Binary Image Analysis (III)

---- Morphological Operators

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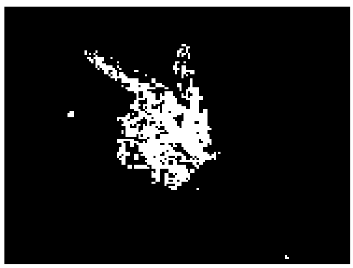
What we've learnt …

- Calculate region area and centroid
- Calculate region orientation
- Connected component labeling
- Simple region boundary → not good (WHY?)

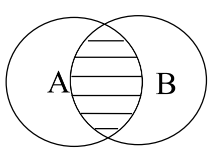
Questions

- Better boundary algorithms?
- What if the segmentation is noisy, i.e, the segments contain pepper noise, gaps, breaks?
  - A segment is not a solid segment
  - CCL will not work well
  - Examples:



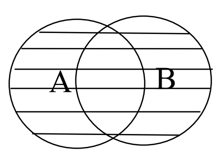
- Objective: a tool for noisy binary images

Preliminaries



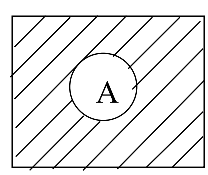
$A \cap B$

intersection



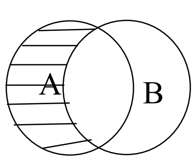
$A \cup B$

union



A^c

complement



difference:

$$\mathrm{A - B} = \{w \mid w \in A, w \notin B\} = A \cap B^c$$

$$\text{reflection}: \quad \hat{\mathrm{B}} = \{w \mid w \in -b, \ \forall b \in B\}$$

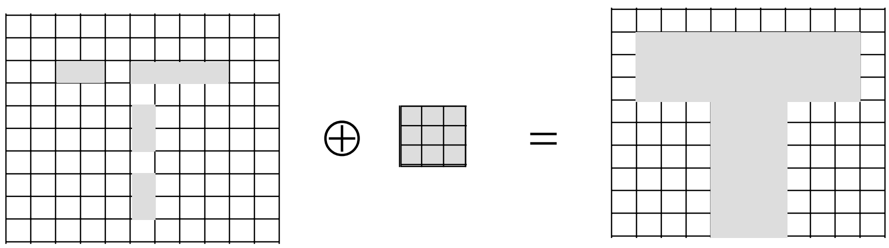
$$\text{translation}: \mathrm{A}_z = \{c \mid c = a + z, \ \forall a \in A\}$$

Dilation

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \phi\} = \bigcup_{a_i \in A} B_{a_i}$$

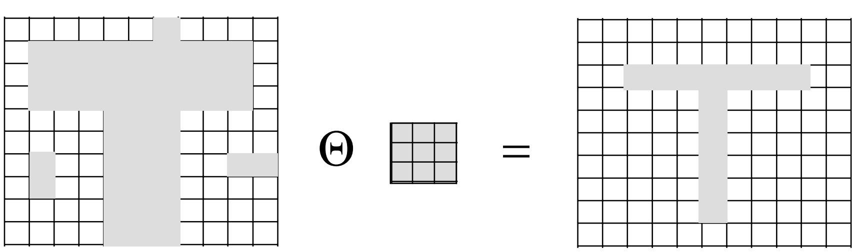
Structuring element, SE

Example: use of dilation for bridging gaps



Erosion

$$A \Theta B = \{z \mid B_z \subseteq A\}$$



Question : $(A \Theta B) \subseteq A$???

Dual

- Theorem: Dilation and erosion are duals

$$(A \Theta B)^c = A^c \oplus \hat{B}$$

- Proof:

$$\begin{aligned}(A \Theta B)^c &= \{z \mid B_z \subseteq A\}^c \\ &= \{z \mid B_z \cap A^c = \phi\}^c \\ &= \{z \mid B_z \cap A^c \neq \phi\} \\ &= A^c \oplus \hat{B}\end{aligned}$$

Opening and Closing

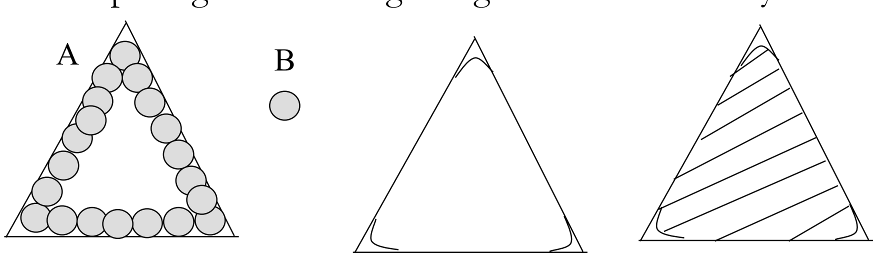
- Opening $A \circ B = (A \Theta B) \oplus B$
- Closing $A \bullet B = (A \oplus B) \Theta B$

Concepts

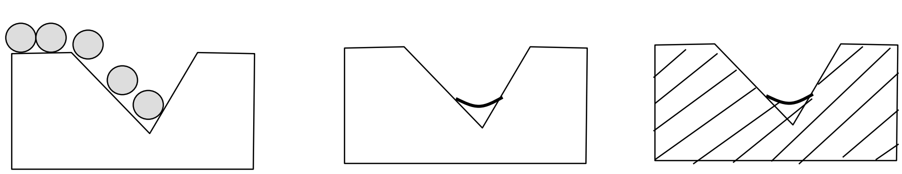
- Dilation
  - Expand an image
- Erosion
  - Shrink an image
- Opening
  - Smooth contour, break narrow isthmuses, eliminate thin protrusion
- Closing
  - Smooth contour, fuses narrow breaks and long thin gulfs, eliminate small holes, fill in gaps on contours

Geometric Explanation

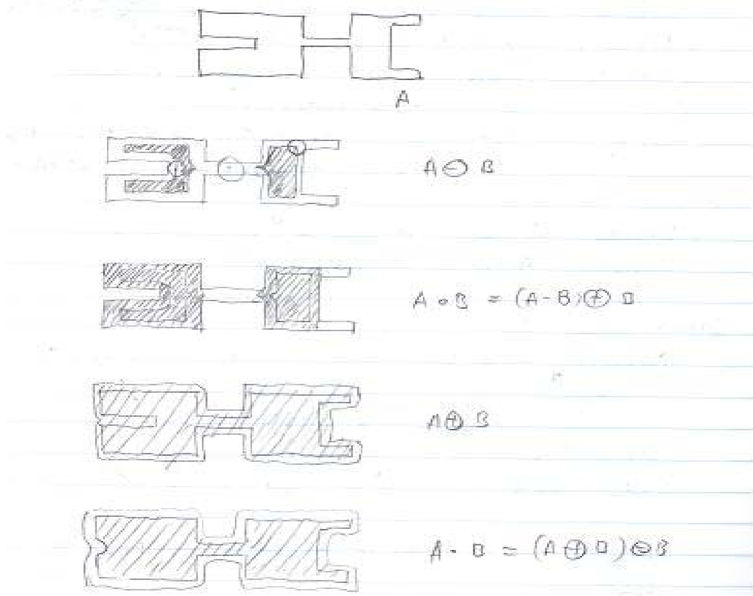
- Opening: SE B rolling along the inner boundary of A



- Closing: SE B rolling on the outer boundary of A



More examples



Properties

- Opening

1. $(A \circ B) \subseteq A$
2. if $C \subseteq D$

   then $C \circ B \subseteq D \circ B$
3. $(A \circ B) \circ B = A \circ B$

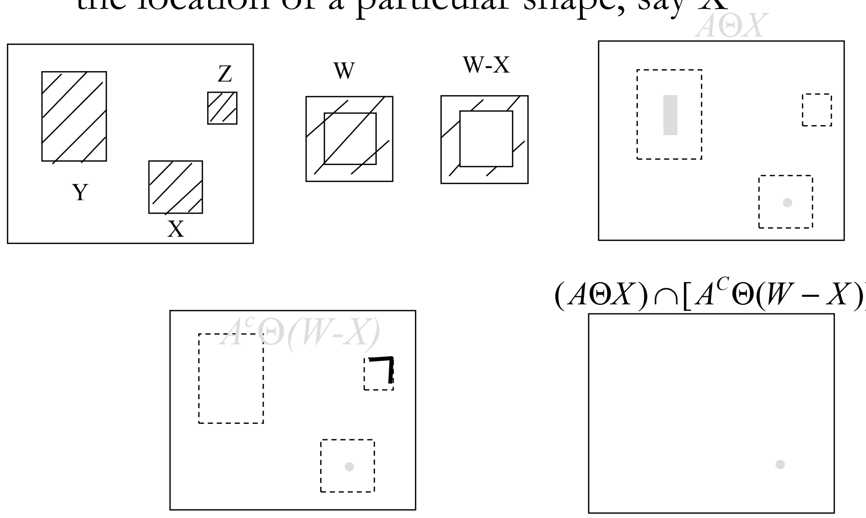
- Closing

1. $(A \bullet B) \supseteq A$
2. if $C \subseteq D$

   then $C \bullet B \subseteq D \bullet B$
3. $(A \bullet B) \bullet B = A \bullet B$

Hit-or-Miss transformation

- It is a basic tool for shape detection that finds the location of a particular shape, say X



Application

- Optical Character Recognition (OCR)
  – Assume the characters are of the same font & size
- Model construction
  1. Extract the character to be recognized
  2. Using opening and closing to fill holes and cavities
  3. Shrink the character image to remove unwanted region to reduce the size, s.t., it fits inside an instance of the character

Issues

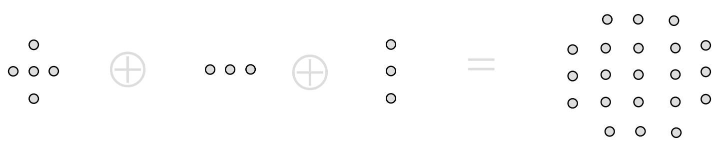
- Boundary extraction

$$\beta(A) = A - (A \Theta B)$$

- Computational issue

$$\text{if } \; B = B_1 \oplus B_2 \oplus \cdots \oplus B_k$$

$$\text{then } \quad A \oplus B = \{(A \oplus B_1) \oplus B_2 \oplus \cdots \oplus B_k\}$$



Question: why can this decomposition save computation?