Università degli Studi di Milano
Dipartimento di Fisica
In collaboration with: Stefano Forte

# THRESHOLD RESUMMATION OF *RAPIDITY DISTRIBUTIONS*: A TRANSLATION OF THE STATE OF THE ART FROM *SCET* TO *dQCD*

Davide Maria Tagliabue

# THE CONTEXT

HIGH-ENERGY COLLISIONS AMONG HADRONS

THRESHOLD RESUMMATION THEORY

dQCD

SCET

## Aim of the Work

- translate the actual state of the art from SCET to dQCD
- analytical comparison with the previous state of the art

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HIGH-ENERGY COLLISIONS AMONG HADRONS → THRESHOLD RESUMMATION THEORY → dQCD, SCET

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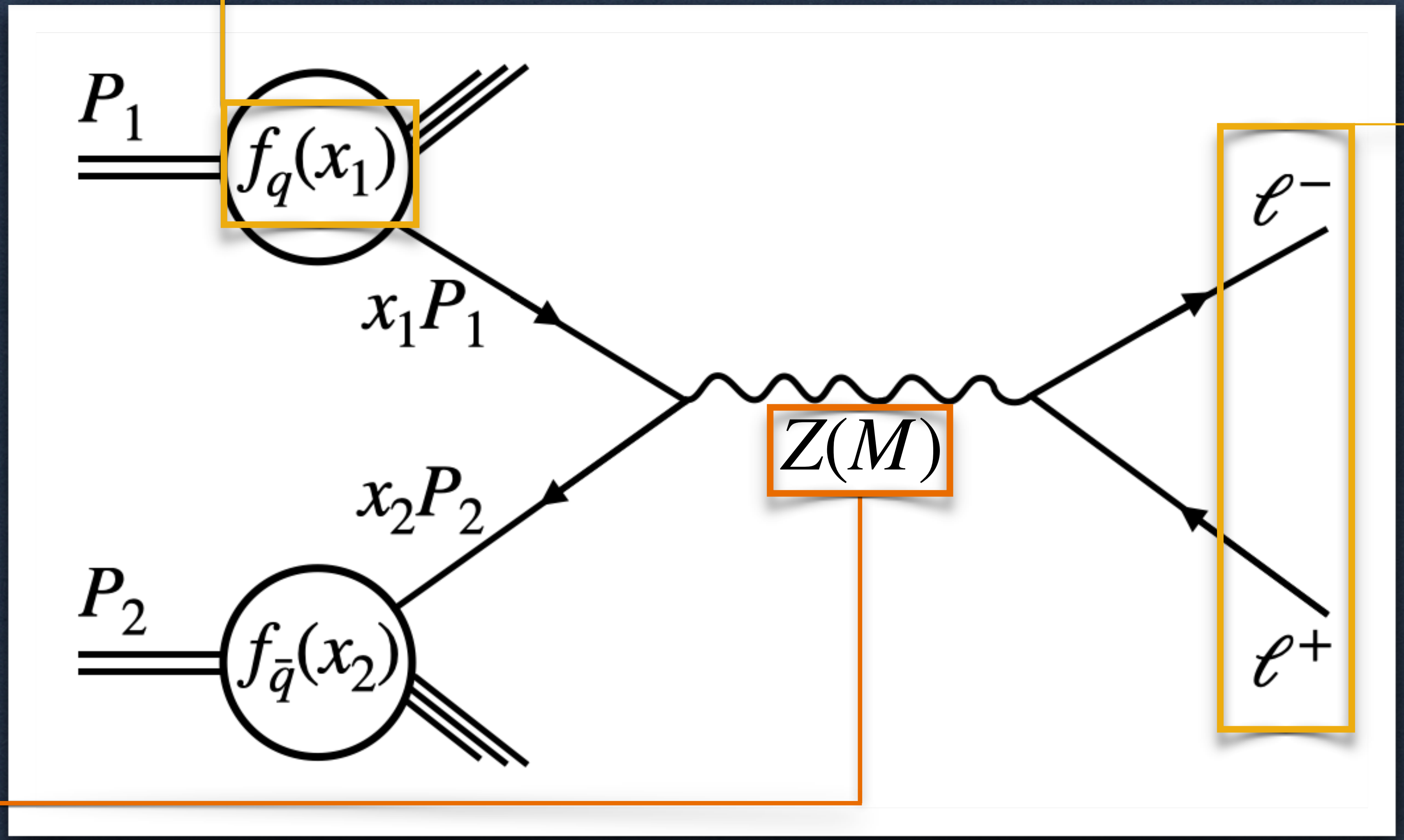
# DRELL-YAN PROCESS

PARTON DISTRIBUTION FUNCTION (PDF)




NON-STRONGLY INTERACTING FINAL STATES

INVARIANT MASS OF THE FINAL STATE

$Y$ = boost w.r.t. the C.M. of the collision

## FACTORIZATION THEOREM

$$\frac{d\sigma}{dM^2dY} = \boxed{f^{(1)} \otimes f^{(2)}} \otimes \boxed{\frac{d\hat{\sigma}}{dM^2dY}}$$

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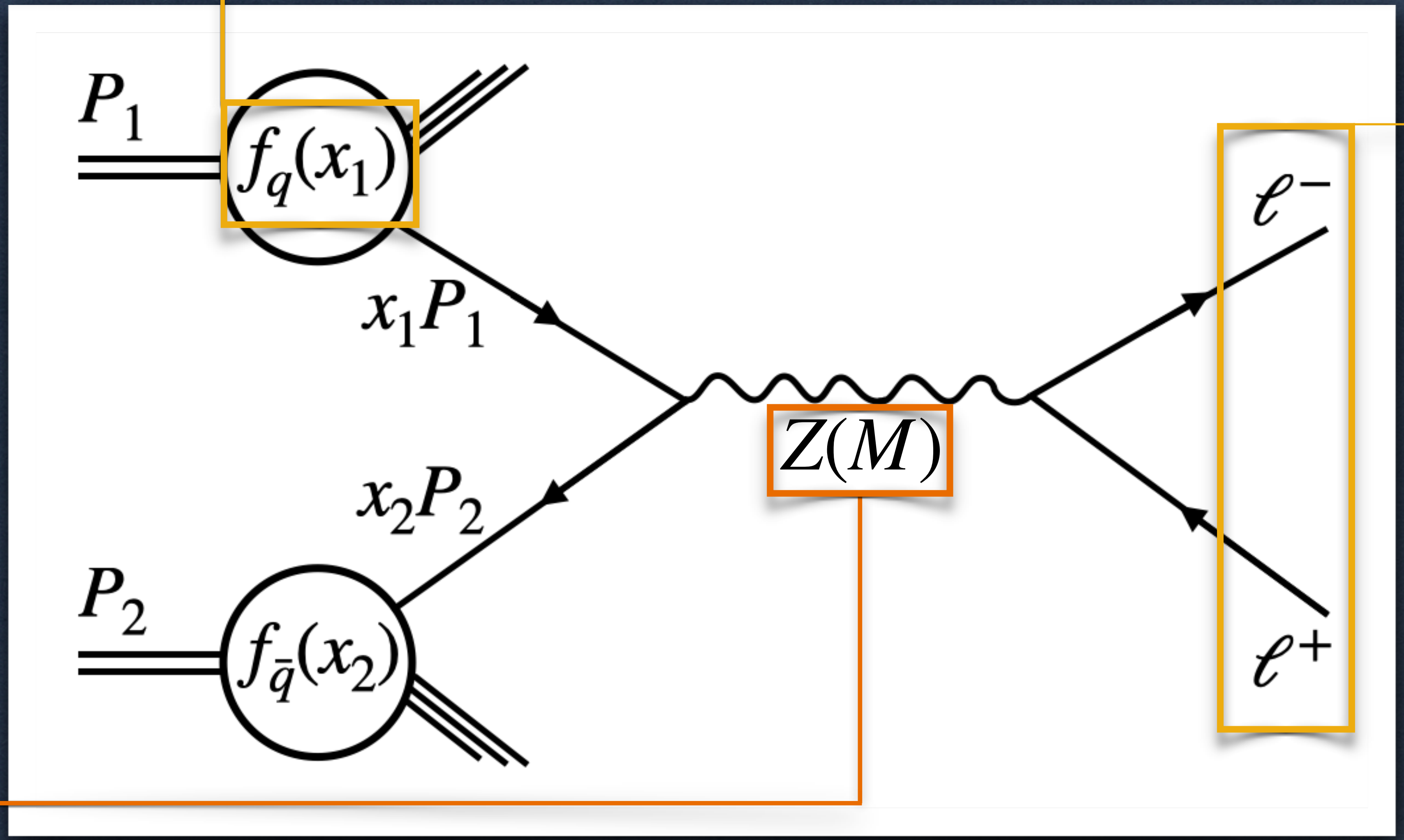


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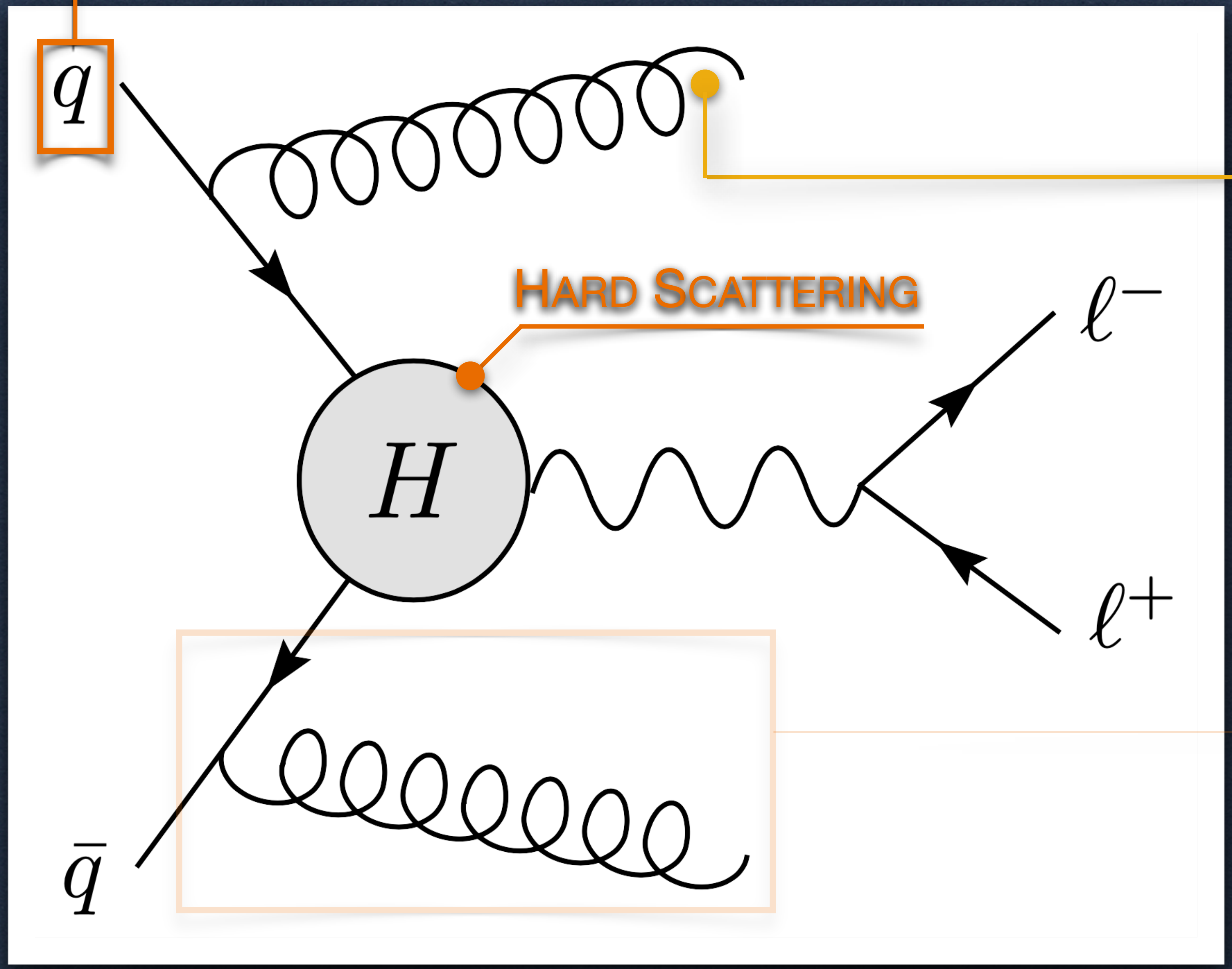
# THRESHOLD RESUMMATION

Incoming Parton



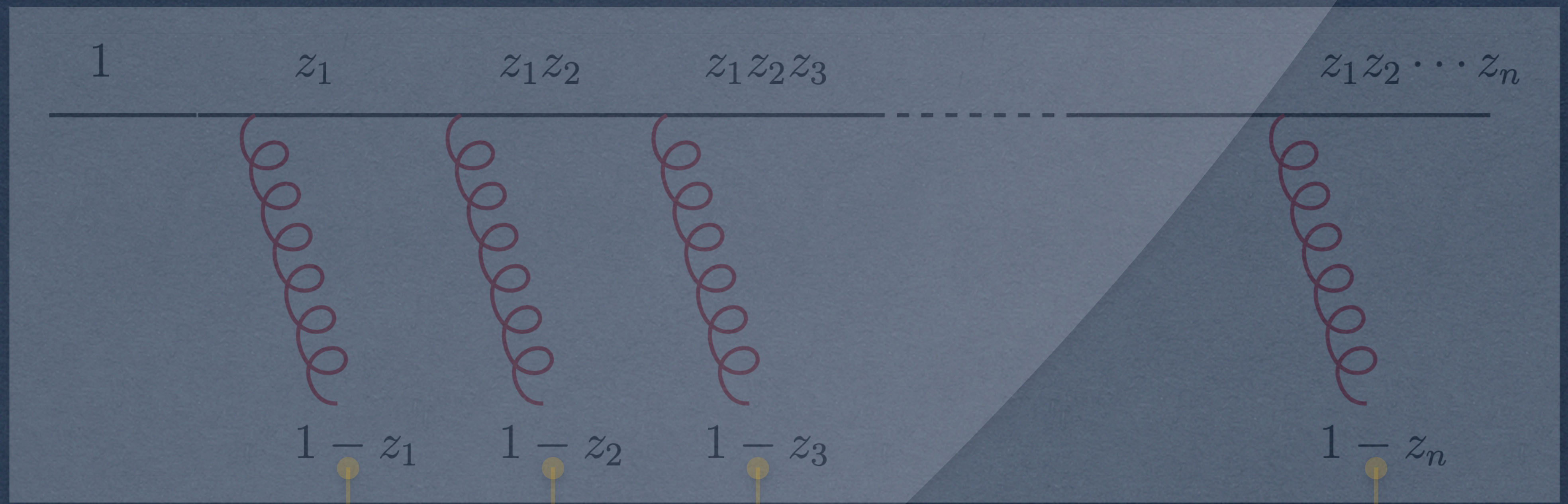

Soft Gluon Emitted

## Birth of resummation

$$\log\left(1-\frac{M^2}{\hat{s}}\right)$$

if $\hat{s} \sim M^2$: $\rightarrow$ threshold limit

1 emission + … + $n$ emissions + … = RESUMMATION




large log | large log | large log | large log

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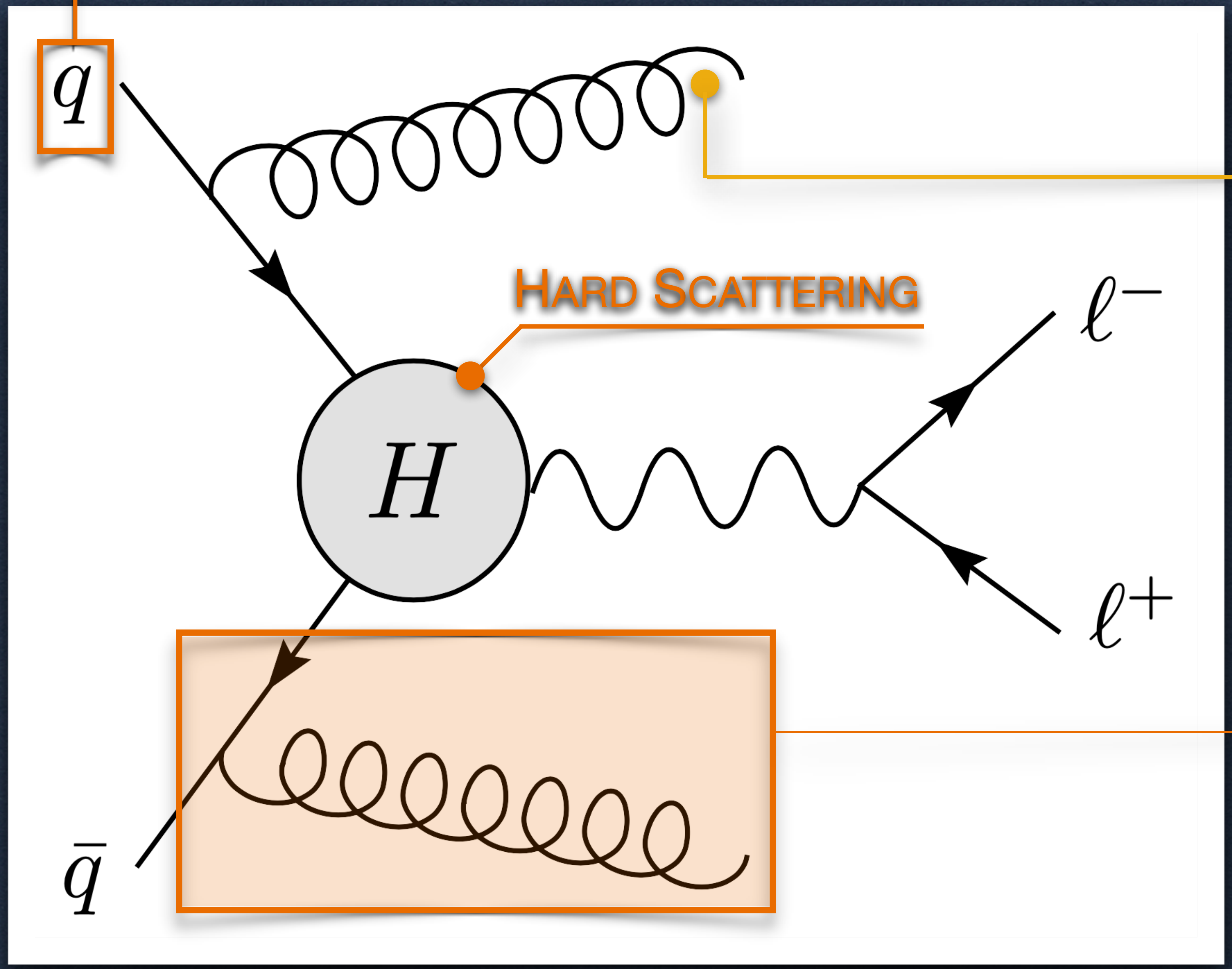


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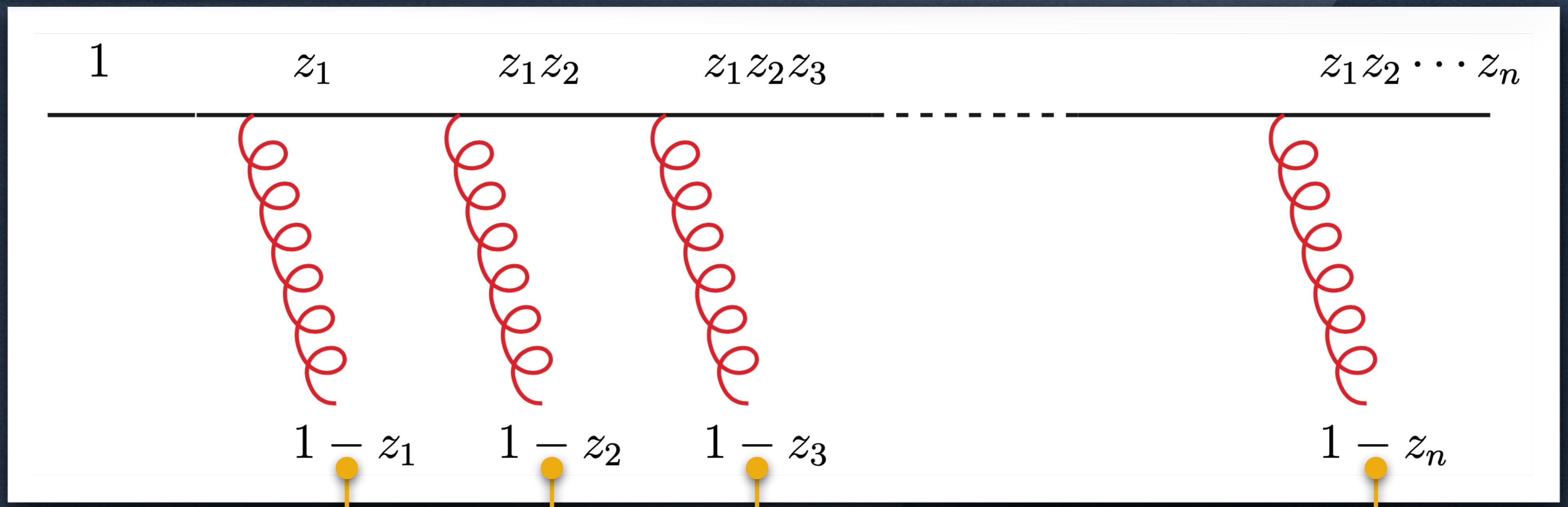


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## INTRODUCTION OF MELLIN VARIABLES

$$\frac{M^2}{\hat{s}} \to 1 \quad \Rightarrow \quad \boxed{N_{a,b} \to \infty}$$

### Bonvini - Forte - Ridolfi

- kinematic limit

$$N_{a,b} \to \infty, \quad \frac{N_a}{N_b} = 1$$

- *subleading in rapidity*
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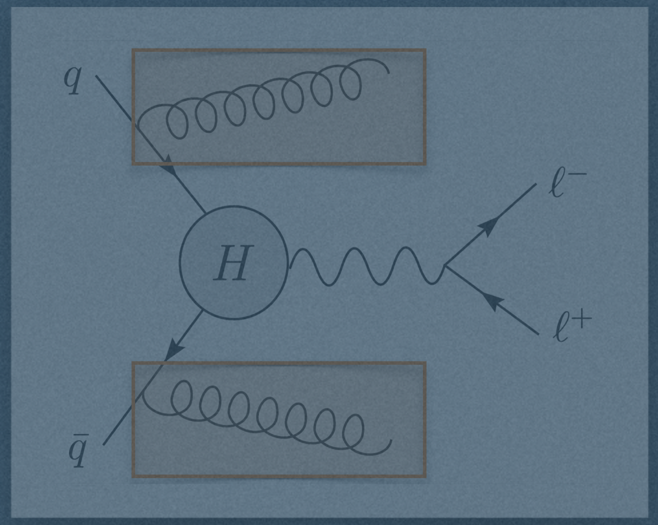


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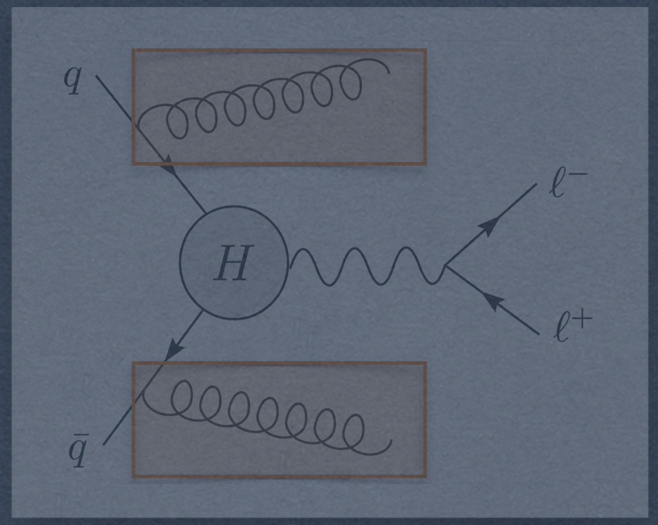


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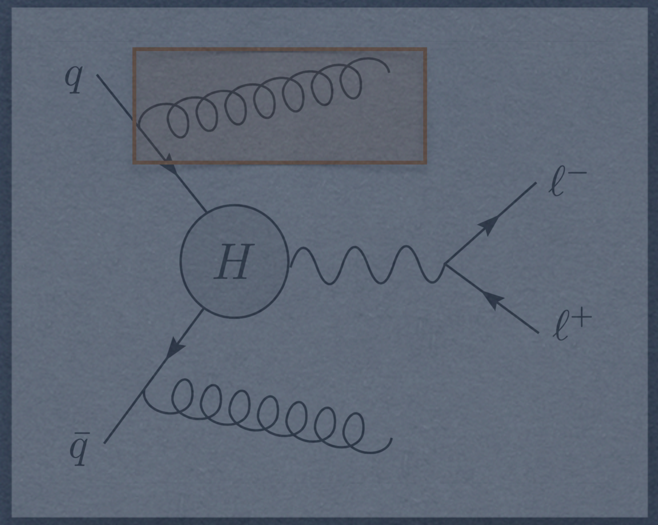


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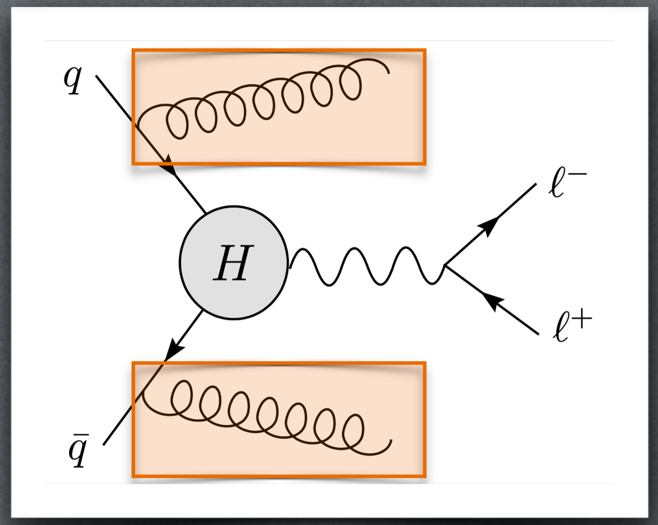


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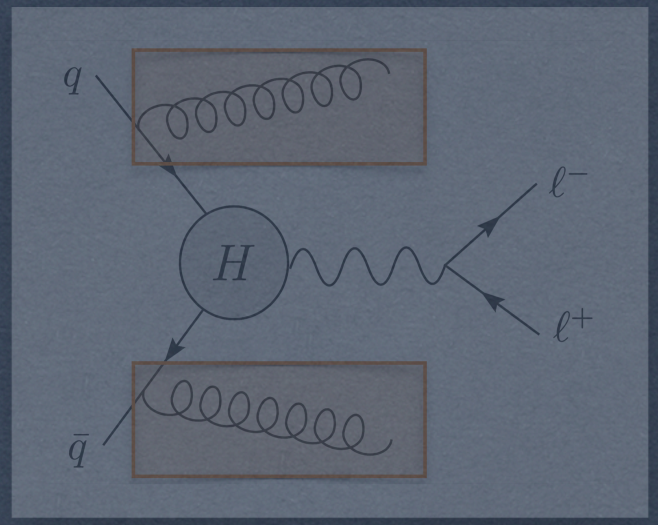


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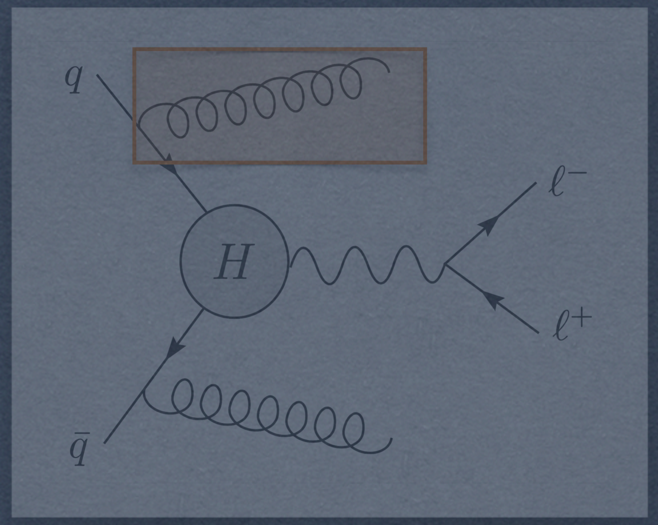


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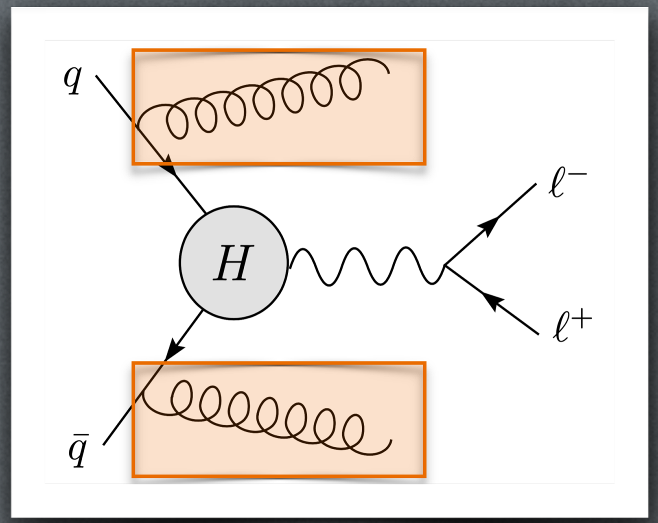


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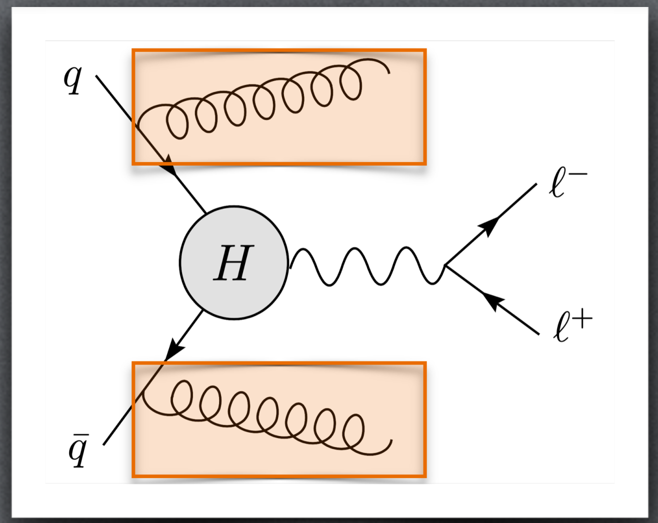


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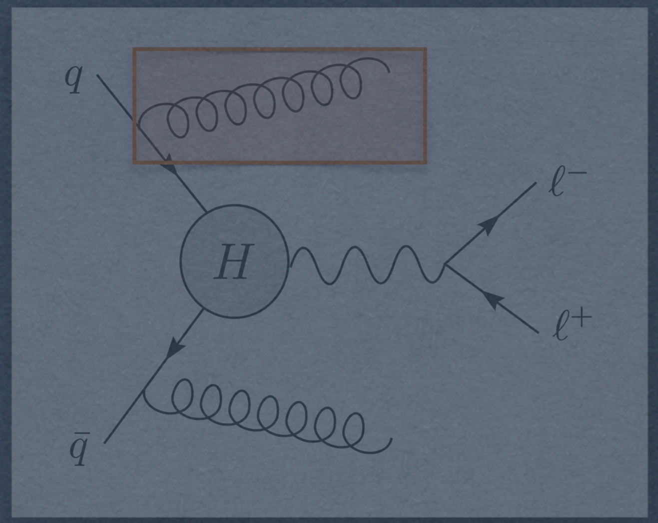


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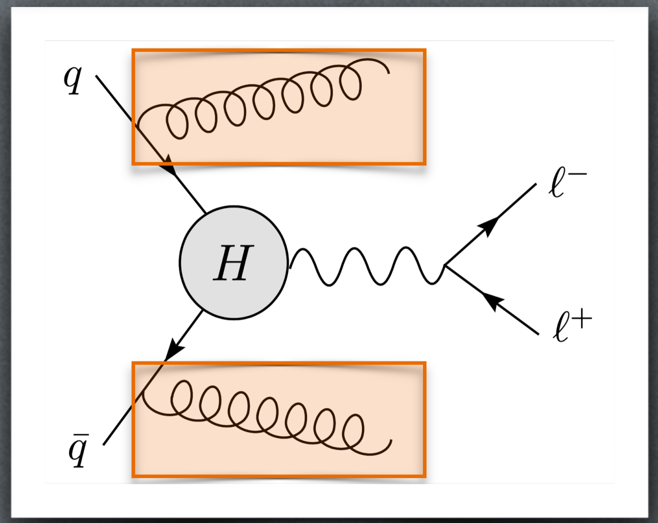


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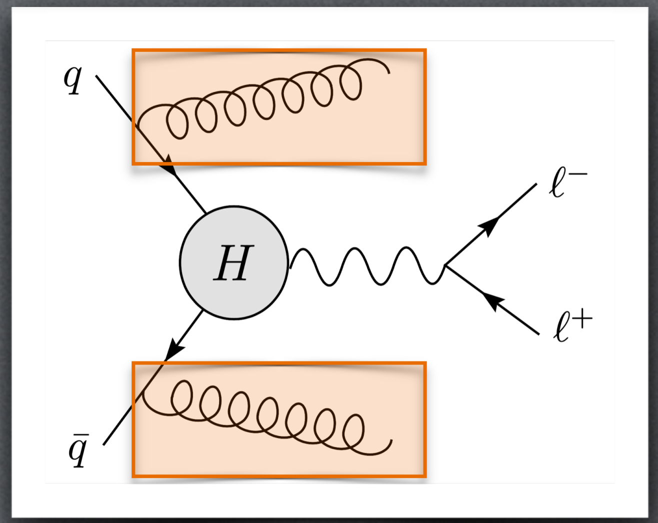


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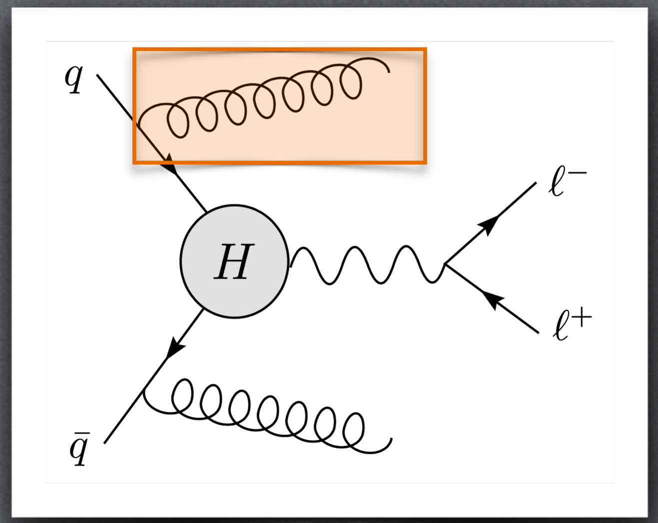


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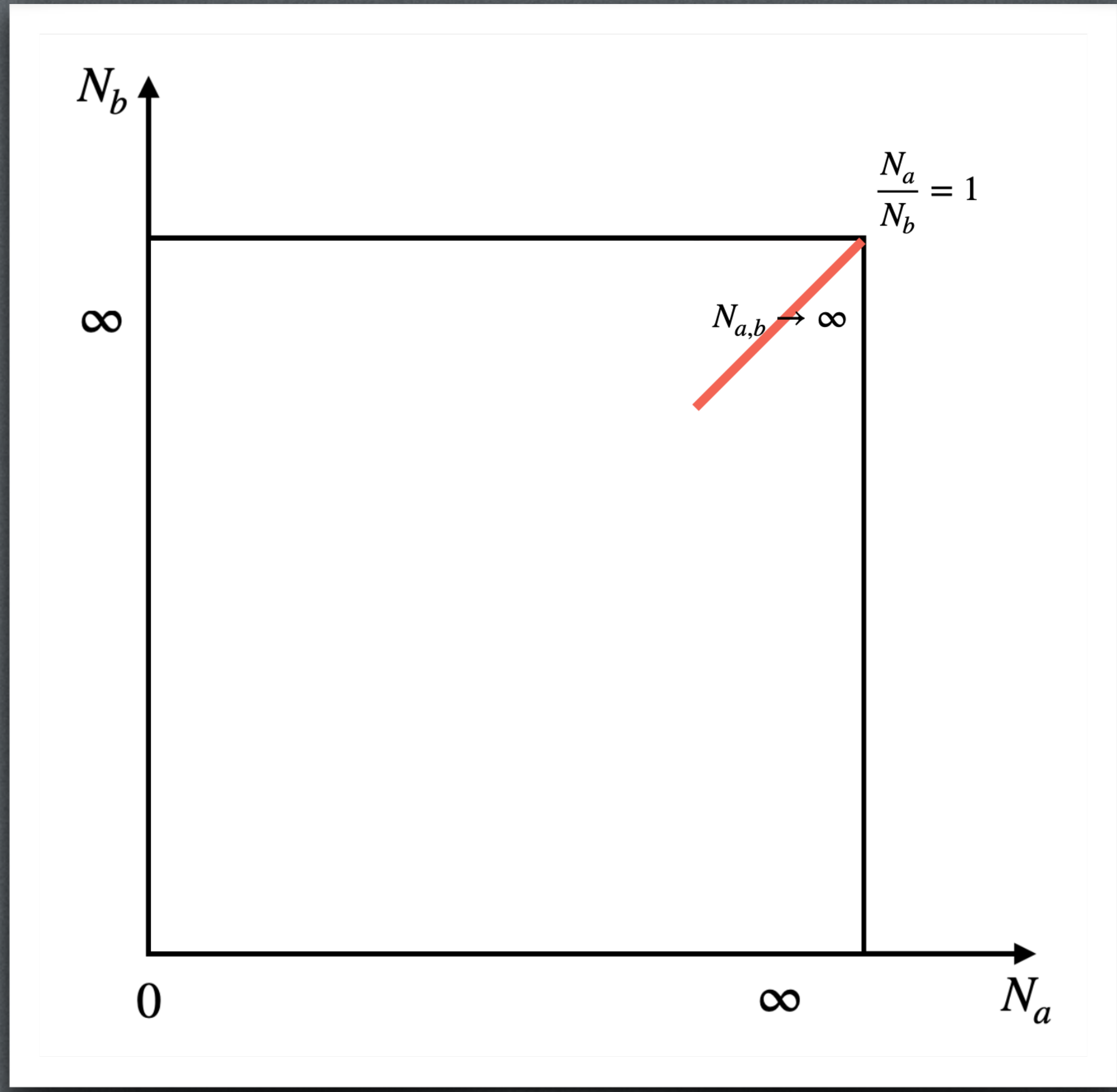


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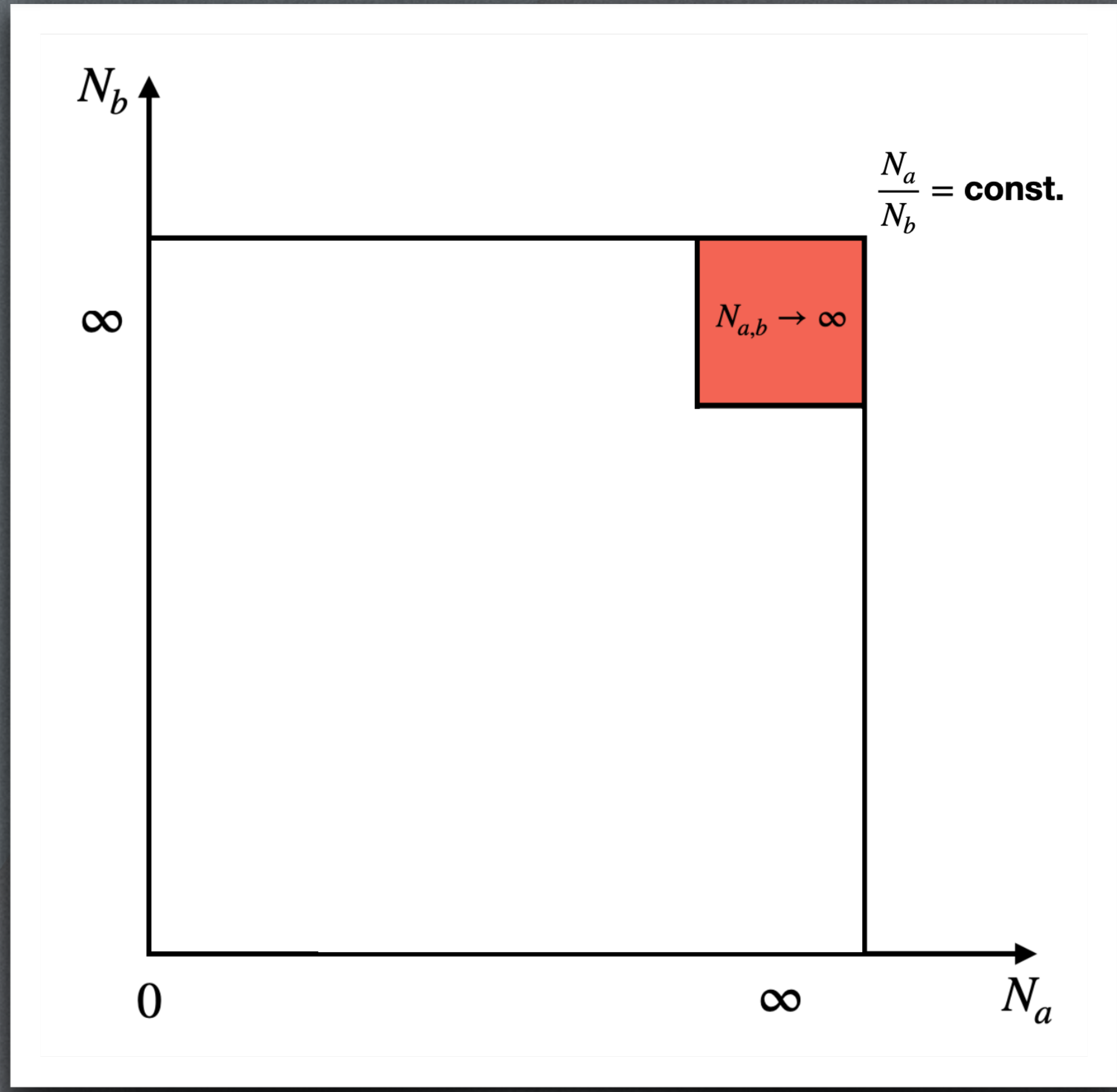


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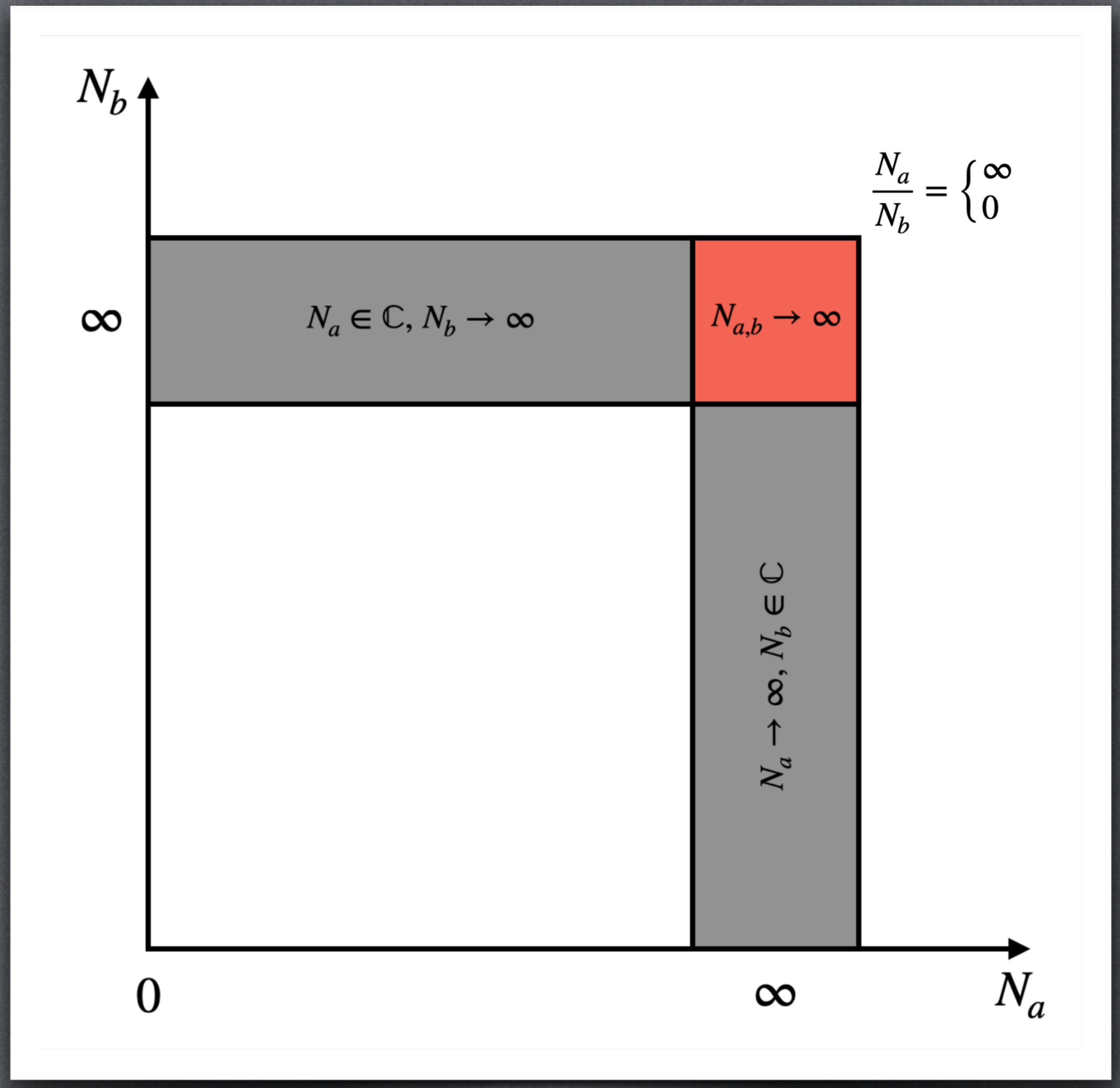


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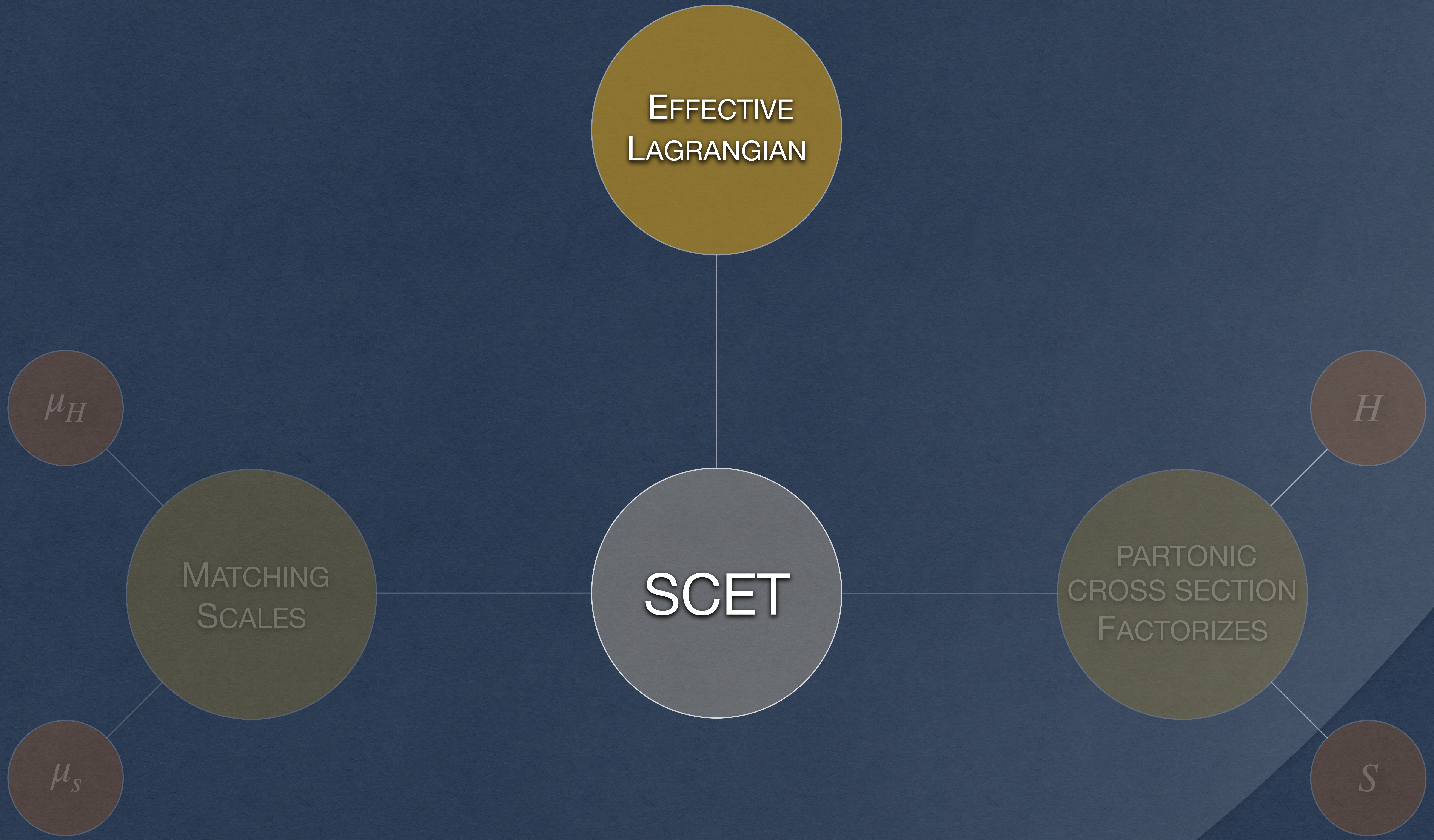
# SCET AND LMT RESULT



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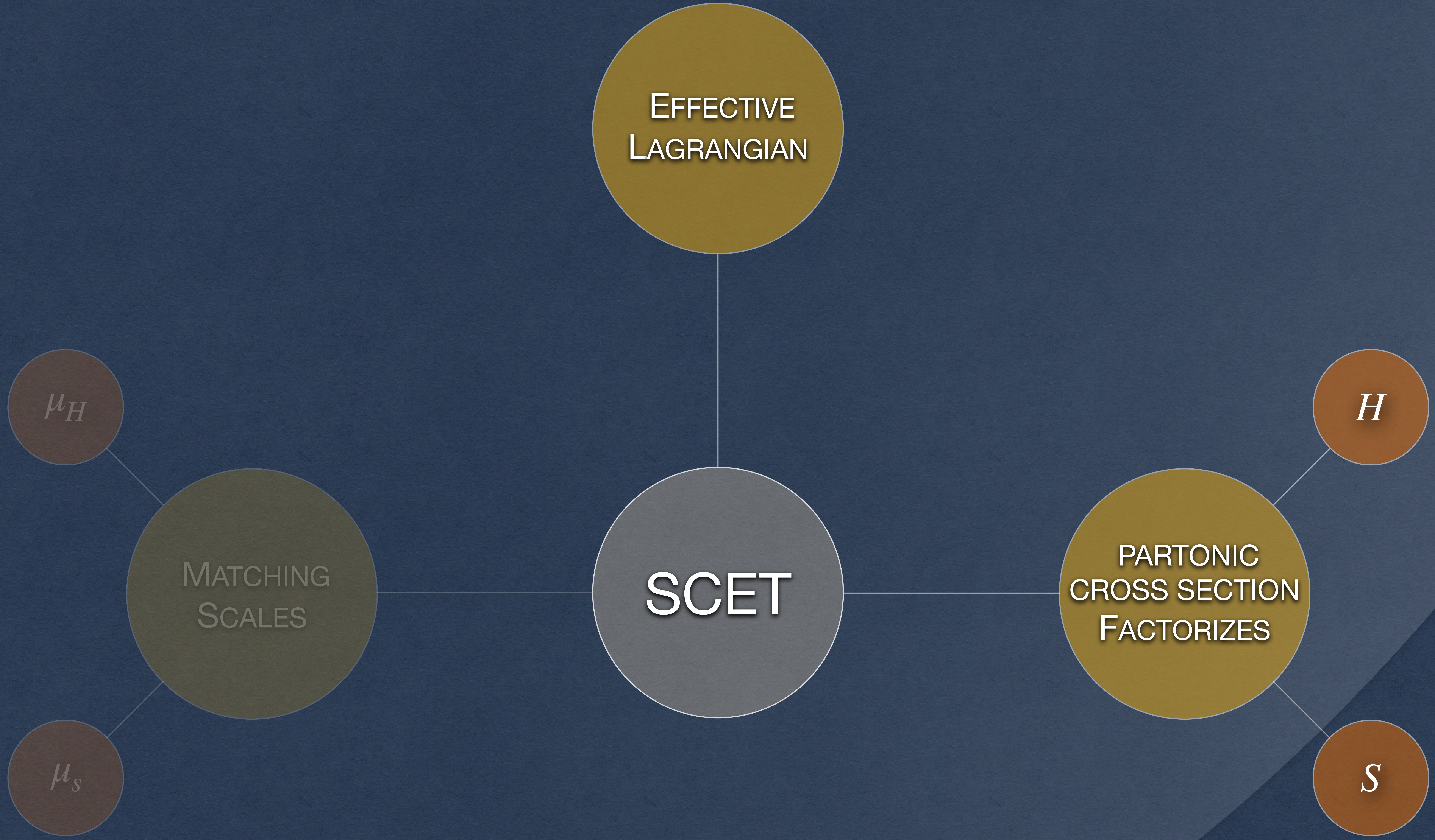

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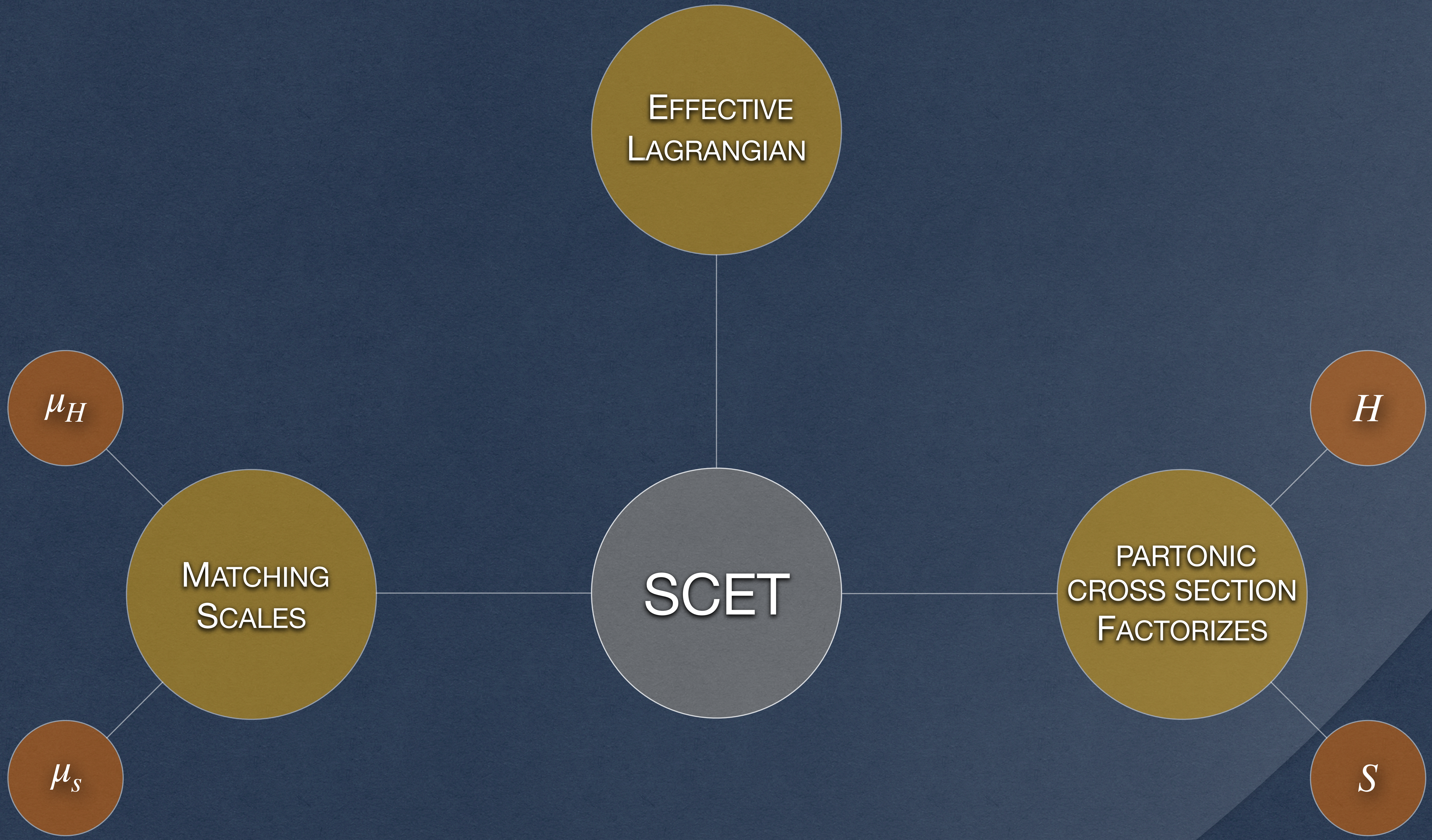

SCET ? dQCD

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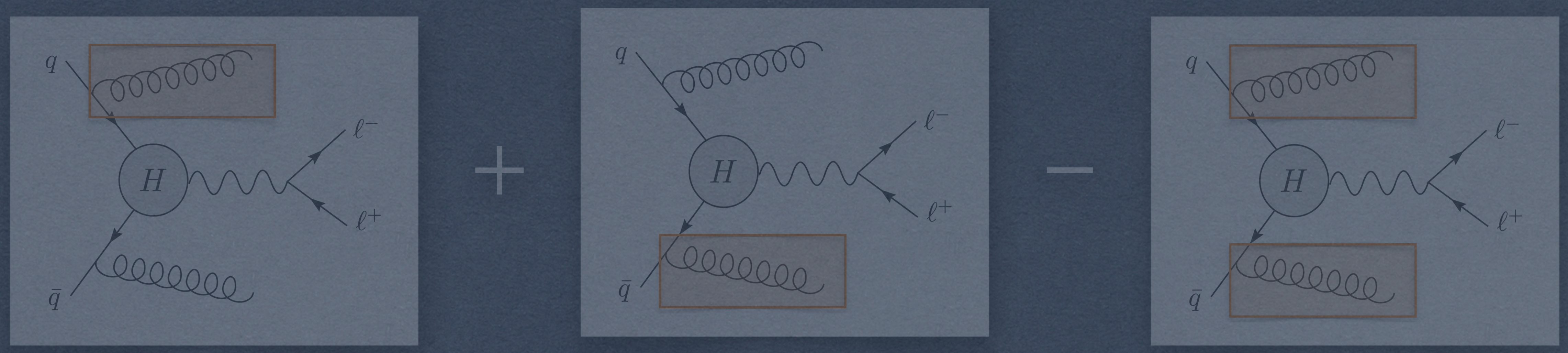
$$B := I \otimes f$$

$$\frac{d\sigma}{dM^2 dY} \propto f_i^{thr} \otimes \tilde{B}_j + \tilde{B}_i \otimes f_j^{thr} - S \otimes f_i^{thr} \otimes f_j^{thr}$$

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- second term

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$$N_{a,b} \to \infty$$

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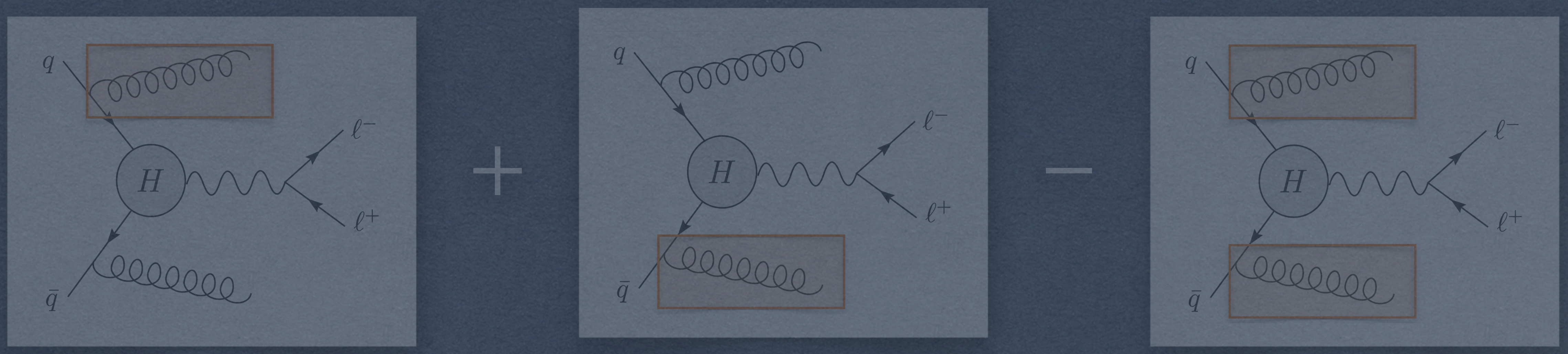


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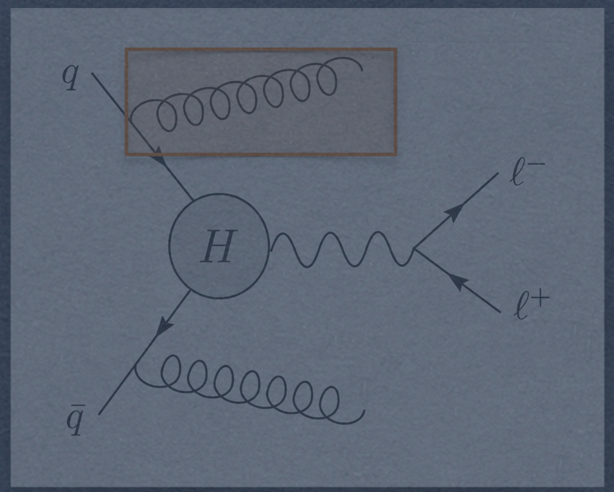


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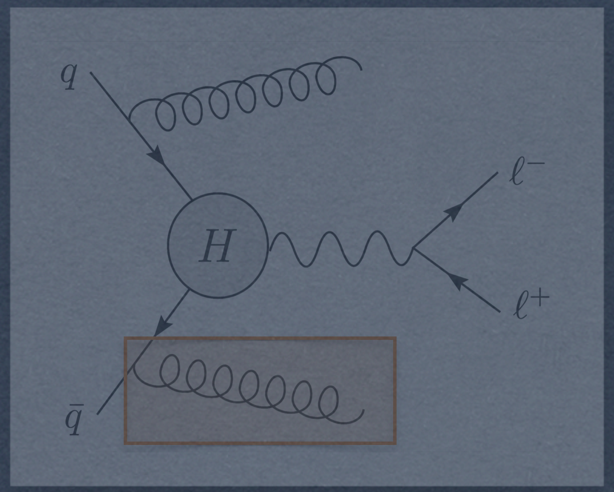


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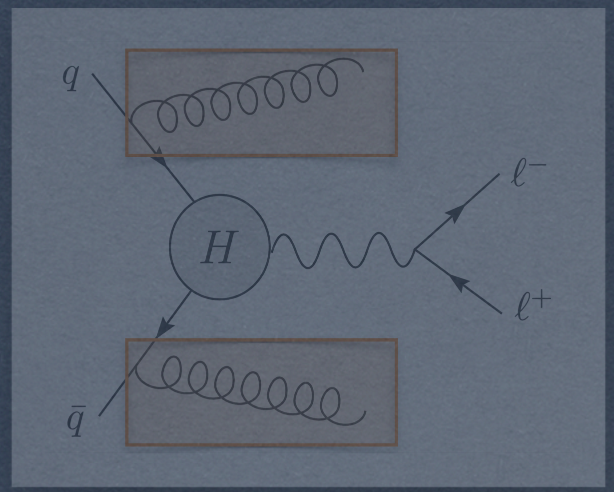


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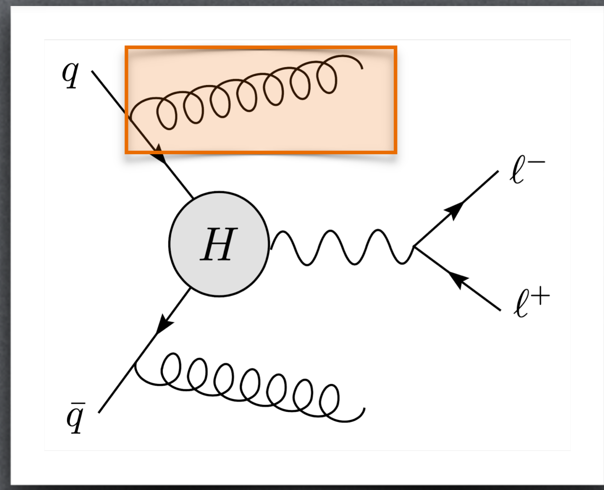


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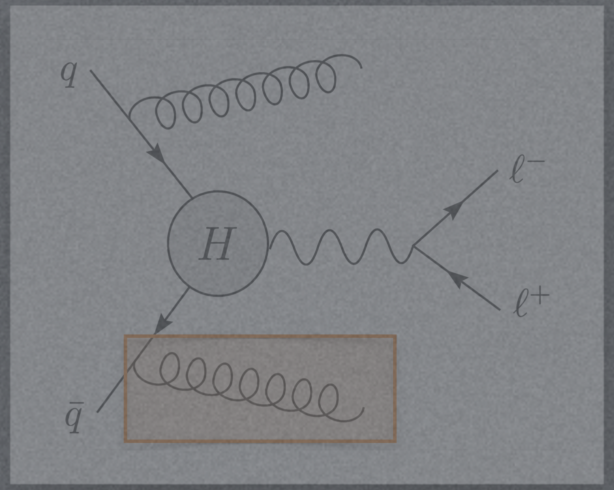


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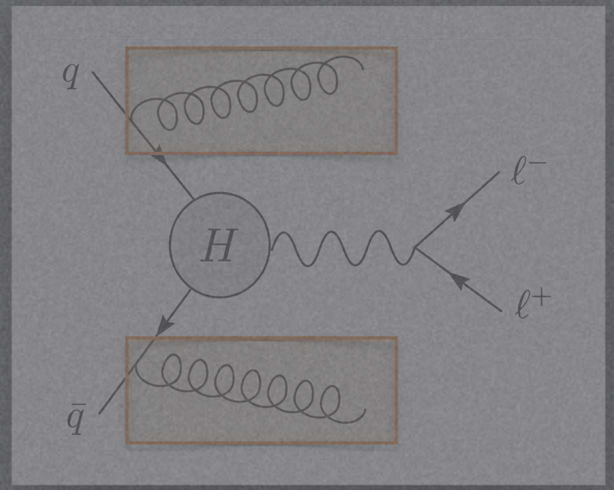


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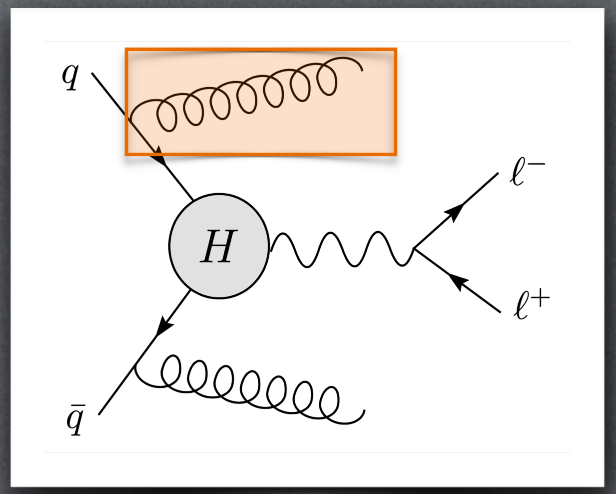


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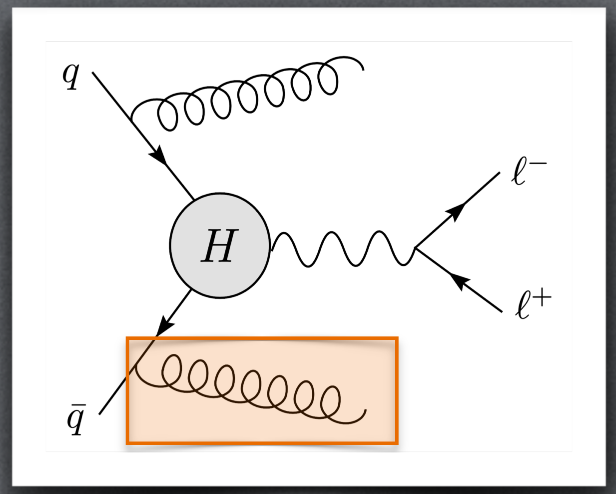


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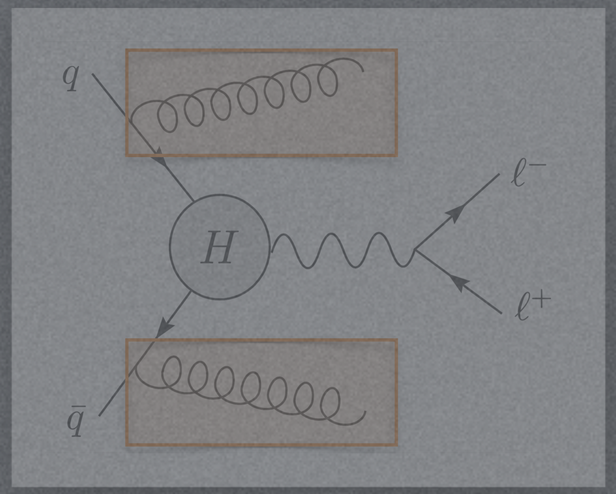


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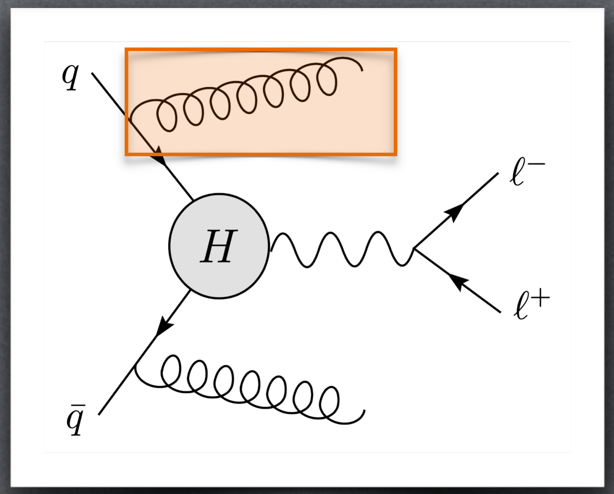


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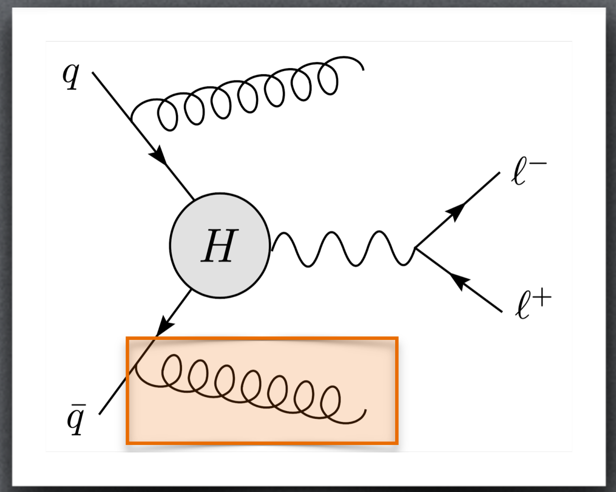


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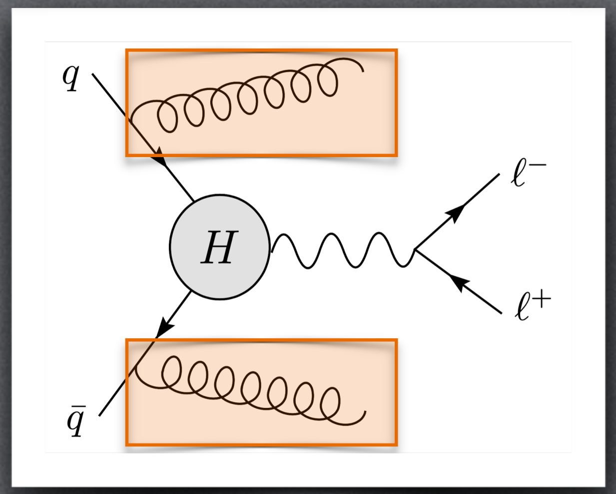


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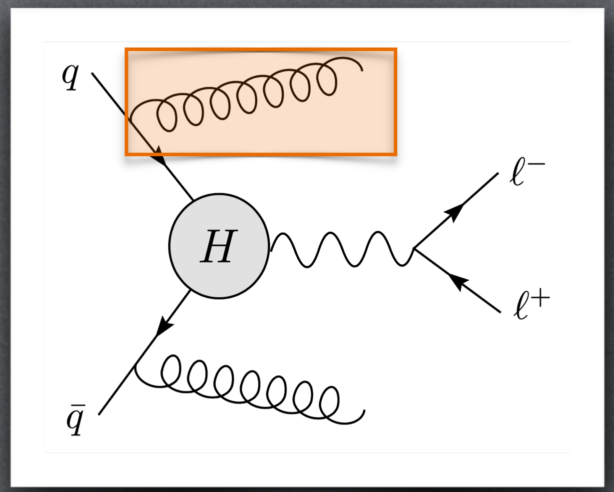


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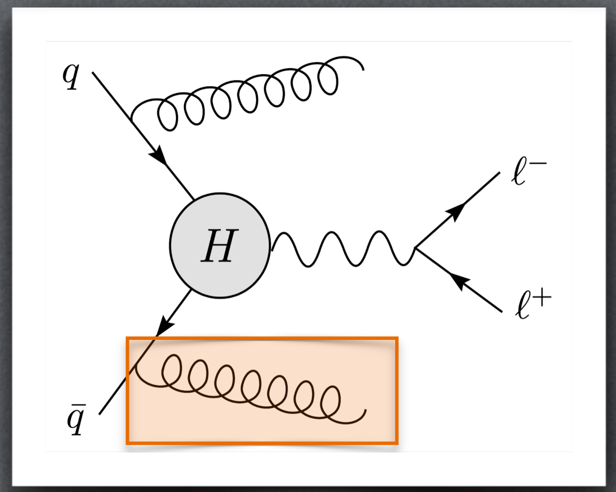


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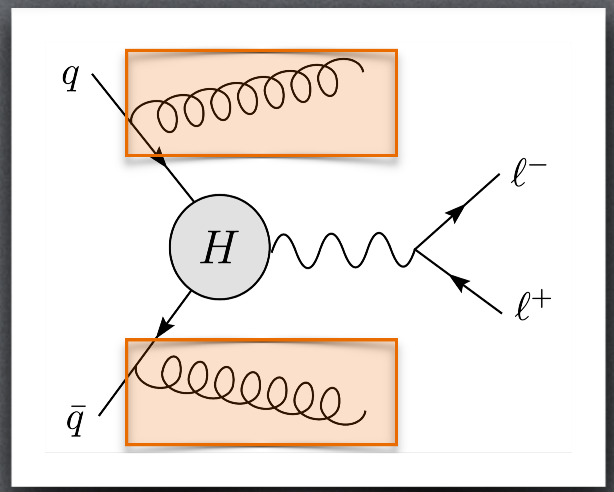


HOW TO WRITE THE LMT RESULT IN dQCD?

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Calculate the LMT Result in Mellin Space → Show the Form of the Extra Terms → Write the Final LMT Result in dQCD

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# LMT RESULT IN dQCD

$$\frac{d\sigma}{dQ^2dY} = H_{ij}\left[f_i^{thr} \otimes \tilde{B}_j + \tilde{B}_i \otimes f_j^{thr} - S \otimes f_i^{thr} \otimes f_j^{thr}\right]$$

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# LMT Result in Mellin Space

## LMT Result in Physical Space

$$\frac{d\sigma}{dM^2dY} = \boxed{H_{ij}\Big[f_i^{thr} \otimes \tilde{B}_j} + \tilde{B}_i \otimes f_j^{thr} - S \otimes f_i^{thr} \otimes f_j^{thr}\Big]$$

$$\frac{d\sigma}{dM^2dY} = H_{ij}(M,\mu)\int d\tilde{t}\, f_i^{thr}\left[x_a\left(1+\frac{\tilde{t}}{M^2},\mu\right)\right]\tilde{B}_j(\tilde{t},x_b,\mu)$$

$$\boxed{\begin{array}{c} N_a \to \infty \\ N_b \in \mathbb{C} \end{array}}$$

# LMT Result in Mellin Space

## RGE of the BEAM FUNCTION

$$\mu \frac{d}{d\mu} \tilde{B}_j(t, x_b, \mu) = \int_0^t \gamma_j^B(t - t', \mu) \tilde{B}_j(t', x_b, \mu)$$

$U_B(t, \mu_s, \mu) =$ EVOLUTION BEAM FUNC.

## Solution of the RGE

$$\tilde{B}_j(t, x_b, \mu) = \int_0^t dt' \boxed{U_B(t - t', \mu_s, \mu)} \tilde{B}_j(t', x_b, \mu_s)$$

# LMT Result in Mellin Space

## Expand $\tilde{I}_{jk}(t, z, \mu_s)$ in Series of $\alpha_s$

$$\tilde{B}_j(t, x_b, \mu_s) := \int_x^1 \frac{dz_b}{z_b} \boxed{\tilde{I}_{jk}(t, z_b, \mu_s)} f_k\left(\frac{x_b}{z_b}, \mu_s\right)$$

$$\tilde{I}_{jk}(t, z_b, \mu_s) := \sum_{n=0}^{\infty} \left[\frac{\alpha_s(\mu_s)}{4\pi}\right]^n \tilde{I}_{jk}^{(n)}(t, z_b, \mu_s) \rightarrow \tilde{I}_{jk} = \tilde{I}_{jk}^{(0)} + \frac{\alpha_s(\mu_s)}{4\pi}\tilde{I}_{jk}^{(1)}$$

# LMT RESULT IN MELLIN SPACE

## LMT RESULT IN MELLIN SPACE

$$\frac{d\sigma}{dQ^2 dY}(N_a, N_b) = \underbrace{\left[U_H(M, \mu_H, \mu) \cdot H_{ij}(M^2, \mu_H)\right]}_{\text{HARD CONTRIBUTION}} \cdot f_i\left(N_a, \mu\right) f_j\left(N_b, \boxed{\mu_s}\right) \cdot \underbrace{\left[U_B(M, \mu_s, \mu) \cdot \tilde{s}_{\mathrm{DY}}(N_a, N_b, \mu_s)\right]}_{\text{SOFT CONTRIBUTION}}$$

Each PDF must be evaluated at the **FACTORIZATION SCALE** $\mu$

# LMT Result in Mellin Space

## HOW THE SOFT EVOLUTION FUNCTION ARISES

- solve the DGLAP equation for the PDF

$$f(N_b, \mu_s) = U_{\mathrm{PDF}}(N_b, \mu_s, \mu) \cdot f(N_b, \mu)$$

- the soft contribution becomes

$$U_B(M, \mu_s, \mu) \cdot U_{\mathrm{PDF}}(N_b, \mu_s, \mu) \cdot \tilde{s}_{\mathrm{DY}}(N_a, N_b, \mu_s)$$

- note that :

$$\boxed{1 = U_{\mathrm{PDF}}(N_b \to \infty) \cdot U^{-1}_{\mathrm{PDF}}(N_b \to \infty)}$$

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$$U_B(M, \mu_s, \mu) \cdot U_{\text{PDF}}(N_b, \mu_s, \mu) \cdot \tilde{s}_{\text{DY}}(N_a, N_b, \mu_s)$$

$$\Longrightarrow \underbrace{\boxed{\left[U_B(M, \mu_s, \mu) \cdot U_{\text{PDF}}(N_b \to \infty, \mu_s, \mu)\right]}}_{\text{Soft Evolution Func. } U_s(M, \mu_s, \mu)} \cdot \underbrace{\left[U_{\text{PDF}}^{-1}(N_b \to \infty, \mu_s, \mu) \cdot U_{\text{PDF}}(N_b, \mu_s, \mu)\right] \cdot \tilde{s}_{\text{DY}}(N_a, N_b, \mu_s)}_{\text{ADDITIONAL CONTRIBUTION}}$$

# FINAL RESULT AT NNLL

$$\frac{d\sigma}{dM^2 dY} \propto \boxed{f_i^{thr} \otimes \tilde{B}_j} + \tilde{B}_i \otimes f_j^{thr} - S \otimes f_i^{thr} \otimes f_j^{thr}$$

## FINAL RESULT WITH $N_a \to \infty$, $N_b \in \mathbb{C}$

$$\frac{d\sigma}{dM^2 dY} \propto \exp\left\{ \int_{M^2}^{\mu_s^2} \frac{d\mu^2}{\mu^2} \left[ A^q[\alpha_s(\mu^2)] \left( \log \frac{1}{\bar{N}_a \bar{N}_b} - \log \frac{\mu^2}{M^2} \right) + \hat{D}_2 \alpha_s^2(\mu^2) \ \boxed{+F(N_b)\alpha_s^2(\mu_s^2)} \right] \right\}$$

$$\cdot \left[ U_{\rm PDF}^{-1}(N_b \to \infty, \mu_s, \mu) \cdot U_{\rm PDF}(N_b \in \mathbb{C}, \mu_s, \mu) \right]$$

COULD IT DISAPPEAR?

$$\tilde{s}_{\rm DY} = 1 + \frac{\alpha_s(\mu_s^2)}{2\pi} \left[ \text{constants} + P_{qq}^{(0)}(z_b) \log\left( \frac{M^2 \cdot F(z_a, z_b)}{\mu_s^2} \right) \right] \rightarrow \boxed{\mu_s^2 = M^2 \cdot F(z_a, z_b)}$$

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$$\cdot \left[ U_{\mathrm{PDF}}^{-1}(N_b \to \infty, \mu_s, \mu) \cdot U_{\mathrm{PDF}}(N_b, \mu_s, \mu) \right]$$

## WHAT HAPPENS IF BOTH $N_{a,b} \to \infty$?

$$\boxed{\mu_s^2 = \frac{M^2}{N_a N_b}}$$

$$\frac{d\sigma}{dM^2dY} \propto \exp\left\{ \int_{M^2}^{\frac{M^2}{N_a N_b}} \frac{d\mu^2}{\mu^2} \left[ A^q[\alpha_s(\mu^2)] \left( \log \frac{1}{\bar{N}_a \bar{N}_b} - \log \frac{\mu^2}{M^2} \right) + \hat{D}_2 \alpha_s^2(\mu^2) \right] \right\}$$

# CONCLUSIONS AND OUTLOOK

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- analytical study of the LMT result
- rewriting of the LMT result in Mellin space
- comparison with previous results known in literature, with explicit expression of the extra terms
- final expression of the LMT result in the dQCD formalism

## OUTLOOK

- find the general soft-scale $\mu_s$
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# THANKS FOR THE ATTENTION