DERIVATE

10 feb '21

ex $\quad y = \dfrac{\sqrt{-x^2+5x-6}}{x}$

CE $\begin{cases} x \neq 0 \\ -x^2+5x-6 \geq 0 \end{cases}$

$x \in [2;3]$

limiti

$\lim_{x \to 2} f(x) = 0$

$\lim_{x \to 3} f(x) = 0$

segno $\quad f(x) > 0 \quad \forall x \in [2;3]$

zeri $\quad f(x) = 0 \iff x \in \{2;3\}$

derivata prima

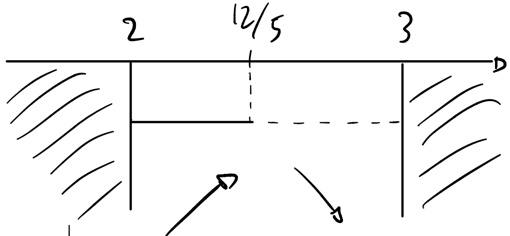
$$y' = \frac{D\left[\sqrt{-x^2+5x-6}\right]\cdot x - D(x)\sqrt{-x^2+5x-6}}{x^2} =$$

$$= \frac{\dfrac{(-2x+5)x}{2\sqrt{-x^2+5x-6}} - \sqrt{-x^2+5x-6}}{x^2} =$$

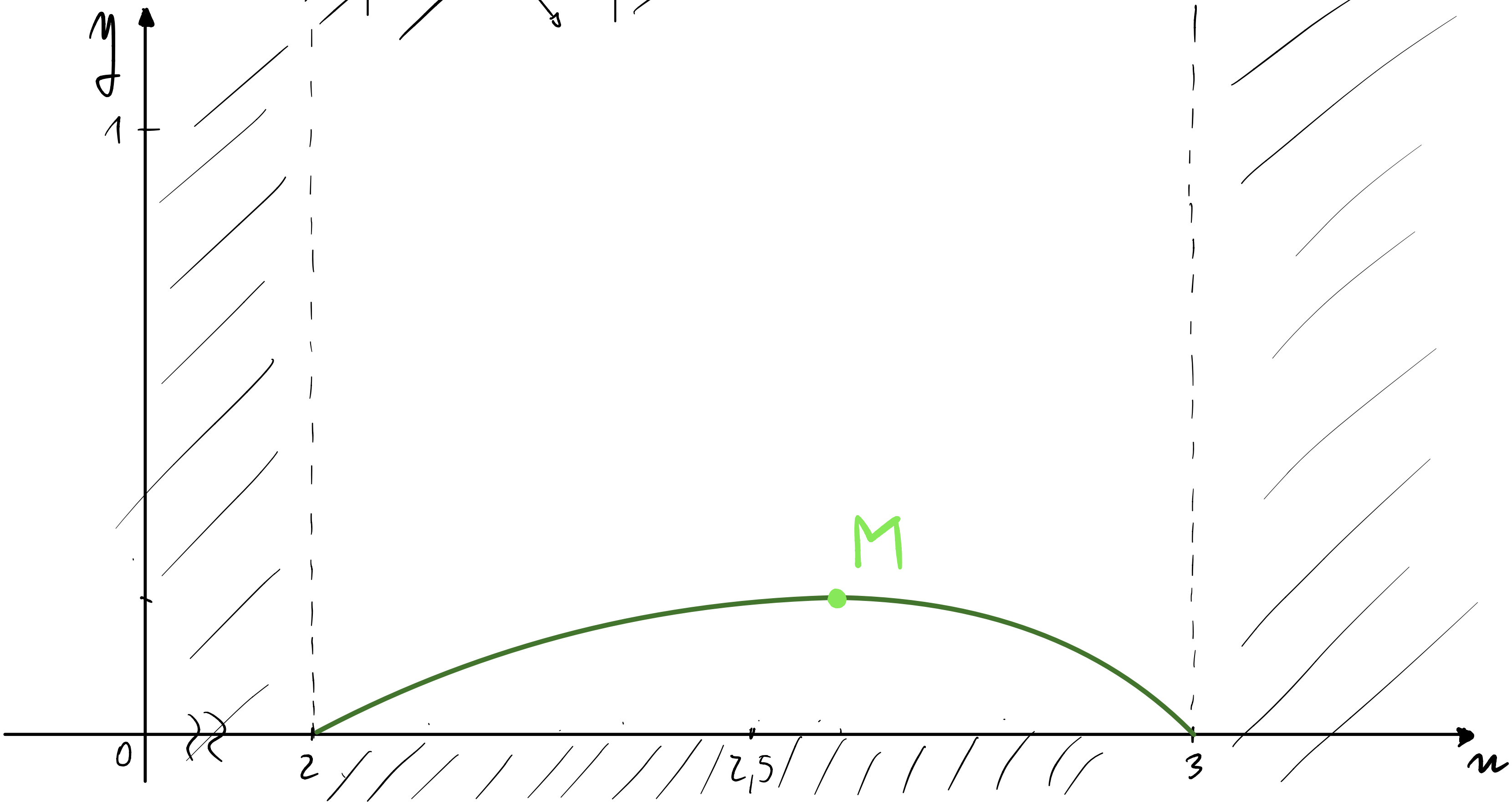
$$= \frac{\cancel{-2x^2}+5x+\cancel{2x^2}-10x+12}{2x^2\sqrt{-x^2+5x-6}} = \frac{-5x+12}{2x^2\sqrt{-x^2+5x-6}}$$

$$y' = 0 \Leftrightarrow x = \frac{12}{5} \rightsquigarrow M\left(\frac{12}{5}; \frac{\sqrt{6}}{12}\right)$$

$y' > 0$



$M \rightsquigarrow$ pto max



ex

$$y = \frac{1}{\tan^2 x} + \frac{2}{\tan x} = \frac{1+2\tan x}{\tan^2 x}$$

CE $\begin{cases} x \neq 0 + k\pi \\ x \neq \frac{\pi}{2} + k\pi \end{cases}$

$x \in \left(0; \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}; \pi\right)$

limiti

$$\lim_{x \to 0^+} \frac{1+2\tan x}{\tan^2 x} = \frac{1}{0^+} = +\infty$$

$x = 0$ A.O. dx

$$\lim_{x \to \frac{\pi}{2}^-} \frac{1+2\tan x}{\tan^2 x} \overset{H}{=} \lim_{x \to \frac{\pi}{2}^-} \left(\frac{\cancel{2} / \cancel{\cos^2 x}}{\cancel{2} \tan x} \cdot \cancel{\cos^2 x} \right) = \frac{1}{+\infty} = 0^+$$

$$\lim_{x \to \frac{\pi}{2}^+} \frac{1+2\tan x}{\tan^2 x} \overset{H}{=} \lim_{x \to \frac{\pi}{2}^+} \left(\frac{\cancel{2} / \cancel{\cos^2 x}}{\cancel{2} \tan x} \cdot \cancel{\cos^2 x} \right) = \frac{1}{-\infty} = 0^-$$

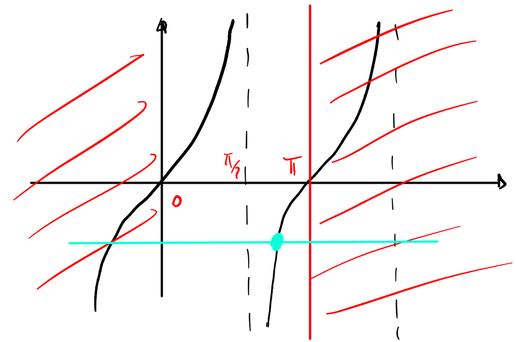
$x = \pi/2$ p.to discontinuità III specie

$$\lim_{x \to \pi^-} \frac{1+2\tan x}{\tan^2 x} = \frac{1}{0^+} = +\infty$$

$x = \pi$ A.O. SX

zeri

$$\frac{1+2\tan x}{\tan^2 x} = 0 \rightsquigarrow \tan x = -\frac{1}{2}$$



$$x = \pi + \arctan\left(-\tfrac{1}{2}\right) \approx 2,68$$

segno

$$1+2\tan x > 0$$

$$2\tan x > -1 \rightsquigarrow \tan x > -\tfrac{1}{2} \Rightarrow x \in \left(0; \tfrac{\pi}{2}\right) \cup \left(\pi + \arctan\left(-\tfrac{1}{2}\right); \pi\right)$$

derivata prima

$$y' = D\left\{\frac{1+2\tan x}{\tan^2 x}\right\} = \frac{D(1+2\tan x)(\tan^2 x) - D(\tan^2 x)(1+2\tan x)}{\tan^4 x} =$$

$$= \frac{(2/\cos^2 x)\tan^2 x - (2\tan x/\cos^2 x)(1+2\tan x)}{\tan^4 x} =$$

$$= \frac{\dfrac{2\sin^2 x}{\cos^4 x} - \dfrac{2\tan x(1+2\tan x)}{\cos^2 x}}{\tan^4 x} =$$

$$= \frac{2\sin^2 x - 2\cos^2 x\tan x(1+2\tan x)}{\sin^4 x} =$$

$$= \frac{2\sin^2 x - 2\cos^2 x\tan x - 4\cos^2 x\tan^2 x}{\sin^4 x} =$$

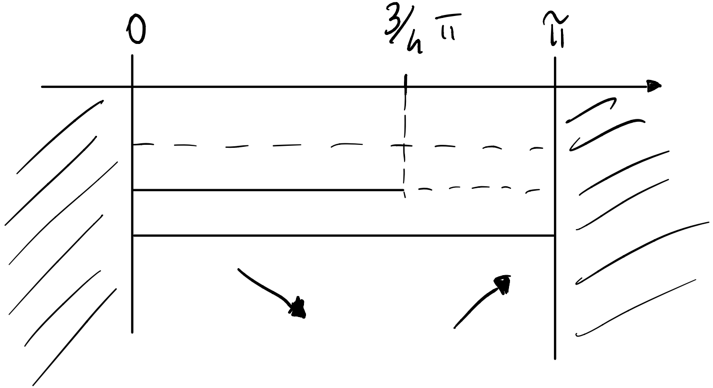
$$= \frac{2\,\mathrm{sen}^2 x - 2\,\mathrm{sen}\,x\cos x - 4\,\mathrm{sen}^2 x}{\mathrm{sen}^4 x} = -2\,\frac{\mathrm{sen}^{\cancel{2}} x + \cancel{\mathrm{sen}\,x}\cos x}{\mathrm{sen}^{\cancel{4}\,3} x} =$$

$$= \frac{-2(\mathrm{sen}\,x + \cos x)}{\mathrm{sen}^3 x} = \frac{-2\sqrt{2}\,\mathrm{sen}(x + \pi/4)}{\mathrm{sen}^3 x}$$

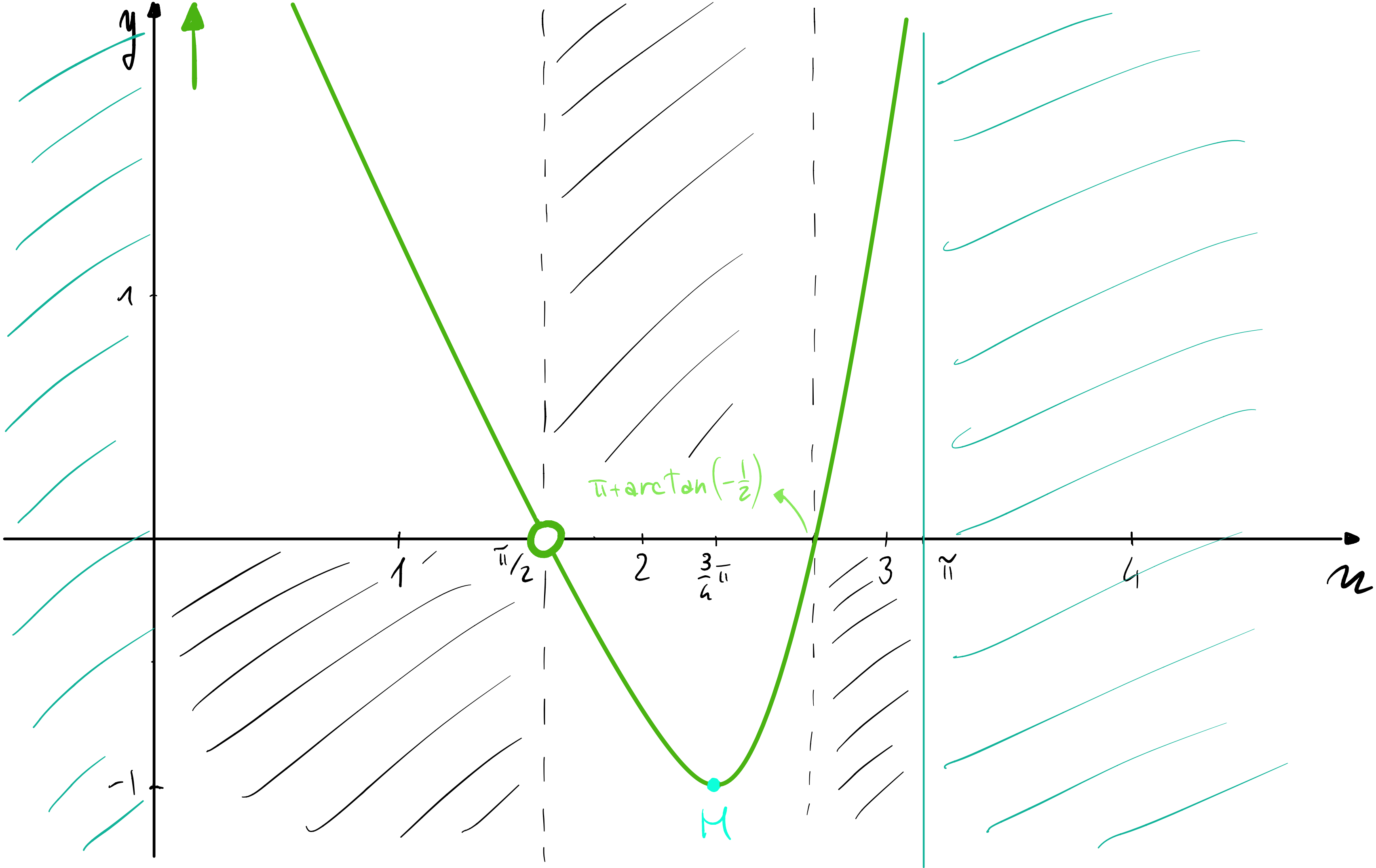
$$y' = 0 \leadsto \mathrm{sen}\left(x + \frac{\pi}{4}\right) = 0$$

$$x + \frac{\pi}{4} = 0 + 2k\pi \;\vee\; x + \frac{\pi}{4} = \pi + 2k\pi \Rightarrow x = \frac{3}{4}\pi$$

$y' > 0$



$M\left(\frac{3}{4}\pi ; -1\right) \leadsto$ p.to min



y
x
1
-1
1
π/2
2
3/4 π
3
π
4
π+arctan(−1/2)
M