

## Assignment 3

### Ex 1

The bigcity.csv dataset contains the population of 49 American cities in 1920 (u) and in 1930 (x). Let's assume that cities are a random sample and that we are observing the couple (U, X), the parameter that we want to estimate is  $\theta = E(X)/E(U)$ , where  $E(\cdot)$  is the expected value operator.

- Propose an estimator for  $\theta$  and produce the estimate based on the sample
- Estimate with the bootstrap the bias and the standard error of the estimate of the previous point
- Get a bootstrap confidence interval for  $\theta$ , 0.90 confidence level

### Ex 2

Consider the Weibull distribution:

$$F(y) = 1 - \exp\{-(y/\mu)^\alpha\}$$

(for  $y > 0$ , otherwise  $F(y) = 0$ ).

- use the Inverse Transform Method to generate pseudo-random values from this distribution, given  $\alpha$  and  $\mu$
- plot the results, comparing them with the theoretical distribution of a Weibull

### Ex 3

Using what already did during the Lecture 6, generalize the EM algorithm in order to estimate also the standard deviations of the two Normal densities of the mixture.

### Ex 4

Consider the distribution X, with density:

$$f(x; \delta) = \begin{cases} \frac{\delta}{x\sqrt{2\pi}} \cosh\{\delta \log(2x)\} \exp\{-\sinh\{\delta \log(2x)\}^2/2\}, & \text{per } x > 0 \\ 0, & \text{altrimenti} \end{cases}$$

- implement a function to compute  $f(x, \delta)$  and plot it with  $\delta = 1.5$
- Implement a function to simulate from X using the Acceptance Rejection sampling, using an Exponential distribution (with  $\lambda = 1$ ) as proposal
- Generate a sample using your function and, with these data, estimate  $E(X^2)$ . Using the bootstrap, give a measure of precision of your estimate.