Limiti

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Limite notevole $\lim_{x\to 0} \frac{\operatorname{sen} x}{x}$; x in gradi

$$\lim_{x_0\to 0} \frac{\operatorname{sen} x_0}{x_0} =$$

sostituisco $x_0 : x_R = 360° : 2\pi$

$$x_0 = \frac{360°}{2\pi} x_R = x_R \frac{180°}{\pi}$$

$$\operatorname{sen} x_0 = \operatorname{sen} x_R$$

$$x_0 \to 0 \Leftrightarrow x_R \to 0$$

$$\lim_{x_R\to 0} \underbrace{\frac{\operatorname{sen} x_R}{x_R}}_{\to 0} \frac{1}{180/\pi} = \frac{\pi}{180°}$$

Forme indeterminate per $\lim_{x\to 0} \frac{\operatorname{sen} x}{x}$

$\boxed{[0\cdot\infty]}$ ▸ $\lim_{x\to\infty} x \cdot \operatorname{sen} 1/x = [0\cdot\infty]$; pongo $X = 1/x \Rightarrow x = 1/X$

$x \to \infty \Rightarrow X \to 0$

$$\lim_{X\to 0} \frac{\operatorname{sen} X}{X} = 1$$

$\boxed{[\frac{0}{0}]}$ ▸ $\lim_{x\to 0} \frac{\arctan x}{x} = [\frac{0}{0}]$; pongo $y = \arctan x \Rightarrow \begin{cases} x = \tan y \\ x\to 0 \Rightarrow y \to 0 \end{cases}$

$$\lim_{y\to 0} \frac{y}{\tan y} = \lim_{y\to 0} \underbrace{\frac{y}{\operatorname{sen} y}}_{\to 1} \cdot \cos y = (\cos 0) = 1$$

Esercizi

- $\lim_{x\to 2} x^{\sqrt{x-1}} = 2$
- $\lim_{x\to 0^+} \left(2x + 1/3\right)^{-2/x} = \left(\frac{1}{3}\right)^{-\frac{2}{0^+}} = \left(\frac{1}{3}\right)^{-\infty} = +\infty$
- $\lim_{x\to +\infty} \left(\frac{1}{x-1}\right)^{x} = 0$
- $\lim_{x\to 0^+} \left(\operatorname{sen}^2 x\right)^{1/x} = (0^+)^{\frac{1}{0^+}} = (0^+)^{+\infty} = 0$

Forme indeterminate esponenziali

sono di 3 tipi:
- $[0^0]$
- $[1^\infty]$
- $[\infty^0]$

$\boxed{[0^0]}$ $y = f(x)^{g(x)}$ ricondur $a = \ln e^a$; $a = e^{\ln a}$

$$y = e^{\ln[f(x)^{g(x)}]} = e^{g(x)\cdot \ln f(x)}$$ → ora basta discutere questo

- $\lim\limits_{n \to +\infty} n^{\frac{1}{\ln n}} = [0^0] = \lim\limits_{n \to \infty} e^{\frac{\ln n}{\ln n}} = e$

- $\lim\limits_{n \to 0^+} \left(\frac{n^2}{4}\right)^{\frac{1}{3\ln n}} = [0^0]$; $\quad y = \left(\frac{n^2}{4}\right)^{\frac{1}{3\ln n}} = e^{\frac{\ln \frac{n^2}{4}}{3\ln n}} = e^{\frac{2\cdot \ln \frac{n}{2}}{3\ln n}} = e^{\frac{2\ln n - \ln 4}{3\ln n}} = e^{\left(\frac{2}{3} - \frac{\ln 4}{\ln n}\right)}$

  $\lim\limits_{n \to 0^+} e^{\left(\frac{2}{3} - \frac{\ln 4}{3\ln n}\right)} = e^{2/3} = \sqrt[3]{e^2}$

- $\lim\limits_{n \to 0^+} n^{-\frac{1}{\ln n^2}} = [0^0] = \lim\limits_{n \to 0^+} e^{\ln n^{-\frac{1}{\ln n^2}}} = \lim\limits_{n \to 0^+} e^{-\frac{\ln n}{\ln n^2}} = \lim\limits_{n \to 0^+} e^{-\frac{\cancel{\ln n}}{2\cancel{\ln n}}} = \lim\limits_{n \to 0^+} e^{-1/2} = 1/\sqrt{e}$

- $\lim\limits_{n \to 0^+} n^{\frac{2}{\ln^2 n}} = [0^0] = \lim\limits_{n \to 0^+} e^{\frac{2\ln n}{\ln^2 n}} = \lim\limits_{n \to 0^+} e^{\frac{2}{\ln n}} = e^{0^-} = 1^-$