

TOWARDS A GENERAL NESTED SOFT-COLLINEAR SUBTRACTION METHOD FOR NNLO CALCULATIONS

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In collaboration with:
F. Devoto, K. Melnikov, R. Röntschi, C. Signorile-Signorile

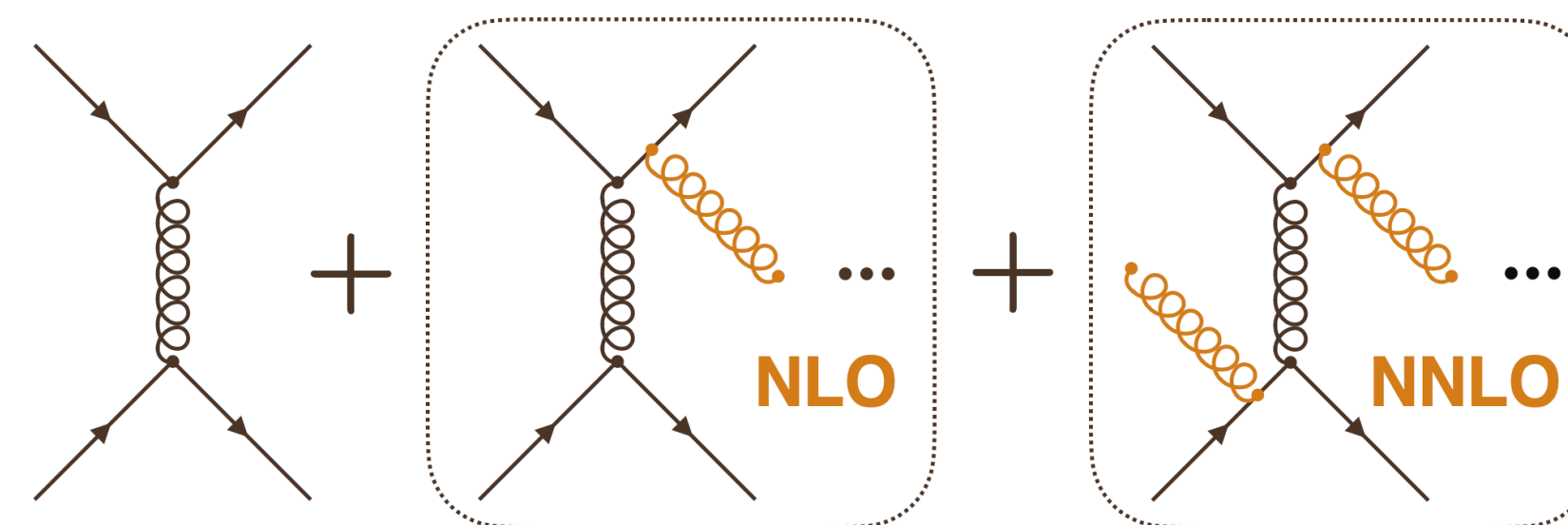


UNIVERSITÀ
DEGLI STUDI
DI MILANO

WHAT IS THE STATUS?

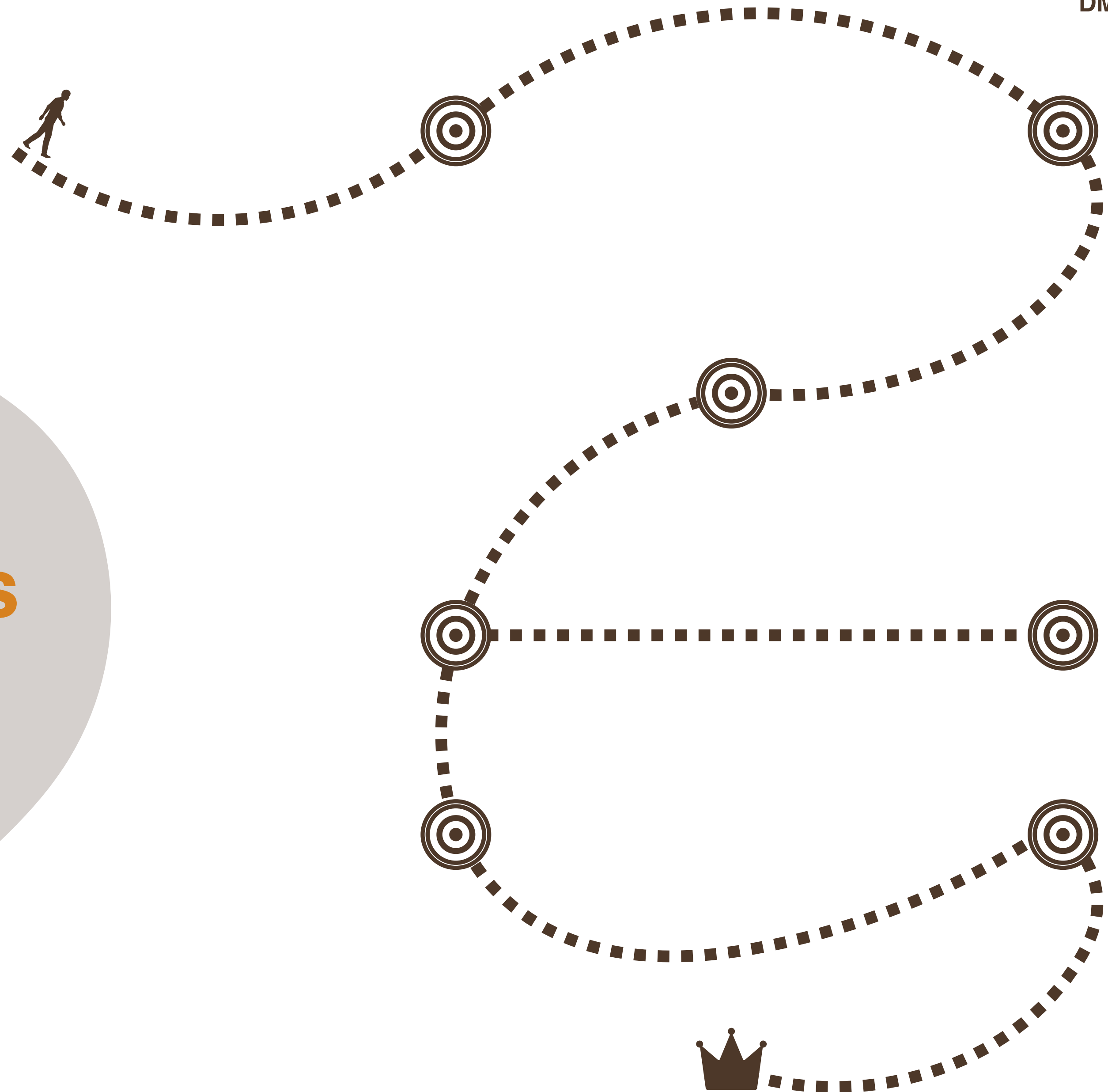
- ✓ Any collider process is characterized by its differential partonic cross section which is often computed in **fixed-order perturbation theory**

- ✓ The orders in the perturbative expansion are referred to as **LO**, **NLO**, **NNLO** and so on



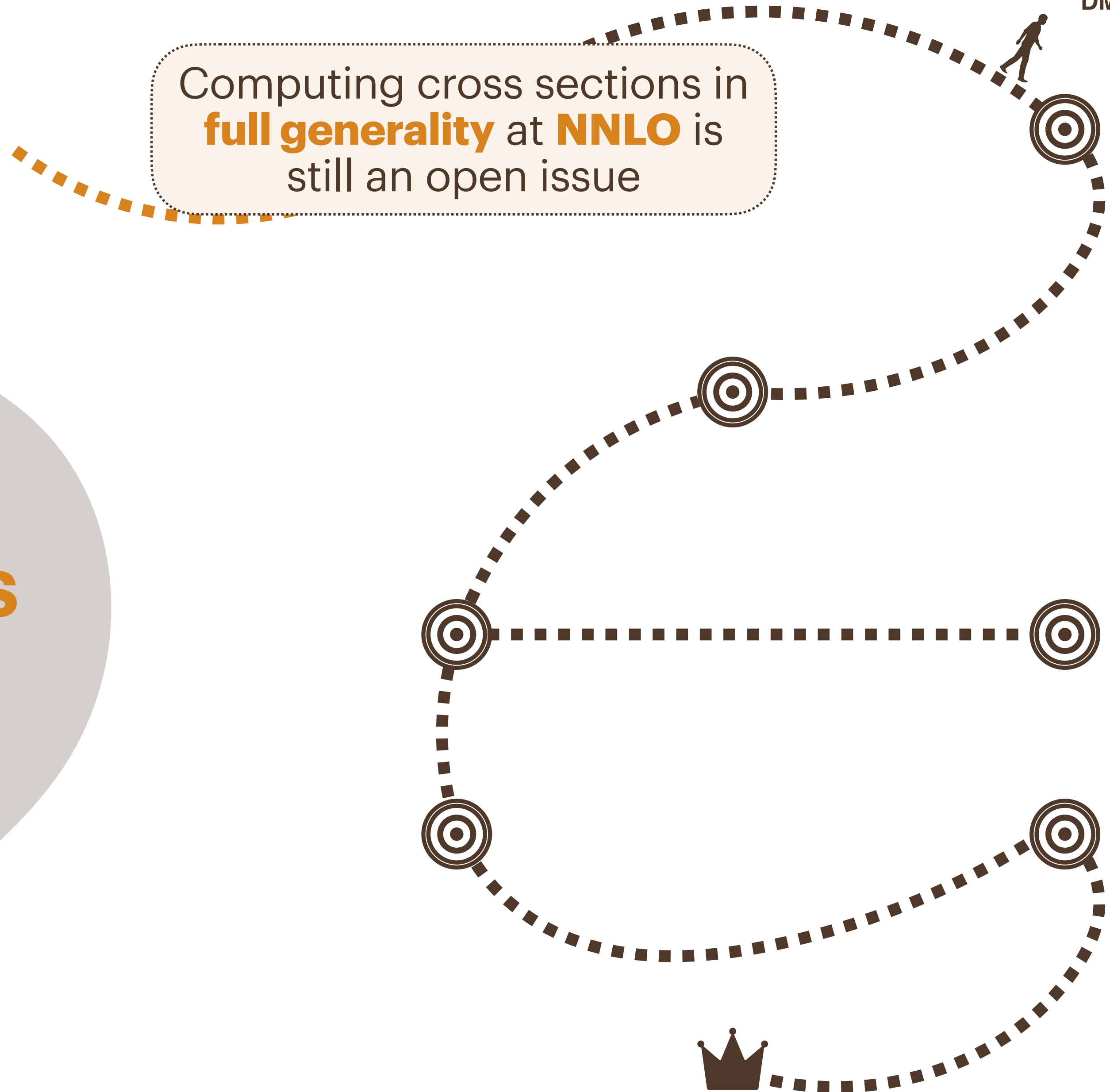
- ✓ **NLO**: solved in full generality almost a decade ago
 - ✓ **NNLO**: noteworthy results have been achieved up to now
 - ✓ **N³LO**: some results are already available
- ✓ Two main difficulties: **IR singularities**, arising from real radiation, and **multi-loop amplitude** calculations
- ✓ About IR singularities: they are unphysical and require specific methods to arrive at a finite physical result. Such methods are referred to as **SUBTRACTION SCHEMES**

WHY WE STUDY
 $P + P \rightarrow X + N \text{ gluons}$
AT NNLO



WHY WE STUDY $P + P \rightarrow X + N \text{ gluons}$ AT NNLO

Computing cross sections in
full generality at **NNLO** is
still an open issue



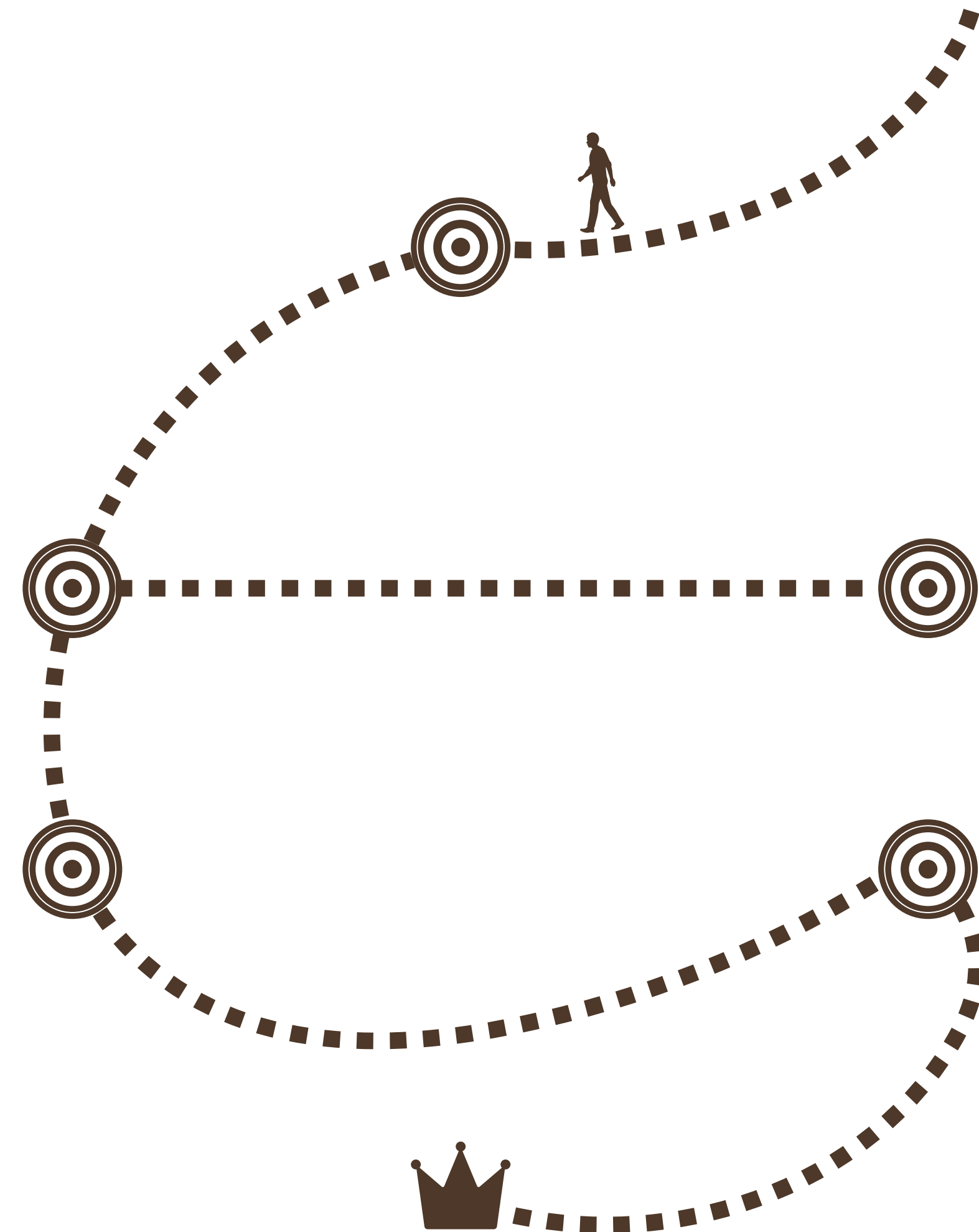
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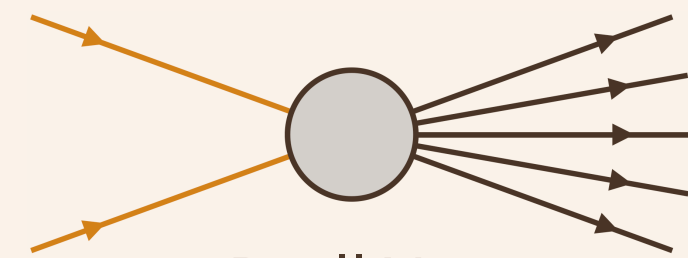
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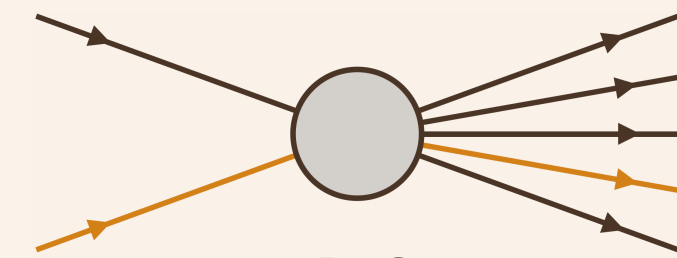
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Simple = limited number of hard partons



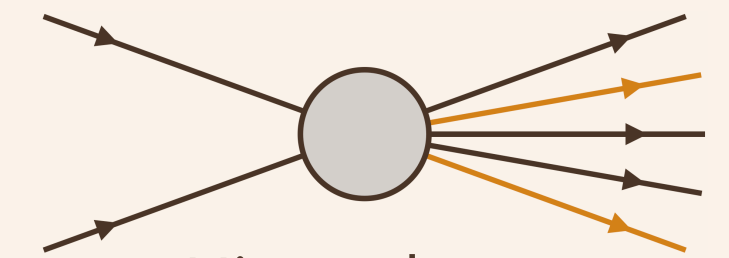
Drell-Yan

[Caola, Melnikov, Rötsch '19]



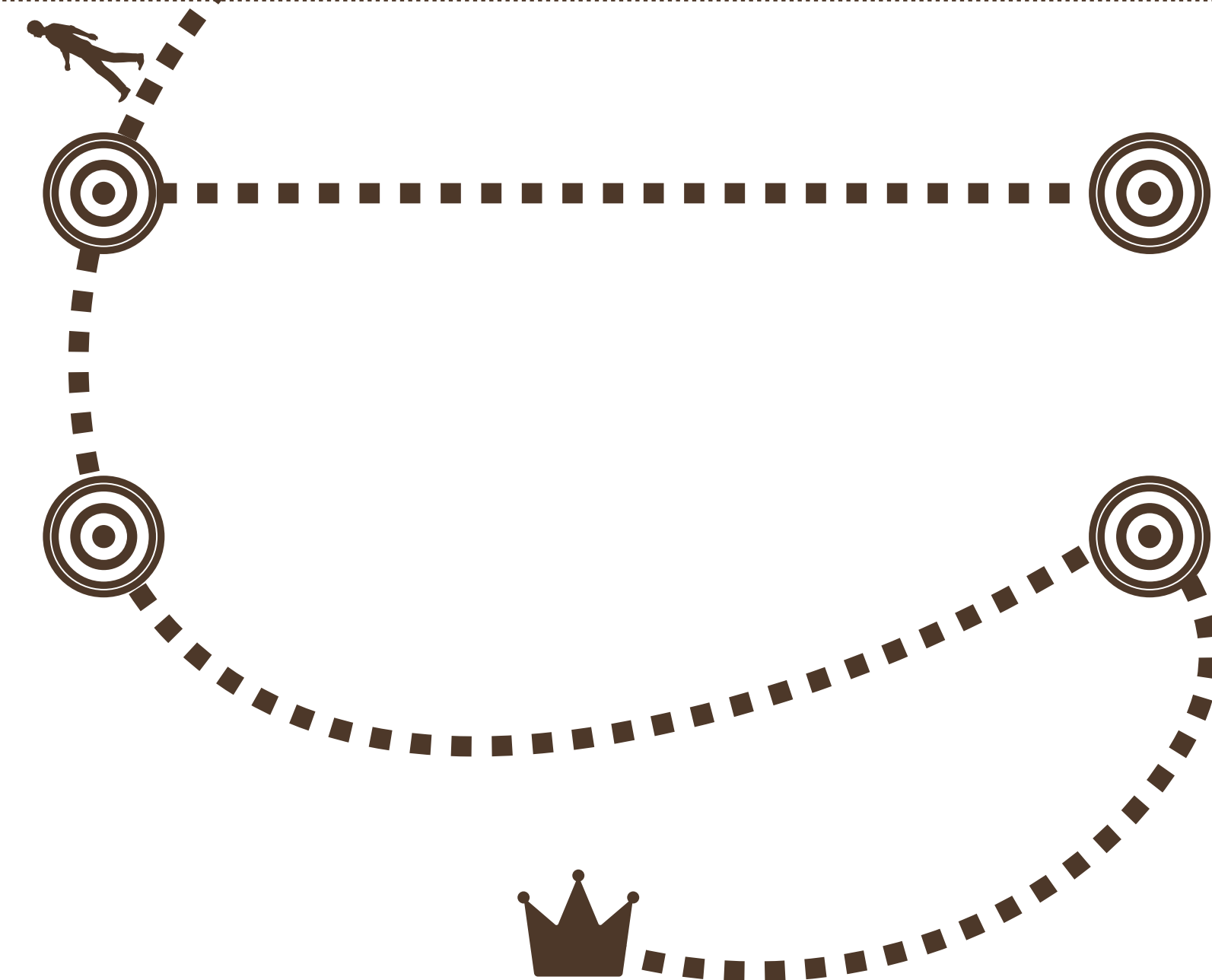
DIS

[Asteriadis, Caola, Melnikov, Rötsch '19]



Higgs decay

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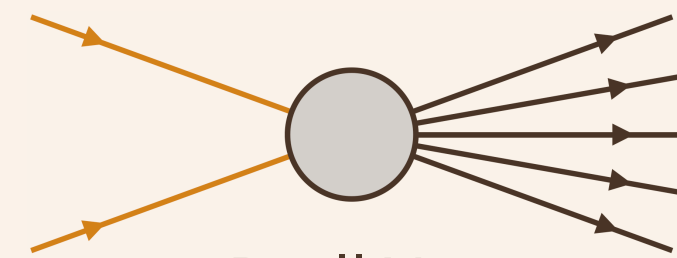
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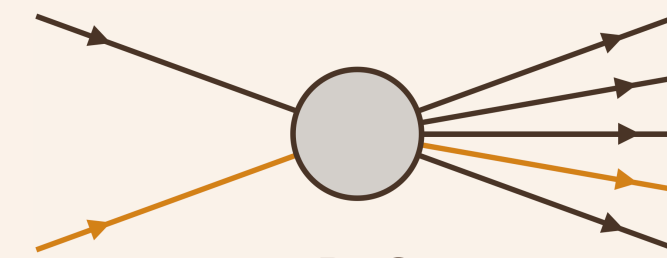
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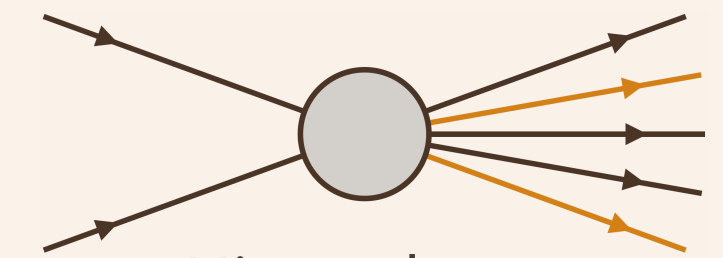
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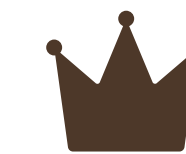


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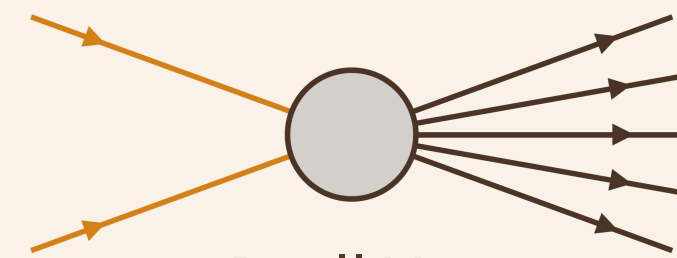
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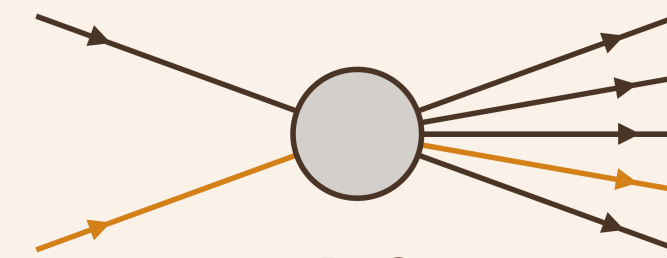
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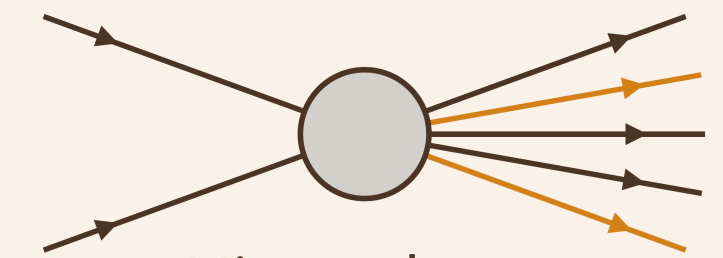
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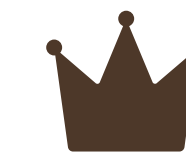
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N → 3

[Czakon et al. '21]
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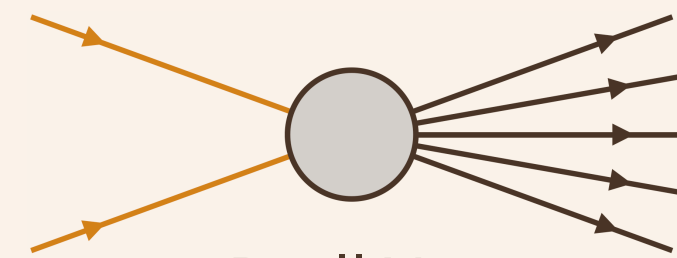
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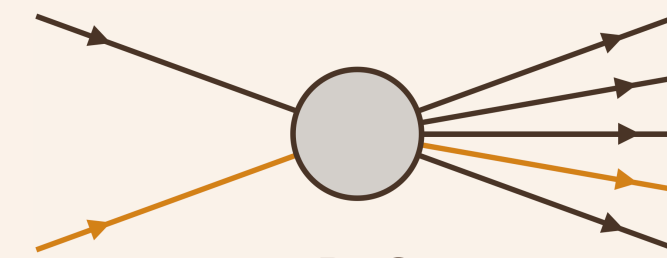
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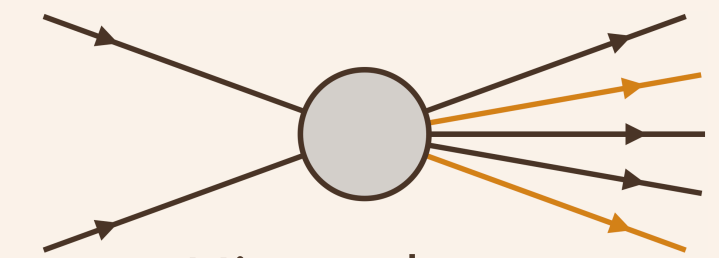
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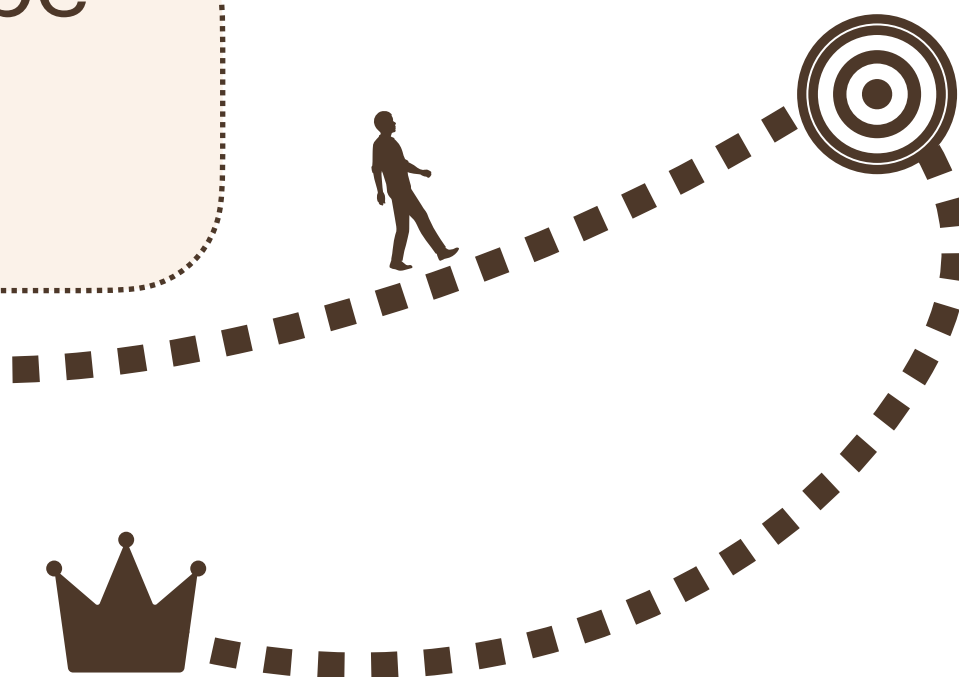
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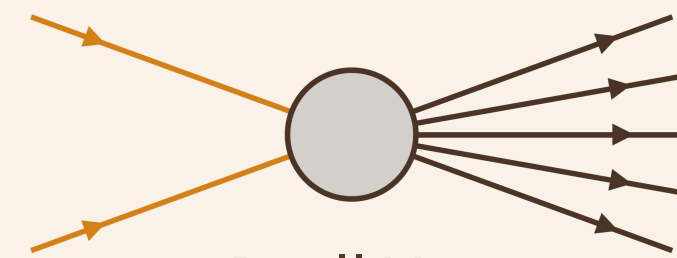
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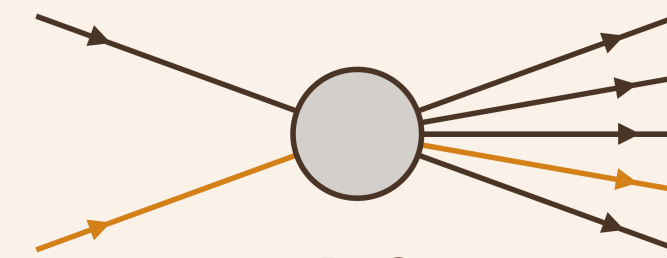
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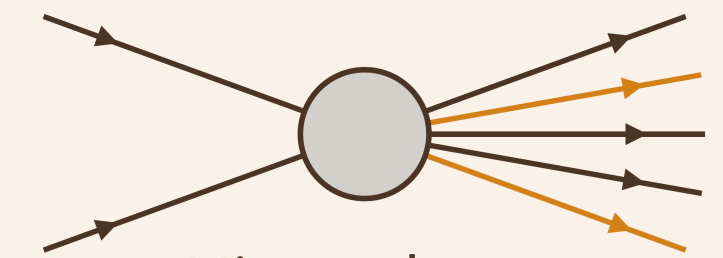
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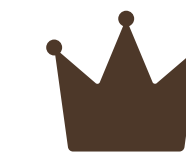
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Remaining bottleneck?

double-loop
amplitudes



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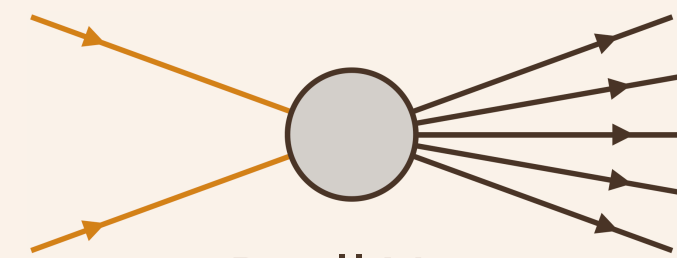
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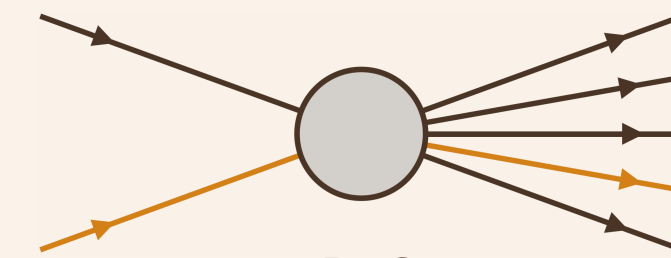
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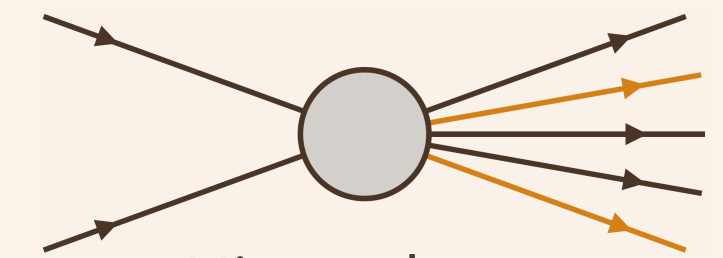
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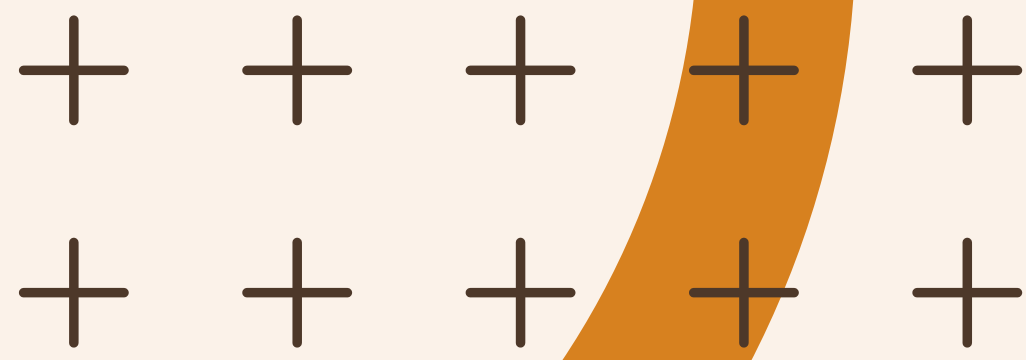
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<< If someone gives me the finite part of the double-loop amplitude of any kind of process, then I can give back the analytical expression of the whole partonic cross section. >>



FOLLOWING CATANI'S SOUL: NLO



Virtual corrections $d\hat{\sigma}^V$: the IR content of virtual amplitude is known [Catani '98]. Through the operator

$$\bar{I}_1(\epsilon) = \frac{1}{2} \sum_{i \neq j}^{N_p} \frac{\mathcal{V}_i^{\text{sing}(\epsilon)}}{T_i^2} (T_i \cdot T_j) \left(\frac{\mu^2}{2p_i \cdot p_j} \right)^\epsilon e^{i\pi\lambda_{ij}\epsilon}$$

$$\mathcal{V}_i^{\text{sing}(\epsilon)} = \frac{T_i^2}{\epsilon^2} + \frac{\gamma_i}{\epsilon}$$

$$N_p = N + 2$$

the divergent part of $d\hat{\sigma}^V$ can be written as

$$I_V(\epsilon) = \bar{I}_1(\epsilon) + \bar{I}_1^\dagger(\epsilon)$$

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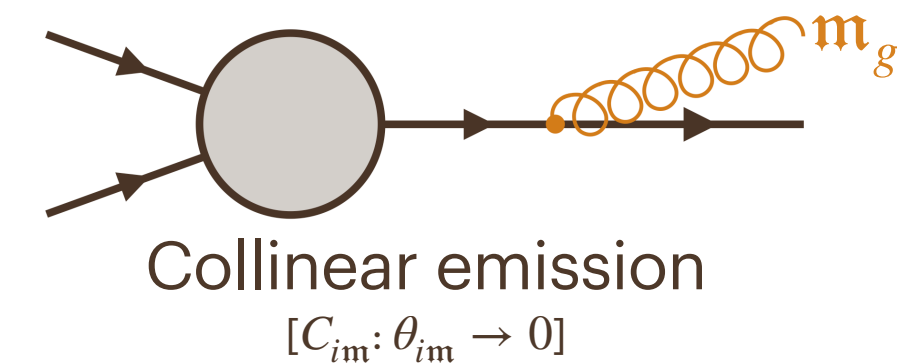
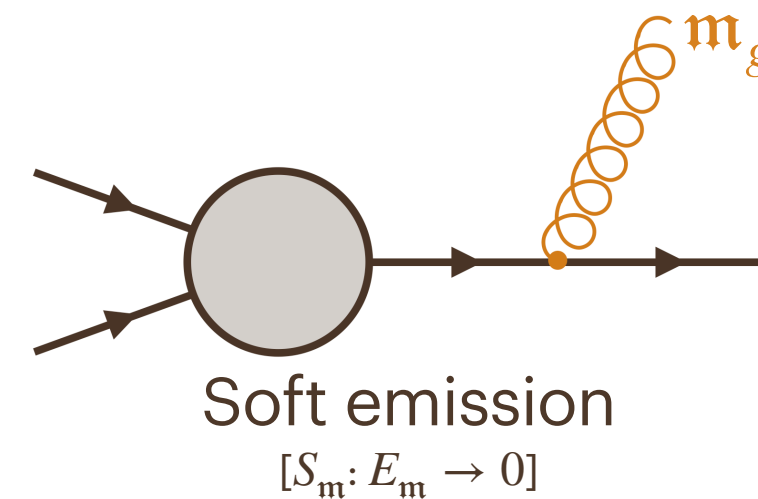
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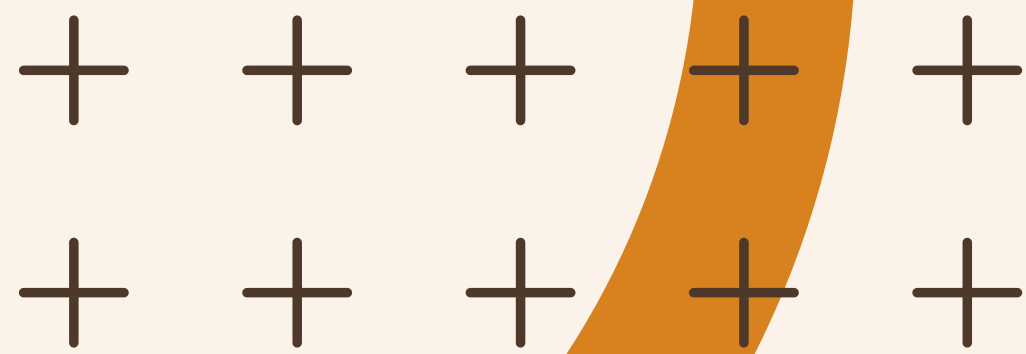


Real corrections $d\hat{\sigma}^R$: we would like something similar



We use **NESTED SOFT-COLLINEAR SCHEME** (FKS at NLO) to regularize these divergences [Caola, Melnikov, Rönsch '17]

$$d\hat{\sigma}^R = \underbrace{\langle S_m F_{LM}(\mathbf{m}) \rangle}_{\text{Soft term } [S_m: E_m \rightarrow 0]} + \sum_{i=1}^{N_p} \underbrace{\langle \bar{S}_m C_{im} \Delta^{(m)} F_{LM}(\mathbf{m}) \rangle}_{\text{Hard-Collinear term } [C_{im}: \theta_{im} \rightarrow 0]} + \langle \mathcal{O}_{\text{NLO}} \Delta^{(m)} F_{LM}(\mathbf{m}) \rangle$$



FOLLOWING CATANI'S SOUL: NLO



It turns out that the **soft term** can be written by means of an **operator** that, at least in its soul, is very **close to** $I_V(\epsilon)$:

$$I_S(\epsilon) = - \frac{(2E_{\text{max}}/\mu)^{-2\epsilon}}{\epsilon^2} \sum_{i \neq j}^{N_p} \eta_{ij}^{-\epsilon} K_{ij} (T_i \cdot T_j)$$

$$\eta_{ij} = (1 - \cos \theta_{ij})/2$$

$$K_{ij} \sim \eta_{ij}^{1+\epsilon} {}_2F_1(1, 1, 1 - \epsilon, 1 - \eta_{ij})$$

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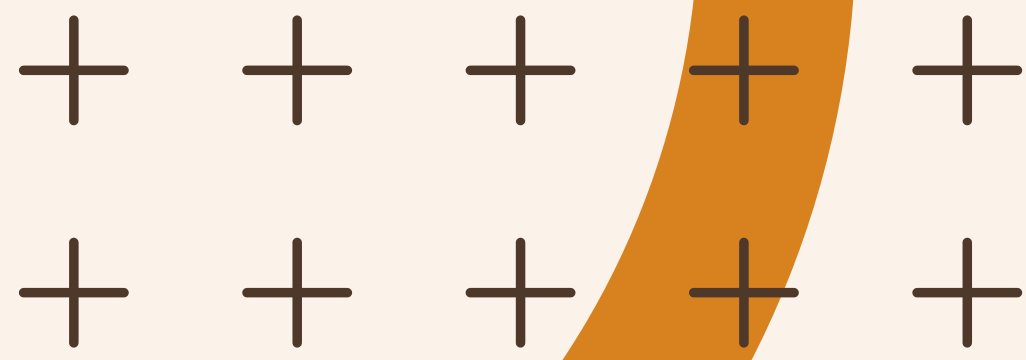
Combination of $I_V(\epsilon) + I_S(\epsilon)$: not only does it **vanishes** the pole $\mathcal{O}(\epsilon^{-2})$, but it makes the pole $\mathcal{O}(\epsilon^{-1})$ free of **color-correlations**

$$I_{V,S}(\epsilon) \sim T_i \cdot T_j \quad T_i = \text{matrices in color space}$$

$$N_p < 4 \Rightarrow d\hat{\sigma}^{\text{NLO}} \sim \frac{C_{A,F}}{\epsilon} \langle M_0 | M_0 \rangle \quad \text{--- NO color-correlations}$$

$$N_p \geq 4 \Rightarrow d\hat{\sigma}^{\text{NLO}} \sim \frac{1}{\epsilon} \langle M_0 | T_i \cdot T_j | M_0 \rangle \quad \text{--- YES color-correlations}$$

This result for $I_V(\epsilon) + I_S(\epsilon)$ is trivially **dependent** on the **number of gluons** in the final state



FOLLOWING CATANI'S SOUL: NLO



What about the **hard-collinear term**? Some parts vanish against the DGLAP contribution, the remaining part **can be collected** within the following **Catani-like operator**

$$I_C(\epsilon) = \sum_{i=1}^{N_p} \frac{\Gamma_{i,f_i}}{\epsilon}$$

$$\Gamma_{a,f_a} = \left[\left(\frac{2E_a}{\mu} \right)^{-2\epsilon} \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \right] \left[\gamma_{f_a} + C_{f_a} \frac{1 - e^{-2\epsilon L_a}}{\epsilon} \right], \quad a = 1, 2$$

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Once more the definition **depends** in a trivial way on N_p

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$I_C(\epsilon)$ cancels perfectly the pole $\mathcal{O}(\epsilon^{-1})$ left by $I_V(\epsilon) + I_S(\epsilon)$. It is thus natural to introduce the **total operator**

$$I_T(\epsilon) = I_V(\epsilon) + I_S(\epsilon) + I_C(\epsilon)$$



pole free



fully general w.r.t. N_p

In this way the final result for the NLO fits in a line:

$$d\hat{\sigma}^{\text{NLO}} = [\alpha_s] \langle I_T(\epsilon) \cdot F_{\text{LM}} \rangle + [\alpha_s] \left[\langle P_{aa}^{\text{NLO}} \otimes F_{\text{LM}} \rangle + \langle F_{\text{LM}} \otimes P_{aa}^{\text{NLO}} \rangle \right] + \langle F_{\text{LV}}^{\text{fin}} \rangle + \langle \mathcal{O}_{\text{NLO}} \Delta^{(\mathbf{m})} F_{\text{LM}}(\mathbf{m}) \rangle$$

$$d\hat{\sigma}^{\text{NNLO}} = d\hat{\sigma}^{\text{VV}} + d\hat{\sigma}^{\text{RV}} + d\hat{\sigma}^{\text{RR}} + d\hat{\sigma}^{\text{pdf}}$$

Double-Virtual Real-Virtual Double-Real PDFs Renor.

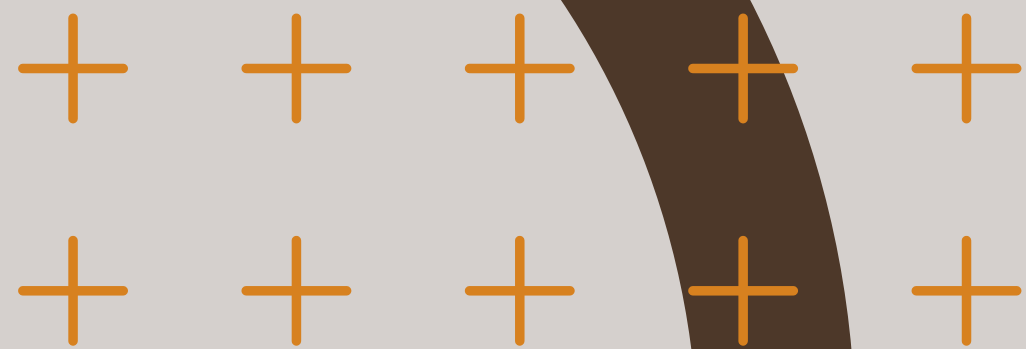
Consider for instance $d\hat{\sigma}^{\text{VV}}$ \Rightarrow it depends **quadratically** on $\bar{I}_1(\epsilon)$ and $\bar{I}_1^\dagger(\epsilon)$

$$\Rightarrow \bar{I}_1, \bar{I}_1^\dagger \sim T_i \cdot T_j$$

$$\Rightarrow d\hat{\sigma}^{\text{VV}} \sim (T_i \cdot T_j) \cdot (T_k \cdot T_l) \quad \text{double color-correlations}$$

The **same** happens for $d\hat{\sigma}^{\text{RV}}$ and $d\hat{\sigma}^{\text{RR}}$. Dealing with such double-color correlated terms (**DCC**) in general makes the **structure of the poles very complicated**

WHAT HAPPENS AT NNLO?



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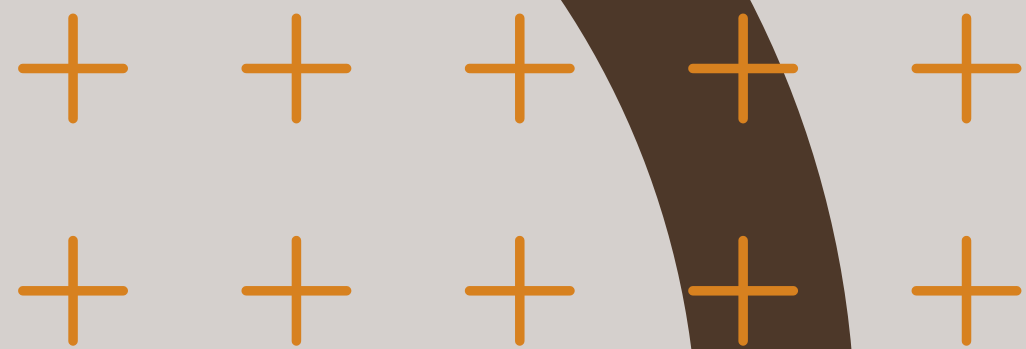


The strategy: **isolate DCC** in $d\hat{\sigma}^{\text{VV}}$ and then **combine** them with **those** contained within $d\hat{\sigma}^{\text{RV}}$ and $d\hat{\sigma}^{\text{RR}}$



The goal: **assemble** all these **DCC** into an expression that we expect being **quadratic** in $I_T(\epsilon)$

WHAT HAPPENS AT NNLO?



WHAT HAPPENS AT NNLO?

Here it is what we find [Devoto, Melnikov, Röntsch, Signorile-Signorile, **D.M.T.**, to appear]

$$Y_{VV} = \frac{[\alpha_s]^2}{2} \langle M_0 | \bar{I}_1^2 + (\bar{I}_1^\dagger)^2 + 2\bar{I}_1^\dagger \bar{I}_1 | M_0 \rangle + \dots$$

$$Y_{RR}^{(ss)} = \frac{[\alpha_s]^2}{2} \langle M_0 | I_S^2 | M_0 \rangle + \dots$$

$$Y_{RR}^{(shc)} = [\alpha_s]^2 \langle M_0 | I_S I_C | M_0 \rangle + \dots$$

$$Y_{RR}^{(cc)} = \frac{[\alpha_s]^2}{2} \langle M_0 | I_C^2 | M_0 \rangle + \dots$$

$$Y_{RV}^{(s)} = \frac{[\alpha_s]^2}{2} \langle M_0 | I_S \bar{I}_1 + \bar{I}_1^\dagger I_S | M_0 \rangle + \dots$$

$$Y_{RV}^{(shc)} = [\alpha_s]^2 \langle M_0 | (\bar{I}_1 + \bar{I}_1^\dagger) I_C | M_0 \rangle + \dots$$




Once combined, these objects return

NB square of NLO

$$Y = \frac{[\alpha_s]^2}{2} \langle M_0 | [I_V + I_S + I_C]^2 | M_0 \rangle + \dots \equiv \langle M_0 | I_T^2 | M_0 \rangle + \dots$$

WHAT HAPPENS AT NNLO?

The benefits of introducing these Catani's like operators:

-  the problem of **double color-correlated poles disappears**, since everything is written in terms of $I_T^2(\epsilon)$, which is $\mathcal{O}(\epsilon^0)$
-  the **definition** of $I_T(\epsilon)$ depends trivially on N_p so the result we got is **fully general w.r.t. the final state**
-  We **do not explicitly calculate** the individual sub-blocks of the process. Instead, we write each of these in terms of $I_V(\epsilon)$, $I_S(\epsilon)$ and $I_C(\epsilon)$, then recombine them to get $I_T(\epsilon)$. The **cancellation of the poles** takes place **automatically**






Once combined, these objects return

$$Y = \frac{[\alpha_s]^2}{2} \langle M_0 | [I_V + I_S + I_C]^2 | M_0 \rangle + \dots \equiv \langle M_0 | I_T^2 | M_0 \rangle + \dots$$

NB square of NLO

WHAT HAPPENS AT NNLO?

The benefits of introducing these Catani's like operators:

-  the problem of **double color-correlated poles disappears**, since everything is written in terms of $I_T^2(\epsilon)$, which is $\mathcal{O}(\epsilon^0)$
-  the **definition** of $I_T(\epsilon)$ depends trivially on N_p so the result we got is **fully general w.r.t. the final state**
-  We **do not explicitly calculate** the individual sub-blocks of the process. Instead, we write each of these in terms of $I_V(\epsilon)$, $I_S(\epsilon)$ and $I_C(\epsilon)$, then recombine them to get $I_T(\epsilon)$. The **cancellation of the poles** takes place **automatically**

↑

Once combined, these objects return

If $N_p \geq 4$

$[\bar{I}_1, \bar{I}_1^\dagger] \neq 0$

$[\bar{I}_1^\dagger, \bar{I}_S] \neq 0 \rightarrow f_{abc} T_i^a T_j^b T_k^c$

$[\bar{I}_1, \bar{I}_S] \neq 0$

$$Y = \frac{[\alpha_s]^2}{2} \langle M_0 | [I_V + I_S + I_C]^2 | M_0 \rangle + \dots \equiv \langle M_0 | I_T^2 | M_0 \rangle + \dots$$

CONCLUSIONS AND OUTLOOK

- 1 We can also **vanish the poles** $\sim f_{abc} T_i^a T_j^b T_k^c$.
How this happens is non trivial
- 2 There are still **terms** that **do not fit** $I_T(\epsilon)$
- 3 The next step is studying the process where we **add** a $q \bar{q}$ couple **in the final state**
- 4 We also intend to study the **asymmetric initial state** $g q$



**MANY THANKS
FOR YOUR
ATTENTION**



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