# MAT 352 Assignment — 2

# Computer Science Department

## Question

**Prove that:**

1. $0 \leq P(A) \leq 1$
2. $P[(A \cap B) \cup (A \cap C) \cup (B \cap C)] = P(A \cap B) + P(A \cap C) + P(B \cap C) - 2P(A \cap B \cap C)$
3. $P(x = x) = q^{x-1}p$ where $x = 1, 2, \ldots$ and $q = 1 - p$

Submitted to Dr. Adinya

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## Name of Students

| S/N | Name | Matric Number |
|---|---|---|
| 1 | Adebowale Joseph Akintomiwa | 214846 |
| 2 | Adedapo Anjorin | 214864 |
| 3 | Adegbola Olatunde Williams | 207186 |
| 4 | Adeleke Sherifdeen Adeboye | 214848 |
| 5 | Adeleke Timothy Toluwani | 214849 |
| 6 | Adelowo Samuel Damilare | 214850 |
| 7 | Adeoti Warith Adetayo | 214851 |
| 8 | Adim Chimaobi Solomon | 222455 |
| 9 | Adisa Inioluwa Christiana | 214853 |
| 10 | Ahmad Animasaun | 214863 |
| 11 | Ajayi Prince Ayokunle | 215221 |
| 12 | Akinade Faith Eniola | 222459 |
| 13 | Akinrinola Akinfolarin | 205526 |
| 14 | Akinrinola, Blessing Opemipo | 214857 |
| 15 | Akinwusi Ifeoluwa | 214858 |
| 16 | Alao Tawakalit Omowunmi | 222461 |
| 17 | Alatise Oluwaseun Abraham | 214860 |
| 18 | Arowolo Ayomide Stephen | 214865 |
| 19 | Brai Daniel | 214868 |
| 20 | Chinedu Promise Okafor | 213930 |
| 21 | Daniel Emmanuel Oghenetega | 224870 |
| 22 | Denedo Oghenetega | 214873 |
| 23 | Emiade James | 214874 |

| | | |
|---|---|---|
| 24 | Farayola Joshua Olatunde | 214878 |
| 25 | Godwin Daniel | 214871 |
| 26 | Ibraheem Nuh Babatunde | 214879 |
| 27 | Ikwuegbu Michael | 214881 |
| 28 | Kareem Mustapha Babatunde | 214883 |
| 29 | Kayode Peter Temitope | 208077 |
| 30 | Kehinde Boluwatife Soyoye | 214916 |
| 31 | Kip Charles Okechukwu Emeka | 215061 |
| 32 | Matric | Number |
| 33 | Kubiat Laura | 214884 |
| 34 | Lawal Uchechukwu Adebayo | 214885 |
| 35 | Matthews Victoria Olayide | 214886 |
| 36 | Nwatu Chidinma Augustina | 214890 |
| 37 | Odulate Oluwatobi Gabriel | 214893 |
| 38 | Ogbolu Precious Chiamaka | 214894 |
| 39 | Oghie Daniel O. | 214895 |
| 40 | Ogunesan Rhoda Oluwatosin | 214897 |
| 41 | Ogunyemi Temidayo Samuel | 214898 |
| 42 | Ojewale Opeoluwa David | 214899 |
| 43 | Okafor Lisa Chisom | 214901 |
| 44 | Okoro Joshua Akachukwu | 214902 |
| 45 | Okumagba Oghenerukevwe Miracle | 222498 |
| 46 | Olagidi Joshua | 222500 |
| 47 | Olalere Khadijat Titilayo | 222502 |
| 48 | Olatunji Michael Oluwayemi | 214903 |
| 49 | Olawale Eniola Emmanuel | 214904 |

| | | |
|---|---|---|
| 50 | Olorogun Ebikabowei Caleb | 214906 |
| 51 | Oluwatade Iyanuoluwa | 214907 |
| 52 | Oluwayelu Oluwanifise | 215257 |
| 53 | Onasoga Oluwapelumi Idris | 214909 |
| 54 | Oyekanmi Eniola | 214913 |
| 55 | Sadiq Peter | 214914 |
| 56 | Salami Lateefat Abimbola | 214915 |
| 57 | Stephen Chidiebere Ivuelekwa | 214882 |
| 58 | Toluwanimi Oluwabukunmi Osuolale | 214912 |
| 59 | Ubaka Amazing-Grace Onyiyechukwu | 214918 |
| 60 | Uchechukwu Ahunanya | 214854 |
| 61 | Wisdom Oyor | 215206 |

# 1 Proof that $0 \leq P(A) \leq 1$

Let $S$ be a sample space and $A$ be an event defined on the sample space $S$

**Recall the Axioms of Probability:**

- Axiom 1: For any event $A$ of a sample space, $P(A) \geq 0$
- Axiom 2: For any sample space $S$, $P(S) = 1$

## Proof:

Let $A^C = S \setminus A$ (Complement of event $A$) and since $A$ and $A^C$ are mutually exclusive (i.e both events cannot occur simultaneously) therefore:

$$S = A \cup A^C \tag{1}$$
$$P(S) = P(A) + P(A^C) \tag{2}$$
$$P(A) = P(S) - P(A^C) \tag{3}$$

From equation (3) above, it can be seen that $P(A) \leq P(S)$. By Axiom 2 ($P(S) = 1$) therefore:

$$P(A) \leq P(S) \tag{4}$$
$$P(A) \leq 1 \tag{5}$$

By Axiom 1 ($P(A) \geq 0$) then:

$$0 \leq P(A) \tag{6}$$

Combining the inequalities (5) and (6) therefore:

$$0 \leq P(A) \leq 1 \tag{7}$$

## Alternatively:

By definition, the probability of an event $A$ is the number of times $m$ it is found (or it occured) within the total number $n$ of possibilities.

$$P(A) = \frac{m}{n} \tag{8}$$

Event $A$ may not be found at in all the total possibilities (i.e $m = 0$). It may also be found any number of times between 0 and $n$ (i.e $0 < m < n$) and finally, it may be found exactly $n$ number of times (i.e $m = n$). Therefore yielding the bound for $m$ as:

$$0 \leq m \leq n \tag{9}$$

Dividing the inequality (9) above through by $n$ gives:

$$\frac{0}{n} \leq \frac{m}{n} \leq \frac{n}{n}$$
$$0 \leq \frac{m}{n} \leq 1 \tag{10}$$

Substituting equation (8) in the inequality (10) above:

$$0 \leq P(A) \leq 1$$

## 2 Proof that $P[(A \cap B) \cup (A \cap C) \cup (B \cap C)] = P(A \cap B) + P(A \cap C) + P(B \cap C) - 2P(A \cap B \cap C)$

The proof can be shown by using the Inclusion-Exclusion Rule for any $n$ number of events which states that:

$$\begin{aligned} P(E_1 \cup E_2 \cup E_3 \cup \cdots \cup E_n) = &\sum_{i=1}^{n} P(E_i) - \sum_{i_1 < i_2} P(E_{i_1} \cap E_{i_2}) + \sum_{i_1 < i_2 < i_3} P(E_{i_1} \cap E_{i_2} \cap E_{i_3}) + \cdots + \\ &(-1)^{r+1} \sum_{i_1 < i_2 < \cdots < i_r} P(E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_r}) + \cdots + \\ &(-1)^{n} \sum_{i_1 < i_2 < \cdots < i_{n-1}} P(E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_{n-1}}) + \\ &(-1)^{n+1} P(E_1 \cap E_2 \cap E_3 \cap \cdots \cap E_n) \end{aligned} \tag{11}$$

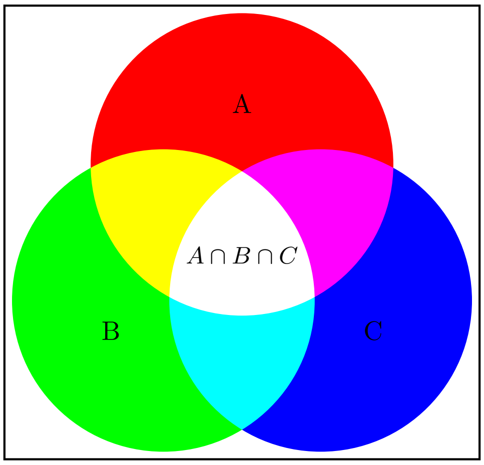
Therefore:

$$\begin{aligned} P([A \cap B] \cup [A \cap C] \cup [B \cap C]) = &P(A \cap B) + P(A \cap C) + P(B \cap C) - P([A \cap B] \cap [A \cap C]) \\ &- P([A \cap B] \cap [B \cap C]) - P([A \cap C] \cap [B \cap C]) \\ &+ P([A \cap B] \cap [A \cap C] \cap [B \cap C]) \end{aligned} \tag{12}$$

From Set Theory:

$$P([A \cap B] \cap [A \cap C]) = P(A \cap B \cap C)$$
$$P([A \cap B] \cap [A \cap C] \cap [B \cap C]) = P(A \cap B \cap C)$$

This is illustrated using the Venn diagram below




- $[A \cap B] \cap [A \cap C]$: $[A \cap B]$ includes the yellow and the white regions and $[A \cap C]$ is the purple and the white regions. The region common to both is the white region $(A \cap B \cap C)$
- $[A \cap B] \cap [B \cap C]$: $[A \cap B]$ includes the yellow and the white regions and $[B \cap C]$ is the cyan and the white regions. The region common to both is the white region $(A \cap B \cap C)$
- $[A \cap C] \cap [B \cap C]$: $[A \cap C]$ includes the purple and the white regions and $[B \cap C]$ is the cyan and the white regions. The region common to both is the white region $(A \cap B \cap C)$
- $[A \cap B] \cap [A \cap C] \cap [B \cap C]$: $[A \cap B]$ = yellow and white region, $[A \cap C]$ = purple and white region, $[B \cap C]$ = cyan and white region. Common to all three regions is the white region $(A \cap B \cap C)$

Equation (12) is the simplified as:

$$\begin{aligned} P([A \cap B] \cup [A \cap C] \cup [B \cap C]) = &P(A \cap B) + P(A \cap C) + P(B \cap C) - P(A \cap B \cap C) \\ &- P(A \cap B \cap C) - P(A \cap B \cap C) + P(A \cap B \cap C) \end{aligned} \tag{13}$$

$$\begin{aligned} = &P(A \cap B) + P(A \cap C) + P(B \cap C) \\ &- P(A \cap B \cap C) - P(A \cap B \cap C) \end{aligned} \tag{14}$$

Finally:

$$P([A \cap B] \cup [A \cap C] \cup [B \cap C]) = P(A \cap B) + P(A \cap C) + P(B \cap C) - 2P(A \cap B \cap C) \tag{15}$$

## 3 Proof that $P(x = x) = q^{x-1}p$ where $x = 1, 2, \ldots$ and $q = 1 - p$

The proof for this formular can be shown by **Axiom 2** of the probability theory that:

$$P(S) = 1 \tag{16}$$

where $S$ is the sample space.

**Given that $X$ is a random variable with $X \in \{1, 2, \ldots\}$ and $q = 1 - p$**

$$\begin{aligned} P(S) &= P(X = 1) + P(X = 2) + P(X = 3) + \cdots && (17)\\ &= q^{1-1}p + q^{2-1}p + q^{3-1}p + \cdots && (18)\\ &= q^0p + q^1p + q^2p + \cdots && (19)\\ &= p + qp + q^2p + \cdots && (20) \end{aligned}$$

The equation above shows an infinite geometric sum where: $a_1 = p$, $a_2 = qp$, $a_3 = q^2p$, $\ldots$ and the common ratio $r = q$

The sum of infinite geometric series is given by:

$$\begin{aligned} S_n = a_1 + a_2 + a_3 + \cdots &= \sum_{i=1}^{\infty} a_i \\ &= \frac{a}{1 - r} \end{aligned}$$

Therefore, we can simplify (20) by using the geometric sum and substituting $q = 1 - p$.

$$\begin{aligned} P(S) &= p + qp + q^2p + \cdots \\ &= \frac{p}{1 - q} \\ &= \frac{p}{1 - (1 - p)} \\ &= \frac{p}{1 - 1 + p} \\ &= \frac{p}{p} \\ &= 1 \end{aligned}$$

So, $P(S) = \sum_{i=1}^{\infty} P[X = x] = \sum_{i=1}^{\infty} q^{x-1}p = 1$.

**Therefore it is true that $P[X = x] = q^{x-1}p$ where $x = 1, 2, \ldots$ and $q = 1 - p$**