INTEGRALI

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$\rightarrow \int \frac{\ln\sqrt{x}}{x}\,dx = \frac{1}{2}\int \ln x \cdot \frac{1}{x}\,dx = \frac{1}{4}\ln^2 x + K = \ln^2\sqrt{x} + K$

$= 2\int \frac{\ln t}{t^2}\, t \cdot dt = 2\int \frac{\ln t}{t}\,dt =$

sia $t = \sqrt{x}$

$x = t^2 \;;\; dx = 2t\,dt$

$= \ln^2 t + K = \ln^2\sqrt{x} + K$

$\rightarrow \int \frac{1}{\sqrt{x}\cdot\sqrt{1-x}}\,dx = 2\int \frac{1}{\cancel{t}\sqrt{1-t^2}} \cdot \cancel{t}\,dt =$

sia $t = \sqrt{x}$

$x = t^2 \;;\; dx = 2t\,dt$

$= 2\arcsen t + K = 2\arcsen\sqrt{x} + K$

$\rightarrow \quad \displaystyle\int \frac{x+3}{\sqrt{x+2}}\,dx = 2\int \frac{t^2+1}{t}\cdot t\,dt =$

sia $t=\sqrt{x+2}$

$x=t^2-2$

$dx = 2t\,dt$

$\displaystyle = 2\int t^2\,dt + 2\int dt = \frac{2}{3}t^3 + 2t + K =$

$\displaystyle = \frac{2}{3}\sqrt{(x+2)^3} + 2\sqrt{x+2} + K = 2\sqrt{x+2}\left[1+\frac{1}{3}(x+2)\right]+K$

$\rightarrow \quad \displaystyle 3\int f\left(\frac{x}{2}\right)dx = 6\int f(t)\,dt$

sia $t=\dfrac{x}{2}$; $x=2t$

$dx = 2dt$

$\rightarrow \quad \displaystyle 6\int f(\sqrt{x})\,dx = 12\int f(t)\cdot t\,dt$

sia $t=\sqrt{x}$; $x=t^2$

$dx = 2t\,dt$

Sostituzione con formule parametriche

$$\rightarrow \int \frac{1}{1+\operatorname{sen} x}\,dx =$$

$$= \int \frac{1}{1+\frac{2t}{1+t^2}} \cdot \frac{2}{1+t^2}\,dt =$$

$$= 2\int \frac{\cancel{1+t^2}}{(t^2+2t+1)\cancel{(1+t^2)}}\,dt =$$

$$= 2\int (t+1)^{-2} \cdot D(t+1)\,dt =$$

$$= 2\cdot(-1)\cdot(t+1)^{-1} + K =$$

$$= \frac{-2}{t+1} + K = \frac{-2}{\tan x/2 + 1} + K$$

<u>nota:</u>

$$\operatorname{sen} x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$t = \tan \frac{x}{2}$$

<u>sia</u>

$$\operatorname{sen} x = \frac{2t}{1+t^2}; \quad t = \tan \frac{x}{2}$$

$$\cancel{x = \arcsin \frac{2t}{1+t^2}}; \quad x = 2 \arctan t$$

$$dx = \frac{2}{1+t^2}\,dt$$

$$\rightarrow \int \frac{2}{\operatorname{sen} x}\,dx = 2\int \frac{1}{\frac{2t}{1+t^2}} \cdot \frac{2}{1+t^2}\,dt =$$

sia $\operatorname{sen} x = \frac{2t}{1+t^2}$; $t = \tan\frac{x}{2}$

$x = 2\arctan t \Rightarrow dx = \frac{2}{1+t^2}\,dt$

$$= 2\int \frac{\cancel{1+t^2}}{t} \cdot \frac{1}{\cancel{1+t^2}}\,dt = 2\ln|t| + K$$

$$= \boxed{2\ln\left|\tan \frac{x}{2}\right| + K}$$

Sostituzione con funzioni irrazionali

$$\rightarrow \int \sqrt{a^2 - x^2}\,dx$$

$\hookrightarrow$ $y = \sqrt{a^2 - x^2}$ semi circonferenza

$\rightarrow$ per gli integrali definiti si usa questo, non la formula