CONTINUITÀ

6 nov 2020

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ex. $f(x) = \begin{cases} x^2 - 2b & x < -1 \\ 2x - b & -1 \leq x < 3 \\ \sqrt{2x + a} & x \geq 3 \end{cases}$ • $a, b \mid f(x)$ continua

i $\quad x = -1 \quad \lim_{x \to -1} f(x) = f(-1) \Rightarrow \lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) = f(-1)$

$$\underbrace{\lim_{x \to -1^-} (x^2 - 2b)}_{1-2b} = \underbrace{\lim_{x \to -1^+} (2x - b)}_{-2-b} = \underbrace{2 \cdot (-1) - b}_{-2-b}$$

$1 - 2b = -2 - b \Rightarrow b = 3$

$$f(x) = \begin{cases} x^2 - 6 & x < -1 \\ 2x - 6 & -1 \leq x < 3 \\ \sqrt{2x + a} & x \geq 3 \end{cases}$$

ii $\quad x = 3 \quad \lim_{x \to 3} f(x) = f(3) \Rightarrow \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$

$$\underbrace{\lim_{x \to 3^-} (2x - 3)}_{3} = \lim_{x \to 3^+} \sqrt{2x + a} = \sqrt{6 + a}$$

$3 = \sqrt{6 + a} \Rightarrow a = 3$

ex

$$f(x) = \begin{cases} \dfrac{\operatorname{sen} ax}{x} & x < 0 \\ x^2 + 2a + 1 & x \geq 0 \end{cases}$$

• $a \mid f(x)$ continua $\mathbb{R}$

• i $f(x)$ continua per $x \neq 0$, in quanto composta da funzioni continue

ii $\lim_{x \to 0} f(x) = f(0) \Rightarrow \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$

$$\lim_{x \to 0^-} \frac{\operatorname{sen} ax}{x} = \lim_{x \to 0^+} (x^2 + 2a + 1) = 2a + 1$$

$$\|$$

$$\lim_{x \to 0^-} \frac{a \cdot \operatorname{sen} ax}{a x} = a = 2a + 1 \Rightarrow a = -1$$

ex

$$f(x) = \begin{cases} \ln(1 - x) - 2a & x \leq 0 \\ \dfrac{\cos x - e^x}{2ax} & x > 0 \end{cases}$$

• $a \mid f(x)$ continua in CE

i CE $1 - x > 0 \Rightarrow x < 1$ okay

$x \neq 0$ okay

CE: $\mathbb{R}$

ii $f(x)$ continua per $x \neq 0$ in quanto composta da funzioni continue

iii $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$

$$\lim_{x\to0^-}\left[\ln(1-x)-2a\right] = \lim_{x\to0^+}\frac{\cos x - e^x}{2ax} = \ln 1 - 2a$$

$$-2a = \lim_{x\to0^+}\frac{\cos x - e^x}{2ax} = \left[\frac{0}{0}\right]$$

NOTA

$$\lim_{x\to0}\frac{1+\cos x}{x} = 0$$

$$\lim_{x\to0}\frac{e^x-1}{x} = 1$$

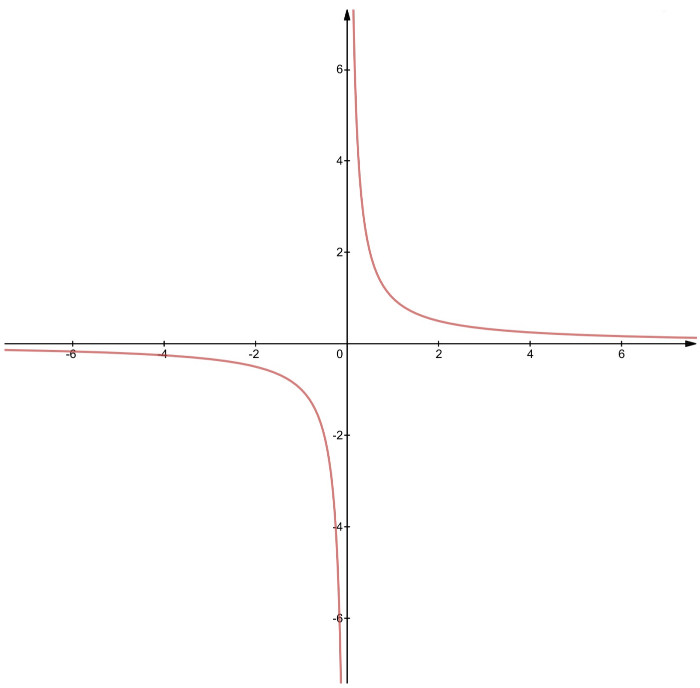
$$-2a = \lim_{x\to0^+}\frac{\cos x - 1 + 1 - e^x}{2ax} =$$

$$-2a = \lim_{x\to0^+}\left(\underbrace{\frac{\cos x - 1}{2ax}}_{\to 0} + \underbrace{\frac{-e^x+1}{2ax}}_{\to -1/2a}\right) =$$

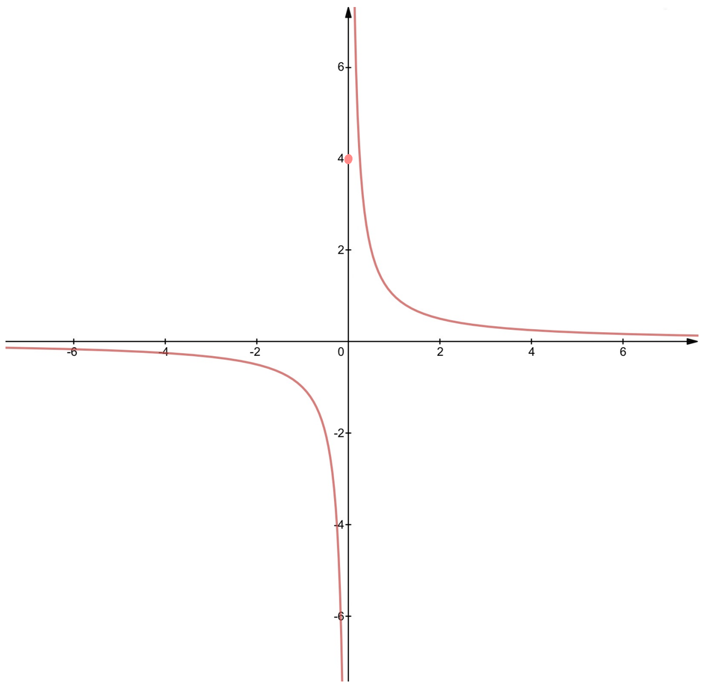
$$-2a = -\frac{1}{2a} \Rightarrow 4a^2 = 1 \Rightarrow a = \pm 1/2$$

OSS. $y = f(x) = \frac{1}{x}$ è continua? ▸ dipende dall'insieme

- CE: $(-\infty;0)\cup(0;+\infty)$ sì, continua
- $\mathbb{R}$ no, non è continua



ex. $y = \begin{cases} 1/x & \text{se } x \neq 0 \\ 4 & \text{se } x = 0 \end{cases}$



CE $\mathbb{R}$

$f(x)$ continua in $\mathbb{R}$?

- $f(x)$ continua per $x \neq 0$

$x = 0$ $\quad \underbrace{\lim_{x \to 0} f(x)}_{\pm\infty} \overset{?}{=} \underbrace{f(0)}_{4}$

NON É CONTINUA

continuità: materiale 2