Limiti notevoli

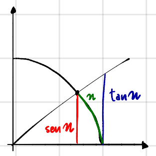
14 ott 2020

Limite notevole: $\lim\limits_{x\to 0} \frac{\operatorname{sen} x}{x} = 1$

Dim $\lim\limits_{x\to 0} \frac{\operatorname{sen} x}{x} = 1$ (x espresso in radianti)

$$\lim_{x\to 0} \frac{\operatorname{sen} x}{x} = 1 \iff \begin{cases} \lim\limits_{x\to 0^+} \frac{\operatorname{sen} x}{x} = 1 & \bullet \\ \lim\limits_{x\to 0^-} \frac{\operatorname{sen} x}{x} = 1 & \bullet\bullet \end{cases}$$

$\bullet$ dim $\lim\limits_{x\to 0^+} \frac{\operatorname{sen} x}{x} = 1$



$x \in (0; x_1)$

$$\operatorname{sen} x \le x_R \le \tan x \qquad / \operatorname{sen} x$$

$$\frac{\operatorname{sen} x}{\operatorname{sen} x} \le \frac{x}{\operatorname{sen} x} \le \frac{\tan x}{\operatorname{sen} x} \Rightarrow 1 \le \frac{x}{\operatorname{sen} x} \le \frac{1}{\cos x}$$

faccio il reciproco

$$1 \ge \frac{\operatorname{sen} x}{x} \ge \cos x$$

$$\left.\begin{array}{l} \cos x \le \frac{\operatorname{sen} x}{x} \le 1 \\ \lim\limits_{x\to 0^+} \cos x = 1 \;;\; \lim\limits_{x\to 0^+} 1 = 1 \end{array}\right] \Rightarrow \lim_{x\to 0^+} \frac{\operatorname{sen} x}{x} = 1$$

(Teorema del confronto)

$\bullet\bullet$ dim. $\lim\limits_{x\to 0^-} \frac{\operatorname{sen} x}{x} = 1$

$$f(x) \overset{?}{=} f(-x) \;;\quad \frac{\operatorname{sen} x}{x} = \frac{\operatorname{sen}(-x)}{-x} \;;\quad \frac{\operatorname{sen} x}{x} = \frac{-\operatorname{sen} x}{-x} = \frac{\operatorname{sen} x}{x}$$

$$f(x) = f(-x) \iff \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) \Rightarrow \lim_{x\to 0^-} \frac{\operatorname{sen} x}{x} = 1$$

$\bullet\bullet\bullet$ $\exists \lim\limits_{x\to 0^+} \frac{\operatorname{sen} x}{x} = 1 \wedge \exists \lim\limits_{x\to 0^-} \frac{\operatorname{sen} x}{x} = 1 \Rightarrow \boxed{\lim\limits_{x\to 0} \frac{\operatorname{sen} x}{x} = 1}$

Limite notevole $\lim\limits_{x\to 0} \frac{1-\cos x}{x}$

$$\lim_{x\to 0} \frac{1-\cos x}{x} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{1-\cos x}{x} \cdot \frac{1+\cos x}{\underbrace{1+\cos x}_{\neq 0}} = \lim_{x\to 0} \frac{1-\cos^2 x}{x(1+\cos x)} = \lim_{x\to 0} \frac{\operatorname{sen}^2 x}{x(1+\cos x)} =$$

$$= \lim_{x\to 0} \underbrace{\frac{\operatorname{sen} x}{x}}_{\to 1} \cdot \underbrace{\frac{\operatorname{sen} x}{1+\cos x}}_{\to 0} = 0$$

Limite notevole $\lim_{x\to0}\frac{1-\cos x}{x^2}$

$$\lim_{x\to0}\frac{1-\cos x}{x}=\left[\frac{0}{0}\right]=\lim_{x\to0}\frac{1-\cos x}{x^2}\cdot\frac{1+\cos x}{\underbrace{1+\cos x}_{\neq0}}=\lim_{x\to0}\frac{1-\cos^2 x}{x^2(1+\cos x)}=\lim_{x\to0}\frac{\operatorname{sen}^2 x}{x^2(1+\cos x)}=$$

$$=\lim_{x\to0}\underbrace{\left(\frac{\operatorname{sen} x}{x}\right)^2}_{\to1}\cdot\underbrace{\frac{1}{1+\cos x}}_{\to1/2}=\frac{1}{2}$$

- $\lim_{x\to0}\frac{\operatorname{sen}(\ell x)}{x}=\left[\frac{0}{0}\right]$ sost. variabile $X=\ell x \rightsquigarrow x=X/\ell$ $\circledast$

  $x\to0 \Rightarrow X\to0$

  $\lim_{X\to0}\underbrace{\frac{\operatorname{sen} X}{X}}_{\to1}\cdot\ell=\ell$

- $\lim_{x\to0}\frac{\tan x-\operatorname{sen} x}{x^3}=\lim_{x\to0}\frac{\operatorname{sen} x}{x}\cdot\frac{\left(1/\cos x-1\right)}{x^2}=\lim_{x\to0}\underbrace{\frac{\operatorname{sen} x}{x}}_{\to1}\cdot\underbrace{\frac{1-\cos x}{x^2}}_{\to1/2}\cdot\underbrace{\frac{1}{\cos x}}_{\to1}=\frac{1}{2}$