A fresh look at the **Nested Soft-Collinear** subtraction scheme: NNLO QCD corrections to N-gluon final state $q\bar{q}$ annihilation

CHRISTMAS MEETING 2023

Davide Maria Tagliabue

In collaboration with:
[F. Devoto, K. Melnikov, R. Röntsch, C. Signorile-Signorile, 2310.17598]

UNIVERSITÀ DEGLI STUDI DI MILANO

PROBLEMS AND SOLUTIONS

- Two main difficulties: **IR singularities**, arising from real and virtual radiation, and **multi-loop amplitude** calculations

- *About IR singularities*: they are unphysical and require specific methods to arrive at a finite physical result. Among those methods, we focus on **SUBTRACTION SCHEMES**

- Some of the many available schemes:

Analytic Sector Subtraction [Magnea et al. 1806.09570, ...]

ColorfullNNLO [Del Duca et al. 1603.08927, ...]

Geometric IR subtraction [Herzog 1804.07949, ...]

Universal Factorization [Anastasiou et al. 2008.12293, ...]

Antenna [Gehermann-De Ridder et al. 0505111, ...]

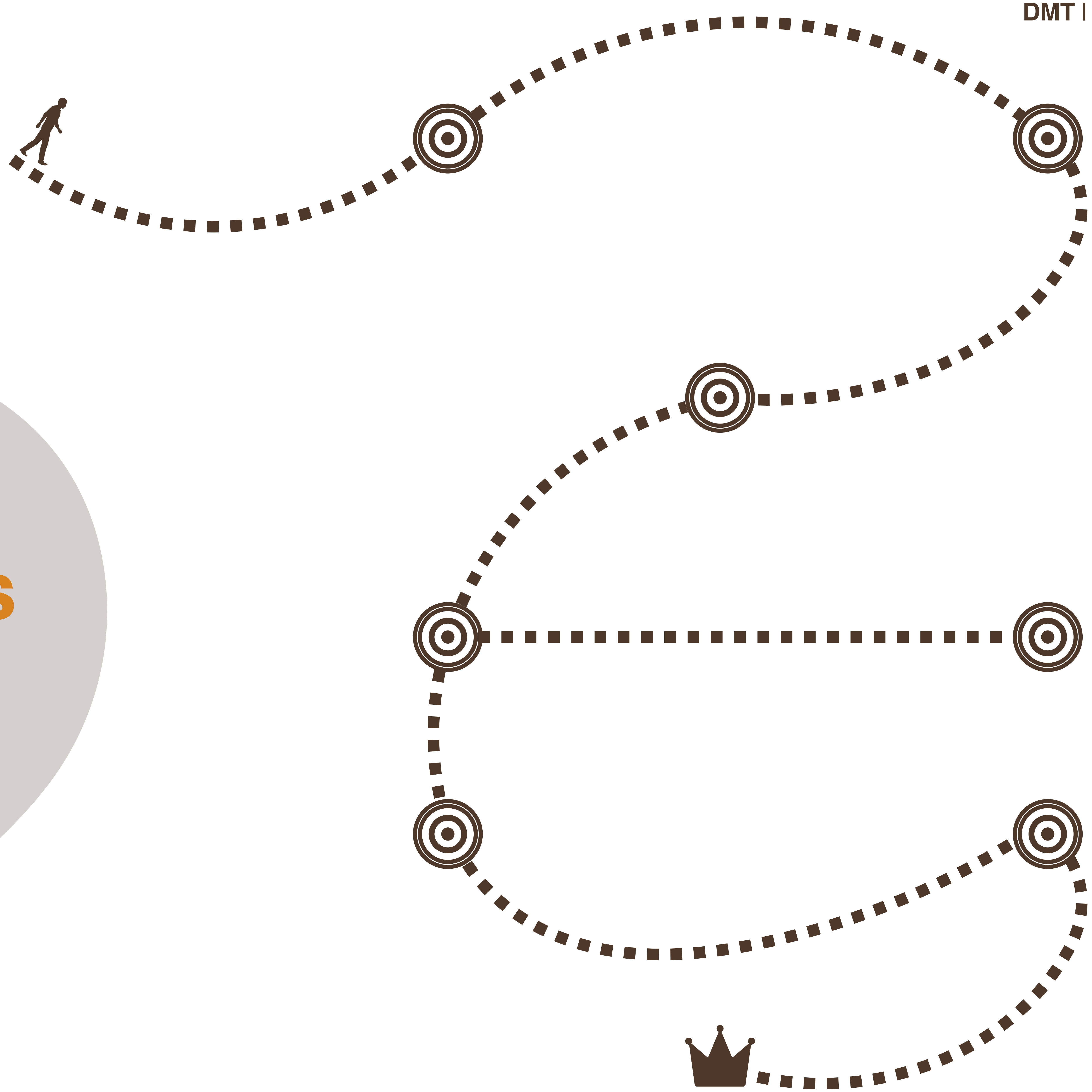
STRIPPER [Czakon 1005.0274, ...]

Unsubtraction [Sborlini et al. 1608.01584, ...]

FDR [Pittau 1208.5457, ...]

Nested Soft-Collinear Subtraction (NSC) [Caola et al. 1702.01352, ...]

WHY WE STUDY
$P + P \to X + N$ gluons
AT NNLO



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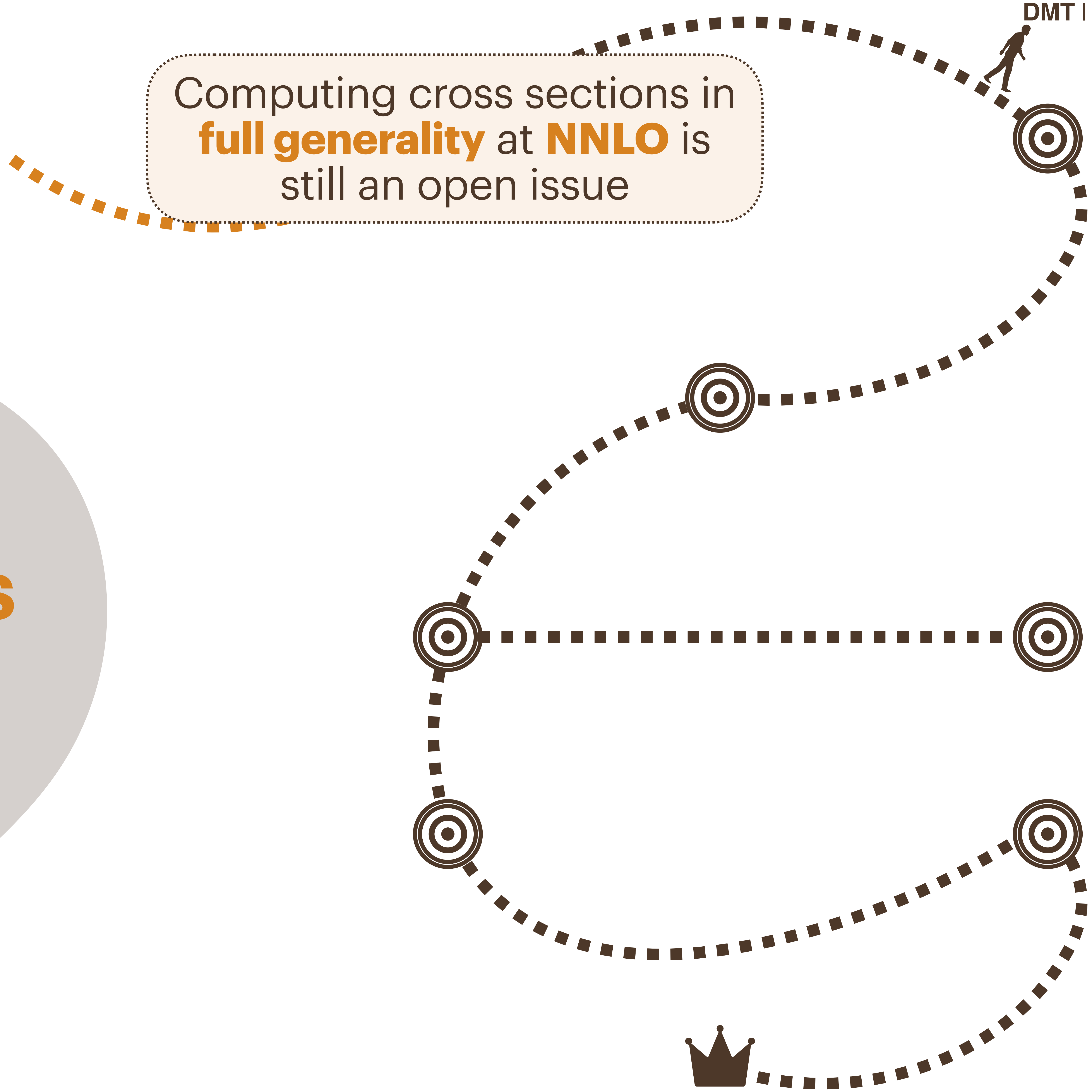
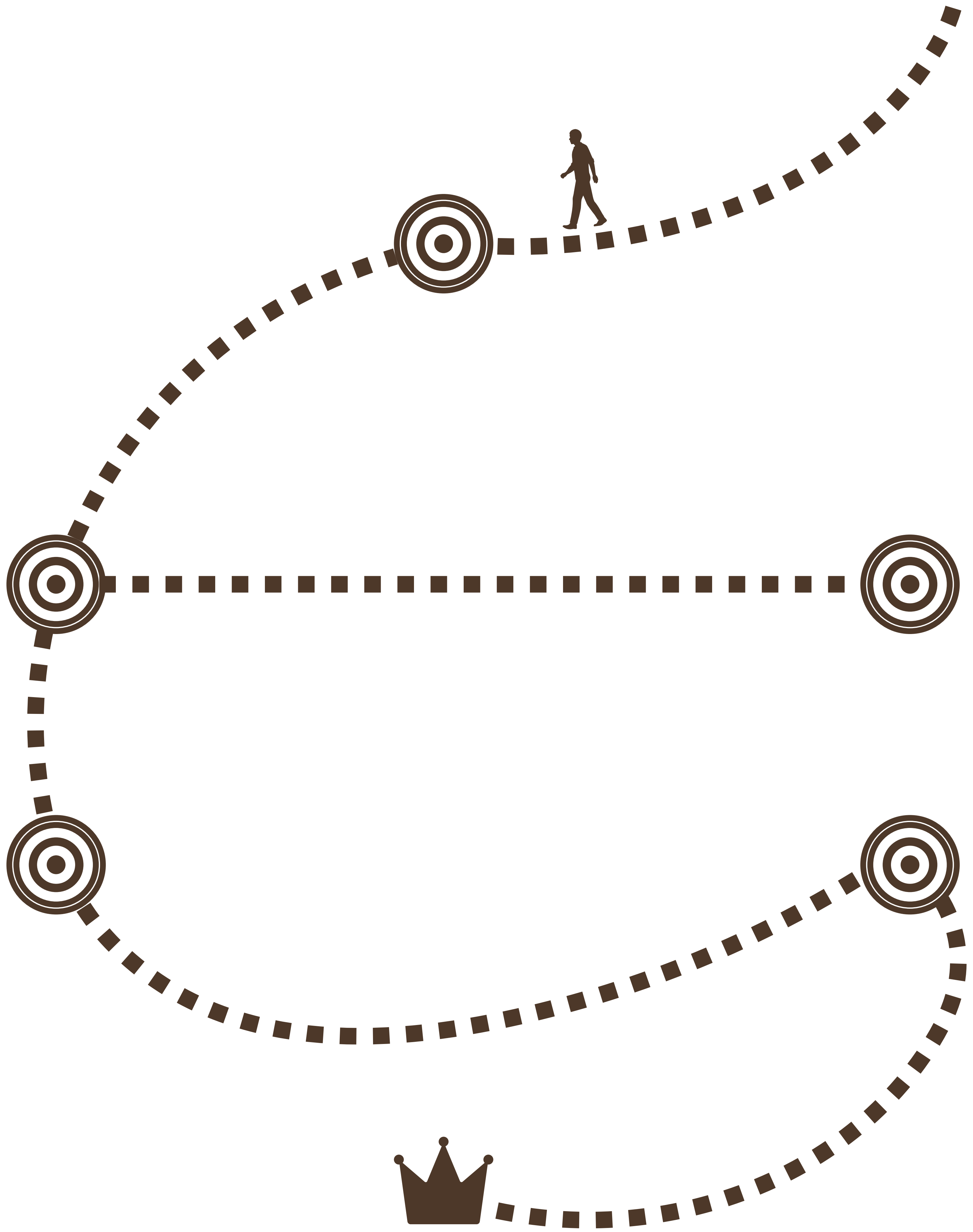


Computing cross sections in **full generality** at **NNLO** is still an open issue

Up to now **NSC** has only been applied to **simple** processes



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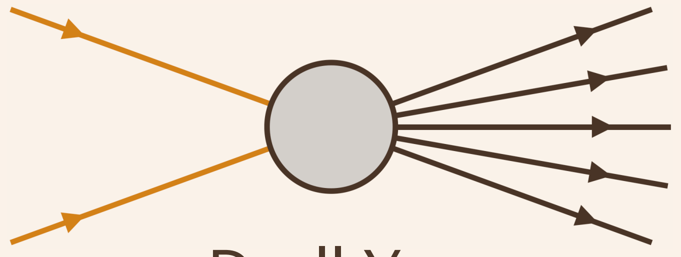
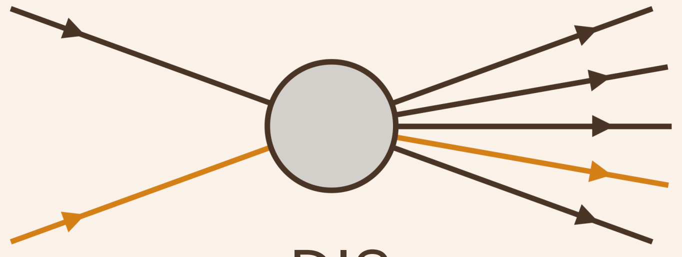


Drell-Yan
[Caola, Melnikov, Röntsch '19]



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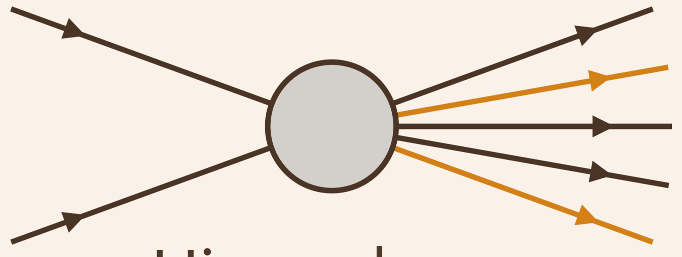

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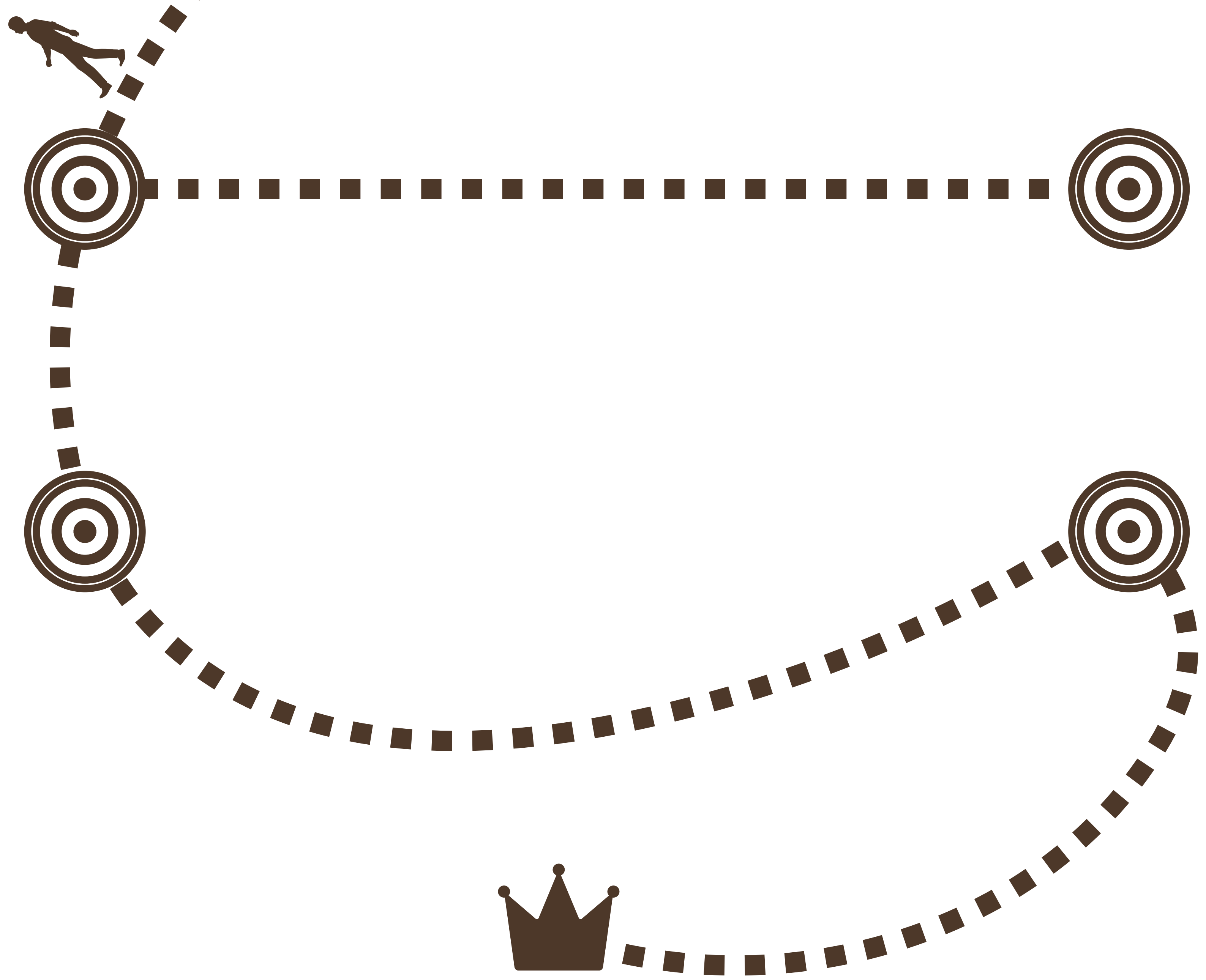

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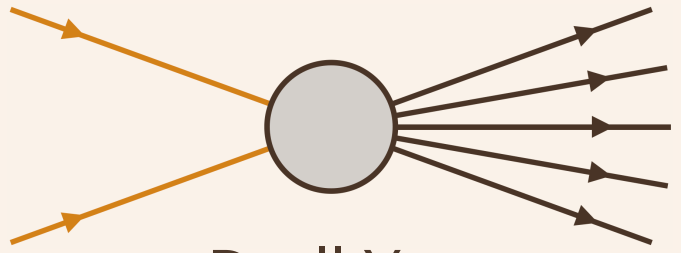

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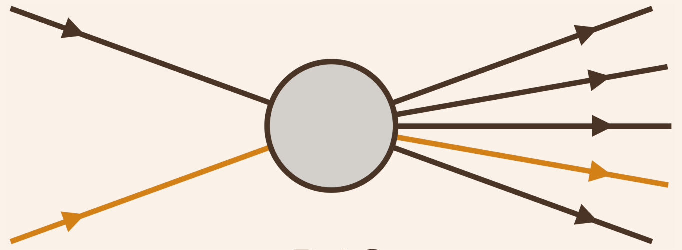

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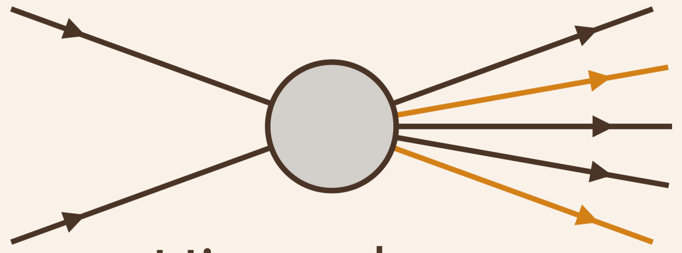

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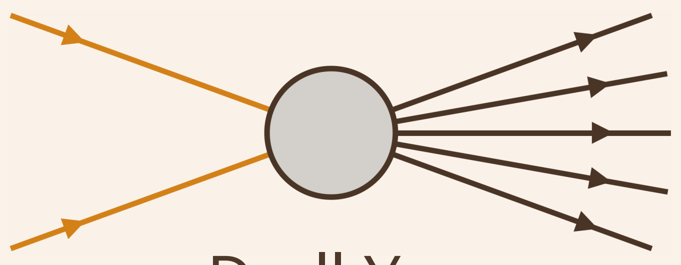

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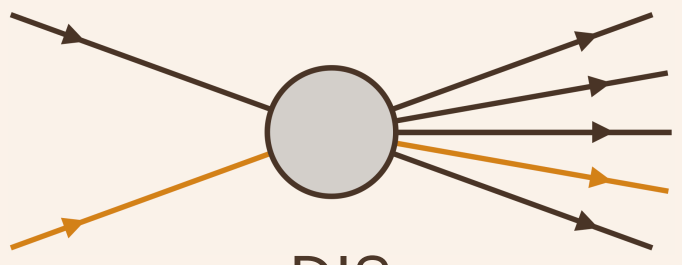

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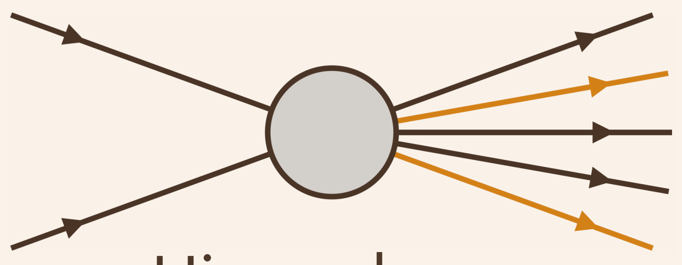

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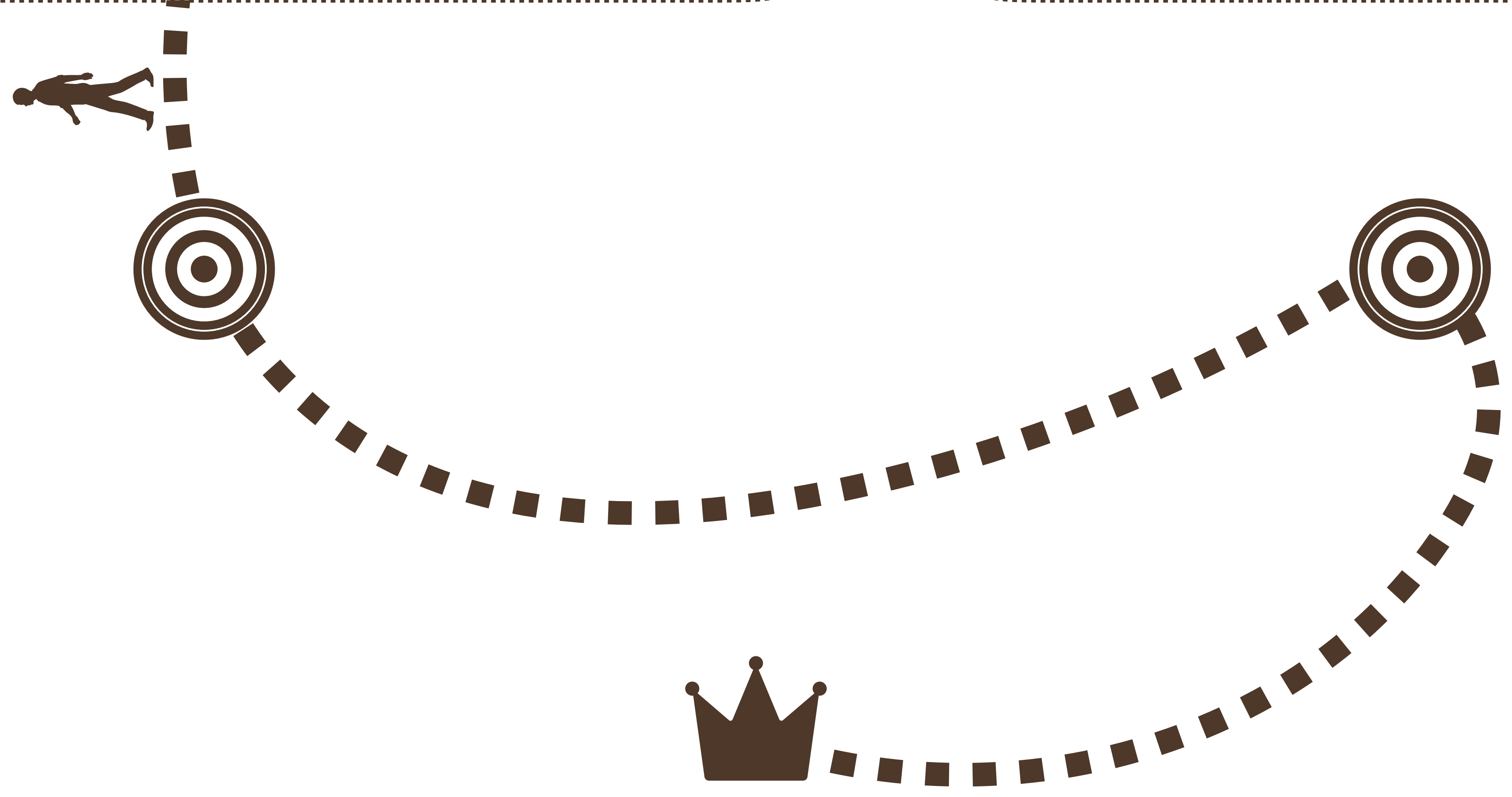

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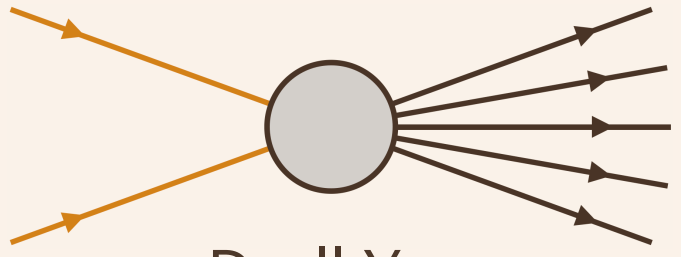

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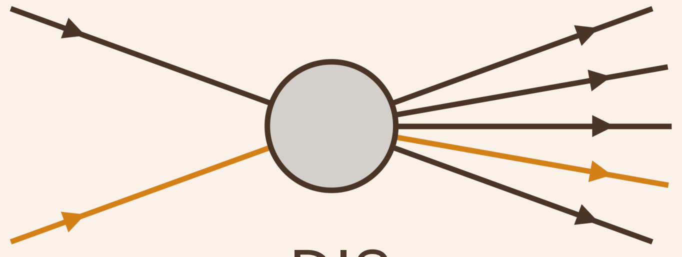

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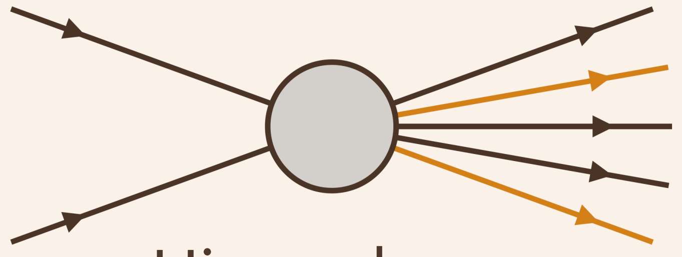

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What is a good prototype of the problem?
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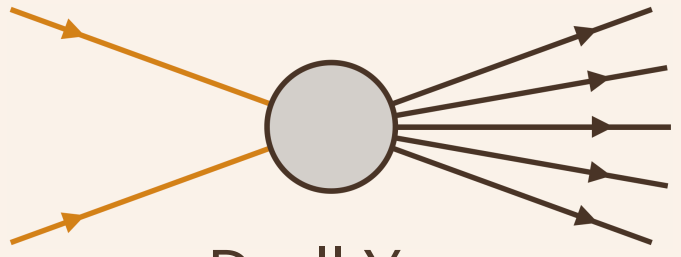
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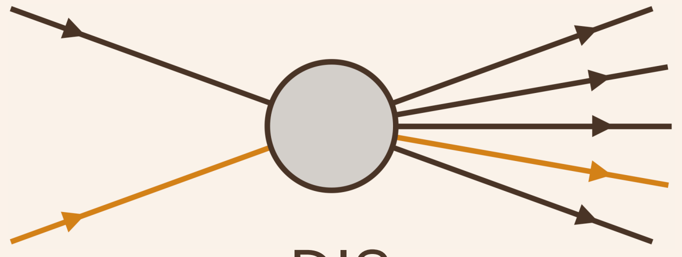
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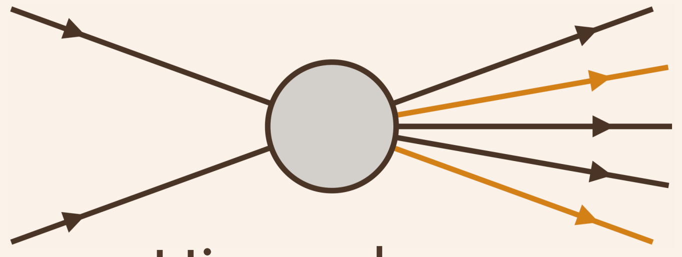
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Remaining bottleneck?
double-loop amplitudes

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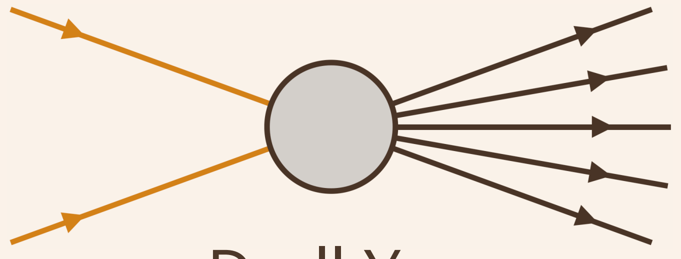

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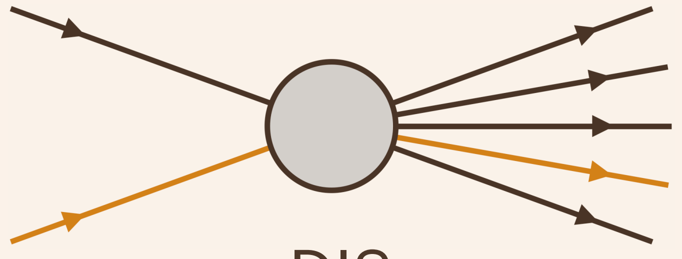

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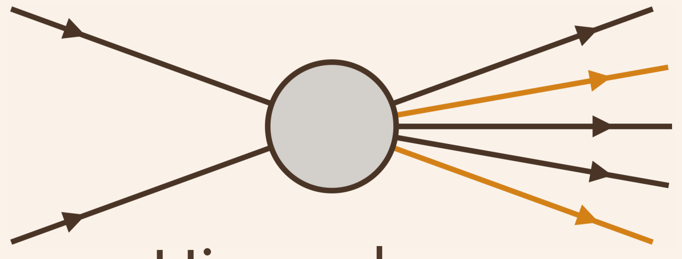

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Remaining bottleneck?
double-loop amplitudes

<< If someone gives me the finite part of the double-loop amplitude of any kind of process, then I can give back the analytical expression of the integrated subtraction terms. >>

HOW THE **NSC** WORKS?

$$\int |\mathcal{M}|^2 F_J \, \mathrm{d}^{(d)}\phi = \boxed{\int \left[|\mathcal{M}|^2 F_J - K\right] \mathrm{d}^{(d)}\phi + \int K \, \mathrm{d}^{(d)}\phi}$$

- fully **local**
- fully **analytic**

Problem of **OVERLAPPING SOFT** and **COLLINEAR** emissions

damping factors $\Delta^{(i)}$ $\Longrightarrow$ tell which parton is unresolved

partition functions ω^{ij} $\Longrightarrow$ select the proper collinear limit

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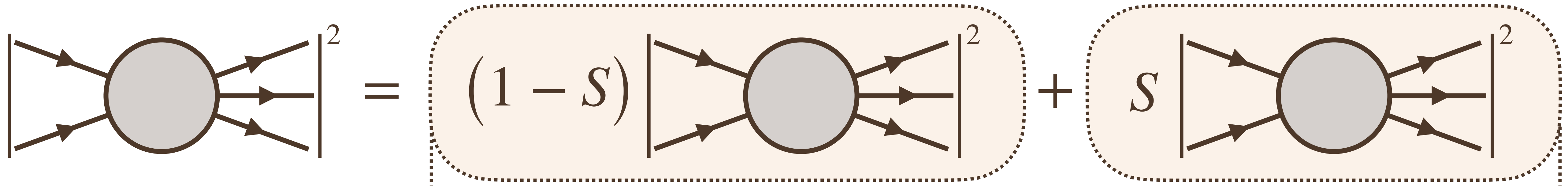
At **NLO** we start by regularizing soft divergences



Soft-regulated
still contains collinear divergences

Soft-counterterm
provides the formula of the soft poles

The **soft-regulated** term then needs a similar treatment for **collinear divergences**: all the singular configurations can be separated out

HOW THE **NSC** WORKS?

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At **NNLO** we follow the same idea of **separating out divergences**

- start from **double-soft** regularization
- regularize also **single-soft** divergences

} The cross section is now soft-regularized

- at this point we have to regularize **collinear** divergences (C_{im}, $C_{jn}C_{im}$, C_{imn}) $\Longrightarrow$ we avoid overlapping thanks to **PARTITIONING** and **SECTORING**

RECURRING OPERATORS AT NLO

Virtual corrections $\mathrm{d}\hat{\sigma}^{\mathrm{V}}$: the IR content of virtual amplitudes is known. Through the operator

$$\bar{I}_1(\epsilon) = \frac{1}{2} \sum_{i \neq j}^{Np} \frac{\mathscr{V}_i^{\mathrm{sing}}(\epsilon)}{\boldsymbol{T}_i^2} (\boldsymbol{T}_i \cdot \boldsymbol{T}_j) \left(-\frac{\mu^2}{s_{ij}} \right)^{\epsilon}$$

$$\mathscr{V}_i^{\mathrm{sing}}(\epsilon) = \frac{\boldsymbol{T}_i^2}{\epsilon^2} + \frac{\gamma_i}{\epsilon}$$

$$N_p = N + 2$$

the divergent part of $\mathrm{d}\hat{\sigma}^{\mathrm{V}}$ can be written as

$$\boldsymbol{I}_{\mathrm{V}}(\boldsymbol{\epsilon}) = \bar{I}_1(\epsilon) + \bar{I}_1^{\dagger}(\epsilon)$$

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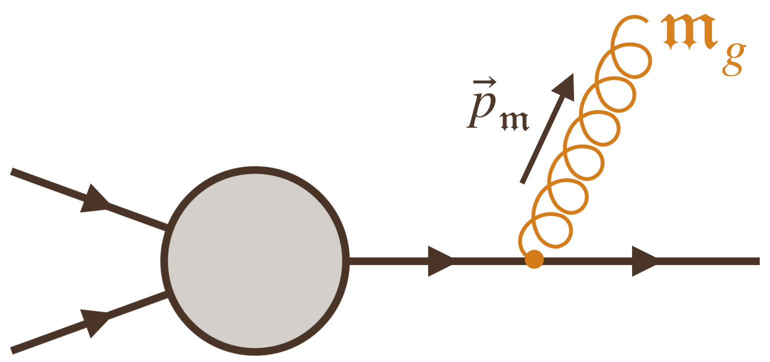
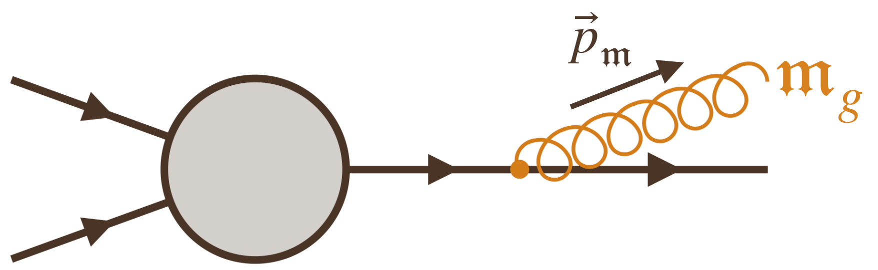
Real corrections $\mathrm{d}\hat{\sigma}^{\mathrm{R}}$: we would like something similar



Soft emission

$S_{\mathfrak{m}}$: $|\vec{p}_{\mathfrak{m}}| \to 0$



Collinear emission

$C_{i\mathfrak{m}}$: $\theta_{i\mathfrak{m}} \to 0$

Making use of **NSC** scheme to regularize this divergences we obtain [Caola, Melnikov, Röntsch '17]

$$\mathrm{d}\hat{\sigma}^{\mathrm{R}} = \underbrace{\left\langle S_{\mathfrak{m}} F_{\mathrm{LM}}(\mathfrak{m})\right\rangle}_{\substack{\text{Soft term} \\ [S_{\mathfrak{m}}:\, E_{\mathfrak{m}}\to 0]}} + \sum_{i=1}^{N_p} \underbrace{\left\langle \bar{S}_{\mathfrak{m}} C_{i\mathfrak{m}} \Delta^{(\mathfrak{m})} F_{\mathrm{LM}}(\mathfrak{m})\right\rangle}_{\substack{\text{Hard-Collinear term} \\ [C_{i\mathfrak{m}}:\, \theta_{i\mathfrak{m}}\to 0]}} + \left\langle \mathscr{O}_{\mathrm{NLO}}\, \Delta^{(\mathfrak{m})} F_{\mathrm{LM}}(\mathfrak{m})\right\rangle$$

RECURRING OPERATORS AT NLO

It turns out that the **soft term** can be written by means of an **operator** that, at least in principle, is very **close to $I_V(\epsilon)$**:

$$I_S(\epsilon) = -\frac{(2E_{\max}/\mu)^{-2\epsilon}}{\epsilon^2}\sum_{i\neq j}^{N_p}\eta_{ij}^{-\epsilon}K_{ij}\,(\boldsymbol{T}_i\cdot\boldsymbol{T}_j)$$

$$\eta_{ij} = (1-\cos\theta_{ij})/2$$

$$K_{ij} = \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\eta_{ij}^{1+\epsilon}\,{}_2F_1(1,1,1-\epsilon,1-\eta_{ij})$$

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- Combination of $I_V(\epsilon) + I_S(\epsilon)$:

$$I_V(\epsilon) + I_S(\epsilon) = -\sum_{i=1}^{N_p}\frac{1}{\epsilon}\left(2\boldsymbol{T}_i^2\,L_i + \gamma_i\right) + \mathcal{O}(\epsilon^0)$$

$$L_i = \log\left(E_{\max}/E_i\right)$$
$$\gamma_q = 3/2\,C_F$$
$$\gamma_g = \beta_0$$

- the pole of $\mathcal{O}(\epsilon^{-2})$ **vanishes**
- has no **color correlations** at $\mathcal{O}(\epsilon^{-1})$
- **trivially dependent on** the number of hard partons N_p

THERE STILL IS A MISSING INGREDIENT

RECURRING OPERATORS AT NLO

$$I_{\mathrm{V}}(\epsilon) + I_{\mathrm{S}}(\epsilon) = -\sum_{i=1}^{N_p} \frac{1}{\epsilon}\left(2T_i^2\, L_i + \gamma_i\right) + \mathcal{O}(\epsilon^0)$$

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Last ingredient: **hard-collinear term**. Some parts vanish against the DGLAP contribution, the remaining one **can be collected** within the **COLLINEAR OPERATOR**

$$I_{\mathrm{C}}(\epsilon) = \sum_{i=1}^{N_p} \frac{\Gamma_{i,f_i}}{\epsilon}$$

$$\Gamma_{i,f_i} = \left[\text{irrelevant prefactor}\right] \times \left[T_i^2 \frac{1-e^{-2\epsilon L_i}}{\epsilon} + \gamma_i\right] \qquad i \in \{1,2\}$$

$$\Gamma_{i,f_i} = \left[\text{irrelevant prefactor}\right] \times \gamma^{22}_{z,g\to gg}(\epsilon, L_i) \qquad i \in [3,N_p]$$

RECURRING OPERATORS AT NLO

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$$I_{\mathrm{C}}(\epsilon) = \sum_{i=1}^{N_p} \frac{1}{\epsilon}\left(\boxed{2T_i^2\, L_i + \gamma_i}\right) + \mathcal{O}(\epsilon^0)$$

$I_{\mathrm{C}}(\epsilon)$ cancels perfectly the pole of $\mathcal{O}(\epsilon^{-1})$ left by $I_{\mathrm{V}}(\epsilon) + I_{\mathrm{S}}(\epsilon)$. It is thus natural to introduce the **total operator**

$$I_{\mathrm{T}}(\epsilon) = I_{\mathrm{V}}(\epsilon) + I_{\mathrm{S}}(\epsilon) + I_{\mathrm{C}}(\epsilon)$$

- pole free
- fully general w.r.t. N_p

RECURRING OPERATORS AT NLO

$$I_V(\epsilon) + I_S(\epsilon) = -\sum_{i=1}^{N_p} \frac{1}{\epsilon}\left(2T_i^2 L_i + \gamma_i\right) + \mathcal{O}(\epsilon^0)$$

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$$I_C(\epsilon) = \sum_{i=1}^{N_p} \frac{1}{\epsilon}\left(2T_i^2 L_i + \gamma_i\right) + \mathcal{O}(\epsilon^0)$$

- In this way the **final result for the NLO fits in a line**:

$$d\hat{\sigma}^{\mathrm{NLO}} = [\alpha_s]\left\langle I_T(\epsilon) \cdot F_{\mathrm{LM}}\right\rangle + [\alpha_s]\left[\left\langle P_{aa}^{\mathrm{NLO}} \otimes F_{\mathrm{LM}}\right\rangle + \left\langle F_{\mathrm{LM}} \otimes P_{bb}^{\mathrm{NLO}}\right\rangle\right] + \left\langle F_{\mathrm{LV}}^{\mathrm{fin}}\right\rangle + \left\langle \mathcal{O}_{\mathrm{NLO}}\, \Delta^{(\mathfrak{m})} F_{\mathrm{LM}}(\mathfrak{m})\right\rangle$$

[FKS, Devoto, Melnikov, Röntsch, Signorile-Signorile, **D.M.T.**, 2310.17598]

WHAT HAPPENS AT NNLO?

$$d\hat{\sigma}^{\mathrm{NNLO}} = \underbrace{d\hat{\sigma}^{\mathrm{VV}}}_{\text{Double-Virtual}} + \underbrace{d\hat{\sigma}^{\mathrm{RV}}}_{\text{Real-Virtual}} + \underbrace{d\hat{\sigma}^{\mathrm{RR}}}_{\text{Double-Real}} + \underbrace{d\hat{\sigma}^{\mathrm{pdf}}}_{\text{PDFs Renor.}}$$

Consider for instance $d\hat{\sigma}^{\mathrm{VV}}$ $\Rightarrow$ it depends **quadratically** on $\bar{I}_1(\epsilon)$ and $\bar{I}_1^\dagger(\epsilon)$

$\Rightarrow$ Identity Operator

$\Rightarrow \bar{I}_1, \bar{I}_1^\dagger \sim \boldsymbol{T}_i \cdot \boldsymbol{T}_j$

$\Rightarrow \bar{I}_1^2 \sim \boxed{(\boldsymbol{T}_i \cdot \boldsymbol{T}_j) \cdot (\boldsymbol{T}_k \cdot \boldsymbol{T}_l)}$ Double color-correlations

$\Rightarrow [\bar{I}_1, \bar{I}_1^\dagger] \sim \boxed{f_{abc} T_k^a T_i^b T_j^c}$ Tripoles

We expect the **same** to happen for $d\hat{\sigma}^{\mathrm{RV}}$ and $d\hat{\sigma}^{\mathrm{RR}}$

First Goal: **isolate DCC** in $d\hat{\sigma}^{\mathrm{RV}}$ and $d\hat{\sigma}^{\mathrm{RR}}$ and **combine** them with **those** contained within $d\hat{\sigma}^{\mathrm{VV}}$

The Strategy: **assemble** all these **DCC** into an expression that we expect to be **quadratic** in $I_{\mathrm{T}}(\epsilon)$

WHAT HAPPENS AT NNLO?

Here it is what we find [Devoto, Melnikov, Röntsch, Signorile-Signorile, **D.M.T.**, 2310.17598]

$$Y_{\rm VV} = \frac{[\alpha_s]^2}{2}\left\langle M_0 \left| \bar{I}_1^2 + (\bar{I}_1^\dagger)^2 + 2\bar{I}_1^\dagger \bar{I}_1 \right| M_0 \right\rangle + \ldots$$

$$Y_{\rm RR}^{(\rm ss)} = \frac{[\alpha_s]^2}{2}\langle M_0 | I_{\rm S}^2 | M_0\rangle + \ldots$$

$$Y_{\rm RR}^{(\rm shc)} = [\alpha_s]^2\langle M_0 | I_{\rm S}\, I_{\rm C} | M_0\rangle + \ldots$$

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$$Y_{\rm RV}^{(\rm shc)} = [\alpha_s]^2\left\langle M_0 \left| (\bar{I}_1 + \bar{I}_1^\dagger)\, I_{\rm C} \right| M_0 \right\rangle + \ldots$$

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Once combined, these objects return

NB square of NLO

$$Y = \frac{[\alpha_s]^2}{2}\left\langle M_0 \left| \left[I_{\mathrm{V}} + I_{\mathrm{S}} + I_{\mathrm{C}} \right]^2 \right| M_0 \right\rangle + \ldots \equiv \boxed{\frac{[\alpha_s]^2}{2}\langle M_0 | I_{\mathrm{T}}^2 | M_0 \rangle} + \ldots$$

WHAT HAPPENS AT NNLO?

The benefits of introducing these Catani-like operators:

- the problem of **double color-correlated poles disappear,** since everything is written in terms of $\mathbf{I}_{\mathrm{T}}^{2}(\epsilon)$, which is $\mathcal{O}(\epsilon^0)$

- the **definition** of $\mathbf{I}_{\mathrm{T}}(\epsilon)$ depends trivially on N_p so the result we got is **fully general w.r.t. the number of final state gluons**

- We **do not explicitly calculate** the individual sub-blocks of the process. Instead, we write each of these in terms of $I_{\mathrm{V}}(\epsilon)$, $I_{\mathrm{S}}(\epsilon)$ and $I_{\mathrm{C}}(\epsilon)$, then recombine them to get $I_{\mathrm{T}}(\epsilon)$. The **cancellation of the poles** takes place **automatically**

$\Uparrow$

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$$Y = \frac{[\alpha_s]^2}{2}\left\langle M_0\right|\left[I_{\rm V}+I_{\rm S}+I_{\rm C}\right]^2\left|M_0\right\rangle + \ldots \equiv \frac{[\alpha_s]^2}{2}\langle M_0|I_{\rm T}^2|M_0\rangle \boxed{+\ldots}$$

WHAT HAPPENS AT NNLO?

TRIPOLE-POLES known in the literature (for $N_{\text{jet}} \geq 2$):

From $\mathrm{d}\hat{\sigma}^{\mathrm{VV}}$

$$H_2(\epsilon) = \frac{i f_{abc}}{384\epsilon}(\gamma_0^{\text{cusp}})^2 \sum_{(i,j,k)}^{N_p} T_i^a T_j^b T_k^c \log\frac{-s_{ij}}{-s_{jk}} \log\frac{-s_{jk}}{-s_{ki}} \log\frac{-s_{ki}}{-s_{ij}} - \frac{i f_{abc}}{128\epsilon}\gamma_0^{\text{cusp}} \sum_{(i,j,k)}^{N_p} T_i^a T_j^b T_k^c \left(\frac{\gamma_0^i}{C_{f_i}} - \frac{\gamma_0^j}{C_{f_j}}\right) \log\frac{-s_{ij}}{-s_{jk}} \log\frac{-s_{ki}}{-s_{ij}} + \frac{\boldsymbol{\Gamma}_1}{16\epsilon} - \frac{\gamma_1^{\text{cusp}}\boldsymbol{\Gamma}_0}{64\epsilon} - \frac{\pi^2\beta_0\Gamma_0'}{128\epsilon}$$

From $\mathrm{d}\hat{\sigma}^{\mathrm{RV}}$

$$S_{\mathrm{m}}^{\text{tri}}\,\mathrm{RV} \sim \sum_{(i,j,k)} \frac{s_{ij}}{s_{im}s_{jm}} \left(\frac{s_{jk}}{s_{jm}s_{km}}\right)^{\epsilon} T_i^a T_j^b T_k^c$$

$\mathcal{O}(\epsilon^{-2})$ ⊖ $\mathcal{O}(\epsilon^{-1})$

Once combined, these objects return

$$Y = \frac{[\alpha_s]^2}{2}\left\langle M_0 \middle| \left[I_{\mathrm{V}} + I_{\mathrm{S}} + I_{\mathrm{C}}\right]^2 \middle| M_0 \right\rangle + \ldots \equiv \frac{[\alpha_s]^2}{2}\langle \boldsymbol{M_0} | \boldsymbol{I}_{\mathrm{T}}^2 | \boldsymbol{M_0}\rangle + \ldots$$

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$\mathcal{O}(\epsilon^{-2})$

$\mathcal{O}(\epsilon^{-1})$

Need to add **other contributions**. But **where** do they come from?

If $N_{\text{jet}} \geq 2$

$[\bar{I}_1, \bar{I}_1^\dagger] \neq 0$

$[\bar{I}_1^\dagger, \bar{I}_S] \neq 0 \rightarrow f_{abc}T_i^a T_j^b T_k^c \Rightarrow$

$[\bar{I}_1, \bar{I}_S] \neq 0$

Combining the commutators

$$\boldsymbol{I}^{\text{tri}} = \frac{1}{2}[I_V + I_S, \bar{I}_1 - \bar{I}_1^\dagger] - \frac{1}{4}[I_V, \bar{I}_1 - \bar{I}_1^\dagger]$$

Once combined with the other triples, this **cancels out** all the **triple-poles**

$$Y = \frac{[\alpha_s]^2}{2}\left\langle M_0\right|\left[I_V + I_S + I_C\right]^2\left|M_0\right\rangle + \ldots \equiv \frac{[\alpha_s]^2}{2}\langle M_0 | I_T^2 | M_0\rangle + \ldots$$

CONCLUSIONS AND OUTLOOK

1. We find **recurring building blocks**, i.e. $I_V(\epsilon)$, $I_S(\epsilon)$, $I_C(\epsilon)$ and $I_T(\epsilon)$, which let us solve the problem of color-correlated poles

2. The **procedure** is (almost) entirely **process independent**

3. The cancellation of the poles is **analytical** and takes place automatically for N_p **gluons**

4. *Work in progress*: next step is a generalization to **asymmetric initial state** and **arbitrary final state**

5. *Outlook*: application of the method to **pheno-studies**