Attacks against hash functions

CATALDO BASILE

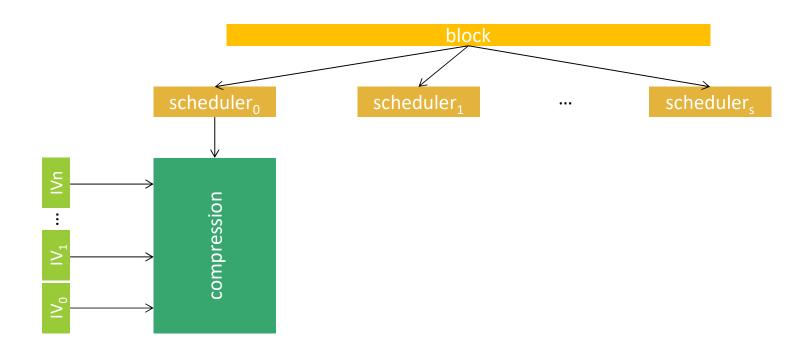
< CATALDO.BASILE@ POLITO.IT >

POLITECNICO DI TORINO

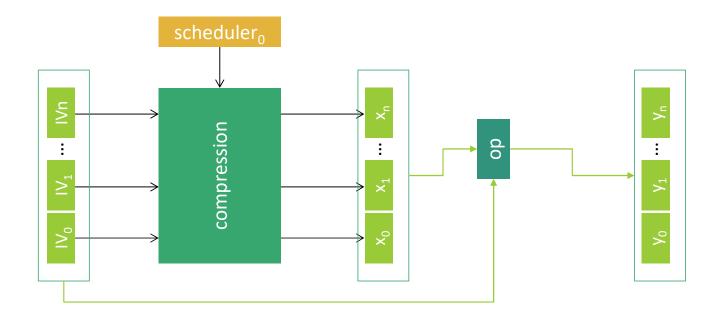
Agenda

- learning objectives
 - weak hash algorithms you can easily find collisions
 - short digests do not help...
 - hash constructions may be intrinsically weak
 - Merkle-Damgard still resists but...
- topics
 - Merkle-Damgard construction
 - seen from an attacker perspective
 - length extension attacks
 - Wang attack against MD4

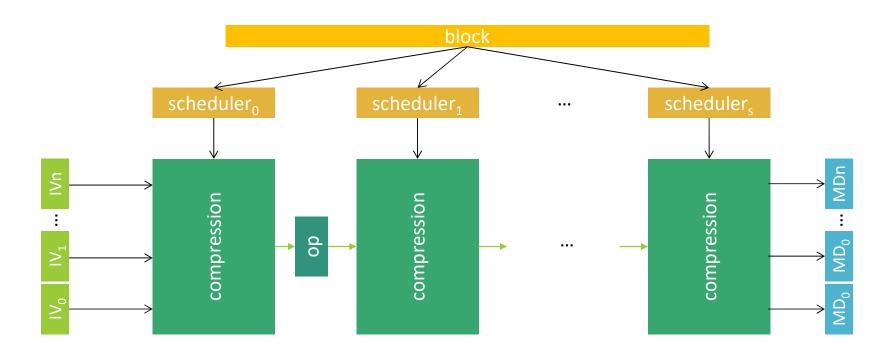
Merkle-Damgard architecture: 1 block



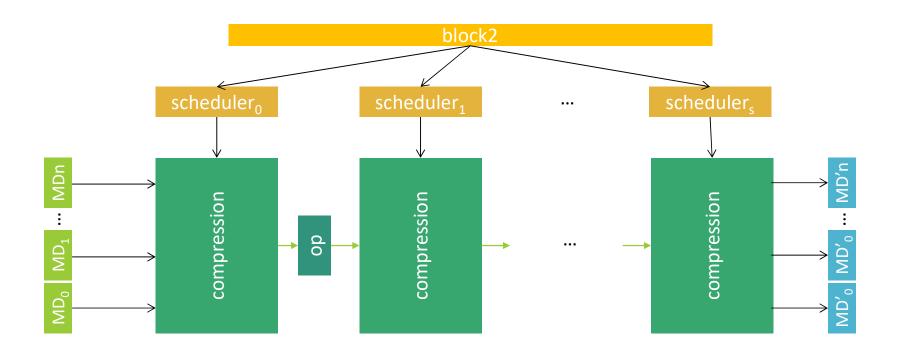
Compression



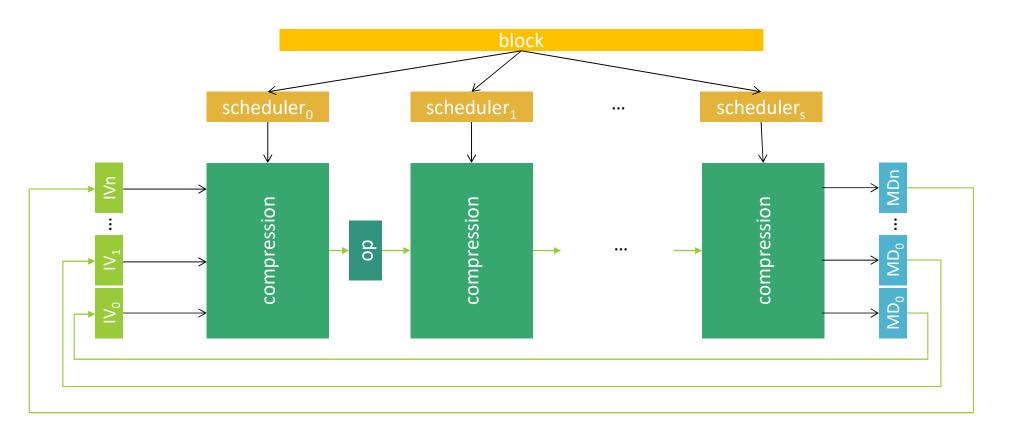
Merkle-Damgard architecture: 1 block



Merkle-Damgard architecture: block 2



Merkle-Damgard architecture



Attacks against the Merkle-Damgard construction

prone to multi-collision attacks

- many messages with the same hash
- i.e., once you find a collision it's computationally easy to generate additional collisions

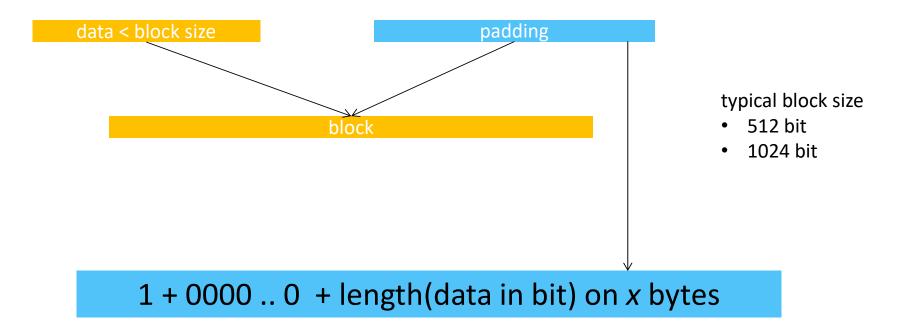
Herding attacks = chosen-prefix collisions

- to find useful collisions in less-than-ideal case
- https://tsapps.nist.gov/publication/get_pdf.cfm?pub_id=150629

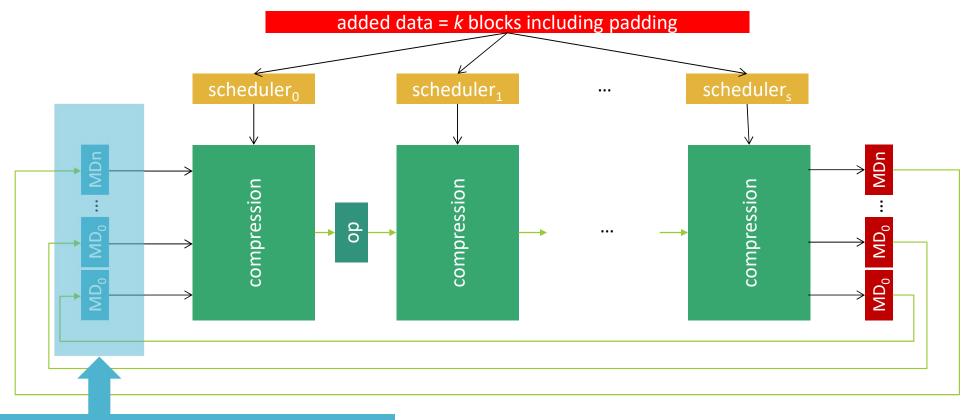
length extension

- digest of the n-th block computed based on the digest of the previous n-1 blocks
 - may break wrongly designed keyed-digests
- worked against commercial web message authentication schemes

Length extension attack



Merkle-Damgard: length extension



this is the (keyed-)hash sniffed by the attacks

A general purpose tool

HashPump automatically implements the attack for several algorithms

- implemented in C++
- https://github.com/bwall/HashPump

Collision attacks

against MD4 based on a paper of Wang

- https://link.springer.com/chapter/10.1007/978-3-642-25243-3_19
- https://eprint.iacr.org/2005/151.pdf
- implementation of the attack is here
 - https://github.com/HMY626/MD4-Collision

another interesting reading (SHAttered, against SHA-1)

https://shattered.io/static/shattered.pdf

visualization of MD5 collisions

https://www.links.org/?p=6

Table 6. A Set of Sufficient Conditions for Collisions of MD4

$\begin{array}{llllllllllllllllllllllllllllllllllll$		
$\begin{array}{l} c_1 & c_{1,7} = 1, \ c_{1,8} = 1, \ c_{1,11} = 0, \ c_{1,26} = d_{1,26} \\ b_1 & b_{1,7} = 1, \ b_{1,8} = 0, \ b_{1,11} = 0, \ b_{1,26} = 0 \\ a_2 & a_{2,8} = 1, \ a_{2,11} = 1, \ a_{2,26} = 0, \ a_{2,14} = b_{1,14} \\ d_2 & d_{2,14} = 0, \ d_{2,19} = a_{2,19}, \ d_{2,20} = a_{2,20}, \ d_{2,21} = a_{2,21}, \ d_{2,22} = a_{2,22}, \ d_{2,26} = 1 \\ c_2 & c_{2,13} = d_{2,13}, \ c_{2,14} = 0, \ c_{2,15} = d_{2,15}, \ c_{2,19} = 0, \ c_{2,20} = 0, \ c_{2,21} = 1, \ c_{2,22} = 0 \\ b_2 & b_{2,13} = 1, \ b_{2,14} = 1, \ b_{2,15} = 0, \ b_{2,17} = c_{2,17}, \ b_{2,19} = 0, \ b_{2,20} = 0, \ b_{2,21} = 0 \\ b_{2,22} = 0 \\ a_3 & a_{3,13} = 1, \ a_{3,14} = 1, \ a_{3,15} = 1, \ a_{3,17} = 0, \ a_{3,19} = 0, \ a_{3,20} = 0, \ a_{3,21} = 0, \\ a_{3,23} = b_{2,23} \ a_{3,22} = 1, \ a_{3,26} = b_{2,26} \\ d_3 & d_{3,13} = 1, \ d_{3,14} = 1, \ d_{3,15} = 1, \ d_{3,17} = 0, \ d_{3,20} = 0, \ d_{3,21} = 1, \ d_{3,22} = 1, \ d_{3,23} = 0, \\ d_{3,26} = 1, \ d_{3,30} = a_{3,30} \\ c_3 & c_{3,17} = 1, \ c_{3,20} = 0, \ c_{3,21} = 0, \ c_{3,22} = 0, \ c_{3,23} = 0, \ c_{3,36} = 0, \ c_{3,30} = 1, \ c_{3,32} = d_{3,32} \\ b_3 & b_{3,20} = 0, \ b_{3,21} = 1, \ b_{3,22} = 1, \ b_{3,23} = c_{3,23}, \ b_{3,26} = 1, \ b_{3,30} = 0, \ b_{3,32} = 0 \\ d_4 & d_{4,23} = 0, \ d_{4,26} = 0, \ a_{4,27} = b_{3,27}, \ a_{4,29} = b_{3,29}, \ a_{4,30} = 1, \ a_{4,33} = 0 \\ d_4 & d_{4,23} = 0, \ d_{4,26} = 0, \ d_{4,27} = 1, \ d_{4,29} = 1, \ d_{4,30} = 0, \ d_{4,30} = 0 \\ d_4 & d_{4,19} = d_{4,19}, \ c_{4,23} = 1, \ c_{4,26} = 1, \ c_{4,27} = 0, \ a_{5,29} = 1, \ a_{5,32} = 1 \\ d_5 & d_{5,19} = c_{4,19}, \ a_{5,26} = 1, \ a_{5,27} = 0, \ a_{5,29} = 1, \ a_{5,32} = 1 \\ d_5 & d_{5,19} = a_{5,19}, \ d_{5,26} = b_{4,26}, \ d_{5,27}, \ c_{5,29} = d_{5,29}, \ c_{5,30} = d_{5,30}, \ c_{5,32} = d_{5,32} \\ b_5 & b_{5,29} = c_{5,29}, \ b_{5,30} = 1, \ b_{5,32} = 0 \\ a_6 & a_{6,29} = 1, \ a_{6,32} = 1 \\ d_6 & d_{6,29} = b_{5,29} \\ c_6 & c_{6,29} = d_{6,29}, \ c_{6,30} = d_{6,30} + 1, \ c_{6,32} = d_{6,32} + 1 \\ b_9 & b_{9,32} = 1 \\ \end{array}$	a_1	$a_{1,7} = b_{0,7}$
$\begin{array}{l} b_1 & b_{1,7} = 1, b_{1,8} = 0, b_{1,11} = 0, b_{1,26} = 0 \\ a_2 & a_{2,8} = 1, a_{2,11} = 1, a_{2,26} = 0, a_{2,14} = b_{1,14} \\ d_2 & d_{2,14} = 0, d_{2,19} = a_{2,19}, d_{2,20} = a_{2,20}, d_{2,21} = a_{2,21}, d_{2,22} = a_{2,22}, d_{2,26} = 1 \\ c_2 & c_{2,13} = d_{2,13}, c_{2,14} = 0, c_{2,15} = d_{2,15}, c_{2,19} = 0, c_{2,20} = 0, c_{2,21} = 1, c_{2,22} = 0 \\ b_2 & b_{2,13} = 1, b_{2,14} = 1, b_{2,15} = 0, b_{2,17} = c_{2,17}, b_{2,19} = 0, b_{2,20} = 0, b_{2,21} = 0 \\ b_{2,22} = 0 \\ a_3 & a_{3,13} = 1, a_{3,14} = 1, a_{3,15} = 1, a_{3,17} = 0, a_{3,19} = 0, a_{3,20} = 0, a_{3,21} = 0, \\ a_{3,23} = b_{2,23} a_{3,22} = 1, a_{3,26} = b_{2,26} \\ d_3 & d_{3,13} = 1, d_{3,14} = 1, d_{3,15} = 1, d_{3,17} = 0, d_{3,20} = 0, d_{3,21} = 1, d_{3,22} = 1, d_{3,23} = 0, \\ d_{3,26} = 1, d_{3,30} = a_{3,30} \\ c_3 & c_{3,17} = 1, c_{3,20} = 0, c_{3,21} = 0, c_{3,22} = 0, c_{3,23} = 0, c_{3,26} = 0, c_{3,30} = 1, c_{3,32} = d_{3,32} \\ b_3 & b_{3,20} = 0, b_{3,21} = 1, b_{3,22} = 1, b_{3,23} = c_{3,23}, b_{3,26} = 1, b_{3,30} = 0, b_{3,32} = 0 \\ a_4 & a_{4,23} = 0, a_{4,26} = 0, a_{4,27} = b_{3,27}, a_{4,29} = b_{3,29}, a_{4,30} = 1, a_{4,32} = 0 \\ d_4 & d_{4,23} = 0, d_{4,26} = 0, d_{4,27} = 1, d_{4,29} = 1, d_{4,30} = 0, d_{4,32} = 1 \\ c_4 & c_{4,19} = d_{4,19}, c_{4,23} = 1, c_{4,26} = 1, c_{4,27} = 0, c_{4,29} = 0, c_{4,30} = 0 \\ b_4 & b_{4,19} = 0, b_{4,26} = c_{4,26} = 1, b_{4,27} = 1, b_{4,29} = 1, b_{4,30} = 0 \\ a_5 & a_{5,19} = c_{4,19}, a_{5,26} = b_{4,26}, d_{5,27} = b_{4,27}, d_{5,29} = b_{4,29}, d_{5,32} = b_{4,32} \\ c_5 & c_{5,26} = d_{5,26}, c_{5,27} = d_{5,27}, c_{5,29} = d_{5,29}, c_{5,30} = d_{5,30}, c_{5,32} = d_{5,32} \\ b_5 & b_{5,29} = c_{5,29}, b_{5,30} = 1, b_{5,32} = 0 \\ a_6 & a_{6,29} = 1, a_{6,32} = 1 \\ d_6 & d_{6,29} = b_{5,29} \\ c_6 & c_{6,29} = d_{6,29}, c_{6,30} = d_{6,30} + 1, c_{6,32} = d_{6,32} + 1 \\ b_9 & b_{9,32} = $	d_1	$d_{1,7} = 0, d_{1,8} = a_{1,8}, d_{1,11} = a_{1,11}$
$\begin{array}{l} a_2 & a_{2,8} = 1, \ a_{2,11} = 1, \ a_{2,26} = 0, \ a_{2,14} = b_{1,14} \\ d_2 & d_{2,14} = 0, \ d_{2,19} = a_{2,19}, \ d_{2,20} = a_{2,20}, \ d_{2,21} = a_{2,21}, \ d_{2,22} = a_{2,22}, \ d_{2,26} = 1 \\ c_2 & c_{2,13} = d_{2,13}, \ c_{2,14} = 0, \ c_{2,15} = d_{2,15}, \ c_{2,19} = 0, \ c_{2,20} = 0, \ c_{2,21} = 1, \ c_{2,22} = 0 \\ b_2 & b_{2,13} = 1, \ b_{2,14} = 1, \ b_{2,15} = 0, \ b_{2,17} = c_{2,17}, \ b_{2,19} = 0, \ b_{2,20} = 0, \ b_{2,21} = 0 \\ b_{2,22} = 0 \\ a_3 & a_{3,13} = 1, \ a_{3,14} = 1, \ a_{3,15} = 1, \ a_{3,17} = 0, \ a_{3,19} = 0, \ a_{3,20} = 0, \ a_{3,21} = 0, \\ a_{3,23} = b_{2,23} \ a_{3,22} = 1, \ a_{3,26} = b_{2,26} \\ d_3 & d_{3,13} = 1, \ d_{3,14} = 1, \ d_{3,15} = 1, \ d_{3,17} = 0, \ d_{3,20} = 0, \ d_{3,21} = 1, \ d_{3,22} = 1, \ d_{3,23} = 0, \\ d_{3,26} = 1, \ d_{3,30} = a_{3,30} \\ c_3 & c_{3,17} = 1, \ c_{3,20} = 0, \ c_{3,21} = 0, \ c_{3,22} = 0, \ c_{3,23} = 0, \ c_{3,26} = 0, \ c_{3,30} = 1, \ c_{3,32} = d_{3,32} \\ b_3 & b_{3,20} = 0, \ b_{3,21} = 1, \ b_{3,22} = 1, \ b_{3,23} = c_{3,23}, \ b_{3,26} = 1, \ b_{3,30} = 0, \ b_{3,32} = 0 \\ a_4 & a_{4,23} = 0, \ a_{4,26} = 0, \ a_{4,27} = b_{3,27}, \ a_{4,29} = b_{3,29}, \ a_{4,30} = 1, \ a_{4,32} = 0 \\ d_4 & d_{4,23} = 0, \ d_{4,26} = 0, \ d_{4,27} = 1, \ d_{4,29} = 1, \ d_{4,30} = 0, \ d_{4,32} = 1 \\ c_4 & c_{4,19} = d_{4,19}, \ c_{4,23} = 1, \ c_{4,26} = 1, \ c_{4,27} = 0, \ c_{4,29} = 0, \ c_{4,30} = 0 \\ b_4 & b_{4,19} = 0, \ b_{4,26} = c_{4,26} = 1, \ b_{4,27} = 1, \ b_{4,29} = 1, \ b_{4,30} = 0 \\ a_5 & a_{5,19} = c_{4,19}, \ a_{5,26} = 1, \ a_{5,27} = 0, \ a_{5,27} = d_{5,29}, \ c_{5,30} = d_{5,30}, \ c_{5,32} = d_{5,32} \\ b_5 & b_{5,29} = c_{5,29}, \ b_{5,30} = 1, \ b_{5,32} = 0 \\ a_6 & a_{6,29} = 1, \ a_{6,32} = 1 \\ b_6 & d_{6,29} = b_{5,29} \\ c_6 & c_{6,29} = d_{6,29}, \ c_{6,30} = d_{6,30} + 1, \ c_{6,32} = d_{6,32} + 1 \\ b_9 & b_{9,32} = 1 \\ \end{array}$	c_1	$c_{1,7} = 1, c_{1,8} = 1, c_{1,11} = 0, c_{1,26} = d_{1,26}$
$\begin{array}{l} d_2 & d_{2,14} = 0, \ d_{2,19} = a_{2,19}, \ d_{2,20} = a_{2,20}, \ d_{2,21} = a_{2,21}, \ d_{2,22} = a_{2,22}, \ d_{2,26} = 1 \\ c_2 & c_{2,13} = d_{2,13}, \ c_{2,14} = 0, \ c_{2,15} = d_{2,15}, \ c_{2,19} = 0, \ c_{2,20} = 0, \ c_{2,21} = 1, \ c_{2,22} = 0 \\ b_2 & b_{2,13} = 1, \ b_{2,14} = 1, \ b_{2,15} = 0, \ b_{2,17} = c_{2,17}, \ b_{2,19} = 0, \ b_{2,20} = 0, \ b_{2,21} = 0 \\ b_{2,22} = 0 \\ a_3 & a_{3,13} = 1, \ a_{3,14} = 1, \ a_{3,15} = 1, \ a_{3,17} = 0, \ a_{3,29} = 0, \ a_{3,20} = 0, \ a_{3,21} = 0, \\ a_{3,23} & b_{2,23} \ a_{3,22} = 1, \ a_{3,26} = b_{2,26} \\ d_3 & d_{3,13} = 1, \ d_{3,14} = 1, \ d_{3,15} = 1, \ d_{3,17} = 0, \ d_{3,20} = 0, \ d_{3,21} = 1, \ d_{3,22} = 1, \ d_{3,23} = 0, \\ d_{3,26} & = 1, \ d_{3,30} = a_{3,30} \\ c_3 & c_{3,17} = 1, \ c_{3,20} = 0, \ c_{3,21} = 0, \ c_{3,22} = 0, \ c_{3,23} = 0, \ c_{3,26} = 0, \ c_{3,30} = 1, \ c_{3,32} = d_{3,32} \\ b_3 & b_{3,20} = 0, \ b_{3,21} = 1, \ b_{3,22} = 1, \ b_{3,23} = c_{3,23}, \ b_{3,26} = 1, \ b_{3,30} = 0, \ b_{3,32} = 0 \\ a_4 & a_{4,23} = 0, \ a_{4,26} = 0, \ a_{4,27} = b_{3,27}, \ a_{4,29} = b_{3,29}, \ a_{4,30} = 1, \ a_{4,32} = 0 \\ d_4 & d_{4,23} = 0, \ d_{4,26} = 0, \ d_{4,27} = 1, \ d_{4,29} = 1, \ d_{4,30} = 0, \ d_{4,32} = 1 \\ c_4 & c_{4,19} = d_{4,19}, \ c_{4,23} = 1, \ c_{4,26} = 1, \ c_{4,27} = 0, \ c_{4,29} = 0, \ c_{4,30} = 0 \\ d_5 & a_{5,19} = c_{4,19}, \ a_{5,26} = 1, \ a_{5,27} = 0, \ a_{5,29} = 1, \ a_{5,32} = 1 \\ d_5 & d_{5,19} = a_{5,19}, \ d_{5,26} = b_{4,26}, \ d_{5,27} = b_{4,27}, \ d_{5,29} = b_{4,29}, \ d_{5,30} = b_{4,32} \\ c_5 & c_{5,26} = d_{5,26}, \ c_{5,27} = d_{5,27}, \ c_{5,29} = d_{5,29}, \ c_{5,30} = d_{5,30}, \ c_{5,32} = d_{5,32} \\ b_5 & b_{5,29} = c_{5,29}, \ b_{5,30} = 1, \ b_{5,32} = 0 \\ a_6 & a_{6,29} = 1, \ a_{6,32} = 1 \\ d_6 & d_{6,29} = b_{5,29}, \\ c_6 & c_{6,29} = d_{6,29}, \ c_{6,30} = d_{6,30} + 1, \ c_{6,32} = d_{6,32} + 1 \\ b_9 & b_{9,32} = 1 \\ \end{array}$	b_1	$b_{1,7} = 1, b_{1,8} = 0, b_{1,11} = 0, b_{1,26} = 0$
$\begin{array}{l} c_2 & c_{2,13} = d_{2,13}, \ c_{2,14} = 0, \ c_{2,15} = d_{2,15}, \ c_{2,19} = 0, \ c_{2,20} = 0, \ c_{2,21} = 1, \ c_{2,22} = 0 \\ \\ b_2 & b_{2,13} = 1, \ b_{2,14} = 1, \ b_{2,15} = 0, \ b_{2,17} = c_{2,17}, \ b_{2,19} = 0, \ b_{2,20} = 0, \ b_{2,21} = 0 \\ \\ b_{2,22} = 0 \\ \\ a_3 & a_{3,13} = 1, \ a_{3,14} = 1, \ a_{3,15} = 1, \ a_{3,17} = 0, \ a_{3,19} = 0, \ a_{3,20} = 0, \ a_{3,21} = 0, \\ \\ a_{3,23} & b_{2,23} \ a_{3,22} = 1, \ a_{3,26} = b_{2,26} \\ \\ d_3 & d_{3,13} = 1, \ d_{3,14} = 1, \ d_{3,15} = 1, \ d_{3,17} = 0, \ d_{3,20} = 0, \ d_{3,21} = 1, \ d_{3,22} = 1, \ d_{3,23} = 0, \\ \\ d_{3,26} & = 1, \ d_{3,30} = a_{3,30} \\ \\ c_3 & c_{3,17} = 1, \ c_{3,20} = 0, \ c_{3,21} = 0, \ c_{3,22} = 0, \ c_{3,23} = 0, \ c_{3,26} = 0, \ c_{3,30} = 1, \ c_{3,32} = d_{3,32} \\ \\ b_3 & b_{3,20} & = 0, \ b_{3,21} = 1, \ b_{3,22} = 1, \ b_{3,23} = c_{3,23}, \ b_{3,26} = 1, \ b_{3,30} = 0, \ b_{3,32} = 0 \\ \\ a_4 & a_{4,23} & = 0, \ a_{4,26} = 0, \ a_{4,27} = b_{3,27}, \ a_{4,29} = b_{3,29}, \ a_{4,30} = 1, \ a_{4,32} = 0 \\ \\ d_4 & d_{4,23} & = 0, \ d_{4,26} = 0, \ d_{4,27} = 1, \ d_{4,29} = 1, \ d_{4,30} = 0, \ d_{4,32} = 1 \\ \\ c_4 & c_{4,19} & = d_{4,19}, \ c_{4,23} = 1, \ c_{4,26} = 1, \ c_{4,27} = 0, \ c_{4,29} = 0, \ c_{4,30} = 0 \\ \\ a_5 & b_{5,19} & = c_{4,19}, \ a_{5,26} = 1, \ a_{5,27} = 0, \ a_{5,29} = 1, \ a_{5,32} = 1 \\ \\ d_5 & d_{5,19} & = a_{5,19}, \ d_{5,26} = b_{4,26}, \ d_{5,27} = b_{4,27}, \ d_{5,29} = b_{4,29}, \ d_{5,32} = b_{4,32} \\ \\ c_5 & c_{5,26} & = d_{5,26}, \ c_{5,27} & = d_{5,27}, \ c_{5,29} = d_{5,29}, \ c_{5,30} = d_{5,30}, \ c_{5,32} = d_{5,32} \\ \\ b_5 & b_{5,29} & = c_{5,29}, \ b_{5,30} = 1, \ b_{5,32} = 0 \\ \\ a_6 & a_{6,29} & = 1, \ a_{6,32} = 1 \\ \\ d_6 & d_{6,29} & = b_{5,29} \\ \\ c_6 & c_{6,29} & = d_{6,29}, \ c_{6,30} & = d_{6,30} + 1, \ c_{6,32} & = d_{6,32} + 1 \\ \\ b_9 & b_{9,32} & = 1 \\ \\ \end{array}$	a_2	$a_{2,8} = 1, a_{2,11} = 1, a_{2,26} = 0, a_{2,14} = b_{1,14}$
$\begin{array}{l} c_2 & c_{2,13} = d_{2,13}, \ c_{2,14} = 0, \ c_{2,15} = d_{2,15}, \ c_{2,19} = 0, \ c_{2,20} = 0, \ c_{2,21} = 1, \ c_{2,22} = 0 \\ \\ b_2 & b_{2,13} = 1, \ b_{2,14} = 1, \ b_{2,15} = 0, \ b_{2,17} = c_{2,17}, \ b_{2,19} = 0, \ b_{2,20} = 0, \ b_{2,21} = 0 \\ \\ b_{2,22} = 0 \\ \\ a_3 & a_{3,13} = 1, \ a_{3,14} = 1, \ a_{3,15} = 1, \ a_{3,17} = 0, \ a_{3,19} = 0, \ a_{3,20} = 0, \ a_{3,21} = 0, \\ \\ a_{3,23} & b_{2,23} \ a_{3,22} = 1, \ a_{3,26} = b_{2,26} \\ \\ d_3 & d_{3,13} = 1, \ d_{3,14} = 1, \ d_{3,15} = 1, \ d_{3,17} = 0, \ d_{3,20} = 0, \ d_{3,21} = 1, \ d_{3,22} = 1, \ d_{3,23} = 0, \\ \\ d_{3,26} & = 1, \ d_{3,30} = a_{3,30} \\ \\ c_3 & c_{3,17} = 1, \ c_{3,20} = 0, \ c_{3,21} = 0, \ c_{3,22} = 0, \ c_{3,23} = 0, c_{3,26} = 0, c_{3,30} = 1, c_{3,32} = d_{3,32} \\ \\ b_3 & b_{3,20} & = 0, \ b_{3,21} = 1, \ b_{3,22} = 1, \ b_{3,23} = c_{3,23}, \ b_{3,26} = 1, \ b_{3,30} = 0, \ b_{3,32} = 0 \\ \\ a_4 & a_{4,23} & = 0, \ a_{4,26} = 0, \ a_{4,27} = b_{3,27}, \ a_{4,29} = b_{3,29}, \ a_{4,30} = 1, \ a_{4,32} = 0 \\ \\ d_4 & d_{4,23} & = 0, \ d_{4,26} = 0, \ d_{4,27} = 1, \ d_{4,29} = 1, \ d_{4,30} = 0, \ d_{4,32} = 1 \\ \\ c_4 & c_{4,19} & = d_{4,19}, \ c_{4,23} = 1, \ c_{4,26} = 1, \ c_{4,27} = 0, \ c_{4,29} = 0, \ c_{4,30} = 0 \\ \\ a_5 & b_{5,19} & = c_{4,19}, \ a_{5,26} = 1, \ a_{5,27} = 0, \ a_{5,29} = 1, \ a_{5,32} = 1 \\ \\ d_5 & d_{5,19} & = a_{5,19}, \ d_{5,26} = b_{4,26}, \ d_{5,27} = b_{4,27}, \ d_{5,29} = b_{4,29}, \ d_{5,32} = b_{4,32} \\ \\ c_5 & c_{5,26} & = d_{5,26}, \ c_{5,27} & = d_{5,27}, \ c_{5,29} = d_{5,29}, \ c_{5,30} = d_{5,30}, \ c_{5,32} = d_{5,32} \\ \\ b_5 & b_{5,29} & = c_{5,29}, \ b_{5,30} = 1, \ b_{5,32} = 0 \\ \\ a_6 & a_{6,29} & = 1, \ a_{6,32} = 1 \\ \\ d_6 & d_{6,29} & = b_{5,29} \\ \\ c_6 & c_{6,29} & = d_{6,29}, \ c_{6,30} & = d_{6,30} + 1, \ c_{6,32} = d_{6,32} + 1 \\ \\ b_9 & b_{9,32} & = 1 \\ \\ \end{array}$	d_2	$d_{2,14}=0,d_{2,19}=a_{2,19},d_{2,20}=a_{2,20},d_{2,21}=a_{2,21},d_{2,22}=a_{2,22},d_{2,26}=1$
$\begin{array}{l} b_2 & b_{2,13} = 1, \ b_{2,14} = 1, \ b_{2,15} = 0, \ b_{2,17} = c_{2,17}, \ b_{2,19} = 0, \ b_{2,20} = 0, \ b_{2,21} = 0 \\ & b_{2,22} = 0 \\ \\ a_3 & a_{3,13} = 1, \ a_{3,14} = 1, \ a_{3,15} = 1, \ a_{3,17} = 0, \ a_{3,19} = 0, \ a_{3,20} = 0, \ a_{3,21} = 0, \\ & a_{3,23} = b_{2,23} \ a_{3,22} = 1, \ a_{3,26} = b_{2,26} \\ \\ d_3 & d_{3,13} = 1, \ d_{3,14} = 1, \ d_{3,15} = 1, \ d_{3,17} = 0, \ d_{3,20} = 0, \ d_{3,21} = 1, \ d_{3,22} = 1, \ d_{3,23} = 0, \\ & d_{3,26} = 1, \ d_{3,30} = a_{3,30} \\ \\ c_3 & c_{3,17} = 1, \ c_{3,20} = 0, \ c_{3,21} = 0, \ c_{3,22} = 0, \ c_{3,23} = 0, c_{3,26} = 0, c_{3,30} = 1, c_{3,32} = d_{3,32} \\ \\ b_3 & b_{3,20} = 0, \ b_{3,21} = 1, \ b_{3,22} = 1, \ b_{3,23} = c_{3,23}, \ b_{3,26} = 1, \ b_{3,30} = 0, \ b_{3,32} = 0 \\ \\ a_4 & a_{4,23} = 0, \ a_{4,26} = 0, \ a_{4,27} = b_{3,27}, \ a_{4,29} = b_{3,29}, \ a_{4,30} = 1, \ a_{4,32} = 0 \\ \\ d_4 & d_{4,23} = 0, \ d_{4,26} = 0, \ d_{4,27} = 1, \ d_{4,29} = 1, \ d_{4,30} = 0, \ d_{4,32} = 1 \\ \\ c_4 & c_{4,19} = d_{4,19}, \ c_{4,23} = 1, \ c_{4,26} = 1, \ c_{4,27} = 0, \ c_{4,29} = 0, \ c_{4,30} = 0 \\ \\ d_5 & b_{4,19} = 0, \ b_{4,26} = c_{4,26} = 1, \ b_{4,27} = 1, \ b_{4,29} = 1, \ b_{4,30} = 0 \\ \\ a_5 & a_{5,19} = c_{4,19}, \ a_{5,26} = 1, \ a_{5,27} = 0, \ a_{5,29} = 1, \ a_{5,32} = 1 \\ \\ d_5 & d_{5,19} = a_{5,19}, \ d_{5,26} = b_{4,26}, \ d_{5,27} = b_{4,27}, \ d_{5,29} = b_{4,29}, \ d_{5,32} = b_{4,32} \\ \\ c_5 & c_{5,26} = d_{5,26}, \ c_{5,27} = d_{5,27}, \ c_{5,29} = d_{5,29}, \ c_{5,30} = d_{5,30}, \ c_{5,32} = d_{5,32} \\ \\ b_5 & b_{5,29} = c_{5,29}, \ b_{5,30} = 1, \ b_{5,32} = 0 \\ \\ a_6 & a_{6,29} = 1, \ a_{6,32} = 1 \\ \\ b_9 & b_{9,32} = 1 \\ \\ \end{array}$	c_2	$c_{2,13} = d_{2,13}, \ c_{2,14} = 0, \ c_{2,15} = d_{2,15}, \ c_{2,19} = 0, \ c_{2,20} = 0, \ c_{2,21} = 1, \ c_{2,22} = 0$
$\begin{array}{l} a_3 & a_{3,13} = 1, \ a_{3,14} = 1, \ a_{3,15} = 1, \ a_{3,17} = 0, \ a_{3,19} = 0, \ a_{3,20} = 0, \ a_{3,21} = 0, \\ & a_{3,23} = b_{2,23} \ a_{3,22} = 1, \ a_{3,26} = b_{2,26} \\ \\ d_3 & d_{3,13} = 1, \ d_{3,14} = 1, \ d_{3,15} = 1, \ d_{3,17} = 0, \ d_{3,20} = 0, \ d_{3,21} = 1, \ d_{3,22} = 1, \ d_{3,23} = 0, \\ & d_{3,26} = 1, \ d_{3,30} = a_{3,30} \\ \\ c_3 & c_{3,17} = 1, \ c_{3,20} = 0, \ c_{3,21} = 0, \ c_{3,22} = 0, \ c_{3,23} = 0, \ c_{3,26} = 0, \ c_{3,30} = 1, \ c_{3,32} = d_{3,32} \\ \\ b_3 & b_{3,20} = 0, \ b_{3,21} = 1, \ b_{3,22} = 1, \ b_{3,23} = c_{3,23}, \ b_{3,26} = 1, \ b_{3,30} = 0, \ b_{3,32} = 0 \\ \\ a_4 & a_{4,23} = 0, \ a_{4,26} = 0, \ a_{4,27} = b_{3,27}, \ a_{4,29} = b_{3,29}, \ a_{4,30} = 1, \ a_{4,32} = 0 \\ \\ d_4 & d_{4,23} = 0, \ d_{4,26} = 0, \ d_{4,27} = 1, \ d_{4,29} = 1, \ d_{4,30} = 0, \ d_{4,32} = 1 \\ \\ c_4 & c_{4,19} = d_{4,19}, \ c_{4,23} = 1, \ c_{4,26} = 1, \ c_{4,27} = 0, \ c_{4,29} = 0, \ c_{4,30} = 0 \\ \\ b_4 & b_{4,19} = 0, \ b_{4,26} = c_{4,26} = 1, \ b_{4,27} = 1, \ b_{4,29} = 1, \ b_{4,30} = 0 \\ \\ a_5 & a_{5,19} = c_{4,19}, \ a_{5,26} = 1, \ a_{5,27} = 0, \ a_{5,29} = 1, \ a_{5,32} = 1 \\ \\ d_5 & d_{5,19} = a_{5,19}, \ d_{5,26} = b_{4,26}, \ d_{5,27} = b_{4,27}, \ d_{5,29} = b_{4,29}, \ d_{5,32} = b_{4,32} \\ \\ c_5 & c_{5,26} = d_{5,26}, \ c_{5,27} = d_{5,27}, \ c_{5,29} = d_{5,29}, \ c_{5,30} = d_{5,30}, \ c_{5,32} = d_{5,32} \\ \\ b_5 & b_{5,29} = c_{5,29}, \ b_{5,30} = 1, \ b_{5,32} = 0 \\ \\ a_6 & a_{6,29} = 1, \ a_{6,32} = 1 \\ \\ d_6 & d_{6,29} = b_{5,29} \\ \\ c_6 & c_{6,29} = d_{6,29}, \ c_{6,30} = d_{6,30} + 1, \ c_{6,32} = d_{6,32} + 1 \\ \\ b_9 & b_{9,32} = 1 \\ \\ \end{array}$	b_2	
$\begin{array}{c} a_{3,23} = b_{2,23} \ a_{3,22} = 1, \ a_{3,26} = b_{2,26} \\ d_3 d_{3,13} = 1, \ d_{3,14} = 1, \ d_{3,15} = 1, \ d_{3,17} = 0, \ d_{3,20} = 0, \ d_{3,21} = 1, \ d_{3,22} = 1, \ d_{3,23} = 0, \\ d_{3,26} = 1, \ d_{3,30} = a_{3,30} \\ c_3 c_{3,17} = 1, \ c_{3,20} = 0, \ c_{3,21} = 0, \ c_{3,22} = 0, \ c_{3,23} = 0, c_{3,26} = 0, c_{3,30} = 1, c_{3,32} = d_{3,32} \\ b_3 b_{3,20} = 0, \ b_{3,21} = 1, \ b_{3,22} = 1, \ b_{3,23} = c_{3,23}, \ b_{3,26} = 1, \ b_{3,30} = 0, \ b_{3,32} = 0 \\ a_4 a_{4,23} = 0, \ a_{4,26} = 0, \ a_{4,27} = b_{3,27}, \ a_{4,29} = b_{3,29}, \ a_{4,30} = 1, \ a_{4,32} = 0 \\ d_4 d_{4,23} = 0, \ d_{4,26} = 0, \ d_{4,27} = 1, \ d_{4,29} = 1, \ d_{4,30} = 0, \ d_{4,32} = 1 \\ c_4 c_{4,19} = d_{4,19}, \ c_{4,23} = 1, \ c_{4,26} = 1, \ c_{4,27} = 0, \ c_{4,29} = 0, \ c_{4,30} = 0 \\ b_4 b_{4,19} = 0, \ b_{4,26} = c_{4,26} = 1, \ b_{4,27} = 1, \ b_{4,29} = 1, \ b_{4,30} = 0 \\ a_5 a_{5,19} = c_{4,19}, \ a_{5,26} = 1, \ a_{5,27} = 0, \ a_{5,29} = 1, \ a_{5,32} = 1 \\ d_5 d_{5,19} = a_{5,19}, \ d_{5,26} = b_{4,26}, \ d_{5,27} = b_{4,27}, \ d_{5,29} = b_{4,29}, \ d_{5,32} = b_{4,32} \\ c_5 c_{5,26} = d_{5,26}, \ c_{5,27} = d_{5,27}, \ c_{5,29} = d_{5,29}, \ c_{5,30} = d_{5,30}, \ c_{5,32} = d_{5,32} \\ b_5 b_{5,29} = c_{5,29}, \ b_{5,30} = 1, \ b_{5,32} = 0 \\ a_6 a_{6,29} = 1, \ a_{6,32} = 1 \\ d_6 d_{6,29} = b_{5,29} \\ c_6 c_{6,29} = d_{6,29}, \ c_{6,30} = d_{6,30} + 1, \ c_{6,32} = d_{6,32} + 1 \\ b_9 b_{9,32} = 1 \\ \end{array}$		
$\begin{array}{l} d_3 & d_{3,13} = 1, \ d_{3,14} = 1, \ d_{3,15} = 1, \ d_{3,17} = 0, \ d_{3,20} = 0, \ d_{3,21} = 1, \ d_{3,22} = 1, \ d_{3,23} = 0, \\ & d_{3,26} = 1, \ d_{3,30} = a_{3,30} \\ \hline c_3 & c_{3,17} = 1, \ c_{3,20} = 0, \ c_{3,21} = 0, \ c_{3,22} = 0, \ c_{3,23} = 0, c_{3,26} = 0, c_{3,30} = 1, c_{3,32} = d_{3,32} \\ b_3 & b_{3,20} = 0, \ b_{3,21} = 1, \ b_{3,22} = 1, \ b_{3,23} = c_{3,23}, \ b_{3,26} = 1, \ b_{3,30} = 0, \ b_{3,32} = 0 \\ \hline a_4 & a_{4,23} = 0, \ a_{4,26} = 0, \ a_{4,27} = b_{3,27}, \ a_{4,29} = b_{3,29}, \ a_{4,30} = 1, \ a_{4,32} = 0 \\ \hline d_4 & d_{4,23} = 0, \ d_{4,26} = 0, \ d_{4,27} = 1, \ d_{4,29} = 1, \ d_{4,30} = 0, \ d_{4,32} = 1 \\ \hline c_4 & c_{4,19} = d_{4,19}, \ c_{4,23} = 1, \ c_{4,26} = 1, \ c_{4,27} = 0, \ c_{4,29} = 0, \ c_{4,30} = 0 \\ \hline b_4 & b_{4,19} = 0, \ b_{4,26} = c_{4,26} = 1, \ b_{4,27} = 1, \ b_{4,29} = 1, \ b_{4,30} = 0 \\ \hline a_5 & a_{5,19} = c_{4,19}, \ a_{5,26} = 1, \ a_{5,27} = 0, \ a_{5,29} = 1, \ a_{5,32} = 1 \\ \hline d_5 & d_{5,19} = a_{5,19}, \ d_{5,26} = b_{4,26}, \ d_{5,27} = b_{4,27}, \ d_{5,29} = b_{4,29}, \ d_{5,32} = b_{4,32} \\ \hline c_5 & c_{5,26} = d_{5,26}, \ c_{5,27} = d_{5,27}, \ c_{5,29} = d_{5,29}, \ c_{5,30} = d_{5,30}, \ c_{5,32} = d_{5,32} \\ \hline b_5 & b_{5,29} = c_{5,29}, \ b_{5,30} = 1, \ b_{5,32} = 0 \\ \hline a_6 & a_{6,29} = 1, \ a_{6,32} = 1 \\ \hline d_6 & d_{6,29} = b_{5,29} \\ \hline c_6 & c_{6,29} = d_{6,29}, \ c_{6,30} = d_{6,30} + 1, \ c_{6,32} = d_{6,32} + 1 \\ \hline b_9 & b_{9,32} = 1 \\ \hline \end{array}$	a_3	$a_{3,13} = 1, \ a_{3,14} = 1, \ a_{3,15} = 1, \ a_{3,17} = 0, \ a_{3,19} = 0, \ a_{3,20} = 0, \ a_{3,21} = 0,$
$\begin{array}{c} d_{3,26}=1,\ d_{3,30}=a_{3,30}\\ \hline\\ c_3\ c_{3,17}=1,\ c_{3,20}=0,\ c_{3,21}=0,\ c_{3,22}=0,\ c_{3,23}=0,\ c_{3,26}=0,\ c_{3,30}=1,\ c_{3,32}=d_{3,32}\\ \hline\\ b_3\ b_{3,20}=0,\ b_{3,21}=1,\ b_{3,22}=1,\ b_{3,23}=c_{3,23},\ b_{3,26}=1,\ b_{3,30}=0,\ b_{3,32}=0\\ \hline\\ a_4\ a_{4,23}=0,\ a_{4,26}=0,\ a_{4,27}=b_{3,27},\ a_{4,29}=b_{3,29},\ a_{4,30}=1,\ a_{4,32}=0\\ \hline\\ d_4\ d_{4,23}=0,\ d_{4,26}=0,\ d_{4,27}=1,\ d_{4,29}=1,\ d_{4,30}=0,\ d_{4,32}=1\\ \hline\\ c_4\ c_{4,19}=d_{4,19},\ c_{4,23}=1,\ c_{4,26}=1,\ c_{4,27}=0,\ c_{4,29}=0,\ c_{4,30}=0\\ \hline\\ b_4\ b_{4,19}=0,\ b_{4,26}=c_{4,26}=1,\ b_{4,27}=1,\ b_{4,29}=1,\ b_{4,30}=0\\ \hline\\ a_5\ a_{5,19}=c_{4,19},\ a_{5,26}=1,\ a_{5,27}=0,\ a_{5,29}=1,\ a_{5,32}=1\\ \hline\\ d_5\ d_{5,19}=a_{5,19},\ d_{5,26}=b_{4,26},\ d_{5,27}=b_{4,27},\ d_{5,29}=b_{4,29},\ d_{5,32}=b_{4,32}\\ \hline\\ c_5\ c_{5,26}=d_{5,26},\ c_{5,27}=d_{5,27},\ c_{5,29}=d_{5,29},\ c_{5,30}=d_{5,30},\ c_{5,32}=d_{5,32}\\ \hline\\ b_5\ b_{5,29}=c_{5,29},\ b_{5,30}=1,\ b_{5,32}=0\\ \hline\\ a_6\ a_{6,29}=1,\ a_{6,32}=1\\ \hline\\ d_6\ d_{6,29}=b_{5,29}\\ \hline\\ c_6\ c_{6,29}=d_{6,29},\ c_{6,30}=d_{6,30}+1,\ c_{6,32}=d_{6,32}+1\\ \hline\\ b_9\ b_{9,32}=1\\ \hline\end{array}$		$a_{3,23} = b_{2,23} \ a_{3,22} = 1, \ a_{3,26} = b_{2,26}$
$\begin{array}{l} c_3 & c_{3,17} = 1, \ c_{3,20} = 0, \ c_{3,21} = 0, \ c_{3,22} = 0, \ c_{3,23} = 0, c_{3,26} = 0, c_{3,30} = 1, c_{3,32} = d_{3,32} \\ b_3 & b_{3,20} = 0, \ b_{3,21} = 1, \ b_{3,22} = 1, \ b_{3,23} = c_{3,23}, \ b_{3,26} = 1, \ b_{3,30} = 0, \ b_{3,32} = 0 \\ a_4 & a_{4,23} = 0, \ a_{4,26} = 0, \ a_{4,27} = b_{3,27}, \ a_{4,29} = b_{3,29}, \ a_{4,30} = 1, \ a_{4,32} = 0 \\ d_4 & d_{4,23} = 0, \ d_{4,26} = 0, \ d_{4,27} = 1, \ d_{4,29} = 1, \ d_{4,30} = 0, \ d_{4,32} = 1 \\ c_4 & c_{4,19} = d_{4,19}, \ c_{4,23} = 1, \ c_{4,26} = 1, \ c_{4,27} = 0, \ c_{4,29} = 0, \ c_{4,30} = 0 \\ b_4 & b_{4,19} = 0, \ b_{4,26} = c_{4,26} = 1, \ b_{4,27} = 1, \ b_{4,29} = 1, \ b_{4,30} = 0 \\ a_5 & a_{5,19} = c_{4,19}, \ a_{5,26} = 1, \ a_{5,27} = 0, \ a_{5,29} = 1, \ a_{5,32} = 1 \\ d_5 & d_{5,19} = a_{5,19}, \ d_{5,26} = b_{4,26}, \ d_{5,27} = b_{4,27}, \ d_{5,29} = b_{4,29}, \ d_{5,32} = b_{4,32} \\ c_5 & c_{5,26} = d_{5,26}, \ c_{5,27} = d_{5,27}, \ c_{5,29} = d_{5,29}, \ c_{5,30} = d_{5,30}, \ c_{5,32} = d_{5,32} \\ b_5 & b_{5,29} = c_{5,29}, \ b_{5,30} = 1, \ b_{5,32} = 0 \\ a_6 & a_{6,29} = 1, \ a_{6,32} = 1 \\ d_6 & d_{6,29} = b_{5,29} \\ c_6 & c_{6,29} = d_{6,29}, \ c_{6,30} = d_{6,30} + 1, \ c_{6,32} = d_{6,32} + 1 \\ b_9 & b_{9,32} = 1 \\ \end{array}$	d_3	
$\begin{array}{l} b_3 b_{3,20} = 0, \ b_{3,21} = 1, \ b_{3,22} = 1, \ b_{3,23} = c_{3,23}, \ b_{3,26} = 1, \ b_{3,30} = 0, \ b_{3,32} = 0 \\ a_4 a_{4,23} = 0, \ a_{4,26} = 0, \ a_{4,27} = b_{3,27}, \ a_{4,29} = b_{3,29}, \ a_{4,30} = 1, \ a_{4,32} = 0 \\ d_4 d_{4,23} = 0, \ d_{4,26} = 0, \ d_{4,27} = 1, \ d_{4,29} = 1, \ d_{4,30} = 0, \ d_{4,32} = 1 \\ c_4 c_{4,19} = d_{4,19}, \ c_{4,23} = 1, \ c_{4,26} = 1, \ c_{4,27} = 0, \ c_{4,29} = 0, \ c_{4,30} = 0 \\ b_4 b_{4,19} = 0, \ b_{4,26} = c_{4,26} = 1, \ b_{4,27} = 1, \ b_{4,29} = 1, \ b_{4,30} = 0 \\ a_5 a_{5,19} = c_{4,19}, \ a_{5,26} = 1, \ a_{5,27} = 0, \ a_{5,29} = 1, \ a_{5,32} = 1 \\ d_5 d_{5,19} = a_{5,19}, \ d_{5,26} = b_{4,26}, \ d_{5,27} = b_{4,27}, \ d_{5,29} = b_{4,29}, \ d_{5,32} = b_{4,32} \\ c_5 c_{5,26} = d_{5,26}, \ c_{5,27} = d_{5,27}, \ c_{5,29} = d_{5,29}, \ c_{5,30} = d_{5,30}, \ c_{5,32} = d_{5,32} \\ b_5 b_{5,29} = c_{5,29}, \ b_{5,30} = 1, \ b_{5,32} = 0 \\ a_6 a_{6,29} = 1, \ a_{6,32} = 1 \\ d_6 d_{6,29} = b_{5,29} \\ c_6 c_{6,29} = d_{6,29}, \ c_{6,30} = d_{6,30} + 1, \ c_{6,32} = d_{6,32} + 1 \\ b_9 b_{9,32} = 1 \\ \end{array}$		$d_{3,26} = 1, d_{3,30} = a_{3,30}$
$\begin{array}{l} a_4 & a_{4,23} = 0, \ a_{4,26} = 0, \ a_{4,27} = b_{3,27}, \ a_{4,29} = b_{3,29}, \ a_{4,30} = 1, \ a_{4,32} = 0 \\ d_4 & d_{4,23} = 0, \ d_{4,26} = 0, \ d_{4,27} = 1, \ d_{4,29} = 1, \ d_{4,30} = 0, \ d_{4,32} = 1 \\ c_4 & c_{4,19} = d_{4,19}, \ c_{4,23} = 1, \ c_{4,26} = 1, \ c_{4,27} = 0, \ c_{4,29} = 0, \ c_{4,30} = 0 \\ b_4 & b_{4,19} = 0, \ b_{4,26} = c_{4,26} = 1, \ b_{4,27} = 1, \ b_{4,29} = 1, \ b_{4,30} = 0 \\ a_5 & a_{5,19} = c_{4,19}, \ a_{5,26} = 1, \ a_{5,27} = 0, \ a_{5,29} = 1, \ a_{5,32} = 1 \\ d_5 & d_{5,19} = a_{5,19}, \ d_{5,26} = b_{4,26}, \ d_{5,27} = b_{4,27}, \ d_{5,29} = b_{4,29}, \ d_{5,32} = b_{4,32} \\ c_5 & c_{5,26} = d_{5,26}, \ c_{5,27} = d_{5,27}, \ c_{5,29} = d_{5,29}, \ c_{5,30} = d_{5,30}, \ c_{5,32} = d_{5,32} \\ b_5 & b_{5,29} = c_{5,29}, \ b_{5,30} = 1, \ b_{5,32} = 0 \\ a_6 & a_{6,29} = 1, \ a_{6,32} = 1 \\ d_6 & d_{6,29} = b_{5,29} \\ c_6 & c_{6,29} = d_{6,29}, \ c_{6,30} = d_{6,30} + 1, \ c_{6,32} = d_{6,32} + 1 \\ b_9 & b_{9,32} = 1 \\ \end{array}$	c_3	$c_{3,17} = 1, c_{3,20} = 0, c_{3,21} = 0, c_{3,22} = 0, c_{3,23} = 0, c_{3,26} = 0, c_{3,30} = 1, c_{3,32} = d_{3,32}$
$\begin{array}{l} d_4 & d_{4,23} = 0, d_{4,26} = 0, d_{4,27} = 1, d_{4,29} = 1, d_{4,30} = 0, d_{4,32} = 1 \\ c_4 & c_{4,19} = d_{4,19}, c_{4,23} = 1, c_{4,26} = 1, c_{4,27} = 0, c_{4,29} = 0, c_{4,30} = 0 \\ b_4 & b_{4,19} = 0, b_{4,26} = c_{4,26} = 1, b_{4,27} = 1, b_{4,29} = 1, b_{4,30} = 0 \\ a_5 & a_{5,19} = c_{4,19}, a_{5,26} = 1, a_{5,27} = 0, a_{5,29} = 1, a_{5,32} = 1 \\ d_5 & d_{5,19} = a_{5,19}, d_{5,26} = b_{4,26}, d_{5,27} = b_{4,27}, d_{5,29} = b_{4,29}, d_{5,32} = b_{4,32} \\ c_5 & c_{5,26} = d_{5,26}, c_{5,27} = d_{5,27}, c_{5,29} = d_{5,29}, c_{5,30} = d_{5,30}, c_{5,32} = d_{5,32} \\ b_5 & b_{5,29} = c_{5,29}, b_{5,30} = 1, b_{5,32} = 0 \\ a_6 & a_{6,29} = 1, a_{6,32} = 1 \\ d_6 & d_{6,29} = b_{5,29} \\ c_6 & c_{6,29} = d_{6,29}, c_{6,30} = d_{6,30} + 1, c_{6,32} = d_{6,32} + 1 \\ b_9 & b_{9,32} = 1 \end{array}$	b_3	$b_{3,20} = 0, b_{3,21} = 1, b_{3,22} = 1, b_{3,23} = c_{3,23}, b_{3,26} = 1, b_{3,30} = 0, b_{3,32} = 0$
$\begin{array}{l} c_4 & c_{4,19} = d_{4,19}, \ c_{4,23} = 1, \ c_{4,26} = 1, \ c_{4,27} = 0, \ c_{4,29} = 0, \ c_{4,30} = 0 \\ b_4 & b_{4,19} = 0, \ b_{4,26} = c_{4,26} = 1, \ b_{4,27} = 1, \ b_{4,29} = 1, \ b_{4,30} = 0 \\ a_5 & a_{5,19} = c_{4,19}, \ a_{5,26} = 1, \ a_{5,27} = 0, \ a_{5,29} = 1, \ a_{5,32} = 1 \\ d_5 & d_{5,19} = a_{5,19}, \ d_{5,26} = b_{4,26}, \ d_{5,27} = b_{4,27}, \ d_{5,29} = b_{4,29}, \ d_{5,32} = b_{4,32} \\ c_5 & c_{5,26} = d_{5,26}, \ c_{5,27} = d_{5,27}, \ c_{5,29} = d_{5,29}, \ c_{5,30} = d_{5,30}, \ c_{5,32} = d_{5,32} \\ b_5 & b_{5,29} = c_{5,29}, \ b_{5,30} = 1, \ b_{5,32} = 0 \\ a_6 & a_{6,29} = 1, \ a_{6,32} = 1 \\ d_6 & d_{6,29} = b_{5,29} \\ c_6 & c_{6,29} = d_{6,29}, \ c_{6,30} = d_{6,30} + 1, \ c_{6,32} = d_{6,32} + 1 \\ b_9 & b_{9,32} = 1 \end{array}$	a_4	
$\begin{array}{llllllllllllllllllllllllllllllllllll$	d_4	
$\begin{array}{l} a_5 & a_{5,19} = c_{4,19}, \ a_{5,26} = 1, \ a_{5,27} = 0, \ a_{5,29} = 1, \ a_{5,32} = 1 \\ d_5 & d_{5,19} = a_{5,19}, \ d_{5,26} = b_{4,26}, \ d_{5,27} = b_{4,27}, \ d_{5,29} = b_{4,29}, \ d_{5,32} = b_{4,32} \\ c_5 & c_{5,26} = d_{5,26}, \ c_{5,27} = d_{5,27}, \ c_{5,29} = d_{5,29}, \ c_{5,30} = d_{5,30}, \ c_{5,32} = d_{5,32} \\ b_5 & b_{5,29} = c_{5,29}, \ b_{5,30} = 1, \ b_{5,32} = 0 \\ a_6 & a_{6,29} = 1, \ a_{6,32} = 1 \\ d_6 & d_{6,29} = b_{5,29} \\ c_6 & c_{6,29} = d_{6,29}, \ c_{6,30} = d_{6,30} + 1, \ c_{6,32} = d_{6,32} + 1 \\ b_9 & b_{9,32} = 1 \end{array}$	c_4	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	b_4	$b_{4,19} = 0, b_{4,26} = c_{4,26} = 1, b_{4,27} = 1, b_{4,29} = 1, b_{4,30} = 0$
$\begin{array}{llll} c_5 & c_{5,26} = d_{5,26}, \ c_{5,27} = d_{5,27}, \ c_{5,29} = d_{5,29}, \ c_{5,30} = d_{5,30}, \ c_{5,32} = d_{5,32} \\ b_5 & b_{5,29} = c_{5,29}, \ b_{5,30} = 1, \ b_{5,32} = 0 \\ \hline a_6 & a_{6,29} = 1, \ a_{6,32} = 1 \\ \hline d_6 & d_{6,29} = b_{5,29} \\ \hline c_6 & c_{6,29} = d_{6,29}, \ c_{6,30} = d_{6,30} + 1, \ c_{6,32} = d_{6,32} + 1 \\ \hline b_9 & b_{9,32} = 1 \\ \hline \end{array}$	a_5	
$\begin{array}{lll} b_5 & b_{5,29} = c_{5,29}, \ b_{5,30} = 1, \ b_{5,32} = 0 \\ a_6 & a_{6,29} = 1, \ a_{6,32} = 1 \\ d_6 & d_{6,29} = b_{5,29} \\ c_6 & c_{6,29} = d_{6,29}, \ c_{6,30} = d_{6,30} + 1, \ c_{6,32} = d_{6,32} + 1 \\ b_9 & b_{9,32} = 1 \end{array}$	d_5	$d_{5,19} = a_{5,19}, \ d_{5,26} = b_{4,26}, \ d_{5,27} = b_{4,27}, \ d_{5,29} = b_{4,29}, \ d_{5,32} = b_{4,32}$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	c_5	
	b_5	$b_{5,29} = c_{5,29}, b_{5,30} = 1, b_{5,32} = 0$
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$		
$b_9 b_{9,32} = 1$	d_6	$d_{6,29} = b_{5,29}$
	c_6	$c_{6,29} = d_{6,29}, c_{6,30} = d_{6,30} + 1, c_{6,32} = d_{6,32} + 1$
$ a_{10} a_{10,32}=1$	_	
-10,02	a_{10}	$a_{10,32} = 1$