Chinese Remainder Theorem: Exercises

1. (a) Which integers leave a reminder of 1 when divided by both 2 and 3?
   (b) Which integers leave a reminder of 1 when divided by 2, 3, and 5?
   (c) Which integers leave a reminder of 1 when divided by 2, 3, 5, and 7?

2. (a) Find all integers that leave a reminder of 1 when divided by either 2 or 5, but which are divisible by 3.
   (b) Find all integers that leave a reminder of 2 when divided by either 3 or 7, but which are divisible by 8.

3. If possible, solve the following system of congruences using either of the two methods of this section.

$$\begin{aligned} x &\equiv 4 \pmod{11} \\ x &\equiv 3 \pmod{17} \\ x &\equiv 6 \pmod{18} \end{aligned}$$

4. If possible, solve the following system of congruences using either of the two methods of this section.

$$\begin{aligned} 2x &\equiv 11 \pmod{23} \\ 9x &\equiv 12 \pmod{31} \end{aligned}$$

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$$\begin{aligned} 2x &\equiv 11 \pmod{23} \\ 9x &\equiv 12 \pmod{33} \end{aligned}$$

6. If possible, solve the following system of congruences using either of the two methods of this section.

$$\begin{aligned} 20x &\equiv 9 \pmod{30} \\ 9x &\equiv 12 \pmod{33} \\ 36x &\equiv 48 \pmod{60} \end{aligned}$$

7. If possible, solve the following system of congruences using either of the two methods of this section.

$$\begin{aligned} 20x &\equiv 10 \pmod{30} \\ 9x &\equiv 12 \pmod{33} \\ 36x &\equiv 48 \pmod{60} \end{aligned}$$

8. A troop of 17 monkeys store their bananas in 11 piles of equal size, each containing more than one banana, with a twelfth pile of 6 left over. When they divide the bananas into 17 equal piles, none are left over. What is the smallest number of bananas they can have?

9. Even though not discussed in this section, you should now be able to solve the system of congruences

$$\begin{aligned} 2x + 5y &\equiv 4 \pmod{11} \\ x + 3y &\equiv 7 \pmod{11} \end{aligned}$$

by using a little bit of high school algebra. Start by eliminating one of the variables, and then use substitution. Find all integer solutions to the system. Is there any sense in which the solution can be said to be unique?

10. If you know about matrices and determinants, look at the coefficient matrix $\left[\begin{smallmatrix} 2 & 5 \\ 1 & 3 \end{smallmatrix}\right]$ of the system. Is the fact that it has determinant 1 relevant?

11. Can the linear system of congruences

$$\begin{aligned} 2x + 3y &\equiv 4 \pmod{6} \\ x + 3y &\equiv 3 \pmod{6} \end{aligned}$$

be solved? Justify your answer. What's different about this system compared with the system in Problem 9?