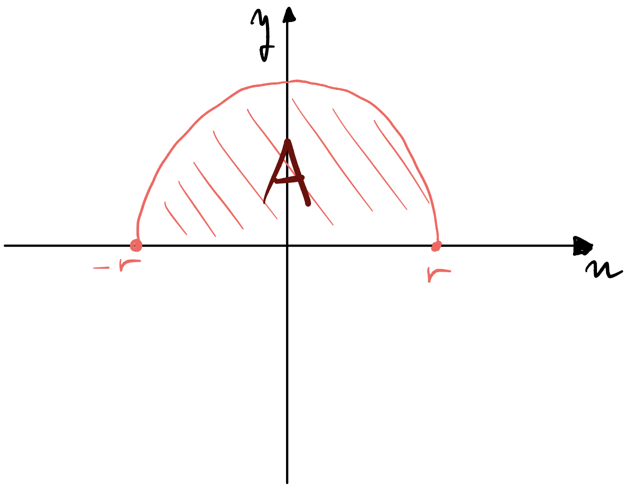
VOLUMI

17 mag '21

Area semicirconferenza e Volume della sfera



$$A = 2\int_0^r \sqrt{r^2 - x^2}\, dx$$

$x = a\,\mathrm{sen}\,t$

p. 1913

...

$$= 2\left[\arcsin\frac{x}{r} + \frac{1}{2}x\sqrt{r^2 - x^2}\right]_0^r = \ldots$$

$$V = 2\pi\int_0^r \left(\sqrt{r^2 - x^2}\right)^2 dx = 2\pi\left[r^2 x - \frac{1}{3}x^3\right]_0^r = 2\pi\left[r^3 - \frac{r^3}{3}\right] = \frac{4}{3}\pi r^3$$

ex

$$y = \frac{x}{2-x} \qquad V \text{ in } [0;1]$$

$$V = \pi \int_0^1 \left(\frac{x}{2-x} \right)^2 dx =$$

$$= \pi \int_0^1 \frac{x^2}{x^2-2x+4} \, dx =$$

$$= \pi \left[x + 2 \ln (x-2)^2 - \frac{4}{x-2} \right]_0^1$$

$$= \pi \left[5 - 2 \ln 4 - 2 \right] =$$

$$\pi \left(3 - \ln 16 \right)$$

$$\to \int \frac{x^2}{x^2-4x+4}\,dx = \int\left(1+\frac{+4x-4}{x^2-4x+4}\right)dx = x + \int\frac{4x-4}{x^2-4x+4}\,dx =$$

$$= x + 2\int\frac{2x-4}{x^2-4x+4}\,dx + 4\int\frac{1}{(x-2)^2}\,dx =$$

$$= x + 2\ln\left(x^2-4x+4\right) - 4\,(x-2)^{-1} + K$$

ex $\quad y = \sqrt{\dfrac{x+1}{x-2}} \qquad V \text{ in } [3;4]$

$$V = \pi \int_3^4 \frac{x+1}{x-2}\,dx =$$

$$= \pi \Big[x + 3 \ln (x-2) \Big]_3^4 =$$

$$= \pi \left(4 + 3 \ln 2 - 3 - 3 \ln 1 \right) =$$

$$= \pi \left(1 + \ln 8 \right)$$

ex $y = e^x$

A in $[0;3]$

V in $[0;3]$ asse x

$$A = \int_0^3 e^x \, dx = \left[e^x\right]_0^3 = e^3 - 1$$

$$V = \pi \int_0^3 e^{2x} \, dx = \frac{\pi}{2} \left[e^{2x}\right]_0^3 = \frac{\pi}{2}\left(e^6 - 1\right)$$

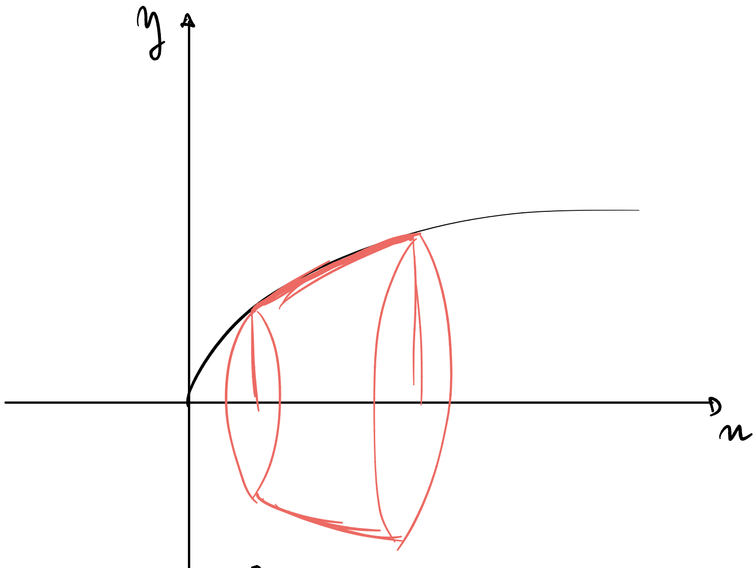
se ruotassi rispetto all'asse y? $\rightarrow$ funzione inversa (ho bisogno di una funzione di y)

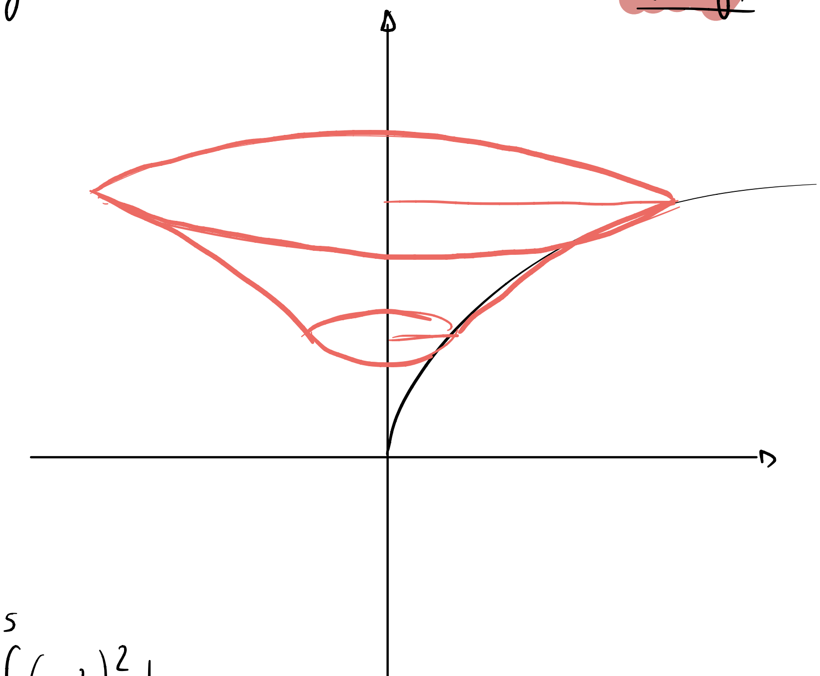
Volumi con rotazione lungo l'asse delle y

FUNZIONE INVERSA

$y = \sqrt{x}$ asse x $x = 2$; $x = 5$

asse y $y = 2$; $y = 5$ ⇝ DEVO AVERE UNA $f(y)$





$$V_x = \pi \int_3^5 x \, dx$$

$$V_y = \pi \int_3^5 (y^2)^2 \, dy$$

ex

$$V = \int_1^e \left(\underbrace{\ln x}_{\text{base}} \cdot \underbrace{3\ln x}_{\text{altezza}}\right) dx =$$

base x altezza ⇝ area della sezione

$$= \int_1^e 3\ln^2 x \, dx = 3\int_0^e \ln^2 x \, dx = 3\left[x\ln^2 x - 2x\ln x + 2x\right]_0^e =$$

$$= 3e - 2e + 2e - 3(2) = 3e - 6$$

$$\to \int \ln^2 x \, dx = x\ln^2 x - 2\int \ln x \, dx =$$

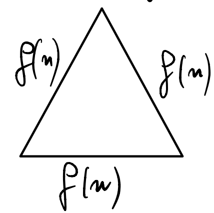
$$= x\ln^2 x - 2x\ln x + 2\int 1 \, dx =$$

$$= x\ln^2 x - 2x\ln x + 2x + K$$

F	D
$\ln^2 x$	$\frac{2\ln x}{x}$
x	1

F	D
$\ln x$	$1/x$
x	1

ex $y = 4\sqrt{x+1}$ $\quad [3;8]$ $\quad$ sezione $\Rightarrow$ triangoli equilateri



$$A = \frac{1}{2} f(x) \cdot \frac{f(x)\sqrt{3}}{2} = \frac{\sqrt{3}}{4}\left[f(x)\right]^2$$

$$V = \int_3^8 4\sqrt{3}\,(x+1)\,dx = 4\sqrt{3}\left[\frac{x^2}{2} + x\right]_3^8 = 4\sqrt{3} \cdot \frac{65}{2} = 130\sqrt{3}$$

ex sia $E = \int_0^1 x e^x \, dx$

$$\int_0^1 x^2 e^x \, dx = \left[\int x^2 e^x \, dx\right]_0^1 = \left[x^2 e^x - 2\int x e^x \, dx\right]_0^1 =$$

$$= \left[x^2 e^x\right]_0^1 - 2\left[\int x e^x \, dx\right]_0^1 = e - 2E$$

INTEGRALI APPLICATI ALLA FISICA

ex

$v(t) = 2\sin 4t$ $\qquad s(t) = ?$

$s(0) = 2m$ $\qquad a(t) = ?$

$$s(t) = \int v(t)\,dt = 2\int \sin 4t\,dt = \frac{1}{2}\int 4\,\text{sen}\,4t\,dt = -\frac{1}{2}\cos 4t + K$$

$$-1/2 + K = 2 \Rightarrow K = 5/2 \qquad s(t) = -\frac{1}{2}\cos 4t + 5/2$$

$$a(t) = D[v(t)] = 8\cos 4t$$