INTEGRALI

16 apr 2021

Sostituzione con funzioni irrazionali

→ $\int \sqrt{a^2-x^2}\,dx$

↳ $y=\sqrt{a^2-x^2}$ semi circonferenza

→ per gli integrali definiti si usa questo, non la formula

metodo di sostituzione

$$\int \sqrt{a^2-x^2}\,dx =$$

$$= \int \sqrt{a^2-a^2\operatorname{sen}^2 t}\cdot a\cos t\,dt =$$

sia $x = a \operatorname{sen} t$ (equivalente ad $a\cos t$)

$dx = a\cos t\,dt$

$t = \arcsin\left(\frac{x}{a}\right)$ $\quad \operatorname{sen} t = \frac{x}{a}$

$$= \int a\sqrt{1-\operatorname{sen}^2 t}\cdot a\cos t\,dt = a^2\int \cos^2 t\,dt = a^2\int \frac{1+\cos 2t}{2}\,dt =$$

$$= a^2\left\{\frac{1}{2}\int dt + \int \frac{\cos 2t}{2}\,dt\right\} = \frac{1}{2}a^2 t + \frac{1}{4}a^2 \operatorname{sen} 2t + K =$$

$$\operatorname{sen} 2t = 2\operatorname{sen} t\cos t =$$

$$= 2\cdot\frac{x}{a}\cdot\sqrt{1-\frac{x^2}{a^2}} =$$

$$= 2\,\frac{x}{a}\cdot\frac{1}{a}\cdot\sqrt{a^2-x^2}$$

$$= \frac{1}{2}a^2\arcsin\frac{x}{a} + \frac{1}{2}x\sqrt{a^2-x^2} + K \qquad \square$$

Funzioni razionali

p. 1884

$$\boxed{\int \frac{f(x)}{g(x)}\,dx}$$

$$\int \frac{N(x)}{D(x)}\,dx$$

- $\deg N(x) \geq \deg D(x)$ — 1
- $\deg D(x) = 1$
  - $\deg N(x) = 0$ — 2a
  - $\deg N(x) = 1$ — 2b
- $\deg D(x) = 2$
  - $N(x) = D'(x)$ — 3a
  - $\Delta > 0$ [di $D(x)$] — 3b
  - $\Delta = 0$ — 3c
  - $\Delta < 0$ — 3d

caso 1

$$\int \frac{4x^2+10x+3}{2x^2+3x}\,dx =$$

$$= \int 2 + \frac{4x+3}{2x^2+3x}\,dx =$$

$$= 2\int dx + \int \frac{4x+3}{2x^2+3x}\,dx =$$

divisione tra polinomi:

$$\begin{array}{rrrr|l} 4x^2 & +10x & +3 & & 2x^2+3x \\ -4x^2 & -6x & & & \hline 2 \\ \hline 0 & +4x & +3 & & \end{array}$$

$$4x^2+10x+3 = 2\cdot(2x^2+3x)+4x+3$$

$$= 2x + K + \int (2x^2+3x)^{-1} \cdot D(2x^2+3x)\,dx =$$

$$= 2x + \ln|2x^2+3x| + K$$

$$\int \frac{x^4+2x-1}{x^2-1}\,dx =$$

$$= \int (x^2+1)\,dx + \int \frac{2x}{x^2-1}\,dx$$

$$= \frac{1}{3}x^3 + x + \ln|x^2-1| + K$$

divisione tra polinomi:

$$\begin{array}{rrrrr|l}
x^4 & +0x^3 & +0x^2 & +2x & -1 & x^2-1 \\ \hline
-x^4 & & +x^2 & & & x^2+1 \\ \hline
 & & x^2 & +2x & -1 & \\
 & & -x^2 & & +1 & \\ \hline
 & & & 2x & &
\end{array}$$

$$x^4+2x-1 = (x^2-1)(x^2+1)+2x$$

caso 2a

$$\int \frac{5}{2x-3}\,dx = 5\cdot\frac{1}{2}\int \frac{2}{2x-3}\,dx = \frac{5}{2}\ln|2x-3| + K$$

Caso 2b

$$\rightarrow \int \frac{x+5}{x+3}\,dx = \int \frac{x+3+2}{x+3}\,dx = \int dx + 2\int \frac{1}{x+3}\,dx =$$

$$= x + 2\ln|x+3| + K$$