

Introdution to LifeV

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- Why LifeV?
- Distribuite Communication
- Mesh
- Linear Algebra
Matrix, Vector, distributor of elements
- Finite Element Space
- ETA: Expression Template Assembler
- Linear Solver
- Post-Processing: Exporter/Importer



LifeV is a open source Finite Element Library for the solution of PDEs developed at MOX, CMCS (EPFL), E(CM)² (Emory) and Estime (INRIA).

- Object oriented, c++ code.
- Trilinos Epetra backend for distribute parallel computation.
- Research code oriented to the development novel numerical methods and algorithms.
- Aim: Solve large-scale, complex multi-physics engineering and scientific problems.
- Scalable library on HPC.
- Licensed under the GNU PBS.
- Release Versions are hosted at public repository <https://github.com/lifev/lifev>
- Google groups for mailing lists (lifev-user and lifev-dev)



Original features of LifeV:

- Type of Mesh: Hexaedra, Tetraedra (3D), Quad and Triangle (2D), Line (1D)
- Internal mesh generator for structured hypercube of dimension 1,2,3.
- Supporting external mesh format: .mesh (MEDIT), .msh (GMSH), .vol (NETGEN), m++
- Different Mesh Partitioner: Parmetis, Zoltan (Trilinos' package); offline-online partitioner to speed up the computation on HPC.
- Parallel I/O based on hdf5 format: internal wrapper of HDF5 library and EpetraExt Trilinos' HDF5 interface.
- Parser: GetPot for datafile and Teuchos (Trilinos) for XML.

- Construction of finite element assembly based on Epetra_FECrsMatrix, Epetra_FEVector and Epetra_Map
- Assembly of Finite Element based on DSEL principle: ETA Expression Template Assembly
- Internal Extension of Epetra Linear Algebra to assembly block matrix and block vector: “MatrixEpetraStructured” and “VectorEpetraStructured”
- Matrix-free methods are derived from Epetra_Operator
 1. LinearOperator
 2. SolverOperator(interface of an Invertible Linear Operator)
 3. ConfinedOperator (interface for restriction to a block part of Linear Block Operator)
- Preconditioner and Linear Solver:
 1. Internal interface for block preconditioner and multiplicative composition of preconditioner: SIMPLE,PCD and Yoshida.
 2. Wrapper of Ifpack (one-level Additive Schwarz Preconditioner), ML (Algebraic Multigrid), Teko (Block Preconditioner- LSC),
 3. Wrapper of Aztec00 (GMRES, CG,BICGSTAB) and Belos (Block GMRES) (Iterative solver)
 4. Wrapper of Anasazi (Eigenvalue) and Amesos (direct linear solver).

Algorithms and Modelling:

1. Geometric Multiscale Method
2. Level-Set
3. Monolithic and segregated algorithms for Fluid-Structure Interaction
4. Electrophysiology for cardiovascular system
5. Structure: Linear and NonLinear Elasticity
6. Fluid: Navier-Stokes
7. Porous Media: Darcy, Non-Darcy and Hyperbolic Problem

Ongoing Research about novel discretizations, methods and models
hosted at <https://cmcsforge.epfl.ch/projects/lifev>

1. Variational MultiScale
2. eXtended Finite Element Method
3. IsoGeometric Analysis on Surface (interface LifeV-LibIGA) and Structural problem (interface FEAP-LifeV)
4. Hierarchical Model reduction
5. Data Assimilation
6. Turbulence Models



Firstly in a own user/developed code in LifeV library, we need to include in header

```
1 #include <lifev/core/LifeV.hpp>
```

"LifeV.hpp" defines a number of types that are used in the library. The most used (in the tutorial) are

```
1 //uint32_type a 32 bit unsigned integer
typedef uint32_type UInt;
3 //! IDs
typedef uint32_type ID;
5 typedef double Real;
```



The Epetra Communicator Objects which encapsulates the general information and services needed to run on serial or parallel our computation.

```
1 #include <Epetra_ConfigDefs.h>
2 #ifdef EPETRA_MPI
3 #include <mpi.h>
4 #include <Epetra_MpiComm.h>
5 #else
6 #include <Epetra_SerialComm.h>
7 #endif
```

Listing 1: Header


```

1  int main ( int argc , char** argv )
   {
3  #ifdef HAVE_MPI
      MPI_Init ( &argc , &argv );
5  //For MPI distributed memory executions
      boost::shared_ptr<Epetra_Comm> Comm ( new Epetra_MpiComm
        (MPI_COMM_WORLD) );
7  #else
      //For serial executions
9      boost::shared_ptr<Epetra_Comm> Comm ( new
        Epetra_SerialComm );
   #endif
11     Int NumProc = Comm->NumProc();
      Int MyPID   = Comm->MyPID();
13     // ... other code follows ...
   #ifdef HAVE_MPI
15     MPI_Finalize();
   #endif
17     return 0;
   }

```



Create Structured Mesh

```
1 #include <lifev/core/mesh/RegionMesh1DStructured.hpp>
   typedef RegionMesh<LinearLine> mesh_Type;
3 meshPtr_Type FullMeshPtr(new mesh_Type(Comm));
   // MarkerID=0; Domain=[-1,2] with 10 Elements along x
5 regularMesh1D (*FullMeshPtr, 0, 10, 3, -1);
```



Create Structured Mesh

```
1 #include <lifev/core/mesh/RegionMesh2DStructured.hpp>
   typedef RegionMesh<LinearTriangle> mesh_Type;
3   meshPtr_Type FullMeshPtr ( new mesh_Type ( Comm ) );
   // MarkerID=0; Domain=[-1,2]x[-1,3] with 8 Elements along x
   // and 11 Elements along y
5   regularMesh2D ( *FullMeshPtr, 0, 8, 11, 2., 3., -1., -1. );
```



Create Structured Mesh

```
1 #include <lifev/core/mesh/RegionMesh3DStructured.hpp>
   typedef RegionMesh<LinearTetra> mesh_Type;
3   meshPtr_Type FullMeshPtr ( new mesh_Type ( Comm ) );
   // MarkerID=0; Domain=[-1,1]x[-1,1]x[-1,1] with 10 Elements
   // along x; 11 Elements along y; 12 Elements along z
5   regularMesh3D ( *FullMeshPtr, 0, 10, 11,12, 1.,1.,1.,
   -1.,-1.,-1 );
```



Declaration of the Method

```
1  template <typename GeoShape, typename MC >
   void regularMesh3D ( RegionMesh<GeoShape,MC >&
       mesh,markerID_Type regionFlag ,
3         const UInt& m_x,
         const UInt& m_y,
5         const UInt& m_z,
         bool verbose = false ,
7         const Real& l_x = 1.0 ,
         const Real& l_y = 1.0 ,
9         const Real& l_z = 1.0 ,
         const Real& t_x = 0.0 ,
11        const Real& t_y = 0.0 ,
         const Real& t_z = 0.0
13    )
```

Geoshape: LinearTriangle, LinearTetra (see ElementShape.hpp)
MC define the policy for the entityFlag treatment

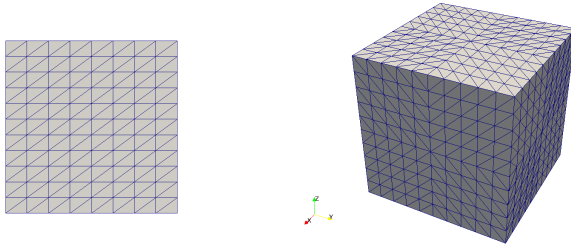


Figure: Triangular Mesh(left) and Tetrahedra Mesh (right)

For 2d Mesh, the Marker(Label) are

- for Edge are: 1 (BOTTOM), 2(LEFT), 3(TOP),4(RIGHT)
- for Corner are: 5(BOTTOM_RIGHT),
6(TOP_RIGHT),7(TOP_LEFT), 8(BOTTOM_LEFT)

For 3d Mesh, see "RegionMesh3DStructured.hpp".



The mesh is build up over the following base properties:

- Markers: label associated to any entity and has a partial order relation. The markers are:
point_marker, edge_marker, face_marker, volume_marker.
- Geometric Entities (GeoShape): Line, Triangle, Quad; their properties (member) are S_dimension, S_numFaces, S_numEdges, S_numVertices, S_shape, S_geometry
- Geometric Entities to define Finite Element: LinearTriangle, QuadTriangle whose properties are: S_numPoints (number of DOF), S_numPointsPerVertex ...
- Basic Entities of Mesh ("MeshEntity.hpp"), whose properties are: identifiers (localId and globalId) and flags to specify geometrical properties (PHYSICAL_BOUNDARY, SUBDOMAIN_INTERFACE, INTERNAL_INTERFACE)

In addition it is provided operation over the Mesh ("meshElementBare.hpp") , mesh Utility ("MeshUtility.hpp") and MeshCheck.hpp, which it is useful when we read external mesh. Finally it also defined the Container ("MeshEntityContainer.hpp") for mesh entity



Change Marker ID according to a function.

```
1 #include <lifev/core/mesh/MeshUtility.hpp>
2 #include <lifev/core/array/VectorSmall.hpp>
3 //Defining a function (outside main)
4 UInt colour_fun ( const VectorSmall<3>& barycentre )
5 {
6     if ( barycentre[0] < 0.5 && barycentre[1] < 0.5 )
7         return 2;
8     return 3;
9 }
10 MeshUtility::assignRegionMarkerID ( *FullMeshPtr ,
11     colour_fun );
12 // How many element have markerID =2 ?
13 const UInt ElementID2=
14     FullMeshPtr->elementList().countElementsWithMarkerID (
15     2, std::equal_to<markerID_Type>() );
```




Extract Extract elements and facets from a mesh based on a functor.

```
1 #include <lifev/core/mesh/MeshEntityContainer.hpp>
```

We need to define the "Predicates" (functors) whose templates are:

1. MeshEntity
2. Comparison Policy which must be a function able to compare two `flag_Type`.

The main method of Predicate is

```
1 bool operator()(const MeshEntity & entity) const
```

Example is provided in `core/testsuite/mesh " entity_selection.cpp"`.



Read External Mesh

The format supported by LifeV are: Mesh++, INRIA mesh, GMSH (.mesh), NETGEN (.vol) and MEDIT (.msh).

```
1 #include <lifev/core/filter/GetPot.hpp>
2 #include <lifev/core/mesh/MeshData.hpp>
3 // Read From dataFile
4     GetPot command_line (argc, argv);
5     const std::string data_file_name =
6         command_line.follow ("data", 2, "-f", "--file");
7     GetPot dataFile (data_file_name);
8     const std::string discretization_section = "mesh";
9     MeshData meshData (dataFile,
10         (discretization_section).c_str() );
11     readMesh (*fullMeshPtr, meshData);
```



```
1 #include <lifev/core/mesh/MeshPartitioner.hpp>
2 boost::shared_ptr<mesh_Type> meshPtr;
3 meshPtr localMesh;
4 {
5     MeshPartitioner< mesh_Type > meshPart;
6     meshPart.setPartitionOverlap ( numOverlap );
7     meshPart.doPartition ( fullMeshPtr, Comm );
8     localMeshPtr = meshPart.meshPartition();
9 }
10 fullMeshPtr.reset();
```



Using HDF5IO.hpp and MeshPartitionerTool.hpp it is possible to divide-et-impera the partitioning in offline(rootprocessor-writer)-online(parallel-read). It is possible to choose:

- MeshPartioner
- Parmetis
- Zoltan

Detail TestCase can be found in "core/testsuite/offline_partition_io".

EpetraMap manage the distribution of elements of matrices or vectors on a parallel machine (wrapper of Map_Epetra of Trilinos).

```
1 #include <lifev/core/array/MapEpetra.hpp>
   MapEpetra ( Int    numGlobalElements, // Num global Elements
3           Int    numMyElements, // Num local Elements
           Int* myGlobalElements, // Array of Id of
           local element
5           const comm_ptrtype& commPtr ); // Pointer
           to Communicator
   MapEpetra ( mapData_Type, // LifeV::Unique or
           LifeV::Repeated
7   const comm_ptrtype& commPtr ); // Pointer to Communicator
```

Repeated is used during assembly, while Unique structured is used for post-processing

Operations: Addition, Assign and Juxtaposition for MapVector

EpetraMatrix manage the Matrix (wrapper of Epetra_FECrsMatrix)

```
1 #include <lifev/core/array/MatrixEpetra.hpp>
   MatrixEpetra ( const MapEpetra& domainmap,
3                 const MapEpetra& rangemap,
                   Int numEntries = 50,
5                 bool ignoreNonLocalValues = false );
   MatrixEpetra ( const MapEpetra& map, const
                   Epetra_FECrsGraph& graph, bool ignoreNonLocalValues =
                   false); // Graph contains the non-zeros pattern
```

Operations: Addition, Subtraction, vector-matrix multiplication

Spy (open in matlab), save in hdf5

To fill-in matrix: setCoefficient, addToCoefficients and sumIntoCoefficients

Close and re-open matrix: openCrsMatrix(), globalAssemble()

EpetraVector manage the Matrix (wrapper of Epetra_FEVector)

```
#include <lifev/core/array/VectorEpetra.hpp>
2 explicit VectorEpetra ( const MapEpetra& map,
                        const MapEpetraType& mapType=Unique ,
4                        const combineMode_Type combineMode = Add );
VectorEpetra& subset ( const VectorEpetra& vector ,
6                        const MapEpetra& map,
                        const UInt offset1 ,
8                        const UInt offset2 );
```

Operations: Addition, Subtraction, division, comparison

To fill-in vector: setCoefficient, add, replace, sumIntoGlobalValues

Close vector, globalAssemble(), and calculating norm norm1(),...



ETFESpace defines a template version of Finite Element Space.

```
2 template<typename MeshType, typename MapType, UInt  
    SpaceDim, UInt FieldDim>  
ETFESpace<MeshType, MapType, SpaceDim, FieldDim>  
(const meshPtr_Type& mesh, const ReferenceFE* refFE, const  
    GeometricMap* geoMap, commPtr_Type& commptr)
```

If GeometricMap is guessed in the constructor, it is used the shape of the element of the mesh. Any information about the quadrature can be recover using QuadratureRuleProvider.hpp or “FESpace.hpp” (using method “qr()”)

```
1 QuadratureRuleProvider::provideExactness (TETRA, 0)
```




Finite Element Space: Scalar and Vector Field

```
1 #include <lifev/eta/fem/ETFESpace.hpp>
```

```
1 boost::shared_ptr<ETFESpace< mesh_Type, MapEpetra, 3, 1 > >  
    ETuSpace  
( new ETFESpace< mesh_Type, MapEpetra, 3, 1 >  
    (localMeshPtr, &feTetraP1, Comm) );
```

```
2 boost::shared_ptr<ETFESpace< mesh_Type, MapEpetra, 3, 3 > >  
    ETuSpace  
( new ETFESpace< mesh_Type, MapEpetra, 3,3 >  
    (localMeshPtr, &feTetraP1, Comm) );
```

Defining before an FESpace:

```
2 #include <lifev/core/fem/FESpace.hpp>
   std::string uOrder ("P1");
4 boost::shared_ptr<FESpace< mesh_Type, MapEpetra > > uSpace
   ( new FESpace< mesh_Type, MapEpetra > (meshPtr, uOrder, 3,
     Comm) );
   boost::shared_ptr<ETFESpace< mesh_Type, MapEpetra, 3, 3 >
     > ETuSpace
6   ( new ETFESpace< mesh_Type, MapEpetra, 3, 3 > (meshPtr, &
     (uSpace->refFE()), & (uSpace->fe().geoMap()), Comm) );
   std::cout<<"Total
     Dof"<<ETuSpace->dof().numTotalDof()<<std::endl;
```



```
1 #include <lifev/core/array/MatrixEpetra.hpp>
2 typedef MatrixEpetra<Real> matrix_Type;
3 boost::shared_ptr<matrix_Type> systemMatrix (new
4     matrix_Type ( ETuSpace->map() ) );
5 *systemMatrix *= 0.0;
6 {" fill in the matrix by Expression Template Assembler"}
7 systemMatrix->globalAssemble();
```



Diffusion-Reaction Equation

Let a domain Ω with a partion of $\partial\Omega = \Gamma_D \cup \Gamma_N \cup \Gamma_R$.

$$\begin{aligned} -\nabla \cdot (K \nabla u) + a_R u &= f \text{ in } \Omega \\ u &= g_D \text{ on } \Gamma_D \\ \frac{\partial u}{\partial n} &= g_N \text{ on } \Gamma_N \\ \alpha u + \frac{\partial u}{\partial n} &= g_R \text{ on } \Gamma_R \end{aligned} \tag{1}$$

Find $u \in H_{\Gamma_D}^1(\Omega)$ such that

$$\int_{\Omega} K \nabla u \nabla v + \int_{\Omega} a_R u + \alpha \int_{\Gamma_R} u v = \int_{\Omega} f v + \int_{\Gamma_N} g_N v + \int_{\Gamma_R} g_R v \quad \forall v \in H_0^1(\Omega) \tag{2}$$

See “lifev/eta/examples/diffusionReaction/main.cpp”



Parameter of Problem

```
Real scalarDiffusion ( const Real& /*t*/, const Real& x ,  
    const Real& y, const Real& z , const ID& /*i*/) 2  
{  
    return 1.; // std::sin (2* pi / y ) * std::cos ( 2*pi /  
    x ) * std::exp ( z ) ;  
} 4
```

```

2  class scalarDiffusionFunctor
3  {
4      public:
5          typedef Real return_Type;
6
7          return_Type operator() ( const VectorSmall<3>
8              spaceCoordinates )
9          {
10             return scalarDiffusion (0, spaceCoordinates[0],
11                 spaceCoordinates[1], spaceCoordinates[2] , 0 ) ;
12         }
13
14         scalarDiffusionFunctor() {}
15         scalarDiffusionFunctor (const scalarDiffusionFunctor&)
16         {}
17         ~scalarDiffusionFunctor() {}
18     };

```

```

2 boost::shared_ptr<scalarDiffusionFct>
  scalarDiffusionFct ( new scalarDiffusionFct );
3 {using namespace ExpressionAssembly;
  integrate ( elements (ETuFESpace->mesh() ),
4             quadRuleTetra4pt ,
5             ETuFESpace ,
6             ETuFESpace ,
7             eval ( scalarDiffusionFct , X) * dot (
8             grad (phi_j) , grad (phi_i) )
9             - eval ( scalarReactionFct , X) *
10             phi_j * phi_i
11             )
12             >> *SystemMatrix;
  }

```

```

2 { using namespace ExpressionAssembly;
   integrate ( elements (ETuFESpace->mesh() ), // Mesh
4
6         uFESpace->qr() , // QR
8         ETuFESpace ,
10        eval ( ScalarFctRhs , X ) * phi_i
12        )
    >> uRhs;
}

```




Impose Robin Boundary Condition:LHS

```
1  QuadratureBoundary myBDQR ( buildTetraBDQR  
   (quadRuleTria4pt) );  
2  integrate ( boundary (ETuFESpace→mesh() ,  
   BCFlags::BOTTOMWALL ) ,  
3           myBDQR ,  
4           ETuFESpace ,  
5           ETuFESpace ,  
6           value ( alpha ) * phi_j * phi_i  
7  
8           )  
9  >> *SystemMatrix;
```



Impose Robin Boundary Condition:RHS

```
integrate ( boundary (ETuFESpace->mesh() ,  
    BCFlags::BOTTOMWALL) , // Mesh  
  
    myBDQR, // QR  
  
    ETuFESpace ,  
  
    eval ( ScalarFctR , X ) * phi_i  
    )  
>> uRhs;
```

Impose Essential Boundary Condition

```
BCHandler bcHandler;  
2  BCFunctionBase dirichletBCFct ( dirichlet );  
   bcHandler.addBC ("Left", BCFlags::LEFTWALL, Essential ,  
   Full , dirichletBCFct , 1);  
4  bcHandler.addBC ("Right", BCFlags::RIGHTWALL,  
   Essential , Full , dirichletBCFct , 1);  
   bcHandler.addBC ("Back", BCFlags::BACKWALL, Essential ,  
   Full , dirichletBCFct , 1);  
6  bcHandler.bcUpdate ( *meshPtr, uFESpace->feBd() ,  
   uFESpace->dof() );  
   bcManage (*SystemMatrix , *uRhsUnique ,  
8         *uFESpace->mesh() , uFESpace->dof() ,  
         bcHandler, uFESpace->feBd() , 1.0 , Real (0.0)  
   );
```

Setting the Algebraic System Solver

```
1    prec_Type* precRawPtr;  
    basePrecPtr_Type precPtr;  
3    precRawPtr = new prec_Type;  
    precRawPtr->setDataFromGetPot ( dataFile , "prec" );  
5    precPtr.reset ( precRawPtr );  
    Teuchos::RCP< Teuchos::ParameterList > solverList =  
    Teuchos::rcp ( new Teuchos::ParameterList );  
7    const std::string solverParam = dataFile  
    ("solver/listName", "SolverParamListBelos.xml");  
    solverList = Teuchos::getParametersFromXmlFile  
    (solverParam );  
9    LinearSolver linearSolver;  
    linearSolver.setCommunicator ( Comm );  
11   linearSolver.setParameters ( *solverList );  
    linearSolver.setPreconditioner ( precPtr );
```



Solving the Algebraic System

```
vectorPtr_Type uSolution ( new vector_Type (
    ETuFESpace->map() , Unique) );
2  linearSolver.setOperator ( SystemMatrix );
   linearSolver.setRightHandSide ( uRhsUnique );
4  linearSolver.solve ( uSolution );
```



Exporter in HDF5

```
std::string const exporterFileName = dataFile (
"exporter/filename", "cube");
2 ExporterHDF5<mesh_Type> exporter ( dataFile, meshPtr,
exporterFileName, Comm->MyPID() );
exporter.setMultimesh ( false );
4 boost::shared_ptr<vector_Type> uExported ( new
vector_Type ( ETuFESpace->map(), exporter.mapType() ) );
exporter.addVariable (
ExporterData<mesh_Type>::ScalarField, "u", uFESpace,
uSolution, UInt (0) );
6 exporter.postProcess ( 1.0 );
exporter.closeFile();
8
```



Time Advance : BDF 1,2,3

```
1 #include <lifev/core/fem/TimeAdvanceBDF.hpp>
   const Real initialTime    = 0.0;
3   const Real endTime       = 100.0;
   const Real timestep       = 1e-1;
5   UInt BDFOrder = 2;
   TimeAdvanceBDF<vector_Type> bdf;
7   bdf.setup (BDFOrder);
   Real currentTime = initialTime - timestep * BDFOrder;
9   bdf.setInitialCondition ( *usolution );
```

11



Time Advance : BDF 1,2,3

```
2  bdf.updateRHSContribution ( timestep );
   *uRhsUnique = *MassMatrix *
       bdf.rhsContributionFirstDerivative();
   SystemMatrix.reset (new matrix_Type ( ETuFESpace->map() )
       );
4  double alphaTime = bdf.coefficientFirstDerivative ( 0 ) /
       timestep;
   *SystemMatrix += *MassMatrix * alphaTime;
6  *SystemMatrix += *BaseMatrix;
   bcManage (*SystemMatrix, *uRhsUnique,
8       *uFESpace->mesh(), uFESpace->dof(),
           bcHandler, uFESpace->feBd(), 1.0, Real (0.0)
       );
10 SystemMatrix->globalAssemble();
```




Tutorial is hosted at Github Portal:

<https://github.com/DavideBaroli/LifeVTutorial> in the folder
“/lifev/eta/example”

- Laplacian with Convergence Test
- Diffusion-Reaction Scalar and Vectorial
- Time-Dependent Diffusion Reaction Scalar
- Darcy Problem in Mixed Formulation
- Stokes
- Complex Coupled Problem: vortex and bi-phasic with different stabilization: VMS,PSPG, SUPG and Mesh Transformation.

Advance examples:

- Structure
- Pseudo-FSI



Efficient and Fast Expression Template Assembler for PDEs discretization

Purpose: Educational Use of LifeV, programming Expression Template, c++11 standard.

Use LifeV Assembler: ETA

1. Mixed-Hybrid and low-Raviart-Thomas formulation for Darcy Problem .
2. Hyperbolic discretization by Godunov Flux for Saturation Equation .
3. Laplace-Beltrami problem applied to porous media flow .
4. Multigrid Space-Time preconditioner for solving Navier-Stokes system.