Problemi con i limiti

21 ott 2020

Limiti

1. $$\lim_{x\to 0^-} 2^{\frac{1}{\arcsin x}} = 2^{\frac{1}{0^-}} = 2^{-\infty} = 0^+$$

2. $$\lim_{x\to 1} \left(x + e^x\right)^{\ln x} = (1+e)^0 = 1$$

3. $$\lim_{x\to 0} \frac{\operatorname{sen} 3x}{\operatorname{sen} 5x} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{\underbrace{\frac{\operatorname{sen} 3x}{3x}}_{\to 1} \cdot 3\cancel{x}}{\underbrace{\frac{\operatorname{sen} 5x}{5x}}_{\to 1} \cdot 5\cancel{x}} = \frac{3}{5}$$

4. $$\lim_{x\to 0} \left(1 - \frac{3}{4}x\right)^{1/x} = \left[1^\infty\right] =$$

sostituisco $X = \frac{-4}{3x} \Rightarrow \begin{cases} x \to 0 \Rightarrow X \to \infty \\ 1/x = -\frac{3}{4}X \end{cases}$

$$= \lim_{X\to 0} \left(1 + \frac{1}{X}\right)^{X\left(-\frac{3}{4}\right)} = e^{-\frac{3}{4}} = \frac{1}{\sqrt[4]{e^3}}$$

5. $$\lim_{x\to 0} \frac{x}{\log_3(1+x)} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \left[\frac{\log_3(1+x)}{x}\right]^{-1} = \lim_{x\to 0} \left\{\log_3\Big[\underbrace{(1+x)^{x}}_{\to e}\Big]\right\}^{-1} = \left(\log_3 e\right)^{-1} = \ldots = \ln 3$$

6. $$\lim_{x\to 0} \frac{2^x - 1}{\log_3(1+x)} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{\underbrace{\frac{2^x-1}{x}}_{\to \ln 2}}{\underbrace{\frac{\log_3(x+1)}{x}}_{\to 1/\ln 3}} = \ln 2 \cdot \ln 3$$