Ripasso

18 gen

ex $y = \dfrac{x}{x^2+9}$ 　 CE 　 $x \in \mathbb{R}$

limiti 　 $\lim_{x \to \infty} \dfrac{x}{x^2+9} = 0 \Rightarrow y = 0$ 　 ASINTOTO OR. COMPLETO

- $x \to -\infty \Rightarrow 0^-$
- $x \to +\infty \Rightarrow 0^+$

intersezione as. 　 $\dfrac{x}{x^2+9} = 0 \Rightarrow O(0;0) \in y = f(x)$

segno 　 $f(x) > 0 \Leftrightarrow x \in (0; +\infty)$

$f(x) < 0 \Leftrightarrow x \in (-\infty; 0)$

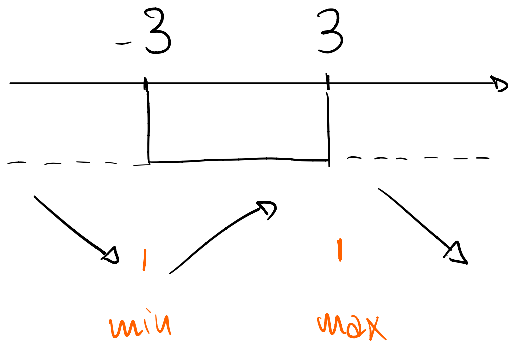
derivata prima

$$y' = \frac{1 \cdot (x^2+9) - x(2x+0)}{(x^2+9)^2} = \frac{-x^2+9}{(x^2+9)^2}$$

$y' = 0 \rightsquigarrow$ punti stazionari

$x = \pm 3$

$y' > 0$



$A\left(-3; -\frac{1}{6}\right)$ minimo

$B\left(3; \frac{1}{6}\right)$ massimo

derivata seconda

$$y'' = \frac{-2x(x^2+9)^{\cancel{2}} - (-x^2+9)\cdot 2\cancel{(x^2+9)}\cdot 2x}{(x^2+9)^{\cancel{4}\,3}} =$$

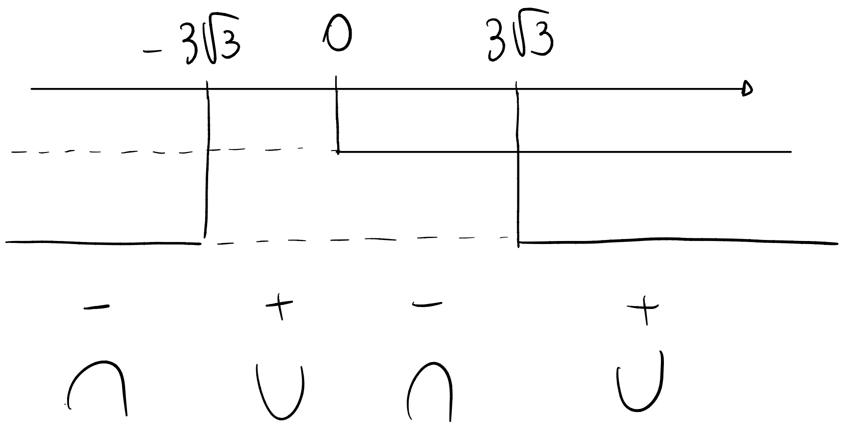
$$= -2x\,\frac{x^2+9+18-2x^2}{(x^2+9)^3} = \frac{-2x(-x^2+27)}{(x^2+9)^3} = \frac{2x(x^2-27)}{(x^2+9)^3}$$

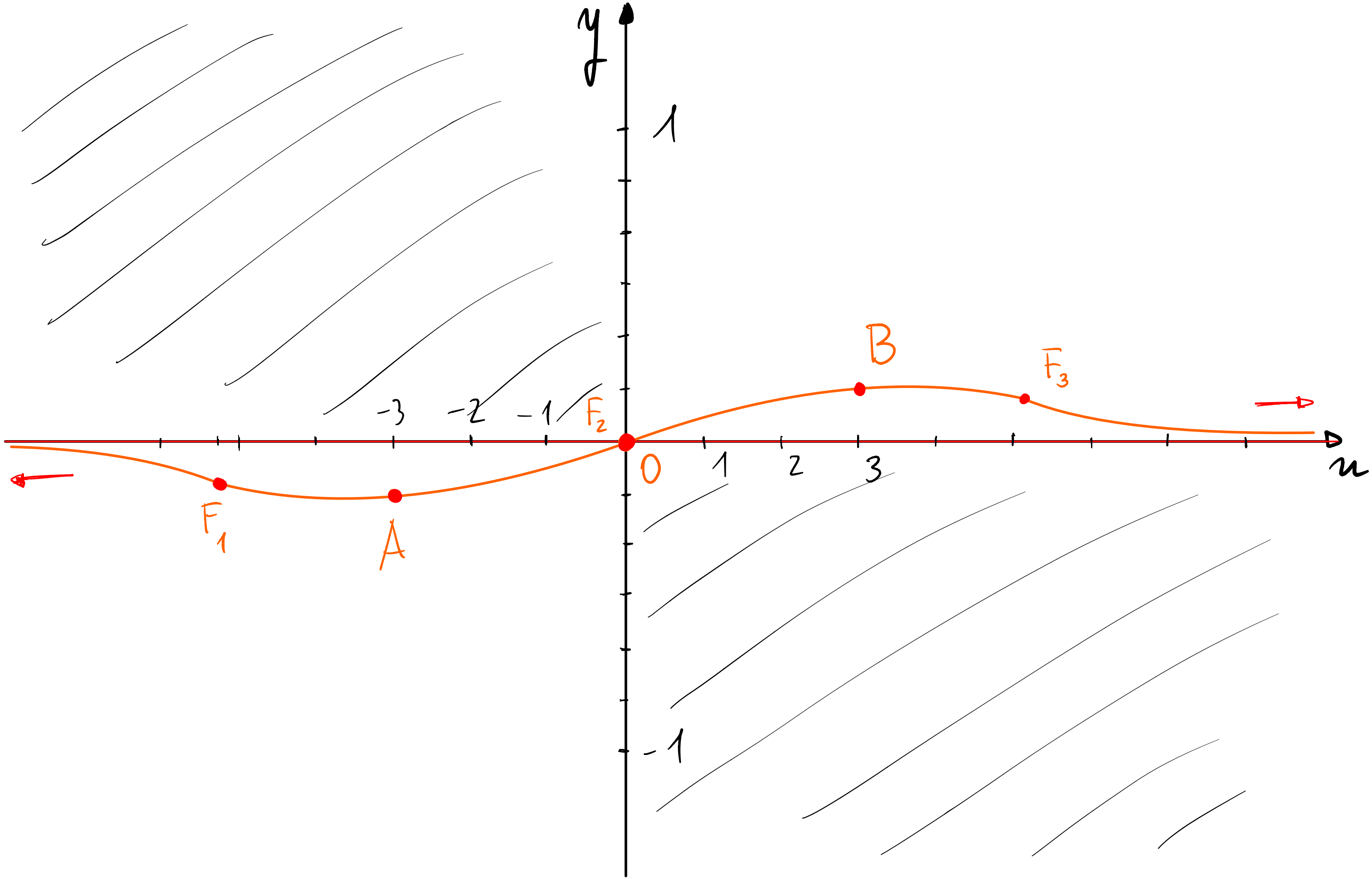
$y'' = 0 \Rightarrow x = 0 \qquad F_2(0;0)$

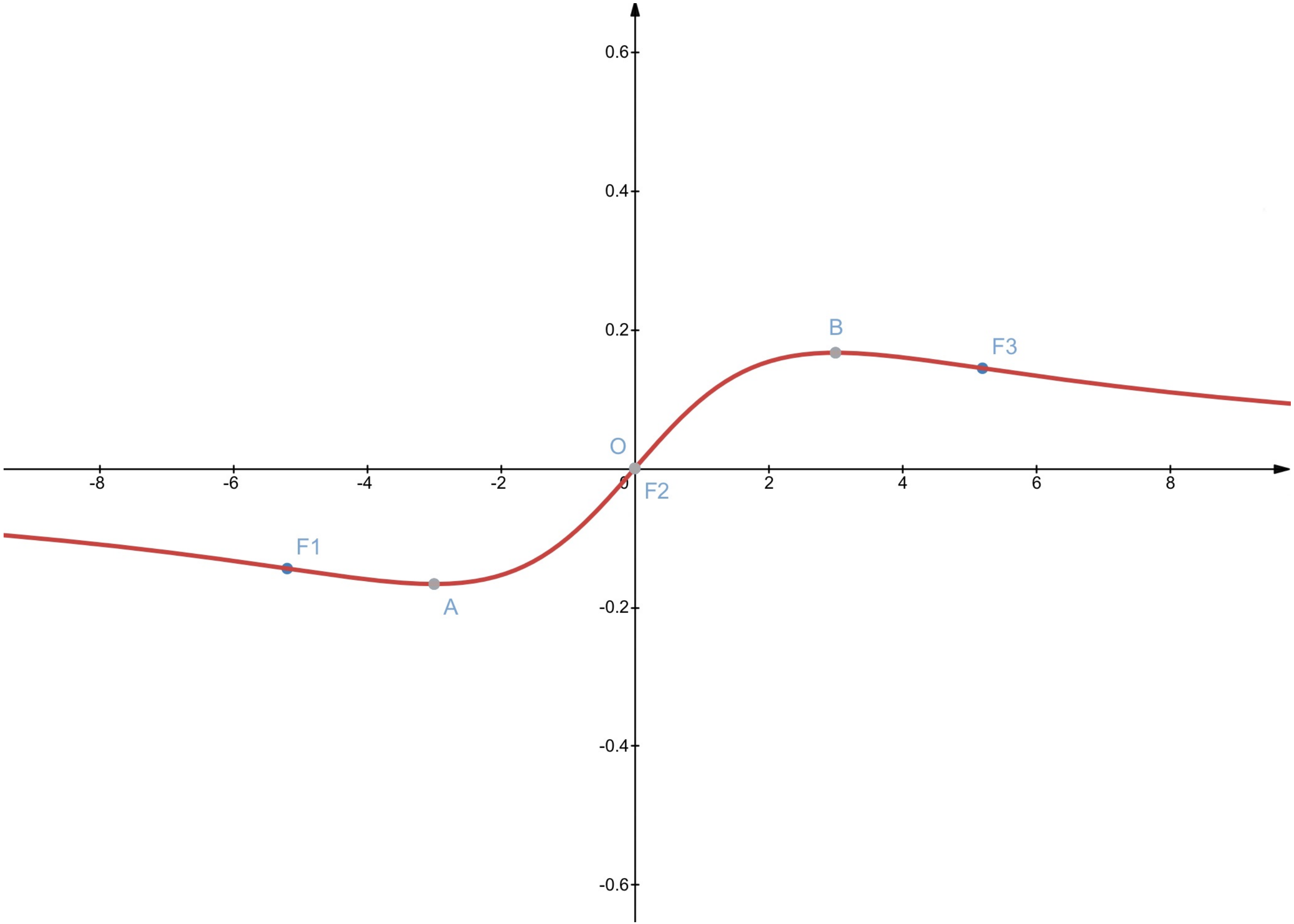
$x = \pm 3\sqrt{3} \qquad F_1\left(-3\sqrt{3}; -\dfrac{\sqrt{3}}{12}\right); \quad F_3\left(3\sqrt{3}; \dfrac{\sqrt{3}}{12}\right)$

flessi

$y'' > 0$




y
1
-1
-3
-2
-1
0
1
2
3
x
A
B
F1
F2
F3


0.6
0.4
0.2
-0.2
-0.4
-0.6
-8
-6
-4
-2
0
2
4
6
8
B
F3
O
F2
F1
A

ex $y=\dfrac{3x-x^2}{2x-8}$ CE $x\neq 4;\quad x\in(-\infty;4)\cup(4;+\infty)$

né pari né dispari

limiti

$$\lim_{x\to-\infty} f(x) = +\infty$$

$$\lim_{x\to 4} f(x) \quad \begin{cases} x\to 4^- \Rightarrow \dfrac{-4}{0^-} = +\infty \\ x\to 4^+ \Rightarrow \dfrac{-4}{0^+} = -\infty \end{cases} \quad x=4 \text{ A.V.}$$

$$\lim_{x\to\infty} f(x) = -\infty$$

asintoto $y=mx+q$

$$m=\lim_{x\to\infty}\frac{3x-x^2}{2x^2-8x}=-\frac{1}{2}$$

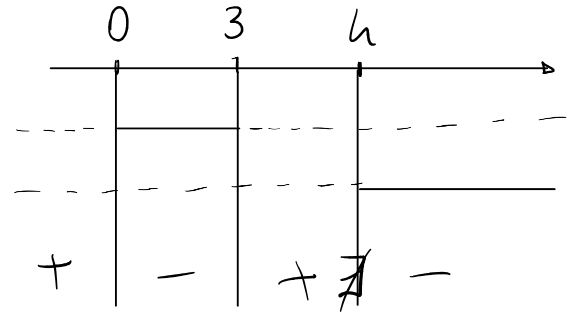
$$y=-\frac{1}{2}x-\frac{1}{2} \quad \text{A.Ob completo}$$

$$q=\lim_{x\to\infty}\left[\frac{-x^2+3x}{2x-8}+\frac{1}{2}x\right]=\lim_{x\to\infty}\frac{-x^2+3x+x^2-4x}{2x-8}=-\frac{1}{2}$$

intersezioni

$$\frac{3x-x^2}{2x-8}=0 \qquad x=0 \qquad x=3$$

segno



asintoto

$$\begin{cases} \dfrac{3x-x^2}{2x-8}=y \\ y=-\dfrac{x}{2}-\dfrac{1}{2} \end{cases}$$

$$\frac{3x-x^2}{2(x-4)}=\frac{-x-1}{2}$$

$$3x-x^2=-(x-4)(x+1) \qquad S=\emptyset$$

NO INTERSEZIONI

derivata prima

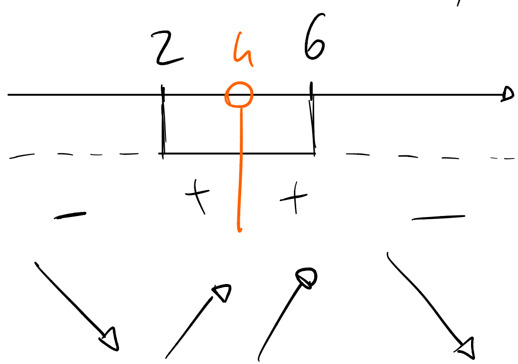
$$y' = \frac{(-2x+3)(\cancel{2}x-\cancel{8}^{\,4}) - \cancel{2}(3x-x^2)}{2\cancel{4}(x-4)^2} = \frac{-2x^2+3x+8x-12+x^2-3x}{2(x-4)^2}$$

$$= \frac{-x^2+8x-12}{2(x-4)^2} = y'$$

$y' = 0$ $\quad x = 6 \quad A\left(6; -\frac{9}{2}\right)$ max

$\quad\quad\quad x = 2 \quad B\left(2; -\frac{1}{2}\right)$ min

$y' > 0$



$x = 2$ min

$x = 6$ max

derivata seconda

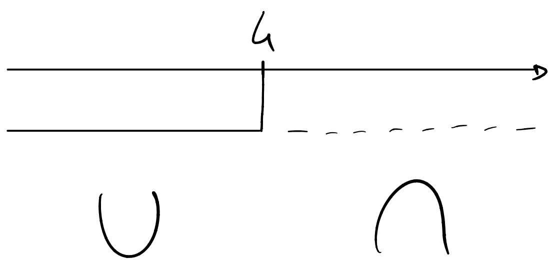
$$y' = \frac{-x^2+8x-12}{2(x-4)^2}$$

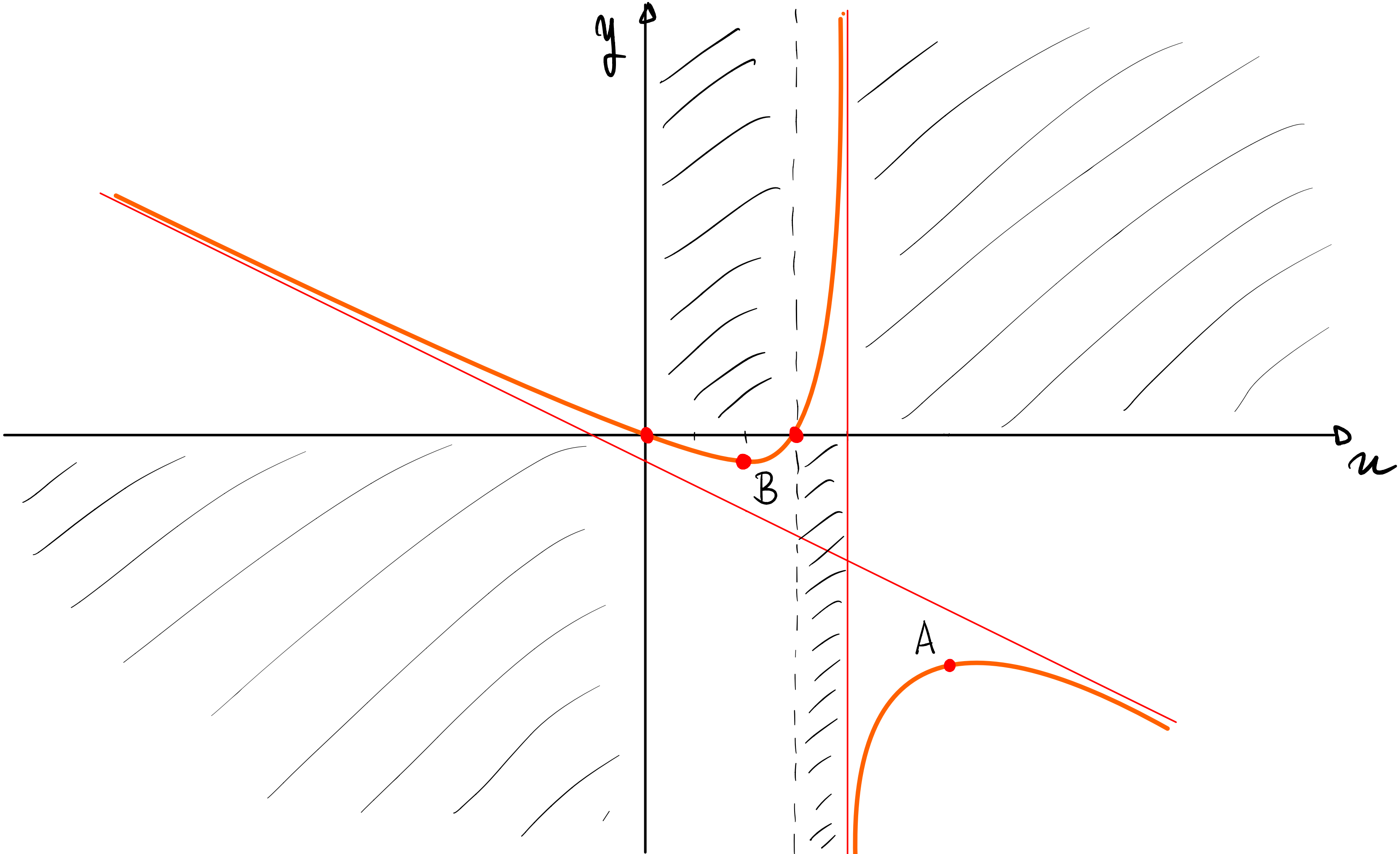
$$y'' = \frac{\cancel{2}(x-4)^{\cancel{2}}\left(-\cancelto{1}{2}x+\cancelto{4}{8}\right) - \left(-x^2+8x-12\right)\left[\cancelto{2}{4}\,\cancel{(x-4)}\right]}{\cancel{2}\,\cancel{4}\,(x-4)^{\cancel{4}\,3}} =$$

$$= \frac{-\cancel{x^2}+\cancel{8x}-16+\cancel{x^2}-\cancel{8x}+12}{(x-4)^3} = \frac{-4}{(x-4)^3} = y''$$

$y''=0 \quad \nexists\, x\in\mathbb{R}$ NON HA FLESSI

$y''>0$





y
x
B
A