

A fresh look at the **Nested Soft-Collinear** subtraction scheme: NNLO QCD corrections to N -gluon final state $q\bar{q}$ annihilation

Oxford - QCD-Lunch - 27/10/2023

Davide Maria Tagliabue

In collaboration with:

[F. Devoto, K. Melnikov, R. Röntsch, C. Signorile-Signorile, 2310.17598]



UNIVERSITÀ
DEGLI STUDI
DI MILANO

PROBLEMS AND SOLUTIONS



Two main difficulties: **IR singularities**, arising from real and virtual radiation, and **multi-loop amplitude** calculations



About IR singularities: they are unphysical and require specific methods to arrive at a finite physical result. Among those methods, we focus on **SUBTRACTION SCHEMES**



Some of the many available schemes:

Analytic Sector Subtraction [Magnea et al. 1806.09570, ...]

Antenna [Gehrmann-De Ridder et al. 0505111, ...]

ColorfullNNLO [Del Duca et al. 1603.08927, ...]

STRIPPER [Czakon 1005.0274, ...]

Geometric IR subtraction [Herzog 1804.07949, ...]

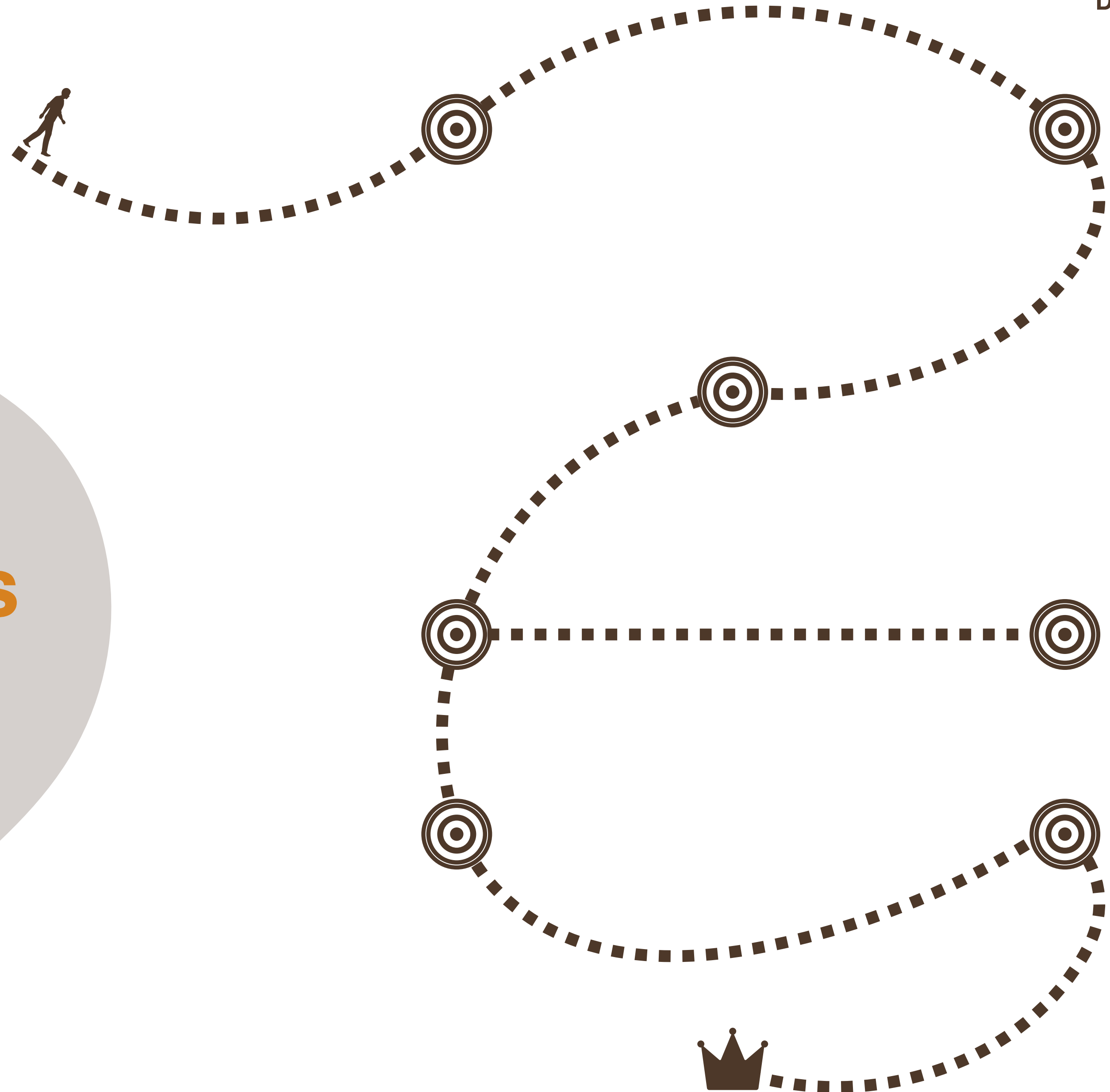
Unsubtraction [Sborlini et al. 1608.01584, ...]

Universal Factorization [Anastasiou et al. 2008.12293, ...]

FDR [Pittau 1208.5457, ...]

Nested Soft-Collinear Subtraction (NSC) [Caola et al. 1702.01352, ...]

WHY WE STUDY
 $P + P \rightarrow X + N \text{ gluons}$
AT NNLO

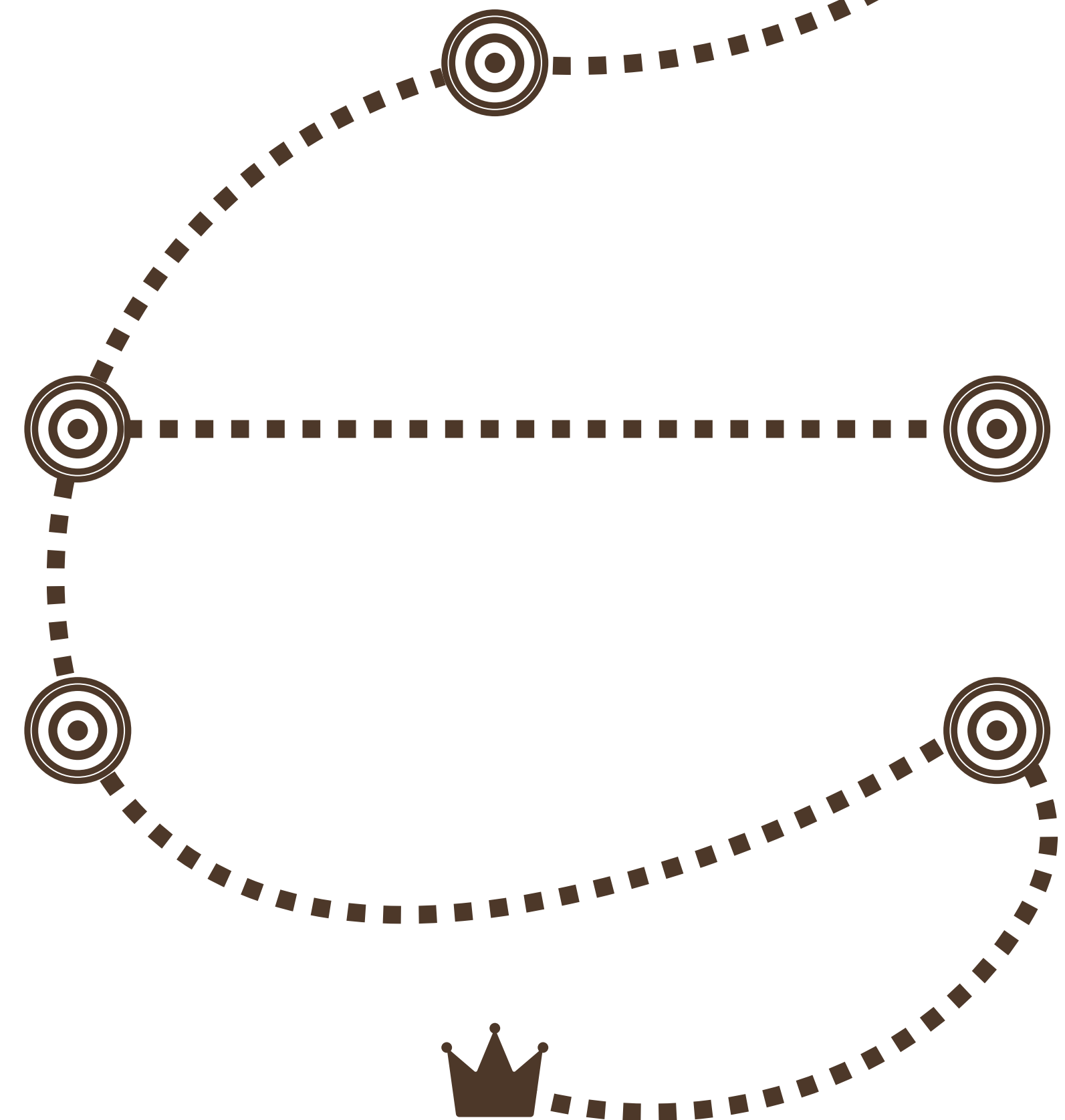


Computing cross sections in
full generality at **NNLO** is
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$P + P \rightarrow X + N$ gluons

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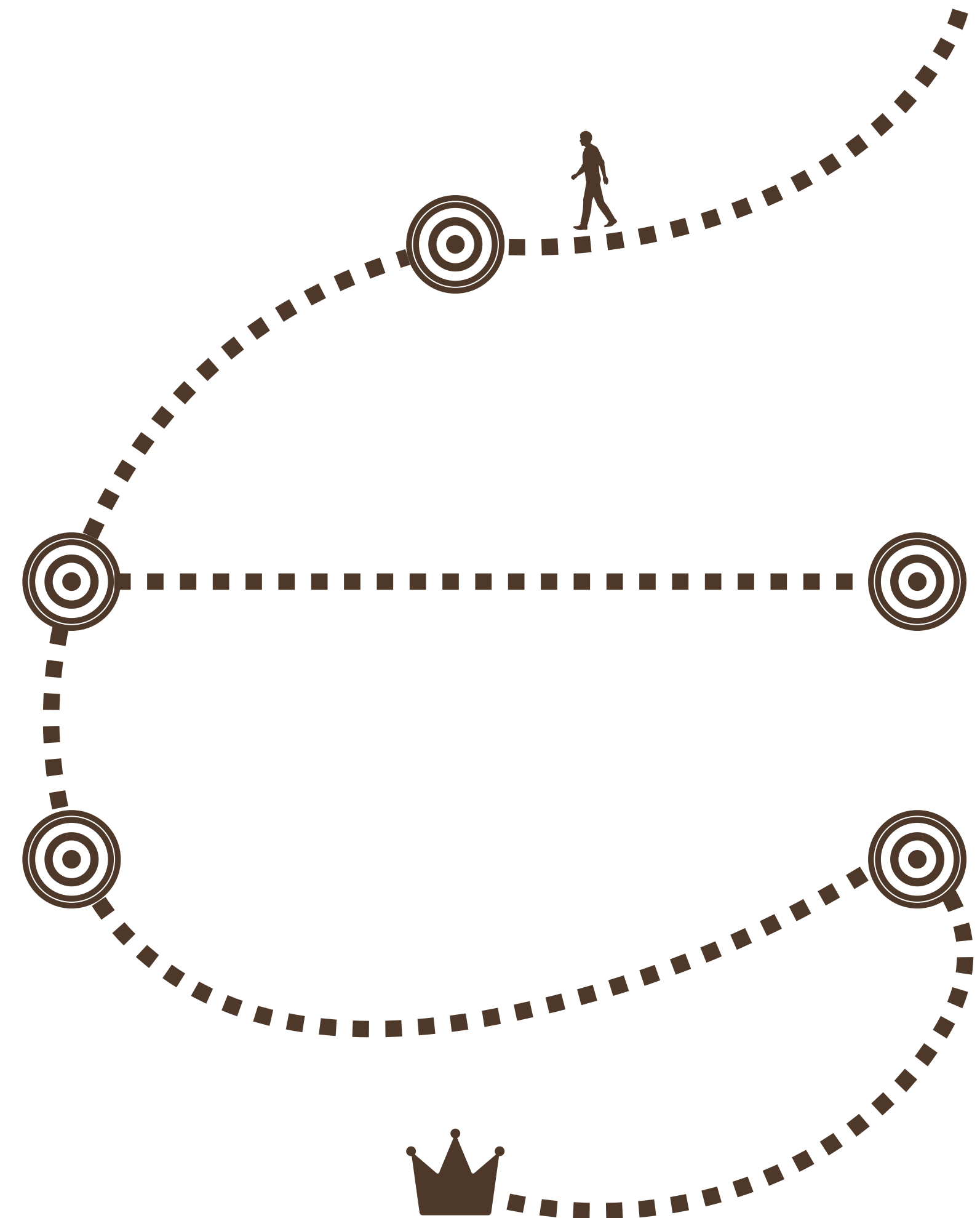
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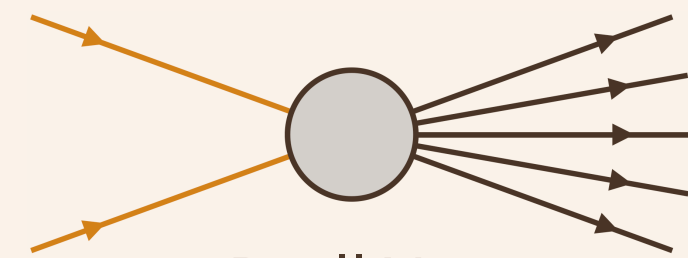
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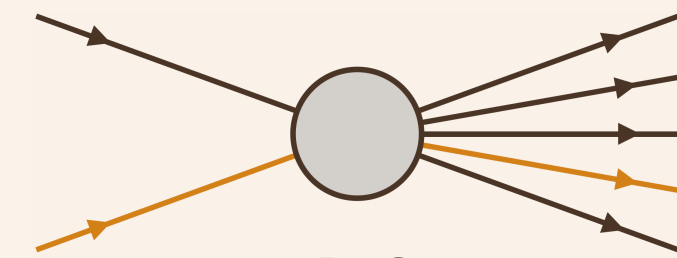
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Simple = limited number of hard partons



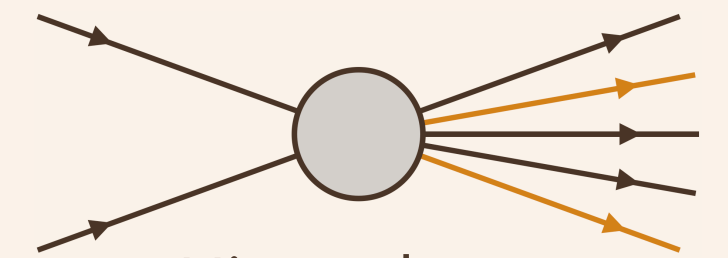
Drell-Yan

[Caola, Melnikov, Rötsch '19]



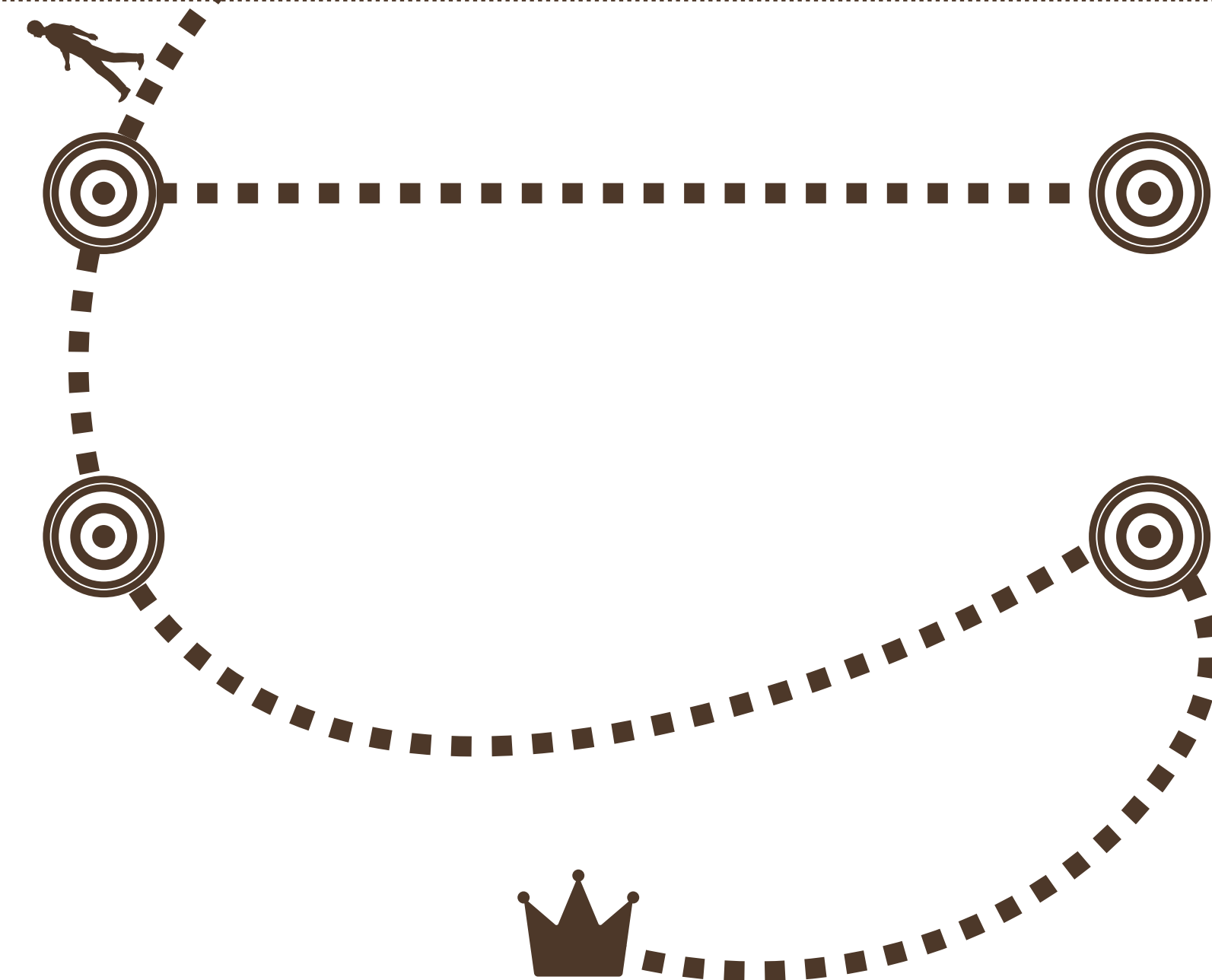
DIS

[Asteriadis, Caola, Melnikov, Rötsch '19]



Higgs decay

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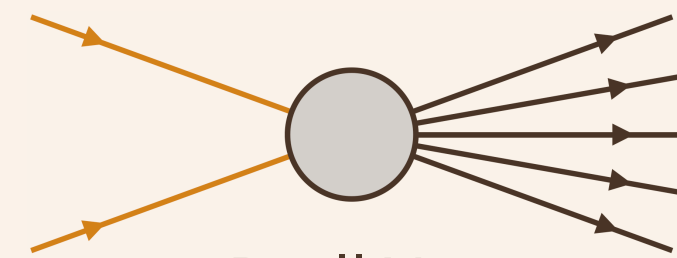
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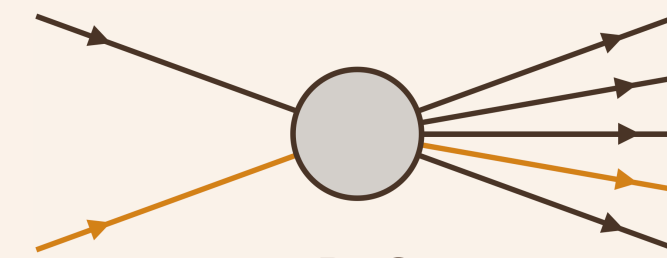
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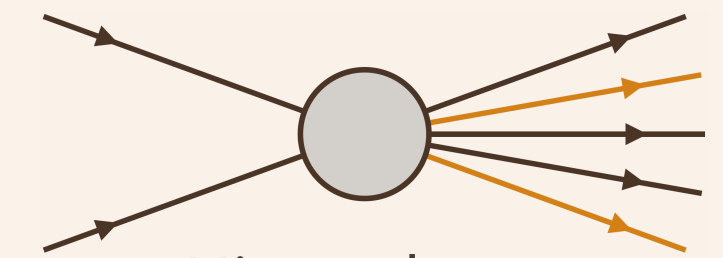
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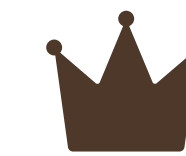


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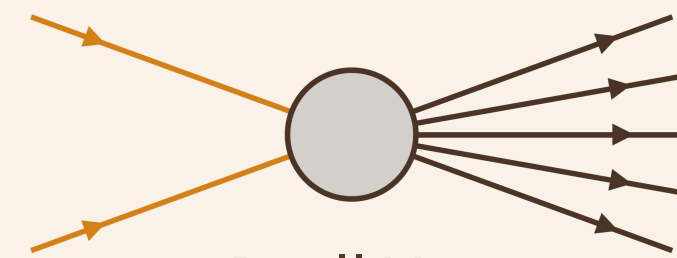
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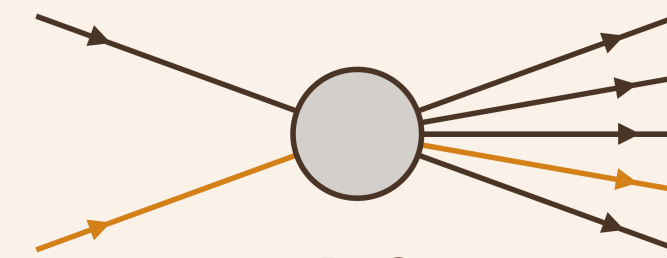
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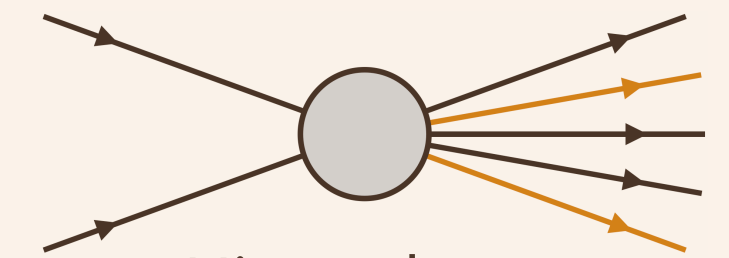
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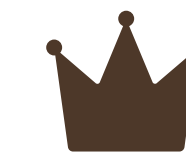
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$P + P \rightarrow X + N \text{ Jets}$

$N \rightarrow 3$

[Czakon et al. '21]



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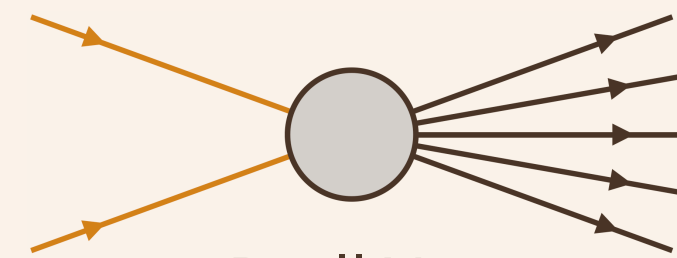
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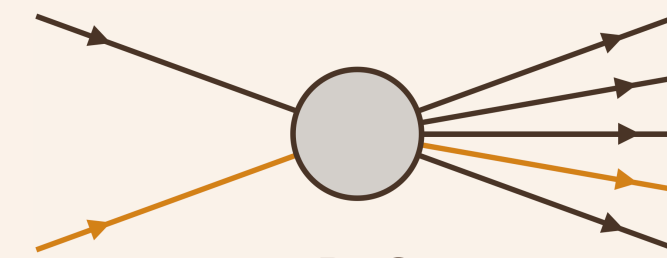
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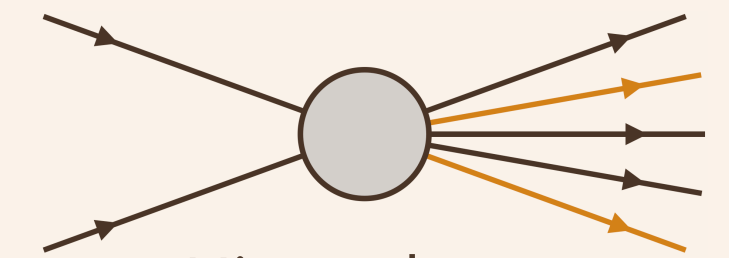
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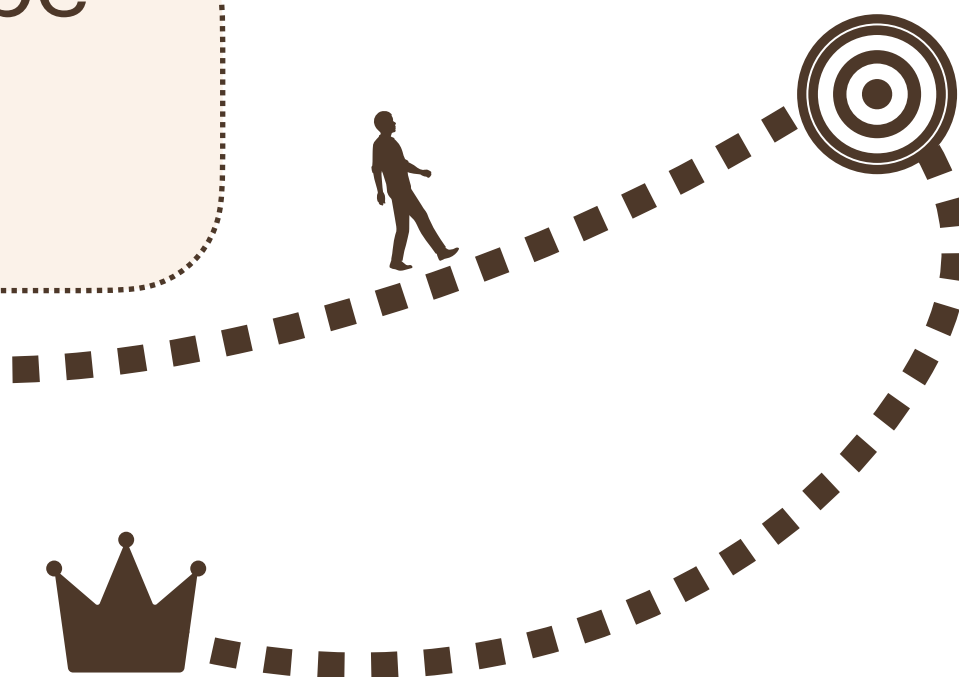
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What is a good prototype of the problem?

$P + P \rightarrow X + N \text{ gluons}$



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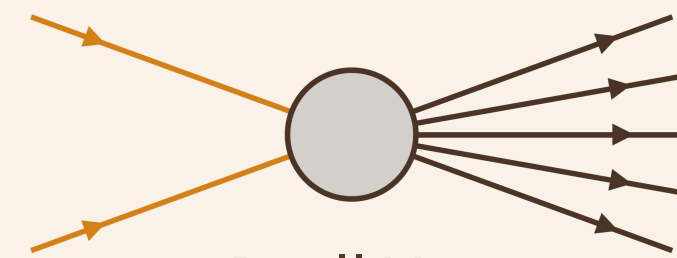
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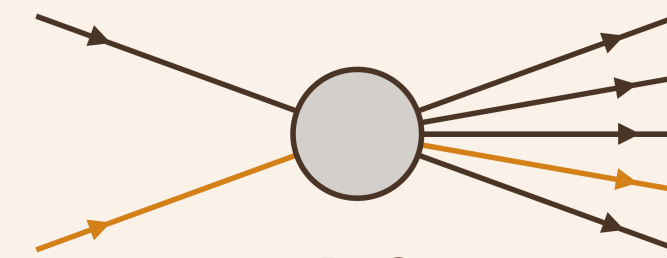
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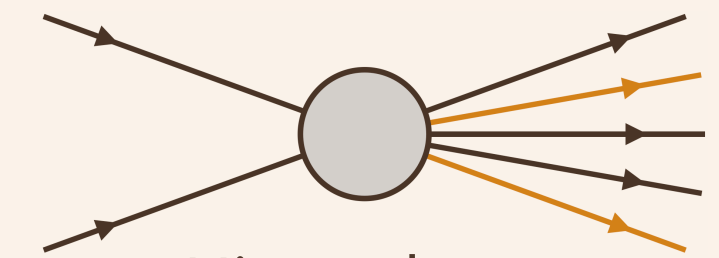
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Remaining bottleneck?

double-loop
amplitudes



WHY WE STUDY $P + P \rightarrow X + N \text{ gluons}$ AT NNLO

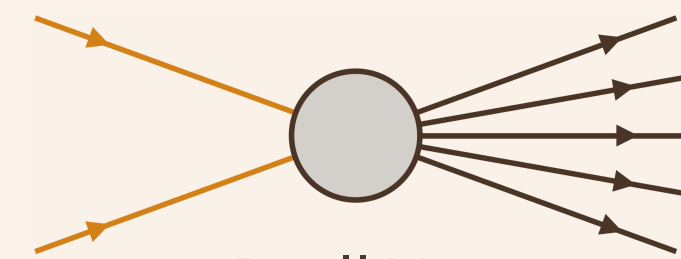
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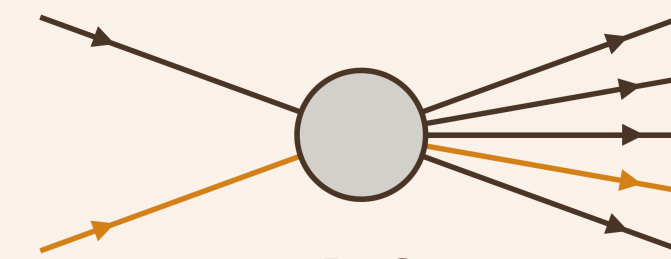
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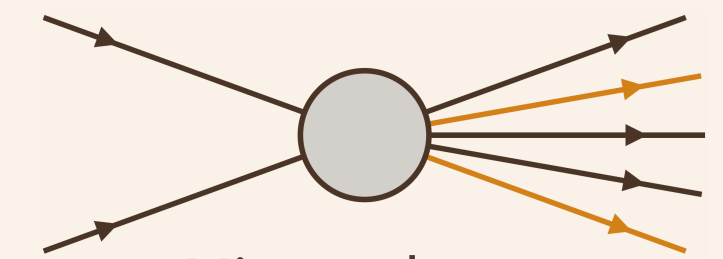
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double-loop
amplitudes

<< If someone gives me the finite part of the double-loop amplitude of any kind of process, then I can give back the analytical expression of the integrated subtraction terms. >>

$$\int \mathcal{M}^2 F_J d^{(d)}\phi = \int [\mathcal{M}^2 F_J - K] d^{(d)}\phi + \int K d^{(d)}\phi$$

👑 fully **local**

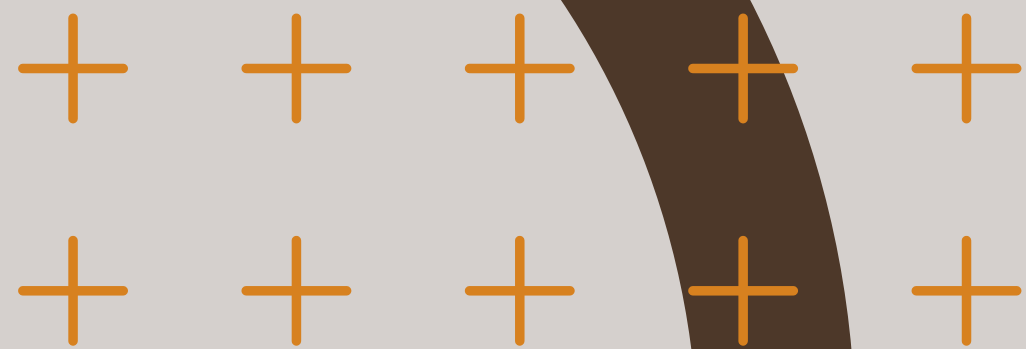
👑 fully **analytic**

Problem of **OVERLAPPING SOFT** and **COLLINEAR** emissions

damping factors $\Delta^{(i)}$ \implies tell which parton is unresolved

partition functions ω^{ij} \implies select the proper collinear limit

HOW THE NSC WORKS?



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HOW THE NSC WORKS?

At **NLO** we start by regularizing soft divergences

$$\left| \text{diagram} \right|^2 = (1 - S) \left| \text{diagram} \right|^2 + S \left| \text{diagram} \right|^2$$

Soft-regulated
still contains collinear
divergences

Soft-counterterm
provides the formula
of the soft poles

The **soft-regulated** term then needs a similar treatment for **collinear divergences**: all the singular configurations can be separated out

$$\int \mathcal{M}^2 F_J d^{(d)}\phi = \int [\mathcal{M}^2 F_J - K] d^{(d)}\phi + \int K d^{(d)}\phi$$

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Problem of **OVERLAPPING SOFT** and **COLLINEAR** emissions

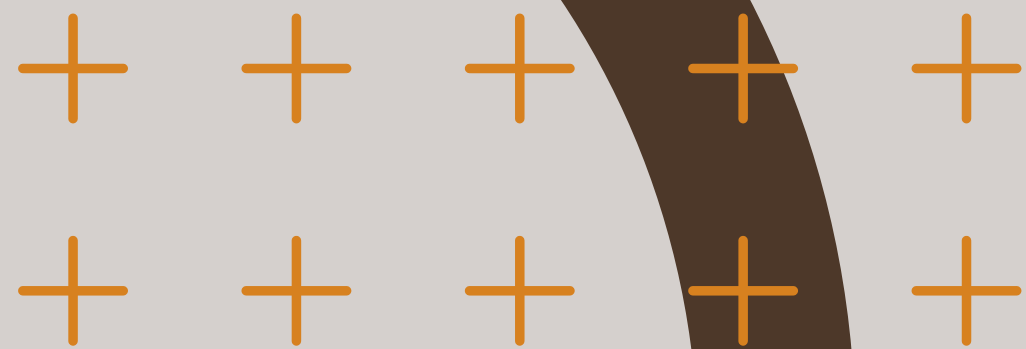
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At **NNLO** we follow the same idea of **separating out divergences**

- start from **double-soft** regularization
 - regularize also **single-soft** divergences
 - at this point we have to regularize **collinear** divergences
 $(C_{im}, C_{jn}C_{im}, C_{imn}) \implies$ we avoid overlapping thanks to
PARTITIONING and **SECTORING**
- } The cross section is now soft-regularized

HOW THE NSC WORKS?



RECURRING OPERATORS AT NLO



Virtual corrections $d\hat{\sigma}^V$: the IR content of virtual amplitudes is known. Through the operator

$$\bar{I}_1(\epsilon) = \frac{1}{2} \sum_{i \neq j}^{N_p} \frac{\mathcal{V}_i^{\text{sing}(\epsilon)}}{T_i^2} (T_i \cdot T_j) \left(-\frac{\mu^2}{s_{ij}} \right)^\epsilon$$

$$\mathcal{V}_i^{\text{sing}(\epsilon)} = \frac{T_i^2}{\epsilon^2} + \frac{\gamma_i}{\epsilon}$$

$$N_p = N + 2$$

the divergent part of $d\hat{\sigma}^V$ can be written as

$$I_V(\epsilon) = \bar{I}_1(\epsilon) + \bar{I}_1^\dagger(\epsilon)$$

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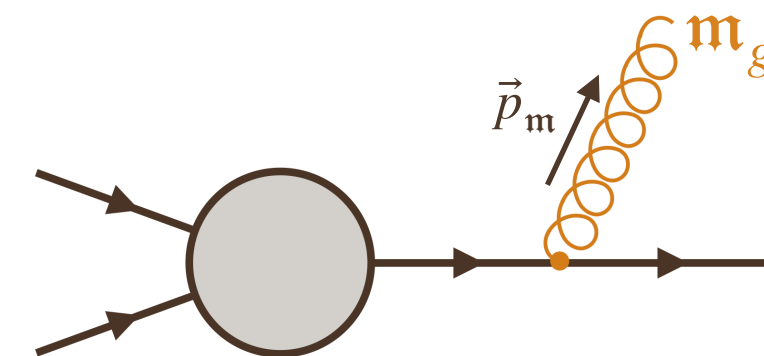
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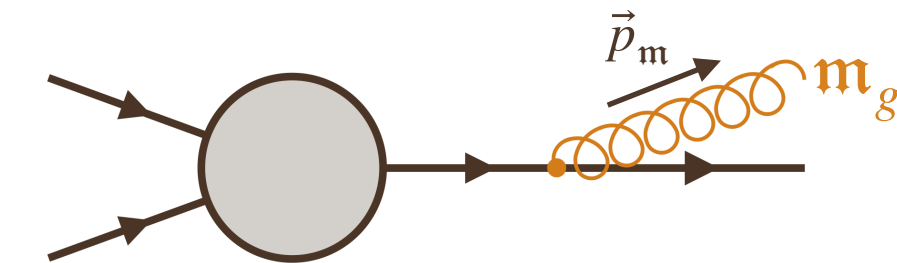
$$I_V(\epsilon) = \bar{I}_1(\epsilon) + \bar{I}_1^\dagger(\epsilon)$$



Real corrections $d\hat{\sigma}^R$: we would like something similar



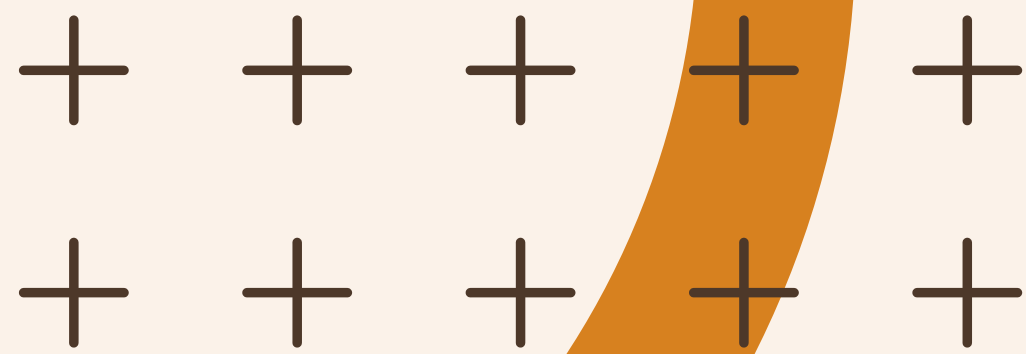
Soft emission
 $S_m: \vec{p}_m \rightarrow 0$



Collinear emission
 $C_{im}: \theta_{im} \rightarrow 0$

Making use of **NSC** scheme to regularize this divergences we obtain [Caola, Melnikov, Rönsch '17]

$$d\hat{\sigma}^R = \underbrace{\langle S_m F_{LM}(\mathbf{m}) \rangle}_{\text{Soft term } [S_m: E_m \rightarrow 0]} + \sum_{i=1}^{N_p} \underbrace{\langle \bar{S}_m C_{im} \Delta^{(m)} F_{LM}(\mathbf{m}) \rangle}_{\text{Hard-Collinear term } [C_{im}: \theta_{im} \rightarrow 0]} + \langle \mathcal{O}_{\text{NLO}} \Delta^{(m)} F_{LM}(\mathbf{m}) \rangle$$



RECURRING OPERATORS AT **NLO**



It turns out that the **soft term** can be written by means of an **operator** that, at least in principle, is very **close to** $I_V(\epsilon)$:

$$I_S(\epsilon) = - \frac{(2E_{\text{max}}/\mu)^{-2\epsilon}}{\epsilon^2} \sum_{i \neq j}^{N_p} \eta_{ij}^{-\epsilon} K_{ij} (T_i \cdot T_j)$$

$$\eta_{ij} = (1 - \cos \theta_{ij})/2$$

$$K_{ij} = \frac{\Gamma^2(1 - \epsilon)}{\Gamma(1 - 2\epsilon)} \eta_{ij}^{1+\epsilon} {}_2F_1(1, 1, 1 - \epsilon, 1 - \eta_{ij})$$

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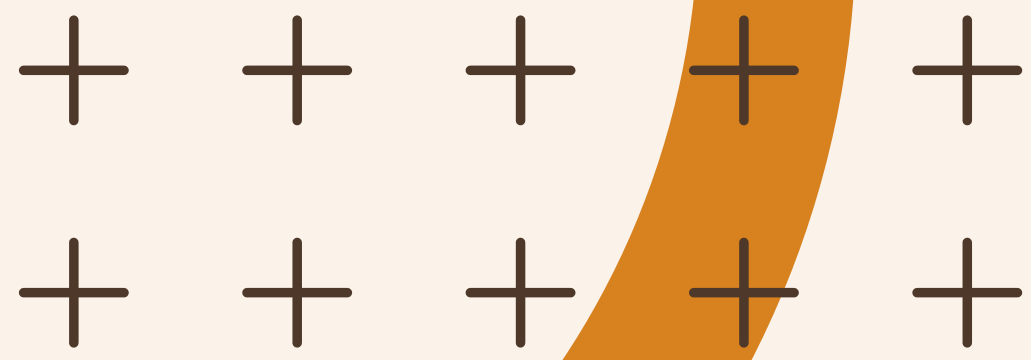
Combination of $I_V(\epsilon) + I_S(\epsilon)$:

$$I_V(\epsilon) + I_S(\epsilon) = - \sum_{i=1}^{N_p} \frac{1}{\epsilon} \left(2T_i^2 L_i + \gamma_i \right) + \mathcal{O}(\epsilon^0)$$

$$\begin{aligned} L_i &= \log(E_{\max}/E_i) \\ \gamma_q &= 3/2 C_F \\ \gamma_g &= \beta_0 \end{aligned}$$

- the pole of $\mathcal{O}(\epsilon^{-2})$ **vanishes**
- has no **color correlations** at $\mathcal{O}(\epsilon^{-1})$
- **trivially dependent on** the number of hard partons N_p

THERE STILL IS A MISSING INGREDIENT



RECURRING OPERATORS AT NLO

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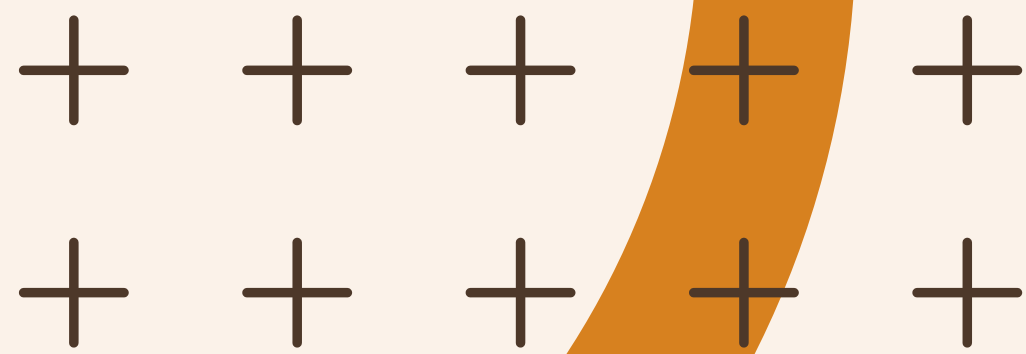


Last ingredient: **hard-collinear term**. Some parts vanish against the DGLAP contribution, the remaining one **can be collected** within the **COLLINEAR OPERATOR**

$$I_C(\epsilon) = \sum_{i=1}^{N_p} \frac{\Gamma_{i,f_i}}{\epsilon}$$

$$\Gamma_{i,f_i} = [\text{irrelevant prefactor}] \times \left[T_i^2 \frac{1 - e^{-2\epsilon L_i}}{\epsilon} + \gamma_i \right] \quad i \in \{1, 2\}$$

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RECURRING OPERATORS AT NLO

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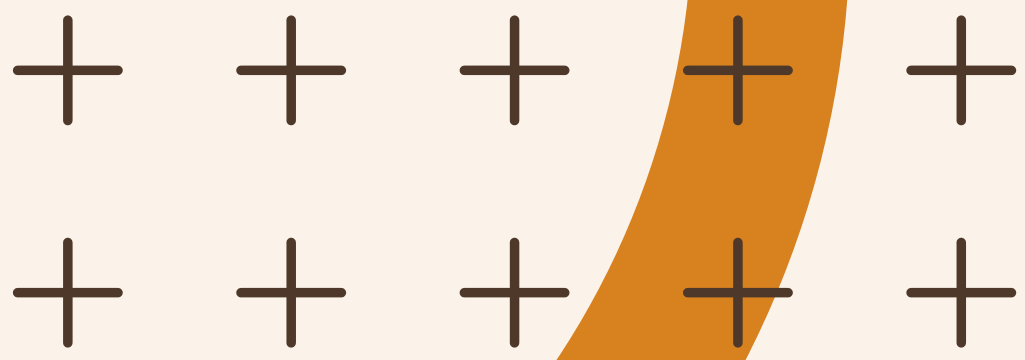
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$I_C(\epsilon)$ cancels perfectly the pole of $\mathcal{O}(\epsilon^{-1})$ left by $I_V(\epsilon) + I_S(\epsilon)$. It is thus natural to introduce the **total operator**

$$I_T(\epsilon) = I_V(\epsilon) + I_S(\epsilon) + I_C(\epsilon)$$

👑 pole free

👑 fully general w.r.t. N_p

RECURRING OPERATORS AT NLO

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$$I_C(\epsilon) = \sum_{i=1}^{N_p} \frac{1}{\epsilon} \left(2T_i^2 L_i + \gamma_i \right) + \mathcal{O}(\epsilon^0)$$



In this way the **final result for the NLO fits in a line**:

$$d\hat{\sigma}^{\text{NLO}} = [\alpha_s] \langle I_T(\epsilon) \cdot F_{\text{LM}} \rangle + [\alpha_s] \left[\langle P_{aa}^{\text{NLO}} \otimes F_{\text{LM}} \rangle + \langle F_{\text{LM}} \otimes P_{bb}^{\text{NLO}} \rangle \right] + \langle F_{\text{LV}}^{\text{fin}} \rangle + \langle \mathcal{O}_{\text{NLO}} \Delta^{(\mathbf{m})} F_{\text{LM}}(\mathbf{m}) \rangle$$

[FKS, Devoto, Melnikov, Rönsch, Signorile-Signorile, **D.M.T.**, 2310.17598]

WHAT HAPPENS AT NNLO?

$$d\hat{\sigma}^{\text{NNLO}} = d\hat{\sigma}^{\text{VV}} + d\hat{\sigma}^{\text{RV}} + d\hat{\sigma}^{\text{RR}} + d\hat{\sigma}^{\text{pdf}}$$

Double-Virtual
Real-Virtual
Double-Real
PDFs Renor.

Consider for instance $d\hat{\sigma}^{\text{VV}} \Rightarrow$ it depends **quadratically** on $\bar{I}_1(\epsilon)$ and $\bar{I}_1^\dagger(\epsilon)$

\Rightarrow Identity Operator

$$\Rightarrow \bar{I}_1, \bar{I}_1^\dagger \sim T_i \cdot T_j$$

$$\Rightarrow \bar{I}_1^2 \sim (T_i \cdot T_j) \cdot (T_k \cdot T_l) \quad \text{Double color-correlations.}$$

$$\Rightarrow [\bar{I}_1, \bar{I}_1^\dagger] \sim f_{abc} T_k^a T_i^b T_j^c \quad \text{Tripoles.}$$

We expect the **same** to happen for $d\hat{\sigma}^{\text{RV}}$ and $d\hat{\sigma}^{\text{RR}}$

First Goal: **isolate DCC** in $d\hat{\sigma}^{\text{RV}}$ and $d\hat{\sigma}^{\text{RR}}$ and **combine** them with **those** contained within $d\hat{\sigma}^{\text{VV}}$

The Strategy: **assemble** all these **DCC** into an expression that we expect to be **quadratic** in $I_T(\epsilon)$

WHAT HAPPENS AT NNLO?

Here it is what we find [Devoto, Melnikov, Röntsch, Signorile-Signorile, **D.M.T.**, 2310.17598]

$$Y_{VV} = \frac{[\alpha_s]^2}{2} \langle M_0 | \bar{I}_1^2 + (\bar{I}_1^\dagger)^2 + 2\bar{I}_1^\dagger \bar{I}_1 | M_0 \rangle + \dots$$

$$Y_{RR}^{(ss)} = \frac{[\alpha_s]^2}{2} \langle M_0 | I_S^2 | M_0 \rangle + \dots$$

$$Y_{RR}^{(shc)} = [\alpha_s]^2 \langle M_0 | I_S I_C | M_0 \rangle + \dots$$

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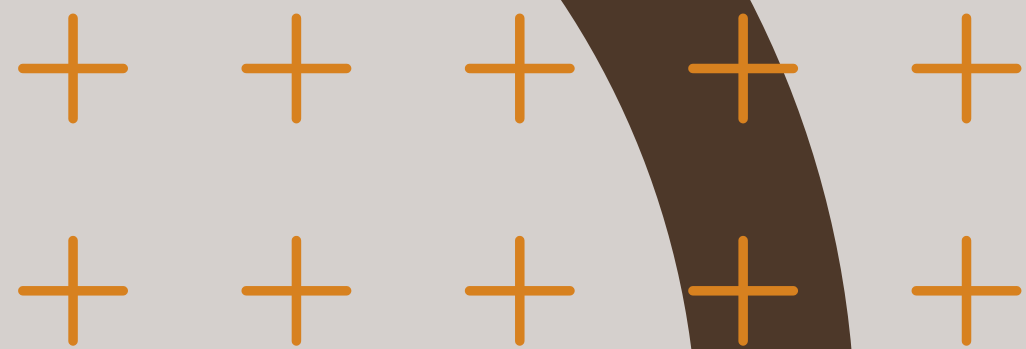
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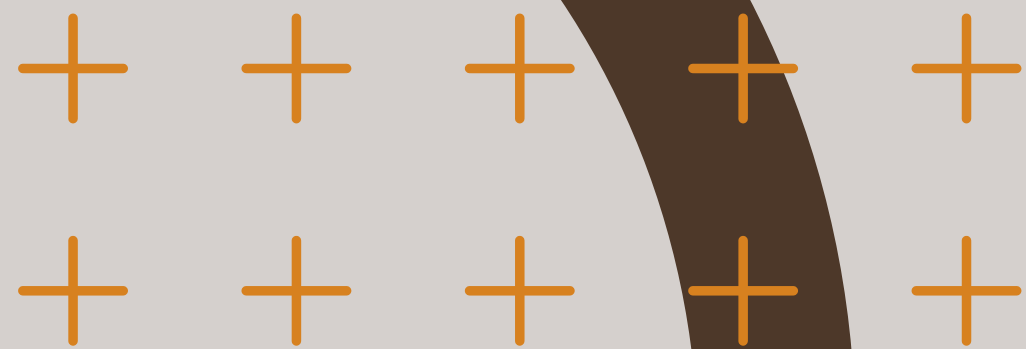
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Once combined, these objects return

NB square of NLO

$$Y = \frac{[\alpha_s]^2}{2} \langle M_0 | [I_V + I_S + I_C]^2 | M_0 \rangle + \dots \equiv \frac{[\alpha_s]^2}{2} \langle M_0 | I_T^2 | M_0 \rangle + \dots$$

WHAT HAPPENS AT NNLO?

The benefits of introducing these Catani-like operators:

-  the problem of **double color-correlated poles disappear**, since everything is written in terms of $I_T^2(\epsilon)$, which is $\mathcal{O}(\epsilon^0)$
-  the **definition** of $I_T(\epsilon)$ depends trivially on N_p so the result we got is **fully general w.r.t. the number of final state gluons**
-  We **do not explicitly calculate** the individual sub-blocks of the process. Instead, we write each of these in terms of $I_V(\epsilon)$, $I_S(\epsilon)$ and $I_C(\epsilon)$, then recombine them to get $I_T(\epsilon)$. The **cancellation of the poles** takes place **automatically**






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?

TRIPOLE-POLES known in the literature (for $N_{\text{jet}} \geq 2$):

From $d\hat{\sigma}^{\text{VV}}$

$$H_2(\epsilon) = \frac{if_{abc}}{384\epsilon} (\gamma_0^{\text{cusp}})^2 \sum_{(i,j,k)}^{N_p} T_i^a T_j^b T_k^c \log \frac{-s_{ij}}{-s_{jk}} \log \frac{-s_{jk}}{-s_{ki}} \log \frac{-s_{ki}}{-s_{ij}} \\ - \frac{if_{abc}}{128\epsilon} \gamma_0^{\text{cusp}} \sum_{(i,j,k)}^{N_p} T_i^a T_j^b T_k^c \left(\frac{\gamma_0^i}{C_{f_i}} - \frac{\gamma_0^j}{C_{f_j}} \right) \log \frac{-s_{ij}}{-s_{jk}} \log \frac{-s_{ki}}{-s_{ij}} \\ + \frac{\Gamma_1}{16\epsilon} - \frac{\gamma_1^{\text{cusp}} \Gamma_0}{64\epsilon} - \frac{\pi^2 \beta_0 \Gamma'_0}{128\epsilon}$$

From $d\hat{\sigma}^{\text{RV}}$

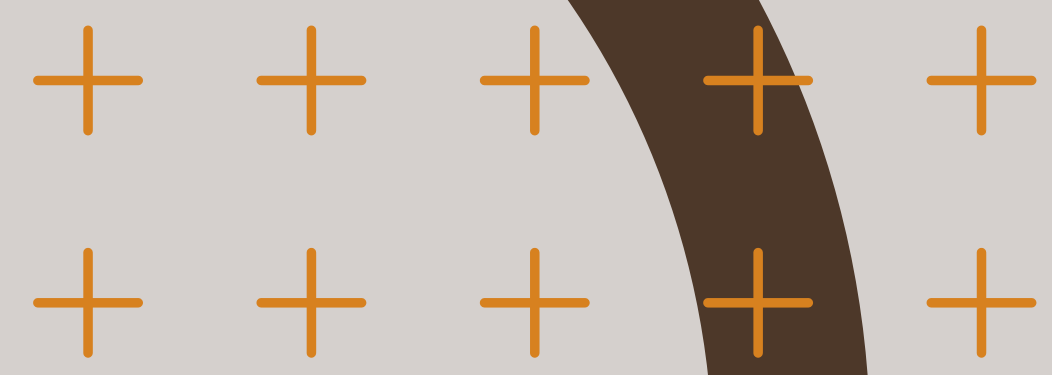
$$S_{\text{m}}^{\text{tri RV}} \sim \sum_{(i,j,k)} \frac{s_{ij}}{s_{im}s_{jm}} \left(\frac{s_{jk}}{s_{jm}s_{km}} \right)^\epsilon T_i^a T_j^b T_k^c \\ \mathcal{O}(\epsilon^{-2}) \\ \ominus \\ \mathcal{O}(\epsilon^{-1})$$

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Once combined, these objects return

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$$\mathcal{O}(\epsilon^{-2})$$



$$\mathcal{O}(\epsilon^{-1})$$

Need to add **other contributions**. But **where** do they come from?

If $N_{\text{jet}} \geq 2$

$$[\bar{I}_1, \bar{I}_1^\dagger] \neq 0$$

$$[\bar{I}_1^\dagger, \bar{I}_S] \neq 0 \rightarrow f_{abc} T_i^a T_j^b T_k^c$$

$$[\bar{I}_1, \bar{I}_S] \neq 0$$

Combining the commutators

$$I^{\text{tri}} = \frac{1}{2} [I_V + I_S, \bar{I}_1 - \bar{I}_1^\dagger] - \frac{1}{4} [I_V, \bar{I}_1 - \bar{I}_1^\dagger]$$

Once combined with the other triples, this **cancels out** all the **triple-poles**

$$Y = \frac{[\alpha_s]^2}{2} \langle M_0 | [I_V + I_S + I_C]^2 | M_0 \rangle + \dots \equiv \frac{[\alpha_s]^2}{2} \langle M_0 | I_{\text{T}}^2 | M_0 \rangle + \dots$$

CONCLUSIONS AND OUTLOOK

- 1 We find **recurring building blocks**, i.e. $I_V(\epsilon)$, $I_S(\epsilon)$, $I_C(\epsilon)$ and $I_T(\epsilon)$, which let us solve the problem of color-correlated poles
- 2 The **procedure** is (almost) entirely **process independent**
- 3 The cancellation of the poles is **analytical** and takes place automatically for N_p **gluons**
- 4 Work in progress: next step is a generalization to **asymmetric initial state** and **arbitrary final state**
- 5 Outlook: application of the method to **pheno-studies**