TEOREMI

10 mag '21

① $\int f(x)\,dx = F(x) + K$ → integrazione indefinita,

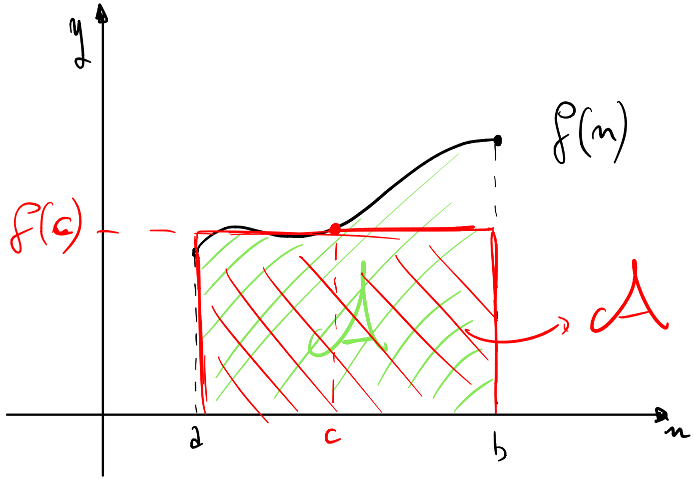
$D[F(x)+K] = f(x)$ operazione INVERSA della derivazione

② $\int_a^b f(x)\,dx =$ numero → integrazione definita, se $f(x)$ continua e derivabile in $[a;b]$ rappresenta l'**area**

nota: $\int_a^b k\,dx = (b-a)\,k$

Teorema del valor medio

p. 1945



$$\exists c \mid \underbrace{\int_a^b f(x)\,dx}_{A} = \underbrace{(b-a)\,f(c)}_{A}$$

hp: $f(x)$ continua in $[a;b]$

th: $\exists\, c \in [a;b] \mid \int_a^b f(x)\,dx = (b-a)\cdot f(c)$ chiamato VALOR MEDIO

corollario

$$f(c) = \frac{\int_a^b f(x)\,dx}{b-a}$$

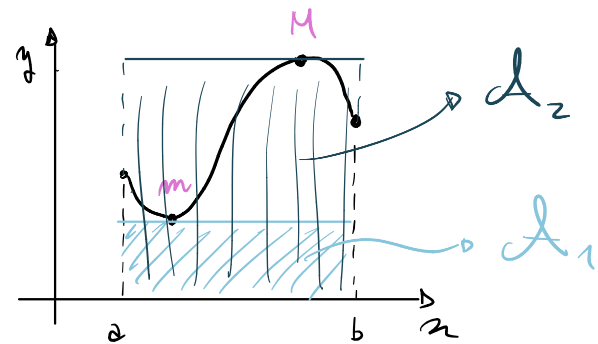
DIM

PER IPOTESI

• $f(x)$ continua $\Rightarrow$ $\exists\, m, M \in [a;b]$ WEIERSTRASS

$$m \leq f(x) \leq M$$ DARBOUX

•• $$\overbrace{\int_a^b m\,dx}^{\mathcal{A}_1} \leq \int_a^b f(x)\,dx \leq \overbrace{\int_a^b M\,dx}^{\mathcal{A}_2}$$



$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$

$$\frac{m\cancel{(b-a)}}{\cancel{b-a}} \leq \frac{\int_a^b f(x)\,dx}{b-a} \leq \frac{M\cancel{(b-a)}}{\cancel{b-a}}$$

$$m \leq \underbrace{\frac{\int_a^b f(x)\,dx}{b-a}}_{A} \leq M$$ $\to$ DARBOUX

$f(x)$ assume in un punto c il valore A

$$\exists\, c \in [a;b] \mid f(c) = \frac{\int_a^b f(x)\,dx}{b-a}$$ □

Sia $F(x) = \int_a^x f(t)\,dt$ → si chiama FUNZIONE INTEGRALE e rappresenta la VARIAZIONE DELL'AREA

Teorema di Torricelli – Barrow

p. 1947

Teorema fondamentale del calcolo integrale

<u>hp</u> $f(x)$ continua in $[a;b]$

$$F(x) = \int_a^x f(t)\,dt$$

<u>th</u> $F(x)$ derivabile in $[a;b]$

$F'(x) = f(x)$; $F(a) = 0$ ⇒ $F(x)$ è una <u>primitiva</u>

DIM

$$F'(x) = \lim_{h\to 0} \frac{F(x+h) - F(x)}{h} = \lim_{h\to 0} f(c) \quad (*)$$

$$\frac{F(x+h)-F(x)}{h} = \frac{\int_a^{x+h} f(t)\,dt - \int_a^{x} f(t)\,dt}{h} =$$

$$= \frac{\cancel{\int_a^{x} f(t)\,dt} + \int_x^{h} f(t)\,dt - \cancel{\int_a^{x} f(t)\,dt}}{h} = \frac{\int_x^{x+h} f(t)\,dt}{h}$$

applico il teorema del valor medio

$$\exists\, c \in [x; x+h] \mid f(c) = \frac{\int_x^{x+h} f(t)\,dt}{h}$$

$(*)$ $\lim_{h\to 0} f(c) = f(x)$ | se $h \to 0$, $c \to x$, in quanto $c \in [x; x+h]$

$\square$

Formula di Newton - Leibniz

p. 1949

$$\int_a^b f(x) = F(a) - F(b)$$

DIM sia $\varphi(x)$ una primitiva generale di $f(x)$

$$\varphi(x) = F(x) + K$$

$\int_a^x f(t)\,dt$ é primitiva di $f(x) \Rightarrow$

$$\varphi(x) = \int_a^x f(t)\,dt$$

$$\varphi(a) = F(a) + K = \int_a^a f(t)\,dt = 0 \, ;$$

$$\varphi(b) = F(b) + K = \int_a^b f(t)\,dt$$

$$\varphi(b) - \varphi(a) = \int_a^b f(t)\,dt - 0 = \left(F(b) + K\right) - \left(F(a) + K\right)$$

$$\int_a^b f(t)\,dt = F(b) - F(a) = \left[F(x)\right]_a^b$$

$\square$