HIGH SCHOOL MATHEMATICAL CONTEST IN MODELING

OUTSTANDING PAPERS

HiMCM

January 2001

The contest offers students the opportunity to compete in a team setting using applied mathematics in the solving of real-world problems.

Major funding provided by the National Science Foundation.

NSF Project Award Number ESI-9708171

Editor's Comments

This is our third HiMCM Special Issue. The growth of HiMCM has again presented a rather happy dilemma: there are too many outstanding papers to print even abridged versions of them all. Thus, herein are edited versions of two papers and the summaries from the others. We emphasize that their selection does not mean these two are superior to the other outstanding papers. We chose them because they are representative and relatively short. They received light editing, although there is insufficient space to include the programming code from the forest fire paper. We also want to emphasize that the papers are not written with publication in mind; students do not have time to revise and polish. Given the one-day time limit, it is remarkable how well written many of the papers are.

We appreciate the outstanding work of students and advisors and the efforts of our contest directors, regional directors, and judges. They have made HiMCM a major success. Because of the enthusiastic response to the contest, the next issue of Consortium will initiate a regular HiMCM column. We welcome suggestions on material that will be helpful to students and advisors as they prepare for future HiMCMs. And we hope that our readers will help spread the word!

Contest Director's Article

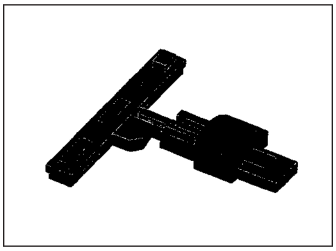
William P. Fox

Department of Mathematics
Francis Marion University
Florence, SC 29501
Wfox@fmarion.edu

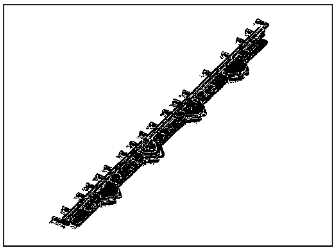
The High School Mathematical Contest in Modeling completed its third year in excellent fashion. It is still moving along in a positive first derivative and consistent with our positive experiences from previous years.

This year the contest consisted of 160 teams from 33 states and three teams from outside the USA: the Hong Kong International School and two Department of Defense schools. The teams accomplished the vision of our founders by providing unique and creative mathematical solutions to complex open-ended real world problems. This year the students had a choice of two problems.

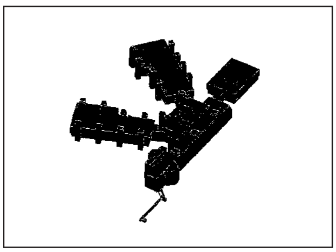
Problem A: The design of airline terminals varies widely. The sketches below show airline terminals from several cities. The designs are quite dissimilar. Some involve circular arcs; others are rectangular; some are quite irregular. Which is optimal for operations? Develop a mathematical model for airport design and operation. Use your model to argue for the optimality of your specified design. Explain how it would operate.



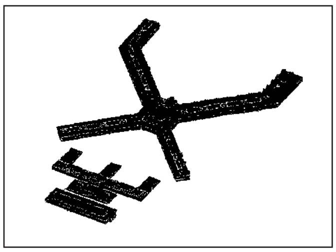
Boston-Logan International



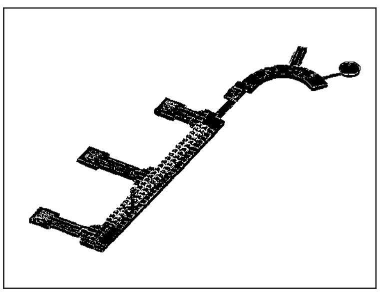
Munich International



Charlotte/Douglas International



Pittsburgh International



Ronald Reagan Washington National

Problem B: Your team has been approached by the Forest Service to help allocate resources to fight wildfires.

In particular, the Forest Service is concerned about wildfires in a wilderness area consisting of small trees and brush in a park shaped like a square with dimensions 80 km on a side. Several years ago, the Forest Service constructed a network of north-south and east-west firebreaks that form a rectangular grid across the interior of the entire wilderness area. The firebreaks were built at 5-km intervals.

Wildfires are most likely to occur during the dry season, which extends from July through September in this particular region. During this season, there is a prevailing westerly wind throughout the day. There are frequent lightning bursts that cause wildfires.

The Forest Service wants to deploy four fire-fighting units to control fires during the next dry season. Each unit consists of 10 firefighters, one pickup truck, one dump truck, one water truck (50,000 liters), and one bulldozer (w/ truck and trailer). The unit has chainsaws, hand tools, and other fire-fighting equipment. The people can be quickly moved by helicopter within the wilderness area, but all the equipment must be driven via the existing firebreaks. One helicopter is on standby at all times throughout the dry season.

Your task is to determine the best distribution of fire-fighting units within the wilderness area. The Forest Service is able to set up base camps for those units at sites anywhere within the area. In addition, you are asked to prepare a damage assessment forecast. This forecast will be used to estimate the amount of wilderness likely to be burned by fire as well as acting as a mechanism for helping the Service determine when additional fire-fighting units need to be brought in from elsewhere.

Commendation: All students and advisors are congratulated for their varied and creative mathematical efforts. The composition of the 160 teams, with 104 of the 160 teams having female participation, shows HiMCM is for both male and female students. As a matter of fact, eight teams were all female. Teams again proved to the judges that they had "fun" with their chosen problems, demonstrating research initiative and creativity in their solutions.

Judging: We ran three regional sites in 2001. Each site judged papers as Outstanding, Meritorious, Honorable Mention, and Successful. All regional Outstanding papers were brought to the national judging. The national judging chooses the "best of the best" as National Outstanding. The national judges commended the regional judges for their efforts and found the results very consistent. We feel that this regional structure provides a good prototype for the future contest structure as HiMCM continues to grow.

JUDGING RESULTS:

National & Regional Combined Results

Problem	National Outstanding	Regional Outstanding	Meritorious	Honorable Mention	Successful	Total
A	**8**	14	22	37	16	**89**
B	**3**	10	21	26	14	**71**
Total	**11**	24	44	63	30	**160**

General Judging Comments: The judge's commentary (written by Rick Jennings) provides comments on the solutions to the two problems. As contest director and head judge, I want to speak generally about student solutions from a judge's point of view.

Papers need to be very coherent, concise, and clear. Students need to restate the problem in their own words so that judges can determine the focus of the paper. Papers that explain the development of the model and its solutions and then support the solutions mathematically generally do well. Assumptions only need to be listed and justified if they affect the solution (this can be part of simplifying the model). Laundry lists of assumptions that are never referred to in the problem solution are not relevant. The model needs to be clearly developed and all variables that are used need to be defined. Thinking outside of the "box" is also considered important by judges. This varies from problem to problem but usually includes model extensions or sensitivity analysis. A clear conclusion and answers to specific scenario questions are all key components. The strengths and weakness section is where the modeling team can reflect on the solution. Paying attention to detail and proof reading are also very important.

CONTEST FACTS:

Facts from the 3rd Annual Contest:

- A wide range of schools competed including a home schooling team.
- 65%, 104 of 160 teams, had female participation. Eight were all female.
- 35% were all male participation.
- There was one all female outstanding team.
- Seven of the outstanding teams had female participants.
- This was an international HIMCM with three teams from outside the U.S.

The Future: The contest, which attempts to give all students an opportunity to compete and achieve success in mathematics endeavors, appears well on its way in meeting this important mission.

We continue to grow. Any school/team can enter, as there are no restrictions on the numbers of schools. A regional judging structure is established based on the number of teams that register.

These are exciting times for our high school students. Mathematics continues to be more than learning skills and operations. Mathematics is a language that involves our daily lives. Applying the mathematical principles that one learns is a key to future success. The ability to recognize problems, formulate a mathematical model, solve, and communicate and reflect on one's work are keys to success. The ability to use technology aggressively to discover, experiment, analyze, resolve, and communicate results is also key to success. Students gain confidence by tackling ill-defined problems and working together to generate a solution. Through team building and team effort solutions are built. Applying mathematics is a team endeavor.

Advisors need only be a motivator and facilitator. They should allow students to be creative and imaginative. Let me encourage high school mathematics faculty to get involved, encourage their students, make mathematics relevant, and open the doors to success.

Mathematical modeling is an art and a science. Through modeling, students learn to think critically, communicate effectively, and be confident, competent problem solvers for the new century.

Contest Dates: Mark your calendars now as the next HiMCM will be held from 7–20 November 2001. Teams will have a consecutive 36-hours block within this window to complete the problem. Teams can register via the worldwide web at www.comap.com.

Judges' Commentary

Rick Jennings

Yakima WA
rickjenn@msm.com

It never ceases to amaze me what high school students can accomplish in such a short time. This year's results demonstrate the resourcefulness and creativity of our students. Congratulations to all teams and their coaches.

I will start with a few general comments about the contest and what tends to separate the top papers from the rest. I will then look specifically at each problem with an eye towards those things that tend to separate the excellent from the good papers.

General Comments

A consistent thread through the judging (both regional and national) is that the summary page of the best papers tells the story of the process that the team went through to arrive at a solution. Particular attention is paid to whether the team went through iterations (of the modeling process) that helped arrive at a final solution and are logical in progression. All of the national outstanding papers had this characteristic.

Another characteristic is clarity. This applies to many areas of the modeling process:

- Summary page - Was the story of the modeling process understandable?
- Formulae - Are they appropriate? Are the parameters and coefficients assigned reasonable values, and was it clear why these values were selected?
- Computer programs/software – If a computer program was created or other software was used, was documentation provided and/or remark statements interwoven within the program so the judges have a sense of what the program and/or software is doing and where decisions are made?

Judges are very interested in assumptions and factors that influence the problem. Some teams gave an itemized list, but many of the listed items were not relevant to the problem. While this is not necessarily bad, it does tend to raise questions as to whether the team understands which assumptions are important and which are just window dressing. Certainly, it is better to be safe and list more, rather than fewer, assumptions, but listing assumptions and characteristics that have no effect on the problem can muddy the waters. It should be noted that the judges are not looking for a set form or a prescription.

Use of the Internet was both a boon and a bust. It was obvious that some teams spent so much time finding information on the Internet that they ran out of time to solve the problem. Some teams used so much Internet information that they were unable to mathematically justify the information they were using. The teams that used the Internet effectively found short, specific information that helped them create a model.

PROBLEM A - DESIGNING AN AIRPORT TERMINAL

The best papers clearly defined what they were trying to optimize. Some chose the distance in the terminal traveled by passengers. Some chose the number of planes that could be loaded/unloaded at a given time. Some chose to optimize over the land area needed for the terminal. Most of the best papers considered more than one criterion, along with safety factors. Some of the weaker papers assumed a certain configuration without clearly showing that it was the best design for their optimization criteria.

The best papers had clear and convincing justifications as to why they selected their configuration. Some used geometric justifications; others used algebraic justifications; and others used a combination of the two.

The very best papers looked at extending their solution to include population growth or growth in the number of flights. Some of the best papers looked at more than one design and selected the one that was the best for their optimization criteria.

PROBLEM B – ALLOCATING RESOURCES FOR A FOREST FIRE RESPONSE

The best papers clearly justified (mathematically and logically) why they placed the resources where they did. Many teams selected locations without verifying that these were indeed the best places for their resources.

The best papers defined "best" by assigning some criterion, i.e., minimizing the response time with the most resources or minimizing the fire-swept area. They then went about finding the best allocation of resources to address that definition.

The best papers included calculations on the effect of the wind in placing their resources and also in finding the area burned by a fire. Many of the teams spoke only in general terms about how much land might be burned and the distances and times it might take to arrive at the fire. The best papers quantified these amounts in some way. Some also looked at the probability of random fires occurring at various locations.

The best papers included all of the above and compared different allocations in quantifiable ways. These papers also included approximations (in percentage of overall or real area) of the damage done with their allocations. Some included analysis for one or more concurrent fires.

Problem A Summary: Chesterfield County Math & Science High School, Midlothian VA

Advisor: Diane Leighty

Team Members: Young Kim, Chia-Shing Yang, Jennifer Dertinger, Daniel Genovese

To create an optimal terminal, one must consider passenger needs: getting to a flight as quickly as possible while covering the smallest distance. Our terminal is based on an international terminal, covers both arrivals and departures, and is self-sufficient with the possibility of being attached to a larger airport. We decided that the maximum number of gates for a given perimeter is a circle. This shape also minimizes walking time and distance. Our model is well equipped to handle routine traffic as well as traffic at maximum capacity. Our model is concerned mainly with efficiency defined as what alterations in the architectural structure would affect the speed at which a passenger can reach a flight.

Problem A Summary: Hong Kong International School

Advisor: William Stork

Team Members: Thomas Lo, Timothy Chen, Sam Cheung, Mamoru Inoue

Our objective was to create a model to determine the efficiency of various airline terminal designs and use the model to find an optimal design. Various shapes and designs were tested, and each design was given three ratings based on running costs and customer satisfaction. These ratings were used to compute an efficiency coefficient for each design. Optimal efficiency was defined as the lowest efficiency coefficient (EC).

From the EC values and a computer simulation of airport operations, the following results were obtained:

1. Regardless of the number of gates, the optimal design is the Half Double-Cross Shape. The disparity in EC between this design and other designs grows as the number of gates increases.

2. The Cross Shape and Double-Cross Shape also have comparatively low EC values regardless of the number of docking gates. In small airports, the Cross Shape is more efficient, but the Double-Cross Shape is better in large airports.

3. The Square, Circle, and Line Shape designs have the highest EC values. This was not as pronounced in small airports.

4. The number of docking gates is directly proportional to EC values. That is, for every design, small airports are more efficient than large airports.

Recommendation: Use a Half Double-Cross Shape regardless of terminal size. If this design is not feasible, then consider a Cross Shape for small airports and either a Cross Shape or a Double-Cross shape for large airports.

Problem A Summary: The Governor's School, Richmond VA

Advisor: Crista Hamilton

Team Members: Konstantin Lantsman, Ben Easter, Devin Yagel, Eric Nielsen

The increasing demand for air transportation and the resulting inability of the air transportation system to address the issue have presented 21st century designers with a dilemma. They must create innovative airports that balance the needs of passengers and planes in an attempt to alleviate some of the problems facing the airline industry. Thus, it is imperative to maximize the number of gates and planes while also reducing walking distances for passengers. Many designs tend to maximize area alone, consequently increasing walking distances and aggravating passengers. Other designs work well on a small scale, but are inefficient in large airports. Therefore, a design composed of elements from a variety of basic layouts is required.

While debating the advantages and disadvantages of possible layouts, ingenuity and innovation came to our aid. We noticed a basket of candy canes and combined four of them into a promising shape. After altering angles and considering additions to the "X" like shape, we fashioned a design that maximized external perimeter, controlled interior size, and kept a 35-foot safety cushion between planes. We then focused on moving passengers through the airport efficiently. Extra walkways were added to avoid excessive distances, and a commercial area was included in the center.

In order to test the model, 20 test cases (10 arrivals and 10 departures) were simulated. To test the model against real life, these cases introduced realistic variables such as party size, delays from airport traffic, and leisure stops. Acceptable time data were observed for all test cases, with no passenger requiring more than 25 minutes to move from gate to gate.

Therefore, our model accounts for the basic flaws of current airport layouts. Remarkably, the compact design accommodates 14,080 people. Current airports simply cannot handle today's volume of air traffic. The new century requires a new breed of airports, which no longer only meet the needs of planes, but also provide for the comfort of passengers.

Problem A Summary: Westminster Schools, Atlanta GA

Advisor: Landy Godbold

Team Members: Alok Deshpande, Anthony Waller, Jeff Huong, Imran Saleh

We focused our efforts on minimizing the time passengers spend traversing the terminal and maximizing the number of gates at the airport. After combining these objectives we got the formula SE = G/T where SE is the structural efficiency, G is the number of gates, and T is the average time required to pass through the airport. We broke each component into its contributing factors. T broke into the times required to use the trains, moving sidewalks, and walkways to get from the entrance to a random gate. As each component expanded, we compared the geometry of certain shapes and the way they interacted with our objectives. Circle-based structures minimized walking time yet also minimized the number of gates. Thus, a combination of a circular center with rectangular arms was created. This compound design was intended to reduce distance traveled by slow modes of transportation such as walking. This translated into faster average passage times and more perimeters for gates. The resulting design is a radial design with trains and moving sidewalks to speed passage through the system. It features two gate sizes to simulate a variety of aircraft while reducing design calculations to a reasonable number. A separate international concourse provides custom services, and a centralized structure houses baggage claim, ground transportation, administration, and building maintenance facilities. The facility has 108 gates, which can service 34,800 people simultaneously. On average, a person departing the security checkpoint can reach a destination gate in 4.5 minutes. The bounding rectangle of the terminal encloses about 20,250,000 ft^2. This terminal represents a move to the future in technology with its innovative curves, and high-speed tram system. Even though the terminal accommodates a large population, the design has future expansion in mind. It seeks to be the traveler's best friend!

Problem A Summary: Chesterfield County Math & Science High School, Midlothian VA

Advisor: Diane Leighty

Team Members: Dena Henderson, Tyler McCall, Jackie Horak, Matthew Lynch

There are many terminal designs used in many parts of the world, but which is best? Our model takes into account the six types of designs that are often used at major airports. It also recognizes several of the most important factors that contribute to airport function, such as plane traffic around a terminal, the number of planes that can fit into a certain area, and the room in the terminal for passengers. The hypothesis was that the one of the six shapes that best fit all of these characteristics would be the best design.

Using six scale models to simulate the designs and construction paper cut to scale to simulate DC-l0s, we made models to test each design's traffic flow, capacity for servicing planes, and room for passengers. Based on their tested capabilities, the six designs were assigned ratios to the theoretical best in each of the three key areas. Based on the level of importance assigned to each of the three areas, the ratio is a point value for each design. At the end of testing, the terminal design that received the most points was called best.

The long, narrow, rectangular design was best. It received the most points and is by far the best at accommodating large numbers of planes. Based on its quantitative comparison against the theoretical best, it can be used in any type of terminal design, therefore allowing for a high level of adaptability.

Problem A Summary: Westminster Schools, Atlanta GA

Advisor: Landy Godbold

Team Members: Tochilin Conor, Michael Miller, Jana Dopson, Koon Ho Cho

We first had to define optimal. We decided that optimal functioning occurs when the airport meets the specific needs of both travelers and airlines most effectively. To consider how an airport might better meet these needs, we simplified to evaluate only three of the most fundamental terminal structures.

We compared what we believed to be the three most promising structures: a single rectangular terminal, a semicircular terminal, and a V-shaped terminal (two rectangles meeting at a right angle) based on the criteria of the number of planes able to dock at each terminal (which we wanted to maximize) and the distance travelers had to walk on average in a given terminal (which we wanted to minimize). Of these three, we found that the V-shape and the rectangle had advantages that would be useful in our effort to design a large airport by combining the different terminal subunits we examined earlier.

To determine how our model could best meet the needs of airlines and relieve passengers of arduous inter-terminal pilgrimages, we assumed two airport categories: hub airports, where single airlines operate many gates in more than one terminal, and non-hub airports, where airlines operate out of only one terminal. In a hub airport, we determined that our design would have to cater to the needs of a large number of passengers with connecting flights, while in a non-hub airport, our design would have to minimize the distance from airport entrances to passenger gates.

For non-hub airports, we designed an airport we dubbed the "rising sun" that allows passengers to depart via a variable number of rectangular terminals protruding from a semicircular concourse, with airport entrances at each terminal-concourse intersection. For hub airports, we offered two similar designs, each with subtle benefits over the other. One of the hub designs consisted of V-shaped terminals aligned along a central underground train system with the tips of the "V" connected by subterranean moving sidewalks. The other hub airport design was simpler, consisting of rectangular terminals connected by an underground train system similar to the one in the V-shaped model.

Problem A Summary: Hunter College High School, New York NY

Advisor: David Hankin

Team Members: J. D. Zamfirescu, Sam Kendig, Nan Gu, Vincent Chen

We devised a novel model that applies fractal geometry to terminal design. Fractals, as applied here, allow for large terminals with many gates, but avoid the problem of excessive passenger walking times. In addition, the fractal design allows for a convenient modular construction that is easily expanded.

One of our goals was to maximize available gate space—the outer perimeter. Our other goal was to minimize the time people spent in the terminals and the distance they had to travel.

The fractal design meets both goals. In standard terminal designs, increasing the number of gates results in a roughly proportional increase in walking distances. In the fractal design, however, when large terminals are built, with repeated branching, significant increases in available gate area (perimeter space) are produced, but without excessive increases in walking distance. Though our model is unnecessarily complex for small airports, it proves more useful as terminal size increases.

Problem B Summary: Illinois Mathematics & Science Academy, Aurora IL

Advisor: Ronald Vavrinek

Team Members: Nicholas Rupprecht, John Carrino, Justin Blanchard, Bradley Kay

Our first stage was to create a realistic model for the spread of an unchecked fire using the given and assumed conditions. To do this, we divided a 5 km square of forested area into 2500 cells. Once a fire started, it could spread from any burning cell to an adjacent non-burning cell. We created a formula to determine the probability of any non-burning cell catching on fire depending on the presence or lack of a firebreak, the temperature and humidity, how long the adjacent cell had been on fire, etc. We then wrote a program to display the progress of the fire. Here we found our program's greatest flaw: modeling the fire in the entire park (more than 300,000 cells) was too complex for our computers. It would also require too many pixels to show the full park on our monitors. We decided instead to view a model of nine 5 km squares surrounded by firebreaks with a fire starting in the center. The location of the center square was chosen randomly. We then assumed that we could find an upper limit for how long firefighters would take to reach the burning area. This allowed us to focus on the immediate surroundings of a lightning strike.

The second stage was to determine the best methods for the units to reach the burning areas and fight fires. We decided that two units would be transported to the 5 km square and build two additional firebreaks to limit the size of the rectangle that would burn. We assumed that with the help of water trucks, the fire would be unable to jump firebreaks. We then worked to find the configuration of base camps that minimized the time it takes to get two units to a fire. We found that there should be one base camp 20 miles from the center of the park in each cardinal direction.

The third stage was to determine the average number of cells burned for each lightning strike and the total expected amount of land burned. We considered all 5 km squares in one quadrant and averaged the amount of forest burned for a fire started in that square over 100 trials. We multiplied this average (3.4789 km^2 per strike) times the expected strikes per season (103) to get the total area of forest destroyed (360 km^2). We also determined that if two fires occurred within a certain period of time, outside help would be needed to go on backup. We determined the length of time to be about 20 hours.

Problem B Summary: Sidwell Friends School

Advisor: Eric Steadman

Team Members: Virginia Harr, Andrea Young, Tristan Kessler, Alison Bloom-Feshbach

The locations of the fire fighter base camps should minimize the distance from the base camps to every point in the park, which maximizes the speed at which any fire is extinguished. Initially, we determined that the ideal locations for the camps are at points (20,20), (20, 60), (60, 20), and (60, 60), without taking into account the westerly wind. (The origin is at the lower left-hand corner of the park.) With our model, the greatest distance from a base camp to a point in the park is 40 km; with other designs, there are always points further than 40 km from any base camp (we assumed that the firebreaks serve as roads, and that the fire trucks move only along firebreaks). Taking into account the constant westerly wind, fires spread toward the east. Thus, fires in the final column of 5 km by 5 km squares (bordered by firebreaks) do not spread. Fires anywhere other than the final column spread eastward to other squares. Therefore, we shifted the base camps left along the x-axis, so that the fire stations moved west by one 5 km by 5 km square. In a sense, we are now working with an 80 km by 75 km region, and this new area can be split into two 35 km by 35 km units and two 40 km by 40 km units. Ideally, the four base camps are located at the centers of each of these units, at points (17.5, 20), (17.5, 60), (55, 20), and (55, 60). We distributed firefighters and fire fighting equipment in strategic locations within each unit.

To make an accurate damage assessment forecast, we needed to find the area of the park destroyed by fire. We made a series of calculations and assumptions to determine the speed and direction of a spreading fire and the time needed for firefighters to reach any point in their unit. We used a wind speed of 30 km/hr, a ratio of wind speed to fire-spreading speed of 3:1, and thus a fire-spreading rate of 10 km/hr. We also assumed the wind deviated from due east by 45°. We set n to be the probability that fires jump over a firebreak. We also determined the probabilities of fires occurring different distances from base camp (assuming an equal probability of lightning striking any point in the park). After assuming a maximum truck speed of 50 km/hr and a reaction time of 10 minutes, we found the time it takes firefighters to reach different distances. Then we found the area destroyed during each of these time spans, in terms of f (f = number of lightning strikes causing fire in one month). Finally, we multiplied the probability of a fire occurring a given distance from base camp by the area destroyed by fire during the time required for the firefighters to travel that distance. Thus, we found that the likely area destroyed in 3 months is $124.5f$ km^2.

Problem A Paper: Phillips Exeter Academy, Exeter NH

Advisor: Joanie Zia

Team Members: Lisa Hardej, Nate Baillargeon, Marissa Lowman, Daniel Graves

We started by making a list of assumptions. The goal was to simplify the problem to an easy starting point, and then develop the problem in a more detailed manner as we progressed.

ASSUMPTIONS AND JUSTIFICATIONS:

1. The airport has only one terminal. This simplifies the problem greatly. We do not need to take into account travel time between terminals, modes of transportation, or confusion between terminals.

2. There are 50 gates in the terminal. This is arbitrary, but we felt the need to assign a number instead of using a variable.

3. There are 5 concourses in the terminal. We decided that dividing the 50 gates evenly among 5 concourses was the best way to organize the terminal.

4. Passengers are arriving, departing, and/or changing planes. This takes into account all possibilities of domestic air travel, and excludes cargo flights.

5. The airport operates 24 hours a day. This is realistic and allows flights to arrive and/or depart at any time.

6. There are no setbacks, delays or technical problems: planes leave on time. This also simplifies the problem and avoids complications and uncertainty in regards to scheduling.

7. There are 5 flights that depart each gate every day (250 flights per day). This is based on information from Boston-Logan airport during March 2000, where there were 39,644 flights. There are 5 terminals, which means roughly 8000 flights per terminal. We assume that the terminals are roughly equal, and so there are about 300 flights each. We decided that our airport would be slightly smaller and run about 250 flights per day; 250 flights over 50 gates gives 5 flights per gate per day.

8. If changing planes, a person exits the plane and boards a different one. This removes the possibility of the shortest route being 0.

9. All concourses are the same length and have the same number of gates. This was another way to simplify the problem and the calculations involved. This also appeared to be optimal for the space involved.

10. The concourses extend from a central focus with equal angles between them. (See 9.)

11. All planes are the same size. This allows for easier calculation of the space and distance needed. The size of the plane is based on the Boeing 747-400 and Boeing 777-300, fairly common planes as well as some of the largest. They have wingspans of about 200 feet and lengths of about 250 feet.

12. All planes go to a gate. This removes the need to account for transportation or walking outside the terminal.

13. All people enter/ exit at the center of the terminal. The center is the ideal place for people to enter because it minimizes walking distance.

14. People walk at a constant 250 feet/minute. This was calculated by timing a person walking a set distance.

15. There are two floors: the top floor is for arrivals and departures; the ground floor houses external transportation, baggage claims, and customs/immigration. Two floors eliminate congestion for the people who are only dropping off or picking up others, and the people actually flying, changing planes, or staying at the airport for any amount of time.

16. There are 50 feet between the wing tips of the planes. This is an arbitrary number that we picked to provide safe spacing.

17. Plane berths are 200 feet wide. This is based on the wingspan of the Boeing 747 and 777.

18. The concourse is 200 feet wide, the walkway is 100 feet wide, and there are seating areas 50 feet deep on both sides of the concourse. These statistics are arbitrary, but picked to resemble an actual concourse.

19. The depth of the first plane berth on each concourse is 400 feet. This is an arbitrary number, but is large enough to allow access to the gates that are closer to the center of the terminal.

20. Each gate entrance is at the middle of the plane's berth. This allows for easy calculations and ease of access.

21. It is 650.5 feet to the first gate and 250 feet between gates after that, with the exception of the gates at the end of the concourse. This is a sum of the 100 feet and the distance from the entrance to the concourse to the first plane berth, which is 550.5 feet. The 550.5 feet is based on the trigonometry of a right angle triangle involving the 400-foot depth of the first plane berth and the 72° angle between concourses (a 36° angle in the right triangle).

ANALYSIS

We understood optimal to mean a design that is efficient in its use of space with respect to its capacity. Our goal was to minimize the walking distance between gates, while maximizing the overall surface area. A large surface area means more gates and flight operations.

We experimented with several models before deciding on our final answer. Our first attempt was a very basic design: a circle. People enter at the center, and the gates are equally spaced around the circumference. We decided against this model because the diameter of the building was slightly over a mile, and we felt that this was too big. Other models we experimented with were deemed too large or inefficient as well.

We returned to the circle and made a more detailed model, so that the terminal now looked like a wheel with spokes extending from the center to the outer edge where all the gates were. We realized that we were wasting space between the spokes and sought to use that space. We concluded that the model that made the most efficient use of space had arms/concourses extending from a central focus, instead of a solid geometric figure, e.g. circle, pentagon, rectangle, etc. We decided to eliminate the outer wheel, but keep the spokes. We immediately realized that this was a lot better in terms of minimizing walk time and maximizing flight operations as well as being a more cost-efficient building plan. We concluded that dividing the 50 gates evenly among a number of concourses was the best design and the easiest in terms of calculations.

One of our first considerations was how much space was needed, and we built the terminal around the airplanes. By using large aircraft (e.g. Boeing 747-400 and 777-300) as the basis for calculations, we were able to accommodate any size airliner. We also added a buffer zone of 50 feet between planes.

We were aiming for a concourse length of less than 2000 feet. This was to make sure that our model was more efficient than our others. The total length of a concourse from the central focus to its tip is 1550.5 feet (1000 feet of which is for airplane berths and gates), and the time required to walk this distance is a mere six minutes and twelve seconds (using our tested 250 feet/min walking speed).

The terminal consists of two floors and a basement level: the ground floor contains baggage claims, external transportation, customs and immigration. Under the ground floor is a parking garage and driveway to load and unload passengers. All of the gates are located on the first floor.

There was some minor trigonometry involved in calculating the length of the concourse. The depth of the first plane berth for each concourse is 400 feet, an arbitrary number that we chose to give enough room to maneuver behind planes. The concourse space needed for just the airplanes is 1000 feet (200 feet per berth, 4 berths along the length of the concourse, with 50 feet between each berth and at the end of the concourse = 1000 feet). But the distance from the central focus to the first berth is needed. It can be found as such: the distance from the center to the first plane is x. If the 72° angle between concourses is bisected to give 36°, then we can form a right triangle using the angle bisector, the 400-foot berth, and x. In this case, $x = 400/\tan 36°$, or 550.5 feet.

RESULTS AND TESTING

Using a random number generator, we produced gate numbers between 1 and 50 to simulate probability in a passenger's destination and walking distance. For arrivals/departures, we generated one number, and calculated the distance from that gate to the central focus of the terminal. For passengers changing planes for connecting flights, we used two numbers to simulate their travel between flights. **Tables 1** and **2** are some of the results that we used to calculate travel time for random passengers to reach their destination within the airport, and then determine if the times were reasonable.

Gates	Distance between Gates (ft)	Time
1–2	250	1 min
35–50	2709	10 min 50 sec
32–16	1813	7 min 15 sec
5–10	447	1 min 47 sec
47–3	2310	9 min 14 sec
6–35	2110.5	8 min 27 sec
50–32	2212	8 min 51 sec
29–9	3107.5	12 min 26 sec

Table 1: Connecting Flights

Gates	Distance between Gates (ft)	Time
22	650.5	2 min 36 sec
13	900.5	3 min 36 sec
45	1150.5	4 min 36 sec
27	1400.5	5 min 36 sec
49	1550.5	6 min 12 sec

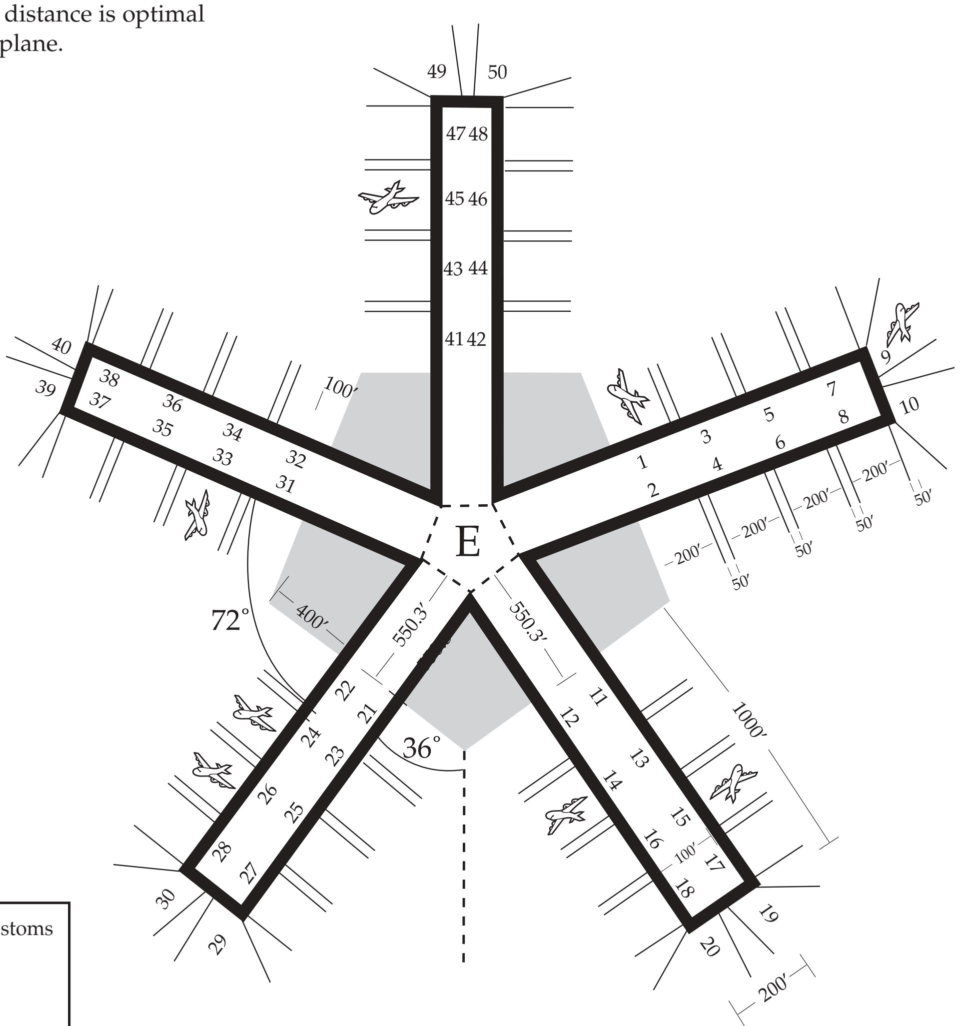
Table 2: Arriving/Departing Flights

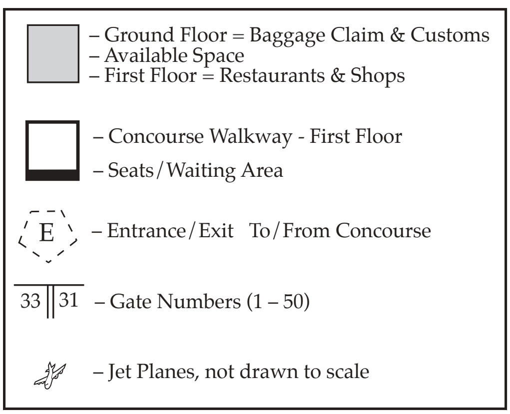
STRENGTHS

- The longest time that a person changing planes walks is 12 min 26 sec.
- The longest time that a person arriving or departing walks is 2 min 36 sec.
- The concourses are less congested because they have ten gates each, and therefore minimize the number of people traveling through each one. The model allows for many simultaneous flight operations.
- Our model is space efficient because it makes use of space in the center that would otherwise be unoccupied.
- We calculated the distance for the largest possible wingspan a plane could have. Thus, the wingspan distance is optimal because it accounts for every possible plane.

WEAKNESSES

- Our model only has one center for people leaving the airport, which might cause congestion.
- The maximum walking time calculation assumed that there are no obstacles and that people walk in straight lines. Because there will probably be other people walking at different speeds, all times are likely to be slightly longer than calculated.
- We assumed that all planes run on time. However, in reality, there are usually delays that cause problems at the gates for the next plane and the people who are waiting for the plane.





Problem B Paper: Arkansas School for Mathematics and Science

Advisor: Bruce Turkal

Team Members: Margery Hunt, Victor Kostyuk, John Broomfield, Beth Ann Pierce

RESTATEMENT OF THE PROBLEM

During the dry season in the northwestern United States, wildfires are an ecological problem. In order to keep the fire problem to a minimum, fire-fighting units are stationed in areas of elevated risk. In this park, four units are available to cover a land area of 6400 km^2. The objective of this particular venture is to place the locations of the units in the most efficient positions. After the units are in position, an assessment of the possible fires during the next dry season is to be made in order to estimate the amount of fire damage and to determine the point at which additional fire-fighting units should be brought in from other sites.

The Forest Service provides many characteristics of the land area and the fire fighting units. The square park is covered with small trees and brush, except for firebreaks that cover the land like a grid with five km^2 units. The dry season of the park is from July to September. During this time interval, there is a constant westward wind and lightning strikes frequently. The firefighting units are equipped with ten people, a pickup truck, a dump truck, a 50,000-liter water truck, and a bulldozer complete with its own truck and trailer. The unit also has the necessary fire-fighting gear, such as chainsaws and hand tools. There is always a helicopter on standby, available to transport the fire fighters, but ground vehicles must transport all equipment.

ASSUMPTIONS AND JUSTIFICATIONS

- The park is in the northwestern United States due to the corresponding months of the dry season.
- The park is relatively flat.
- Because the average rainfall in many northwestern states is less than a third of an inch per month during the dry season, precipitation doesn't help extinguish fires.
- Fire vehicles travel at an average speed of 72 km/hr. This relatively slow speed was chosen for safety reasons: the narrow firebreaks are not paved, and the bulky vehicles must make 90° turns.
- All vehicles travel at the same speed.
- All injured fire fighters are replaced immediately.
- All damaged or broken equipment is replaced immediately.
- The average wind speed is about 12.87 km/hr. This was determined by estimating the average wind speeds of Idaho and Utah over the months of July, August, and September.
- The average size of the fires in the park is about 0.2023 km^2 – 0.4047 km^2 (50 – 100 acres). This parallels the fire range of Yellowstone National Park. Yellowstone information is appropriate because it is in the northwest region.
- The helicopter is a UH1-Huey capable of transporting 6 passengers and an external load of 4000 pounds at speeds up to 225.3 km/hr. It was chosen because of its relatively mid-size and capabilities.
- The water truck has three hoses, each releasing 473 liters of water per minute the entire time it is in use. This in turn means that the three hoses can be used simultaneously for a maximum of 35 minutes. In this amount of time 0.0243 km^2 (6 acres) of fire can be extinguished.
- A lone fighter with heavy equipment can put out one acre in 20 minutes. A lone fighter with light equipment can put out an acre in 50 minutes.
- There is enough heavy equipment in each firefighting unit to boost the unit's firefighter effectiveness to an acre in 20 minutes. The effectiveness of firefighters in putting out a fire is directly proportional to the number of firefighters fighting the fire.
- A firebreak is a strip of cleared or plowed land.
- Each firebreak is 4.572 meters wide, the size of an average road.
- Each firebreak will contain all fires within its originating grid.
- The average fire spread is 12.87 km/hr west, 0.5-km/hr north and south, and 0 km/hr east. On average, fires spread as fast as the wind is blowing.
- Burned land does not catch fire again. In the model, a unit of land (0.25 km^2) where fire was put out is considered burned.
- It takes about 10 minutes for 0.0041 km^2 (an acre) to burn.
- The water truck does not need to return to base in order to refill its water tank, or alternatively, its absence during refilling does not affect the firefighters' performance, due to the presence of other equipment.

MODEL DESIGN

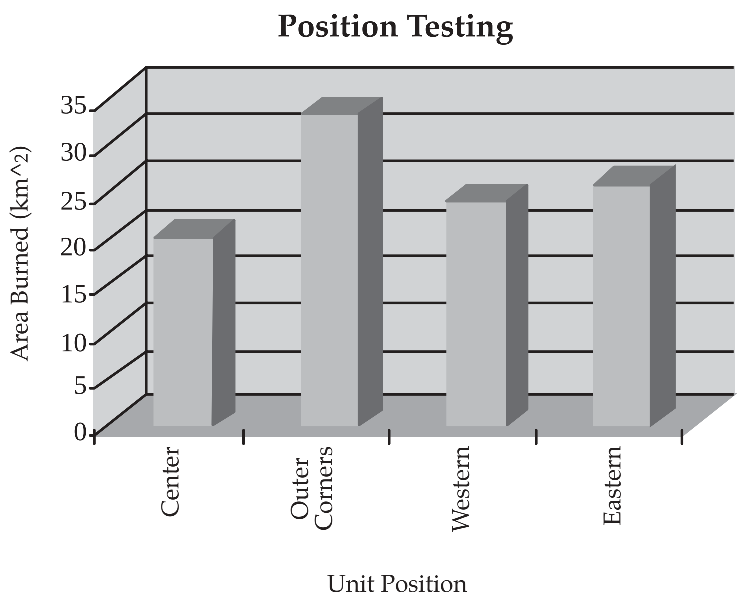
The goal of the model, which employed a number of strong assumptions to simplify the problem, was to simulate a month during the dry season in the park. Some of the assumptions are realistic, such as that fires start at relatively random locations and spread mainly to the West. Other assumptions are less warranted, such as that fires never cross the firebreaks, and the equipment (not to mention the people) does not break.

Nevertheless, the model provides a rather accurate picture of how the positioning of the bases influences the effectiveness of the firefighting efforts, since many of our stronger assumptions do not play a role in that decision. On the other hand, almost all our assumptions exercise great influence on the amount of damage the park incurs. Thus, although our model is comparatively robust when it comes to base positioning, it most certainly does not employ the real–world data that is needed for making accurate damage assessment predictions, instead relying on statistical approximations from existing conditions.

We constructed a program in Visual C++ to simulate the model. Initial conditions specifying the locations of the four camps were given, and then an iterating loop was used to simulate the flow of time. A single iteration represents one minute; hence, 40,320 iterations cover a month. From this, results for the entire dry season were extrapolated under the assumption that all three months have similar conditions. The area burned during that month, then, is simply multiplied by 3, resulting in the amount of land damaged in the entire season. Fires were assumed to start in random locations according to a set probability based on statistical data for fires in Yellowstone, the dry season of which occurs at the time specified. After a fire starts, it could be detected with a probability depending on the fire's size with a cutoff point at 108.08 acres (7 grid squares), at which the probability of detection reaches 1. On detection of a fire, its location was added to a queue list according to which fire-fighter units were dispatched. Fire fighters were then sent to the locations recorded. The helicopter was used to transport the first wave of six fire fighters from a single camp to the site of the fire; the remaining four fire fighters drove the trucks to the fire site. Thus the helicopter was used for general fire fighter transport whenever possible. When multiple fires occurred simultaneously, the units were split among the fires. In such a pace, the firefighters moved from fire to fire, until all fires were put out (or the simulation ended).

VERIFICATION AND TESTING

In order to test the model, we ran the program several times, using a different setup for the units each time. First, we positioned one unit in the middle of each of the four units of the main grid. From this we found that approximately 20.05 km^2 of land would burn. Next, we placed one unit in each of the four corners of the main grid. This positioning allowed 33 km^2 of land to burn. We then placed all four units along the western side of the grid. This allowed 23.63 km^2 of land to burn. Finally, we positioned the units along the eastern side of the grid. This positioning allowed 25.69 km^2 of land to burn. The following bar graph depicts the amount of area that burned for each positioning.



The simulation showed that bases positioned at the centers of the four 40 km squares that make up the park provide the best fire protection. This setup has resulted in only 6.688 km^2 of parkland burned in one month. Thus, during the dry season 3 x 6.688 = 20.064 km^2 of land burns.

DISCUSSION

Our program, which was designed to simulate a month during the dry season, is dependent on the accuracy of our assumptions. If our estimates concerning the rate at which fire spreads or how much acreage a firefighting unit can contain are skewed, then our damage report is considerably affected. However, most of our assumptions have no affect on determining the most effective base locations.

Through verification and testing, we found that the best position for the units is at the center of each grid unit, despite our hypothesis that the best position would be more toward the West due to the winds.

However, an argument can be made that the model neglects the fact that the bulldozer, which each firefighting unit possesses, can be effectively used to cut off the fire from the West, thus practically eliminating its spread.