Assignment 5

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I use MATLAB to calculate the result. Here is the result and code:

1. Use predictor-corrector method(predictor: Euler method; corrector: modified Euler method) to solve this problem. Choose step size 0.001 to ensure five digits correct.

```
f = @(y, x) sin(x) + y;

x0 = 0;
y0 = 2;

h = 0.001;

y01 = modified_euler(f, y0, x0, h, 0.1);
y05 = modified_euler(f, y0, x0, h, 0.5);

fprintf('y(0.1) = %.5f\n', y01);
fprintf('y(0.5) = %.5f\n', y05);

function y = modified_euler(f, y0, x0, h, xn)
    y = y0;
    for i = x0:h:xn-h
        % Predicxor:
        y_next = y + h*f(y, i);
        % Correcxor
        y_next = y + 0.5*h*f(y, i) + 0.5*h*f(y_next, i+h);

        y = y_next;
    end
end
```

```
>> Q1
y(0.1) = 2.21551
y(0.5) = 3.44330
```

2.

$$\frac{d}{dt}x(t) = f(x,t) \qquad \int_{t_n}^{t_{n+1}} f(x,t)\,dt = \int_{t_n}^{t_{n+1}} \frac{dx(t)}{dt}\,dt = x(t_{n+1}) - x(t_n) = c_0 f_{n-1} + c_1 f_n$$

Chose $t = -h,\ 0$ and $f(x,t) = 1,\ f(x,t) = t$

$$\begin{bmatrix} 1 & 1 \\ -h & 0 \end{bmatrix}\begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} \int_0^h 1\,dt \\ \int_0^h t\,dt \end{bmatrix} = \begin{bmatrix} h \\ \frac{h^2}{2} \end{bmatrix} \qquad \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = h\begin{bmatrix} -\frac{1}{2} \\ \frac{3}{2} \end{bmatrix}$$

$$x(t_{n+1}) = x(t_n) + \frac{h}{2}\left(-f(x_{n-1}, t_{n-1}) + 3f(x_n, t_n)\right)$$

3. a. Use ODE45 to solve

```
% a
% Define the system of differential equations
f = @(t, y) [y(2); y(3); 2*y(1) - t*y(2) + t];

% Initial conditions
y0 = [0; 1; 0];

% Solve the system using ode45
[~, y] = ode45(f, [0, 0.2], y0);
y02 = y(end, :)';
[~, y] = ode45(f, [0, 0.4], y0);
y04 = y(end, :)';
[~, y] = ode45(f, [0, 0.6], y0);
y06 = y(end, :)';

% Display the results
fprintf('y(0.2) = %.5f\n', y02(1));
fprintf('y(0.4) = %.5f\n', y04(1));
fprintf('y(0.6) = %.5f\n', y06(1));
```

```
y(0.2) = 0.20013
y(0.4) = 0.40213
y(0.6) = 0.61078
```

b. Use the results from (a) and the initial condition for the first 4 steps. Apply the Adams-Moulton method to derive y (0.8). Then, use the same method to derive y (1.0).

```
% b
% Step size
h = 0.2;

yn = y06;
yn_1 = y04;
yn_2 = y02;
yn_3 = y0;

% Solve the system using the Adams-Moulton method
for i = 0.6:h:0.8
    % Predictor
    y_next = yn + h/24 * (55*f(i, yn) - 59*f(i-h, yn_1) + 37*f(i-2*h, yn_2) - 9*f(i-3*h,yn_3));
    % Corrector
    y_next = yn + h/24 * (9*f(i+h, y_next) + 19*f(i, yn) - 5*f(i-h, yn_1) + 1*f(i-2*h,yn_2));

    yn_3 = yn_2;
    yn_2 = yn_1;
    yn_1 = yn;
    yn = y_next;
end

fprintf('y(1.0) = %.5f\n', yn(1));
```

```
y(1.0) = 1.08256
```

4.

$$x''(t_i) = \frac{x_{i+1} - 2x_i + x_{i-1}}{h^2}, \quad x'(t_i) = \frac{x_{i+1} - x_{i-1}}{2h}$$

4 sub intervals

x_{-1} x_0 x_1 x_2 x_3 x_4 x_5

0 $\frac{\pi}{8}$ $\frac{\pi}{4}$ $\frac{3}{8}\pi$ $\frac{\pi}{2}$ $h = \frac{\pi}{8}$

$$y'' + y = 0$$

$$\Rightarrow x_{i+1} - 2x_i + x_{i-1} - h^2(-x_i) = 0$$

$$\Rightarrow x_{i-1} + (h^2-2)x_i + x_{i+1} = 0$$

$$y(0) + y'(0) = 2 \qquad \Rightarrow x_0 + \frac{1}{2h}(x_1 - x_{-1}) = 2$$

$$y\left(\frac{\pi}{2}\right) + y'\left(\frac{\pi}{2}\right) = -1 \qquad \Rightarrow x_4 + \frac{1}{2h}(x_5 - x_3) = -1$$

$$\begin{pmatrix} 1 & h^2-2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & h^2-2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & h^2-2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & h^2-1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & h^2-1 & 1 \\ \frac{-1}{2h} & 1 & \frac{1}{2h} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{2h} & 1 & \frac{1}{2h} \end{pmatrix} \begin{pmatrix} x_{-1} \\ x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ -1 \end{pmatrix}$$

```
h = pi/8;

A = [1, h^2-2, 1, 0, 0, 0, 0;
     0, 1, h^2-2, 1, 0, 0, 0;
     0, 0, 1, h^2-2, 1, 0, 0;
     0, 0, 0, 1, h^2-2, 1, 0;
     0, 0, 0, 0, 1, h^2-2, 1;
     -1/(2*h), 1, 1/(2*h), 0, 0, 0, 0;
     0, 0, 0, 0, -1/(2*h), 1, 1/(2*h)];

b = [0; 0; 0; 0; 0; 2; -1];

y = A\b;

for i = -1:5
    fprintf('y(%.5f) = %.5f\n', i*h, y(i+2, 1));
end
```

```
y(0.00000) = 1.50931
y(0.39270) = 1.58563
y(0.78540) = 1.41742
y(1.17810) = 1.03063
y(1.57080) = 0.48490
```