
# Metropolis

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17 Luglio 2018

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# Table of contents

# Introduzione

# Obiettivi

- E' il Machine Learning applicabile per calcolare l'energia di ground state di un Condensato di Bose-Einstein (BEC)?
- Possiamo imparare qualcosa in più sulla Restricted Boltzmann Machine?

# Machine Learning

lol

# Condensato di Bose-Einstein

- Stato della materia in cui gas diluiti di bosoni vengono sottoposti a una transizione di fase quando vengono raffreddati fino a basse temperature ($T \to 0$ K). La maggior parte dei bosoni condensa nel ground state. Questo causa nuovi fenomeni quantistici come superfluidità e coerenza di fase.
- Predetto da A. Einstein nel 1925 seguendo il lavoro del fisico indiano S. N. Bose (1924) sulla statistica dei bosoni.
- Verificati sperimentalmente nel 1995 dal team di Cornell e Wieman a Boulder con $^{87}$Rb e da Ketterle al MIT con $^{23}$Na raffreddati a temperature di 100 nK attraverso laser cooling e evaporative cooling in trappole magneto-ottiche. Premio Nobel 2001.

# Gross-Pitaevskii equation

Negli esperimenti sono stati utilizzati gas diluiti (e.g. atomi alcalini) e non uniformi perchè confinati in trappole magneto-ottiche. L'equazione di riferimento è l'equazione di Gross-Pitaevskii (GP):

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) + g|\Psi(\mathbf{r},t)|^2\right)\Psi(\mathbf{r},t)$$

dove $i = \sqrt{-1}, \hbar$ è la costante di Planck ridotta, $t$ è il tempi, $\Psi$ è la funzione d'onda del condensato, $\mathbf{r}$ rappresenta la posizione dei bosoni e $V$ è il potenziale esterno che intrappola i bosoni. La quantità $g$ è la costante di accoppiamento

$$g = \frac{4\pi\hbar^2 a}{m}$$

dove $a$ is la lunghezza di scattering dell'onda s. Confronteremo le sue soluzioni in [**DalfString**] con i nostri risultati.

# Motivazioni

La condizione di diluizione è data dal parametro $na^3$, dove $n$ è la densità del sistema. Quando il parametro è piccolo, GP funziona molto bene. Tuttavia ci sono esperimenti in cui il parametro supera tale valore. A qual punto è importante studiare il sistema con un approccio many-body. In letterature, troviamo molti papers in cui vengono utilizzati metodi Monte Carlo nel range diluito fino alla densità dell'$^4$He. Per esempio:

1. in [**vmcarticle**], Dubois e Glyde usano Variational Monte Carlo (VMC);
2. in [**Giorgini**], Giorgini et al. usano Diffusion Monte Carlo (DMC);
3. in [**Gruter**], Grüter et al. usano the path-integral Monte Carlo.

In this thesis, we use ML and we compare the results with GP equation since we consider a diluite system. Nevertheless, out of the diluite range, we note that ML results should be compared to the Monte Carlo ones which we have briefly mentioned above.

# Metodi

# Il sistema

We consider a gas of $N_p$ bosons in spherical and elliptical harmonic oscillator potentials. The interaction between bosons in modeled by the hard-core model as in [**vmcarticle**]. The Hamiltonian of the system is

$$H = \sum_{i=1}^{N_p} \left[ -\frac{1}{2}\frac{\hbar^2}{m}\nabla_i^2 + V_{ext}(\mathbf{r}_i) \right] + \sum_{i \neq k} V_{int}(r_{ik}), \tag{1}$$

where $V_{ext}(\mathbf{r})$ is the harmonic oscillator potential given by

$$V_{ext}(\mathbf{r}) = \begin{cases} \frac{1}{2} m \omega_{ho}^2 r^2 & \text{Spherical} \\ \frac{1}{2} m \left[ \omega_{ho}^2 \left( x^2 + y^2 \right) + \omega_z^2 z^2 \right] & \text{Elliptical} \end{cases}$$

and $V_{int}(r_{ik})$ is the hard-shell interaction potential

$$V_{int}(r_{ik}) = \begin{cases} 0, & r_{ik} > a \\ \infty, & r_{ik} < a. \end{cases}$$

The quantity $r_{ik} = |\mathbf{r}_i - \mathbf{r}_k|$ represents the distance between particle $i$ and $k$, while $a$ is the size of the interaction between particles.

# No interaction - spherical trap

When we set the interaction to be zero ($V_{int} = 0$), we are left with a harmonic oscillator potential, where we consider a spherical shape for simplicity. In this case the solutions are known analytically. In general the energy is given by $E_n = \hbar\omega_{ho}(n + \frac{1}{2})$. The ground state is

$$E(N_p, D) = \frac{1}{2}\mathrm{D}N_p \ \hbar\omega_{ho}.$$

where D is the dimension of the system and $N_p$ is the number of particles. This case is useful to benchmark our code at the beginning.

# No interaction - elliptic trap

The interaction is still null ($V_{int} = 0$), the trap now is considered to be elliptic.
Let us introduce lengths in unit of $a_{ho} = \sqrt{\hbar/(m\omega_{ho})}$, $r \to r/a_{ho}$ and energy in units of $\hbar\omega_{ho}$. The Hamiltonian can be rearranged as

$$H = \sum_{k=1}^{N_p} \frac{\hbar\omega_{ho}}{2}\left( - a_{ho}\nabla_k^2 + a_{ho}\hbar\left[x_k^2 + y_k^2 + \frac{\omega_z^2}{\omega_{ho}^2} z_k^2\right]\right).$$

We set $\lambda = \omega_z/\omega_{ho}$, we get

$$H = \sum_{k=1}^{N_p} \frac{1}{2}\left( - \nabla_k^2 + V_{ext}(\mathbf{r}_k)\right) \tag{2}$$

where $V_{ext} = x_k^2 + y_k^2 + \lambda^2 z_k^2$. As in [**DalfString**], we set $\lambda = \sqrt{8}$. In [**vmcarticle**], the energy of non-interacting bosons in this trap is shown to be

$$\frac{E}{N} \to E_{ho} = \hbar\omega_{ho}\left(1 + \frac{\lambda}{2}\right) = 2.414\ \hbar\omega_{ho}.$$

# Interaction - elliptic trap

At this point, we turn on the interaction $V_{int} \neq 0$ and we consider an elliptic trap. The Hamiltonian is

$$H = \sum_{k=1}^{N_p} \frac{1}{2}\Big( -\nabla_k^2 + V_{ext}(\mathbf{r}_k)\Big) + \sum_{k<i}^{N_p} V_{int}(\mathbf{r}_k, \mathbf{r}_i). \tag{3}$$

where $V_{ext} = x_k^2 + y_k^2 + \lambda^2 z_k^2$ and $\lambda = \sqrt{8}$ as above.

# all small caps

This frame uses the `allsmallcaps` titleformat.

## Potential problems

As this titleformat also uses smallcaps you face the same problems as with the `smallcaps` titleformat. Additionally this format can cause some other problems. Please refer to the documentation if you consider using it.

As a rule of thumb: Just use it for plaintext-only titles.

# ALL CAPS

This frame uses the `allcaps` titleformat.

## Potential Problems

This titleformat is not as problematic as the `allsmallcaps` format, but basically suffers from the same deficiencies. So please have a look at the documentation if you want to use it.

# Analisi dati

# TYPOGRAPHY

```
The theme provides sensible defaults to
\emph{emphasize} text, \alert{accent} parts
or show \textbf{bold} results.
```

becomes

The theme provides sensible defaults to *emphasize* text, accent parts or show **bold** results.

# FONT FEATURE TEST

- Regular
- *Italic*
- SmallCaps
- **Bold**
- ***Bold Italic***
- **Bold SmallCaps**
- `Monospace`
- *`Monospace Italic`*
- **`Monospace Bold`**
- ***`Monospace Bold Italic`***

# LISTS

Items

- Milk
- Eggs
- Potatos

Enumerations

1. First,
2. Second and
3. Last.

Descriptions

**PowerPoint** Meeh.

**Beamer** Yeeeha.

# ANIMATION

- This is important

# ANIMATION

- This is important
- Now this

# ANIMATION

- This is important
- Now this
- And now this

# ANIMATION

- This is really important
- Now this
- And now this

# FIGURES

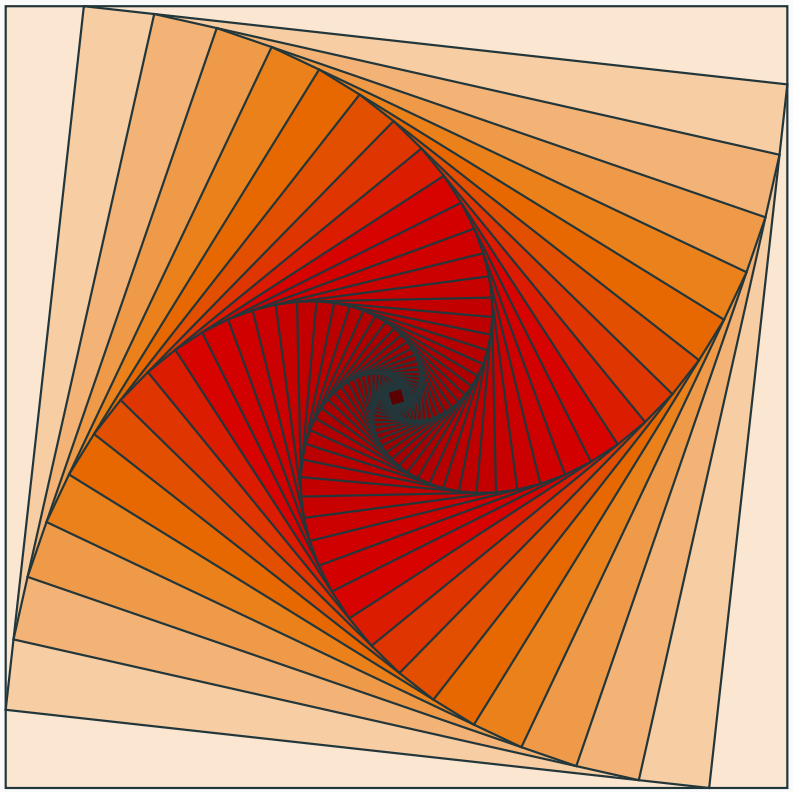

**Figure 1:** Rotated square from texample.net.

# TABLES

**Table 1:** Largest cities in the world (source: Wikipedia)

| City | Population |
|---|---|
| Mexico City | 20,116,842 |
| Shanghai | 19,210,000 |
| Peking | 15,796,450 |
| Istanbul | 14,160,467 |

# BLOCKS

Three different block environments are pre-defined and may be styled with an optional background color.

**Default**

Block content.

**Alert**

Block content.

**Example**

Block content.

**Default**

Block content.

**Alert**

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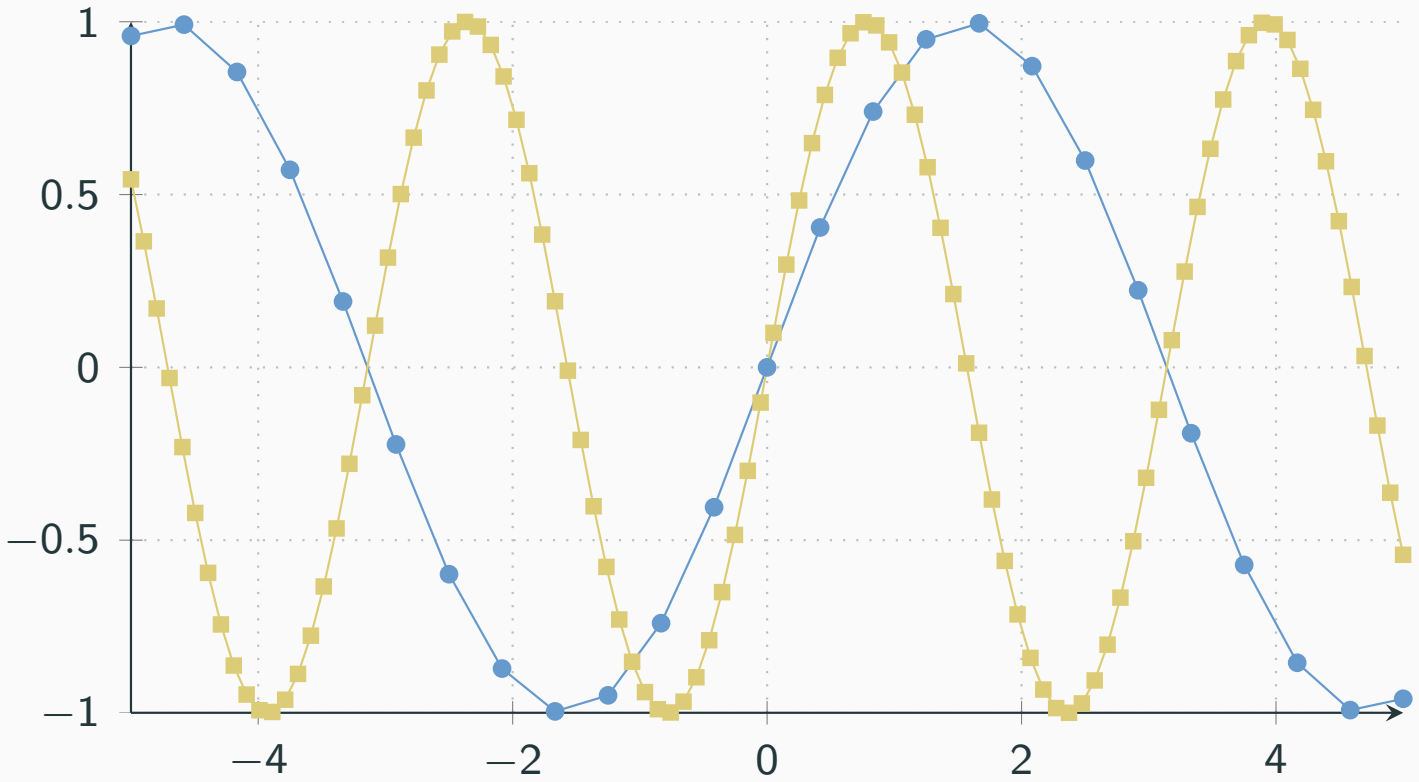
**Example**

Block content.

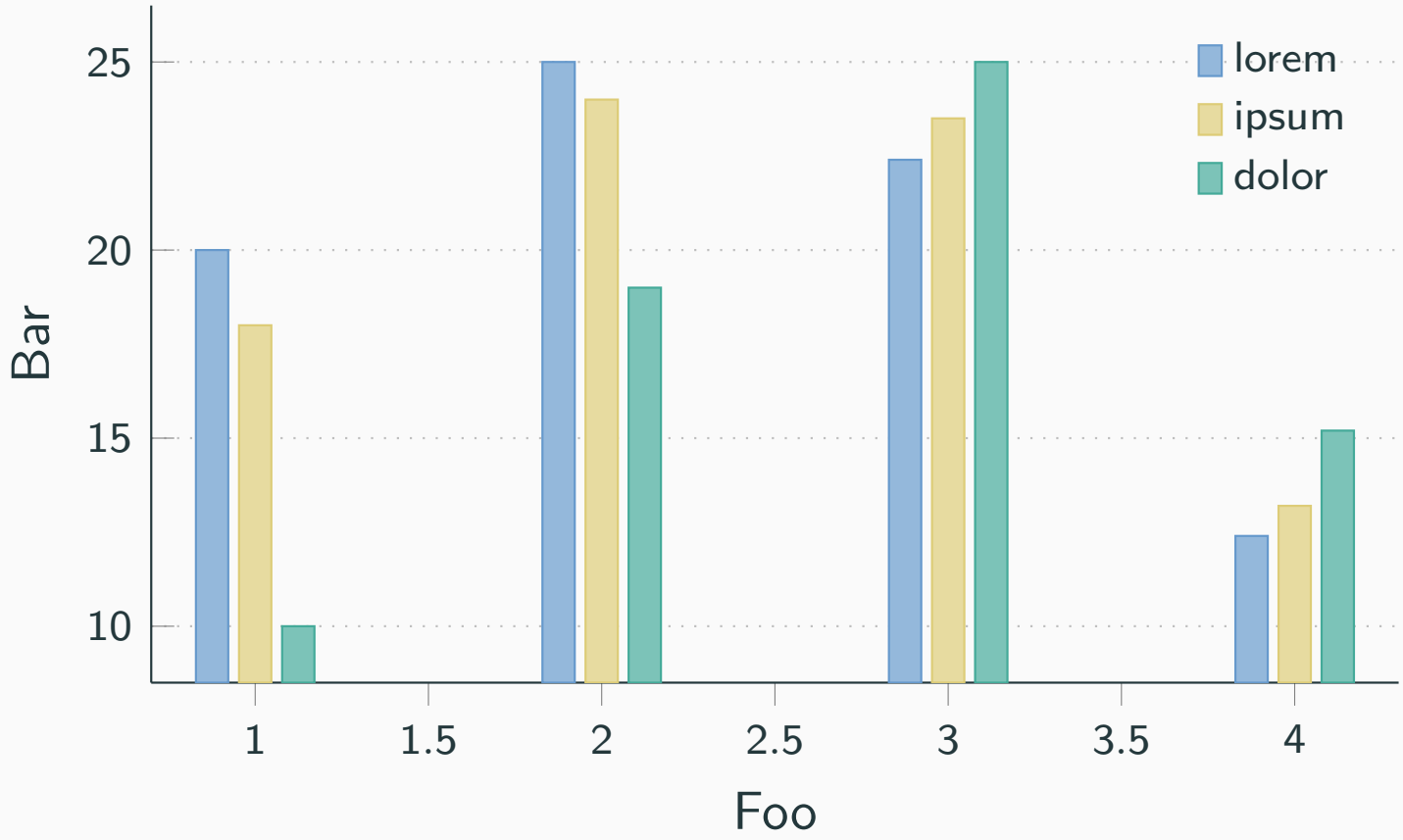
# MATH

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

# LINE PLOTS



# BAR CHARTS



# QUOTES

*Veni, Vidi, Vici*

# FRAME FOOTER

**metropolis** defines a custom beamer template to add a text to the footer. It can be set via

```
\setbeamertemplate{frame footer}{My custom footer}
```

# REFERENCES

Some references to showcase [allowframebreaks] [**knuth92**, **ConcreteMath**, **Simpson**, **Er01**, **greenwade93**]

# Conclusione

# SUMMARY

Get the source of this theme and the demo presentation from

`github.com/matze/mtheme`



# Questions?

# BACKUP SLIDES

Sometimes, it is useful to add slides at the end of your presentation to refer to during audience questions.

The best way to do this is to include the `appendixnumberbeamer` package in your preamble and call `\appendix` before your backup slides.

**metropolis** will automatically turn off slide numbering and progress bars for slides in the appendix.

# REFERENCES I