MAT 352 Assignment

Computer Science Department

March 2023

Question

1. Prove that
   (a) $0 \leq P(E) \leq 1$
   (b) $P[(A \cap B) \cup (A \cap C) \cup (B \cap C)] = P(A \cap B) + P(A \cap C) + P(B \cap C) - 2P(A \cap B \cap C)$
   (c) $P(x = x) = q^{x-1}p$ where $x = 1, 2$ and $q = 1 - p$

a. Prove that $0 \leq P(E) \leq 1$

Proof:

Let S be the sample space, and let A be any event.

Remember that:

- **Axiom 1:** $P(A) \geq 0$
- **Axiom 2:** $P(S) = 1$

Note that by Axiom 1, $P(A) \geq 0$.
Then $S = A \cup (S \setminus A)$, where $(S \setminus A)$ means everything in S but not in A.

NB: $P(A) + P(S \setminus A)$ means A and $(S \setminus A)$ are mutually exclusive.

By Axiom 1: $P(A) > 0$

$P(A) + P(S \setminus A) \geq P(A) + 0$

$P(S) \geq P(A) + P(S \setminus A)$

$P(S) \geq P(A)$

NB: $P(S) = 1$ i.e Axiom 2 probability of simple space= 1

Thus, $P(S) \geq P(A)$

$I > P(A)$

$P(A) \leq 1$

Thus, since Axiom 1: $P(A) \geq 0$

And by Axiom 2: $P(A) \leq 1$

Therefore $0 \leq P(A) \leq I$

b. Prove that $P[(A \cap B) \cup (A \cap C) \cup (B \cap C)] = P(A \cap B) + P(A \cap C) + P(B \cap C) - 2P(A \cap B \cap C)$

Since $(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Then

$P((A \cap B) \cup (A \cap C) \cup (B \cap C)) = P(A \cap B) + P(A \cap C) + P(B \cap C) - P((A \cap B) \cap (A \cap C)) - P((A \cap B) \cap (B \cap C)) - P(A \cap C \cap B \cap C) + P((A \cap B) \cap (A \cap C) \cap (B \cap C))$

Recall

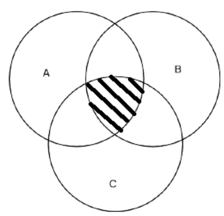

Figure 1: Venn Diagram

instead of labeling the shaded area, you can do the below!
Let the shaded area be T
From the Venn diagram, it can be seen that

- $(A \cap B) \cap (A \cap C) = T$
- $(B \cap B) \cap (B \cap C) = T$
- $(A \cap C) \cap (B \cap C) = T$
- $(A \cap B \cap C) = T$

Therefore,
$(A \cap B) \cap (A \cap C) = (A \cap B) \cap (B \cap C) = (A \cap C) \cap (B \cap C) = A \cap B \cap C$
$= P(A \cap B) + P(A \cap C) + P(B \cap C) - P(A \cap B \cap C) - P(A \cap B \cap C) - P(A \cap B \cap C) + P(A \cap B \cap C)$

$$= P(A \cap B) + P(A \cap C) + P(B \cap C) - 2P(A \cap B \cap C)$$