# 前置代码

```
namespace pre{
  using namespace std;
 #define io ios::sync_with_stdio(false);cin.tie(0);
  typedef long long 11;
  typedef long double ld;
 const int inf=(1<<30);</pre>
 const long double eps=1e-8;
 const long double pi=acos((long double)-1.0);
 typedef long long 11;
 const 11 mod=1e9+7;
 const 11 mod2=998244353;
 const 11 mod3=1004535809;
 const int maxn=1e6+5;
 const ll linf=1LL<<62;</pre>
 inline bool _{zero(double x)\{return (((x)>0?(x):-(x))\leq ps);\}}
 inline double _sq(double x){return x*x;}
  inline ll sqr(ll a){return a*a;}
 inline int _sign(double x){return ((x)>eps?1:((x)<-eps?2:0));}</pre>
}
```

## 点

note: 旋转的函数需要注意精度问题

```
using namespace pre;
  struct Point(//point(x,y)
      double x,y;
      Point():x(0),y(0){}
      Point(double x,double y): x(x),y(y) {}
      friend istream& operator >>(istream &in, Point &p) {
      in>>p.x>>p.y;
      return in;
    }
    friend ostream& operator <<(ostream &out,const Point &p){
      out<<p.x<<" "<<p.y;
      return out;
    }
      Point operator + (Point b) const {return Point(x+b.x,y+b.y);}
      Point operator - (Point b) const {return Point(x-b.x,y-b.y);}
      Point operator * (double b) const {return Point(b*x,b*y);}
      double operator * (Point b) const {return x*b.y-y*b.x;} //叉乘
      double operator & (Point b) const {return x*b.x+y*b.y;} //点乘
```

```
double distance() const {return sqrt( sq(x)+ sq(y));}
     Point unit() const {return Point(x/distance(),y/distance());}
     bool operator < (const Point &b) const{//极角排序, [-pi,pi)开始逆时针排序
     return (*this)*b>eps | zero((*this)*b) && distance()<b.distance();
    }
   static bool comp(const Point &a,const Point &b){//坐标排序,下到上,左到右
     return a.y<b.y | a.y==b.y && a.x<b.x;
     double distance(Point p) const {
     return (*(this)-p).distance();
     double distance(Point p1,Point p2) const {
     return (p2-p1).distance();
     double xmult(Point p1,Point p2) const {//向量this->p1叉乘this->p2
     return (p1-(*this))*(p2-(*this));
     double dmult(Point p1, Point p2) const {//向量this->p1点乘this->p2
     return (p1-(*this))&(p2-(*this));
     bool dots_inline(Point p1,Point p2) const{//三点共线
         return zero(xmult(p1,p2));
    }
     Point rotate(Point p, double angle, double scale) { //p绕逆时针旋转angle, 放大或缩小至
scale
         Point ret=p;
         x=p.x, y=p.y;
         p.x=scale*cos(angle);
         p.y=scale*sin(angle);
         ret.x+=x*p.x-y*p.y;
         ret.y+=x*p.y+y*p.x;
         return ret;
     }
     double rag(Point a, Point b)const{//a->this->b夹角
     return acos((_sq(distance(a))+_sq(distance(b))-
_sq(a.distance(b)))/(2*distance(a)*distance(b)));
   }
   Point pedal() const{ //顺时针转90度
     return Point(y,-x);
    }
  };
```

#### 极角定义

```
struct Polar//polar(radius, theta)
{
    double radius, theta;
    Polar():radius(0), theta(0){}
```

```
Polar(double theta):radius(1),theta(theta){}
     Polar(double radius, double theta):radius(radius), theta(theta){}
     Polar(Point p):radius(p.distance()),theta(atan2(p.y,p.x)){}
     friend istream& operator >>(istream &in,Polar &p){
     in>>p.radius>>p.theta;
     return in;
    }
   friend ostream& operator <<(ostream &out,const Polar &p){
     out<<p.radius<<" "<<p.theta;</pre>
     return out;
    }
     bool operator < (const Polar &b)const{return theta<b.theta | theta==b.theta &&
radius<b.radius;}
     Point topoint(){ //和x正半轴夹角, (0,pi)在y>0区域, (-pi,0)在y<0区域
     return Point(radius*cos(theta),radius*sin(theta));
   }
     Polar unit(){ //unit circle
     return Polar(theta);
   }
  };
```

## 线

```
struct Line{//line
      Point a,b;
      Line(): a(Point()),b(Point()){}
      Line(Point a, Point b): a(a),b(b) {}
      friend istream& operator >>(istream &in,Line &l){
      in>>1.a>>1.b;
      return in;
    }
    friend ostream& operator <<(ostream &out,const Line &1){
      out<<l.a<<" "<<l.b;
     return out;
      Point midpoint() const {//mid point
      return (a+b)*0.5;
      double distance() const {//length
      return a.distance(b);
      bool dot_inline(Point p) {//if point in line
      return zero(p.xmult(a,b));
    }
```

```
bool dot online(Point p) {//if point in segment
     return zero(p.xmult(a,b)) \& (a.x-p.x)*(b.x-p.x) < eps & (a.y-p.y)*(b.y-p.y) < eps;
   }
     bool dot online in(Point p){//if point in segment except two point
     return dot_online(p)&&(!_zero(p.x-a.x)||!_zero(p.y-a.y))&&(!_zero(p.x-
b.x) | ! zero(p.y-b.y));
   }
  //提前判断点在线上
     bool same side (Point p1, Point p2) {//if two points on same side except line
     return a.xmult(b,p1)*a.xmult(b,p2)>eps;
     bool opposite_side(Point p1,Point p2){//if two points on opposite side except
line
     return a.xmult(b,p1)*a.xmult(b,p2)<-eps;</pre>
    }
     bool parallel(Line v){//if two lines parallel
     return _zero(Point().xmult(a-b,v.a-v.b));
   }
   bool parallel_ex(Line v){//if two lines parallel and not coincidence 不重合
     return zero(Point().xmult(a-b,v.a-v.b))&&!dot inline(v.a);
   }
     bool perpendicular(Line v){//if two lines perpendicular 两线垂直
     return _zero(Point().dmult(a-b,v.a-v.b));
   }
     bool intersect(Line v){//if two segments have same points 线段是否相交
          return !same side(v.a,v.b)&&!v.same side(a,b);
      }
     bool intersect ex(Line v){//if two segments have only same point except end
points
     return opposite_side(v.a,v.b)&&v.opposite_side(a,b);
     Point intersection(Line v){//calculate the intersection of two lines, please
notice they are not parallel
          Point ret=a;
          double t=((a.x-v.a.x)*(v.a.y-v.b.y)-(a.y-v.a.y)*(v.a.x-v.b.x))
                  /((a.x-b.x)*(v.a.y-v.b.y)-(a.y-b.y)*(v.a.x-v.b.x));
         return a+(b-a)*t;
      }
     Point pedal(Point p){//calculate the nearest point to the line, pedal 直线到点最短距
离
          Point t=p;
          t.x+=a.y-b.y,t.y+=b.x-a.x;
          if(dot inline(p)) return p;
          return intersection(Line(p,t));
      }
     double distance(Point p){//calculate the distance from point to line
      return fabs(b.xmult(p,a))/distance();
```

```
Point ptoseg(Point p){//calculate the nearest point to the segment
                                                                             线段到点最短距
离
          Point t=p;
          t.x+=a.y-b.y,t.y+=b.x-a.x;
          if (p.xmult(a,t)*p.xmult(b,t)>eps)
              return a.distance(p) < b.distance(p)?a:b;</pre>
          return pedal(p);
      double disptoseg(Point p){//calculate the distance from point to segment 点到线段
距离
          Point t=p;
          t.x+=a.y-b.y,t.y+=b.x-a.x;
          if (p.xmult(a,t)*p.xmult(b,t)>eps)
              return Line(p,a).distance()<Line(p,b).distance()?</pre>
Line(p,a).distance():Line(p,b).distance();
          return fabs(b.xmult(p,a))/distance();
      Line midperpendicular(){
                                  //中垂线
        return Line(midpoint(), midpoint()+(b-a).pedal());
   Point definite point(double rate) { //a往b方向rate倍的点
      Point v=b-a;
      return a+v*rate;
  };
```

#### 一般式表示

```
struct General line// ax+by+c=0 for general line expression exp
  {
      double a,b,c;
      General_line():a(0),b(0),c(0){}
      General_line(double a,double b,double c):a(a),b(b),c(c){}
      General line(Line 1):a((1.a-1.b).y),b(-(1.a-1.b).x),c(-a*1.a.x-b*1.a.y){}
      friend istream& operator >>(istream &in, General line &1){
      in>>1.a>>1.b>>1.c;
      return in;
    friend ostream& operator <<(ostream &out,const General_line &1){
      out<<l.a<<" "<<l.b<<" "<<l.c;
      return out;
  //点到直线对称点
      Point symmetricalPointofLine(Point p){//calculate symmetric point of the line
          double d=_sq(a)+_sq(b);
          return Point((\_sq(b)*p.x-\_sq(a)*p.x-2*a*b*p.y-2*a*c)/d,(\_sq(a)*p.y-2*a*b*p.y-2*a*c)/d)
_sq(b)*p.y-2*a*b*p.x-2*b*c)/d);
```

```
};
```



```
struct Circle(/circle(point, radius)
      Point center;
      double radius;
      Circle():center(Point()),radius(1){}
      Circle(Point center, double radius):center(center), radius(radius){}
      Circle(Line 1): center(1.midpoint()),radius(1.distance()/2){}//diameter for a
circle
      friend istream& operator >>(istream &in,Circle &c){
      in>>c.center>>c.radius;
      return in;
    }
    friend ostream& operator <<(ostream &out,const Circle &c){
      out << c.center << " " << c.radius;
      return out;
    }
      Circle(Point p1,Point p2,Point p3){//three points for a circle
        Line u, v;
          u=Line(p1,p2).midperpendicular();
          v=Line(p1,p3).midperpendicular();
      center=u.intersection(v);
      radius=center.distance(p1);
    }
      double area() const{//the area
      return pi* sq(radius);
      double perimeter(){//the perimeter 周长
        return 2.0*pi*radius;
      bool inside circle(Point p){//if point in circle
      return center.distance(p)<radius+eps;</pre>
      bool intersect(Line 1) {//if line and circle have intersection
      return l.distance(center)<radius+eps;</pre>
    }
      bool intersect_ex(Line 1) {//if segment and circle have intersection
          double t1=center.distance(1.a)-radius,t2=center.distance(1.b)-radius;
          Point t=center;
          if (t1<eps||t2<eps)
              return t1>-eps | |t2>-eps;
```

```
t.x+=1.a.y-1.b.y;
          t.y+=1.b.x-1.a.x;
          return t.xmult(1.a,center)*t.xmult(1.b,center)<eps&&l.distance(center)-
radius<eps;
      }
     bool intersect(Circle c2){//if two circles have intersection
     return center.distance(c2.center)
<radius+c2.radius+eps&&center.distance(c2.center)>fabs(radius-c2.radius)-eps;
     int circleposition(Circle c2){    //relative location of two circles
          double d=center.distance(c2.center),rs=radius+c2.radius,rd=radius-c2.radius;
          if ( zero(d)&& zero(rd)) return -2;//coincidence 重合
          if (_zero(rs-d)) return 4;//circumscribed
          if ( zero(rd-d)) return 3;//inscribe and c2 in c
                                                              两种内切
          if ( zero(rd+d)) return -3;//inscribe and c in c2
          if (d>rs) return 0;//separation of two circles from outside 外离
          if (intersect(c2)) return 2;//circle intersection
          if (rd>0) return 1;//separation of two circles and c2 in c 两种内离
          return -1; //separation of two circles and c in c2
     }
   double intersection_area(Circle c2){
                                            //圆交面积
     double dis=center.distance(c2.center);
      if(radius+c2.radius<=dis)
            return 0.0;
        if(radius-c2.radius>=dis)
            return c2.area();
        if(c2.radius-radius>=dis)
            return area();
        double angle1 =acos((_sq(radius)+_sq(dis)-_sq(c2.radius))/(2*dis*radius));
        double angle2 =acos(( sq(c2.radius)+ sq(dis)- sq(radius))/(2*dis*c2.radius));
        return _sq(radius)*angle1+_sq(c2.radius)*angle2-sin(angle1)*radius*dis;
   Point intersection(Circle c2){//use if have intersection and return one of it
     double dis=center.distance(c2.center);
     Point p1=(center+c2.center)*0.5,p2=(c2.center-center)*0.5;
     double i1=( sq(radius) - sq(c2.radius))/( sq(dis)),i2=sqrt(2*
(\_sq(radius)+\_sq(c2.radius))/(\_sq(dis))-\_sq(i1)-1);
      //另外一点为p1+p2*i1-Point(p2.y,p2.x)*i2, 可以配合circlepostion使用
     return p1+p2*i1+Point(p2.y,p2.x)*i2;
    }
    double distance(Point p){
     double dis=p.distance(center)-radius;
     if(dis<0) return 0.0;
     else return dis;
    }
   double distance(Line 1){
     double dis=1.distance(center)-radius;
     if(dis<0) return 0.0;
      else return dis;
```

```
double disptoseg(Line 1) {
   double dis=1.disptoseg(center)-radius;
   if(dis<0) return 0.0;
   else return dis;
}
</pre>
```

# 三角形

```
struct Triangle{//triangle
                 vector<Point> p;
                 Triangle(){p.resize(3);}
                 Triangle(Point a, Point b, Point c) {p.resize(3);p[0]=a;p[1]=b;p[2]=c;}
                 double area()//the area
                  {
                             return fabs(p[0].xmult(p[1],p[2]))/2;
                 Point perpencenter()//the perpencenter, orthocenter of a triangle 垂心, 三条高的交点
                             Line u, v;
                             u.a=p[2];
                             u.b=Line(p[0],p[1]).pedal(p[2]);
                             v.a=p[1];
                             v.b=Line(p[0],p[2]).pedal(p[1]);
                             return u.intersection(v);
                 }
            //重心, 三条中线交点
                 Point barycenter()//the barycenter of triangle, it's the point of lowest sum of
square of distance to three points, it's the point of largest product of square of
distance to three points in triangle
                  {
                             Line u, v;
                             u.a=Line(p[0],p[1]).midpoint();
                             u.b=p[2];
                             v.a=Line(p[0],p[2]).midpoint();
                             v.b=p[1];
                             return u.intersection(v);
                  }
                 Point circumcenter()//excenter of a triangle 外心, 三条中垂线的交点, 外接圆圆心
                  {
                             Point tmp;
                 tmp.x=p[0].x+(0.5*(\_sq(p[1].x-p[0].x)+\_sq(p[1].y-p[0].y))*(p[2].y-p[0].y)-0.5*
(\_sq(p[2].x-p[0].x) + \_sq(p[2].y-p[0].y)) * (p[1].y-p[0].y)) / ((p[1].x-p[0].x) * (p[2].y-p[0].y) + \_sq(p[2].y-p[0].y) + \_sq(p[2].y-p[0].y-p[0].y) + \_sq(p[2].y-p[0].y-p[0].y) + \_sq(p[2].y-p[0].y-p[0].y-p[0].y) + \_sq(p[2].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p[0].y-p
p[0].y)-(p[2].x-p[0].x)*(p[1].y-p[0].y));
```

```
tmp.y=p[0].y+(0.5*(sq(p[2].x-p[0].x)+sq(p[2].y-p[0].y))*(p[1].x-p[0].x)-0.5*
 (\_sq(p[1].x-p[0].x) + \_sq(p[1].y-p[0].y)) * (p[2].x-p[0].x)) / ((p[1].x-p[0].x) * (p[2].y-p[0].x) ) 
p[0].y)-(p[2].x-p[0].x)*(p[1].y-p[0].y));
                         return tmp;
               }
               Point incenter()//incenter of a triangle 内心, 内接圆圆心
                         Line u,v;
                         double m,n;
                         u.a=p[0];
                         m=atan2(p[1].y-p[0].y,p[1].x-p[0].x);
                         n=atan2(p[2].y-p[0].y,p[2].x-p[0].x);
                         u.b.x=u.a.x+cos((m+n)/2);
                         u.b.y=u.a.y+sin((m+n)/2);
                         v.a=p[1];
                         m=atan2(p[0].y-p[1].y,p[0].x-p[1].x);
                         n=atan2(p[2].y-p[1].y,p[2].x-p[1].x);
                         v.b.x=v.a.x+cos((m+n)/2);
                         v.b.y=v.a.y+sin((m+n)/2);
                         return u.intersection(u);
               Point fermetpoint(){//fermet point, the point of lowest sum of distance to three
points 费马点
                         Point u, v;
                         double
step=fabs(p[0].x)+fabs(p[0].y)+fabs(p[1].x)+fabs(p[1].y)+fabs(p[2].x)+fabs(p[2].y);
                         int i, j, k;
                         u.x=(p[0].x+p[1].x+p[2].x)/3;
                         u.y=(p[0].y+p[1].y+p[2].y)/3;
                         while (step>eps)
                                   for (k=0; k<10; step/=2, k++)
                                              for (i=-1;i<=1;i++)
                                                        for (j=-1; j<=1; j++) {
                                                                   v.x=u.x+step*i;
                                                                   v.y=u.y+step*j;
(u.distance(p[0])+u.distance(p[1])+u.distance(p[2])>v.distance(p[0])+v.distance(p[1])+v
.distance(p[2]))
                                                                             u=v;
                                                         }
                        return u;
               }
               Circle circumcircle(){//the circumcircle of the triangle 外接圆
                         Circle tmp;
                \\  \text{tmp.center.x=p[0].x+} \\  (0.5*(\_sq(p[1].x-p[0].x)+\_sq(p[1].y-p[0].y))*(p[2].y-p[0].y)) \\  \\  (0.5*(\_sq(p[1].x-p[0].x)+\_sq(p[1].y-p[0].y)) \\  (0.5*(\_sq(p[1].x-p[0].x)+\_sq(p[1].x)+\_sq(p[1].x)) \\  (0.5*(\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)) \\  (0.5*(\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)) \\  (0.5*(\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)) \\  (0.5*(\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)) \\  (0.5*(\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)) \\  (0.5*(\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x) \\  (0.5*(\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x) \\  (0.5*(\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x) \\  (0.5*(\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x) \\  (0.5*(\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x) \\  (0.5*(\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1].x)+\_sq(p[1
p[0].y)-0.5*(_sq(p[2].x-p[0].x)+_sq(p[2].y-p[0].y))*(p[1].y-p[0].y))/((p[1].x-p[0].x)*
(p[2].y-p[0].y)-(p[2].x-p[0].x)*(p[1].y-p[0].y));
```

# 多边形

### 类定义

注意多边形退化为线

```
struct Polygon{ //polygon
      int n;
      vector<Point> p;
      Polygon(): n(0){}
      Polygon(int n,Point* p0): n(n) {for (int i=0;i<n;i++) p.push_back(p0[i]);}</pre>
      Polygon(int n,vector<Point> p0): n(n) {p.assign(p0.begin(),p0.end());}
      Polygon(vector<Point> p0): n(p0.size()) {p.assign(p0.begin(),p0.end());}
      friend istream& operator >>(istream &in, Polygon &x){
      in>>x.n;
      x.p.resize(x.n);
      for(int i=0;i< x.n;i++)
        in>>x.p[i];
      return in;
    }
    friend ostream& operator <<(ostream &out,const Polygon &x){
      for(int i=0;i<x.p.size();i++) cout<<x.p[i]<<endl;</pre>
      return out;
      bool is_convex(){//if this is convex,edge collinear //判断非严格凸包
          int i,s[3]=\{1,1,1\};
          for (i=0;i<n&&s[1]|s[2];i++)
              s[sign(p[i].xmult(p[(i+1)%n],p[(i+2)%n]))]=0;
          return s[1] s[2];
      }
      bool is_convex_ex(){//if this is convex,no edge collinear //判断严格凸包
          int i,s[3]=\{1,1,1\};
          for (i=0; i < n\&\&s[0]\&\&s[1] | s[2]; i++)
              s[sign(p[i].xmult(p[(i+1)%n],p[(i+2)%n]))]=0;
          return s[0]&&s[1]|s[2];
      }
      bool inside_convex(Point q){ //if point in convex
          int i,s[3]=\{1,1,1\};
          for (i=0; i<n&&s[1]|s[2]; i++)
              s[_sign(p[i].xmult(p[(i+1)%n],q))]=0;
```

```
return s[1]|s[2];
      }
      bool inside convex ex(Point q){//if point in convex, except edge
          int i,s[3]=\{1,1,1\};
          for (i=0; i < n\&\&s[0]\&\&s[1] | s[2]; i++)
              s[_sign(p[i].xmult(p[(i+1)%n],q))]=0;
          return s[0]&&s[1]|s[2];
      }
      int inside polygon(Point q,int on edge=1){//if point in polygon,if on edge return
on_edge
          Point q2;
          int i=0,count;
          while (i<n)
              for (count=i=0,q2.x=rand()+10000,q2.y=rand()+10000;i<n;i++)
                  if (_zero(p[(i+1)%n].xmult(q,p[i]))%&(p[i].x-q.x)*(p[(i+1)%n].x-q.x)
eps&&(p[i].y-q.y)*(p[(i+1)*n].y-q.y)eps)
                      return on edge;
                  else if (_zero(p[i].xmult(q,q2)))
                      break;
                  else if (q2.xmult(q,p[i])*q2.xmult(q,p[(i+1)%n])<-
eps\&\&p[(i+1)&n].xmult(p[i],q)*p[(i+1)&n].xmult(p[i],q2)<-eps)
                      count++;
          return count&1;
      }
      int inside polygon(Line 1)///if segments in convex include edge
      {
          Point 11=1.a,12=1.b,tt;
          vector<Point> t;
          int i,j;
          if (!inside polygon(11)||!inside polygon(12))
              return 0;
          for (i=0; i< n; i++)
(Line(p[i],p[(i+1)%n]).opposite\_side(l1,l2)&l.opposite\_side(p[i],p[(i+1)%n]))
                  return 0;
              else if (Line(p[i],p[(i+1)%n]).dot_online_in(l1))
                  t.push_back(11);
              else if (Line(p[i],p[(i+1)%n]).dot online in(12))
                  t.push_back(12);
              else if (Line(11,12).dot_online_in(p[i]))
                  t.push_back(p[i]);
          for (i=0;i<t.size();i++)</pre>
              for (j=i+1; j<t.size(); j++){</pre>
                  tt.x=(t[i].x+t[j].x)/2;
                  tt.y=(t[i].y+t[j].y)/2;
                  if (!inside polygon(tt))
                      return 0;
              }
          return 1;
```

```
Point barycenter(){//the barycenter of the polygon 重心
          Point ret,t;
          double t1=0,t2;
          int i;
          ret.x=ret.y=0;
          for (i=1;i<n-1;i++)
               if (fabs(t2=p[i+1].xmult(p[0],p[i]))>eps){
                   t=Triangle(p[0],p[i],p[i+1]).barycenter();
                  ret.x+=t.x*t2;
                  ret.y+=t.y*t2;
                   t1+=t2;
              }
          if (fabs(t1)>eps)
              ret.x/=t1,ret.y/=t1;
          return ret;
      }
      double area(){//the area of the polygon
          double s1=0, s2=0;
          for(int i=0;i<n;i++)</pre>
               s1+=p[(i+1)%n].y*p[i].x,s2+=p[(i+1)%n].y*p[(i+2)%n].x;
          return fabs(s1-s2)/2;
      }
      double perimeter(){//the perimeter of the polygon 周长
      double pm=0;
      if(n>1) pm=p[n-1].distance(p[0]);
      for(int i=0;i<n-1;i++)
        pm+=p[i].distance(p[i+1]);
      return pm;
      Polygon graham(){//Calculate the convex of polygon or points 凸包
          Polygon g=(*this);
          if(g.n<3) return g;
          sort(g.p.begin(),g.p.end(),Point().comp);
          Point bp=g.p[0];
          for(int i=0;i<g.n;i++) g.p[i]=g.p[i]-bp;</pre>
          sort(g.p.begin(),g.p.end());
          Polygon gra;
          gra.p.push_back(g.p[0]);
          gra.p.push_back(g.p[1]);
      int i=2;
      for(i=2;i<n;i++)
        while(gra.p.size()>1&\&gra.p[gra.p.size()-2].xmult(gra.p[gra.p.size()-1],g.p[i])
<eps) //<eps æ\partial'\tilde{O}\pi\omega <-eps \forall 180\Omega\cdot\infty\)</pre>
          gra.p.pop back();
        gra.p.push_back(g.p[i]);
      }
      gra.n=gra.p.size();
```

```
for(int i=0;i<gra.n;i++) gra.p[i]=gra.p[i]+bp;</pre>
          return gra;
      }
    Circle mincircle(){//calculate the min circle to cover polygon or points 最小覆盖圆
      Polygon g=*this;
        random_shuffle(g.p.begin(),g.p.end());
        Circle ans;
        ans=Circle(g.p[0],0);
      Point &c=ans.center;
      double &r=ans.radius;
        for(int i=1;i<n;i++)</pre>
      {
            if(c.distance(g.p[i])>r+eps)
        {
                ans=Circle(g.p[i],0);
                for(int j=0;j<i;j++)</pre>
          {
                    if(c.distance(g.p[j])>r+eps)
            {
 ans=Circle(Line(g.p[i],g.p[j]).midpoint(),g.p[i].distance(g.p[j])/2);
                        for(int k=0; k<j; k++) {
                             if(c.distance(g.p[k])>r+eps)
                {
                               ans=Triangle(g.p[i],g.p[j],g.p[k]).circumcircle();
                             }
                        }
                    }
                }
            }
        }
        return ans;
    double rotate_calipers(){ //求凸多边形宽度, 凸多边形的宽度定义为平行切线间的最小距离
      Polygon convex=graham();
      if(convex.n<2) return 0;</pre>
      double ans=convex.p[0].distance(convex.p[1]);
      int q=1;
      for(int i=0;i<convex.n;i++)</pre>
          while(convex.p[(i+1)%convex.n].xmult(convex.p[q],convex.p[i])
<convex.p[(i+1)%convex.n].xmult(convex.p[(q+1)%convex.n],convex.p[i]))</pre>
               q=(q+1) %convex.n;
 ans=max(ans,max(convex.p[q].distance(convex.p[i]),convex.p[(q+1)%convex.n].distance(co
nvex.p[(i+1)%convex.n])));
        }
        return ans;
    }
```