MAT 352 Assignment — 2

Computer Science Department

Question

Prove that:

1. $0 \leq P(A) \leq 1$
2. $P[(A \cap B) \cup (A \cap C) \cup (B \cap C)] = P(A \cap B) + P(A \cap C) + P(B \cap C) - 2P(A \cap B \cap C)$
3. $P(x = x) = q^{x-1}p$ where $x = 1, 2, \ldots$ and $q = 1 - p$

Submitted to Dr. Adinya

March 31, 2023

Name of Students

S/N	Name	Matric Number
1	Adebowale Joseph Akintomiwa	214846
2	Adedapo Anjorin	214864
3	Adegbola Olatunde Williams	207186
4	Adeleke Sherifdeen Adeboye	214848
5	Adeleke Timothy Toluwani	214849
6	Adelowo Samuel Damilare	214850
7	Adeoti Warith Adetayo	214851
8	Adim Chimaobi Solomon	222455
9	Adisa Inioluwa Christiana	214853
10	Ahmad Animasaun	214863
11	Ajayi Prince Ayokunle	215221
12	Akinade Faith Eniola	222459
13	Akinrinola Akinfolarin	205526
14	Akinrinola, Blessing Opemipo	214857
15	Akinwusi Ifeoluwa	214858
16	Alao Tawakalit Omowunmi	222461
17	Alatise Oluwaseun Abraham	214860
18	Arowolo Ayomide Stephen	214865
19	Brai Daniel	214868
20	Chinedu Promise Okafor	213930
21	Daniel Emmanuel Oghenetega	224870
22	Denedo Oghenetega	214873
23	Emiade James	214874

24	Farayola Joshua Olatunde	214878
25	Godwin Daniel	214871
26	Ibraheem Nuh Babatunde	214879
27	Ikwuegbu Michael	214881
28	Kareem Mustapha Babatunde	214883
29	Kayode Peter Temitope	208077
30	Kehinde Boluwatife Soyoye	214916
31	Kip Charles Okechukwu Emeka	215061
32	Matric	Number
33	Kubiat Laura	214884
34	Lawal Uchechukwu Adebayo	214885
35	Matthews Victoria Olayide	214886
36	Nwatu Chidinma Augustina	214890
37	Odulate Oluwatobi Gabriel	214893
38	Ogbolu Precious Chiamaka	214894
39	Oghie Daniel O.	214895
40	Ogunesan Rhoda Oluwatosin	214897
41	Ogunyemi Temidayo Samuel	214898
42	Ojewale Opeoluwa David	214899
43	Okafor Lisa Chisom	214901
44	Okoro Joshua Akachukwu	214902
45	Okumagba Oghenerukevwe Miracle	222498
46	Olagidi Joshua	222500
47	Olalere Khadijat Titilayo	222502
48	Olatunji Michael Oluwayemi	214903
49	Olawale Eniola Emmanuel	214904

50	Olorogun Ebikabowei Caleb	214906
51	Oluwatade Iyanuoluwa	214907
52	Oluwayelu Oluwanifise	215257
53	Onasoga Oluwapelumi Idris	214909
54	Oyekanmi Eniola	214913
55	Sadiq Peter	214914
56	Salami Lateefat Abimbola	214915
57	Stephen Chidiebere Ivuelekwa	214882
58	Toluwanimi Oluwabukunmi Osuolale	214912
59	Ubaka Amazing-Grace Onyiyechukwu	214918
60	Uchechukwu Ahunanya	214854
61	Wisdom Oyor	215206

1 Proof that $0 \leq P(A) \leq 1$

Let S be a sample space and A be an event defined on the sample space S

Recall the Axioms of Probability:

- Axiom 1: For any event A of a sample space, $P(A) \geq 0$
- Axiom 2: For any sample space S, $P(S) = 1$

Proof:

Let $A^C = S \setminus A$ (Complement of event A) and since A and A^C are mutually exclusive (i.e both events cannot occur simultaneously) therefore:

$$S = A \cup A^C \tag{1}$$
$$P(S) = P(A) + P(A^C) \tag{2}$$
$$P(A) = P(S) - P(A^C) \tag{3}$$

From equation (3) above, it can be seen that $P(A) \leq P(S)$. By Axiom 2 ($P(S) = 1$) therefore:

$$P(A) \leq P(S) \tag{4}$$
$$P(A) \leq 1 \tag{5}$$

By Axiom 1 ($P(A) \geq 0$) then:

$$0 \leq P(A) \tag{6}$$

Combining the inequalities (5) and (6) therefore:

$$0 \leq P(A) \leq 1 \tag{7}$$

Alternatively:

By definition, the probability of an event A is the number of times m it is found (or it occured) within the total number n of possibilities.

$$P(A) = \frac{m}{n} \tag{8}$$

Event A may not be found at in all the total possibilities (i.e $m = 0$). It may also be found any number of times between 0 and n (i.e $0 < m < n$) and finally, it may be found exactly n number of times (i.e $m = n$). Therefore yielding the bound for m as:

$$0 \leq m \leq n \tag{9}$$

Dividing the inequality (9) above through by n gives:

$$\frac{0}{n} \leq \frac{m}{n} \leq \frac{n}{n}$$
$$0 \leq \frac{m}{n} \leq 1 \tag{10}$$

Substituting equation (8) in the inequality (10) above:

$$0 \leq P(A) \leq 1$$

2 Proof that $P[(A \cap B) \cup (A \cap C) \cup (B \cap C)] = P(A \cap B) + P(A \cap C) + P(B \cap C) - 2P(A \cap B \cap C)$

The proof can be shown by using the Inclusion-Exclusion Rule for any n number of events which states that:

$$\begin{aligned} P(E_1 \cup E_2 \cup E_3 \cup \cdots \cup E_n) = & \sum_{i=1}^{n} P(E_i) - \sum_{i_1<i_2} P(E_{i_1} \cap E_{i_2}) + \sum_{i_1<i_2<i_3} P(E_{i_1} \cap E_{i_2} \cap E_{i_3}) + \cdots + \\ & (-1)^{r+1} \sum_{i_1<i_2<\cdots<i_r} P(E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_r}) + \cdots + \\ & (-1)^{n} \sum_{i_1<i_2<\cdots<i_{n-1}} P(E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_{n-1}}) + \\ & (-1)^{n+1} P(E_1 \cap E_2 \cap E_3 \cap \cdots \cap E_n) \end{aligned} \tag{11}$$

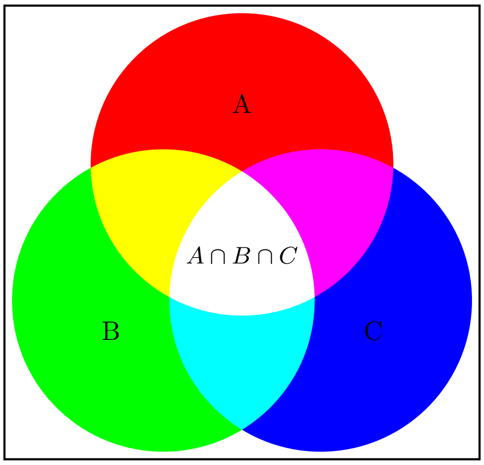
Therefore:

$$\begin{aligned} P([A \cap B] \cup [A \cap C] \cup [B \cap C]) = & P(A \cap B) + P(A \cap C) + P(B \cap C) - P([A \cap B] \cap [A \cap C]) \\ & - P([A \cap B] \cap [B \cap C]) - P([A \cap C] \cap [B \cap C]) \\ & + P([A \cap B] \cap [A \cap C] \cap [B \cap C]) \end{aligned} \tag{12}$$

From Set Theory:

$$P([A \cap B] \cap [A \cap C]) = P(A \cap B \cap C)$$
$$P([A \cap B] \cap [A \cap C] \cap [B \cap C]) = P(A \cap B \cap C)$$

This is illustrated using the Venn diagram below



- $[A \cap B] \cap [A \cap C]$: $[A \cap B]$ includes the yellow and the white regions and $[A \cap C]$ is the purple and the white regions. The region common to both is the white region $(A \cap B \cap C)$
- $[A \cap B] \cap [B \cap C]$: $[A \cap B]$ includes the yellow and the white regions and $[B \cap C]$ is the cyan and the white regions. The region common to both is the white region $(A \cap B \cap C)$
- $[A \cap C] \cap [B \cap C]$: $[A \cap C]$ includes the purple and the white regions and $[B \cap C]$ is the cyan and the white regions. The region common to both is the white region $(A \cap B \cap C)$
- $[A \cap B] \cap [A \cap C] \cap [B \cap C]$: $[A \cap B]$ = yellow and white region, $[A \cap C]$ = purple and white region, $[B \cap C]$ = cyan and white region. Common to all three regions is the white region $(A \cap B \cap C)$

Equation (12) is the simplified as:

$$\begin{aligned} P([A \cap B] \cup [A \cap C] \cup [B \cap C]) = & P(A \cap B) + P(A \cap C) + P(B \cap C) - P(A \cap B \cap C) \\ & - P(A \cap B \cap C) - P(A \cap B \cap C) + P(A \cap B \cap C) \end{aligned} \tag{13}$$

$$\begin{aligned} = & P(A \cap B) + P(A \cap C) + P(B \cap C) \\ & - P(A \cap B \cap C) - P(A \cap B \cap C) \end{aligned} \tag{14}$$

Finally:

$$P([A \cap B] \cup [A \cap C] \cup [B \cap C]) = P(A \cap B) + P(A \cap C) + P(B \cap C) - 2P(A \cap B \cap C) \tag{15}$$

3 Proof that $P(x = x) = q^{x-1}p$ where $x = 1, 2, \ldots$ and $q = 1 - p$

The proof for this formular can be shown by **Axiom 2** of the probability theory that:

$$P(S) = 1 \tag{16}$$

where S is the sample space.

Given that X is a random variable with $X \in \{1, 2, \ldots\}$ and $q = 1 - p$

$$\begin{aligned} P(S) &= P(X = 1) + P(X = 2) + P(X = 3) + \cdots && (17)\\ &= q^{1-1}p + q^{2-1}p + q^{3-1}p + \cdots && (18)\\ &= q^0 p + q^1 p + q^2 p + \cdots && (19)\\ &= p + qp + q^2 p + \cdots && (20) \end{aligned}$$

The equation above shows an infinite geometric sum where: $a_1 = p$, $a_2 = qp$, $a_3 = q^2p$, $\ldots$ and the common ratio $r = q$

The sum of infinite geometric series is given by:

$$\begin{aligned} S_n = a_1 + a_2 + a_3 + \cdots &= \sum_{i=1}^{\infty} a_i \\ &= \frac{a}{1 - r} \end{aligned}$$

Therefore, we can simplify (20) by using the geometric sum and substituting $q = 1 - p$.

$$\begin{aligned} P(S) &= p + qp + q^2 p + \cdots \\ &= \frac{p}{1 - q} \\ &= \frac{p}{1 - (1 - p)} \\ &= \frac{p}{1 - 1 + p} \\ &= \frac{p}{p} \\ &= 1 \end{aligned}$$

So, $P(S) = \sum_{i=1}^{\infty} P[X = x] = \sum_{i=1}^{\infty} q^{x-1}p = 1$.

Therefore it is true that $P[X = x] = q^{x-1}p$ where $x = 1, 2, \ldots$ and $q = 1 - p$