

AUTOMATIC CONTROL

Computer Engineering, Electronic and Communications Engineering

Laboratory practice n. 1

Objectives: computation of the time response of LTI dynamical systems through the solution of the system state equations (in Laplace domain), state space and transfer function representation of electric circuits.

Problem 1

Consider the following LTI system

$$\dot{x}(t) = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} x(t) + \begin{bmatrix} 5 \\ 8 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} -1 & 3 \end{bmatrix} x(t) + 8u(t)$$

Compute the state $x(t)$ and the output $y(t)$ responses when $x(0)=[2,2]^T$ and the input $u(t)$ is a step signal with amplitude 2.

$$(\text{Result: } x(t) = \begin{bmatrix} 3.0\bar{6}e^{5t} - 0.\bar{6}e^{-t} - 0.4 \\ 6.1\bar{3}e^{5t} + 0.\bar{6}e^{-t} - 4.8 \end{bmatrix} \varepsilon(t) \quad y(t) = (15.\bar{3}e^{5t} + 2.\bar{6}e^{-t} + 2)\varepsilon(t))$$

Problem 2

Given the LTI system

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 10 & 1 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Compute the output $y(t)$ response when $x(0)=[0,0]^T$, and $u_1(t) = 0$, $u_2(t) = 2 \delta(t)$.

$$(\text{Result: } y(t) = (2.3094e^{-0.5t} \cos(0.866t - 1.5708))\varepsilon(t))$$

Problem 3

Given the LTI system

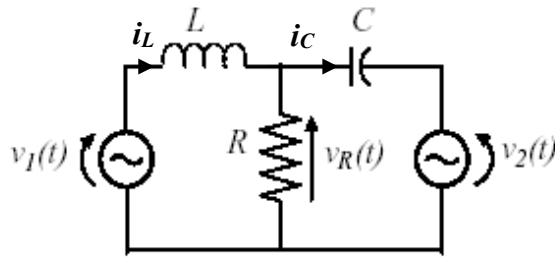
$$\dot{x}(t) = \begin{bmatrix} 0 & 6 \\ -1 & -5 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

compute the exponential matrix e^{At} of the system; (hint: recall that $e^{At} = \mathcal{L}^{-1}\{(sI - A)^{-1}\}$).

$$(\text{Result: } e^{At} = \begin{bmatrix} 3e^{-2t} - 2e^{-3t} & 6e^{-2t} - 6e^{-3t} \\ -e^{-2t} + e^{-3t} & -2e^{-2t} + 3e^{-3t} \end{bmatrix})$$

Problem 4

In the electric dynamical system below, generator voltages $v_1(t)$ e $v_2(t)$ are the inputs while the voltage $v_R(t)$ is the output:



Using basic circuit equations derive matrices A, B, C and D of the state space representation. (Hint. Use $i_L(t)$ e $v_C(t)$ as state variables)

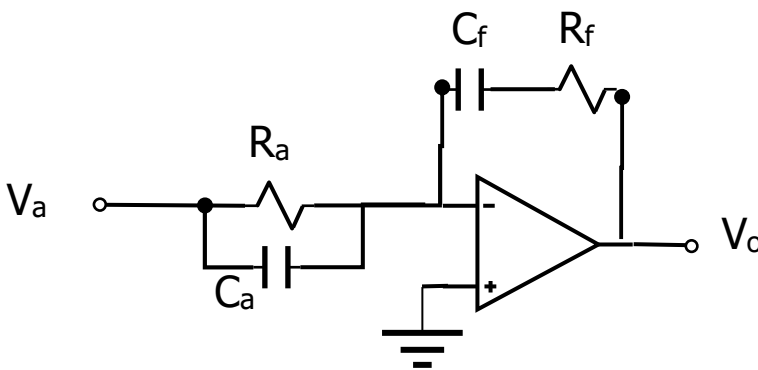
(Answer:

$$u(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \quad x(t) = \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad y(t) = v_R(t) = x_2(t) + u_2(t)$$

$$A = \begin{bmatrix} 0 & -1/L \\ 1/C & -1/RC \end{bmatrix} \quad B = \begin{bmatrix} 1/L & -1/L \\ 0 & -1/RC \end{bmatrix} \quad C = [0 \quad 1] \quad D = [0 \quad 1]$$

Problem 5

Show that the transfer function $H(s) = V_o(s)/V_a(s)$ of the circuit below is not proper.



(Answer:

$$H(s) = - \frac{(1 + R_f C_f s)(1 + R_a C_a s)}{R_a C_f s}$$