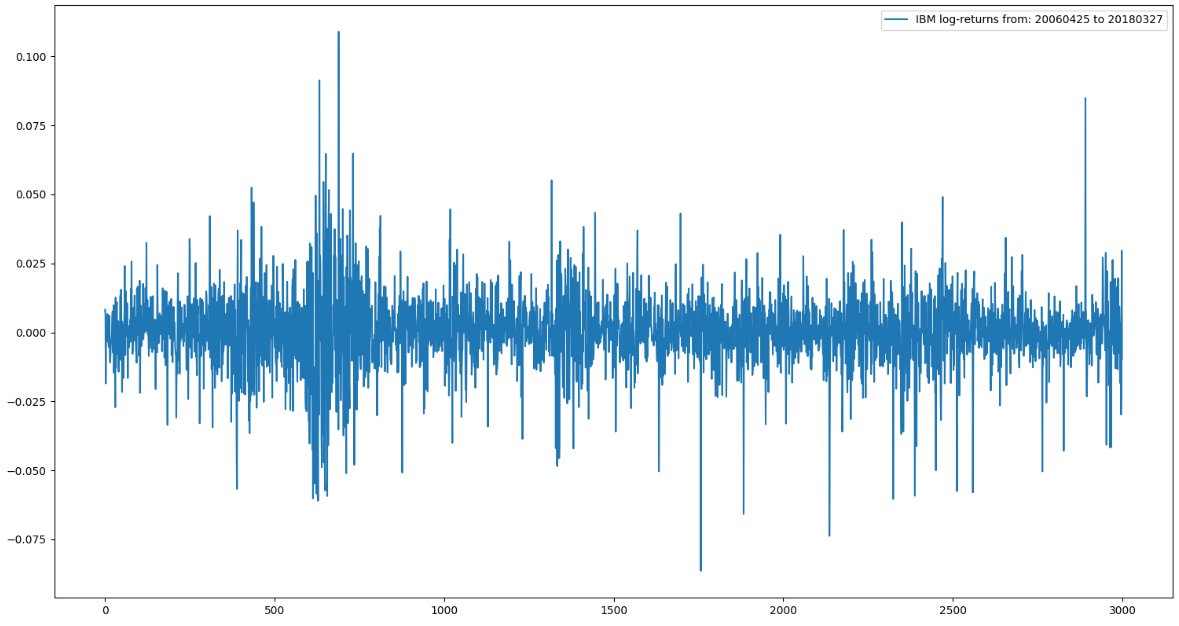
Parameters inference and filtering in stocks time series

The data

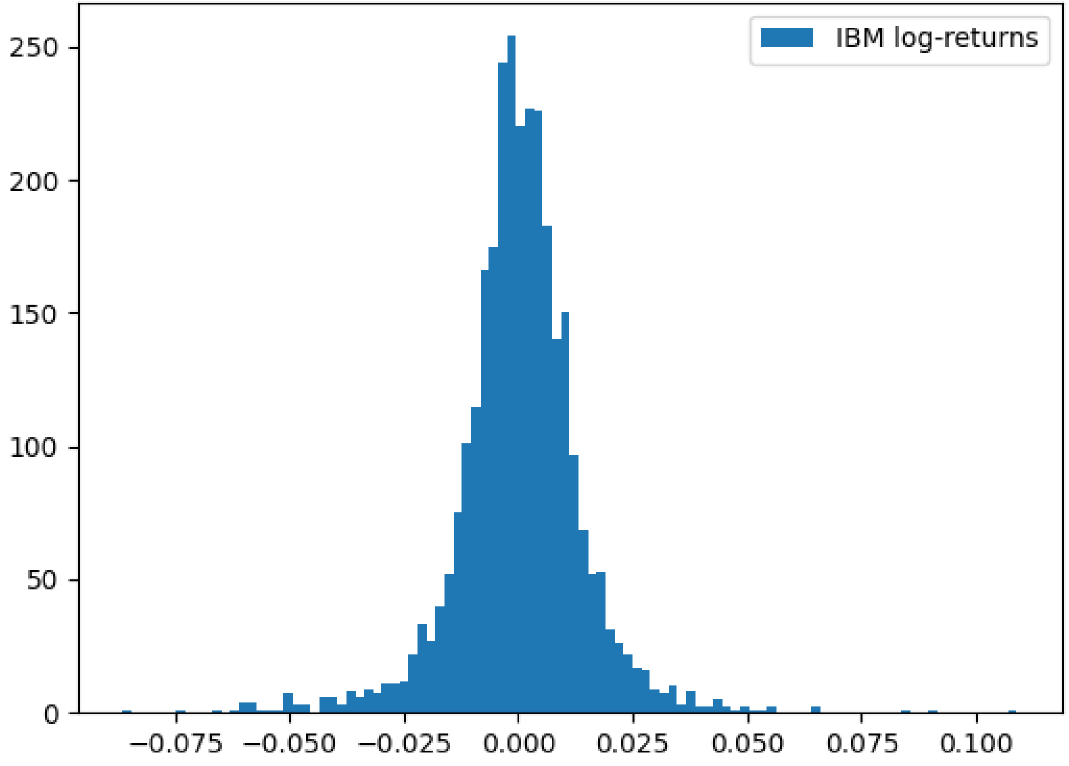
- IBM stock values from 25/04/2006 to 27/03/2018

Log-Returns: $y_t = \ln(\frac{s_t}{s_{t-1}}) = \ln(s_t) - \ln(s_{t-1})$

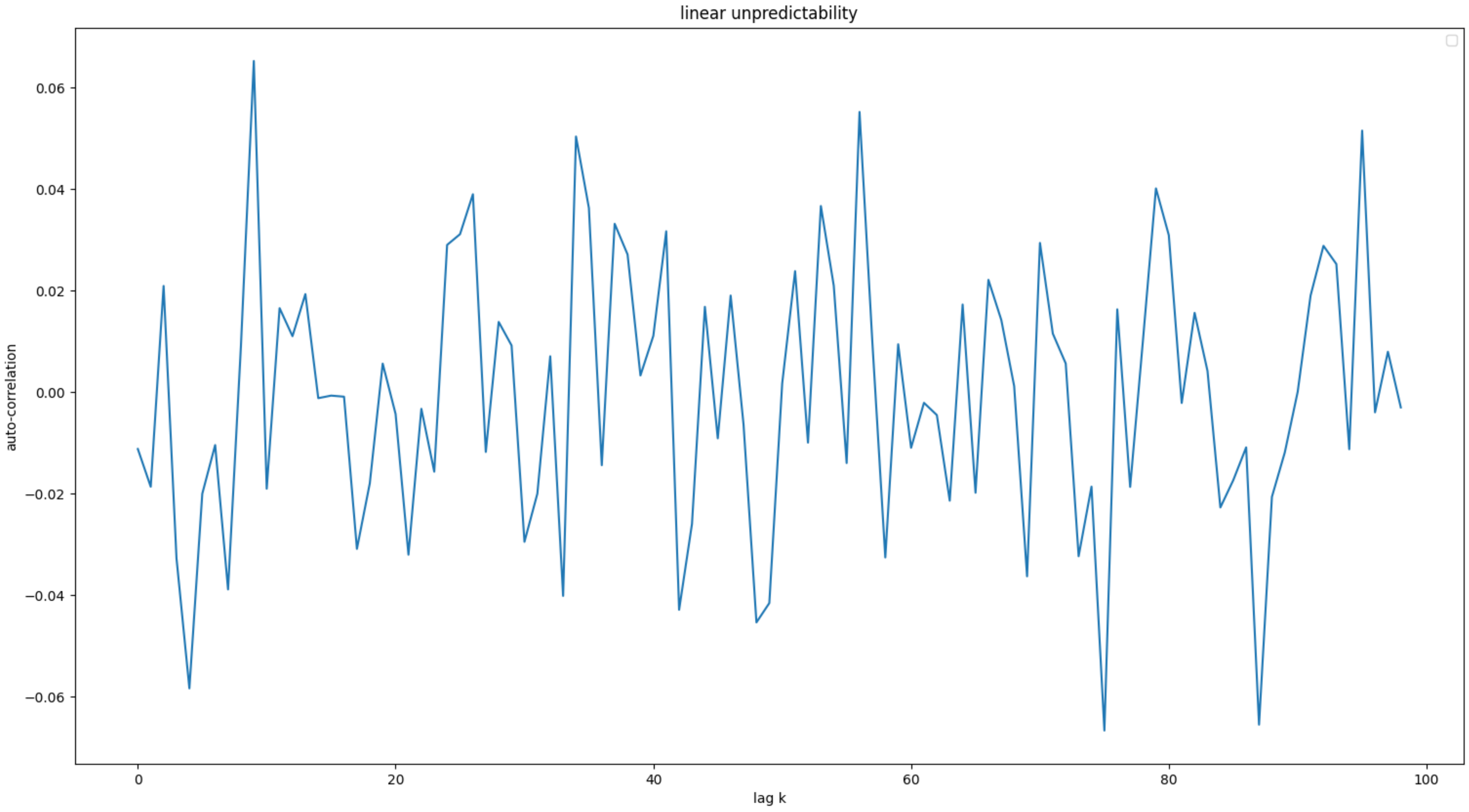
Log-returns series



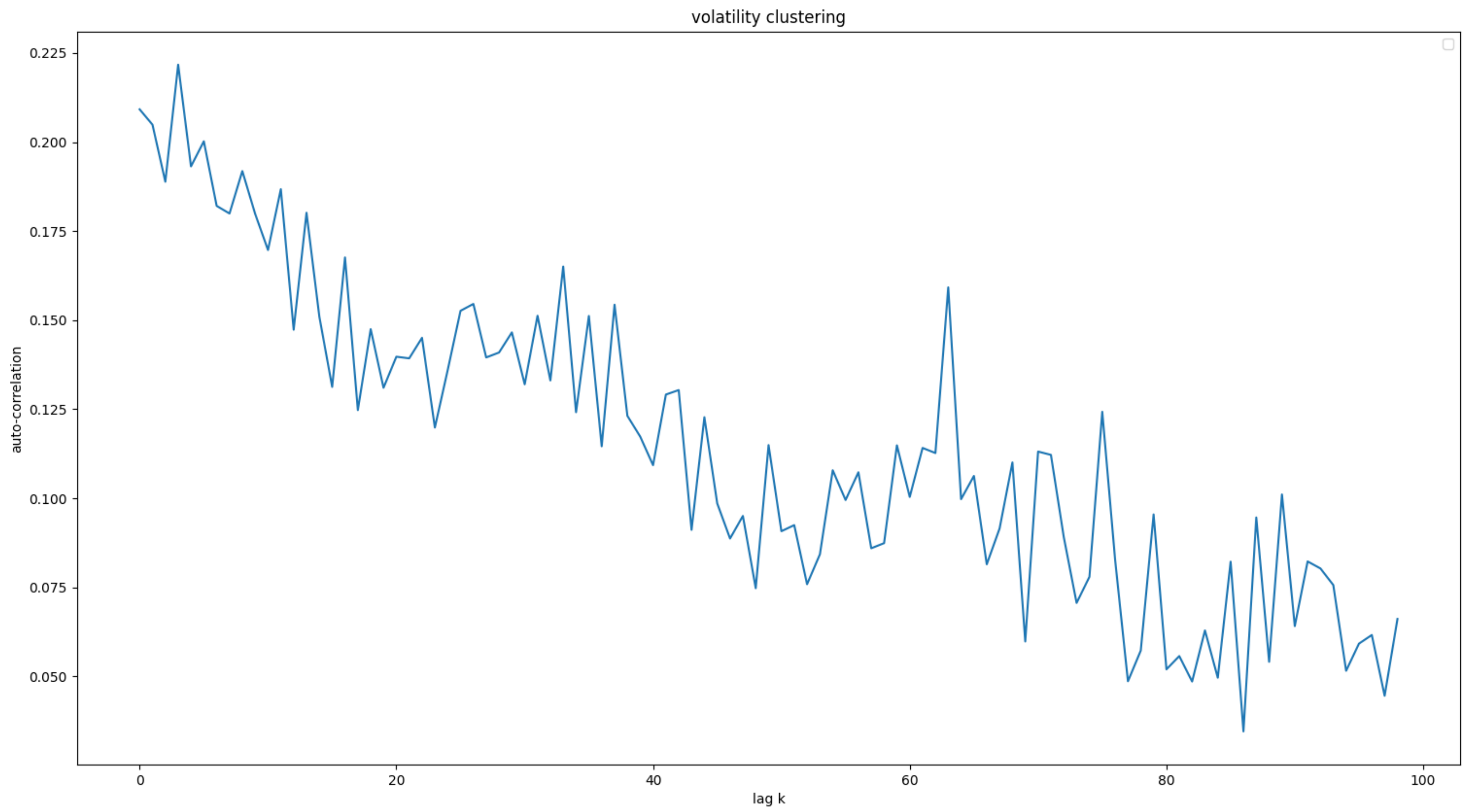
Log-returns distribution



There is no autocorrelation in the series of Log-returns



But if we consider $|y_t|$ then there is some correlation

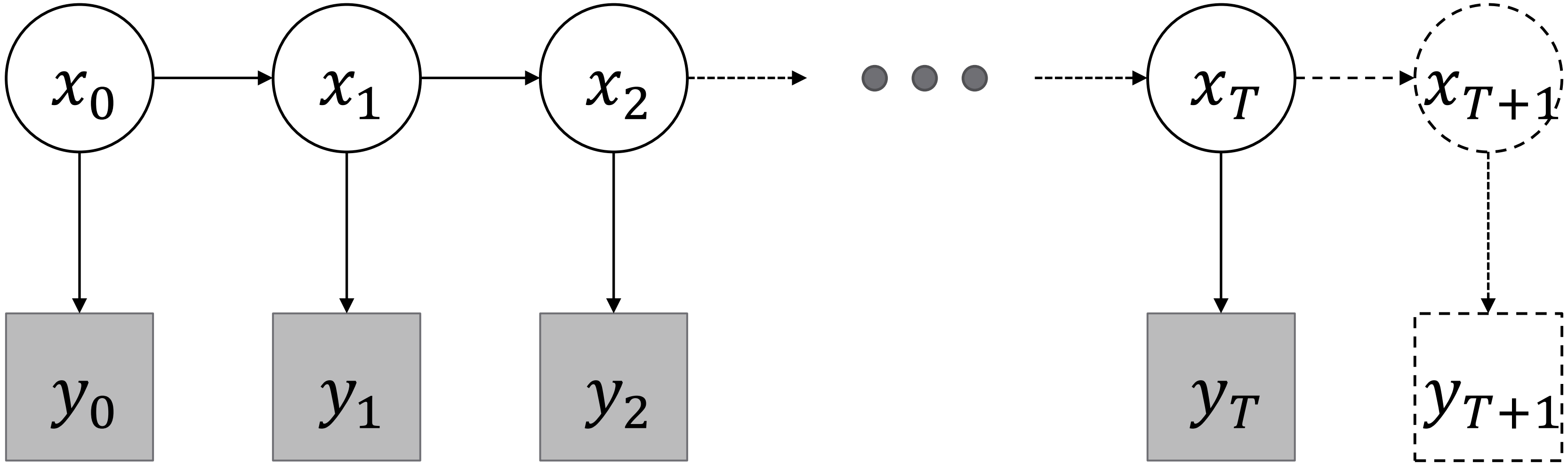


- This fenomenon is known as volatility clustering
- Periods of high variability tends to cluster
- Long range dependency

- The behavior of the time series depends on the ‘volatility’ associated to a certian period.

- A series can be modeled with an Hidden Markov Model, with y_t as the observed variable and the hidden variables x_t representing the volatility.

HMM



- y_t are observed
- x_t are hidden variables
- x_t is a Markov process: $\forall t \; p(x_t | x_0 \ldots x_{t-1}) = f(x_t, x_{t-1})$
- Transition probability: $p(x_t \mid x_{t-1})$
- Observation probability: $p(y_t \mid x_t)$

Inference

We are interested in:

- The distribution of the last hidden state given the observations $p(x_T|y_0, \dots, y_T)$ (**filtering problem**)
- The distribution of hidden states given the observations $p(x_0, \dots, x_T|y_0, \dots, y_T)$
- The distribution of the next observation given the previous observations $p(y_{T+1}|y_0, \dots, y_T)$

An analitical solution is not feasible because of multidimensional integrals.

So we rely on approximate methods.

The model

x_t: volatility at time t

y_t: log-return at time t

$$x_t = \phi x_{t-1} + \epsilon_1$$

$$y_t = \exp\left(\frac{\mathrm{x_t}}{2}\right)\epsilon_2$$

where $\epsilon_1 \sim N(\mu, \sigma)$, $\epsilon_2 \sim N(0,1)$

The parameters $\theta = (\phi, \mu, \sigma)$ are unknown

Objectives

- Estimate the volatility x_t over time
- Estimate $\theta = (\phi, \mu, \sigma)$
- evaluate $p(y_T | y_0, \ldots, y_{T-1})$
- Possibly online

Methods

- MCMC can be used for all these inference problems.
- Not an online algorithm, adding a new observation y_{T+1} would need to restart it.

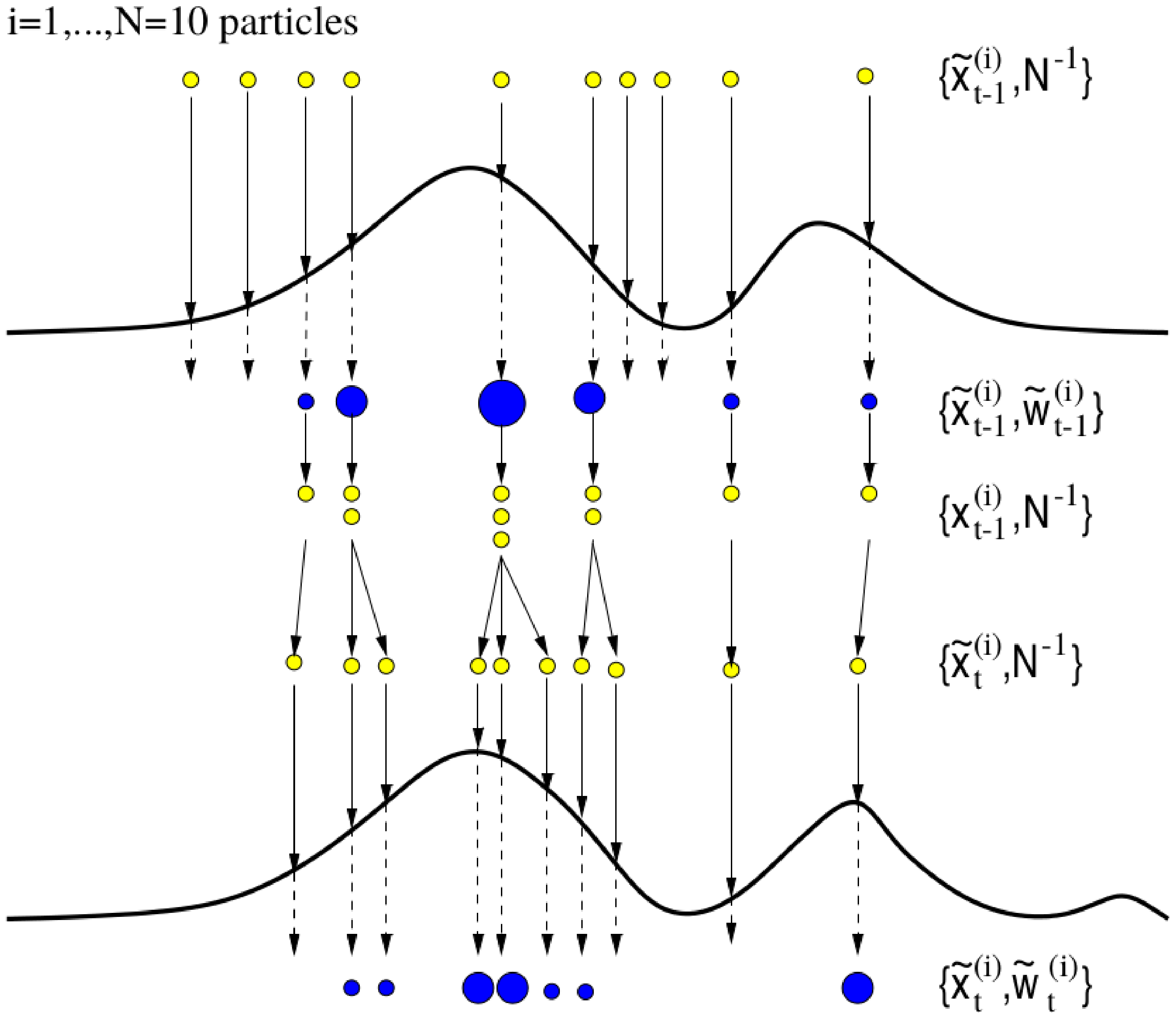
- Sequential Monte Carlo (Particle Filters) methods can be a solution.

Particle Filters

- Particle filters permits to do approximate inference on the current hidden state x_t given all the observations $y_0, y_1, \dots, y_t$ on-line.
- Based on the idea of using a set of points (particles) and weights to approximate a distribution : $\sum_i w_i \delta_{x_i}$

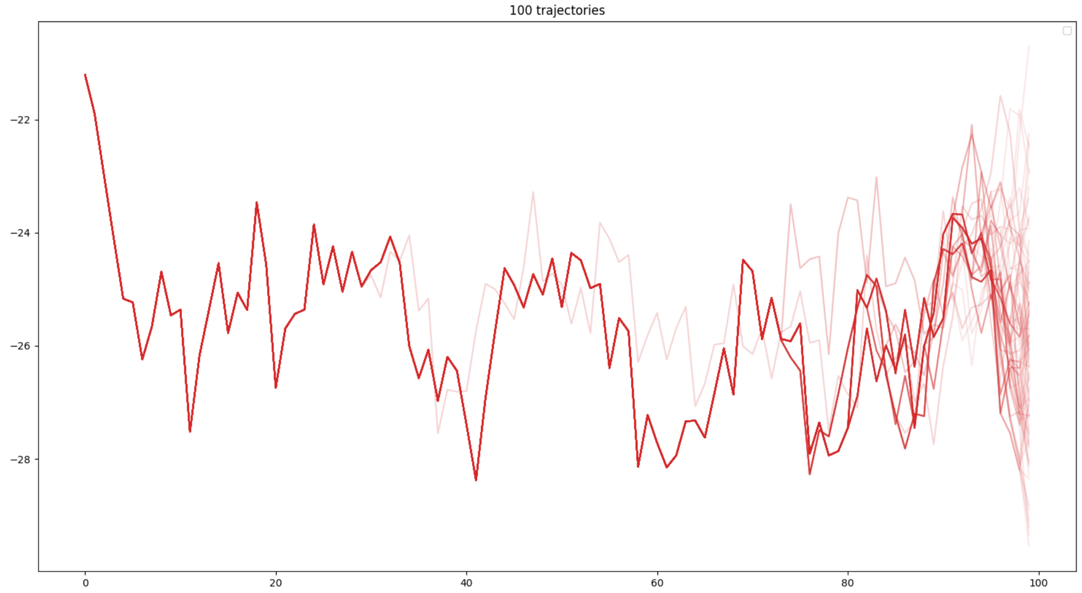
Basic SMC algorithm

- Start with a set of N points: $\tilde{\boldsymbol{x}}_{\boldsymbol{t}}$
- Compute weights: $w_i = p\left(y_t \middle| \tilde{x}_{i,t}\right), \quad w_i := \frac{w_i}{\sum_i w_i}$ (weights normalization)
- Now $\quad p(x_t) \sim \sum_i w_i \delta_{\tilde{x}_{i,t}}$
- The new set of points $\tilde{\boldsymbol{x}}_{\boldsymbol{t+1}}$ for the next iteration is given by:
  - Resample $\hat{\boldsymbol{x}}_{\boldsymbol{t}}$ from $\tilde{\boldsymbol{x}}_{\boldsymbol{t}}$ according to the weights w_i
  - Sample $\tilde{x}_{i,t+1}$ according to the transition probability distribution $p(x_{t+1}|\hat{x}_{i,t})$


i=1,...,N=10 particles
$\{\tilde{x}^{(i)}_{t-1}, N^{-1}\}$
$\{\tilde{x}^{(i)}_{t-1}, \tilde{w}^{(i)}_{t-1}\}$
$\{x^{(i)}_{t-1}, N^{-1}\}$
$\{\tilde{x}^{(i)}_{t}, N^{-1}\}$
$\{\tilde{x}^{(i)}_{t}, \tilde{w}^{(i)}_{t}\}$

Drawbacks

- All particles quickly end-up in having the same ancestors
- This implies that for ‘old’ hidden state the distribution is approximated by just one point



- Often not an issue if we are interested just in the current hidden state
- but what about fixed parameters estimation ?

Fixed parameters estimation

- Naïve approach: include θ in the hidden state, with 'static' transition distribution

- θ is not evolving, after few iterations all particles will have the same ancestor and so the same θ
  - Cannot have a distribution on θ
  - Cannot 'learn' θ
  - A good θ has to be randomly found in the first sample (not feasible in high dimensions)

Possible Solutions:

- Start with θ sampled with MCMC
- Change the model and make θ dynamic
  - Loss of information
- Liu West filter

Liu and West filter

- Main idea: use a mixture of Gaussian to approximate $p(\theta|y_0, \dots, y_t)$ in order to generate fresh samples.

- With shrinkage in order to compensate the loss of information

- $p(x_{t+1}, \theta|y_0, \dots, y_{t+1}) \propto p(y_{t+1}|x_{t+1}, \theta)\, p(x_{t+1}|\theta, y_0, \dots, y_t)\, p(\theta|y_0, \dots, y_t)$

$$p(\theta|y^t) \approx \sum_{j=1}^{N} \omega_t^{(j)} N(\boldsymbol{\theta}|\boldsymbol{m}_t^{(j)}, h^2 \boldsymbol{V}_t)$$

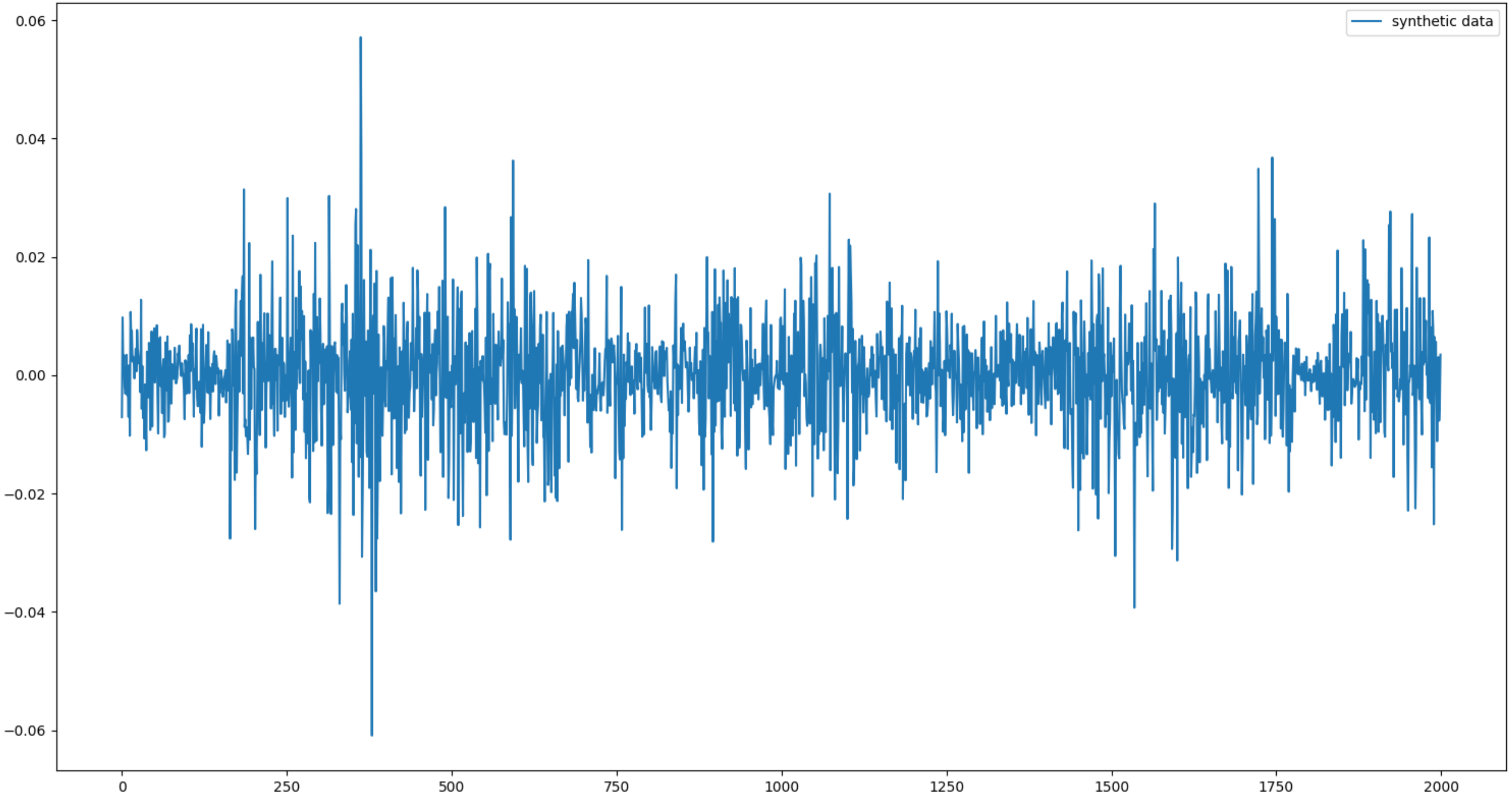
$$V_t = \sum_{j=1}^{N} (\theta_t^{(j)} - \bar{\theta}_t)(\theta_t^{(j)} - \bar{\theta}_t)'/N$$

$$m^{(j)} = a\theta_t^{(j)} + (1-a)\tilde{\theta}_t, \tilde{\theta}_t = \sum_{j=1}^{N} \theta_t^{(j)}/N$$

$$p(\boldsymbol{\theta}_{t+1}|\boldsymbol{\theta}_t) \sim N(\boldsymbol{\theta}_{t+1}|a\boldsymbol{\theta}_t + (1-a)\overline{\boldsymbol{\theta}}_t, h^2 \boldsymbol{V}_t)$$

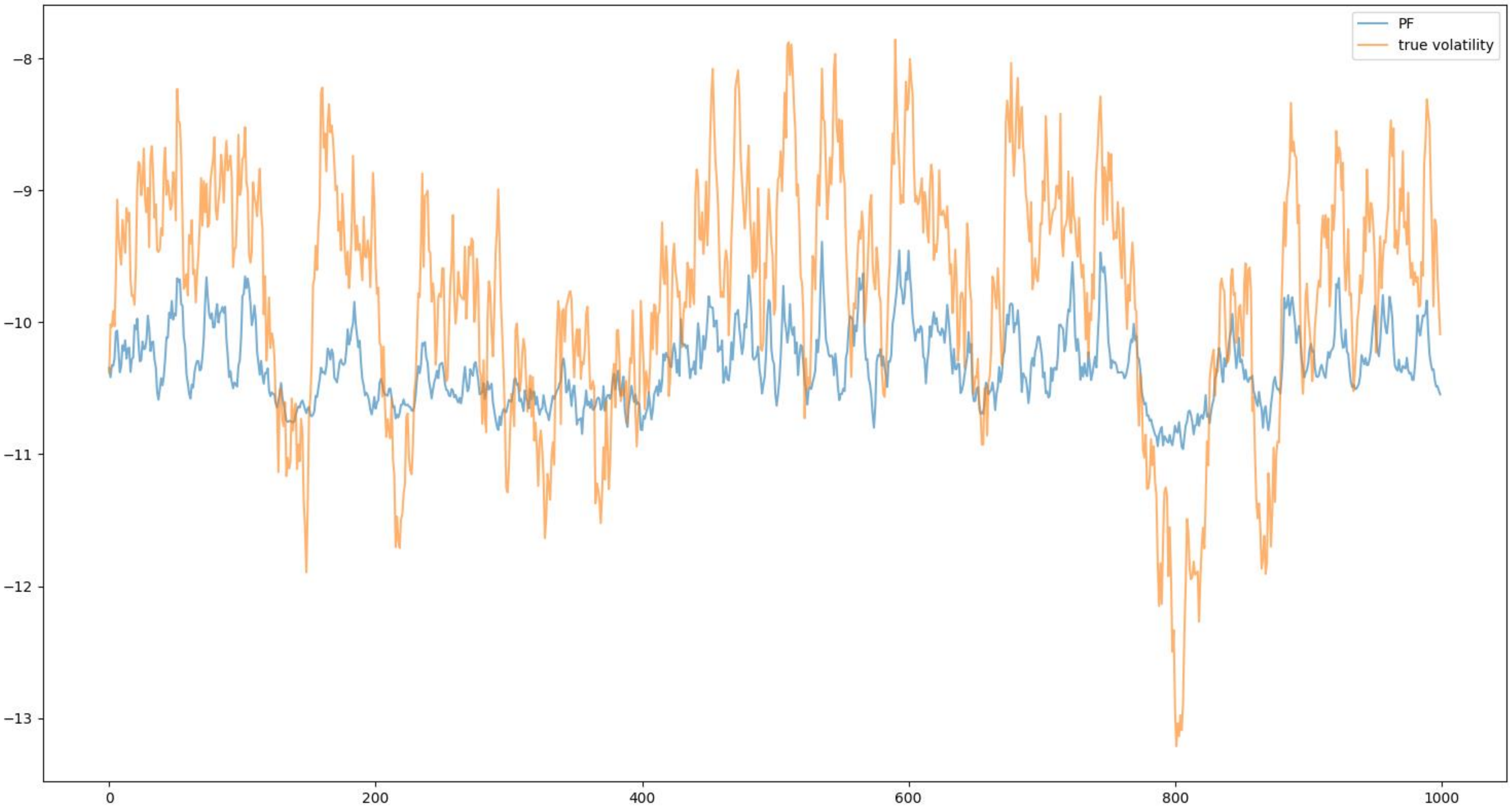
Test on synthetic data

Simulated series



$$\phi = 0.95$$
$$\mu = -0.5$$
$$\sigma = 0.3$$

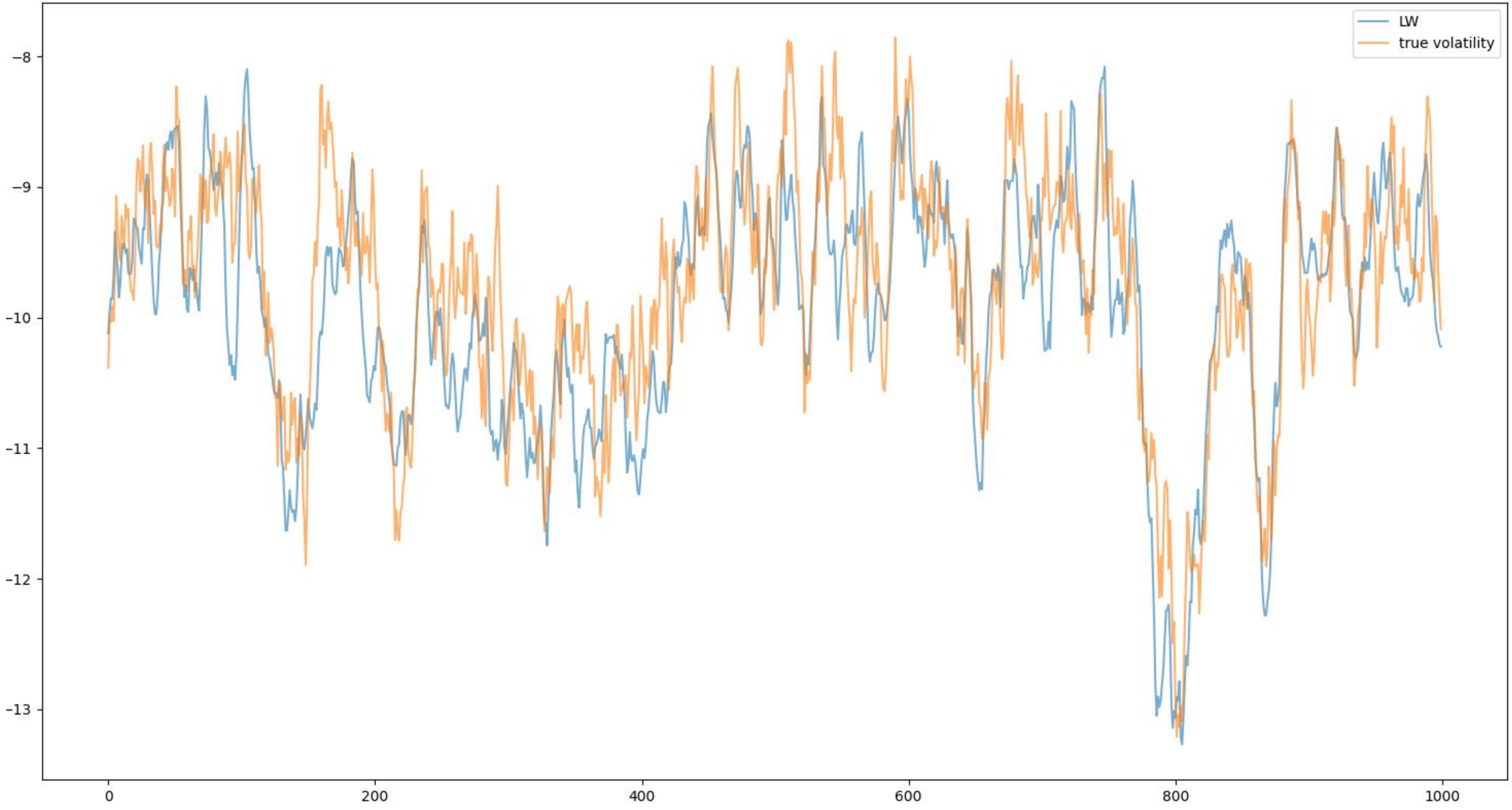
Basic sequential Monte Carlo



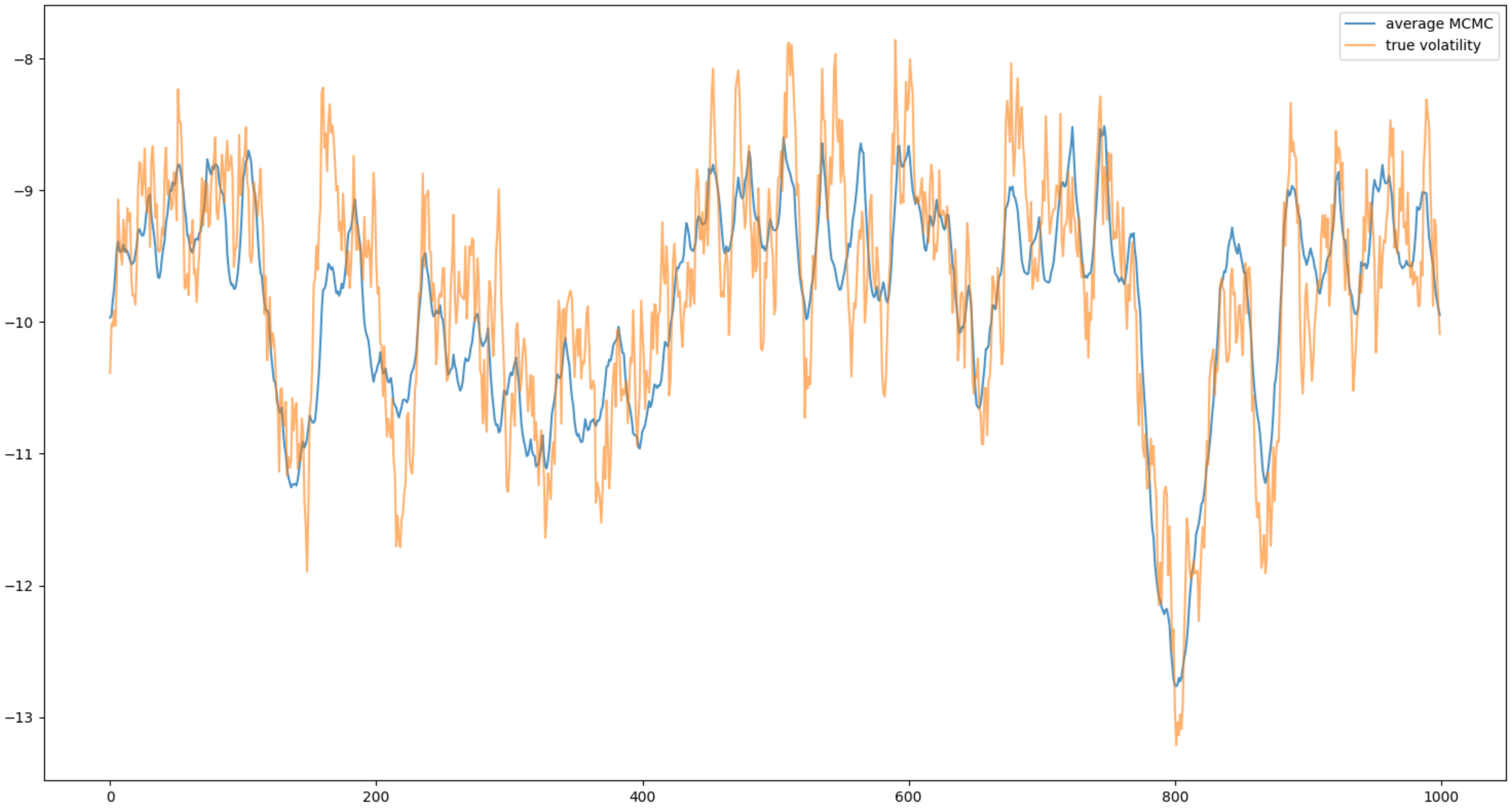
Learnt params:

$$\phi = 0.678$$
$$\mu = -3.40$$
$$\sigma = 0.237$$

Liu West Particle filter



MCMC

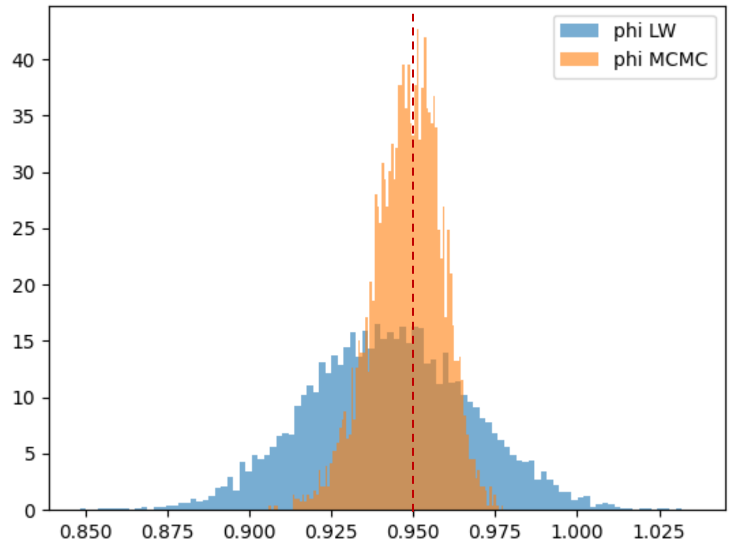


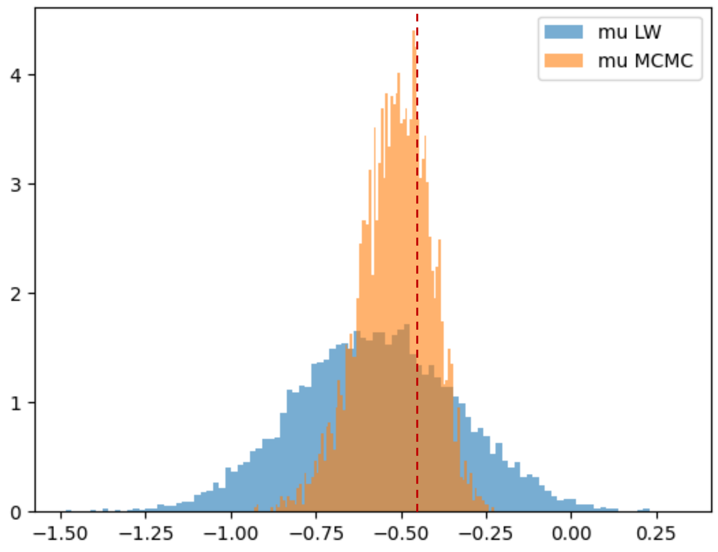
average MCMC
true volatility
−8
−9
−10
−11
−12
−13
0
200
400
600
800
1000

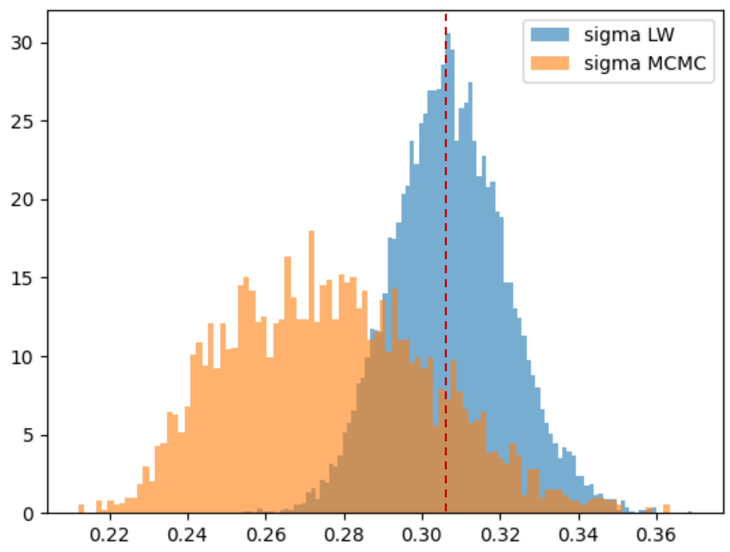
Standard Errors:

SMC: 0.730
LWPF: 0.556
MCMC: 0.496

LW and MCMC learnt parameters:

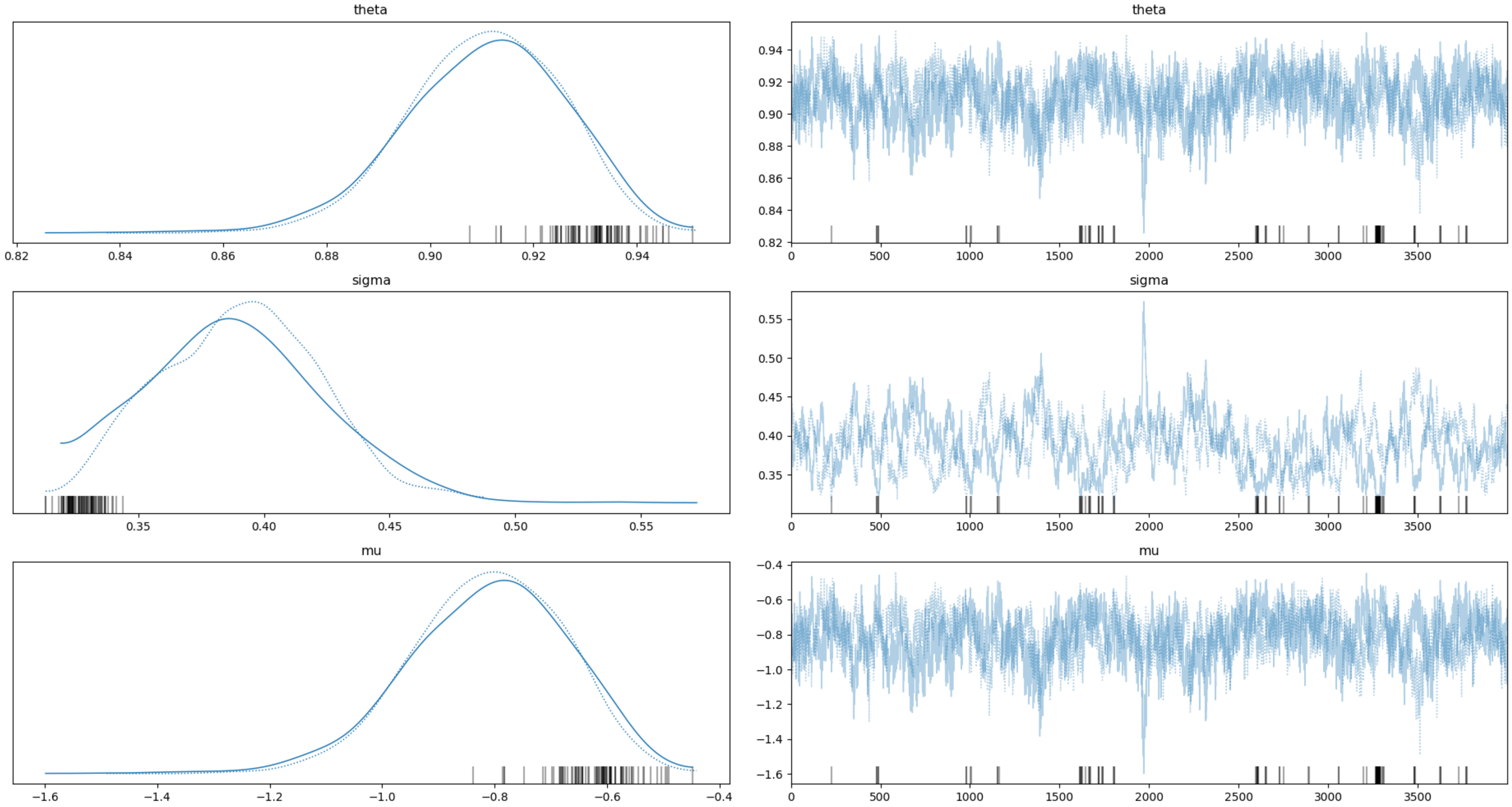




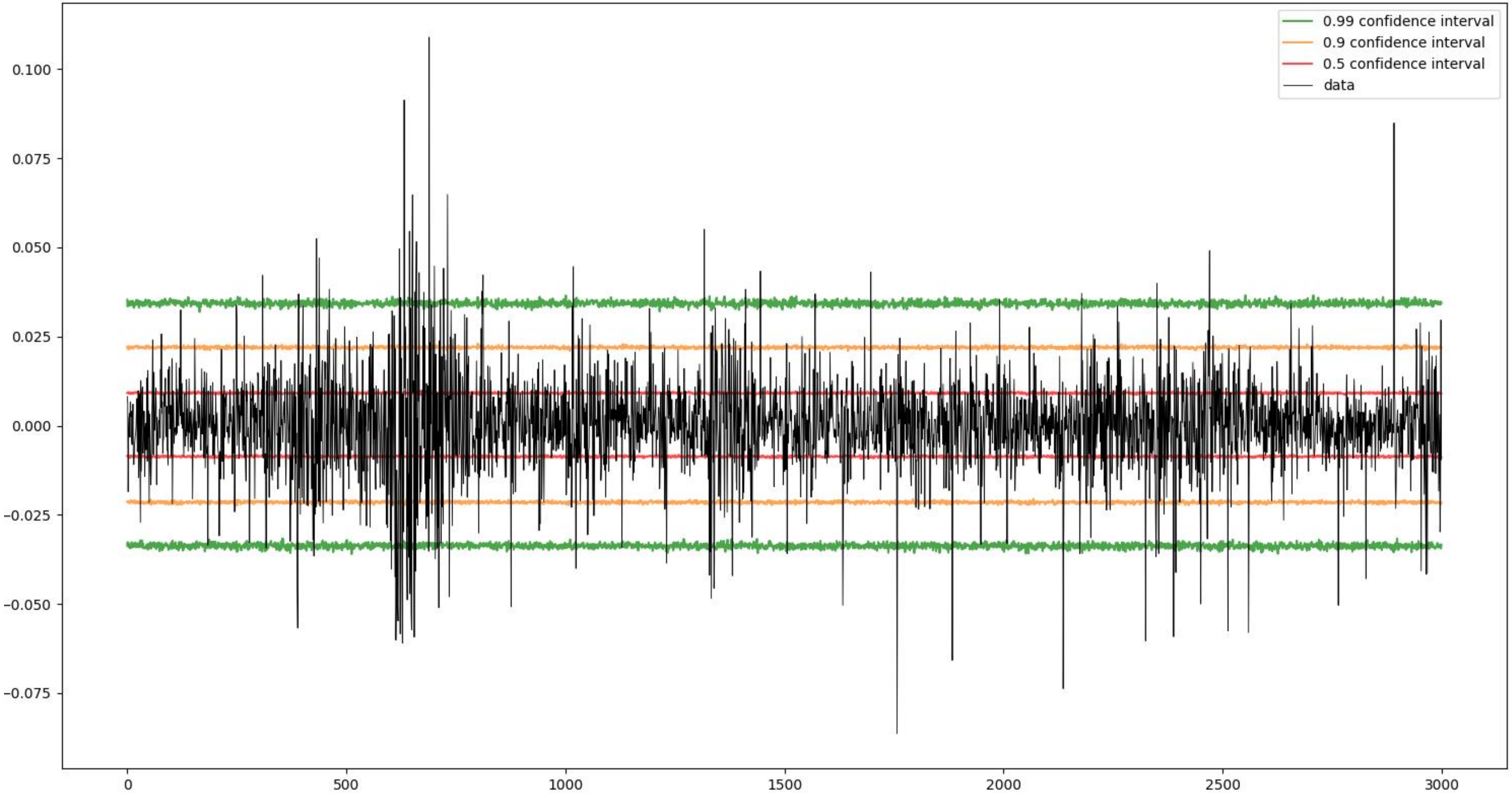


Real data

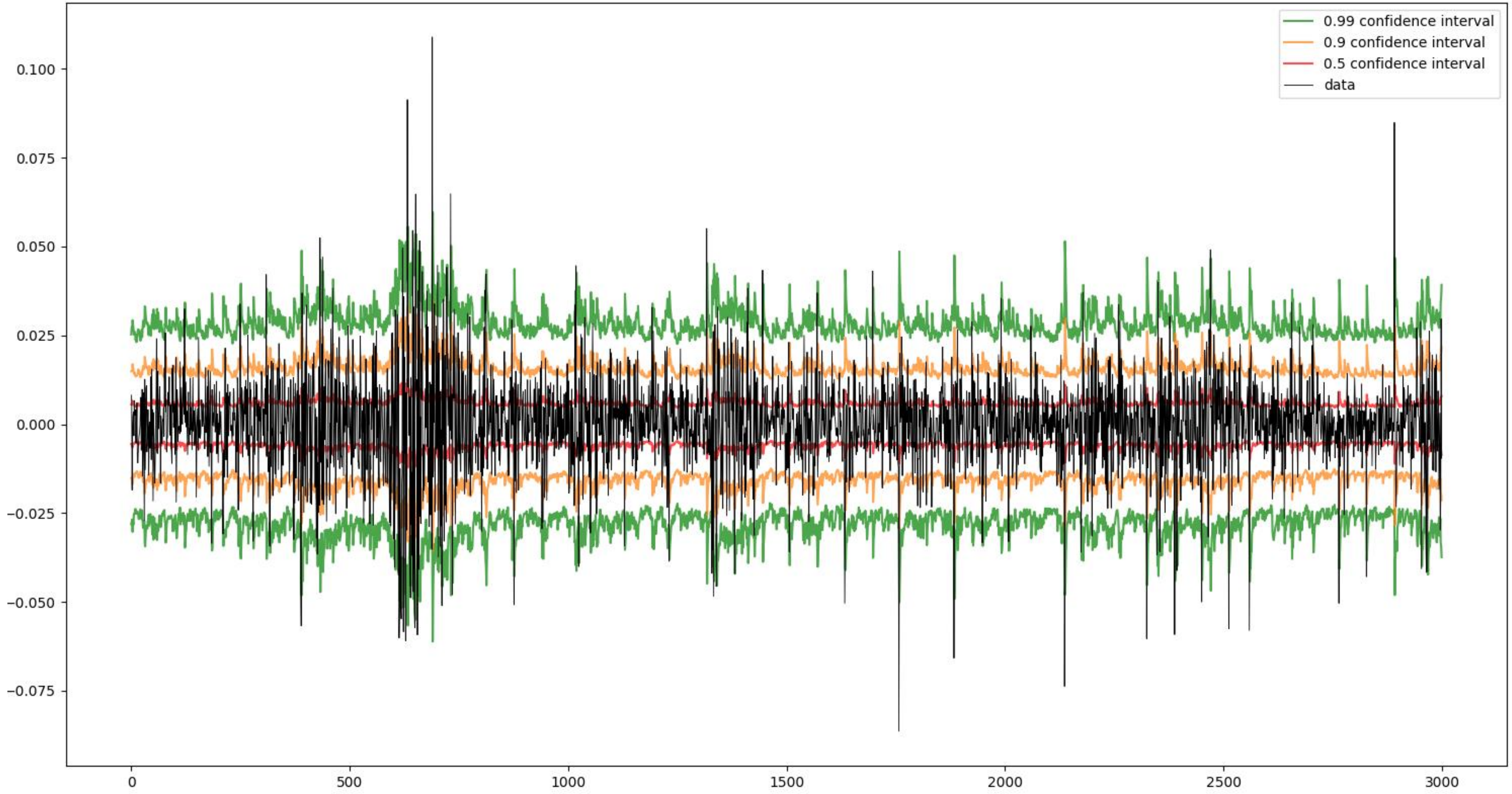
MCMC θ distribution



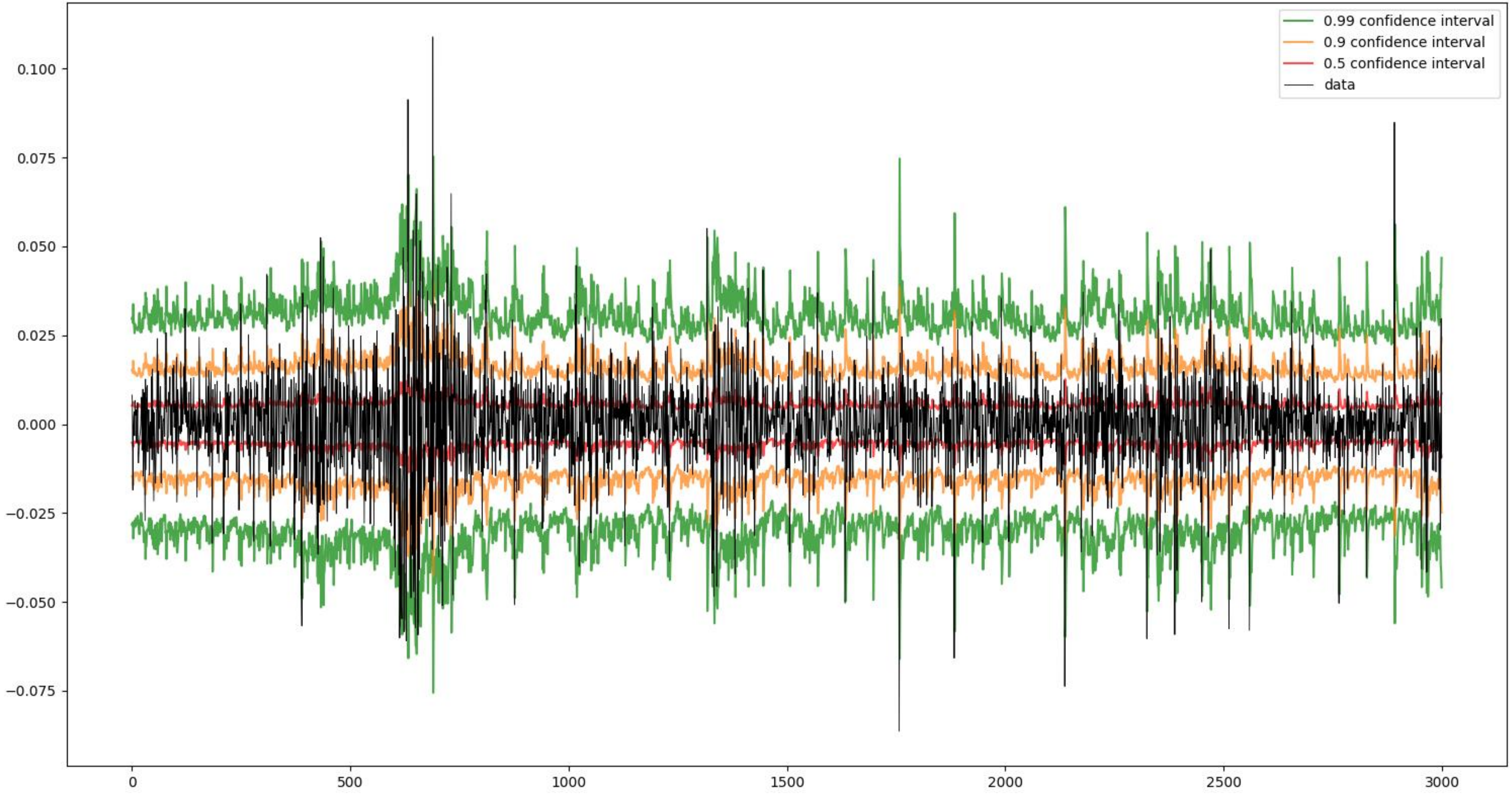
Gaussian fit



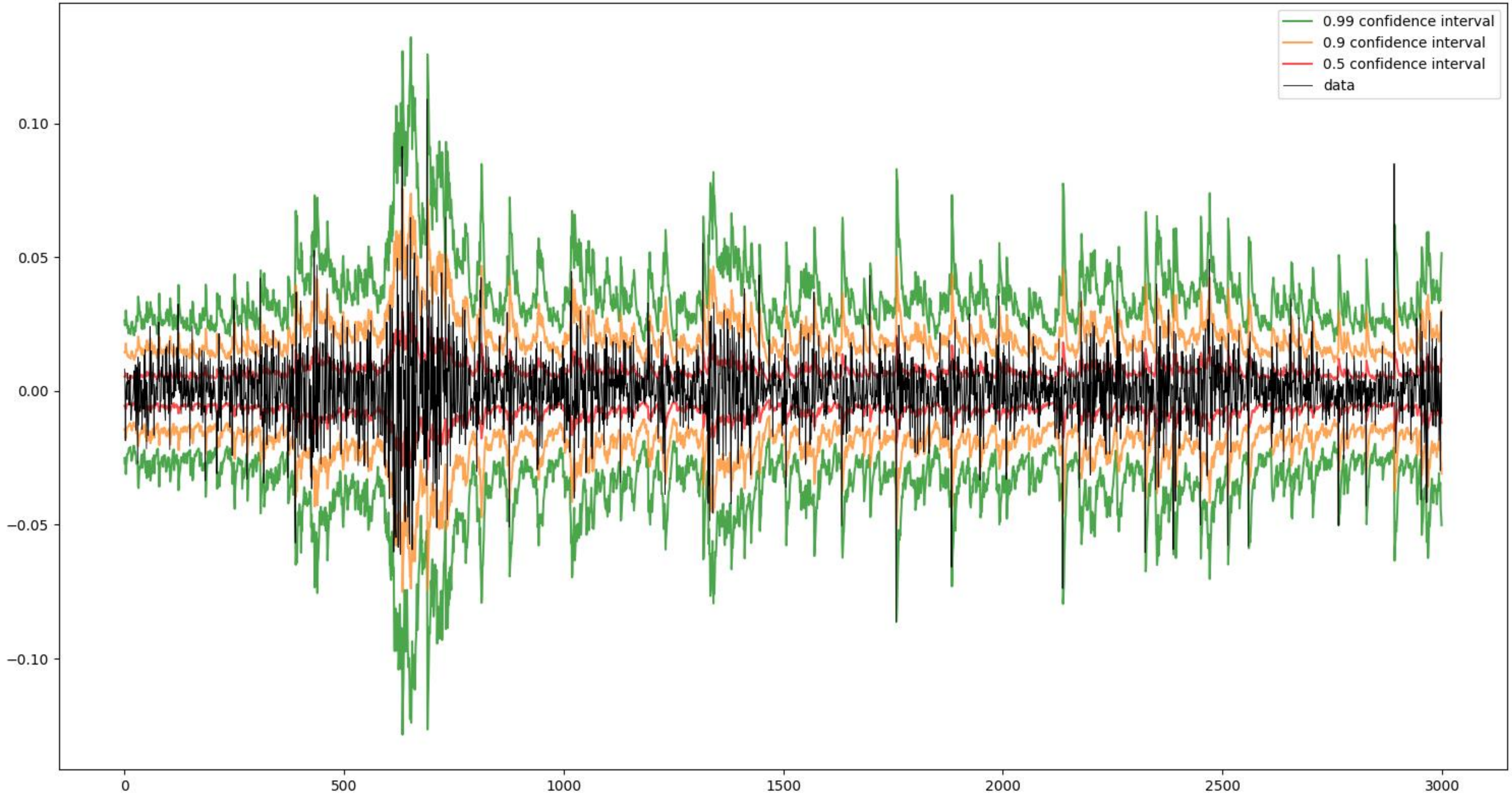
Basic sequential Monte Carlo



MCMC initialized Particle filter



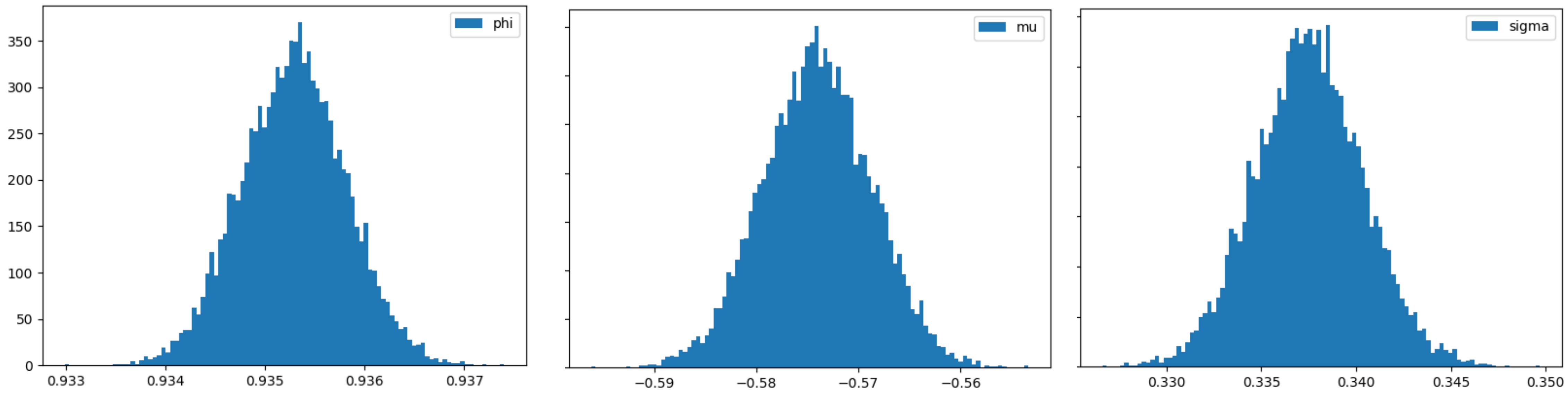
LW particle filter

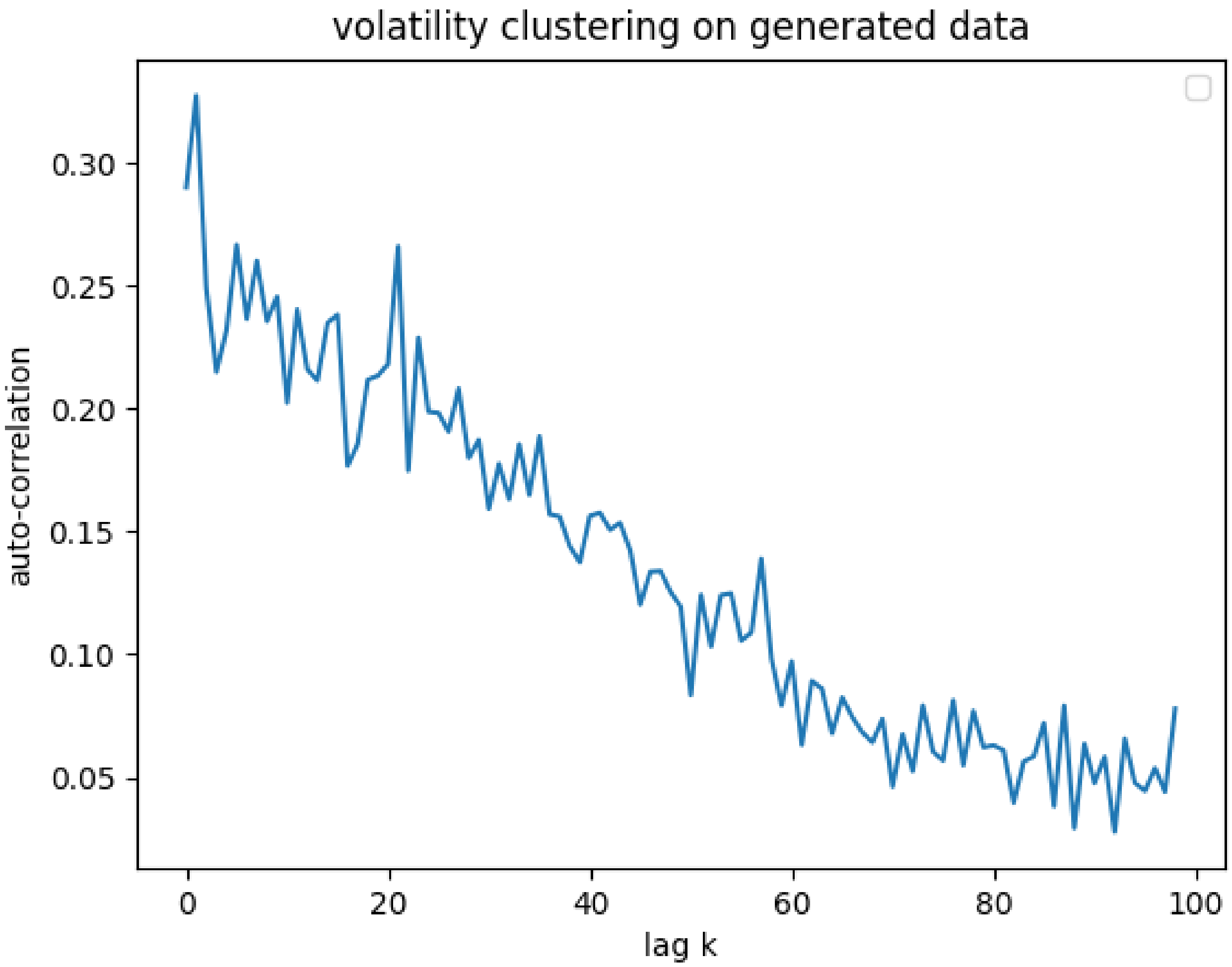


Mean log-likelihood:

Gaussian: 2.91
SMC: 2.97
MCMC initialized SMC: 2.99
LWPF: 3.018

LW learnt parameters:





volatility clustering on generated data
auto-correlation
0.30
0.25
0.20
0.15
0.10
0.05
0
20
40
60
80
100
lag k

References

- Carvalho, Carlos M.; Johannes, Michael S.; Lopes, Hedibert F.; Polson, Nicholas G. Particle Learning and Smoothing. Statist. Sci. 25 (2010), no. 1, 88--106. doi:10.1214/10-STS325. https://projecteuclid.org/euclid.ss/1280841735
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