GRAFICI QUALITATIVI

30 ott 2020

① $y = \sqrt[3]{x^2(x-1)}$

a) CE: $D: x \in \mathbb{R}$ $\rightsquigarrow$ NO ASINTOTI VERTICALI

b) $\lim_{x \to -\infty} \sqrt[3]{x^2(x-1)} = -\infty$

$\lim_{x \to +\infty} \sqrt[3]{x^2(x-1)} = +\infty$

$\rightsquigarrow$ NO ASINTOTI ORIZZONTALI

ASINTOTI OBLIQUI

$$m = \lim_{x \to \infty} \frac{\sqrt[3]{x^3 - x^2}}{x} = \left[\frac{\infty}{\infty}\right] = \lim_{x \to \infty} \frac{\cancel{x}\sqrt{1 - \frac{1}{x}}}{\cancel{x}} = 1$$

$\frac{1}{x} \to 0$

$$q = \lim_{x \to \infty} \left[\sqrt[3]{x^3 - x^2} - x\right] = [\infty - \infty] =$$

razionalizzo

$(a^3 - b^3) = (a-b)(a^2 + ab + b^2)$

$$= \lim_{x \to \infty} \frac{\left(\sqrt[3]{x^2(x-1)} - x\right) \cdot \left[\left(\sqrt[3]{x^2(x-1)}\right)^2 + x\sqrt{x^2(x-1)} + x^2\right]}{\left[\left(\sqrt[3]{x^2(x-1)}\right)^2 + x\sqrt{x^2(x-1)} + x^2\right]} =$$

$$= \lim_{x \to \infty} \frac{\left(\sqrt[\cancel{3}]{x^2(x-1)}\right)^{\cancel{3}} - x^3 \ (*)}{\left[\left(\sqrt[3]{x^2(x-1)}\right)^2 + x\sqrt{x^2(x-1)} + x^2\right]} = \left[\frac{\infty}{\infty}\right] =$$

$(*)$ $x^2(x-1) - x^3 = x^2(x - 1 - x) = -x^2$

$$= \lim_{n\to\infty} \frac{-n^2}{\sqrt[3]{(n^3-n^2)^2} + n\sqrt[3]{n^3-n^2} + n^2} =$$

$$= \lim_{n\to\infty} \frac{-n^2}{\sqrt[3]{n^6-2n^5+n^4} + n^2\sqrt[3]{1-\frac{1}{n}} + n^2} =$$

$$= \lim_{n\to\infty} \frac{-n^2}{n^2\sqrt[3]{1-\frac{2}{n}+\frac{1}{n^2}} + n^2\sqrt[3]{1-\frac{1}{n}} + n^2} =$$

$$= \lim_{n\to\infty} \frac{-\cancel{n^2}}{\cancel{n^2}\left\{\sqrt[3]{1-\underbrace{\frac{2}{n}}_{\to 0}+\underbrace{\frac{1}{n^2}}_{\to 0}} + \sqrt[3]{1-\underbrace{\frac{1}{n}}_{\to 0}} + 1\right\}} = -\frac{1}{3}$$

$$y = mn + q \Rightarrow \boxed{y = n - \frac{1}{3}}$$

c) ZERI e $\cap$

$$\begin{cases} \sqrt[3]{x^2(x-1)} = y \\ y = 0 \end{cases}$$

$$\sqrt[3]{x^2(x-1)} = 0 \Rightarrow x^2 = 0 \lor x - 1 = 0;$$

$x = 0$ doppio $\rightsquigarrow$ la curva é tangente all'asse

$x = 1$

$A(0;0)$; $B(1;0)$

$$\begin{cases} \sqrt[3]{x^2(x-1)} = y \\ y = x - \frac{1}{3} \end{cases} \Rightarrow \left(\sqrt[3]{x^3 - x^2}\right)^3 = \left(x - \frac{1}{3}\right)^3$$

$$\cancel{x^3} - \cancel{x^2} = \cancel{x^3} - \cancel{x^2} + \frac{1}{3}x - \frac{1}{27}$$

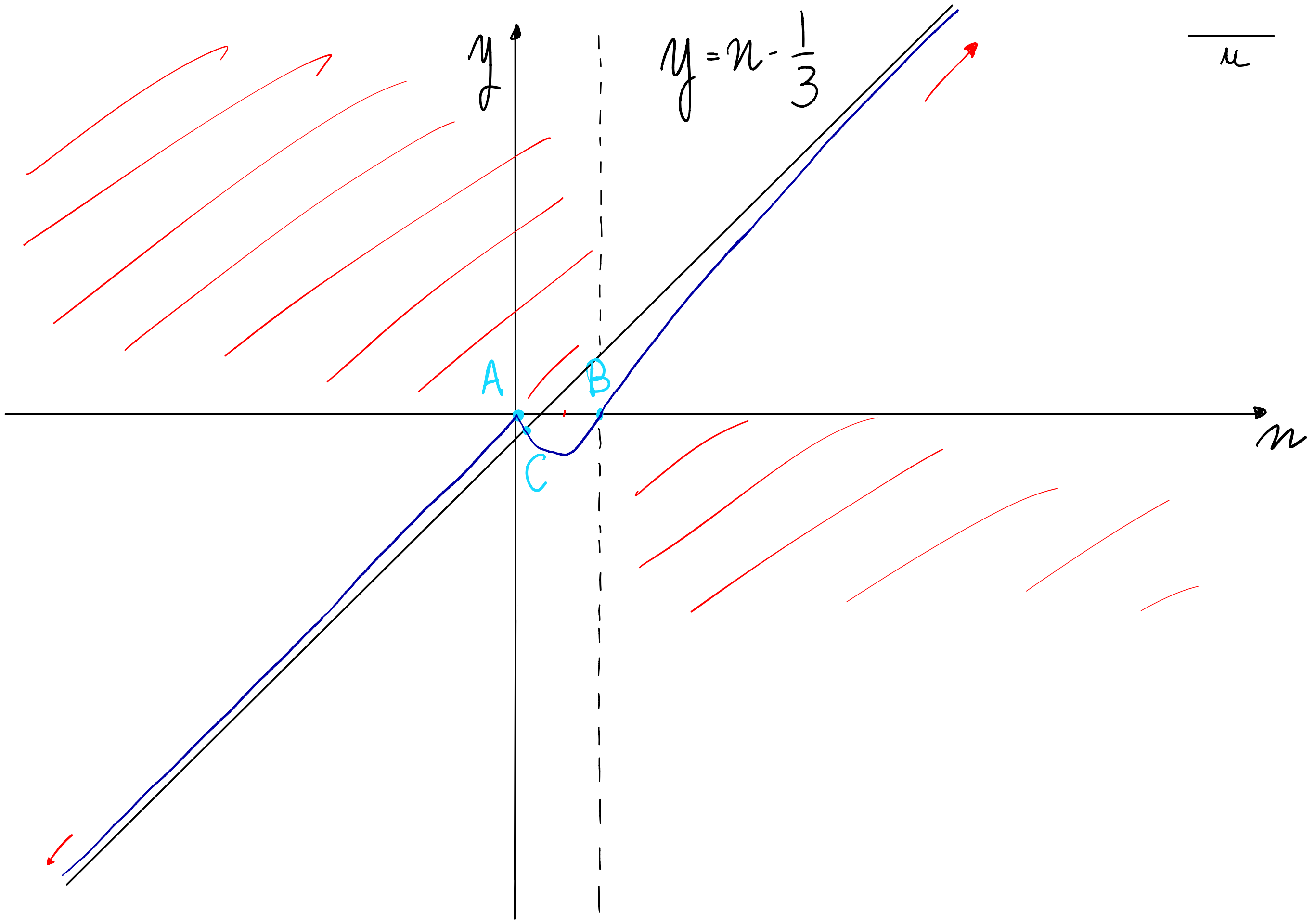
$$\frac{1}{3}x = \frac{1}{27} \Rightarrow x = \frac{1}{9} \Rightarrow y = -\frac{2}{9}$$

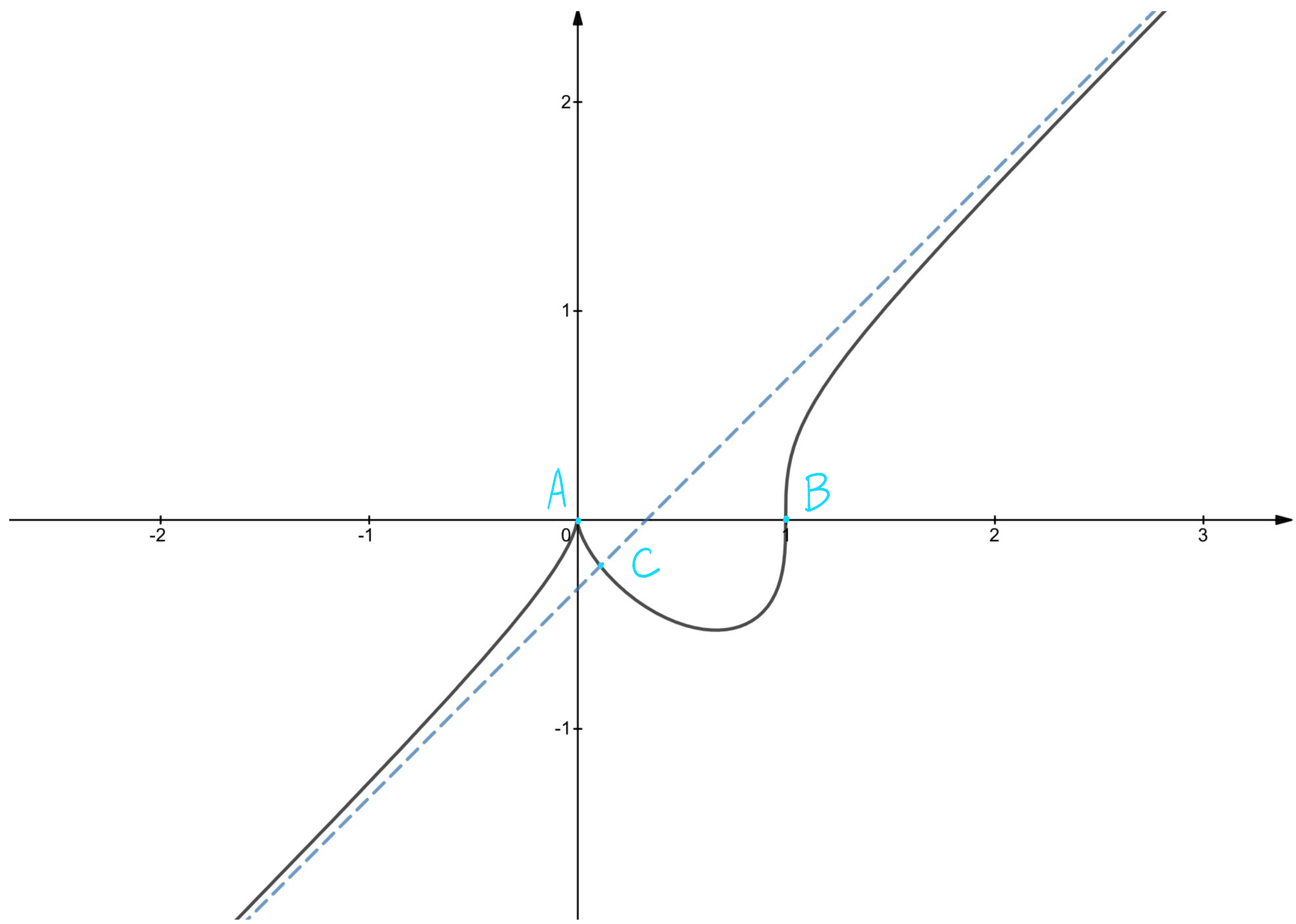
$C\left(\frac{1}{9}; -\frac{2}{9}\right)$

d) SEGNI

$$\sqrt[3]{x^2(x-1)} \geq 0 \iff x - 1 \geq 0 \lor x = 0$$

		0		1	
x^2	+	0	+	+	+
$(x-1)$	−		−	0	+
	−	0	−	0	+


y
$y = x - \frac{1}{3}$
x
A
B
C


2
1
-1
-2
-1
0
1
2
3
A
B
C