Cap 3. Stochastic calculus

3.1 Introduction

In mathematical finance many problems involve handling random variables over infinitesimal time intervals: the field of mathematics covering this subject is known as *stochastic calculus*, whose rules are different in some aspects compared to "standard" calculus.

3.3 Functions

In deterministic calculus a function is defined $f: R => R$: from one input (usually called x) there can be only one output (usually defined y). In stochastic calculus this is not the case: for example the future stock price varies according to the state of the world we will be in. We then write $f: R\ x\ \Omega => R$. Given $\omega\ \epsilon\ \Omega$ the function depends only on the value of x and is deterministic. This is illustrated in the figure below.

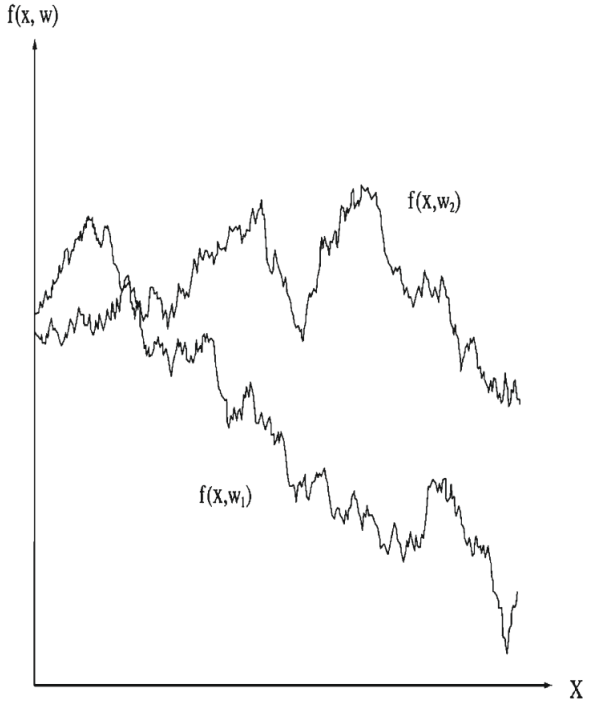

FIGURE 3.1 Plot of the function $f(x, \omega)$ for two different values of ω.

$f(x, \omega)$ can be called **random function** or **stochastic process**. Usually x represents the time and we are therefore interested in the process for $x \geq 0$. The randomness of a stochastic process is in terms of the trajectory as a whole, rather than a particular value at a specific point in time.

In the book there are much more technical details and an overview of deterministic calculus. We leave the technical aspects about deterministic calculus to the reader, as in the summary of this chapter we just want to give a very brief about the difficulties posed by stochastic calculus.

Source: *An introduction to the Mathematics of Financial Derivatives, Neftci Salih N.*