VERIFICA MATEMATICA

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Problema 2

$s = t^2 \cdot e^{-\frac{1}{3}t}$

$t \geq 0$

a. $v(t)$; $a(t)$

b. $t \mid v(t) = 0$; $t \mid a(t) = 0$

c. $s(t)$ max

a) $v(t) = D[s(t)] = D[t^2 \cdot e^{-\frac{1}{3}t}] = D(t^2) \cdot e^{-\frac{1}{3}t} + D(e^{-\frac{1}{3}t})\, t^2 =$

$= 2t\, e^{-\frac{1}{3}t} - \frac{1}{3} e^{-\frac{1}{3}t} t^2 = e^{-\frac{1}{3}t} \cdot t \left(2 - \frac{t}{3}\right) =$

$v(t) = \frac{1}{3} t \cdot e^{-\frac{1}{3}t} (6 - t)$

$$a(t) = D[v(t)] = D\left[\frac{1}{3} t e^{-1/3 t}(6-t)\right] = \frac{1}{3} D\left[(-t^2+6t)\, e^{-\frac{1}{3}t}\right] =$$

$$= \frac{1}{3}\left\{(-2t+6)\, e^{-\frac{1}{3}t} + (-t^2+6t)\left(-\frac{1}{3}\right) e^{-\frac{1}{3}t}\right\} =$$

$$= \frac{1}{3} \cdot e^{-\frac{1}{3}t}\left\{-2t+6+\frac{1}{3}t^2-2t\right\} = \frac{1}{9} e^{-\frac{1}{3}t}\left(t^2-12t+18\right) = a(t)$$

b) $t \mid v(t) = 0$

$$\frac{1}{3} t \cdot e^{-1/3 t}(6-t) = 0 \leadsto e^{-1/3 t} > 0 \quad \forall t \geq 0$$

$$t = 0 \vee 6-t = 0 \Rightarrow t = 0 \vee t = 6$$

$$S = \{0;\ 6\}$$

$t \mid a(t) = 0$

$$\frac{1}{9} e^{-\frac{1}{3}t}\left(t^2 - 12t + 18\right) = 0 \rightsquigarrow e^{-1/3 t} > 0 \quad \forall\, t \geq 0$$

$$t^2 - 12t + 18 = 0 \rightsquigarrow t = 6 \pm \sqrt{3}$$

$$S = \{6 - \sqrt{3};\ 6 + \sqrt{3}\}$$

c) $s(t)$ max $\rightarrow$ é una funzion continua e derivabile $\forall t \geq 0$

quindi ha massimi e minimi solo quando $s'(t) = 0$

$s'(t) = v(t) = 0 \rightsquigarrow t = 0 \vee t = 6$ (per soluzion precedente)

$s(0) = 0$; $s(6) = 36 e^{-2} \rightsquigarrow s(t)$ max