AUTOMATIC CONTROL

Computer Engineering, Electronic and Communications Engineering

Laboratory practice n. 2

Objectives: Modal analysis, internal stability, BIBO stability of LTI systems. Equilibrium computation and stability (INF only)

Problem 1 (modal analysis of LTI systems)
Given the LTI system

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 10 & 1 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Perform the modal analysis.
(Answer $e^{-0.5t}\cos(0.866t - 1.5708) \rightarrow$ exponentially convergent)

Problem 2 (modal analysis of LTI systems)
Given the LTI system

$$\dot{x}(t) = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} x(t) + \begin{bmatrix} 5 \\ 8 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} -1 & 3 \end{bmatrix} x(t) + 8u(t)$$

Perform the modal analysis.
(Answer $e^{5t} \rightarrow$ exponentially divergent $e^{-t} \rightarrow$ exponentially convergent)

Problem 3 (internal stability of LTI systems)
Perform the modal analysis and study the internal stability properties of an LTI system characterized by the following dynamic matrix *A*.

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(Answer: $e^{-2t} \rightarrow$ exponentially convergent $\varepsilon(t) \rightarrow$ bounded constant $t \rightarrow$ polynomially divergent ;
the system is unstable)

Problem 4 (Internal and BIBO stability of LTI systems)
An LTI system is described by the following state space representation:

$$\dot{x}(t) = \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0.5 & -0.5 \end{bmatrix} x(t)$$

Study its internal and BIBO stability properties.

(Answer: internal stability → unstable, BIBO stability → stable)

Problem 5 (Internal of LTI systems)

Suppose that $p \in \mathbb{R}$, then discuss the internal stability properties of the LTI system characterized by the following dynamic matrix A.

$$A = \begin{bmatrix} p^2 - 1 & 0 & 0 \\ 0 & p - 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(Hint: notice that matrix A is diagonal…
Answer: $-1 < p < 1$ → asymptotically stable, $p = \pm 1$ → stable, $p > 1$, $p < -1$ → unstable)

Problem 6 (BIBO stability of LTI systems)

Analyse the BIBO stability properties of an LTI system having the following dynamic transfer function in the presence of variations of the real parameter p:

$$H(s) = \frac{4}{s^2 + (p+1)s + 4p - 2}$$

(Hint: apply Descartes' rule of sign to the denominator polynomial…
Answer: the system is BIBO stable for $p > 0.5$)

Problem 7 (INF only - Equilibrium solution computation and stability)

For the pendulum system with state equation

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -\dfrac{g}{l}\sin(x_1(t)) - \dfrac{\beta}{ml^2}x_2(t) + \dfrac{1}{ml^2}u(t) \end{cases}$$

$m = 0.1\,kg$, $l = 0.5\,m$, $\beta = 0.1\,Nsrad^{-1}$, $g = 9.81\,ms^{-2}$

Compute the equilibrium input $\bar{u}$ that corresponds to the equilibrium state $\bar{x} = \begin{bmatrix} \frac{\pi}{2} \\ 0 \end{bmatrix}$ and study its stability properties through the linearized model. (Answer: $\bar{u} = mgl \approx 0.49\ \mathrm{Nm}$, no conclusion can be drawn about the equilibrium stability)

Problem 8 (INF only - Equilibrium solution computation)

Consider the nonlinear dynamical system below

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -19.62\sin(x_1(t)) - 4x_1(t) - 4x_2(t) + 40u(t) \end{cases}$$

Compute the equilibrium states $\bar{x}$ corresponding to the equilibrium input $\bar{u} = 0$ and study their stability properties through the linearized model. (Hint: the solving equation for equilibrium computation can be trivially solved graphically through a MatLab plot. Answer: $\bar{x}' = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \bar{x}'' = \begin{bmatrix} -1.5708 \\ 0 \end{bmatrix}, \bar{x}''' = \begin{bmatrix} 1.5708 \\ 0 \end{bmatrix}$, all the equilibrium points are asymptotically stable)