INTEGRALI IMPROPRI

12 mag

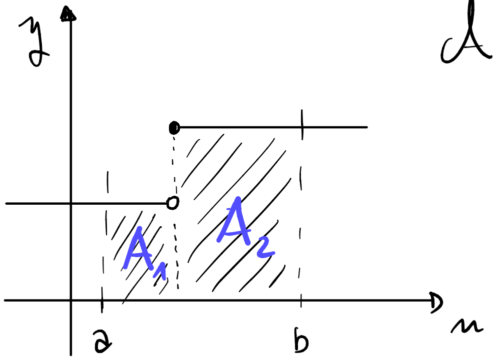
DEF $f(x)$ è integrabile $\overset{def}{\Longleftrightarrow}$ $\exists \int_a^b f(x)\,dx$

- $\left.\begin{array}{l} \text{se } f(x) \text{ continua in } [a;b] \\ \text{e } f(x) \geq 0 \ \forall x \in [a;b] \end{array}\right\} \Rightarrow f(x)$ è integrabile

- sufficiente | $f(x)$ derivabile in $[a;b] \Rightarrow f(x)$ è integrabile
  $f(x)$ monotona in $[a;b] \Rightarrow f(x)$ è integrabile

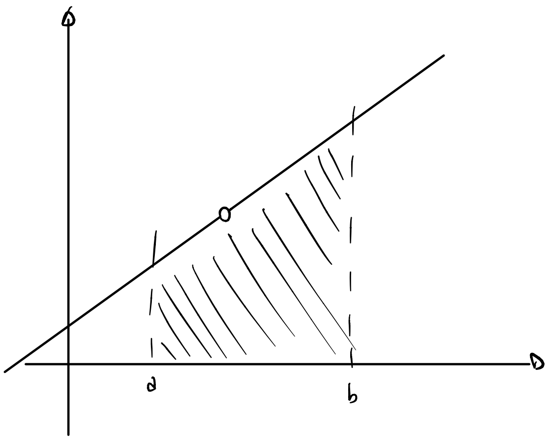
- funzione non continua? **PUNTI DI DISCONTINUITÀ**

I specie



$\mathcal{A} = A_1 + A_2$ (non ci sono problemi)

III specie



Area facilmente calcolabile

II specie $\rightarrow$ Boh

PROBLEMI DI $\int_a^b f(x)\,dx$

→ p.ti discontinuità di II specie

→ $(-\infty; b]$

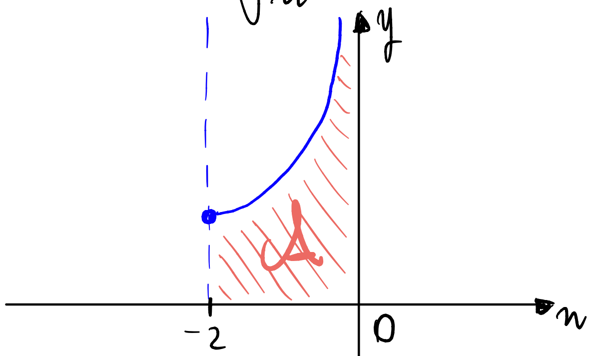
$[a; +\infty)$

$(-\infty; +\infty)$

↓

Integrali impropri

ex $\quad y = \dfrac{1}{\sqrt[3]{x^2}}$ in $[-2; 0]$ $\qquad$ CE $x \neq 0$



$$\mathcal{A} = \int_{-2}^{0} \frac{1}{\sqrt[3]{x^2}}\,dx = \lim_{t\to 0^-} \int_{-2}^{t} \frac{1}{\sqrt[3]{x^2}}\,dx =$$

$$= \lim_{t\to 0^-} \left\{ 3\left[\sqrt[3]{t} + \sqrt[3]{2}\right]\right\} = 3\sqrt[3]{2}$$

AREA FINITA

$\Rightarrow$ INTEGRALE

CONVERGENTE

$$\rightarrow \int x^{-2/3}\,dx = 3\sqrt[3]{x} + K$$

$$\rightarrow \int_{-2}^{t} f(x)\,dx = 3\left[\sqrt[3]{x}\right]_{-2}^{t} =$$

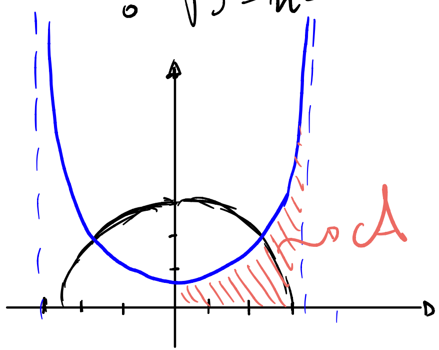
$$= 3\left[\sqrt[3]{t} + \sqrt[3]{2}\right]$$

ex

$$\int_{0}^{3} \frac{1}{\sqrt{9-x^2}}\,dx = \lim_{t\to 3^-} \int_{0}^{t} \frac{1}{\sqrt{9-x^2}}\,dx =$$

$$= \lim_{t\to 3^-} \arcsin t/3 =$$

$$= \arcsin 1 = \pi/2$$



$$\rightarrow \int \frac{1}{\sqrt{9-x^2}}\,dx = \int \frac{1}{3} \cdot \frac{1}{\sqrt{1-\left(\frac{x}{3}\right)^2}} =$$

$$= \arcsin\left(\frac{x}{3}\right) + K$$

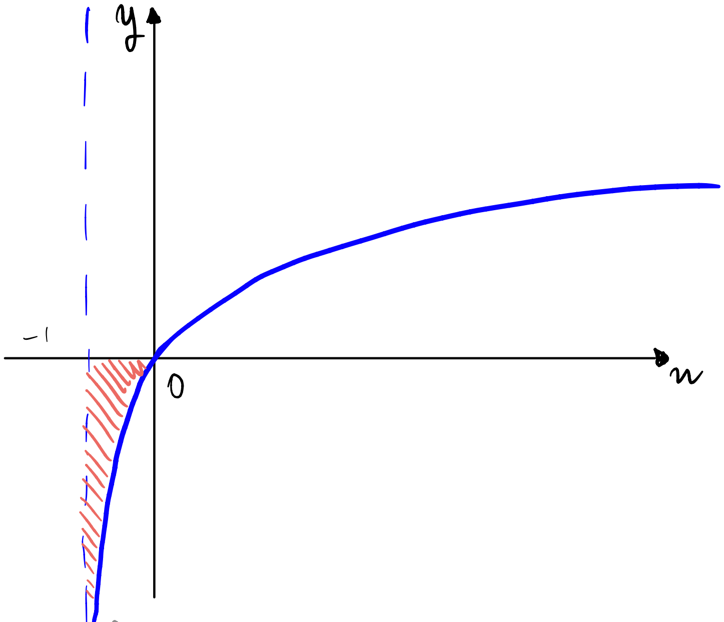
$$\rightarrow \int_{0}^{t} \frac{1}{\sqrt{9-x^2}} = \left[\arcsin\left(\frac{x}{3}\right)\right]_{0}^{t} =$$

$$= \arcsin t/3 - \arcsin 0 =$$

$$\arcsin t/3$$

ex $\int_{-1}^{0} \ln(x+1)\,dx = \lim_{t\to -1^+} \int_{t}^{0} \ln(x+1)\,dx = \lim_{t\to -1^+} \left[t - \ln(t+1)(t+1)\right] =$

$$= \lim_{t\to -1^+} \left[t - \ln(t+1)^{(t+1)}\right] = \dots = -1$$



$\to \int \ln(x+1)\,dx =$

F	D
$\ln(x+1)$	$\frac{1}{x+1}$
x	1

$= x\ln(x+1) - \int \frac{x}{x+1}\,dx$

$= x\ln(x+1) - \left[\int \left(1 - \frac{1}{x+1}\right)dx\right] =$

$= x\ln(x+1) - x + \ln(x+1) + K =$

$= (x+1)\ln(x+1) - x + K$

$\to \int_{t}^{0} f(x)\,dx = \left[(x+1)\ln(x+1) - x\right]_{t}^{0} = -(t+1)\ln(t+1) + t$

ex $\displaystyle\int_0^3 \frac{1}{x^2-9}\,dx = \lim_{t\to 3^-} \int_0^t \frac{1}{x^2-9}\,dx = \frac{1}{6}\lim_{t\to 3^-} \ln\left|\frac{t-3}{t+3}\right| = -\infty$

INTEGRALE DIVERGENTE, AREA INFINITA

$\to \displaystyle\int \frac{1}{x^2-9}\,dx = \dots = \frac{1}{6}\ln\left|\frac{x-3}{x+3}\right| + K$

$\to \displaystyle\int_0^t f(x)\,dx = \frac{1}{6}\left[\ln\left|\frac{x-3}{x+3}\right|\right]_0^t = \frac{1}{6}\left[\ln\left|\frac{t-3}{t+3}\right| - \ln\left|\frac{-3}{3}\right|\right] =$

$\displaystyle = \frac{1}{6}\ln\left|\frac{t-3}{t+3}\right|$

ex $$\int_0^1 \frac{1+x^3}{x}\,dx = \lim_{t\to 0^+} \int_t^1 \frac{1+x^3}{x} = \lim_{t\to 0^+} \left[\frac{1}{3} - \ln|t| - \frac{t^3}{3} \right] = +\infty$$

$$\rightarrow \int \frac{1+x^3}{x}\,dx = \ln|x| + \frac{x^3}{3} + K$$

$$\rightarrow \int_t^1 f(x)\,dx = \left[\ln|x| + \frac{x^3}{3} \right]_t^1 = \frac{1}{3} - \ln|t| - \frac{t^3}{3}$$

ALTRI INTEGRALI IMPROPRI

$$\int_a^{+\infty} f(x)\,dx = \lim_{t \to +\infty} \int_a^t f(x)\,dx$$

$$\int_{-\infty}^b f(x)\,dx = \lim_{t \to -\infty} \int_t^b f(x)\,dx$$

$$\int_{-\infty}^{+\infty} f(x)\,dx = \lim_{a \to -\infty} \left[\lim_{b \to +\infty} \int_a^b f(x)\,dx \right] = \lim_{\substack{a \to -\infty \\ b \to +\infty}} \int_a^b f(x)\,dx$$