

Interplanetary Trajectories Optimization by Deep Networks

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1 Introduction

In this bibliographic research, various methodologies in the domain of interplanetary trajectory optimization will be compared and contrasted, highlighting their advantages, drawbacks, and potential opportunities. The utilization of deep networks and artificial intelligence techniques has emerged as a promising avenue for addressing the complex challenges inherent in space exploration and mission planning. The existing literature will be analyzed and evaluated based on specific criteria, such as the type of problem addressed, the employed solving methodologies, and other relevant aspects.

One prevalent area of investigation pertains to the generation of optimal spacecraft trajectories from a single nominal trajectory. This approach has garnered attention due to its potential to create millions of optimal trajectories without the need for solving additional optimal control problems. The advantages of this method, including computational efficiency, feasibility, and adherence to critical constraints like Pontryagin's minimum principle and relevant transversality conditions, will be explored.

Another significant aspect of interest involves fast low-thrust transfer cost approximators, which play a crucial role in enhancing mission design efficiency and accommodating more complex missions. The analysis will focus on methodologies proposing online frameworks capable of adaptively adjusting deep neural networks using newly acquired low-thrust transfer data. Understanding the advantages and opportunities presented by these approximators is essential in devising effective and dynamic mission planning strategies.

Moreover, the application of deep artificial neural networks to approximate optimal state-feedback control for continuous-time, deterministic, and non-linear systems will be investigated. Assessing the strengths and limitations of these networks in representing optimal control solutions will provide valuable insights into their potential across diverse problem domains. A critical aspect of interplanetary missions is achieving precise soft landings on asteroids for surface exploration and resource exploitation. In this context, real-time optimal control approaches utilizing deep neural networks to enhance landing control autonomy and improve landing site selection accuracy will be explored. Identifying the benefits and drawbacks of these approaches will contribute to refining landing strategies and ensuring mission success. Furthermore, the comparative analysis will encompass articles addressing trajectory optimization under different problem settings, solving methodologies, and cost functions. By examining the advantages and limitations of each methodology, a comprehensive understanding of their applicability and potential in various mission scenarios will be gained.

Lastly, the concept of evolutionary neurocontrol, a novel approach integrating artificial neural networks and evolutionary algorithms for global low-thrust trajectory optimization, will be explored. Evaluating this intelligent method will reveal its efficacy in finding optimal trajectories, particularly when starting far from the training area. This research aims to provide valuable insights into the advancements and challenges in interplanetary trajectory optimization using deep networks and artificial intelligence.

2 Literature Analysis

In this section, a literature analysis on the interplanetary trajectory optimization methodologies found in the research will be presented. Some relevant articles will be cited, and their main ideas will be summarized, providing a broad perspective on the approaches used in this field.

The selection of methodologies showcased in this section aims to offer a representative sample, capturing key aspects of the problem. It is essential to acknowledge that other methodologies have also been explored to gain a more comprehensive understanding of the challenges and opportunities in this area. By comparing and contrasting the findings from different articles, similarities and differences in their approaches will be highlighted.

2.1 Interplanetary Transfers via Deep Representations of the Optimal Policy and of the Value Function

In the study by Dario Izzo, Ekin Öztürk, and Marcus Märten, titled *Interplanetary Transfers via Deep Representations of the Optimal Policy and/or of the Value Function*, a novel method for generating optimal spacecraft trajectories efficiently is explored. The primary objective is to create a large-scale dataset of trajectories without the need to solve specific optimal control problems, except for the nominal trajectory.

Traditionally, computing such datasets using brute-force approaches involves solving the optimal control problem for a predefined set of initial states, resulting in significant computational complexities. To overcome this limitation, previous works, including the contributions by Sanchez and Izzo, introduced a continuation approach. However, this approach still incurs substantial computational costs due to solving the problem for each new set of initial conditions. In contrast, the proposed efficient approach involves backward propagation in time of the Hamiltonian dynamics equations (shown below) from suitably perturbed final values of the state and co-states of the nominal trajectory.

$$\begin{cases} \dot{\mathbf{x}} = \frac{\partial \mathcal{H}}{\partial \mathbf{x}} = \frac{c_1 u(t)}{m} \mathbf{B} \hat{\mathbf{i}}_r(t) + \mathbf{D} \\ \dot{m} = \frac{\partial \mathcal{H}}{\partial \lambda_m} = -c_2 u(t) \\ \dot{\lambda} = -\frac{\partial \mathcal{H}}{\partial \mathbf{x}} \\ \dot{\lambda}_m = -\frac{\partial \mathcal{H}}{\partial m} \end{cases}$$

The perturbations are carefully chosen to ensure that the transversality conditions and the condition on the Hamiltonian are satisfied. This technique yields new trajectories that fulfill all of Pontryagin's necessary conditions for optimality, except for the Hamiltonian condition. By finding new final values for certain parameters, a valid optimal trajectory is obtained for learning purposes. Due to the small perturbations applied to the final co-states, the timing and direction of thrust maneuvers remain close to the nominal trajectory during the backward integration. As a result, the final state of the newly generated trajectory is unlikely, if not impossible, to deviate significantly from the nominal optimal solution.

The research's conclusion highlights the success of the proposed method in providing suitable datasets for training deep neural networks. The approach's low computational cost and reliability make it a promising alternative to complex solvers for optimal control problems. By applying small disturbances to the final co-states, the researchers obtained a diverse dataset, facilitating reliable control policy learning through imitation learning.

The policy network demonstrates promising precision, bringing the team closer to achieving on-board optimal guidance and control using neural networks. Additionally, the value networks accurately estimate the value function, providing valuable approximations of the mass-budget needed for optimal transfers from various starting points. While the preliminary results are encouraging, further investigation is necessary. Exploring larger perturbation values of the final co-states and alternative sampling strategies will enhance the method's applicability to more complex and challenging missions. Moreover, the approach holds the potential to refine the controller even when co-state information is unavailable. Overall, the study offers valuable insights into the efficient generation of optimal trajectories and their application in deep neural

network training, contributing to the advancement of interplanetary exploration and mission planning.

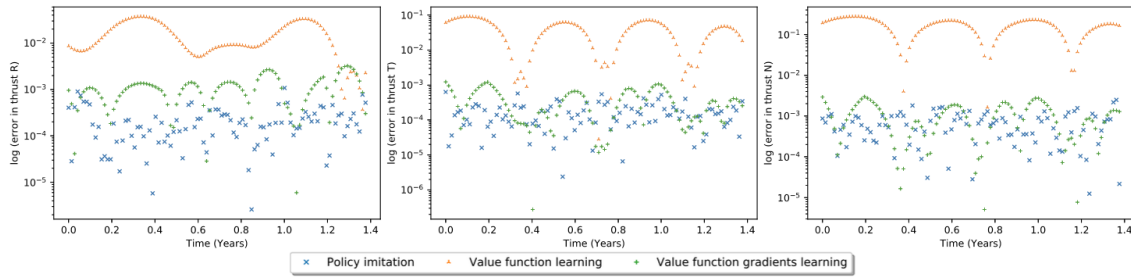


Figure 4: Absolute error for predicting the optimal thrust direction $\hat{\mathbf{i}}_T$ of the nominal trajectory (log-scale).

2.2 An on-line deep learning framework for low-thrust trajectory optimisation

The article *On-line deep learning framework for low-thrust trajectory optimization* by Ruida Xie and Andrew G. Dempster presents an approach for efficient low-thrust trajectory optimization using an on-line deep learning framework. The core idea is to overcome the computational challenges and reliance on initial guesses in traditional methods by leveraging Deep Neural Networks (DNNs) and online learning. The framework consists of three main steps: low-thrust data generation, DNN construction, and DNN optimization.

In the data generation step, a large-scale dataset of low-thrust transfer data is randomly generated, including starting epoch and flight time. Initially, no reliable classification model is available, so the data undergoes direct optimization, resulting in convergence or divergence outcomes, which are stored and labeled. Data augmentation techniques are then applied to rebalance the imbalanced dataset. The DNN-classifier, with a series structure of fully connected layers, is constructed to predict the convergence or divergence of the low-thrust optimization problem. The model is trained batch by batch using Cross-entropy loss, aiming to minimize the loss on mini-batches. Parameter optimization is critical for the DNN-classifier's performance, and hyper-parameters, such as the number of layers, neurons, activation function, learning rate, and others, are determined through a search in predefined spaces. The evaluation metrics, including accuracy and F1-score, measure the effectiveness of the DNN-classifier in predicting convergence or divergence.

In the image below, the process is schematized.

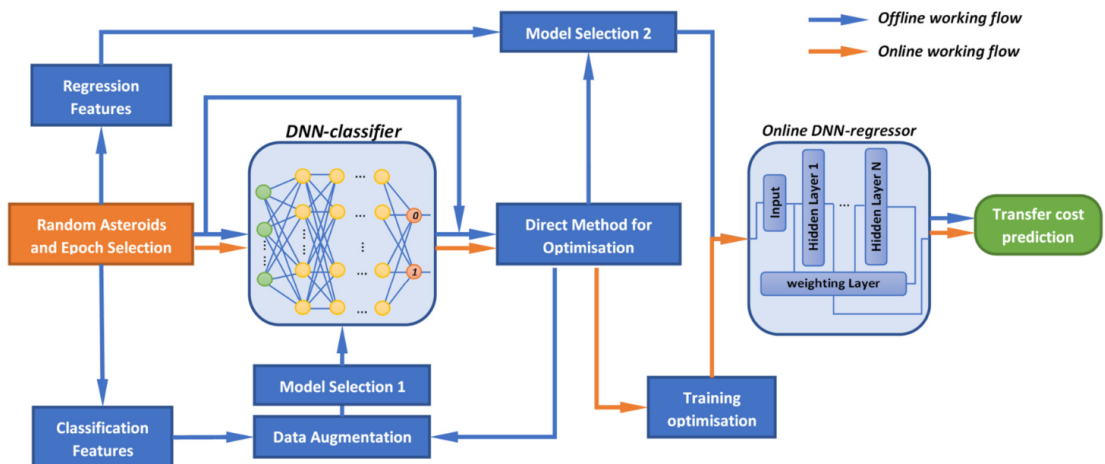


Fig. 2. The general on-line DNN framework. (For interpretation of the colours in the figure(s), the reader is referred to the web version of this article.)

The online characteristic of the framework enables continuous updates when new asteroids are added. The data goes through the DNN classifier, and if predicted to be converging, it is sent to direct optimization for higher convergence rates. The model is then updated using the newly added data.

The proposed on-line deep learning framework demonstrates promising results in efficiently identifying low-thrust trajectory optimization convergence, leading to more efficient and autonomous mission planning in space exploration. The ability to adapt to new data and handle imbalanced datasets further enhances its practicality and applicability in real-world scenarios. By significantly reducing computational complexity and improving convergence rates, this innovative approach opens new possibilities for trajectory optimization in future deep space missions.

2.3 Learning the Optimal State-Feedback Using Deep Neural Networks

In the paper titled *Learning the Optimal State-Feedback Using Deep Neural Networks*, Carlos Sánchez-Sánchez, Dario Izzo, and Daniel Hennes investigate the potential of deep artificial neural networks (DNNs) to approximate the optimal state-feedback control of continuous-time, deterministic, non-linear systems. The main objective is to train DNNs in a supervised manner using trajectories obtained by solving the optimal control problem through the Hermite-Simpson transcription method.

The authors demonstrate that DNNs can successfully represent the optimal state-feedback with high accuracy, not just within the training data, but also well beyond it, indicating the ability of the networks to generalize effectively. This suggests that the DNNs effectively learn the underlying model described by the Hamilton-Jacobi-Bellman (HJB) equations, making them powerful tools for optimal control in complex systems.

To assess the performance of DNNs in various scenarios, the study considers three different domains of increasing complexity: the inverted pendulum swing-up and stabilization, a multicopter pin-point landing, and a spacecraft free landing problem. Additionally, they explore different cost functions resulting in both smooth and discontinuous (bang-bang) optimal control solutions. In the figure, it can be seen that in the case of the deep network, a safe landing condition is achieved.

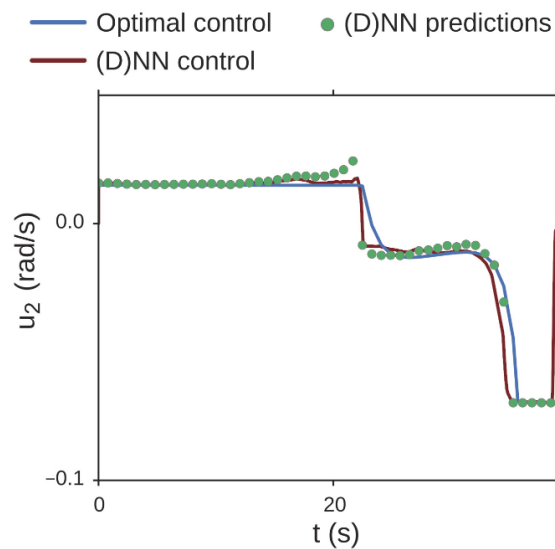


Figure 1: The optimal control, the (D)NN predictions for each state along the optimal trajectory and the full trajectory driven by the (D)NN are included. Deep network (4 hidden layers, 64 units, 12992 parameters).

The results highlight the significance of the depth of the DNNs in achieving accurate approximations of the optimal state-feedback. Shallow networks struggle to capture the complexity of the control, while deeper networks prove more effective in representing the optimal control. The use of activation functions such as rectified linear units contributes to the networks' performance, allowing them to avoid issues like vanishing gradients and sparse representations, leading to improved generalization capabilities.

2.4 Low-thrust trajectory optimization and interplanetary mission analysis using evolutionary neurocontrol

Low-thrust propulsion systems have the potential to significantly enhance or even enable such missions due to their higher total impulse capability. However, finding optimal low-thrust trajectories concerning transfer time or propellant consumption is challenging and time-consuming. Traditional optimizers based on numerical optimal control methods often require an adequate initial guess, making them difficult to converge to a global optimum. To address these limitations, a novel smart global trajectory optimization method, termed "InTrance" (Intelligent Trajectory Optimization using Neurocontroller Evolution), was introduced. This approach combines artificial neural networks (ANNs) and evolutionary algorithms (EAs) into so-called evolutionary neurocontrollers (ENCs). ENC is inspired by natural processes of information processing and optimization found in animal nervous systems. The concept of evolutionary neurocontrol stems from the idea that if natural evolutionary neurocontrollers can optimally steer organisms like houseflies to perform complex tasks, artificial evolutionary neurocontrollers might be capable of steering spacecraft optimally on interplanetary trajectories. ENC allows for autonomous learning of optimal control strategies for spacecraft trajectory optimization without the need for an initial guess or the expertise of a trajectory optimization expert.

Machine learning, particularly reinforcement learning problems, forms the basis for evolutionary neurocontrol. In this context, ANNs are used as neurocontrollers, and they learn to provide control outputs based on inputs that capture relevant information for the control task. The evolution process in EAs simulates the natural selection of individuals (solutions) in a population based on a fitness function. The fittest individuals reproduce and undergo genetic transformations like mutation and recombination, leading to improved generations. Eventually, EAs converge to a solution that, ideally, is the globally optimal trajectory. In the following figure, it is shown how the evolutionary neurocontrol is implemented.

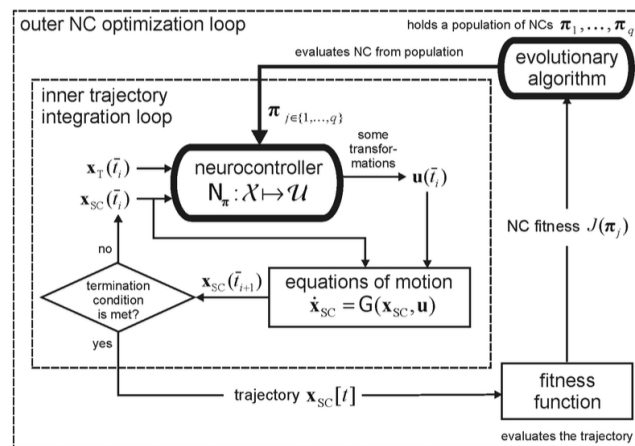


Figure 4: Low-thrust trajectory optimization using evolutionary neurocontrol

By applying evolutionary neurocontrol to low-thrust trajectory optimization, the paper demonstrates its effectiveness in finding near-globally optimal spacecraft steering strategies. The

method was tested for an exemplary mission to a near-Earth asteroid, where it outperformed traditional local trajectory optimization methods, reducing transfer time by 74%. Evolutionary neurocontrollers explore the search space more extensively than human experts using conventional optimal control methods, leading to better trajectories closer to the global optimum.

The advantage of evolutionary neurocontrol lies in its problem-independence, making it applicable to a wide range of optimal control problems. Furthermore, the method runs without requiring an initial guess and without continuous expert guidance, allowing for autonomous and efficient trajectory optimization.

2.5 Comparison

The literature analysis introduces several methodologies with a shared goal of optimizing interplanetary trajectories, employing the power of deep neural networks. Each approach comes with its own advantages and addresses specific challenges in the trajectory optimization domain.

To start with, Izzo, Öztürk, and Märten propose an efficient method for generating optimal trajectories based on a single nominal trajectory. Their approach reduces computational complexity significantly by applying slight perturbations to the final co-states. This allows for the creation of diverse datasets, facilitating accurate learning of control policies through imitation learning. While the focus lies on efficient dataset generation and control policy approximation, further exploration may be needed to handle more complex missions and cope with co-state information unavailability.

Moving on, Xie and Dempster's online deep learning framework prioritizes the optimization of low-thrust trajectories with a specific emphasis on predicting convergence. By leveraging deep neural networks, this framework predicts whether a trajectory optimization process will converge or diverge, enabling the use of direct optimization for improved convergence rates. The online nature of this approach allows for continuous updates and adaptability to new data. It shows promising potential in autonomously planning efficient space exploration missions. However, to ensure its applicability across diverse scenarios, further validation is required.

Lastly, Sánchez-Sánchez, Izzo, and Hennes explore the application of deep neural networks in approximating optimal state-feedback control for continuous-time, deterministic, and non-linear systems. Their research demonstrates that deep neural networks effectively represent the underlying model described by the Hamilton-Jacobi-Bellman equations. This approach outperforms shallow networks in accurately representing optimal control in various domains, making it a promising tool for real-time optimal control in complex systems. However, it's worth noting that the evaluation primarily focuses on specific scenarios, and a more extensive investigation covering a wider range of missions would enhance its applicability.

3 Conclusion

The integration of deep neural networks into trajectory optimization opens up exciting possibilities for space exploration, ushering in enhanced efficiency and autonomy in mission planning. Whether it is generating optimal trajectories, predicting convergence in low-thrust transfer optimization, or approximating state-feedback control, these approaches underscore the power of artificial intelligence in tackling the intricacies of interplanetary missions. While each methodology demonstrates promising results within its specific domain, further research is necessary to address their limitations and broaden the applicability to more diverse and complex mission scenarios. Moreover, exploring the synergies between these methodologies or incorporating insights from different studies may pave the way for groundbreaking advancements in interplanetary trajectory optimization. The future of space exploration appears bright as AI continues to contribute to solving the challenges of exploring the cosmos.

References

- [1] Carlos Sánchez-Sánchez, Dario Izzo, Daniel Hennes. *Learning the optimal state-feedback using deep networks*. IEEE Symposium Series on Computational Intelligence. 1–8. 2016.
- [2] Dario Izzo, Ekin Öztürk, Marcus Mörtens. *Interplanetary Transfers via Deep Representations of the Optimal Policy and/or of the Value Function*. <https://arxiv.org/abs/1904.08809>. 1-9. 2019.
- [3] Ruida Xie, Andrew G. Dempster. *An on-line deep learning framework for low-thrust trajectory optimisation*. Aerospace Science and Technology. 1-15. 2021.
- [4] Bernd Dachwald. *Low-thrust trajectory optimization and interplanetary mission analysis using evolutionary neurocontrol*. Institut für Raumsimulation. 1-10. 2004.
- [5] Roberto Furfaro, Ilaria Bloise, Marcello Orlandelli, Pierluigi DiLizia. *Deep Learning for Autonomous Lunar Landing*. AAS/AIAA Astrodynamics Specialist Conference. 1–22. 2018.
- [6] Lin Cheng, Zhenbo Wang, Yu Song, Fanghua Jiang. *Real-Time Optimal Control for Irregular Asteroid Landings Using Deep Neural Networks*. arXivpreprint arXiv:1901.02210. 1-27. 2019.
- [7] Lin Cheng, Zhenbo Wang, Fanghua Jiang, Chengyang Zhou. *Real-time optimal control for spacecraft orbit transfer via multiscale deep neural networks*. Aerospace Electronic Systems. 1-15. 2019.

Keywords

Artificial neural networks; Trajectory optimization; Deep learning; Trajectory design; DNN; Machine learning; Interplanetary trajectories; Autonomous mission planning; Global trajectory optimization; Continuous-time systems; Deterministic systems; Surface exploration; Value function learning; Interplanetary transfers; Transfer time optimization; Hamilton-Jacobi-Bellman equation; Mass-optimal trajectories; Pontryagin's minimum principle; Trajectory generation; Neural network training; Supervised learning; Smart global trajectory optimization; Optimal control strategies.