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Case study:
Image deblur and super-resolution

Computational imaging
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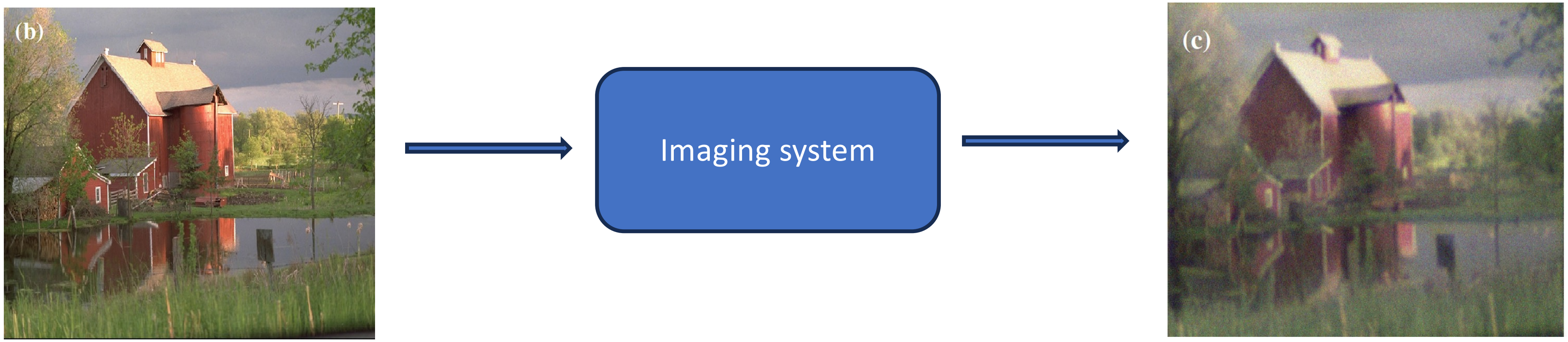
Image deblur (or image deconvolution)

Bibliography:
Hansen, Nagy, O’Leary, Deblurring images: matrices, spectra and filtering.



Blurring in images can depend on several factors, such as limitations of the optical system, camera and Object motion, astigmatism, and enviromental effects.

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Image deblur (or image deconvolution)

In many photo editing softwares there are filters to deblur images. However, they work for Mild blurs but they cannot overcome severe blurs.

We consider a *model-based* approach, where the cause of blur is mathematically modelled, and we use this model to recover a sharper, visually appealing image.

In this model, the blurred image is obtained as:

$$y = Ax$$

Where A is the matrix describing the action of the imagng system on the scanned object.

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Matehmatical tools: the convolution operator

To mathematically define the convolutional operation we need some preliminary concepts and definitions.

Definition (extended matrix)

Let A a matrix of size $M \times N$ and $k \in N$, we say that a matrix B of size $(M+2k) \times (N+2k)$ is *an extension of* A if and only if it holds:

$\forall\ (i,j) \in \{1,\ldots,M\} \times \{1,\ldots,N\}$

$$A_{(i,j)} = B_{(i+k,j+k)}$$

For example, if A is:

7	8	9
12	13	14
17	18	19

Then this can be an extension of A

(k=1)

$$\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{matrix}$$

Convolution operator

Definition (center of a matrix)

Let K be a matrix of size $D \times D$, with D odd. We define the *center of the matrix* K as the element of coordinates $(\frac{D+1}{2}, \frac{D+1}{2})$.

For example, for the matrix represented below with size 5×5, the center is the coefficient of indices $(\frac{5+1}{2}, \frac{5+1}{2})$: 13.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Convolution operator

Definition (convolution submatrices of A with respect to B).

Let A a matrix of size MxN, D an odd number.Let B a matrix of size (M+(D-1))x (N+(D-1)) an extension of A.

We define for each (i,j), i=1, ….M , j=1 ….N the convolution submatrices matrices of A $W_{(i,j)}$ as Sumbamtrices of B of size DxD, with center $B(i+\frac{D-1}{2}, j+\frac{D-1}{2})$

For example, if A is the following 3x3 matrix:

7	8	9
12	13	14
17	18	19

and D=3, to apply the previous definition I need a (3+2)x(3+2) matrix B extension of A. For example:

B=

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

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Convolution operator

For example that the matrix in red rectangle is a sub-matrix W(1,3) with center B(2,4).

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

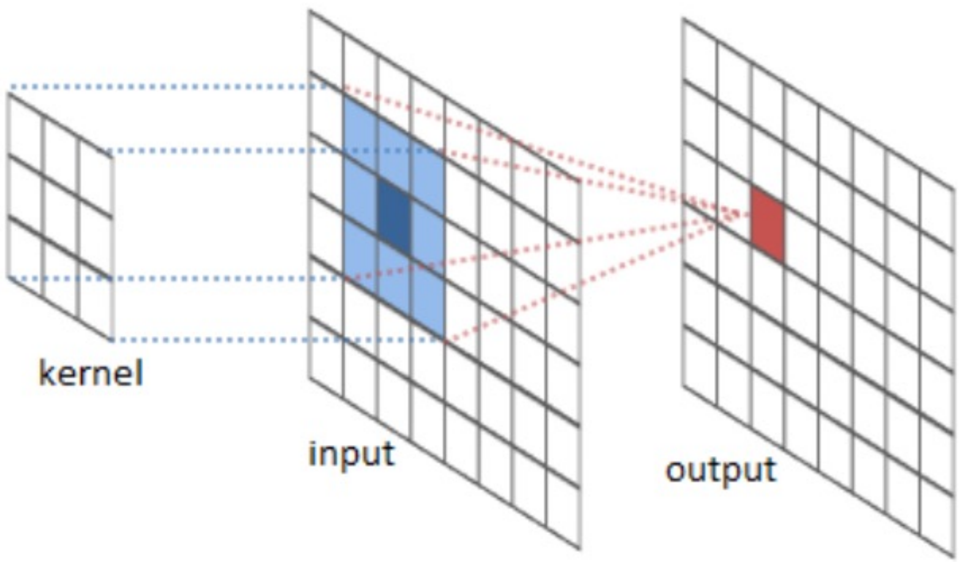
Definition. (convolution of K and A, K*A)
Let A a matrix of size $M \times N$ and K a matrix of size $D \times D$,(D odd).
Let B a matrix of size $(M + (D-1)) \times (N + (D-1))$ extension of A.
$\forall\ (i,j) \in \{1,\ldots,M\} \times \{1,\ldots,N\}$, Let $W_{(i,j)}$ the convolution sub-matrices $D \times D$ of A rwith respect to the extension B centered in $\cancel{B(i,j)}$.
WE define the matrix C of size $M \times N$,as follows:
$\forall\ (i,j) \in \{1,\ldots,M\} \times \{1,\ldots,N\}$:

$$C_{(i,j)} = \sum_{m=1}^{(D-1)/2} \sum_{n=1}^{(D-1)/2} K(m,n) W_{i,j}(m,n)$$

C will be denoted as $C = [K * A \mid B]$, and called *convolution of K and A with respect to the extension B.*

Convolution operator

K is the **kernel** of the convolution.



Convolution operator

To define the convolution is necessaryto extend the image matrix.

Possible extensions of the image matrix:

Zero boundary conditions

Periodic conditions

Reflective conditions

Convolution kernels: examples

- Esempio:

0	¼	0
¼	0	¼
0	¼	0

matrice di convoluzione

100	100	200	200
120	120	220	100
140	140	220	80
160	200	252	40

convoluzione →

100	100	200	200
120	145	160	100
140	170	173	80
160	200	252	40

$100\times0+100\times¼+200\times0+$
$120\times¼+120\times0+220\times¼+$
$140\times0+140\times¼+220\times0 \quad =145$

0	¼	0
¼	0	¼
0	¼	0

matrice di convoluzione

convoluzione →

Convolution kernels: examples

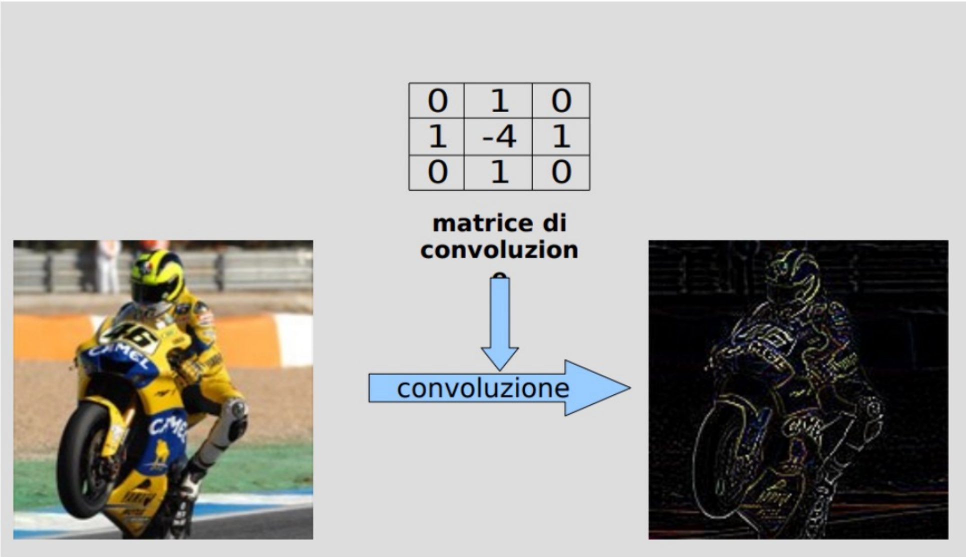


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Image blur as a convolution

The blurring process is a convolution between the matrix representing the object and a blurring kernel called Point Spread Function (PSF).

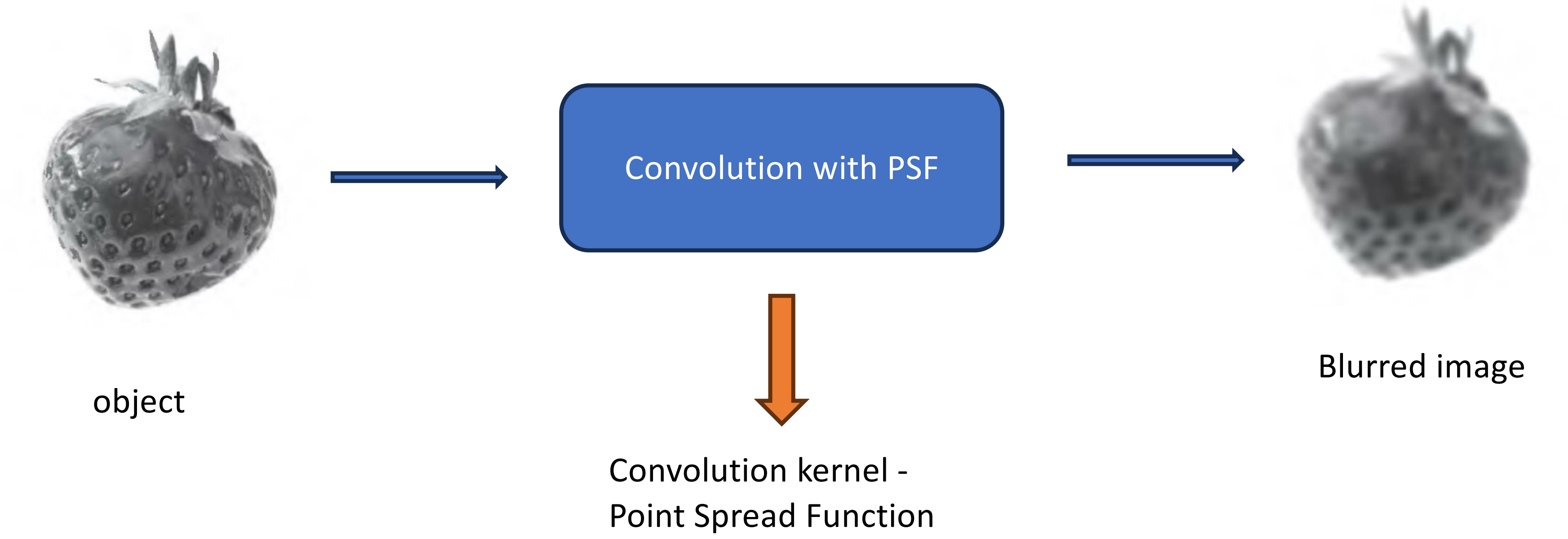

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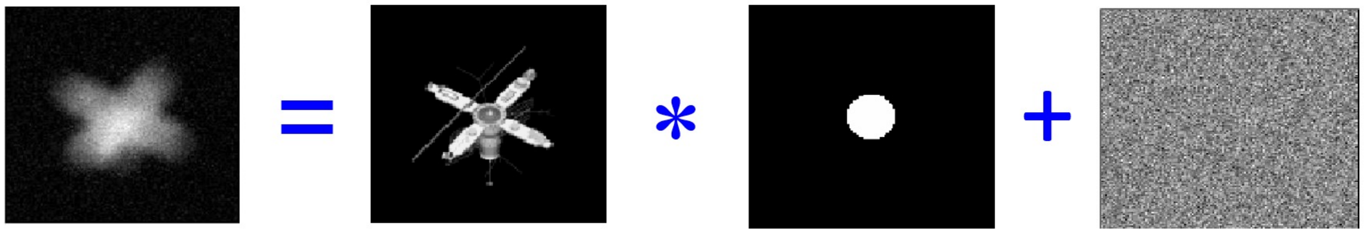
Image blur as a convolution



Observed image (known) = Original image (unknown) * Point Spread Function (known/unknown) + Realization of Random Noise (unknown)

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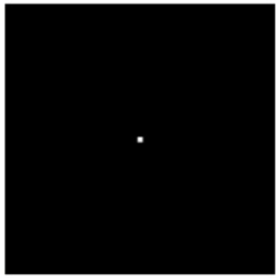
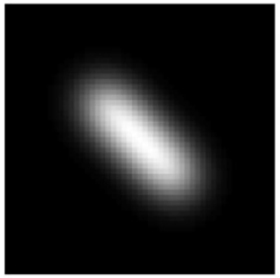
Image blur as a convolution

Single pixel image

Point Spread Function

$\mathcal{A}(X) = \mathcal{A}\Big($  $\Big) =$ 

In optical systems, the PDF represents the deformation due to the lens imperfection.

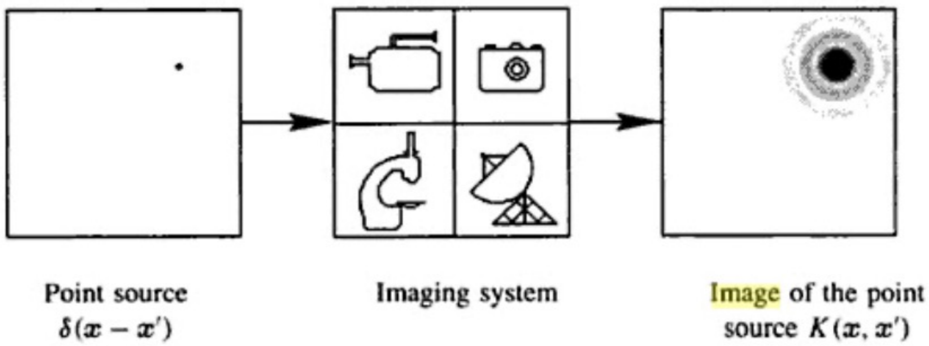


Figure 3.1. Schematic representation of the point spread function.

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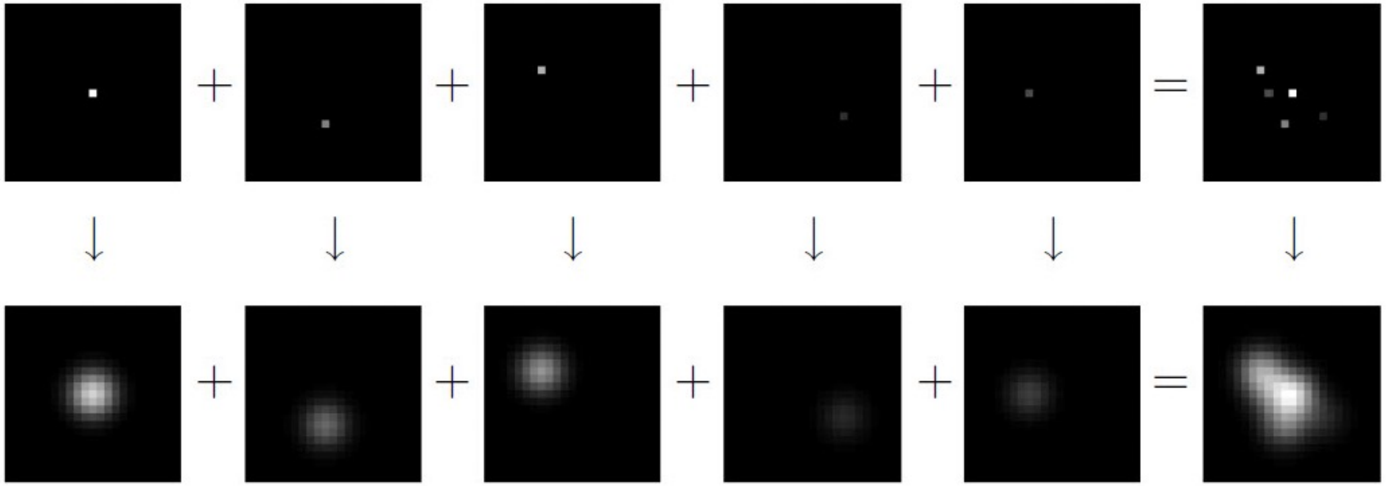
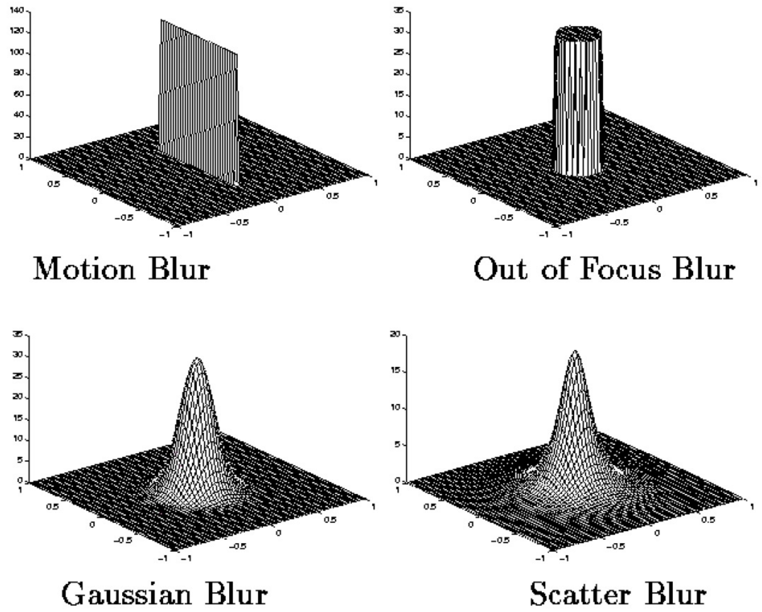
Space invariant PSF



The same PSF (convolution kernel) is applied to every pixel of the exact image (or object).
The final degraded image is the sum of the images obtained by applying the PSF to each pixel.
In this case we say that the PSF is space invariant and the mathematical model is a convolution.

Point Spread Functions: examples



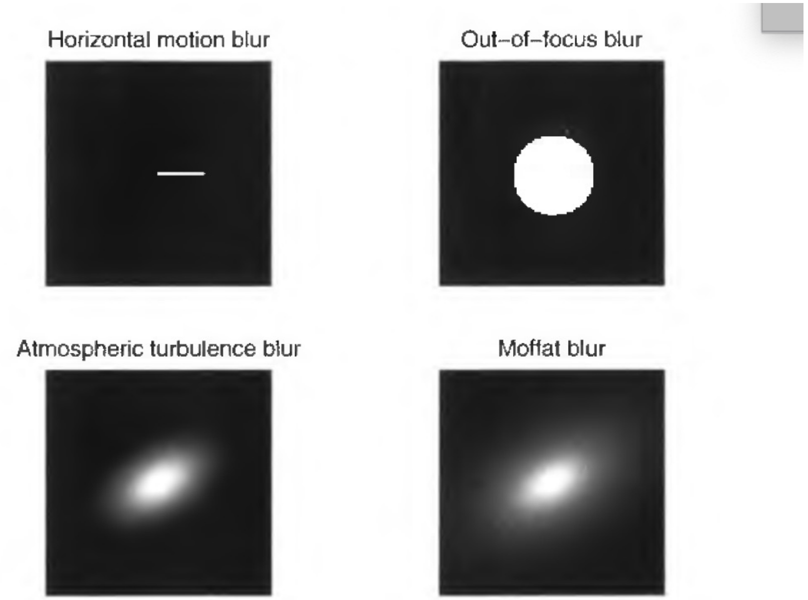


Figure 3.3. *Examples of four PSFs. In all four cases the center of the PSF coincides with the center of the PSF array.*

Point Spread Functions: examples

Usually the PSF is described by a mathematical function. The PSF array visualized in the previous figures and applied to each pixel of the exact image to generate the blurred one

$$\text{PSF}= \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

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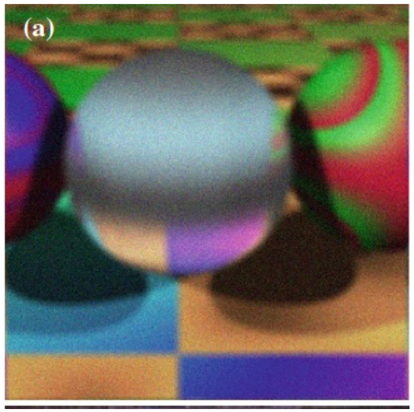
Various blurs

Gaussian blur



Motion blur

Color image blur

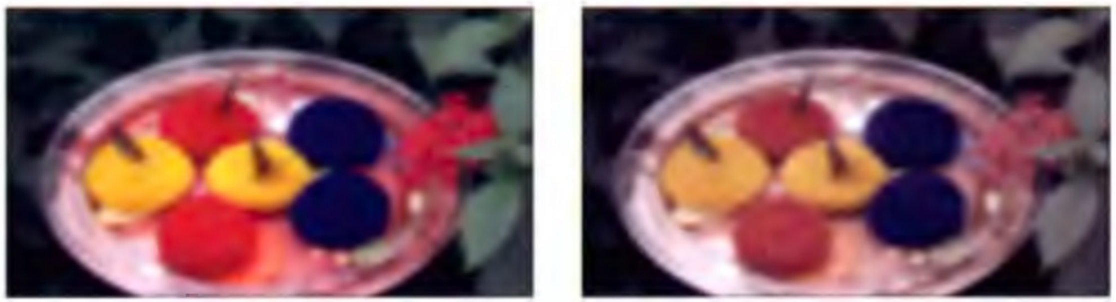


Figure 7.3. *Two types of blurred color images. Left: within-channel blurring only. Right: both within-channel blurring and cross-channel blurring with* $(a_{rr}, a_{rg}, a_{rb}) = (0.7, 0.2, 0.1)$, $(a_{br}, a_{gg}, a_{gb}) = (0.25, 0.5, 0.25)$, *and* $(a_{br}, a_{bg}, a_{bb}) = (0.15, 0.1, 0.75)$.

Various blurs

Camera shake (Camera motion blur)

Object movement (Object motion blur)

Out of focus (Defocus blur)

Combinations (vibration & motion, ...)

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Image blurring process

Imaging system

+

noise

Types of added noise

In addition to blurring, observed images are usually contaminated with noise.

Noise can arise from different sources and can be linear or non linear, additive or multiplicative.

WE consider in this course the noise commonly introduced that essentially comes from
One of these two sources:

- Background photons corrupt each pixel value measured by the sensor. This noise is modelled by a Poisson process and it is commonly called *Poisson noise*.

- The analog-to-digital conversion of measured values form sensors. This noise is usually called *white noise* and it is assumed to be drawn from a gaussian (normal) distribution with mean 0 and standard deviation proportional to the amplitude of the noise.

In our model, we incorporate the noise in the additive term:

$$Ax = y + e$$

And we approximate the Poisson noise with gaussian additive noise.

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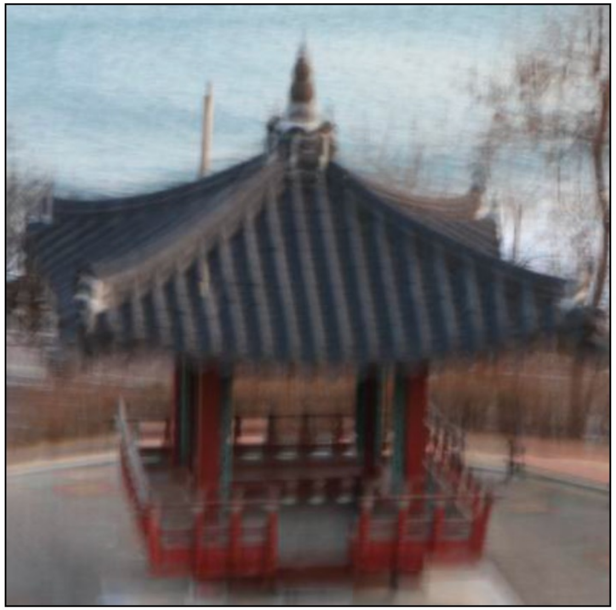
DEBLUR as inverse problem



from a given blurred image

find its latent sharp image

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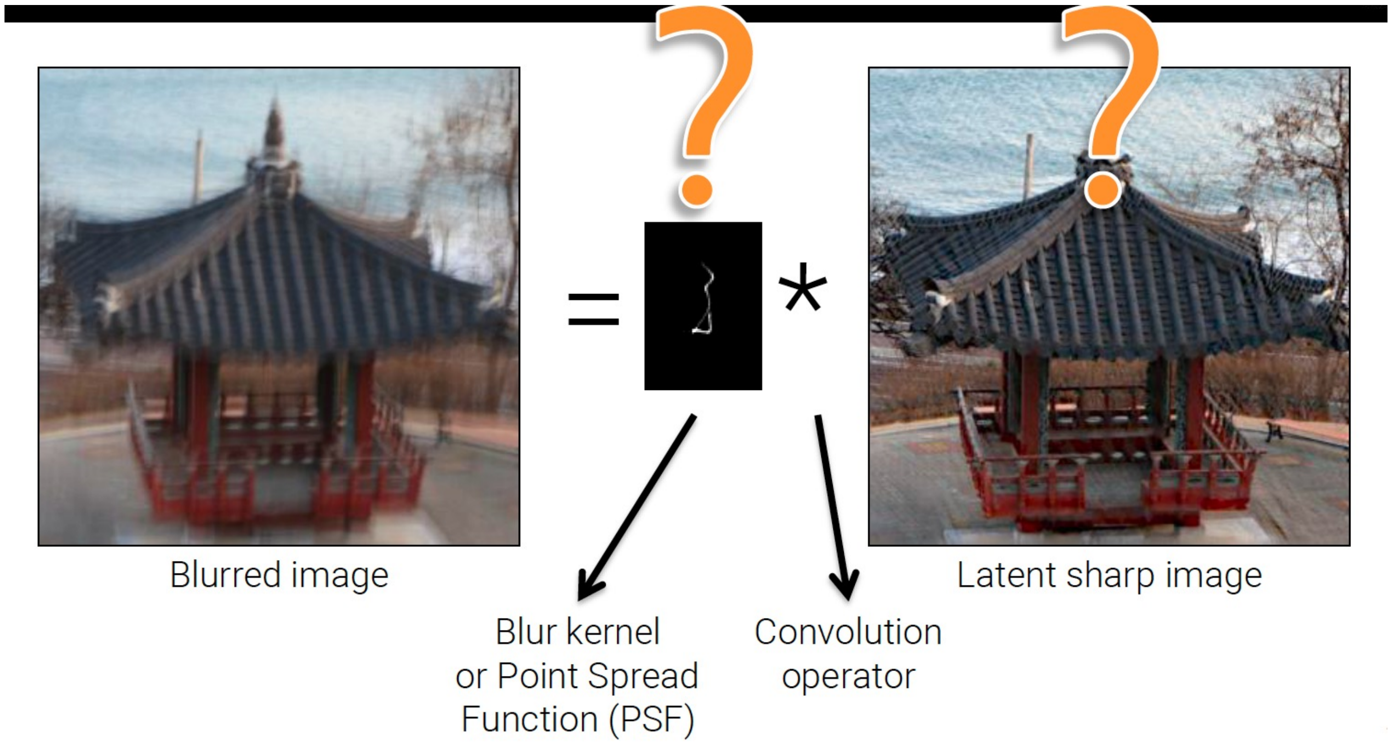
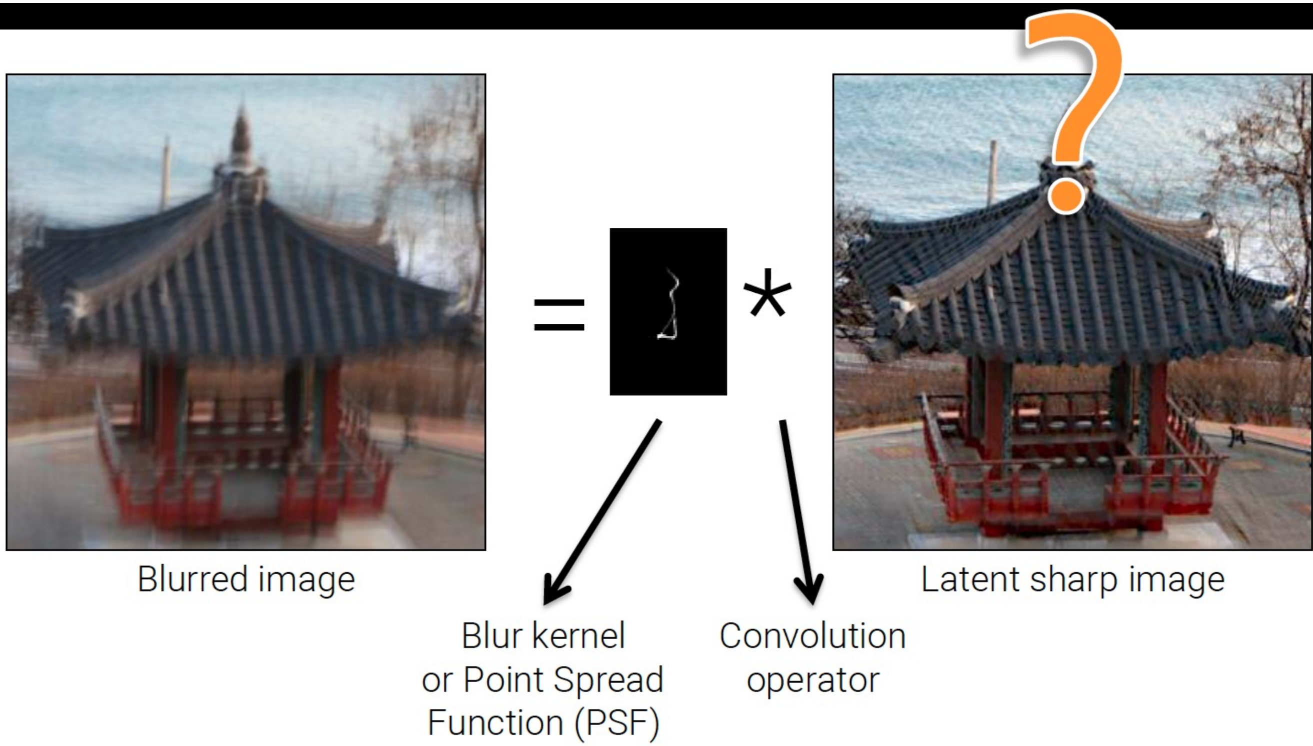
BLIND DECONVOLUTION



NON BLIND DECONVOLUTION

[redacted]



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NON BLIND DECONVOLUTION

If we simply solve the inverse problem:

$$min_x \|Ax - (y + e)\|_2^2$$

We obtain an image dominated by noise.

- Even if we know the true blur kernel, we cannot restore the latent image perfectly, because:

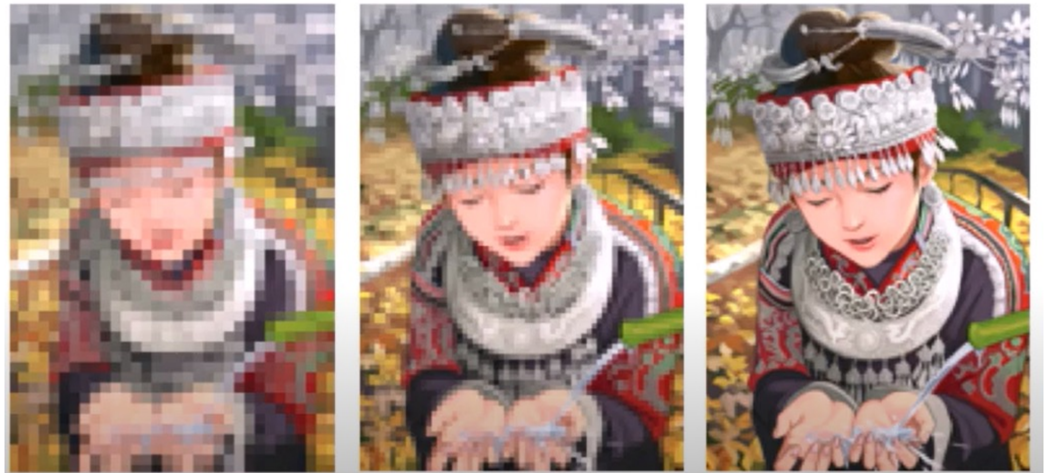
- Loss of high-freq info & noise ≈ denoising & super-resolution

SUPER-RESOLUTION

Super-resolution (SR) can have different meanings:

- temporal resolution
- Spectral resolution
- **Spatial** resolution

We consider the case of spatial resolution.



46 × 32 | 91 × 63 (2X) | 181 × 125 (4X)

SUPER-RESOLUTION

- Birth of SR: Blade Runner movie (1982):

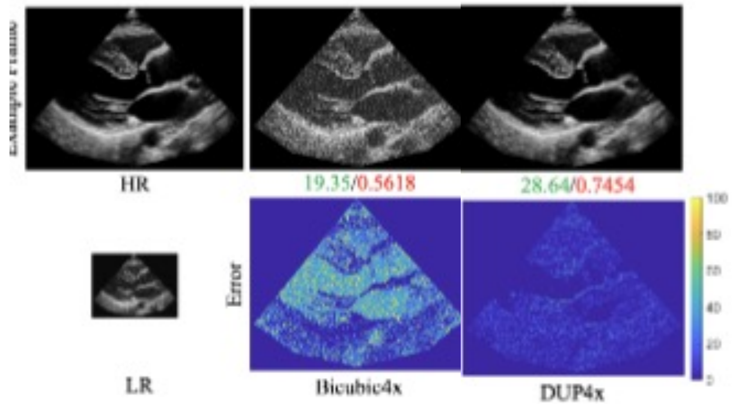
[Super-Resolution: Computational and Deep Learning-Based Approaches]

- SR refers to obtaining an image at a higher resolution than that of the camera sensor.
- SR from a single image (SISR) is a highly ill-posed problem.
- The problem bemoes easier when you have a sequence of low-resolution frames of the same scene, acquired for example by a camera.
- In this case the algorithm reconstructs a single high resolution image from the sequence by also removing blurring.

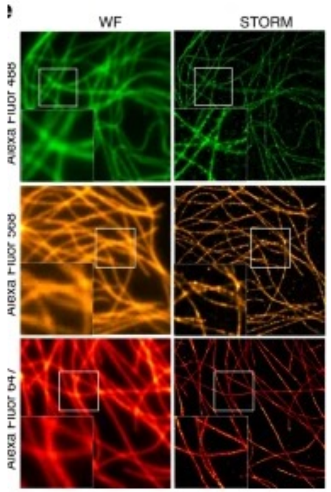
SUPER-RESOLUTION APPLICATIONS

Satellite images

Medical imaging



Surveillance



Microscopy images

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SUPER-RESOLUTION MODEL

$$x \qquad B_k \quad D + n_k \quad = \quad y_k$$

$$y_k = DB_k x + n_k = W_k x + n_k$$

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SUPER-RESOLUTION MODEL

y_k observed low-resolution noisy images (*M* captures - known)

x original high-resolution image (Needs to be calculated)

B_k blur model: optical, motion, sensor pixel size, etc. (known or estimated based on application)

D downsampling matrix dictated by required resolution ratio (known)

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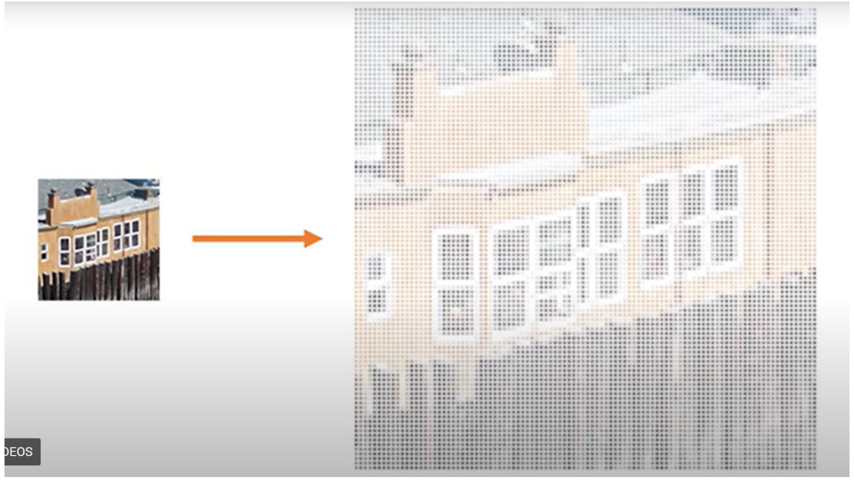
BASIC SR ALGORITHMS



The simplest way is to copy the values from the Adjacent pixels **(Nearest Neighbor Upscaling)**

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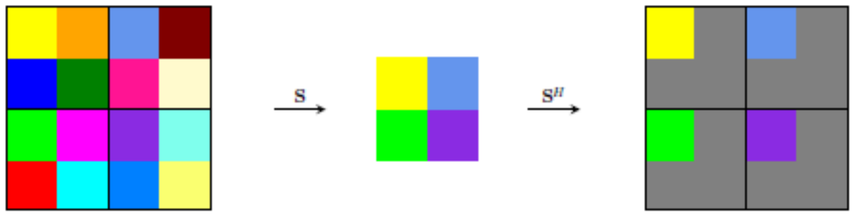
BASIC SR ALGORITHMS

Another possible approach is to interpolate the Values of adjacent pixels
(bilinear or bicubic interpolation upscaling)

SR AS INVERSE PROBLEM

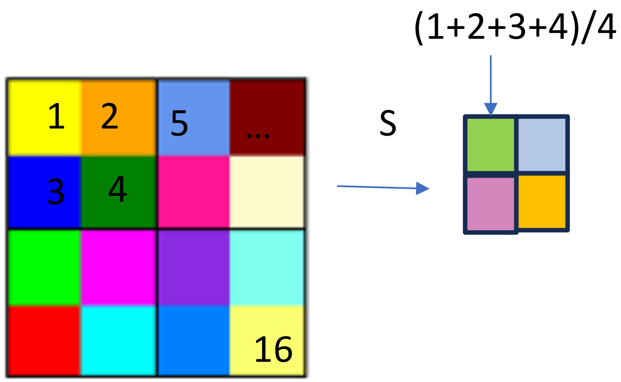
Downsampling operators

1.



Zero padding

2.



Average