INTEGRALI

26 mar '21

$\int f(x)\,dx = F(x) + K$

integrazione indefinita → restituisce un insieme di funzioni

$F'(x) = f(x)$

$\int_a^b f(x) = \mathcal{A}$

b estremo superiore

a estremo inferiore

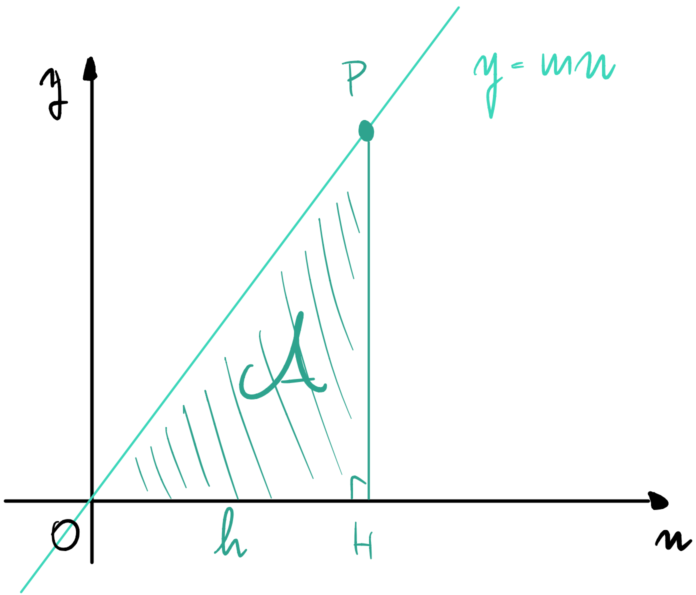
integrazione definita → restituisce un'area

$f(x)$ continua in $[a; b]$

$f(x) \geq 0$

$a < b$

Applicazioni



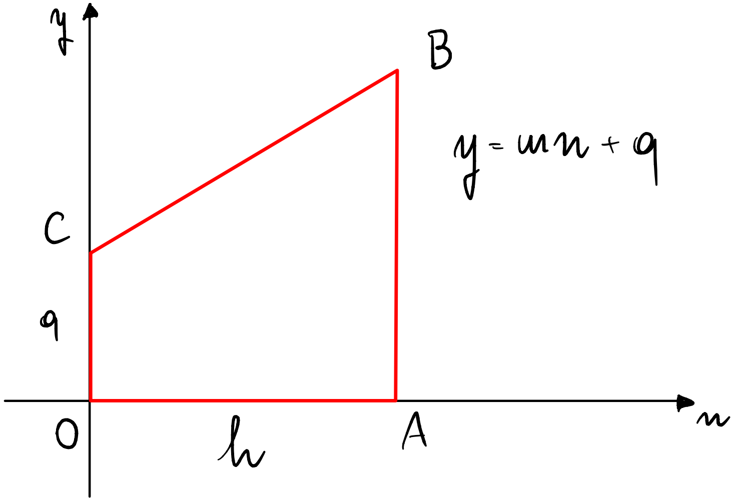
$$\mathcal{A} = \frac{1}{2}\,OH \cdot PH = \frac{1}{2}\,h \cdot mh = \frac{1}{2}\,mh^2$$

$$\mathcal{A}_{OPH} = \int_0^h mx\,dx = m\int_0^h x\,dx =$$

$$= m\left[\frac{x^2}{2}\right]_0^h = \frac{1}{2}\,m\left[x^2\right]_0^h =$$

$$= \frac{1}{2}\,m\left(h^2 - 0^2\right) = \frac{1}{2}\,mh^2$$

TRAPEZIO



$O(0;0)$

$A(h;0)$

$B(h;mh+q)$

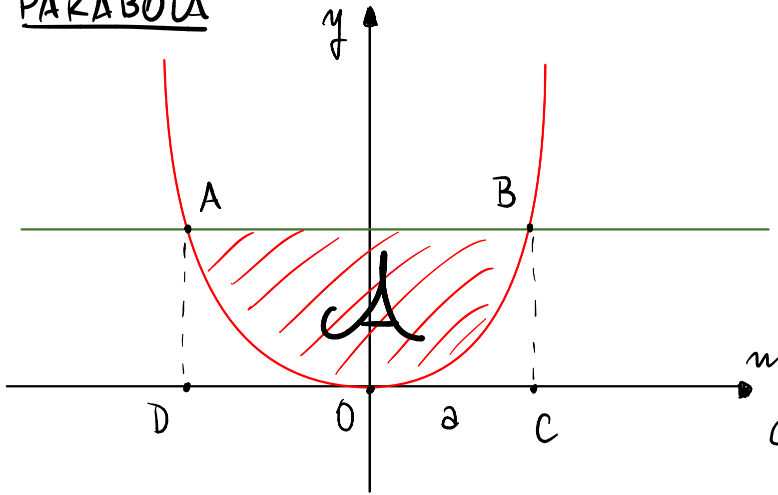
$C(0;q)$

$$\mathcal{A}(OABC) = \frac{1}{2}(q + mh + q)h = \frac{1}{2}mh^2 + qh$$

$$\mathcal{A} = \int_0^h (mx+q)\,dx = \left[m\frac{x^2}{2} + qx\right]_0^h$$

$$= \frac{mh^2}{2} + qh$$

PARABOLA



$A(-a; a^2)$ $\quad$ $C(a; 0)$

$B(a; a^2)$ $\quad$ $D(-a; 0)$

$$\mathcal{A} = \frac{2}{3}\mathcal{A}(ABCD) = \frac{2}{3}\,2a \cdot a^2 = \frac{4}{3}a^3$$

$$\mathcal{A} = \mathcal{A}_{RETTANGOLO} - \mathcal{A}_{PARABOLA}\,(x=-a;\ x=a)$$

$$\mathcal{A}_{PARABOLA} = \int_{-a}^{a} x^2\,dx = \left[\frac{x^3}{3}\right]_{-a}^{a} = \frac{a^3}{3} - \frac{(-a)^3}{3} = \frac{2}{3}a^3$$

$$\mathcal{A}_{SEG\ PARABOLICO} = (2a \cdot a^2) - \frac{2}{3}a^3 = 2a^3 - \frac{2}{3}a^3 = \boxed{\frac{4}{3}a^3}$$

Se la retta non é orizzontale si fa

$A(\text{trapezio}) - A(\text{parabola})$

$\rightarrow \int_1^2 \left(x^2 + 1/x^2\right) dx =$

controllo se $f(x) \geq 0$ in $[1;2]$

$x^2 + 1/x^2 > 0 \quad \forall x \in \mathbb{R} - \{0\}$

$= \left[\frac{x^3}{3} - \frac{1}{x}\right]_1^2 = F(2) - F(1) =$

$= \left(\frac{8}{3} - \frac{1}{2}\right) - \left(\frac{1}{3} - 1\right) = 17/6$ $\square$

$\rightarrow \int_0^2 \frac{4x}{1+x^2}\, dx = (*)$

$f(x) \geq 0$ in $[0;2]$

$(*)\ 2\int (x^2+1)^{-1} \cdot 2x\, dx =$

$= 2\ln(x^2+1) + K$

$= \left[2\ln(x^2+1)\right]_0^2 =$

$= 2\ln 5 - 2\ln 1 = 2\ln 5$ $\square$