

# Downward Nominal Wage Rigidity and State-Dependent Government Spending Multipliers<sup>☆</sup>

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## Abstract

Despite much empirical evidence on business cycle-dependent government spending multipliers, the theoretical channels underlying such results are uncertain. In an environment with involuntary unemployment, this paper shows that downward nominal wage rigidity, which arises only in recessions, can generate business cycle-dependent government spending multipliers. In line with Keynesian views, a demand stimulus reduces unemployment in recessions and may not drive up inflation and wages as in expansions. Thus, the positive income effects from reduced unemployment and weaker crowding-out effects from a smaller increase in the real interest rate enhance the expansionary spending effects in recessions. The theoretical implications are largely consistent with existing empirical evidence on business cycle-dependent government spending effects on key macroeconomic variables.

*Keywords:* business cycle-dependent multiplier, state dependent government spending effects, downward nominal wage rigidity, fiscal policy effects, New Keynesian models

*JEL codes:* E31, E62, H30

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## 1. Introduction

Do government spending multipliers differ between recessions and expansions? A burgeoning empirical literature generally indicates bigger multipliers in recessions than in expansions (e.g., Auerbach and Gorodnichenko, 2012, 2013; Bachmann and Sims, 2012; Baum et al., 2012; Caggiano et al., 2015; Fazzari et al., 2015; Furceri and Li, 2017).<sup>1</sup>

Despite largely consistent empirical support, the theoretical channels through which

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<sup>1</sup>Caggiano et al. (2015) find significant differences in multipliers when comparing deep recessions to strong expansions, but little difference in multipliers between mild recessions and expansions.

government spending has bigger multipliers in recessions than in expansions are uncertain.<sup>2</sup> Canzoneri et al. (2016) propose that countercyclical variation in bank intermediation costs can generate business cycle-dependent multipliers, as a government spending increase reduces interest rate spreads, facilitating private borrowing in recessions. Also, Michailat (2014) shows that increasing public employment in expansions raises labor costs, thus dampening its effectiveness to raise aggregate employment.

We propose an alternative theoretical channel through which downward nominal wage rigidity (DNWR) in recessions can contribute to the business cycle-dependent multipliers. Using a simplified New Keynesian (NK) model, we first examine its log-linearized equilibrium to illustrate the key mechanisms analytically. Next, we quantify the multipliers with an NK model that is solved fully nonlinearly. Consistent with most empirical evidence, we obtain robust simulation results that output multipliers are bigger in recessions than in expansions. Moreover, the theoretical implications for consumption, unemployment, real wages, and inflation are largely in line with existing empirical evidence on business cycle-dependent government spending effects (see Tagkalakis, 2008; Auerbach and Gorodnichenko, 2013; Fazzari et al., 2015).

DNWR is well documented and prevalent. Using micro-level data of U.S. and European countries, Dickens et al. (2007) estimate that, on average 28 percent of the wage cuts that would have taken place under flexible wage setting were averted by DNWR.<sup>3</sup> To see whether DNWR is discernible in macro-level data, we plot employment, nominal hourly compensation, and labor productivity for the U.S. from 1969Q4 to 2015Q4.<sup>4</sup> The top plot in Figure 1 shows that except for the 1969 and 2001 recessions, labor productivity fell in each of the seven recessions since 1955, yet the nominal hourly compensation largely increased

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<sup>2</sup>A competing view posits that multipliers may not differ much across business cycles states; see Ramey and Zubairy (forthcoming) and Sims and Wolff (forthcoming).

<sup>3</sup>Other papers presenting evidence on DNWR include Nickell and Quintini (2003), Messina et al. (2010), Abbritti and Fahr (2013), and Holden and Wulfsberg (2009). See Kim and Ruge-Murcia (2009) for a survey of earlier evidence on DNWR.

<sup>4</sup>All the data come from the Bureau of Labor Statistics (BLS) and are seasonally adjusted: labor productivity (measured by output per hour) is series PRS85006093; nominal hourly compensation is series PRS85006103; employment is converted from total nonfarm employment (series CES0000000001) to an index series. Shading in Figure 1 indicates the recession periods identified by the NBER.

even for much of the Great Recession. The average productivity decline over these seven recessions is 1.2% from the beginning quarter to the quarter of the lowest productivity within a recession, yet the corresponding nominal wage rate grows by 3% and the real wage rate grows by 0.1%.<sup>5</sup> The bottom plot in Figure 1 compares the quarterly growth rates of nominal hourly compensation and labor productivity. Despite negative labor productivity growth rates in most recessions, growth rates of hourly nominal compensation (solid line) are seldom negative; most incidents of negative growth rates occurred within the Great Recession (2007Q4-2009Q2) and in the subsequent slow recovery period.

Intuitively, business cycle-dependent multipliers arise because higher government spending in recessions reduces unemployment and does not drive up the real interest rate as much as in expansions. In expansions, a spending increase has the usual crowding out effect. Firms hire more labor in response to higher goods demand, increasing wages and inflation. Monetary authorities then raise the nominal rate and indirectly the real rate, crowding out private demand, partially offsetting the expansionary effects of government spending.

In recessions, if DNWR binds—resulting from a high nominal wage rate in the prior period combined with a severe enough recession, then DNWR prevents nominal wages from falling beyond the floor despite weak private demand. Under such circumstances, higher government spending (if not sufficiently large) does not increase nominal wages immediately, whereas inflation rises because of higher goods demand from more government spending. With higher inflation, the binding nominal wage floor translates into a lower real wage floor than otherwise without a spending increase. Thus, with DNWR, a government spending increase does not immediately drive up firms’ marginal cost in recessions, as it does in expansions, which produces smaller increases in inflation and the real interest rate, generating less crowding out than in expansions.

Moreover, DNWR enhances the expansionary spending effects through reduced unemployment. As the effective real wage—the wage subject to DNWR—is higher than the labor market-clearing wage, DNWR creates involuntary unemployment. Additional gov-

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<sup>5</sup>Real wage changes are computed by subtracting inflation from nominal wage growth rates, and inflation is calculated from the consumer price index published by the BLS.

ernment demand reduces unemployment, resulting from the falling effective wage (inflation lowers the real wage floor) and the rising market-clearing wage (higher goods demand). Reduced unemployment boosts income and consumption, making government spending more expansionary in recessions.

Under the baseline simulation in the quantitative analysis, we find that the impact multiplier in the recession state is 1.37, versus 0.54 in the expansion state. We also explore different initial conditions with respect to recession severity and degrees of DNWR. Output multipliers are larger with a more severe recession and a higher degree of DNWR. Also, a larger spending increase generates weaker expansionary effects in the recession state than a smaller spending increase, because of stronger crowding-out effects.

Our findings extend the literature on state-dependent multipliers. Aside from business cycle states studied here, the states examined include monetary policy (e.g., Davig and Leeper, 2011; Leeper et al., 2017) and government indebtedness (e.g., Ilzetzki et al., 2013; Nickel and Tudyka, 2014; Bi et al., 2016). In particular, Christiano et al. (2011) and Erceg and Lindé (2014) show that a government spending under the binding zero lower bound can be much bigger than under normal monetary policy. Our specification of monetary policy allows an occasionally binding zero lower bound (ZLB). In an alternative simulation, where a recession is sufficiently severe to trigger the binding ZLB, we find that the impact output multiplier can rise to 2.32, in line with the finding in Christiano et al. (2011). This result shows that both DNWR and the ZLB work in the same direction, to enlarge government spending effects in recessions.

## 2. The Model

We use a simple NK model with elastic labor supply and inelastic capital. The central frictions are price rigidities and DNWR, as in Schmitt-Grohe and Uribe (2016). DNWR generates involuntary unemployment in the model, crucial to yield business cycle-dependent multipliers. Our baseline specification features the GHH preference (Greenwood et al., 1988), which does not have a wealth effect on labor supply, motivated by Schmitt-Grohe

and Uribe (2012).<sup>6</sup>

### 2.1. Households

A representative household with a GHH preference chooses composite consumption ( $c_t$ ), labor ( $n_t$ ), and nominal bonds ( $B_t$ ) to maximize utility over an infinite time horizon:

$$\max \sum_{t=0}^{\infty} d_t \frac{[c_t - \chi(n_t)^\varphi]^{1-\sigma}}{1-\sigma}, \quad (1)$$

where  $\sigma$  is the inverse of the intertemporal elasticity of substitution and  $\varphi$  governs the Frisch elasticity of labor supply. Define  $d_t = \prod_{j=1}^t \beta_j$  for  $t > 0$ , and  $d_0 = 1$ , where  $\beta_j$  is the time-varying discount factor in period  $j$ . The composite consumption is aggregated from differentiated goods  $c_t(i)$  with the Dixit and Stiglitz (1977) aggregator

$$c_t = \left( \int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \quad (2)$$

where  $\theta > 1$  is the intratemporal elasticity of substitution between varieties. The demand function for each good  $i$  is

$$c_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} c_t, \quad (3)$$

where  $P_t(i)$  is the nominal price for  $c_t(i)$  and  $P_t$  is the aggregate price level. The household's budget constraint is

$$c_t + \frac{B_t}{P_t} + z_t = \frac{W_t n_t}{P_t} + \frac{R_{t-1} B_{t-1}}{P_t} + \int_0^1 \Gamma(i) di, \quad (4)$$

where  $W_t$  is the nominal wage rate,  $B_t$  is the one-period nominal bond holding in zero net supply,  $R_{t-1}$  is the nominal interest rate between  $t-1$  and  $t$ ,  $\Gamma(i)$  is the profit from firm  $i$ , and  $z_t$  is lump-sum taxes. The transversality condition for bonds must hold, implying

$$\lim_{T \rightarrow \infty} E_t q_{t,T} \frac{B_T}{P_T} = 0, \quad (5)$$

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<sup>6</sup>In a DSGE model that accommodates various degrees of wealth effects on labor supply, Schmitt-Grohe and Uribe's (2012) estimation finds the wealth effects on labor supply is almost nonexistent.

where  $q_{t,T} \equiv \frac{R_{t-1}}{(P_T/P_t)}$ .

To model DNWR, we assume that

$$W_t \geq W_{t-1}, \quad (6)$$

so that  $W_{t-1}$  is a price floor for nominal wages in period  $t$ . Our specification is embedded in the general specification proposed by Schmitt-Grohe and Uribe (2016), where  $W_t \geq \gamma W_{t-1}$ . Even though  $\gamma = 1$  appears special, it does not affect the analytical linear results, because  $\gamma$  is dropped in the linearized equilibrium. In the quantitative analysis with a fully nonlinear solution, we explore the implication of  $0 < \gamma < 1$ .

Unemployment is cyclical; it arises when the economy is hit by contractionary shocks that lower the labor market clearing wage below the wage floor. A binding DNWR generates involuntary unemployment. The unemployment rate is computed as

$$u_t \equiv \frac{n_t^s - n_t}{n_t^s} \times 100\%, \quad (7)$$

where  $n_t$  is actual labor worked, and  $n_t^s$  is desired labor supplied obtained from the household's optimality condition for labor

$$\chi \varphi(n_t^s)^{\varphi-1} = \frac{W_t}{P_t} \equiv w_t, \quad (8)$$

where  $n_t^s$  depends on the real wage rate,  $w_t$ . The household, however, may be unable to work the desired hours. In each period, the nominal wage and labor satisfy the slackness condition

$$(n_t^s - n_t)(W_t - W_{t-1}) = 0, \quad (9)$$

which says that either the labor market clears ( $n_t^s = n_t$ ) or DNWR binds ( $W_t = W_{t-1}$ ).

## 2.2. Firms

The economy has two types of firms: a representative competitive final goods producer and monopolistically competitive intermediate goods producers who produce a continuum

of differentiated goods, indexed by  $i$ . The final goods producer produces composite goods using the technology

$$y_t = \left[ \int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}. \quad (10)$$

The intermediate goods firm  $i$  produces using labor and linear technology:

$$y_t(i) = a n_t(i), \quad (11)$$

where  $a$  is the common technology. Cost minimization implies that each intermediate firm has the same real marginal cost:

$$mc_t = \frac{w_t}{a}. \quad (12)$$

Following Calvo (1983), a fraction  $1 - \omega$  of intermediate firms can optimally choose their nominal prices each period. Firms which get a chance to reset their prices in period  $t$  choose their price to maximize the expected sum of discounted future real profits:

$$\max_{P_t(i)} E_t \sum_{j=0}^{\infty} (\omega\beta)^j \frac{\lambda_{t+j}}{\lambda_t} \left[ \frac{P_t(i)}{P_{t+j}} - mc_{t+j} \right] y_{t+j}(i), \quad (13)$$

subject to

$$y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} y_t. \quad (14)$$

The first order condition to determine the optimal price  $P_t^*$  is given by

$$\frac{P_t^*}{P_t} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{j=0}^{\infty} (\omega\beta)^j \lambda_{t+j} y_{t+j} mc_{t+j} \left( \frac{P_t}{P_{t+j}} \right)^{-\theta}}{E_t \sum_{j=0}^{\infty} (\omega\beta)^j \lambda_{t+j} y_{t+j} \left( \frac{P_t}{P_{t+j}} \right)^{1-\theta}}, \quad (15)$$

which can be rewritten as

$$\frac{P_t^*}{P_t} = \frac{\theta}{\theta - 1} \frac{k_{1t}}{k_{2t}}, \quad (16)$$

where  $k_{1t} = \lambda_t y_t mc_t + \omega\beta E_t k_{1t+1} \pi_{t+1}^{\theta}$  and  $k_{2t} = \lambda_t y_t + \omega\beta E_t k_{2t+1} \pi_{t+1}^{\theta-1}$ . Using the aggregate

price index  $P_t^{1-\theta} = (1-\omega)(P_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta}$ , inflation is solved for as

$$\pi_t = \left[ \frac{1}{\omega} - \frac{1-\omega}{\omega} \left( \frac{P_t^*}{P_t} \right)^{1-\theta} \right]^{\frac{1}{\theta-1}} = \left[ \frac{1}{\omega} - \frac{1-\omega}{\omega} \left( \frac{\theta}{\theta-1} \frac{k_{1t}}{k_{2t}} \right)^{1-\theta} \right]^{\frac{1}{\theta-1}}. \quad (17)$$

Compute aggregate labor as  $n_t = \int_0^1 n_t(i) di$ . Linear aggregation of individual market clearing conditions implies the aggregate production function is given by

$$y_t = \frac{a n_t}{\Delta_t}, \quad (18)$$

where  $\Delta_t$  is the relative price dispersion, defined as  $\Delta_t \equiv \int_0^1 \left[ \frac{P_t(i)}{P_t} \right]^{-\theta} di$ . Using the aggregate price index,  $\Delta_t$  evolves according to

$$\Delta_t = (1-\omega) \left( \frac{P_t^*}{P_t} \right)^{-\theta} + \omega \pi_t^\theta \Delta_{t-1}. \quad (19)$$

### 2.3. Fiscal and Monetary Policy

In the analytical model, we assume that government spending,  $g_t$ , follows a simple stochastic process:

$$g_t = g e^{\varepsilon_t^g}, \quad (20)$$

where  $\varepsilon_t^g \sim_{i.i.d.} N(0, \sigma_g^2)$ . Throughout the paper, a variable without a time subscript indicates its steady state value. The government budget constraint is

$$g_t = z_t. \quad (21)$$

Monetary authorities adopt a simple Taylor rule, adjusting the nominal interest rate,  $R_t$ , in response to the inflation rate according to

$$R_t = \max \left( R \left( \frac{\pi_t}{\pi} \right)^\alpha, 1 \right), \quad (22)$$

with  $\alpha > 1$ , implying that the interest rate responds to inflation more than one for one. We impose the ZLB restriction at  $R_t = 1$ , implying a net interest rate of zero. Our baseline



analysis does not have the binding ZLB, as we intend to focus on the equilibrium of an active monetary policy and passive fiscal policy in the sense of Leeper (1991). In the quantitative analysis, we investigate the circumstance under which the ZLB binds.

The aggregate resource constraint is

$$y_t = c_t + g_t. \quad (23)$$

The equilibrium conditions of the analytical model consist of equations (A.1) to (A.12) in Appendix A.

### 3. Analytics of State-Dependent Multipliers

Before launching into the quantitative analysis with a complicated nonlinear equilibrium, we begin with a simpler linear approximation that yields an analytical solution for government spending multipliers. This demonstrates how DNWR aggravates a recession and generates business cycle-dependent multipliers. As in Christiano et al. (2011), a discount factor shock generates business cycles in the model. Also, to simplify the derivation, we assume that the steady-state inflation rate ( $\pi$ ) and labor ( $n$ ) are both 1. A variable with a “^” indicates the percent deviation from its steady-state value.

To derive the “IS” equation, substitute the monetary policy rule (22) into the intertemporal Euler equation (A.3). Log-linearization yields

$$\hat{\lambda}_t = E_t \hat{\lambda}_{t+1} + \alpha \hat{\pi}_t - E_t \hat{\pi}_{t+1} + E_t \hat{\beta}_{t+1}. \quad (24)$$

Also, log-linearizing the marginal utility of consumption (A.1) yields

$$\hat{\lambda}_t = -\frac{\sigma c}{c - \chi n^\varphi} \hat{c}_t + \frac{\sigma \chi \varphi n^\varphi}{c - \chi n^\varphi} \hat{n}_t. \quad (25)$$

Define the steady-state government spending-to-output ratio as  $s_g \equiv \frac{g}{y}$ ; steady-state consumption is then  $c = (1 - s_g)y$ . Solving for the model-implied parameter  $\chi$  as  $\frac{\theta-1}{\theta\varphi}$ ,

equation (25) becomes

$$\hat{\lambda}_t = -\frac{\sigma\varphi\theta(1-s_g)}{\theta\varphi(1-s_g)-\theta+1}\hat{c}_t + \frac{\sigma\varphi(\theta-1)}{\theta\varphi(1-s_g)-\theta+1}\hat{n}_t. \quad (26)$$

The production function (18) and the aggregate resource constraint (23) imply

$$\hat{c}_t = \frac{1}{1-s_g}\hat{y}_t - \frac{s_g}{1-s_g}\hat{g}_t \quad (27)$$

$$\hat{n}_t = \hat{y}_t. \quad (28)$$

Substituting (25), (27), and (28) into (24) yields the IS equation:

$$\hat{y}_t = E_t\hat{y}_{t+1} - \Psi(\alpha\hat{\pi}_t - E_t\hat{\pi}_{t+1}) + \theta s_g(\hat{g}_t - E_t\hat{g}_{t+1}) - \Psi E_t\hat{\beta}_{t+1}, \quad (29)$$

where  $\Psi = \frac{\theta\varphi(1-s_g)-\theta+1}{\sigma\varphi}$ .

Combining (16) and (17) and log-linearizing yield the Phillips curve:

$$\hat{\pi}_t = \frac{(1-\omega)(1-\omega\beta)}{\omega}\hat{m}c_t + \beta E_t\hat{\pi}_{t+1}. \quad (30)$$

The IS equation, (29), and the Phillips curve, (30), fully characterize the equilibrium. We now define the business cycle states in the linearized equilibrium.

**Definition 1.** *In period  $t$ , the economy is in the expansion state if  $\hat{\beta}_t = b_L < 0$  and is in the recession state if  $\hat{\beta}_t = b_H > 0$ , and  $b_L = -b_H$ .*

When  $\hat{\beta}_t = b_L$ , households are less patient than in the steady state, which makes them consume more. This increases the demand of intermediate goods firms, generating the expansion, and vice versa for  $\hat{\beta}_t = b_H$ , which generates the recession.

Empirically, the duration of an expansion typically exceeds that of a recession.<sup>7</sup> Thus, we assume that households in the expansion state are more likely to expect that the state

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<sup>7</sup>Among the 11 recessions since 1948 in the U.S., the average duration of recessions is 11 months and of expansions is 58 months (based on the NBER's business cycle dates).

will continue in the next period than in the recession state. To capture this idea and to make the solution tractable, we make the following assumption about the expectations on future discount factors in the two states.

**Assumption 1.** *When the economy is in the expansion state,  $P(E_t\hat{\beta}_{t+1} = b_L|\hat{\beta}_t = b_L) = 1$ ,  $P(E_t\hat{\beta}_{t+2} = b_L|E_t\hat{\beta}_{t+1} = b_L) = 0.5$ , and  $P(E_t\hat{\beta}_{t+2} = b_H|E_t\hat{\beta}_{t+1} = b_L) = 0.5$ . When the model economy is in the recession state,  $P(E_t\hat{\beta}_{t+1} = b_H|\hat{\beta}_t = b_H) = 1$ ,  $P(E_t\hat{\beta}_{t+2} = b_H|E_t\hat{\beta}_{t+1} = b_H) = 0$ , and  $P(E_t\hat{\beta}_{t+2} = b_L|E_t\hat{\beta}_{t+1} = b_H) = 1$ .*

Assumption 1 says that, when the economy is in the expansion state at time  $t$ , households expect that expansion will continue at  $t+1$  with probability 1 and switch to recession at  $t+2$  with probability 0.5. On the other hand, if the economy is in the recession state at  $t$ , they also expect that the economy would continue to be in recession in  $t+1$  with probability 1 but will switch to expansion at  $t+2$  with probability 1. Later we adopt general AR(1) specifications for the shocks. In that environment, households form expectations based on the stochastic processes of shocks.

Under Definition 1 and Assumption 1, we obtain Proposition 1:

**Proposition 1.** *Without DNWR, the government spending multiplier is given by*

$$M_y = \frac{\omega\theta}{\omega + \Psi\alpha(1-\omega)(1-\omega\beta)(\varphi-1)}, \quad (31)$$

*which takes on the same value in both the expansion and recession states.*

*Proof.* See Appendix B.1. □

Under the common values calibrated for the Frisch elasticity of labor supply ( $\varphi \geq 2$ , implying the Frisch elasticity is less than or equal to 1) and the government-to-output share ( $s^g \ll 0.5$ ),  $\Psi = \frac{\theta\varphi(1-s^g)-\theta+1}{\sigma\varphi} > 0$  and  $M_y > 0$ . Since the model economy without DNWR does not have unemployment in either state, the government spending multiplier is positive under full employment in our model.

Note that with flexible prices ( $\omega = 0$ ), equation (31) implies a government spending multiplier of zero. Under flexible prices and the GHH preference which eliminates the negative

wealth effect on labor supply, a government spending increase leaves the labor supply curve unchanged and thus has no stimulus power. On the other hand, nominal price stickiness combined with the GHH preference can produce positive government spending multipliers. In response to higher aggregate demand (as prices do not rise fully immediately), the labor demand curve shifts right, leading to a higher wage rate and labor that generate positive spending multipliers.<sup>8</sup>

Next, we consider a scenario where a contractionary discount factor shock (a big value of  $b_H$ ) hits at time  $t$  that triggers binding DNWR, similar to the assumption made in Christiano et al. (2011).

**Assumption 2.** *Suppose that the economy at  $t - 1$  has full employment (i.e., DNWR does not bind) and  $\hat{w}_{t-1} > 0$ . At time  $t$ , the economy is hit by a contractionary discount factor shock,  $b_H$ . Then, from Assumption 1,  $E_t \hat{\beta}_{t+1} = b_H$  and  $E_t \hat{\beta}_{t+2} = b_L$ . Let  $b_H$  be sufficiently high to make DNWR bind at  $t$ , while  $b_L$  is sufficiently low to generate time- $t$  expectation of full employment for  $t + 2$ .*

By assumption 2, binding DNWR lasts for one period at time  $t$ , and the economy goes back to full employment state at time  $t + 1$ .<sup>9</sup> At time  $t$ , DNWR results in real wage rigidities. To see this, rewrite (6) in real terms,

$$w_t \geq \frac{w_{t-1}}{\pi_t}, \quad (32)$$

which indicates that inflation at  $t$  can lower the real wage floor. Log-linearizing (32) yields

$$\hat{w}_t \geq \hat{w}_{t-1} - \hat{\pi}_t. \quad (33)$$

Let  $*$  denote the labor market clearing equilibrium values for endogenous variables; i.e.,

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<sup>8</sup>Monacelli and Perotti (2008) show that, with nominal price stickiness, the size of the multiplier increases as the degree of complementarity between hours and consumption rises. Such complementarity is the highest without the wealth effect on labor supply, as in the GHH preference. Under this setup, a wide range of parameterizations can generate multipliers exceeding 1.

<sup>9</sup>Later in the quantitative analysis, the periods of binding DNWR are endogenously determined.

the equilibrium values as if there were no DNWR. Then  $\hat{w}_t^*$  is the labor market clearing wage rate, and DNWR binds when

$$\hat{w}_t^* < \hat{w}_{t-1} - \hat{\pi}_t. \quad (34)$$

Under this circumstance, the effective real wage rate is  $\hat{w}_t = \hat{w}_{t-1} - \hat{\pi}_t > \hat{w}_t^*$ . In the recession state, a contractionary discount factor shock decreases goods demand and labor demand, which lowers inflation ( $\hat{\pi}_t$ ) and the market clearing real wage ( $\hat{w}_t^*$ ). Given the real wage from the prior period ( $\hat{w}_{t-1}$ ), equation (34) implies that a lower  $\hat{w}_t^*$  and a lower  $\hat{\pi}_t$  make DNWR more likely to bind.<sup>10</sup>

In our model (and likely empirically), DNWR causes labor market distortions and aggravates a recession. A negative demand shock discourages consumption and suppresses aggregate demand. If there were no DNWR, the real wage rate would fall until the labor market clears. The decreased real wage also implies that goods prices adjust downward despite nominal price rigidities, which mitigates part of the negative effect on goods demand from the demand shock.

With DNWR, as the real wage cannot fall below its floor, the effective real wage decreases by less, creating unemployment. The increased unemployment generates a negative income effect, further lowering consumption. Also, a smaller decline in the real wage means that price cannot fall as much as without DNWR; thus the negative impact on private demand from the original discount factor shock cannot be offset by lower goods prices. As a result, the output loss is larger in the recession with DNWR.

**Proposition 2.** *Under Assumption 2, the government spending multiplier in the recession state with DNWR,  $M_y^{DNWR}$ , is bigger than that in the expansion state:*

$$M_y^{DNWR} = \theta > M_y. \quad (35)$$

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<sup>10</sup>To see explicitly how a contractionary discount factor shock lowers the labor market clearing wage, note that in the recession state  $E_t \hat{\beta}_{t+1} = b_H$  from Assumption 1. By combining (B.1) and (B.4) in Appendix B.1, we have the labor market clearing wage  $\hat{w}_t^* = (\varphi - 1)A_y^R \hat{g}_t + (\varphi - 1)B_y^R b_H$ , implying  $\frac{\partial \hat{w}_t^*}{\partial b_H} = (\varphi - 1)B_y^R < 0$ .

*Proof.* See Appendix B.2. □

As mentioned, in the expansion state, more government spending adds to goods demand and generates expansionary effects. At the same time, high labor demand raises the real wage. An increase in firms' real marginal cost pushes up inflation. Under active monetary policy and sticky prices, the nominal interest rate and hence the real interest rate increase, crowding out private demand.

A bigger government spending multiplier in the recession state occurs because DNWR deepens a recession. In the recession state with DNWR, an increase in government spending mitigates the labor market distortion caused by DNWR and amplifies the expansionary effects through lower unemployment and smaller crowding out effects. As DNWR is binding, high goods demand increases labor demand, shrinking unemployment. Also, with *i.i.d.* government spending shocks that are not big enough to dissipate DNWR at  $t$  (as assumed here), a spending increase does not drive up the real wage. As households do not expect higher future demand under *i.i.d.* shocks, a one-time government spending increase in the recession state does not drive up real marginal costs and, hence, inflation. As a result, monetary authorities do not raise the nominal interest rate, leaving the real rate unchanged. Unlike a government spending increase in the expansion state, the real wage and the real interest rate both do not rise in response to a government spending increase, eliminating the crowding out effect. This result is formalized in Proposition 3.

**Proposition 3.** *In the expansion state, a government spending increase has no effect on unemployment ( $\frac{\partial \hat{u}_t}{\partial \hat{g}_t} = 0$ ) but raises the real interest rate ( $\frac{\partial \hat{r}_t}{\partial \hat{g}_t} > 0$ ). In the recession state with binding DNWR, a government spending increase reduces unemployment ( $\frac{\partial \hat{u}_t}{\partial \hat{g}_t} < 0$ ) but does not increase the real interest rate ( $\frac{\partial \hat{r}_t}{\partial \hat{g}_t} = 0$ ).*

*Proof.* See Appendix B.3. □

The result that the real interest rate does not rise in response to a government spending increase in recession is a special case. Later when a general government spending process is adopted (like an AR(1)), current inflation can rise because of expected higher future

government spending. The result that government spending multiplier is bigger in the recession state than in the expansion state, however, remains.

#### 4. Quantitative Analysis

The above analysis highlights the mechanisms through which DNWR generates business cycle-dependent multipliers. That analysis, while illuminating, does not produce empirically sensible results as many strong assumptions (such as *i.i.d.* spending shocks and stylized expectations for discount factors) are necessary to produce the analytical solution.

We now relax these assumptions to obtain a fully nonlinear solution under rational expectations. We first quantify government spending multipliers under the baseline simulations and study how various initial conditions—i.e. recession severity—affect multipliers. One of the initial conditions simulated is a deep recession with the binding ZLB, a condition of particular interest after the Great Recession. Next, we investigate how the degree of DNWR and the size of the government spending matter for government spending effects in recessions. Finally, to further check the robustness of business cycle-dependent multipliers, Appendix D explores the commonly used KPR preference (King et al., 1988).

The quantitative model modifies the setup in Section 3. We assume that both discount factor and government spending shocks follow AR(1) processes:

$$\ln \frac{\beta_t}{\beta} = \rho^\beta \ln \frac{\beta_{t-1}}{\beta} + \varepsilon_t^\beta; \text{ and} \quad (36)$$

$$\ln \frac{g_t}{g} = \rho^g \ln \frac{g_{t-1}}{g} + \varepsilon_t^g, \quad (37)$$

where  $\varepsilon_t^\beta \sim_{i.i.d.} N(0, \sigma_\beta^2)$  and  $\varepsilon_t^g \sim_{i.i.d.} N(0, \sigma_g^2)$ . We inject sufficiently contractionary discount factor shocks (i.e., positive  $\varepsilon_t^\beta$ ) to generate a recession state where DNWR binds. Also, we use the general specification for DNWR as laid out in Schmitt-Grohe and Uribe (2016),

$$W_t \geq \gamma W_{t-1}, \quad (38)$$

where  $0 \leq \gamma \leq 1$ , and the price floor for the nominal wage is  $\gamma W_{t-1}$ . When  $\gamma = 0$ , nominal

wages are perfectly flexible; when  $\gamma = 1$ , they are absolutely downward rigid as assumed earlier. Dividing both sides of equation (38) by the price,  $P_t$ , we obtain the nominal wage constraint in real terms:

$$w_t \geq \gamma \frac{w_{t-1}}{\pi_t}. \quad (39)$$

Lastly, we adopt a general monetary policy rule to allow the nominal interest rate to respond to inflation and output

$$R_t = \max \left( R \left( \frac{\pi_t}{\pi} \right)^{\alpha_\pi} \left( \frac{y_t}{y} \right)^{\alpha_y}, 1 \right), \quad (40)$$

where  $\alpha_\pi > 1$  and  $\alpha_y > 0$ .<sup>11</sup>

#### 4.1. Calibration and the Solution Method

We use the following baseline parameter values, typical of those in the macro literature. The quarterly real interest rate is set to 1% so  $\beta = 0.99$ . Preferences over consumption are logarithmic, so  $\sigma = 1$ . A Frisch labor elasticity of 0.5 implies that  $\varphi = 3$ , and  $\chi$  is determined by the assumption that the steady-state labor is  $n = 1$ . We assume that intermediate goods firms' price markup is 15 percent, so  $\theta = 7.67$ . The degree of price stickiness,  $\omega$ , is set to be 0.75, implying average price rigidities of one year. We assume zero steady-state inflation so  $\pi = 1$ . The steady-state government spending-to-output ratio is 0.2. To calibrate the stochastic government spending process, we estimate an AR(1) process using detrended U.S. data, which yields  $\rho_g = 0.81$  and  $\sigma_g = 0.0096$ .<sup>12</sup> Given a wide range of values on the persistence in the discount factor used in the literature—e.g., 0.80 in Fernández-Villaverde et al. (2015a) and 0.18 in Fernández-Villaverde et al. (2015b), we set  $\rho_\beta = 0.6$  and  $\sigma_\beta = 0.0008$ .<sup>13</sup> Monetary policy follows a Taylor principle with  $\alpha_\pi = 1.5$  and  $\alpha_y = 0.04$ .

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<sup>11</sup>We also conduct the simulation using a Taylor rule with interest rate smoothing, and the key result that the output multiplier is bigger in recessions than in expansions does not change qualitatively. This result is available upon request.

<sup>12</sup>Government spending data are the sum of government consumption expenditure and gross government investment (NIPA Table 3.1, lines 21 and 39) less consumption of fixed capital (NIPA Table 3.1, line 42), deflated by the GDP deflator. The constructed government spending series is detrended by the HP filter.

<sup>13</sup>A relatively small  $\sigma_\beta$  is chosen such that the expected frequency of ZLB events is not too high to ensure determinacy. The size of  $\sigma_\beta$  in general does not matter much for the results.



To calibrate the degree of DNWR,  $\gamma$ , we follow Schmitt-Grohe and Uribe’s (2016) method. Since  $\gamma$  is a constraint on the lower bound of the nominal wage, we resort to the largest economic downturn in the postwar U.S. history—the Great Recession—to calibrate  $\gamma$ . In this recession, the peak nominal hourly compensation occurs in 2008Q4 (an index number of 100.2) and the trough occurs in 2009Q1 (97.7), implying a ratio of 0.975. Since changes in nominal wages also reflect real economic growth, we divide this ratio by the long-run average quarterly growth rate in real GDP from 1947 to 2015 to obtain  $\gamma = 0.96$ .

The model is solved using the monotone mapping method, a numerical algorithm based on Coleman (1991) and Davig (2004), to get a fully nonlinear solution under rational expectations. This method discretizes the state space, which requires a set of initial guesses and finds a fixed point in decision rules for each point in the state space. Let  $\mathbf{S}_t$  denote the state space at time  $t$ . The solutions converge to functions that map the minimum set of state variables—the current discount factor, government spending, lagged relative price dispersion, and the real wage rate ( $\mathbf{S}_t = [\beta_t, g_t, \Delta_{t-1}, w_{t-1}]$ )—into values for the endogenous variables ( $n_t$ ,  $k_{1t}$ , and  $k_{2t}$  in (16)). Appendix C describes the methodology in detail.

#### 4.2. Quantifying Business Cycle-Dependent Multipliers: the Baseline Simulation

To derive an analytical solution, the earlier analysis for the expansion state focuses on one particular discount factor value ( $\hat{\beta}_L = b_L < 0$ , see Assumption 1), which drives output above the steady state. Since our comparison of spending multipliers in different business cycle states can be thought of as multiplier differences when DNWR binds or not, in the quantitative analysis we broadly define that expansion states to be those in which DNWR does not bind, including the steady state. In the analysis hereinafter, the expansionary state analyzed is the steady state.

In Figure 2, the recession state (solid lines) is generated by injecting a contractionary discount factor shock at time 0, which leads the discount factor to rise by 2% above its steady-state level ( $\beta$  from 0.99 to 1.01).<sup>14</sup> To generate a recession where DNWR binds at

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<sup>14</sup>A discount factor larger than one implies that households value future consumption more than current consumption. This is one way to generate large negative demand shocks, as in Christiano et al. (2011) and Fernández-Villaverde et al. (2015a).

time 0, we need to have a high real wage in the previous period,  $w_{t-1}$ , as shown in (39). To obtain this, from  $t = -5$  to  $-1$  we inject a series of expansionary discount factor shocks (which take negative values), so that the real wage is 6.07% above the steady state at  $t = -1$ .<sup>15</sup> Combining a high real wage rate at  $t = -1$  and a contractionary discount factor shock at time 0 produces DNWR that binds at time 0, with an unemployment rate of 5.14% and output that is 4.44% below the steady state. These initial macroeconomic conditions are close to those of the Great Recession: the Congressional Budget Office estimates that the average output gap from 2008 to 2010 is about 4% (Congressional Budget Office, 2017). Also, the 2010 unemployment rate increases by 5 percentage points from the 2007 level.<sup>16</sup>

In both states, government spending increases by 1% of the steady-state level at time 0, following the process of (37). Figure 2 shows that the main differences between the two states lie in unemployment, output, the real wage rate, inflation, and the real interest rate, as in the analytical model. As the initial states are different, the units are measured by gaps between paths with and without the government spending shock, scaled by the stochastic steady-state values, except for unemployment and the discount factor: the unemployment rate is in level difference in percent (see equation (7)) between the paths with and without the spending shock, and the discount factor is in percent deviation from the steady state. Since the expansion state has full employment with or without the spending shock, the difference in the unemployment rates stays at zero throughout the horizon. Table 1 reports the cumulative government spending multipliers for output and consumption in the two states, computed as

$$\frac{\sum_{i=1}^k r_{t+i-1}^{-1} \Delta x_{t+i-1}}{\sum_{i=1}^k r_{t+i-1}^{-1} \Delta g_{t+i-1}}, \quad x \in \{y, c\}, \quad (41)$$

where  $\Delta$  denotes level changes relative to a path without a government spending increase, and  $r_t \equiv E_t \frac{R_t}{\pi_{t+1}}$  is the real interest rate.

In either state, government spending increases labor and output, but it is much more expansionary in recession than in expansion. Output rises by 0.27% of their steady-state

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<sup>15</sup>We set  $\left\{\hat{\beta}_t\right\}_{t=-5}^0 = [-0.87\%, -1.38\%, -1.69\%, -1.88\%, -1.99\%, 2.01\%]$ .

<sup>16</sup>Based on the BLS data, the average unemployment rate is 4.6% in 2007 and is 9.6% in 2010.

values in recession, compared to only 0.11% in expansion. The impact multiplier for output (consumption) is 1.37 (0.37) in recession, versus 0.54 (−0.46) in expansion. This quantitative assessment is roughly in line with the estimates by Auerbach and Gorodnichenko (2012). Their baseline results have the short-run (the first year) government spending multipliers between 0 and 0.5 in expansions, and between 1 and 1.5 in recessions.<sup>17</sup>

Why do impact multipliers for consumption have opposite signs in different business cycle states? The positive short-run consumption responses in recession can be explained from the real interest rate channel and the unemployment channel, consistent with the analytics section. In expansion, the consumption decline is driven mainly by the rising real interest rate and the resulting crowding out. In recession, the interest rate channel still operates, but the crowding out effect is smaller. With persistent positive spending shocks, the expected inflation rate rises and hence the current inflation rate. The magnitude of this inflation increase is, however, smaller than in expansion (as shown in Figure 2). Because of binding DNWR, the nominal wage does not rise in recession in the first two quarters. The inflation lowers the real wage rate ( $\gamma \frac{w_{t-1}}{\pi_t}$  implied by (39)) and hence the real marginal cost. As a result, firms do not raise goods price as much as in expansion, generating less inflation. The smaller increase in inflation leads to a smaller increase in real interest rates, and less crowding out in recession than in expansion.

For the unemployment channel, a spending increase has positive income effects from reduced unemployment in recession, absent in expansion. More government spending raises the market-clearing wage because of higher demand, and lowers the real wage floor because of inflation. As the real wage decreases, labor demand rises and labor supply falls, shrinking unemployment and boosting household income. An increase in private demand leads to a further rise in labor demand, a further decline in unemployment, and a further increase in consumption. Given the positive income effect from reduced unemployment on consumption and the small crowding out effects from a modest increase in the real interest

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<sup>17</sup>Our longer term multipliers are not directly comparable to those in Auerbach and Gorodnichenko (2012), as their empirical estimates are periodic multipliers and we have cumulative multipliers. Moreover, by assuming the recessions and expansions are absorbing states, the estimated multipliers in the longer horizon in Auerbach and Gorodnichenko (2012) are considered as the upper bound of multipliers.

rate, consumption responds positively in recession, in contrast to the negative response in expansion.

Our finding of a positive consumption response in recession is consistent with Dupor et al. (2017) that a government spending increase can generate positive consumption when the nominal wage gets stuck at a level above the market clearing wage. This provides an alternative explanation for the empirical VAR evidence that a government spending increase crowds in consumption (Fatas and Mihov, 2001; Blanchard and Perotti, 2002; Bouakez and Rebei, 2007). This explanation differs from hand-to-mouth consumers (Gali et al., 2007; Colciago, 2007; Furlanetto, 2011), complementarity between consumption and government spending (Bouakez and Rebei, 2007), or deep habits (Zubairy, 2014). The importance of these competing channels remains to be verified empirically.<sup>18</sup>

A notable difference between the quantitative analysis and the analytical results is the real interest rate response in the recession state: the real rate rises here but remains unchanged in the earlier analytical results. The difference arises because the quantitative analysis assumes that the government spending increase is persistent, versus an *i.i.d* shock in the analytical analysis. A persistent spending increase implies higher future goods demand and inflation. From equation (30), we see that expected higher future inflation drives up current inflation and the real interest rate. Despite different response patterns of the real interest rate, both the analytical results and the quantitative simulation have smaller increases in the real rate in recession than in expansion, producing less crowding out in recession. Thus, a government spending increase remains more expansionary in recession regardless of the extent of the spending shock persistence.

#### 4.3. *Different Initial Recession States and the Binding ZLB*

Our analysis of fiscal multipliers in the recession state has a spending shock occur in an initial state other than the steady state. A natural question is how different initial states, recession severity, matter for multipliers? In the analysis below, we conduct simulations

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<sup>18</sup>Since our simulation has negative consumption responses in expansion, we recognize that the other channels identified in the literature as mentioned here may still be important to the empirical positive consumption response to a government spending increase.

for mild and deep recessions, relative to the baseline recession state. Since a sufficiently deep recession can make the ZLB bind, we also analyze an initial condition that combines a deep recession with the binding ZLB.

Figure 3 compares the recession state in the baseline simulation to two other initial recession conditions: a mild recession (dashed lines) and a deep recession (dotted-dashed lines). The mild and deep recessions have output gaps of  $-2.49\%$  and  $-7.01\%$  at  $t = 0$ , compared to the baseline's  $-4.44\%$ .<sup>19</sup> A more severe recession generates higher unemployment as goods demand shrinks more, lowering firms' labor demand further. With higher unemployment, a given government spending increase generates a bigger positive income effect by reducing unemployment more: the spending increase in the deep recession reduces unemployment by  $7.76\%$  from  $t = 0$  to  $t = 2$ , while the same size increase in government spending only reduces unemployment by  $3.11\%$  from  $t = 0$  to  $t = 1$  in the mild recession. In Figure 3, we see that the bigger positive income effect induces a more positive consumption response in the deep recession.

In addition to the unemployment channel, the crowding out effect also differs via the real interest rate channel. In the mild recession, DNWR binds for one period, while it binds for two periods with the deep recession (as well as the baseline recession). As the real wage rebounds sooner in the mild recession, the production cost and the goods price also rises sooner, leading monetary authorities to raise the nominal interest rate earlier, at  $t = 1$ . Consequently, the real interest rate also increases earlier, producing a stronger and earlier crowding output effect in the mild recession. In the case of the mild recession, the impact consumption response is barely positive, suggesting that the response is dominated by the crowding-out effect from higher real interest rate. This is very different from the much positive consumption response in the deep recession, which is dominated by the positive income effect from reduced unemployment.

Monetary authorities conduct monetary policy based on the Taylor principle (see (40)).

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<sup>19</sup>We set the discount factor to  $1.01\%$  above the steady-state value at  $t = 0$  for the mild recession and  $3.3\%$  for the deep recession, compared to  $2.01\%$  in the baseline recession state. A higher discount factor suppresses households' goods demand more as households become more patient, generating a more severe recession.

When a negative demand shock is sufficiently large, deflation and a large negative output gap can push the nominal rate to its lower bound, 1. Figure 4 depicts government spending effects in the initial state of a deep recession with the binding ZLB (dashed lines), compared to the baseline recession (solid lines). When the economy is at the ZLB, the crowding out effect via the real interest rate channel is muted if the government spending increase does not increase output sufficiently to lift the economy out of the ZLB as simulated here. With the nominal rate trapped at the ZLB plus higher inflation expectation from the spending increase, the real interest rate in this simulation falls relative to the level without the spending increase, whereas it rises under the baseline recession. With a fall in the real interest rate, consumption is higher, and hence the output multiplier is also higher: the impact output multiplier is much bigger at 2.32, compared to 1.37 in the baseline recession.

Table 2 compares output and consumption multipliers for various initial recession conditions. One pattern emerges: the deeper a recession is, the bigger the output and consumption multipliers are throughout the horizon. Since the corresponding expansion state at the steady state is the same across all the recessions, our results show that the differences in business cycle-dependent multipliers are more pronounced when the recession under comparison is more severe.

#### *4.4. Degrees of Downward Nominal Wage Rigidity*

Our baseline simulation is conditional on a specific DNWR degree with a particular size increase in government spending. In this and next subsections, we investigate how DNWR degrees and spending sizes affect government spending multipliers. This cannot be explored with typical linearized equilibrium systems, which implies that multipliers are invariant to spending sizes and DNWR degrees.

Figure 5 compares the baseline simulation ( $\gamma = 0.96$ , solid lines) to that with a higher degree of DNWR ( $\gamma = 0.98$ , dashed lines) in recession. When  $\gamma$  rises, the government spending multiplier on impact increases from 1.37 to 1.65, and it is persistently higher relative to the baseline. The middle row of each panel in Table 3 summarizes the output and consumption multipliers. The consumption multiplier is also persistently higher with  $\gamma = 0.98$  throughout the horizon.

Prior to a government spending increase, a contractionary discount factor shock generates a deeper recession with  $\gamma = 0.98$  because the real wage rate is binding at a higher floor than the baseline with  $\gamma = 0.96$ . When  $\gamma = 0.98$ , the initial unemployment rate is 7.67% versus 5.14% in the baseline recession, and the involuntary unemployment is present for three quarters, rather than 2 quarters in the baseline, implying a higher degree of labor market distortion. Under these circumstances, a given size of government spending increase is more effective in closing the unemployment gap with a higher  $\gamma$ , as shown in Figure 5. A larger decrease in unemployment brings forth a stronger positive income effect, which makes consumption and output rise more than in the baseline. Table 3 shows that a higher degree of DNWR ( $\gamma = 0.98$ ) produces a larger impact consumption multiplier of 0.65, compared to 0.37 in the baseline recession.

In addition to the stronger income effect, a higher degree of DNWR implies a weaker crowding out effect, as seen by the smaller increase in the real interest rate than in the baseline. The longer binding duration of DNWR also means that the real wage rises later, thus the smaller inflationary pressure during the binding period. This triggers a smaller response from monetary authorities to raise the nominal interest rate, leading to a smaller increase in the real interest rate relative to the baseline. As a result, the crowding out effect, which offsets the positive income effect on consumption, is weaker with  $\gamma = 0.98$  than with  $\gamma = 0.96$  in the baseline recession. Combining a more positive income effect with weaker crowding-out means that government spending effects are more expansionary in recession when the downward nominal wage is more rigid.

#### 4.5. *Size of Government Spending Shocks*

The size of government spending also impacts the multipliers. Figure 6 compares the responses of two simulations that start with the same recession state as in the baseline simulation but are injected with different spending stimuli at time 0.<sup>20</sup> On impact, a

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<sup>20</sup>Unlike previous figures, Figure 6 does not plot the difference with and without a government spending shock. Instead, it plots the combined responses to a contractionary discount factor shock plus a government spending shock so we can see how stimulus size matters for recovery speed. Thus, output and consumption fall below the steady state mainly because of the contractionary discount factor shock, even though government spending has a positive effect on output and consumption.

5% increase lowers unemployment by 1.33 percentage points more than a 1% spending increase. With a 5% spending increase, however, output returns to the steady state only slightly faster than a 1% increase, and the recovery paths of output and consumption of the two scenarios are very similar. In other words, the benefit of a much bigger fiscal stimulus is small in terms of facilitating economic recovery, despite a bigger decline in the unemployment rate.

To understand this counterintuitive result, consider again the unemployment and real interest rate channels. A bigger spending increase should add more to aggregate demand and labor demand than a smaller increase, and hence be more effective in reducing unemployment. A larger reduction in unemployment ought to induce a stronger income effect, leading to a larger private consumption increase. While the income effect from reduced unemployment is stronger with a bigger spending increase, the offsetting crowding-out effect is also more severe. A bigger spending increase implies higher future government spending (through the autoregressive process), which exerts more inflationary pressure. Thus, the real interest rate rises more as monetary authorities raise the nominal rate more to combat inflation. The higher real interest rate in turn crowds out consumption more, offsetting the stronger positive income effect on consumption from reduced unemployment.

Table 3 shows that a 5% increase in government spending has slightly smaller output and consumption multipliers than a 1% increase. In recession, the impact output (consumption) multiplier is slightly bigger at 1.20 (0.20) with the 5% increase, versus 1.37 (0.37) with a 1% increase. This suggests that crowding out dominates the income effect from reduced unemployment with the 5% spending increase.

Although a bigger stimulus can help the economy return to the steady state sooner than a smaller one, this benefit must be weighed against the costs. Even with lump-sum financing (as assumed here), the consumption and output multipliers of a 5% stimulus are slightly smaller than for a 1% increase. When deficit financing is considered, the multipliers for the bigger spending increase can decrease further, because distorting fiscal adjustments, such as future income tax rate hikes or government spending reversals, are necessary to maintain debt sustainability.



## 5. Empirical Importance of Business Cycle-Dependent Multipliers via DNWR

The key predictions from the simulations can be summarized as follows. In response to a positive government spending shock, 1) the real wage and inflation increase less in recessions than in expansions; 2) consumption increases in recessions but decreases in expansions; and 3) unemployment falls in recessions but does not change in expansions. To see if the DNWR proposed in this paper is an empirically important driver of business cycle-dependent government spending effects, we compare these theoretical predictions to estimates found in the literature.

While the majority of the empirical papers on business cycle dependent multipliers focus only on output responses (see Section 1), Auerbach and Gorodnichenko (2013) follow the methodology in Auerbach and Gorodnichenko (2012)—regime-switching SVARs—to estimate business cycle-dependent government spending effects for a large number of OECD countries on other key macroeconomic variables. They find that mean private consumption responds positively in recessions and negatively in expansions within the three years after the government spending shock. The consumption responses are significant at 1% in recessions but are insignificant in expansions. Also, the mean unemployment rate declines in recessions (significant at 10%), while its response is not significant in expansions. Moreover, the mean real wage does not rise in recessions but it increases strongly in expansions, significant at 10%. Lastly, the mean inflation responses, whether measured by the consumer price index or the GDP deflator, are not significantly different from zero, but both point estimates have lower inflation responses in recessions than in expansions. These estimation results are largely consistent with our theoretical predictions.

In addition to Auerbach and Gorodnichenko (2013), additional evidence on consumption and unemployment responses is also available. Using U.S. data, Fazzari et al. (2015) find that private consumption responds positively in both states of low and high capacity utilization, but it is much more positive in the low-utilization state (usually in recessions). They also find that unemployment declines more in the low-utilization state than high-utilization state. Also, using OECD data, Tagkalakis (2008) finds that tax cuts or government spending increases are more effective in boosting private consumption in recessions

than in expansions especially for countries with less developed credit markets.

The largely consistent findings in the above empirical papers support our key theoretical predictions suggest DNWR is an important channel in driving business cycles-dependent multipliers. The literature exists competing explanations—such as countercyclical variation in bank intermediation costs (Canzoneri et al., 2016) or procyclical public employment spending (Michaillat, 2014). The empirical relevance of these other two channels, however, is yet to be established. Verifying the intermediation cost channel requires finding evidence that a government spending increase causes intermediation costs to fall more in recessions than in expansions, and verifying the public employment channel requires finding evidence that public employment systematically rises more and leads to a larger increase in the overall wage rate in expansions than in recessions.<sup>21</sup>

## 6. Conclusion

This paper proposes DNWR as a theoretical explanation for business cycle-dependent government spending multipliers. To this end, we build a simple NK model in which the economy has involuntary unemployment due to binding DNWR in recessions and full employment in expansions. We first obtain an analytical solution from a simplified model to show that government spending is more expansionary in recessions via two channels. First, government spending reduces unemployment, raising household income and private demand, more so in recessions than in expansions. Second, government spending induces a smaller increase in inflation and, hence, a smaller increase in nominal and real interest rates, generating less crowding out in recessions than in expansions.

The baseline simulation generates an impact output multiplier of 1.37 in recession and of 0.54 in expansion. Alternative simulations find that more severe macroeconomic conditions or a higher degree of DNWR make the output multipliers bigger in recessions than in expansions. In particular, a sufficiently severe recession with the binding ZLB further

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<sup>21</sup>Canzoneri et al. (2016) only provide reduced-form evidence that higher government spending shares are associated with higher interest rate spread levels in recessions than in expansions. Michaillat (2014) conducts a quantitative assessment on public employment multipliers using a calibrated model, without testing the predictions empirically.

amplifies the expansionary effect of government spending. Also, a larger increase in government spending only slightly improves recovery speed, as it induces stronger crowding out, producing a smaller output multiplier, than a smaller government spending increase.

This paper contributes to the literature by offering a new theoretical explanation for business cycle-dependent government spending effects. The theoretical implications are consistent with the business-cycle dependent empirical responses of consumption, output, unemployment, the real wage, and inflation to a government spending increase in the literature. This suggests that DNWR is an empirically important channel in driving business cycles-dependent multipliers.

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Business cycle states	impact	4 quarters	20 quarters
	<b>output multiplier:</b> $\frac{PV(\Delta y)}{PV(\Delta g)}$		
Expansion	0.54	0.54	0.54
Recession	1.37	0.95	0.78
	<b>consumption multiplier:</b> $\frac{PV(\Delta c)}{PV(\Delta g)}$		
Expansion	−0.46	−0.46	−0.46
Recession	0.37	−0.05	−0.22

Table 1: Baseline simulation: output and consumption multipliers.

Business cycle states	impact	4 quarters	20 quarters
	<b>output multiplier</b>		
Baseline recession	1.37	0.95	0.78
Mild recession	1.06	0.69	0.62
Deep recession	1.67	1.13	0.89
Deep recession with ZLB	2.32	1.70	1.23
	<b>consumption multiplier</b>		
Baseline recession	0.37	−0.05	−0.22
Mild recession	0.06	−0.31	−0.38
Deep recession	0.67	0.13	−0.11
Deep recession with ZLB	1.32	0.70	0.23

Table 2: Various initial recession conditions: output and consumption multipliers.

Business cycle states	impact	4 quarters	20 quarters
	<b>output multiplier</b>		
Baseline	1.37	0.95	0.78
$\gamma = 0.98$	1.65	1.35	1.02
5% g shock	1.20	0.80	0.69
	<b>consumption multiplier</b>		
Baseline	0.37	−0.05	−0.22
$\gamma = 0.98$	0.65	0.35	0.02
5% g shock	0.20	−0.20	−0.31

Table 3: Alternative simulations—more rigid DNWR ( $\gamma = 0.98$ ) or a bigger spending increase (5% government spending shock): output and consumption multipliers.

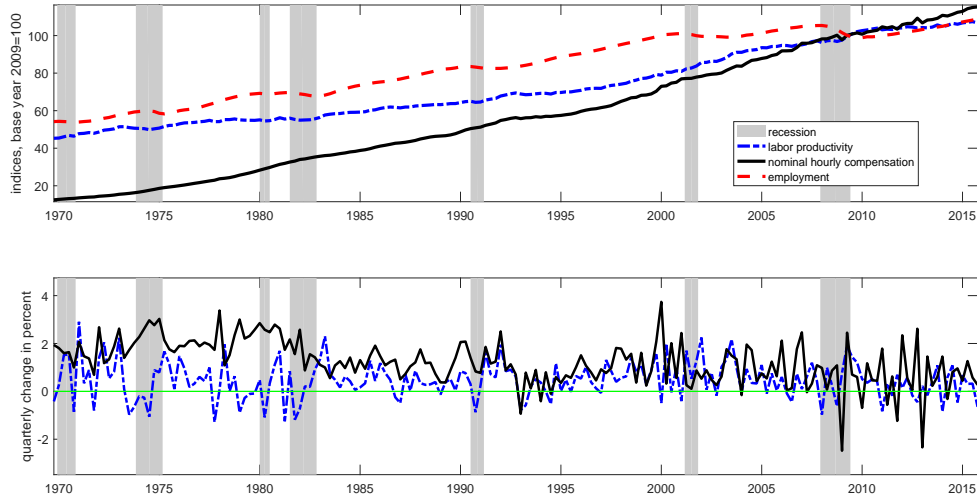


Figure 1: Downward nominal wage rigidity in the U.S. See data description in footnote 4.

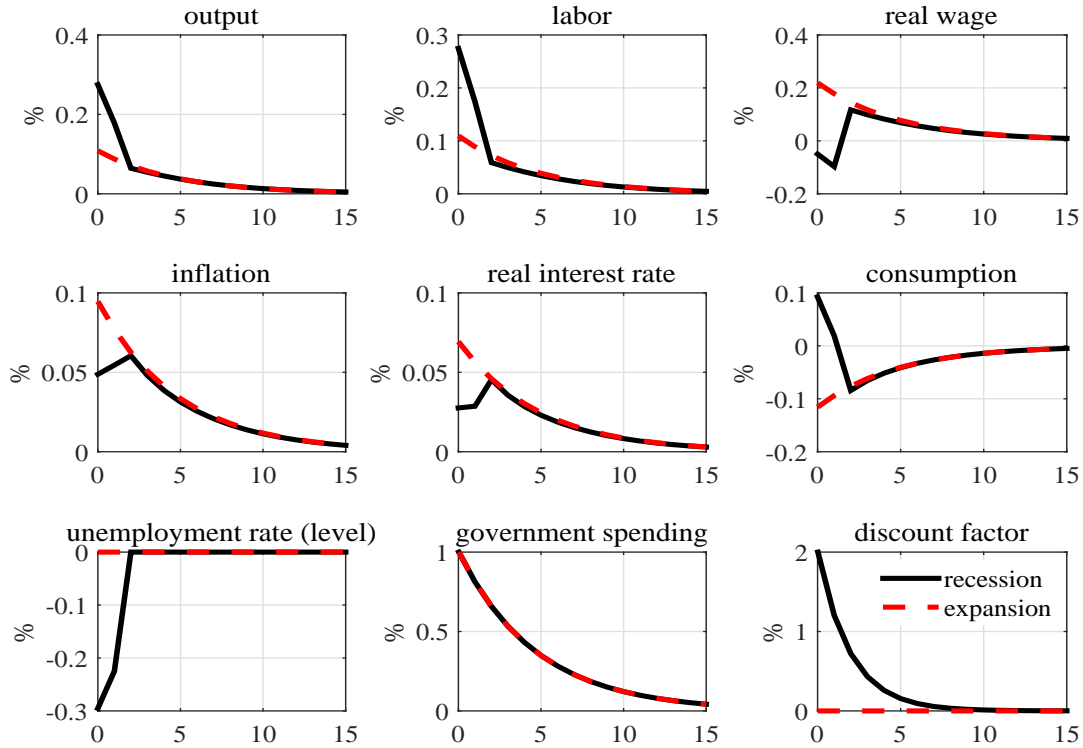


Figure 2: Impulse responses to a government spending increase in recession and in expansion. Except for unemployment and the discount factor, the y-axis is the response differences between a path with and without a government spending shock, scaled by the stochastic steady-state values. The unemployment rate is level difference in percent (see (7)), and the discount factor is in percent deviation from the steady state.



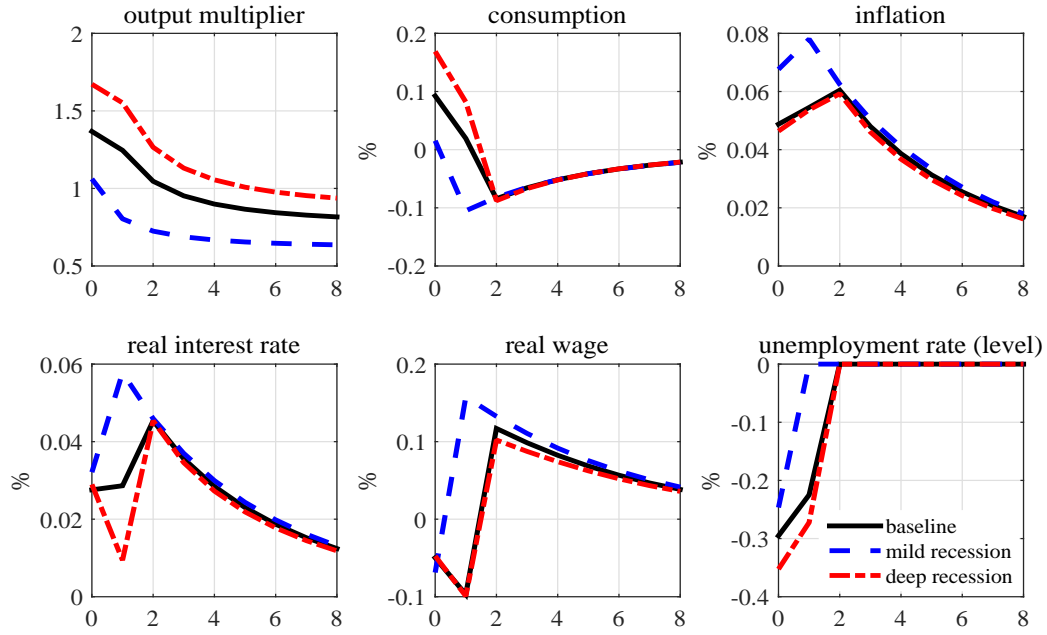


Figure 3: Responses in recession: different initial conditions. Except for the multiplier plot, see Figure 2 for axis units of other plots.

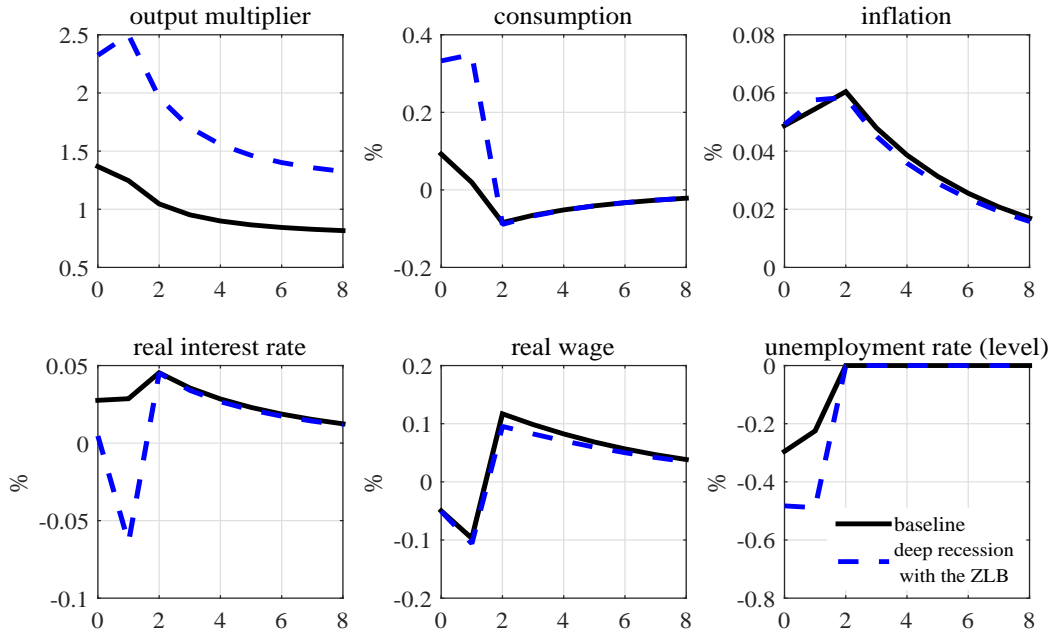


Figure 4: A deep recession with the binding ZLB.

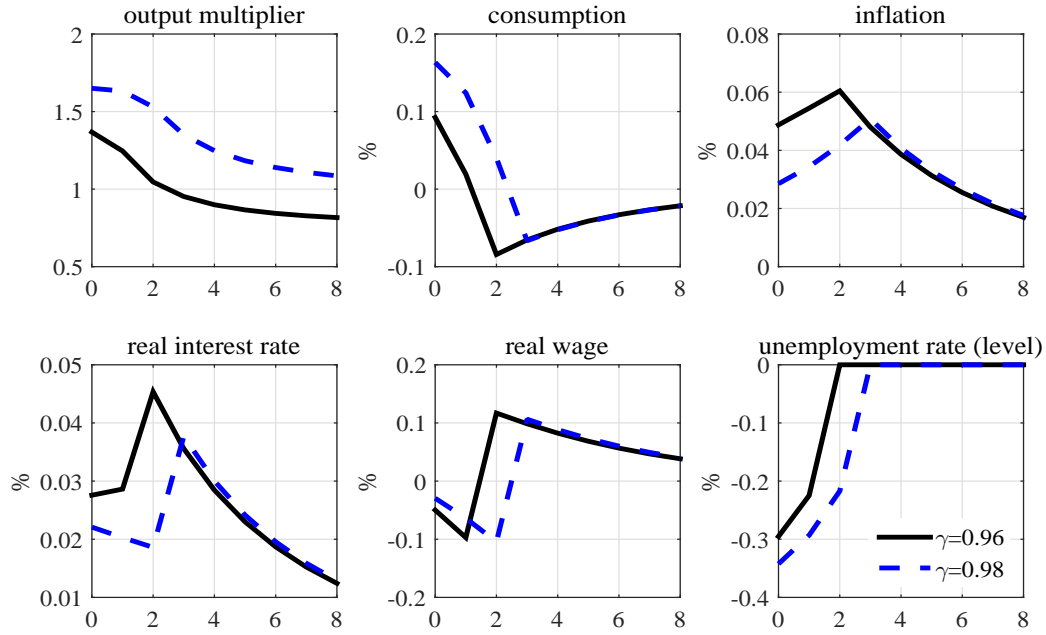


Figure 5: Responses in recession: different degrees of DNWR.

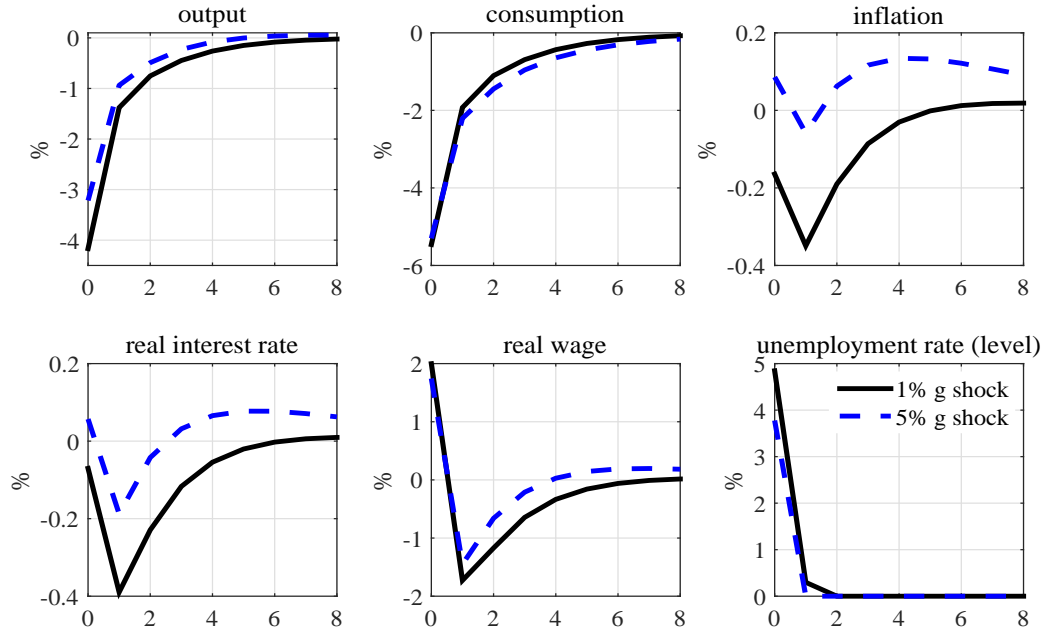


Figure 6: Responses in recession: different sizes of government spending increases. All variables (except unemployment) are in percent deviations from the steady state under both a contractionary discount factor shock and a government spending shock, and the unemployment rate is in percent as defined in (7).

## Appendix A. Equilibrium Conditions

The equilibrium for the model in Section 2 is characterized by equations (A.1)-(A.12), and the equilibrium for the quantitative analysis in Section 4 is characterized by equations (A.1)-(A.15) below.

$$\lambda_t = (c_t - \chi n_t^\varphi)^{-\sigma} \quad (\text{A.1})$$

$$\chi \varphi n_t^{\varphi-1} = w_t \quad (\text{A.2})$$

When DNWR binds, the above equation is replaced by  $\chi \varphi (n_t^s)^{\varphi-1} = w_t$ , and  $u_t = n_t^s - n_t$ .

$$\lambda_t = R_t E_t \frac{\beta_{t+1} \lambda_{t+1}}{\pi_{t+1}} \quad (\text{A.3})$$

$$m c_t = \frac{w_t}{a} \quad (\text{A.4})$$

$$\frac{P_t^*}{P_t} = \frac{\theta}{\theta - 1} \frac{k_{1t}}{k_{2t}} \quad (\text{A.5})$$

$$k_{1t} = \lambda_t y_t m c_t + \omega E_t \beta_{t+1} k_{1t+1} \pi_{t+1}^\theta \quad (\text{A.6})$$

$$k_{2t} = \lambda_t y_t + \omega E_t \beta_{t+1} k_{2t+1} \pi_{t+1}^{\theta-1} \quad (\text{A.7})$$

$$\pi_t = \left[ \frac{1}{\omega} - \frac{1 - \omega}{\omega} \left( \frac{P_t^*}{P_t} \right)^{1-\theta} \right]^{\frac{1}{\theta-1}} \quad (\text{A.8})$$

$$y_t = \frac{a n_t}{\Delta_t} \quad (\text{A.9})$$

$$y_t = c_t + g_t \quad (\text{A.10})$$

$$\Delta_t = (1 - \omega) \left( \frac{P_t^*}{P_t} \right)^{-\theta} + \omega \pi_t^\theta \Delta_{t-1} \quad (\text{A.11})$$

$$R_t = \max \left( R \left( \frac{\pi_t}{\pi} \right)^\alpha, 1 \right) \quad (\text{A.12})$$

In the quantitative analysis, the monetary policy rule is replaced by the following:

$$R_t = \max \left( R \left( \frac{\pi_t}{\pi} \right)^{\alpha_\pi} \left( \frac{y_t}{y} \right)^{\alpha_y}, 1 \right) \quad (\text{A.13})$$

Also, the discount factor and government spending follow the AR(1) processes:

$$\ln \frac{\beta_t}{\beta} = \rho^\beta \ln \frac{\beta_{t-1}}{\beta} + \varepsilon_t^\beta \quad (\text{A.14})$$

$$\ln \frac{g_t}{g} = \rho^g \ln \frac{g_{t-1}}{g} + \varepsilon_t^g \quad (\text{A.15})$$

## Appendix B. Analytics of the Linear Model

### Appendix B.1. Proof of Proposition 1

When DNWR does not bind, an economy has full employment; i.e.  $n_t^s = n_t$  or  $u_t = 0$  in (7). We can use (A.2), (A.4), and (A.9) to derive the marginal cost:

$$\hat{m}c_t = \hat{w}_t = (\varphi - 1)\hat{n}_t = (\varphi - 1)\hat{y}_t. \quad (\text{B.1})$$

Then, using (B.1) in (30), the equilibrium system can be summarized by

$$\hat{y}_t = E_t \hat{y}_{t+1} - \Psi(\alpha \hat{\pi}_t - E_t \hat{\pi}_{t+1}) + \theta s_g(\hat{g}_t - E_t \hat{g}_{t+1}) - \Psi E_t \hat{\beta}_{t+1} \quad (\text{B.2})$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \omega)(1 - \omega\beta)(\varphi - 1)}{\omega} \hat{y}_t. \quad (\text{B.3})$$

Let the labor market clearing solution (denoted by “\*”) take the form:

$$\begin{aligned} \hat{y}_t^* &= A_y^j \hat{g}_t + B_y^j E_t \hat{\beta}_{t+1} \\ \hat{\pi}_t^* &= A_\pi^j \hat{g}_t + B_\pi^j E_t \hat{\beta}_{t+1}, \end{aligned} \quad (\text{B.4})$$

where  $j \in \{\text{E}, \text{R}\}$  and E (R) indicates the expansion (recession) state.

Given an *i.i.d* process of government spending (see (20)), the expected output and inflation are given by,

$$\begin{aligned} E_t \hat{y}_{t+1}^* &= B_y^j E_t \hat{\beta}_{t+2} \\ E_t \hat{\pi}_{t+1}^* &= B_\pi^j E_t \hat{\beta}_{t+2}. \end{aligned} \quad (\text{B.5})$$

According to Definition 1 and Assumption 1,  $E_t \hat{\beta}_{t+1} = b_L$  and  $E_t \hat{\beta}_{t+2} = 0.5b_L + 0.5b_H = 0$  in expansions, and  $E_t \hat{\beta}_{t+1} = b_H$  and  $E_t \hat{\beta}_{t+2} = b_L$  in the recession state. Apply these assumptions and substitute (B.4) and (B.5) to (B.2) and (B.3). Then, we obtain the solution in the following form:

$$\begin{aligned} A_y &= A_y^{\text{E}} = A_y^{\text{R}} = \frac{\omega \theta s_g}{\omega + \Psi \alpha (1 - \omega)(1 - \omega\beta)(\varphi - 1)} > 0 \\ A_\pi &= A_\pi^{\text{E}} = A_\pi^{\text{R}} = \frac{\theta s_g (1 - \omega)(1 - \omega\beta)(\varphi - 1)}{\omega + \Psi \alpha (1 - \omega)(1 - \omega\beta)(\varphi - 1)} > 0 \\ B_y^{\text{E}} &= \frac{-\omega}{\omega + \Psi \alpha (1 - \omega)(1 - \omega\beta)(\varphi - 1)} < 0 \\ B_y^{\text{R}} &= \frac{-\omega}{2\omega(1 + \beta) + \Psi(\alpha + 1)(1 - \omega)(1 - \omega\beta)(\varphi - 1)} < 0 \\ B_\pi^{\text{E}} &= \frac{-(1 - \omega)(1 - \omega\beta)(\varphi - 1)}{\omega + \Psi \alpha (1 - \omega)(1 - \omega\beta)(\varphi - 1)} < 0 \\ B_\pi^{\text{R}} &= \frac{-(1 - \omega)(1 - \omega\beta)(\varphi - 1)}{2\omega(1 + \beta) + \Psi(\alpha + 1)(1 - \omega)(1 - \omega\beta)(\varphi - 1)} < 0. \end{aligned} \quad (\text{B.6})$$

The government spending multiplier is:

$$M_y = \frac{dy_t^*}{dg_t} = \frac{\hat{y}_t^* Y}{\hat{g}_t G} = \frac{A_y^j}{s_g} = \frac{\omega\theta}{\omega + \Psi\alpha(1-\omega)(1-\omega\beta)(\varphi-1)}, \quad j \in \{E, R\}. \quad (\text{B.7})$$

Equation (B.7) shows that without DNWR binding, the fiscal multiplier in the expansion and recession states are the same.

### Appendix B.2. Proof of Proposition 2

To derive the government spending multiplier in the recession state where DNWR binds, we maintain Assumption 2 that the economy at  $t-1$  has full employment and  $\hat{w}_{t-1} > 0$ , and it is hit by a sufficiently large contractionary discount factor shock ( $b_H$ ) such that DNWR binds at  $t$ . To solve the equilibrium with DNWR, substitute  $\hat{m}c_t$  with  $\hat{w}_t$  ( $= \hat{w}_{t-1} - \hat{\pi}_t$ ) in (30). This yields the equilibrium conditions with DNWR:

$$\hat{y}_t = E_t \hat{y}_{t+1} - \Psi(\alpha \hat{\pi}_t - E_t \hat{\pi}_{t+1}) + \theta s_g (\hat{g}_t - E_t \hat{g}_{t+1}) - \Psi E_t \hat{\beta}_{t+1} \quad (\text{B.8})$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1-\omega)(1-\omega\beta)}{\omega} (\hat{w}_{t-1} - \hat{\pi}_t). \quad (\text{B.9})$$

In contrast to the equilibrium conditions that characterize the expansion state (where the labor market clears), the equilibrium for the recession state where DNWR binds has the lagged real wage ( $\hat{w}_{t-1}$ ) enter the state space at  $t$ ; comparing (B.9) to (B.3), we see that the lagged real wage now matters for current inflation. Thus, the solution takes the form of

$$\begin{aligned} \hat{y}_t &= E_y \hat{w}_{t-1} + F_y \hat{g}_t + H_y b_H \\ \hat{\pi}_t &= E_\pi \hat{w}_{t-1} + F_\pi \hat{g}_t + H_\pi b_H. \end{aligned} \quad (\text{B.10})$$

From Assumptions 1 and 2, once  $E_t \hat{\beta}_{t+1} = b_H$ ,  $E_t \hat{\beta}_{t+2} = b_L$  with probability 1, and  $b_L$  does not trigger DNWR. As a result, the expected output and inflation are given by the full employment solution

$$\begin{aligned} E_t \hat{y}_{t+1} &= B_y^E b_L \\ E_t \hat{\pi}_{t+1} &= B_\pi^E b_L. \end{aligned} \quad (\text{B.11})$$

Substitute (B.10) and (B.11) to (B.8) and (B.9), we obtain the coefficients of interest

$$\begin{aligned} E_y &= -\frac{\Psi\alpha(1-\omega)(1-\omega\beta)}{\omega + (1-\omega)(1-\omega\beta)} < 0 \\ E_\pi &= \frac{(1-\omega)(1-\omega\beta)}{\omega + (1-\omega)(1-\omega\beta)} > 0 \\ F_y &= \theta s_g > 0 \\ F_\pi &= 0 \end{aligned} \quad (\text{B.12})$$

In recessions with DNWR, the government spending multiplier is

$$M_y^{DNWR} = \frac{dy_t}{dg_t} = \frac{\hat{y}_t Y}{\hat{g}_t G} = \frac{F_y}{s_g} = \theta. \quad (\text{B.13})$$

To compare the two multipliers in the state with and without binding DNWR, we subtract  $M_y$  as implied in (B.7) from  $M_y^{DNWR}$  and get

$$M_y^{DNWR} - M_y = \theta - \frac{\omega\theta}{\omega + \Delta} = \theta \left( 1 - \frac{\omega}{\omega + \Delta} \right) > 0, \quad (\text{B.14})$$

where  $\Delta \equiv \Psi\alpha(1 - \omega)(1 - \omega\beta)(\varphi - 1) > 0$ . Thus, the government spending multiplier in the recession state with DNWR is greater than the multiplier in the expansion state.

### Appendix B.3. Proof of Proposition 3

Unemployment, defined as the difference between the desired labor supply and actual labor hours worked, is given by (7). From the labor supply equation, (8), and the production function, (18), the desired linearized labor supply is  $\hat{n}_t^s = \frac{1}{\varphi-1}\hat{w}_t$ , but the actual linearized labor worked is  $\hat{n}_t = \hat{y}_t$ . Using the solution in (B.12), we can solve for unemployment to get

$$\begin{aligned} \hat{u}_t &= \frac{1}{\varphi - 1}(\hat{w}_{t-1} - \hat{\pi}_t) - \hat{y}_t \\ &= \Gamma_w^u \hat{w}_{t-1} + \Gamma_g^u \hat{g}_t + \Gamma_\beta^u b_H, \end{aligned} \quad (\text{B.15})$$

where

$$\begin{aligned} \Gamma_w^u &= \frac{1 - E_\pi}{\varphi - 1} - E_y = \frac{\omega + \Psi\alpha(1 - \omega)(1 - \omega\beta)(\varphi - 1)}{(\varphi - 1)[\omega + (1 - \omega)(1 - \omega\beta)]} > 0 \\ \Gamma_g^u &= -\frac{F_\pi}{\varphi - 1} - F_y = -\theta s_g < 0 \\ \Gamma_\beta^u &= -\frac{H_\pi}{\varphi - 1} - H_y. \end{aligned} \quad (\text{B.16})$$

Thus, in the recession state with binding DNWR, a government spending increase decreases unemployment, as  $\frac{d\hat{u}_t}{d\hat{g}_t} = \Gamma_g^u < 0$ . In the expansion state, where DNWR does not bind and the economy is in full employment, a government spending increase has no effect on unemployment.

Next, we solve for the real interest rate using  $\hat{r}_t = \alpha\hat{\pi}_t - E_t\hat{\pi}_{t+1}$ . In the recession state with binding DNWR, substitute (B.11) and (B.12) to get

$$\begin{aligned} \hat{r}_t &= \alpha\hat{\pi}_t - E_t\hat{\pi}_{t+1} \\ &= \Gamma_w^r \hat{w}_{t-1} + \Gamma_g^r \hat{g}_t + \Gamma_\beta^r b_H, \end{aligned} \quad (\text{B.17})$$

where

$$\begin{aligned}\Gamma_w^r &= \alpha E_\pi = \frac{\alpha(1-\omega)(1-\omega\beta)}{\omega + (1-\omega)(1-\omega\beta)} > 0 \\ \Gamma_g^r &= \alpha F_\pi = 0 \\ \Gamma_\beta^r &= \alpha H_\pi + B_\pi^E.\end{aligned}\tag{B.18}$$

From (B.18), we see that a government spending increase does not increase the real interest rate in the recession state with binding DNWR,

$$\frac{d\hat{r}_t}{d\hat{g}_t} = \Gamma_g^r = 0.\tag{B.19}$$

In the expansion state, we use the solutions from (B.6) to solve for the real interest rate,

$$\begin{aligned}\hat{r}_t &= \alpha\hat{\pi}_t - E_t\hat{\pi}_{t+1} \\ &= \Gamma_g^r\hat{g}_t + \Gamma_\beta^r b_L,\end{aligned}\tag{B.20}$$

where

$$\begin{aligned}\Gamma_g^r &= \alpha A_\pi = \frac{\theta s_g(1-\omega)(1-\omega\beta)(\varphi-1)}{\omega + \Psi\alpha(1-\omega)(1-\omega\beta)(\varphi-1)} > 0 \\ \Gamma_\beta^r &= B_\pi^E < 0.\end{aligned}\tag{B.21}$$

From (B.21), we see that a government spending increase drives up the real interest rate in the expansion state:

$$\frac{d\hat{r}_t}{d\hat{g}_t} = \Gamma_g^r > 0.\tag{B.22}$$

### Appendix C. The Numerical Solution Method

When solving the nonlinear model, the state space is  $\mathbf{S}_t = \{\beta_t, g_t, \Delta_{t-1}, w_{t-1}\}$ . Define the decision rules for hours as  $n_t = f^n(\mathbf{S}_t)$ ,  $k_{1t} = f^{k_1}(\mathbf{S}_t)$ , and  $k_{2t} = f^{k_2}(\mathbf{S}_t)$ . The decision rules are solved as follows.

1. Define the grid points by discretizing the state space. Make initial guesses for  $f_0^n$ ,  $f_0^{k_1}$ , and  $f_0^{k_2}$  over the state space.
2. At each grid point, solve the nonlinear model and obtain the updated rules  $f_i^n$ ,  $f_i^{k_1}$ , and  $f_i^{k_2}$  using the given rules  $f_{i-1}^n$ ,  $f_{i-1}^{k_1}$ , and  $f_{i-1}^{k_2}$ :
  - a) Derive  $\frac{P_t^*}{P_t}$  and  $\pi_t$  using (A.5) and (A.8).
  - b) Given  $w_{t-1}$  and  $\pi_t$ , compute  $\gamma \frac{w_{t-1}}{\pi_t}$ , and compare it with the full employment wage  $w_t^*$ . If  $\gamma \frac{w_{t-1}}{\pi_t} \leq w_t^*$ ,  $w_t = w_t^*$ ; if  $\gamma \frac{w_{t-1}}{\pi_t} < w_t^*$ ,  $w_t = \gamma \frac{w_{t-1}}{\pi_t}$ .
  - c) Compute  $\Delta_t$ ,  $y_t$ ,  $c_t$ ,  $mc_t$ , and  $\lambda_t$  using (A.1), (A.4), (A.9), (A.10), and (A.11).
  - d) Derive the desired labor supply  $n_t^s$  from (A.2), and then the unemployment rate is defined as  $\frac{n_t^s - n_t}{n_t^s} \times 100\%$ .

- e) Given  $f_{i-1}^\pi$ , obtain the nominal interest rate  $R_t$  from equation (A.13). If  $R_t < 1$ , use 1 as the nominal interest rate.
  - f) Use linear interpolation to obtain  $f_{i-1}^n(\mathbf{S}_{t+1})$ ,  $f_{i-1}^{k_1}(\mathbf{S}_{t+1})$ , and  $f_{i-1}^{k_2}(\mathbf{S}_{t+1})$ , where  $\mathbf{S}_{t+1} = (\beta_{t+1}, g_{t+1}, \Delta_t, w_t)$ , and  $\beta_{t+1}$  and  $g_{t+1}$  are updated according to the AR(1) processes from (A.14) and (A.15). Then follow the above steps to solve  $\lambda_{t+1}$ ,  $\pi_{t+1}$ ,  $k_{1t+1}$ , and  $k_{2t+1}$ .
  - g) Update the decision rules  $f_i^n$ ,  $f_i^{k_1}$ , and  $f_i^{k_2}$  using (A.3), (A.6), and (A.7).
3. Check convergence of the decision rules. If  $|f_i^n - f_{i-1}^n|$ , or  $|f_i^{k_1} - f_{i-1}^{k_1}|$ , or  $|f_i^{k_2} - f_{i-1}^{k_2}|$  are above the desired tolerance (set to  $1e - 7$ ), go back to step 2; otherwise,  $f_i^n$ ,  $f_i^{k_1}$ , and  $f_i^{k_2}$  are the decision rules.

## Appendix D. Wealth Effects on Labor Supply

The baseline specification has a GHH preference, which does not have a wealth effect on labor supply. The alternative, the KPR preference (King et al., 1988), however, is common in macroeconomic models. The appendix explores whether business cycle-dependent multipliers are robust when the wealth effect on labor supply is present.

To proceed, the preference (1) is modified as:

$$\max \sum_{t=0}^{\infty} d_t \frac{[c_t(1 - \chi n_t^\varphi)]^{1-\sigma}}{1 - \sigma}.$$

To make the current preference comparable to the previous one, we re-calibrate  $\varphi$  such that the elasticity of labor supply is 0.5 under the two preferences. Also, to make the initial recession state at  $t = 0$  similar to the baseline, we inject discount factor shocks from  $t = -5$  to 0 such that the unemployment rate is 5.7% at  $t = 0$  and the real wage is 6.4% above the steady state at  $t = -1$ , close to the baseline.

Table 4 presents the consumption and output multipliers under the KPR preference. The impact output multiplier falls to 0.60 under KPR, compared to 1.37 under GHH. It remains the case that the output multiplier is smaller in the expansion state with the impact output multiplier of 0.49.

The much weaker stimulative effect in the recession state under KPR shows that the wealth effect on labor supply plays an important role in the strength of the expansionary effect in recessions. Under KPR, a positive government spending shock indicates higher future taxes, creating a negative wealth effect that shifts the labor supply curve to the right. An increase in labor supply weakens the effectiveness of a spending increase in reducing unemployment. Thus, under KPR an increase in the desired labor supply generates a smaller positive income effect relative to the case under the baseline GHH case. With a much weaker effect in reducing unemployment, consumption responds negatively to a spending increase in the recession state and thus the impact consumption multiplier is much more negative in the recession state compared to that under GHH ( $-0.40$  vs  $0.37$ ).



Business cycle states	impact	4 quarters	20 quarters
<b>output multiplier</b>			
Expansion	0.49	0.49	0.49
Recession	0.60	0.54	0.52
<b>consumption multiplier</b>			
Expansion	−0.51	−0.51	−0.51
Recession	−0.40	−0.46	−0.48

Table 4: The multipliers under the KPR preference.