Quantum optical correlations in the absence of intensity correlations

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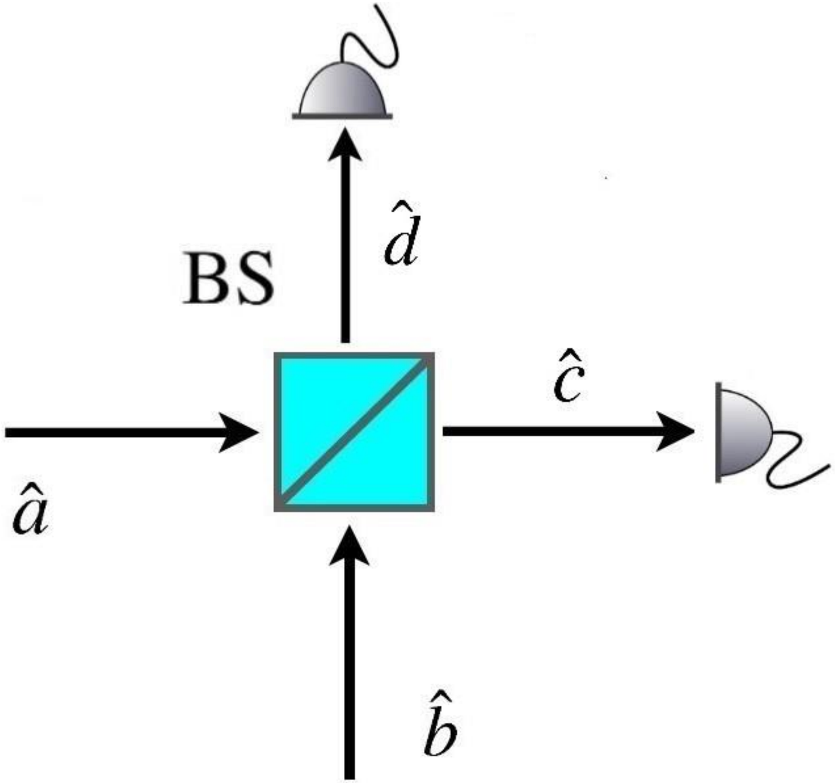
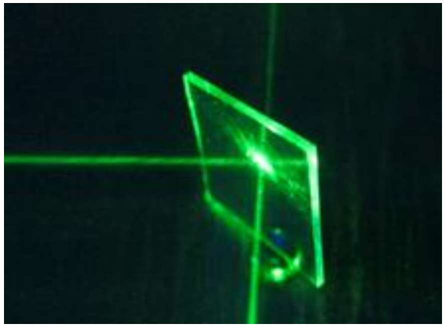
Introduzione

Beam splitter



$$\hat{c} = \hat{\mathcal{U}}^{\dagger}_{\mathrm{BS}}(\xi)\ \hat{a}\ \hat{\mathcal{U}}_{\mathrm{BS}}(\xi)$$

$$\hat{d} = \hat{\mathcal{U}}^{\dagger}_{\mathrm{BS}}(\xi)\ \hat{b}\ \hat{\mathcal{U}}_{\mathrm{BS}}(\xi)$$



$$\hat{\mathcal{U}}_{\mathrm{BS}}(\xi) = e^{\xi \hat{a}^{\dagger}\hat{b} - \xi^{*}\hat{a}\hat{b}^{\dagger}}$$

Correlazioni Ottiche

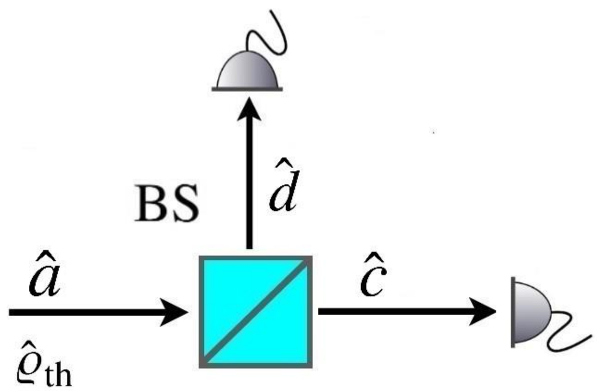
Intensità

$$\Gamma = \frac{\langle \hat{n}_c \hat{n}_d \rangle - \langle \hat{n}_c \rangle \langle \hat{n}_d \rangle}{\sqrt{\mathrm{Var}(\hat{n}_c)\mathrm{Var}(\hat{n}_d)}}$$

$$\hat{\varrho}_{\text{th}} \otimes |0\rangle \langle 0|$$

$$= \frac{\mathrm{Var}(\hat{n}_a) + \mathrm{Var}(\hat{n}_b) - 2\langle \hat{n}_a \rangle \langle \hat{n}_b \rangle - \langle \hat{n}_a \rangle - \langle \hat{n}_b \rangle}{\mathrm{Var}(\hat{n}_a) + \mathrm{Var}(\hat{n}_b) + 2\langle \hat{n}_a \rangle \langle \hat{n}_b \rangle + \langle \hat{n}_a \rangle + \langle \hat{n}_b \rangle}$$

$$= \frac{N_{\text{th}}}{N_{\text{th}} + 2}$$



$$\langle \hat{n} \rangle = N_{\text{th}}$$

$$\mathrm{Var}(\hat{n}) = N_{\text{th}}(N_{\text{th}} + 1)$$

Stato termico

$$\hat{\varrho}_{\text{th}} = \frac{1}{1+N_{\text{th}}} \sum_{n=0}^{+\infty} \left(\frac{N_{\text{th}}}{1+N_{\text{th}}} \right)^n |n\rangle \langle n|$$

$$\langle \hat{n} \rangle = N_{\text{th}}$$

$$\text{Var}(\hat{n}) = N_{\text{th}}(N_{\text{th}}+1)$$

- Stato classico
- Disco di Arecchi

Correlazioni Ottiche

Intensità

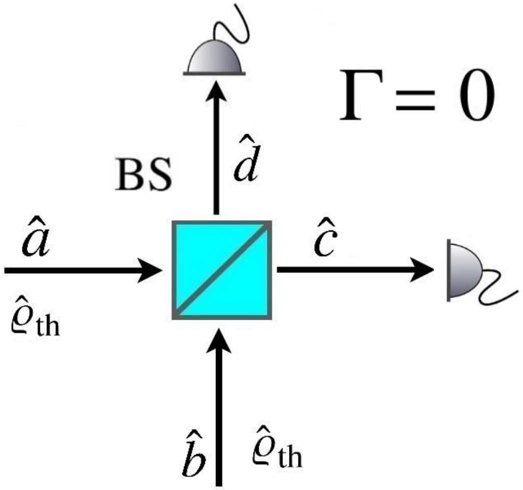
$$\hat{\varrho}_{\text{th}} \otimes \hat{\varrho}_{\text{th}}$$

$$\Gamma = \frac{\langle \hat{n}_c \hat{n}_d \rangle - \langle \hat{n}_c \rangle \langle \hat{n}_d \rangle}{\sqrt{\text{Var}(\hat{n}_c)\text{Var}(\hat{n}_d)}}$$

$$= \frac{\text{Var}(\hat{n}_a) + \text{Var}(\hat{n}_b) - 2\langle \hat{n}_a \rangle \langle \hat{n}_b \rangle - \langle \hat{n}_a \rangle - \langle \hat{n}_b \rangle}{\text{Var}(\hat{n}_a) + \text{Var}(\hat{n}_b) + 2\langle \hat{n}_a \rangle \langle \hat{n}_b \rangle + \langle \hat{n}_a \rangle + \langle \hat{n}_b \rangle} = 0$$

$$\langle \hat{n} \rangle = N_{\text{th}}$$

$$\text{Var}(\hat{n}) = N_{\text{th}}(N_{\text{th}} + 1)$$



Modello

Mistura di stati di Fock

$$\hat{\varrho}_m = p_0|0\rangle\langle 0| + p_1|1\rangle\langle 1| + p_2|2\rangle\langle 2|$$

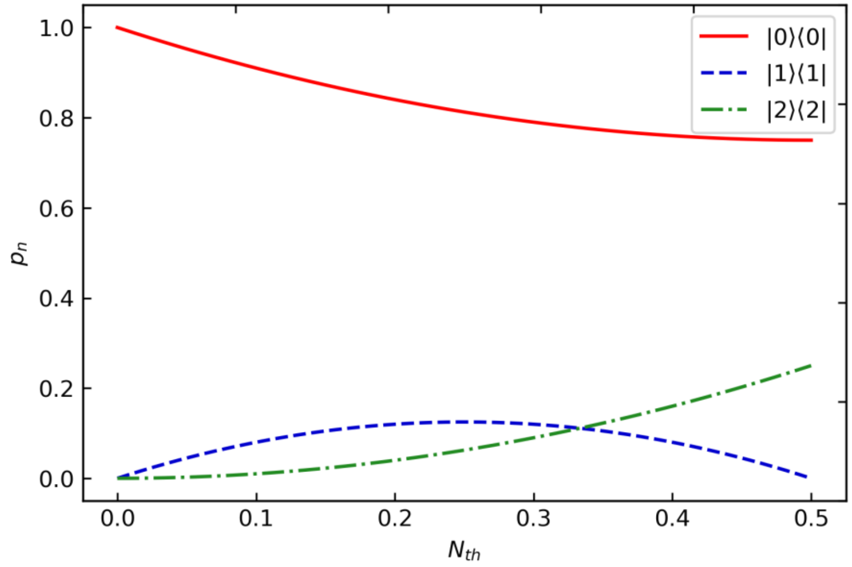
$$\hat{\varrho}_m(N_{\text{th}}) = \left(N_{\text{th}}^2 - N_{\text{th}} + 1\right)|0\rangle\langle 0| + \left(N_{\text{th}} - 2N_{\text{th}}^2\right)|1\rangle\langle 1| + N_{\text{th}}^2\ |2\rangle\langle 2|$$

$$p_0 + p_1 + p_2 = 1$$

$$\langle \hat{n} \rangle = N_{\text{th}}$$

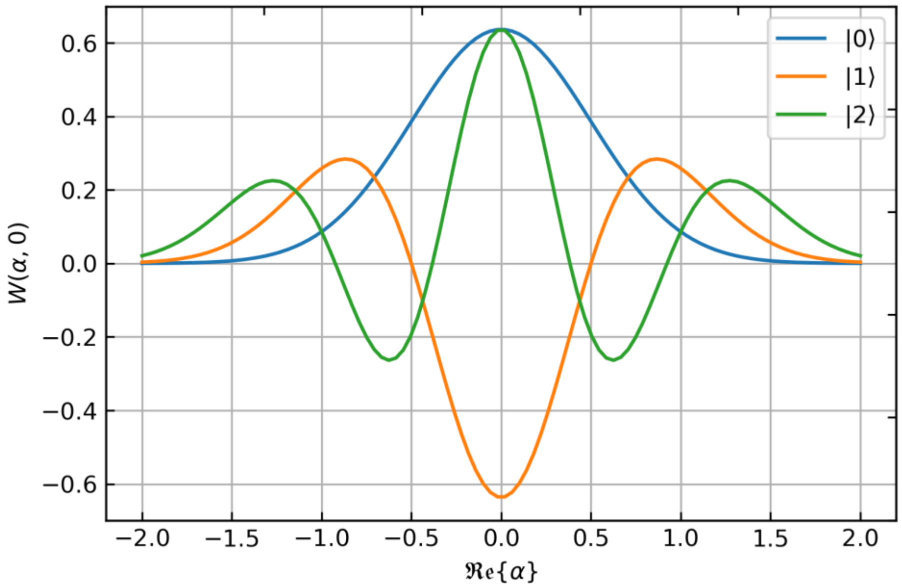
$$\text{Var}(\hat{n}) = N_{\text{th}}(N_{\text{th}} + 1)$$

$$p_n > 0 \leftrightarrow 0 < N_{\text{th}} < \frac{1}{2}$$



Nonclassicalità

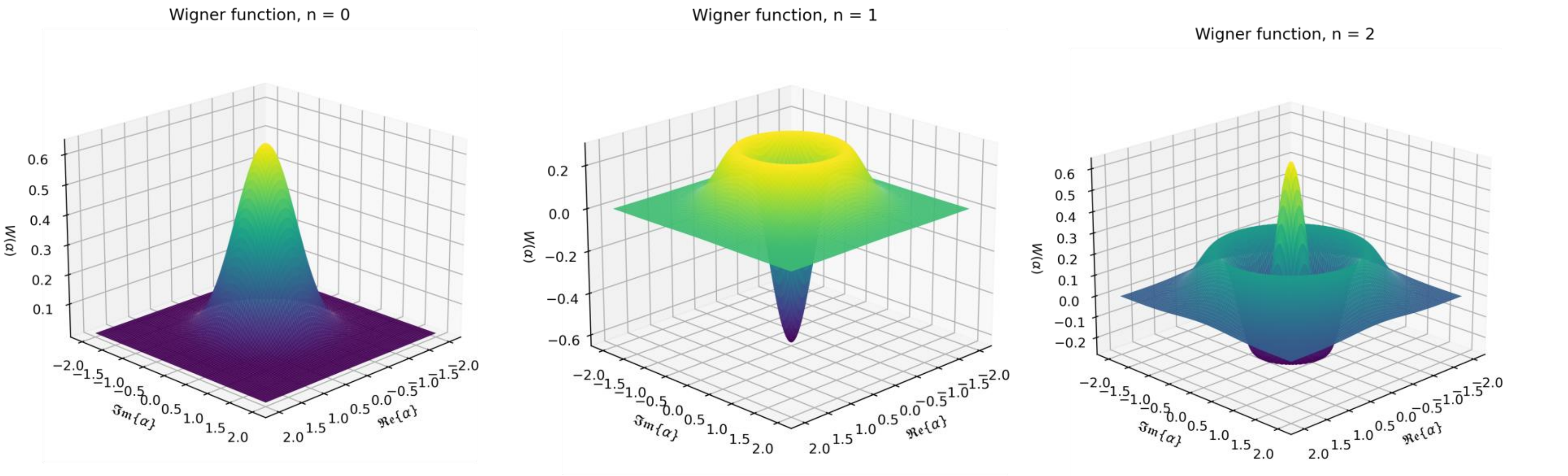
Funzione di Wigner – Stati di Fock



$$W_n(\alpha) = \frac{2}{\pi}(-1)^n e^{-2|\alpha|^2} L_n\left(4|\alpha|^2\right)$$

Nonclassicalità

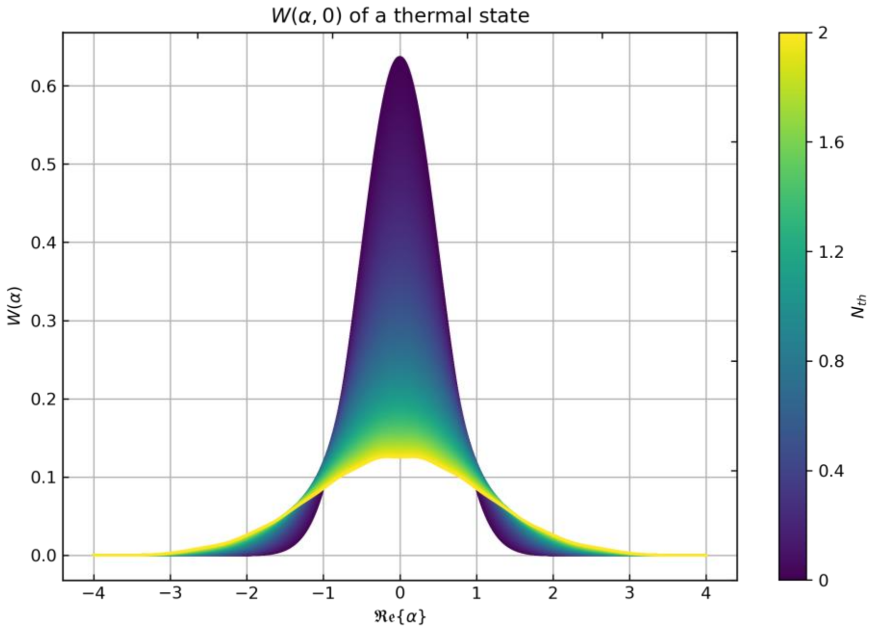
Funzione di Wigner – Stati di Fock



Nonclassicalità

Funzione di Wigner

$$W(\alpha) = \frac{2}{\pi} e^{-2|\alpha|^2} \sum_{n=0}^{2} p_n (-1)^n L_n(4|\alpha|^2)$$

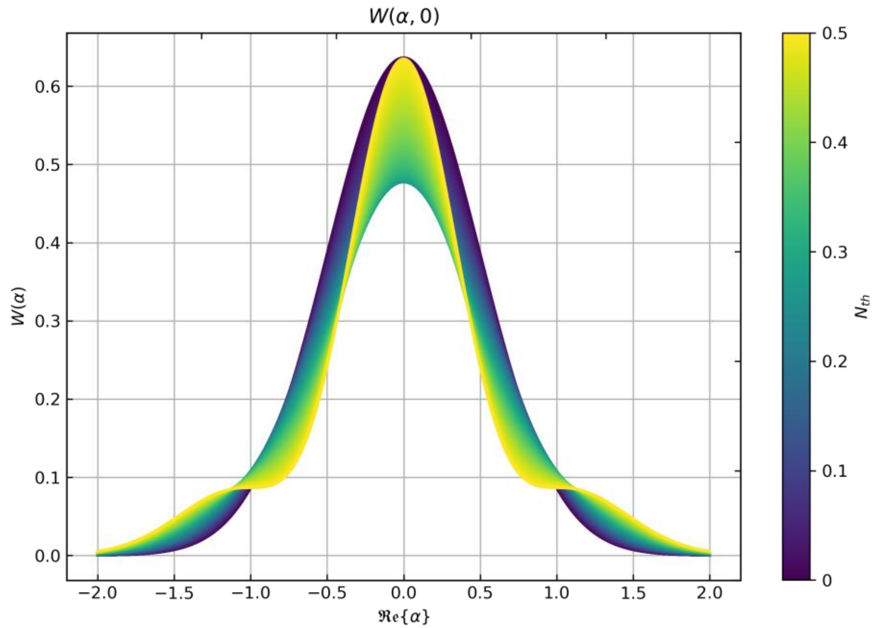


$$\hat{\varrho}_{\text{th}} = \frac{1}{1 + N_{\text{th}}} \sum_{n=0}^{+\infty} \left(\frac{N_{\text{th}}}{1 + N_{\text{th}}} \right)^n |n\rangle \langle n|$$

Nonclassicalità

Funzione di Wigner

$$W(\alpha) = \frac{2}{\pi} e^{-2|\alpha|^2} \sum_{n=0}^{2} p_n (-1)^n L_n(4|\alpha|^2)$$



$$\hat{\varrho}_m(N_{\rm th}) = \left(N_{\rm th}^2 - N_{\rm th} + 1\right) |0\rangle\langle 0| + \left(N_{\rm th} - 2N_{\rm th}^2\right) |1\rangle\langle 1| + N_{\rm th}^2\ |2\rangle\langle 2|$$

Nonclassicalità

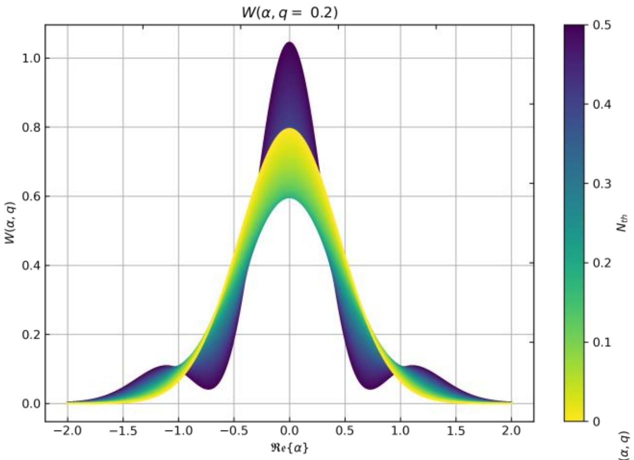
Funzione di quasi-probabilità

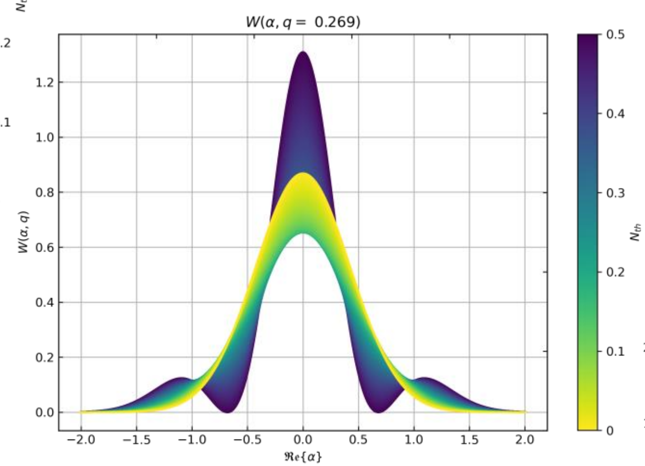
$$W(\alpha, q) = \frac{2}{\pi(1-q)} \int_{\mathbb{C}} d^2\beta \, P(\beta) \; \exp\left[-\frac{2|\alpha-\beta|^2}{1-q}\right]$$

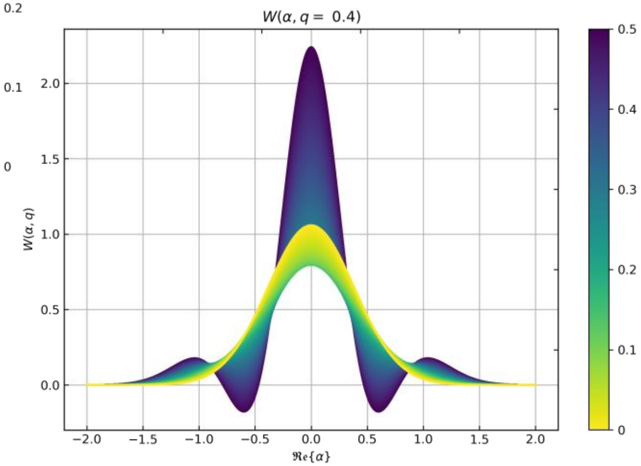
$$P(\beta) = \sum_{n=0}^{2} p_n \; \sum_{m=0}^{n} \binom{n}{m} \frac{1}{m!} \left(\frac{\partial^2}{\partial\beta\partial\beta^*}\right)^m \delta^{(2)}(\beta)$$

Nonclassicalità

Funzione di quasi-probabilità



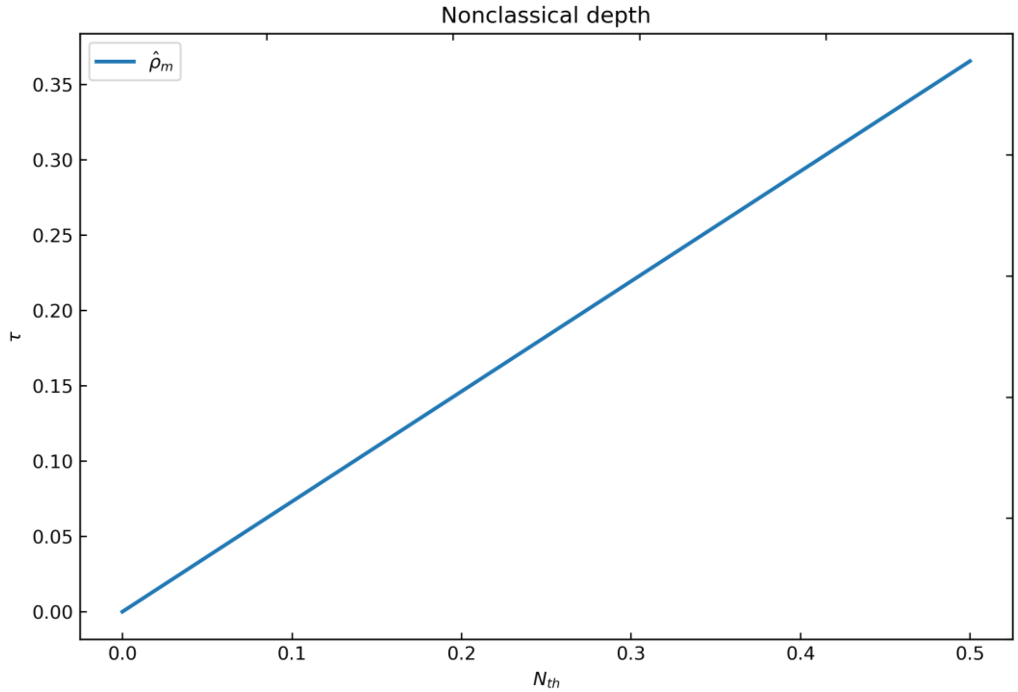




Nonclassicalità

Nonclassical depth

$$\tau = \frac{1-q}{2}$$



Nonlocalità

- Disuguaglianze di Bell e disuguaglianza CHSH

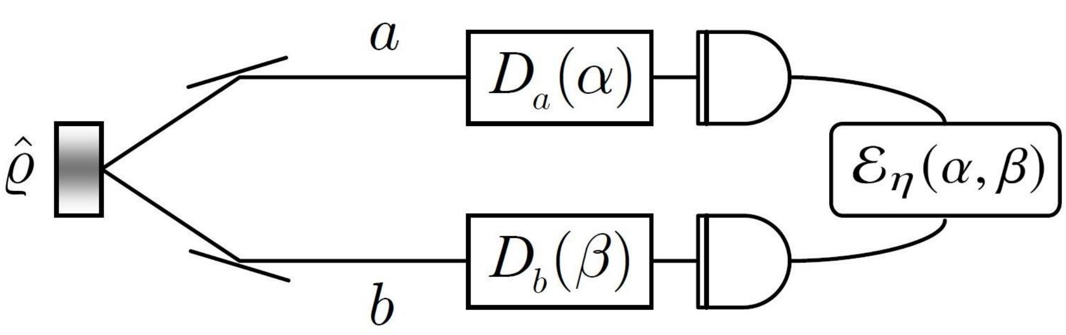
$$\mathcal{B} = \mathcal{E}_\eta(\alpha,\beta) + \mathcal{E}_\eta(\alpha,\beta') + \mathcal{E}_\eta(\alpha',\beta) - \mathcal{E}_\eta(\alpha',\beta')$$

$$|\mathcal{B}| < 2$$

[1] M. S. Kim, W. Son, V. Bužek, and P. L. Knight. Entanglement by a beam splitter: Nonclassicality as a prerequisite for entanglement. *Physical Review A*, 65(3), Feb 2002. ISSN 1094-1622.

Nonlocalità

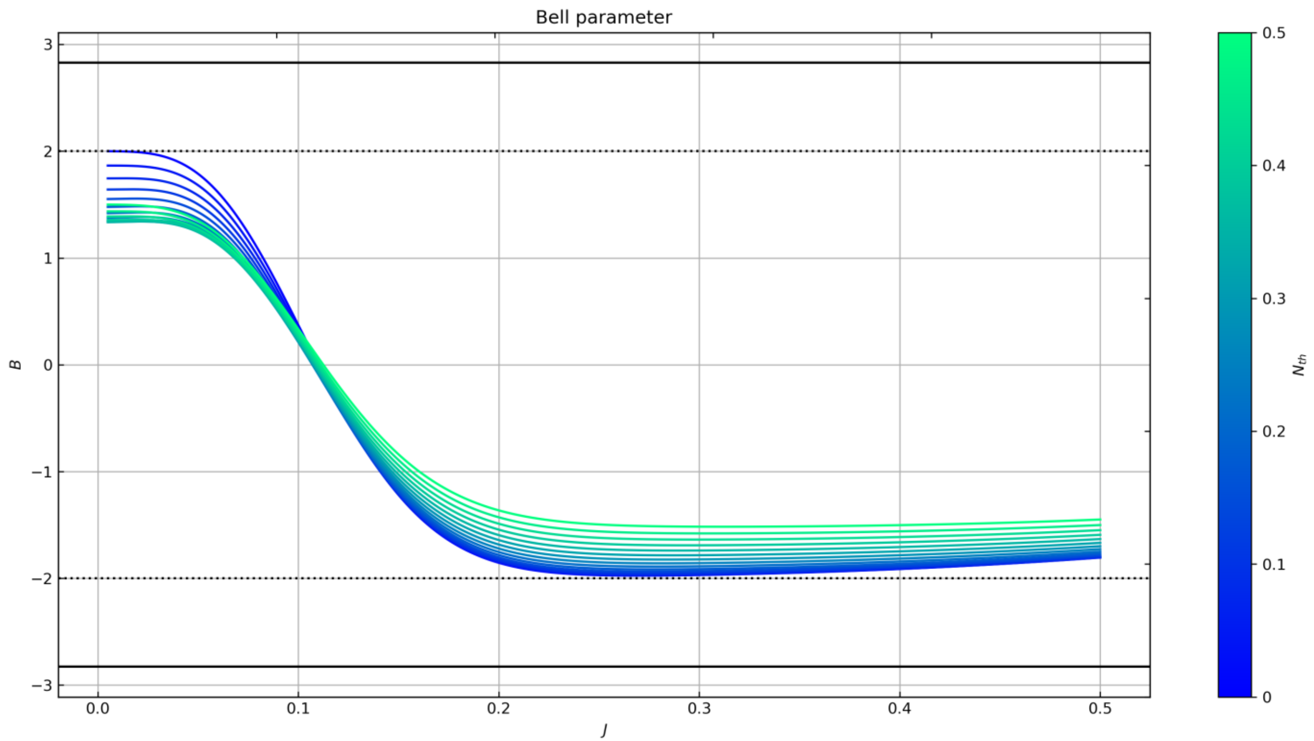
- Misura on/off

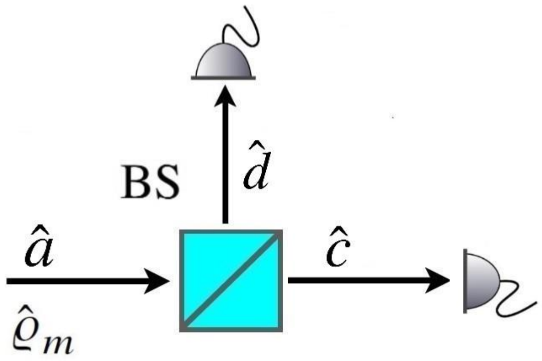


$$\hat{D}(\alpha) = e^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}}$$

Nonlocalità

$$\hat{\varrho}_m \otimes |0\rangle\langle 0|$$

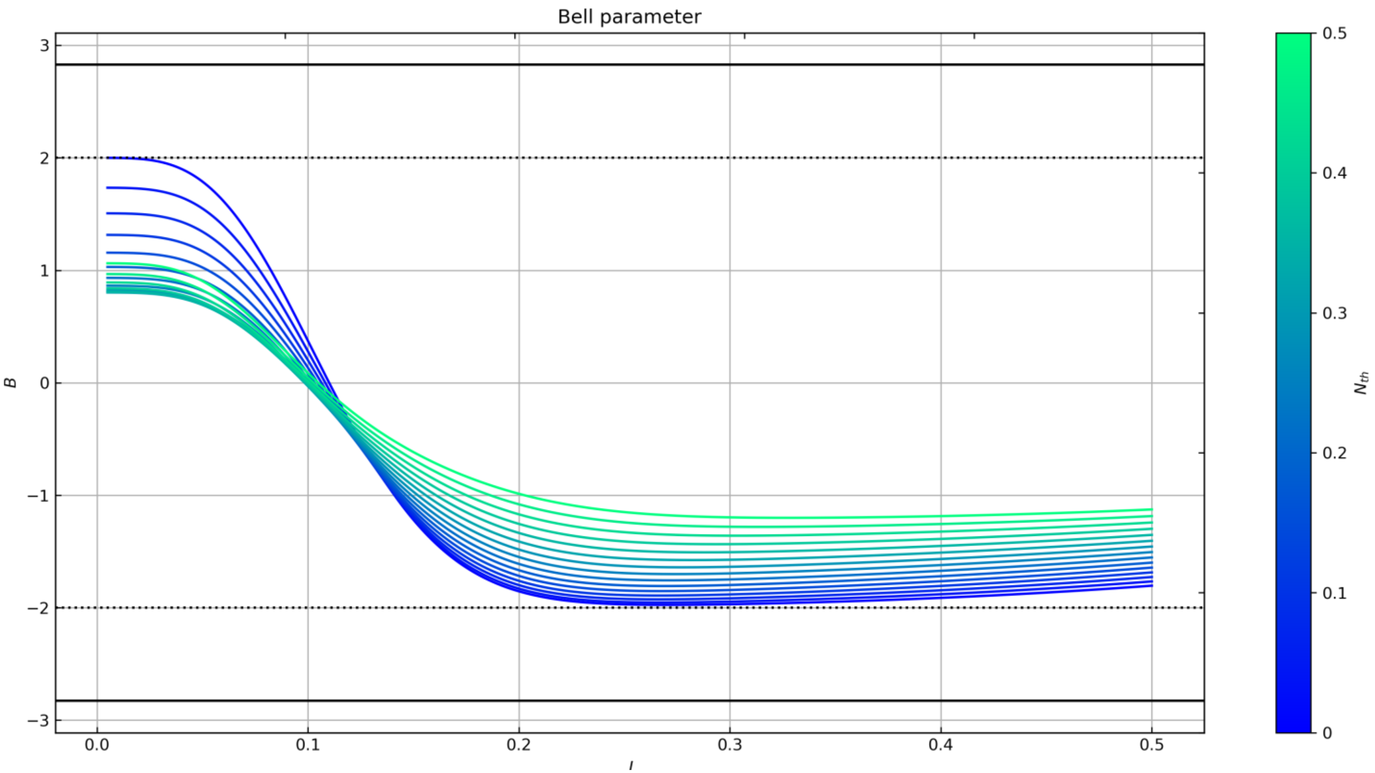


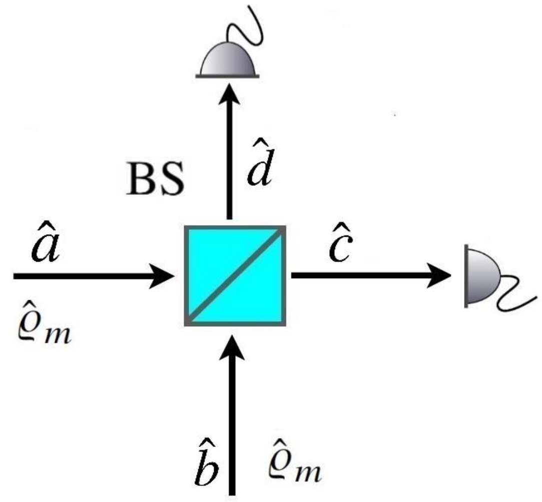


$$J = |\alpha| = |\beta| = +10|\alpha'| = +10|\beta'|$$

Nonlocalità

$$\hat{\varrho}_m \otimes \hat{\varrho}_m$$



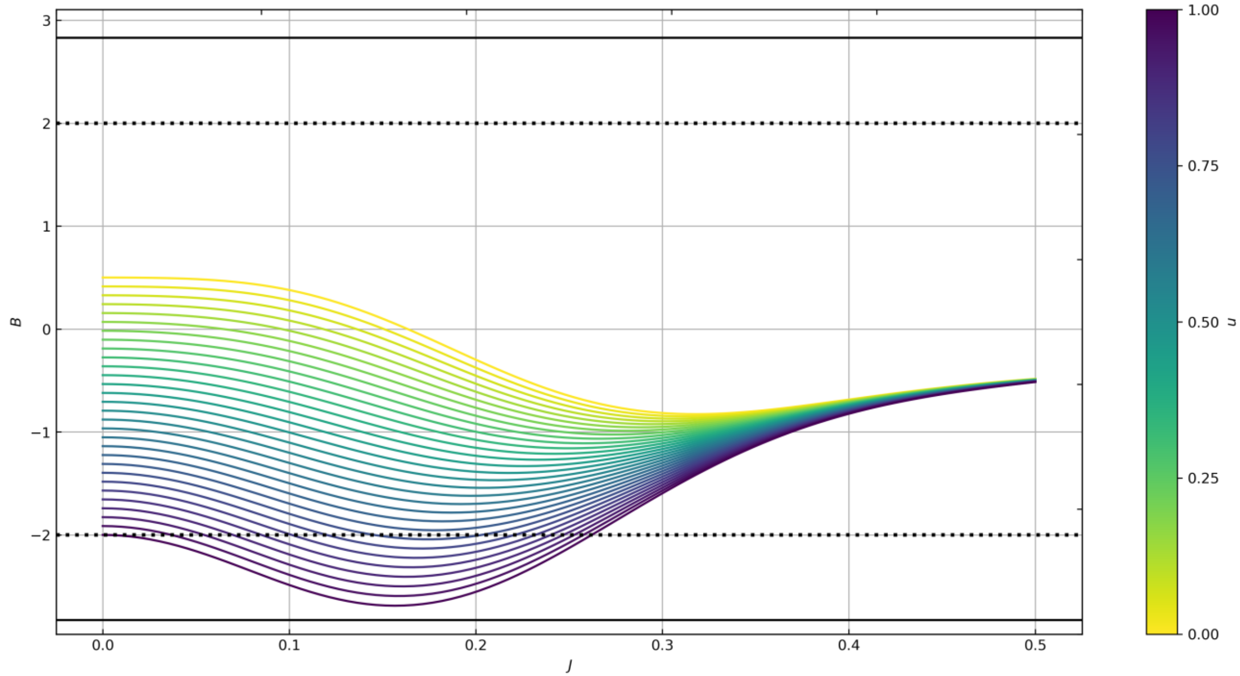


$$J = |\alpha| = |\beta| = -10|\alpha'| = -10|\beta'|$$

Conclusioni

Sviluppi futuri

$$\hat{\varrho}_u = \frac{1-u}{2}|0\rangle\langle 0| + u|1\rangle\langle 1| + \frac{1-u}{2}|2\rangle\langle 2|$$



$$J = |\alpha| = -|\beta| = -3.6|\alpha'| = 3.6|\beta'|$$

Grazie per l'attenzione!

Separabilità

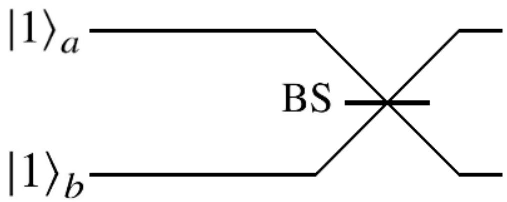
Stato puro: $$|\psi\rangle_{AB} = |\psi\rangle_A \otimes |\psi\rangle_B$$

Miscela: $$\hat{\varrho}_{AB} = \sum_k p_k \hat{\varrho}_A^{(k)} \otimes \hat{\varrho}_B^{(k)}$$

Stato non entangled → separabile

Entanglement

- Effetto Hong-Ou-Mandel



$$|\psi\rangle = \frac{|20\rangle - |02\rangle}{\sqrt{2}}$$

Funzioni di quasi-probabilità

$$W_n(\alpha, 1) = P_n(\alpha) = \sum_{m=0}^{n} \binom{n}{m} \frac{1}{m!} \left(\frac{\partial^2}{\partial\alpha\partial\alpha^*} \right)^m \delta^{(2)}(\alpha)$$

$$W_n(\alpha, 0) = W_n(\alpha) = \frac{2}{\pi}(-1)^n e^{-2|\alpha|^2} L_n\left(4|\alpha|^2\right)$$

$$W_n(\alpha, -1) = Q_n(\alpha) = \frac{1}{\pi} e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!}$$

$$W(\alpha, p) = \sum_n p_n W_n(\alpha, p)$$

Disuguaglianze di Bell

$$\mathcal{B} = \mathcal{E}_\eta(\alpha,\beta) + \mathcal{E}_\eta(\alpha,\beta') + \mathcal{E}_\eta(\alpha',\beta) - \mathcal{E}_\eta(\alpha',\beta')$$

$$\begin{aligned}
\mathcal{E}_\eta(\alpha,\beta) &= \sum_{h,k} hk\, P_{h,k}(\alpha,\beta), \\
&= \sum_{h,k=0}^{1} (-1)^{h+k} \langle \Pi^\eta_{h,k}(\alpha,\beta) \rangle, \\
&= 1 + 4\langle \Pi^\eta_{0,0}(\alpha,\beta)\rangle - 2\left[\langle \Pi^\eta_0(\alpha)\otimes \mathbb{I}\rangle + \langle \mathbb{I}\otimes \Pi^\eta_0(\beta)\rangle\right] \\
&\equiv 1 + 4\mathcal{I}(\alpha,\beta) - 2\left[\mathcal{G}(\alpha) + \mathcal{Y}(\beta)\right]
\end{aligned}$$

Disuguaglianze di Bell

$$\mathcal{I}(\alpha,\beta) = \langle \Pi^{\eta}_{0,0}(\alpha,\beta)\rangle = \mathrm{Tr}[\hat{\varrho}\Pi^{\eta}_{0}(\alpha)\otimes\Pi^{\eta}_{0}(\beta)]$$

$$\mathcal{G}(\alpha) = \langle \Pi^{\eta}_{0}(\alpha)\otimes\mathbb{I}\rangle = \mathrm{Tr}[\hat{\varrho}\Pi^{\eta}_{0}(\alpha)\otimes\mathbb{I}_b]$$

$$\mathcal{Y}(\beta) = \langle \mathbb{I}\otimes\Pi^{\eta}_{0}(\beta)\rangle = \mathrm{Tr}[\hat{\varrho}\mathbb{I}_a\otimes\Pi^{\eta}_{0}(\beta)]$$

$$\mathcal{I}(\alpha,\beta) = \pi^2\int_{\mathbb{C}^2} d^2z\, d^2w\, W[\hat{\varrho}](z,w)W[\Pi_{0,\eta}](z-\alpha)W[\Pi_{0,\eta}](w-\beta),$$
$$\mathcal{G}(\alpha) = \pi^2\int_{\mathbb{C}^2} d^2z\, d^2w\, W[\hat{\varrho}](z,w)W[\Pi_{0,\eta}](z-\alpha)W[\mathbb{I}](w-\beta),$$
$$\mathcal{Y}(\beta) = \pi^2\int_{\mathbb{C}^2} d^2z\, d^2w\, W[\hat{\varrho}](z,w)W[\mathbb{I}](z-\alpha)W[\Pi_{0,\eta}](w-\beta),$$

Disuguaglianze di Bell

$$\Pi_{0,\eta} = \sum_{n=0}^{+\infty} (1-\eta)^n |n\rangle \langle n|$$

$$\Pi_{1,\eta} = \mathbb{I} - \Pi_{0,\eta}$$

$$W[\Pi_{0,\eta}](\alpha) = \frac{2}{\pi} \frac{1}{2-\eta} e^{\frac{2\eta}{2-\eta}|\alpha|^2}$$

$$W[\Pi_{1,\eta}](\alpha) = W[\mathbb{I}](\alpha) - W[\Pi_{0,\eta}](\alpha)$$

Correlazione

$$\begin{aligned}\langle \hat{n}_c \, \hat{n}_d \rangle &= \mathrm{Tr}[\hat{\varrho}_c \otimes \hat{\varrho}_d \; \hat{n}_c \otimes \hat{n}_d] = \\ &= \mathrm{Tr}[\hat{\varrho}_c \; \hat{n}_c]\mathrm{Tr}[\hat{\varrho}_d \; \hat{n}_d] = \\ &= \langle \hat{n}_c \rangle \, \langle n_d \rangle\end{aligned}$$

$$\Gamma = \frac{\langle \hat{n}_c \hat{n}_d \rangle - \langle \hat{n}_c \rangle \langle \hat{n}_d \rangle}{\sqrt{\mathrm{Var}(\hat{n}_c)\mathrm{Var}(\hat{n}_d)}}$$

Correlazione

$\Gamma = 0$ $$\mathcal{U}_{\mathrm{BS}}\ \hat{\varrho}_{\mathrm{th}} \otimes \hat{\varrho}_{\mathrm{th}}\ \mathcal{U}_{\mathrm{BS}}^{\dagger} \to \hat{\varrho}_{\mathrm{th}} \otimes \hat{\varrho}_{\mathrm{th}}$$

$$\Gamma = \frac{\langle \hat{n}_c \hat{n}_d \rangle - \langle \hat{n}_c \rangle \langle \hat{n}_d \rangle}{\sqrt{\mathrm{Var}(\hat{n}_c)\mathrm{Var}(\hat{n}_d)}}$$