Limiti

23 ott 2020

Problemi

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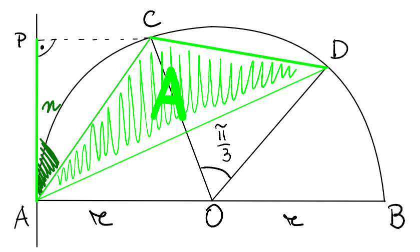

• $y = f(x) = \dfrac{\overline{AP} \cdot \overline{CD}}{A(ACD)}$

•• $\lim_{C \to A} f(x) = ?$

$P\hat{A}C = x$

$0 < x \leq \frac{\pi}{3}$

• $P\hat{A}C = x$ ~ angolo alla circonferenza sull'arco $\overset{\frown}{AC}$

$\overline{CD} = 2r \cdot \operatorname{sen} \frac{\pi/3}{2} = r$

$\overline{AP} = AC \cdot \cos x = 2r \operatorname{sen} x \cos x$

• $AC = 2r \cdot \operatorname{sen} A\hat{B}C = 2r \operatorname{sen} x$

$A = \frac{1}{2} \overline{AC} \cdot \overline{AD} \cdot \operatorname{sen} \frac{\pi}{6} = \frac{1}{\cancel{2}} \cdot \cancel{2}r \operatorname{sen} x \cdot \cancel{2}r \operatorname{sen}\left(x + \frac{\pi}{6}\right) \cdot \frac{1}{\cancel{2}} = \operatorname{sen} x \operatorname{sen}\left(x + \frac{\pi}{6}\right) \cdot r^2$

→ angolo alla circonferenza che insiste su CD

• $AC = 2r \operatorname{sen} x$

• $AD = 2r \cos\left[\frac{\pi}{2} - x - \frac{\pi}{6}\right] =$
$= 2r \operatorname{sen}\left(x + \frac{\pi}{6}\right)$

$f(x) = \dfrac{\overline{AP} \cdot \overline{CD}}{A(ACD)} = \dfrac{2\cancel{r} \cancel{\operatorname{sen} x} \cos x \cdot \cancel{r}}{\cancel{\operatorname{sen} x} \cdot \operatorname{sen}\left(x + \frac{\pi}{6}\right) \cancel{r^2}} = \dfrac{2 \cos x}{\operatorname{sen}\left(x + \frac{\pi}{6}\right)}$ □

•• $\lim_{C \to A} f(x) = \lim_{x \to 0} \dfrac{2 \cos x}{\operatorname{sen}\left(x + \frac{\pi}{6}\right)} = \dfrac{2}{1/2} = 4$ □

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$\gamma_1: \ y = x^2 - 4x$

$\gamma_{11}: \ y = -x^2 + 6x$

$\gamma_1 \cap \gamma_{11} = \{A, B\} \mid x_A < x_B$

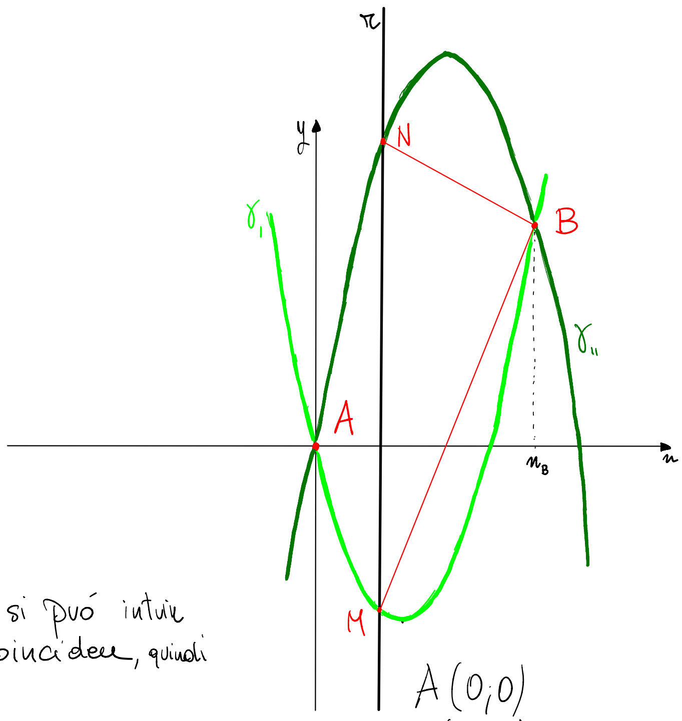
$r: \ x = k; \ k \in (x_A; x_B)$

$r \cap \gamma_1 = M$

$r \cap \gamma_{11} = N$

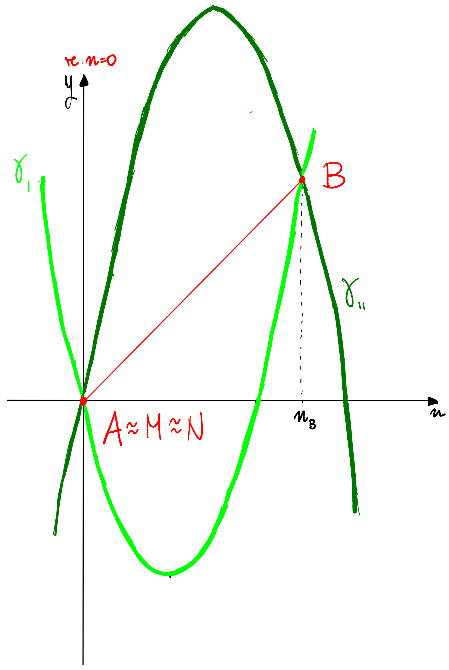
• $\lim_{k \to x_A} \dfrac{\overline{MB}}{\overline{NB}}$

i. facendo un ragionamento geometrico si può intuire che i punti M e N tenderanno a coincidere, quindi il limite sarà uguale a 1



$A(0;0)$
$B(5;5)$

DEDUZIONE GEOMETRICA



ii calcoliamo il limite algebricamente, trovando prima la funzione

$$f(k)=\frac{\overline{MB}}{\overline{NB}}$$

$M:\begin{cases} x=k \\ y=x^2-4x \end{cases}$ $\qquad$ $N:\begin{cases} x=k \\ y=-x^2+6x \end{cases}$

$M(k; k^2-4k)$ $\qquad$ $N(k; -k^2+6k)$

$MB=\sqrt{(5-k)^2+(k^2-4k-5)^2}$ $\qquad$ $NB=\sqrt{(k-5)^2+(k^2-6k+5)^2}$

$$f(k)=\frac{\sqrt{(5-k)^2+(k^2-4k-5)^2}}{\sqrt{(k-5)^2+(k^2-6k+5)^2}}=\sqrt{\frac{(5-k)^2+(k^2-4k-5)^2}{(k-5)^2+(k^2-6k+5)^2}}$$

$$\lim_{k\to 0}\sqrt{\frac{(5-k)^2+(k^2-4k-5)^2}{(k-5)^2+(k^2-6k+5)^2}}=\sqrt{\frac{(5-0)^2+(0^2-4\cdot 0-5)^2}{(0-5)^2+(0^2-6\cdot 0+5)^2}}=\sqrt{\frac{25+25}{25+25}}=$$

$$=1$$