

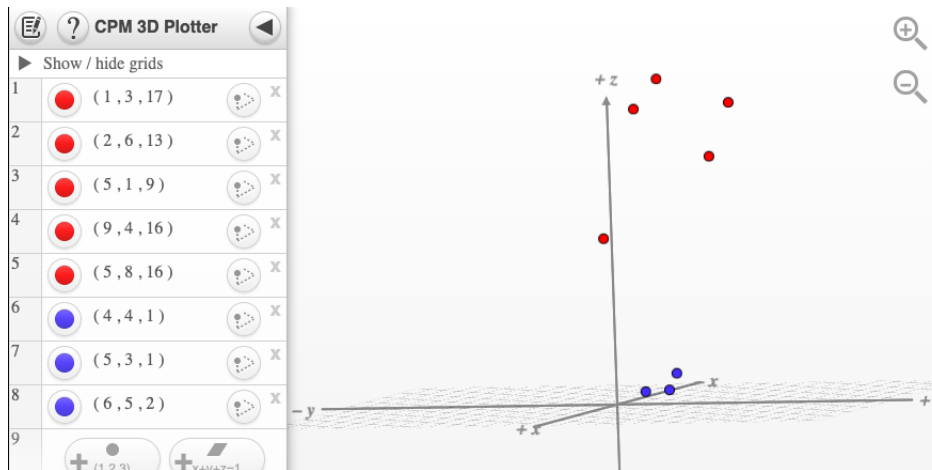
## Homework 5

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- You will need to submit your solutions in PDF to UVA-Collab.

- The maximum (hard) margin separating line is  $y = x + 1$ . The margin is  $\sqrt{2}$ . I know that this is the maximum margin because the graph is simple enough to see that the separating line can not be rotated or translated to give a larger margin.
  - The line  $y = x + 1$  would no longer be optimal because a soft margin classifier would allow certain extreme points to be classified incorrectly. I would estimate that an optimal soft margin classifier would something like a vertical line at  $x = 4$ . This is because the point at (6,8) seems to be a bit far from the rest of the (-) points.
  - The data transformation could be  $\phi((x, y)) = (x, y, (x - 5)^2 + (y - 4)^2)$ . I choose this function because the (+) points seem to be enclosed in a circle that is centered on (5,4) by the (-) points



- Let us add a new variable  $x_3$  that we will maximize under the following constraints

$$\begin{aligned}
 x_1 + x_2 &\geq 5 \\
 x_2 &\leq 2 \\
 2x_1 - x_2 &\geq x_3 \\
 -2x_1 + x_2 &\geq x_3
 \end{aligned}$$

- This problem can be reduced to a linear programming problem because the absolute value function is convex as adding convex functions to itself is still convex.
- If each player is playing randomly, they will have a  $1/3$  chance of selecting rock, a  $1/3$  chance of selecting paper, and a  $1/3$  chance of selecting scissors. The utility for selecting rock = the utility for selecting paper = utility for selecting scissors =  $1/3$  \*

$0 + 1/3 * 1 + 1/3 * -1 = 0$ . If one player changes their strategy while the other keep the strategy, the outcomes will stay the same because of the randomness of the other player.

b) If the column player cannot play scissors, the Nash equilibrium will be changed. We can eliminate the right column of the the row. We can also make a good guess that the row player will not play rock because it would either make him/her tie or lose. Given this information, we can try solving for the new Nash Equilibrium. Let  $P$  = the probability that Column Player plays Rock.

$$\begin{aligned} P * (-1) + (1 - P) * 0 &= -P \\ P * (1) + (1 - P) * -1 &= 2P - 1 \\ P &= 1/3 \end{aligned}$$

Now, let  $q$  = probability that Row Player plays Paper.

$$\begin{aligned} q * (1) + (1 - q) * -1 &= 2q - 1 \\ q * (0) + (1 - q) * 1 &= 1 - q \\ q &= 2/3 \end{aligned}$$

The new Nash equilibrium is Row Player: (rock: 0, paper: 2/3, scissors: 1/3) and Column Player: (rock: 1/3, paper: 2/3, scissors: 0).

4. a)  $(3000+4000)/2 * (1/3) + 4000 * (2/3) = 3833.33$  The expected revenue at equilibrium would be 3833.33
- b) i.  $5000 * (1/3) = 1666.67$ . There is a  $\frac{1}{3}$  chance for the buyer's value to be 5000.
- ii. The probability of the car being sold to the stranger at a certain price be  $P(x) = (6000-x)/3000$ . The expected value will be  $f(x) = x * P(x)$ . The max expected value from the other buyer be  $\max f(4000 < x \leq 6000)$  which is the derivative  $\frac{x}{3000} + \frac{x-3000}{x} = 0$  and  $x = 3000$  which is less than 4000 meaning that does not matter. Since we can sell it for 4000, 4000 is the best price to post.