EQUAZIONI DIFFERNZIALI

21 mag '21

A VARIABILI SEPARABILI

ex

$$y'/x = 1 + y^2$$

$$\frac{dy}{x\,dx} = 1 + y^2 \Rightarrow \frac{dy}{1+y^2} = x\,dx$$

$$\int \frac{1}{1+y^2}\,dy = \int x\,dx$$

$$\arctan y = \frac{x^2}{2} + K \Rightarrow y = \tan\left(\frac{x^2}{2} + K\right)$$

LINEARI DI I ORDINE

$$y' = a(x) \cdot y + b(x)$$

se $b(x) = 0$ $\leadsto$ omogenee

$$y' = a(x) \cdot y \quad \Longrightarrow \quad \frac{dy}{dx} = a(x) \cdot y$$

$$\frac{dy}{y} = a(x) \cdot dx$$

$$\int \frac{1}{y}\, dy = \int a(x)\, dx$$

$$\ln|y| = \int a(x)\, dx + c \quad \Longrightarrow \quad |y| = e^{\int a(x)\, dx + c}$$

$$y = \pm\, e^{\int a(x)\, dx + c} = \underbrace{\pm\, e^{c}}_{\text{costante}} \cdot e^{\int a(x)\, dx} = K \cdot e^{\int a(x)\, dx} \quad ; \quad K \in \mathbb{R}$$

se $b(x) \neq 0 \rightsquigarrow$ complete

p. 2034

$$y' = a(x) \cdot y + b(x)$$

PREM 1 Sia $A(x) = \int a(x)\, dx$

PREM 2 $D\left[e^{-A(x)} \cdot y\right] = e^{-A(x)} \cdot y' + D\left[e^{-A(x)}\right] \cdot y =$

$$= \left[y' \cdot e^{-A(x)} - y \cdot a(x) \cdot e^{-A(x)}\right]$$

DIM $y' - a(x) \cdot y = b(x)$

$$e^{-A(x)} \cdot \left[y' - a(x) \cdot y\right] = \underbrace{e^{-A(x)}}_{\neq 0 \ \forall x} b(x) \Rightarrow y' \cdot e^{-A(x)} - y \cdot a(x) \cdot e^{-A(x)} = e^{-A(x)} b(x)$$

$$D\left[e^{-A(x)}\cdot y\right] = e^{-A(x)}\, b(x)$$

$$e^{-A(x)}\cdot y + K = \int e^{-A(x)}\cdot b(x)\,dx \Rightarrow \quad y = e^{A(x)}\left[\int e^{-A(x)}\cdot b(x)\,dx + K\right]$$

$$y = e^{\int a(x)\,dx}\left[\int e^{-\int a(x)\,dx}\cdot b(x)\,dx + K\right]$$

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ex $y' = -\frac{y}{x} + x \; ; \; x > 0$

$$y' = -\frac{1}{x} \cdot y + x$$

$a(x) = -1/x$

$b(x) = x$

$$\rightarrow \int a(x)\,dx = -\int 1/x\,dx = -\ln|x| + K$$

$$\rightarrow e^{-\int a(x)\,dx} = e^{-(-\ln|x|)} = e^{\ln|x|} = x$$

$$\rightarrow \int b(x) \cdot e^{-\int a(x)\,dx}\,dx = \int x \cdot x \cdot dx = \frac{x^3}{3} + K$$

$$\rightarrow y = e^{-\ln|x|} \cdot \left[\frac{x^3}{3} + K\right] = \frac{1}{x} \cdot \left[\frac{x^3}{3} + K\right] = x^2/3 + K/x$$