi + 1} \quad (42)$$

where the minimum partial gains  $\gamma_{\theta m}$  and  $\gamma_{\phi m}$  are defined as

$$\begin{aligned} \gamma_{\theta m} &= \int G_\theta(\theta, \theta_m, \phi, \phi_m) p_\theta(\theta, \phi) d\Omega \\ \gamma_{\phi m} &= \int G_\phi(\theta, \theta_m, \phi, \phi_m) p_\phi(\theta, \phi) d\Omega \end{aligned} \quad (43)$$

where

$$[\theta_m \quad \phi_m] = \arg_{\theta', \phi'} \min G_c(\theta', \phi') \quad (44)$$

Hence, the rotation angles  $\theta_m$  and  $\phi_m$  minimise the MEG of the rotated antenna given by (38).

The minimum, the maximum and average MEG against the channel XPR are shown in Fig. 4. The depicted plots apply to the half-wavelength dipole with AoAs distributed according to the 3D-uniform probability density distribution (29). Clearly, when the XPR equals 0 dB, MEG equals -3 dB for all the dipole orientations. Hence, the variability of the link is minimised.

Another interesting result is obtained by defining the infimum and supremum MEG. Namely, for directive antennas, these two magnitudes can serve as a 'quick and dirty' estimate of the variability of MEG which is independent from the PAS but still takes the channel XPR into account.

We will show that this supremum MEG bounds the maximum MEG from above.

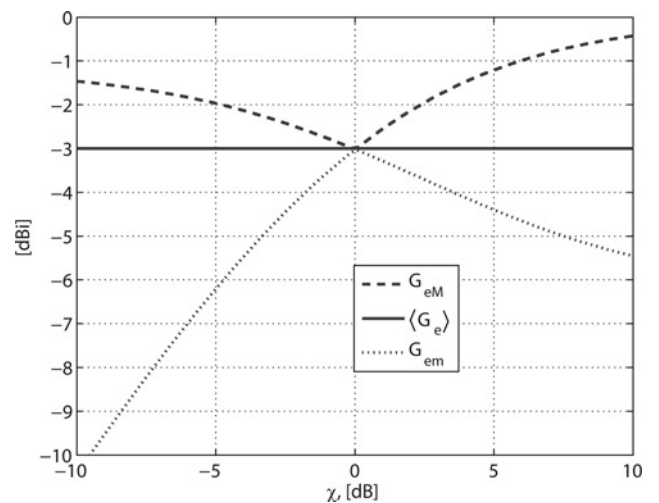
Consider the MEG equation

$$G_c = \int \frac{\chi}{\chi+1} G_\theta(\Omega) p_\theta(\Omega) + \frac{1}{\chi+1} G_\phi(\Omega) p_\phi(\Omega) d\Omega$$

Now, since  $G_\theta(\Omega)$ ,  $p_\theta(\Omega)$ ,  $G_\phi(\Omega)$  and  $p_\phi(\Omega)$  are all non-negative over the sphere of unit radius, it is valid to write

$$\begin{aligned} & \int \frac{\chi}{\chi+1} G_\theta(\Omega) p_\theta(\Omega) + \frac{1}{\chi+1} G_\phi(\Omega) p_\phi(\Omega) d\Omega \\ & \leq \sup G_\theta(\Omega) \frac{\chi}{\chi+1} \int p_\theta(\Omega) d\Omega \\ & \quad + \sup G_\phi(\Omega) \frac{1}{\chi+1} \int p_\phi(\Omega) d\Omega \\ & = \frac{\chi}{\chi+1} \sup G_\theta(\Omega) + \frac{1}{\chi+1} \sup G_\phi(\Omega) \end{aligned} \quad (45)$$

By using similar arguments for the infimum of the partial gain and for the minimum and the supremum of MEG



**Figure 4** Average, minimum and maximum MEG as a function of XPD

given in Proposition 3, the following inequality is valid

$$\begin{aligned}
 & \min \{ \inf G_\theta(\Omega), \inf G_\phi(\Omega) \} \\
 & \leq G_{e\inf} \\
 & = \frac{\chi}{\chi+1} \inf G_\theta(\Omega) + \frac{1}{\chi+1} \inf G_\phi(\Omega) \\
 & \leq G_e \\
 & \leq \frac{\chi}{\chi+1} \sup G_\theta(\Omega) + \frac{1}{\chi+1} \sup G_\phi(\Omega) \\
 & = G_{e\sup} \leq \max \{ \sup G_\theta(\Omega), \sup G_\phi(\Omega) \} \quad (46)
 \end{aligned}$$

We now establish the following 'MEG inequalities'

**Proposition 6:** The MEG of an antenna satisfies the following inequalities

$$\begin{aligned}
 \text{(I)} \quad & \min \{ \inf G_\theta(\Omega), \inf G_\phi(\Omega) \} \\
 & \leq \min \{ \gamma_\theta, \gamma_\phi \} \\
 & \leq G_e \\
 & \leq \max \{ \gamma_\theta, \gamma_\phi \} \\
 & \leq \max \{ \sup G_\theta(\Omega), \sup G_\phi(\Omega) \} \quad (47)
 \end{aligned}$$

$$\begin{aligned}
 \text{(II)} \quad & \min \{ \inf G_\theta(\Omega), \inf G_\phi(\Omega) \} \\
 & \leq G_{em} \leq G_e \leq G_{eM} \\
 & \leq \max \{ \sup G_\theta(\Omega), \sup G_\phi(\Omega) \} \quad (48)
 \end{aligned}$$

The inequality (I) follows from  $\inf G_\theta(\Omega) \leq \gamma_\theta \leq \sup G_\theta(\Omega)$  and  $\inf G_\phi(\Omega) \leq \gamma_\phi \leq \sup G_\phi(\Omega)$ . The inequality (II) follows from  $G_{em} \geq \min \{ \gamma_{\theta m}, \gamma_{\phi m} \} \geq \min \{ \inf G_\theta(\Omega), \inf G_\phi(\Omega) \}$  and  $G_{eM} \leq \max \{ \gamma_{\theta M}, \gamma_{\phi M} \} \leq \max \{ \sup G_\theta(\Omega), \sup G_\phi(\Omega) \}$ .

The physical interpretation is straightforward: the MEG of the antenna is always bounded by the infimum (the smallest) of the partial antenna gains and the supremum (the largest) of the partial antenna gains, that is, when the AoA of a single plane wave coincides with the direction of the smallest and the largest partial antenna gain, respectively. Equality is achieved in the LOS scenario with only one deterministic wave impinging at the antenna.

### 4.3 Maximum effective gain

In the previous sections, we were interested in analysing the MEG of an antenna in a given propagation environment. The total power received by the antenna was compared with the total available power averaged over the small-scale fading. In this sense, we were in fact estimating the mean effective performance (gain) of the antenna. However, from the communication point of view, it is also relevant to evaluate the maximum link quality, or more precisely, the optimum total power received by the antenna in a random field. We will show below that the maximum effective gain

is achieved when channel knowledge is available and the antenna can be adapted to the incident field (e.g. beamforming), that is, the maximum is obtained when the antenna far-field equals the conjugate of the complex amplitudes of the incident waves. This means that both the polarisation, the DoAs of incoming waves and the mobile speed must be known to the receiver in order to maximise the link gain, that is, the received power. The gain defined now refers to an instantaneous effective gain from which an average or 'mean maximum effective gain' (MMEG) can still be inferred.

**Proposition 7:** In a multipath environment characterised by uncorrelated random electromagnetic fields only, the maximum effective gain of an antenna is a symmetric function of the antenna effective XPD,  $\kappa \geq 0$  and the instantaneous channel XPR in the isotropic environment,  $\chi_i \geq 0$  and directly proportional to the radiation efficiency  $\eta_{\text{rad}}$  of the antenna and is independent from the PAS of the incoming waves, that is

$$G_o = 4\pi\eta_{\text{rad}} \frac{(\sqrt{\chi_i\kappa} + 1)^2}{(\chi_i + 1)(\kappa + 1)} \quad (49)$$

Proposition 7 is derived in Appendix 2.

It should be observed that (49) is an instantaneous effective gain in a multipath environment and therefore a stochastic parameter that depends on the short-term fading statistics (small-scale fading) through the instantaneous channel XPR  $\chi_i$ . On the other hand, the MEG (24) depends on the long-term statistics (large-scale fading or shadowing) through the channel XPR  $\chi$ .

**Proposition 8:** In a multipath environment characterised by uncorrelated random electromagnetic fields only, the maximum effective gain of an antenna is upper bounded by the area of the unit sphere times the radiation efficiency  $\eta_{\text{rad}}$ , that is

$$G_o \leq 4\pi\eta_{\text{rad}} \quad (50)$$

Equality is achieved iff  $\chi_i = \kappa$ , where  $\kappa \geq 0$  is the antenna effective XPD in the isotropic environment and  $\chi_i \geq 0$  is the channel XPR.

Proposition 8 clearly follows by means of the first and second derivative tests of the maximum effective gain (49) relative to the antenna effective XPD  $\kappa \geq 0$  for fixed channel XPR  $\chi \geq 0$ .

The physical interpretation is that in the 'maximum effective regime' (that is beamforming), 'perfect' polarisation matching in multipath environments is achieved if and only if the effective XPD of the antenna equals the XPR of the channel, and not only for purely vertically or horizontally polarised channels and antennas as in the case of MEG. In all other cases, there will be a

polarisation mismatch loss quantified by the term  $(\sqrt{\chi_i \kappa} + 1)^2 / (\chi_i + 1)(\kappa + 1)$ . Furthermore, in this case, the maximum effective gain equals the integral of the total gain over the unit sphere

$$\max\{G_o\} = 4\pi\eta_{\text{rad}} = \int G_\theta(\Omega) + G_\phi(\Omega) d\Omega \quad (51)$$

**Proposition 9:** In a multipath environment characterised by uncorrelated random electromagnetic fields only, the maximum MEG of an antenna is bounded by

$$\begin{aligned} 16\pi\eta_{\text{rad}} \frac{\sqrt{\chi\kappa}}{(\chi+1)(\kappa+1)} &\leq G_{\text{co}} \\ &\leq 8\pi\eta_{\text{rad}} \frac{\chi\kappa+1}{(\chi+1)(\kappa+1)} \end{aligned} \quad (52)$$

where  $\kappa \geq 0$  is the instantaneous channel XPR in the isotropic environment,  $\chi_i \geq 0$  and directly proportional to the radiation efficiency  $\eta_{\text{rad}}$  of the antenna and is independent from the PAS of the incoming waves. Equality is achieved iff  $\chi = \kappa = 1$ .

Proposition 9 is derived in Appendix 3.

## 5 Summary

In this paper, fundamental properties of the MEG of antennas were presented. The MEG is a measure of the interplay of the antenna with the propagation channel. Therefore the results of this paper are of value when assessing the in-network performance of wireless handsets. New closed-form formulae for the MEG in mixed fields, that is, fields with both stochastic and the deterministic components, are provided with corresponding physical interpretation. We showed that the MEG in uncorrelated random fields with deterministic components can be expressed as the sum of two terms, each denoting the contribution of each component to the MEG. We then showed that the MEG computed by Taga, that is, MEG in uncorrelated fields, is a special case of the mixed fields case. In the uncorrelated case, MEG is a symmetric function in the channel XPR (XPR,  $\chi$ ) and the antenna effective XPD (effective XPD,  $\kappa$ ), which is an expression of the channel/antenna duality or equivalence under these conditions. We showed further that the MEG in uncorrelated random fields is upper bounded by the largest of the two average partial gains in  $\theta$ - and  $\phi$ -polarisations. We also showed that when either the channel or the antenna is power-balanced in polarisation, that is,  $\chi = 1$  or  $\kappa = 1$ , the MEG is always one-half of the radiation efficiency. We defined and analysed the infimum, minimum, average, maximum and supremum MEG with the objective of characterise the span of variability of MEG as a function of the antenna orientation in space and the long-term statistics of the channel variability that affect the XPR. We showed that in an environment characterised by uncorrelated random fields the average over both the XPR

and the antenna orientation equals the half of the radiation efficiency of the antenna. We proved the MEG inequalities that showed the lower and upper bounds of MEG, that is, the span of variation of MEG. Finally, we showed that the maximum effective gain is achieved with 'beamforming' and equals  $4\pi\eta_{\text{rad}}$ , where  $\eta_{\text{rad}}$  is the radiation efficiency of the antenna, when  $\kappa = \chi_i$ , where  $\kappa$  is the instantaneous XPR of the channel. We also provided bounds for the average of the maximum effective gain and showed that the bound is achieved.

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## 7 Appendix 1: computation of an integral

By making use of the McLaurin series expansion

$$\frac{1}{1+f(x)} = \sum_{n=0}^{\infty} (-1)^n f(x)^n$$

where  $|f(x)| < 1$ , the integral is obtained as follows

$$\begin{aligned} & \int \frac{1}{\chi+1} p_{\chi}(\chi) d\chi \\ &= \frac{1}{\sigma_{\chi} \sqrt{2\pi}} \int_0^{\infty} \frac{1}{(\chi+1)\chi} e^{-(\ln(\chi)-\mu_{\chi})^2/2\sigma_{\chi}^2} d\chi \\ &= \frac{1}{\sigma_{\chi} \sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{-t}}{1+e^{-t}} e^{-(t-\mu_{\chi})^2/2\sigma_{\chi}^2} dt \\ &= \frac{1}{\sigma_{\chi} \sqrt{2\pi}} \sum_{n=0}^{\infty} (-1)^n \int_{-\infty}^{\infty} e^{-(n+1)t} e^{-(t-\mu_{\chi})^2/2\sigma_{\chi}^2} dt \end{aligned}$$



$$= \sum_{n=1}^{\infty} (-1)^{n+1} e^{-n\mu_{\chi} + n^2 \sigma_{\chi}^2 / 2}$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} e^{-2n\mu_{\chi}} \mu_{\chi}^n$$

where  $\mu_{\chi}^n = e^{n\mu_{\chi} + n^2 \sigma_{\chi}^2 / 2}$  is the  $n$ th moment of  $\chi$ .

## Appendix 2: proof of proposition 7

Consider (3). Let us compute the signal power

$$|V_{oc}(t)|^2 = \left| \int F_r(\Omega) \cdot E_i(\Omega) e^{-i(2\pi/\lambda)u \cdot e_r(\Omega)t} d\Omega \right|^2$$

By the triangle inequality

$$|V_{oc}(t)|^2 \leq \left( \int |F_r(\Omega) \cdot E_i(\Omega)| d\Omega \right)^2$$

By the triangle inequality

$$|V_{oc}(t)|^2 \leq \left( \int |F_{r\theta}(\Omega) E_{i\theta}(\Omega)| d\Omega + \int |F_{r\phi}(\Omega) E_{i\phi}(\Omega)| d\Omega \right)^2$$

By the Cauchy–Schwartz–Buniakowsky inequality and observing that equality is achieved for  $F_{r\theta}(\Omega) = c_{\theta} E_{i\theta}^*(\Omega) e^{i(2\pi/\lambda)u \cdot e_r(\Omega)t}$  and  $F_{r\phi}(\Omega) = c_{\phi} E_{i\phi}^*(\Omega) e^{i(2\pi/\lambda)u \cdot e_r(\Omega)t}$

$$|V_{oc}(t)|^2 \leq \left( \sqrt{\int |F_{r\theta}(\Omega)|^2 d\Omega} \sqrt{\int |E_{i\theta}(\Omega)|^2 d\Omega} \right. \\ \left. + \sqrt{\int |F_{r\phi}(\Omega)|^2 d\Omega} \sqrt{\int |E_{i\phi}(\Omega)|^2 d\Omega} \right)^2$$

$$= \left( \sqrt{\int G_{\theta}(\Omega) d\Omega} \sqrt{\int |E_{i\theta}(\Omega)|^2 d\Omega} \right. \\ \left. + \sqrt{\int G_{\phi}(\Omega) d\Omega} \sqrt{\int |E_{i\phi}(\Omega)|^2 d\Omega} \right)^2$$

$$= \left( \sqrt{\gamma_{\theta}} \sqrt{\int |E_{i\theta}(\Omega)|^2 d\Omega} + \sqrt{\gamma_{\phi}} \sqrt{\int |E_{i\phi}(\Omega)|^2 d\Omega} \right)^2$$

which gives us the maximum received signal  $|V_{oc}(t)|_{opt}^2$  in the ‘beamforming’ sense since for each time  $t$  the far-field amplitude must satisfy the following conditions

$$F_{r\theta}(\Omega, t) = E_{i\theta}^*(\Omega) e^{i(2\pi/\lambda)u \cdot e_r(\Omega)t}$$

$$F_{r\phi}(\Omega, t) = E_{i\phi}^*(\Omega) e^{i(2\pi/\lambda)u \cdot e_r(\Omega)t}$$

Under this condition, the maximum effective gain is defined relative to the instantaneous available power of the

electromagnetic field

$$G_o = \frac{\left( \sqrt{\gamma_{\theta}} \sqrt{\int |E_{i\theta}(\Omega)|^2 d\Omega} + \sqrt{\gamma_{\phi}} \sqrt{\int |E_{i\phi}(\Omega)|^2 d\Omega} \right)^2}{\int |E_{i\theta}(\Omega)|^2 d\Omega + \int |E_{i\phi}(\Omega)|^2 d\Omega}$$

$$= \frac{(\sqrt{\gamma_{\theta} P_{i\theta}} + \sqrt{\gamma_{\phi} P_{i\phi}})^2}{P_{i\theta} + P_{i\phi}}$$

$$= 4\pi\eta_{rad} \frac{(\sqrt{\chi_i \kappa} + 1)^2}{(\chi_i + 1)(\kappa + 1)}$$

where  $P_{i\theta} = (1/2) \int |E_{i\theta}(\Omega)|^2 d\Omega$ ,  $P_{i\phi} = (1/2) \int |E_{i\phi}(\Omega)|^2 d\Omega$  and  $\chi_i = P_{i\theta}/P_{i\phi}$ , where  $E_{i\theta}(\Omega)$  and  $E_{i\phi}(\Omega)$  are the instantaneous complex amplitudes of the random electromagnetic field incident at the antenna and the instantaneous XPR of the channel.

## Appendix 3. proof of proposition 9

The upper bound on the maximum MEG can be derived from

$$\langle |V_{oc}(t)|_{opt}^2 \rangle = \left\langle \left( \sqrt{\gamma_{\theta}} \sqrt{\int |E_{i\theta}(\Omega)|^2 d\Omega} \right. \right. \\ \left. \left. + \sqrt{\gamma_{\phi}} \sqrt{\int |E_{i\phi}(\Omega)|^2 d\Omega} \right)^2 \right\rangle$$

$$= \left\langle \gamma_{\theta} \int |E_{i\theta}(\Omega)|^2 d\Omega + \gamma_{\phi} \int |E_{i\phi}(\Omega)|^2 d\Omega \right. \\ \left. + 2\sqrt{\gamma_{\theta}\gamma_{\phi}} \sqrt{\int |E_{i\theta}(\Omega)|^2 d\Omega \int |E_{i\phi}(\Omega)|^2 d\Omega} \right\rangle$$

By Cauchy’s mean theorem (arithmetic mean-geometric mean inequality)

$$\langle |V_{oc}(t)|_{opt}^2 \rangle \leq \left\langle \gamma_{\theta} \int |E_{i\theta}(\Omega)|^2 d\Omega + \gamma_{\phi} \int |E_{i\theta}(\Omega)|^2 d\Omega \right. \\ \left. + \gamma_{\phi} \int |E_{i\phi}(\Omega)|^2 d\Omega + \gamma_{\theta} \int |E_{i\phi}(\Omega)|^2 d\Omega \right\rangle$$

By the Jensen’s inequality for convex functions

$$\leq 2 \left( \gamma_{\theta} \int \langle |E_{i\theta}(\Omega)|^2 \rangle d\Omega + \gamma_{\phi} \int \langle |E_{i\phi}(\Omega)|^2 \rangle d\Omega \right)$$

$$= 4 \left( \gamma_{\theta} P_{\theta} \int p_{\theta}(\Omega) d\Omega + \gamma_{\phi} P_{\phi} \int p_{\phi}(\Omega) d\Omega \right)$$

$$= 4(\gamma_{\theta} P_{\theta} + \gamma_{\phi} P_{\phi})$$

and therefore the upper bound on the maximum effective gain is given by

$$G_{eo} \leq \frac{2(\gamma_\theta P_\theta + \gamma_\phi P_\phi)}{P_\theta + P_\phi} = 8\pi\eta_{rad} \frac{\chi\kappa + 1}{(\chi + 1)(\kappa + 1)}$$

Observe that for  $\chi = \kappa = 1$ ,  $G_{eo} \leq \max\{G_o\}$ .

The lower bound on the maximum MEG can now be derived from

$$\langle |V_{oc}(t)|_{opt}^2 \rangle = \left\langle \left( \sqrt{\gamma_\theta} \sqrt{\int |E_{i\theta}(\Omega)|^2 d\Omega} + \sqrt{\gamma_\phi} \sqrt{\int |E_{i\phi}(\Omega)|^2 d\Omega} \right)^2 \right\rangle$$

By the Jensen's inequality

$$\langle |V_{oc}(t)|_{opt}^2 \rangle \geq 4\sqrt{\gamma_\theta \gamma_\phi} \sqrt{\int |E_{i\theta}(\Omega)|^2 d\Omega \int |E_{i\phi}(\Omega)|^2 d\Omega}$$

By the Jensen's inequality for concave functions

$$\begin{aligned} \langle |V_{oc}(t)|_{opt}^2 \rangle &\geq 4\sqrt{\gamma_\theta \gamma_\phi} \sqrt{\int |E_{i\theta}(\Omega)|^2 d\Omega \int |E_{i\phi}(\Omega)|^2 d\Omega} \\ &= 8\sqrt{\gamma_\theta \gamma_\phi} \sqrt{P_\theta \int p_\theta(\Omega) d\Omega P_\phi \int p_\phi(\Omega) d\Omega} \\ &= 8\sqrt{\gamma_\theta \gamma_\phi P_\theta P_\phi} \end{aligned}$$

Hence

$$G_{eo} \geq \frac{4\sqrt{\gamma_\theta \gamma_\phi P_\theta P_\phi}}{P_\theta + P_\phi} = 16\pi\eta_{rad} \frac{\sqrt{\chi\kappa}}{(\chi + 1)(\kappa + 1)}$$

Observe that for  $\chi = \kappa = 1$ ,  $G_{eo} \geq \max\{G_o\}$ .