Dynamical Systems formulations in Siconos.

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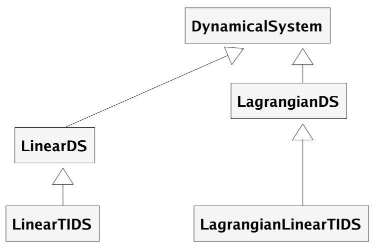
1 Class Diagram

There are five possible formulations for dynamical systems in Siconos, three for first order systems and two for second order Lagrangian systems. The main class is DynamicalSystem, all other derived from this one, as shown in the following diagram:



2 General non linear first order dynamical systems $\rightarrow$ class *DynamicalSystem*

This is the top class for dynamical systems. All other systems classes are derived from this one.

A general dynamical systems is described by the following set of n equations, completed with initial conditions:

$$\begin{aligned} \dot{x} &= f(x,t) + T(x)u(x,t) + r \qquad (1) \\ x(t_0) &= x_0 \qquad (2) \end{aligned}$$

- x: state of the system - Vector of size n.
- $f(x,t)$: sometimes called vector field - Vector of size n.
- $u(x,t)$: control term - Vector of size $uSize$.
- $T(x)$: $n \times uSize$ matrix, related to control term.
- r: input due to non-smooth behavior - Vector of size n.

The Jacobian matrix, $\nabla_x f(x,t)$, of f according to x, $n \times n$ square matrix, is also a member of the class.

Initial conditions are given by the member x_0, vector of size n. This corresponds to x value when simulation is starting, ie after a call to strategy->initialize().

There are plug-in functions in this class for f, its Jacobian, $jacobianXF$, u and T. All of them can handle a vector of user-defined parameters:

Main functions of the class:

- *computeRhs(time)*: to compute right hand side of equation (1) (saved in member rhs)
- *computeJacobianXRhs(time)*: to compute the Jacobian according to x of the right-hand side.
- *computeF(time)*: to compute $f(x,t)$ and save it into rhs member. Plug-in function must be set with *setComputeFFunction(pluginPath, functionName).*
- *computeJacobianXF(time): to compute $\nabla_x f(x,t)$, and save it into member JacobianXF. Plug-in function must be set with setComputeJacobianXFFunction(pluginPath, functionName).*

With *pluginPath* the name of the library that contains your plugin and functionName the name you give to the function. (for example if you defined your plug-in for f in the file MyPlugin.cpp and call the function computeMyF, then pluginPath ="MyPlugin.so" and functionName = "computeMyF".) Warning: the name of the plugin file must end with the string "Plugin", and the length of the string before "Plugin" must not exceed 6 letters.
The signature of each function (ie the number and type of arguments) must be exactly the same as the one given in Kernel/src/plugin/DefaultPlugin.cpp for the corresponding function.

3 First order linear dynamical systems → class *LinearDS*

Derived from DynamicalSystem, described by the set of n equations and initial conditions:

$$\dot{x} = A(t)x(t) + Tu(t) + b(t) + r \tag{3}$$
$$x(t_0) = x_0 \tag{4}$$

With:

- $A(t)$: $n \times n$ matrix, state independent but possibly time-dependent.
- $b(t)$: Vector of size n, possibly time-dependent.

Other variables are those of DynamicalSystem class.
A and B have corresponding plug-in functions.

Links with vectorField and its Jacobian are:

$$f(x,t) = A(t)x(t) + b(t) \tag{5}$$
$$jacobianXF = \nabla_x f(x,t) = A(t) \tag{6}$$

Main functions of the class:

- *computeRhs(time)*: to compute $Ax + b + Tu$ (saved in member rhs)
- *computeJacobianXRhs(time)*: compute $A(t)$.
- *computeA(time)*: to compute $A(t)$. Plug-in function must be set with *setComputeAFunction(pluginPath, functionName)*.
- *computeB(time)*: to compute $b(t)$. Plug-in function must be set with *setComputeBFunction(pluginPath, functionName)*.

4 First order time-invariant linear dynamical systems → class *LinearTIDS*

Derived from DynamicalSystem, described by the set of n equations and initial conditions:

$$\dot{x} = Ax(t) + Tu(t) + b + r \tag{7}$$
$$x(t_0) = x_0 \tag{8}$$

With:

- $A(t)$: $n \times n$ constant matrix
- $b(t)$: constant vector of size n

Other variables are those of DynamicalSystem class.

Links with vectorField and its Jacobian are:

$$f(x,t) = Ax(t) + b \tag{9}$$
$$jacobianXF = \nabla_x f(x,t) = A \tag{10}$$

Main functions of the class:

- *computeRhs(time)*: to compute $Ax + b + Tu$ (saved in member rhs)
- *computeJacobianXRhs(time)*: compute $A(t)$.

5 Second order non linear Lagrangian dynamical systems → class *LagrangianDS*

Lagrangian second order non linear systems are described by the following set of $nDof$ equations + initial conditions:

$$M(q)\ddot{q} + NNL(\dot{q},q) + F_{Int}(\dot{q},q,t) = F_{Ext}(t) + p \tag{11}$$
$$q(t_0) = q0 \tag{12}$$
$$\dot{q}(t_0) = velocity0 \tag{13}$$

With:

- $M(q)$: $nDof \times nDof$ matrix of inertia.
- q: state of the system - Vector of size $nDof$.
- $\dot{q}$ or $velocity$: derivative of the state according to time - Vector of size $nDof$.
- $NNL(\dot{q},q)$: non linear terms, time-independent - Vector of size $nDof$.
- $F_{Int}(\dot{q},q,t)$: time-dependent linear terms - Vector of size $nDof$.
- $F_{Ext}(t)$: external forces, time-dependent BUT do not depend on state - Vector of size $nDof$.
- p: input due to non-smooth behavior - Vector of size $nDof$.

The following Jacobian are also member of this class:

- jacobianQFInt = $\nabla_q F_{Int}(t,q,\dot{q})$ - $nDof \times nDof$ matrix.
- jacobianVelocityFInt = $\nabla_{\dot{q}} F_{Int}(t,q,\dot{q})$ - $nDof \times nDof$ matrix.

- jacobianQNNL = $\nabla_q NNL(q,\dot{q})$ - $nDof \times nDof$ matrix.
- jacobianVelocityNNL = $\nabla_{\dot{q}} NNL(q,\dot{q})$ - $nDof \times nDof$ matrix.

There are plug-in functions in this class for F_{int}, F_{Ext}, M, NNL and the four Jacobian matrices. All of them can handle a vector of user-defined parameters.
Call *computeOperator(...)* to compute value for operator = $FInt$, $FExt$, *Mass*, NNL, $JacobianQFInt$, $JacobianVelocityFInt$, $JacobianQNNL$,$JacobianVelocityNNL$. For any of them, link with plug-in function must be set using *setComputeOperatorFunction(pluginPath,pluginName)*.

Links with first order dynamical system are:

$$\begin{aligned}
n &= 2nDof && (14)\\
x &= \begin{bmatrix} q \\ \dot{q} \end{bmatrix} && (15)\\
f(x,t) &= \begin{bmatrix} \dot{q} \\ M^{-1}(F_{Ext} - F_{Int} - NNL) \end{bmatrix} && (16)\\
& && (17)\\
\nabla_x f(x,t) &= \begin{bmatrix} 0_{nDof\times nDof} & I_{nDof\times nDof} \\ \nabla_q(M^{-1})(F_{Ext} - F_{Int} - NNL) - M^{-1}\nabla_q(F_{Int} + NNL) & -M^{-1}\nabla_{\dot{q}}(F_{Int} + NNL) \end{bmatrix} && (18)\\
r &= \begin{bmatrix} 0_{nDof} \\ p \end{bmatrix} && (19)\\
u(x,\dot{x},t) &= u_L(\dot{q},q,t) \text{ (not yet implemented)} && (20)\\
T(x) &= \begin{bmatrix} 0_{nDof} \\ T_L(q) \end{bmatrix} \text{ (not yet implemented)} && (21)\\
& && (22)
\end{aligned}$$

With 0_n a vector of zero of size n, $0_{n\times m}$ a $n \times m$ zero matrix and $I_{n\times n}$, identity $n \times n$ matrix.

Warning: control terms (Tu) are not fully implemented in Lagrangian systems. This will be part of future version.

6 Second order linear and time-invariant Lagrangian dynamical systems → class *LagrangianLinearTIDS*

$$M\ddot{q} + C\dot{q} + Kq = F_{Ext}(t) + p \tag{23}$$

With:

- C: constant viscosity $nDof \times nDof$ matrix
- K: constant rigidity $nDof \times nDof$ matrix

And:

$$\begin{aligned}
F_{Int} &= C\dot{q} + Kq && (24)\\
NNL &= 0_{nDof} && (25)
\end{aligned}$$

7 How to handle parameters in plug-in functions for Dynamical Systems

7.1 User management

All plug-in in DynamicalSystem class, or in its derived classes, have a possible user-defined parameter argument, usually the last one in the list (see DefaultPlugin.cpp file). This argument consists in a pointer

to SimpleVector, which can be defined by user in the following way:
In the main input file of your sample (python or cpp),

- first declare and define a SimpleVector
- assign this SimpleVector to the DynamicalSystem plug-in, using one of the two following:
  - *setParameter(yourVector, "id")*, where *"id"* is the plug-in name (see table 1 below for the list of available ids for each class). In that case, yourVector is a SimpleVector (not a pointer!). This will set yourVector values as an input list for parameters in plug-in function *"id"*.
  - *setParameterPtr(yourVector, "id")*, where *"id"* is the plug-in name (see table 1 below for the list of available ids for each class). Here, yourVector is a pointer to SimpleVector. This will link parameters vector in plug-in function *"id"* to yourVector. (Warning: this means that any change to one of them (yourVector and parameter) will affect the other).
- then in the corresponding yourPlugin.cpp file, the variable *param* corresponds to the vector you defined.

Example: suppose that you defined a LagrangianDS named lds, and want to set two parameters in the external forces, say *mu* and *lambda*. Then cpp input file looks like:

```
// In the main file:

double mu;
double lambda;

// ... give mu and lambda the required value
// ... declare and built your dynamical system
DynamicalSystem * lds = new LagrangianDS(...)
// === First way, with setParameter function ===
// declare and built a SimpleVector of size 2
SimpleVector parameters(2);
parameters(0) = mu;
parameters(1) = lambda;

lds→setParameter(parameters,"fExt");

// In this case, if parameters values are change after this step, this wonÂt't affect param values inside the
dynamical system.

// === Second way, with setParameterPtr function ===
// declare and built a pointer to SimpleVector of size 2
SimpleVector * parameters = new SimpleVector(2);
(*parameters)(0) = mu;
(*parameters)(1) = lambda;

lds→setParameter(parameters,"fExt");

// Warning: in that case, from this point any change in parameters will affect param value in the dynamical
system.
//

// Then in the plug-in file, you have access to the parameter values:

extern "C" void computeFExt(const unsigned int&sizeOfq, const double *time, double *fExt, double
*param)
```

```
{
  for(unsigned int i =0; i<sizeOfq;++i)
    fExt[i] = cos(param[1]*time) + param[0] ;

  // this means that Fext = cos(lambda t) + mu

}
```

Warning: there is no relation between the name you give to your plug-in function (computeFExt in previous example) and the name given when you call $setParameter(..., id)$ $("Fext"$ in previous example).

7.2 List of plug-in parameters id for Dynamical Systems

Parameter id in ...	Corresponding operator
DynamicalSystem class:	
$"f"$	$f(x,t)$
$"jacobianXF"$	$\nabla_x f(x,t)$
$"u"$	$u(x,t)$
$"T"$	$T(x)$
LinearDS class:	
$"A"$	$A(t)$
$"b"$	$b(t)$
LagrangianDS class:	
$"mass"$	$M(q)$
$"fExt"$	$F_{Ext}(t)$
$"fInt"$	$F_{Int}(\dot{q},q,t)$
$"NNL"$	$NNL(\dot{q},q)$
$"jacobianQFInt"$	$\nabla_q F_{Int}(\dot{q},q,t)$
$"jacobianVelocityFInt"$	$\nabla_{\dot{q}} F_{Int}(\dot{q},q,t)$
$"jacobianQNNL"$	$\nabla_q NNL(\dot{q},q)$
$"jacobianVelocityNNL"$	$\nabla_{\dot{q}} NNL(\dot{q},q)$
$"uL"$	$u_L(q,t)$
$"TL"$	$T_L(q)$
LagrangianLinearTIDS class:	
$"fExt"$	$F_{Ext}(t)$

Table 1: List of available parameters id for plug-in functions in Dynamical Systems.

Mind that any item present in a class is also available in its derived classes.

Warning: if you create a new system using the copy constructor, any existing parameters in plug-in functions will also be copied. This means that no link between pointer for SimpleVector parameters will remain. In that case it may be better to re-set properly your parameters for the new created dynamical system.

7.3 More details for developpers

DynamicalSystem class has a member named parameterList which is a $map < string, SimpleVector* >$, ie a list of pointers to SimpleVector*, with a string as a key to identified them. For example, $parametersList["mass"]$ is a SimpleVector*, which corresponds to the last argument given in mass plug-in function.
By default, each parameters vectors must be initialized with a SimpleVector of size 1, as soon as the plug-in is declared. Moreover, to each vector corresponds a flag in isAllocatedIn map, to check if the corresponding vector has been allocated inside the class or not.
For example, in DynamicalSystem, if $isPlugin["vectorField"] == true$, then, during call to constructor or set function, it is necessary to defined the corresponding parameter:
$parametersList["vectorField"] = newSimpleVector(1)$
and to complete the $isAllocatedIn$ flag:
$isAllocatedIn["parameter_for_vectorField"] = true$.