DERIVATE ELEMENTARI

11 dic 2020

Derivata $y = \operatorname{sen} x$

DIM $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\operatorname{sen}(x+h) - \operatorname{sen} x}{h} =$$

$$= \lim_{h\to 0} \frac{\operatorname{sen} x \cos h + \operatorname{sen} h \cos x - \operatorname{sen} x}{h} =$$

$$= \lim_{h\to 0} \left[\operatorname{sen} x \underbrace{\frac{(\cos h - 1)}{h}}_{\to 0} + \underbrace{\frac{\operatorname{sen} h}{h}}_{1} \cos x \right] = \cos x \qquad \square$$

Derivata $y = \cos x$

DIM $$f'(x) = \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h} =$$

$$= \lim_{h\to 0} \frac{\cos x \cos h - \operatorname{sen} x \operatorname{sen} h - \cos x}{h} =$$

$$= \lim_{h\to 0} \left[\cos x \underbrace{\frac{(\cos h - 1)}{h}}_{0} - \operatorname{sen} x \underbrace{\frac{\operatorname{sen} h}{h}}_{1} \right] = -\operatorname{sen} x \qquad \square$$

Derivata di $y = a^x$ $(a > 0)$

DIM $$f'(x) = \lim_{h\to 0} \frac{a^{(x+h)} - a^x}{h} = \lim_{h\to 0} \frac{a^x \cdot a^h - a^x}{h} = \lim_{h\to 0} \left[a^x \underbrace{\frac{a^h - 1}{h}}_{\ln a} \right] =$$

$$= a^x \ln a \qquad \square$$

caso particolare $\quad y = e^x \to y' = e^x \ln e = e^x$

Derivata $y = \log_a x$ $\quad (a>0 \wedge a \neq 1)$

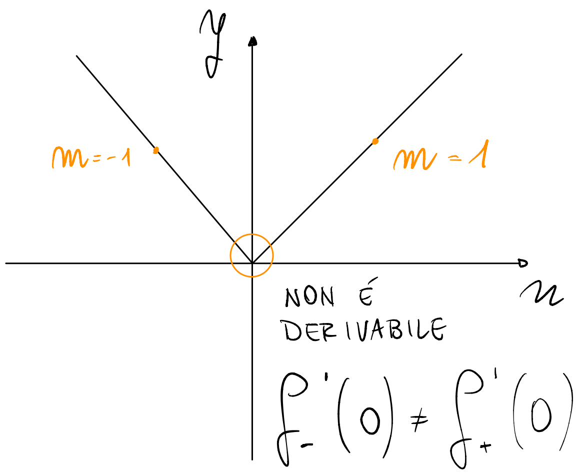
DIM
$$f'(x) = \lim_{h \to 0} \frac{\log_a(x+h) - \log_a x}{h} = \lim_{h\to 0} \frac{1}{h} \log_a \frac{x+h}{x} =$$
$$= \lim_{h\to 0} \log_a \left(1 + \frac{h}{x}\right)^{1/h} =$$
$$= \lim_{t\to\infty} \log_a \left(1 + \frac{1}{t}\right)^{t/x} =$$
$$= \lim_{t\to\infty} \log_a \left[\underbrace{\left(1 + 1/t\right)^{t}}_{e}\right]^{1/x} = \log_a e^{1/x} =$$
$$= \frac{1}{x} \log_a e = \frac{1}{x} \cdot \frac{\ln e}{\ln a} = \frac{1}{x \ln a} \quad \square$$

pongo $h/x = 1/t$

$t = x/h \Rightarrow h \to 0 \Rightarrow t \to \infty$

caso particolare $\quad y = \ln x \to y' = \dfrac{1}{x \ln e} = 1/x$

Derivata del valore assoluto



$y = |x| \to y' = ?$

$$y = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$$

$$y' = \begin{cases} 1 & \text{se } x > 0 \\ -1 & \text{se } x < 1 \end{cases}$$

posso quindi scriverla come

$y = \dfrac{|x|}{x}$, $\quad x = 0$ pt. discontinuitá

Operazioni con le derivate

- $D[K \cdot f(x)] = K \cdot f'(x)$
- $D[f(x) + g(x)] = f'(x) + g'(x)$

ex $\quad y = \frac{1}{4x} = \frac{1}{4} \cdot \frac{1}{x} \rightarrow y' = \frac{1}{4} \cdot \frac{-1}{x^2} = -\frac{1}{4x^2}$

$y = 10x^4 \rightarrow y' = 10 \cdot 4 \cdot x^3 = 40x^3$

$\rightarrow \underbrace{y''}_{\text{derivata seconda}} = 40 \cdot 3x^2 = 120x^2$

derivata seconda ~ derivata della derivata

$\rightarrow y''' = 240x$

$\rightarrow y^{IV} = 240$

$\rightarrow y^{V} = 0$

$y = \frac{1}{4x^3} = \frac{1}{4} x^{-3} \rightarrow y' = -\frac{3}{4} x^{-4} = -\frac{1}{2x^4}$

$\rightarrow y'' = -\frac{3}{4}(-4) x^{-5} = \frac{3}{x^5}$

ex $\quad y = 5x^4 - 3x^3 + 2x^2$

$\rightarrow y' = 20x^3 - 9x^2 + 4x$

$\rightarrow y'' = 60x^2 - 18x$

ex $y = \frac{1+8x^2}{2x^3} = \frac{1}{2x^3} + \frac{8x^2}{2x^3} = \frac{1}{2x^3} + \frac{4}{x}$

$$\longrightarrow y' = \frac{1}{2}(-3)x^{-4} + 4(-1)x^{-2} =$$

$$= -\frac{3}{2x^4} - \frac{4}{x^2} = -\frac{8x^2+3}{2x^4}$$

esercizi p. 1600