INTEGRALI

21 apr 2021

CASO 3a $\quad N(x) = D'(x)$

- $\displaystyle\int \frac{6x}{4+x^2}\,dx = 3\int \frac{1}{4+x^2}\cdot 2x\,dx =$

$= 3\ln(4+x^2) + K$

nota:

$\displaystyle\int \frac{1}{f(x)} f'(x)\,dx =$

$= \ln|f(x)| + K$

CASO 3B $\quad \Delta > 0 \leadsto D(x)$ si scompone

- $\displaystyle\int \frac{5x-1}{x^2-x-2}\,dx = \int \frac{5x-1}{(x-2)(x+1)}\,dx =$

vedasi note pagina successiva

$\displaystyle= \dots = \int \left[\frac{3}{x-2} + \frac{2}{x+1}\right] dx = 3\ln|x-2| + 2\ln|x+1| + K$

$= \ln\left[|x-2|^3 \cdot (x+1)^2\right] + K$

nota:

$$\frac{3}{x+1}+\frac{4}{x-2}=\frac{7x-2}{(x+1)(x-2)}$$

posso fare il processo inverso ⊛

$$\int \frac{7x-2}{(x+1)(x-2)}\,dx=$$

$$=\underbrace{\int \frac{3}{x+1}\,dx}_{\frac{1}{f'(x)}\ \rightsquigarrow\ \text{logaritmo}}+\int \frac{4}{x-2}\,dx$$

nota 2: ⊛

$$\frac{7x-2}{(x+1)(x-2)} \ (*) = \frac{A}{x+1}+\frac{B}{x-2}=$$

$$=\frac{(x-2)A+(x+1)B}{(x+1)(x-2)}$$

$$=\frac{x(A+B)+(2A-B)}{(x+1)(x-2)} \ (**)$$

$$\begin{cases} A+B=7 \\ 2A-B=-2 \end{cases} \quad \ldots$$

il polinomio (*) è uguale a (**)
per il principio d'identità dei polinomi

$$-\int \frac{1}{3x^2-x-2}\,dx = \int \frac{1}{(x-1)(3x+2)}\,dx =$$

$$= \int \frac{1/5}{x-1}\,dx - \int \frac{3/5}{3x+2}\,dx =$$

$$= \frac{\ln|x-1| - \ln|3x+2|}{5} + K =$$

$$= \ln \sqrt[5]{\left|\frac{x-1}{3x+2}\right|} + K$$

calcoli

$$\frac{1}{3x^2-x-2} = \frac{A}{x-1} + \frac{B}{3x+2}$$

$$\frac{(3A+B)x + 2A-B}{(x-1)(3x+2)}$$

$$\begin{cases} 3A+B=0 \\ 2A-B=1 \end{cases} \qquad A=\frac{1}{5} \qquad B=-\frac{3}{5}$$

CASO 3C $\Delta = 0$

- 3C.a $\deg N(x) = 0$
- 3C.b $\deg N(x) = 1$
- 3C.c $\deg N(x) = 2$

3C.a - $\displaystyle\int \frac{2}{x^2-6x+9}\,dx = 2\int (x-3)^{-2}\,dx =$

nota:

$\displaystyle\int [f(x)]^n \cdot f'(x)\,dx =$

...

3C.b - $\displaystyle\int \frac{2x-1}{x^2+2x+1}\,dx = \int \frac{(2x+2)-3}{(x^2+2x+1)}\,dx = \int (x^2+2x+1)^{-1} \cdot Df(x)\,dx$

$\displaystyle - 3\int \frac{1}{(x+1)^2}\,dx$

entrambi casi precedenti ...

$$-\int \frac{3x-1}{x^2+2x+1}\,dx \quad (*)$$

$$(*) = \frac{A}{x+1} + \frac{B}{(x+1)^2} =$$

di grado 0

$$= \frac{A(x+1)+B}{(x+1)^2}$$

situazioni precedenti

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