

# Informatics

## Algorithmic Thinking

Claudio Sartori

Department of Computer Science and Engineering

[claudio.sartori@unibo.it](mailto:claudio.sartori@unibo.it)

<https://www.unibo.it/sitoweb/claudio.sartori/>

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– Ch.10

# Learning Objectives

- Explain similarities and differences among algorithms, programs, and heuristic solutions
- List the five essential properties of an algorithm
- Use the Intersect Alphabetized List algorithm to do the following:
  - Follow the flow of the instruction execution
  - Follow an analysis to pinpoint assumptions
- Demonstrate algorithmic thinking by being able to do the following:
  - Explain the importance of alphabetical order on the solution
  - Explain the importance of the barrier abstraction for correctness

# The Letter Algorithm

- J.D. Bauby was paralyzed and could communicate only by blinking one eyelid
- An assistant would say or point to a letter and Bauby would indicate if it was the right one
- Point to letters until the correct one is reached
- Repeat to spell words and sentences

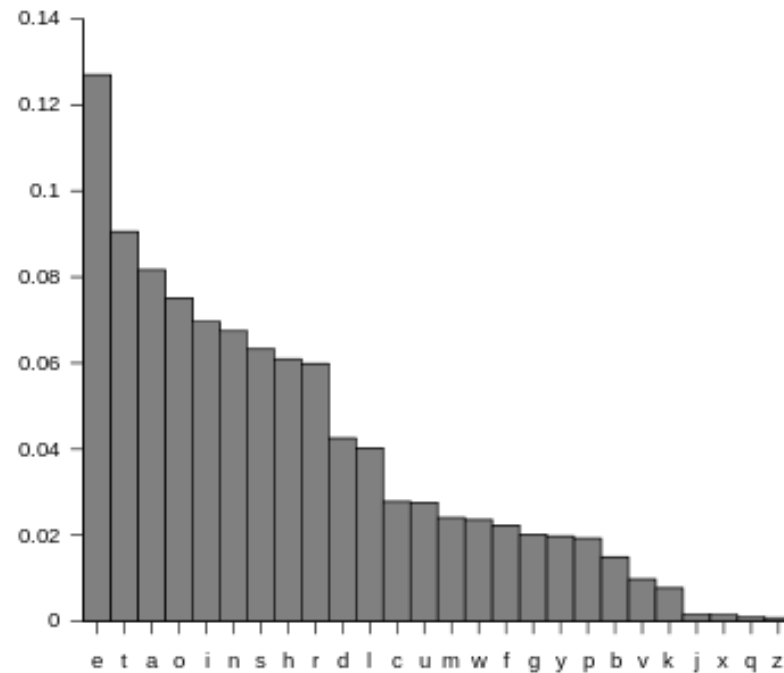
# The Letter Algorithm

- This process is an algorithm: A precise, systematic method for producing a specified result
- We invent algorithms all the time
- An algorithm need not use numbers
- The agent running an algorithm may be a human being, rather than a computer
- There are better and poorer versions of this algorithm

# The Letter Algorithm

- Making the process faster
  - a) **Completion:** The assistant can guess a word before it is all spelled out
  - b) **Ask the letters in frequency order**, and work from the most-frequently-used letter downward
    - \_ What improvement in the process do you expect from the b) solution?

# Relative frequencies of letters in english text



# How can we specify the algorithm?

- It depends on the capabilities of the executor
- The more skilled is the executor, the less detail will be necessary
- Human executor
  - can use logic and experience
- Mechanical executor
  - must be instructed in detail

# Programs are Algorithms

- **programs:** algorithms that have been specialized to a specific set of conditions and assumptions
- sometimes, the words *program* and *algorithm* are used interchangeably, but, in general
  - *algorithm* is a more abstract description of a solution
  - *program* is a precise description of a solution expressed in some *programming language*



# Examples of Algorithms

- Binary to Decimal Conversion
  - if there is a 1, write down the place value for its position in decimal
  - add up those place values
  - *is this description precise enough?*
- Binary Addition
  - add as in decimal but limit digit values to two
  - *is this description precise enough?*

# Another example:

Finding information on the web through a search engine

- begin with a general topic
  - examine the results
- choose descriptive words
  - examine the results
- refine by adding words
  - examine the results
- avoid overconstraining
  - examine the results
- remove specific words
  - examine the results

# Algorithms vs. Heuristic Processes

- not all the description of processes are algorithms
- the process to find information on the web using a search engine was not an algorithm
  - not systematic
  - not guaranteed to find it (process could fail)
  - called a **heuristic process**: helpful procedure for finding a result

# Algorithm Properties

- An algorithm *must* have five properties:
  1. Input specified
  2. Output specified
  3. Definiteness
  4. Effectiveness
  5. Finiteness

# 1. Input Specified

- The **input** is the data to be transformed during the computation to produce the output
- What data do you need to begin to get the result you want?
- Input precision requires that you know what kind of data, how much and what form the data should be

## 2. Output Specified

- The output is the data resulting from the computation (your intended result)
- Output precision also requires that you know what kind of data, how much and what form the output should be (or even if there will **be** any output at all!)

### 3. Definiteness

- Algorithms must specify every step and the order the steps must be taken in the process
- Definiteness means specifying the sequence of operations for turning input into output
- Details of each step must be spelled out (including how to handle errors)
- example:
  - compute  $x$  divided by  $y$
  - input 6,0
  - the result is not defined

## 4. Effectiveness

- For an algorithm to be effective, each of its steps must be doable
- The agent must be able to perform each step without any outside help or extraordinary powers
- the description must be understood by the *executor agent*
- example of non-effectiveness:
  - tell the result of the coin toss I will do in the next minute



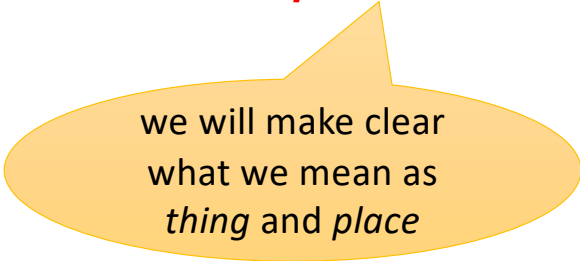
## 5. Finiteness

- The algorithm must stop, eventually!
- *Stopping* may mean that you get the expected output  
**OR** you get a response that no solution is possible
- Finiteness is not usually an issue for non-computer algorithms
- Computer algorithms often repeat instructions with different data and finiteness may be a problem
  - quite easy to write, by mistake, a never ending algorithm/program

# Example 1

- subtract 3 to the current value of x
- repeat and stop only when x is equal to 0
- What if we start with  $x \leftarrow 6$  ?

the symbol  $\leftarrow$  is interpreted as an *arrow right to left* and is read as *store the thing on the right side into the place on the left side*



we will make clear  
what we mean as  
*thing* and *place*

# Example 1 - division

- counter  $\leftarrow$  0
  - repeat if x is greater than 0
    - subtract 3 to the current value of x
    - increment the counter by 1
  - output the value of the counter
- 
- What are the constraints on the inputs?

## Example 2

- compute the result with all the decimals of  $20/3$



endless loop

# Example: Query Evaluation (Brin & Page)

simplified version

1. parse the query
2. convert words into wordIDs
3. seek to the start of the DocList for every word
4. scan through the DocList until there is a document that matches all the words
5. compute the rank of that document in the query
6. if we are not at the end of the doclist, go back to step 4
7. sort the documents found by rank and return the top K

# Query Evaluation

- Makes an ordered list of the pages after a search query
- Not written in a programming language, instead just everyday English
  - but it does use “tech speak”
  - Writing in English instead of a programming language allows to omit details
- Written for people, not for computers

# Algorithm Fact #1

1. Algorithms can be specified at different levels of detail
  - Algorithms use *functions* to simplify the algorithmic description
  - These functions (such as *scan*) may have their own algorithms associated with them

# Algorithm Fact #2

2. Algorithms always build on functionality previously defined and known to the user
  - Assume the use familiar functions and algorithms
  - For example, “scan through” would use the ability to scan a list; b.t.w., what is a list?



# Algorithm Fact #3

3. Different algorithms can solve the same problem differently, and the different solutions can take different amounts of time (or space)

# Correctness

- anyone who creates an algorithm needs to know why it works
  - finding the algorithm's correctness-preserving properties and explaining why they do the job

# Language

- If the executor is a human the algorithm can be specified in natural language?
  - Natural languages can easily be ambiguous
- *Formal languages* have been invented to precisely express concepts in well-defined contexts
  - E.g. the language of mathematics
- The *programming languages* are designed to specify algorithms for computer execution

# Programming language

- The syntax is completely defined
- The semantics is completely explicit and known to the programmer
  - At least to the good one 😊
- Program = algorithm expressed with a programming language
- When a program does not work
  - Bad algorithm → design error
  - Bad translation → programming error

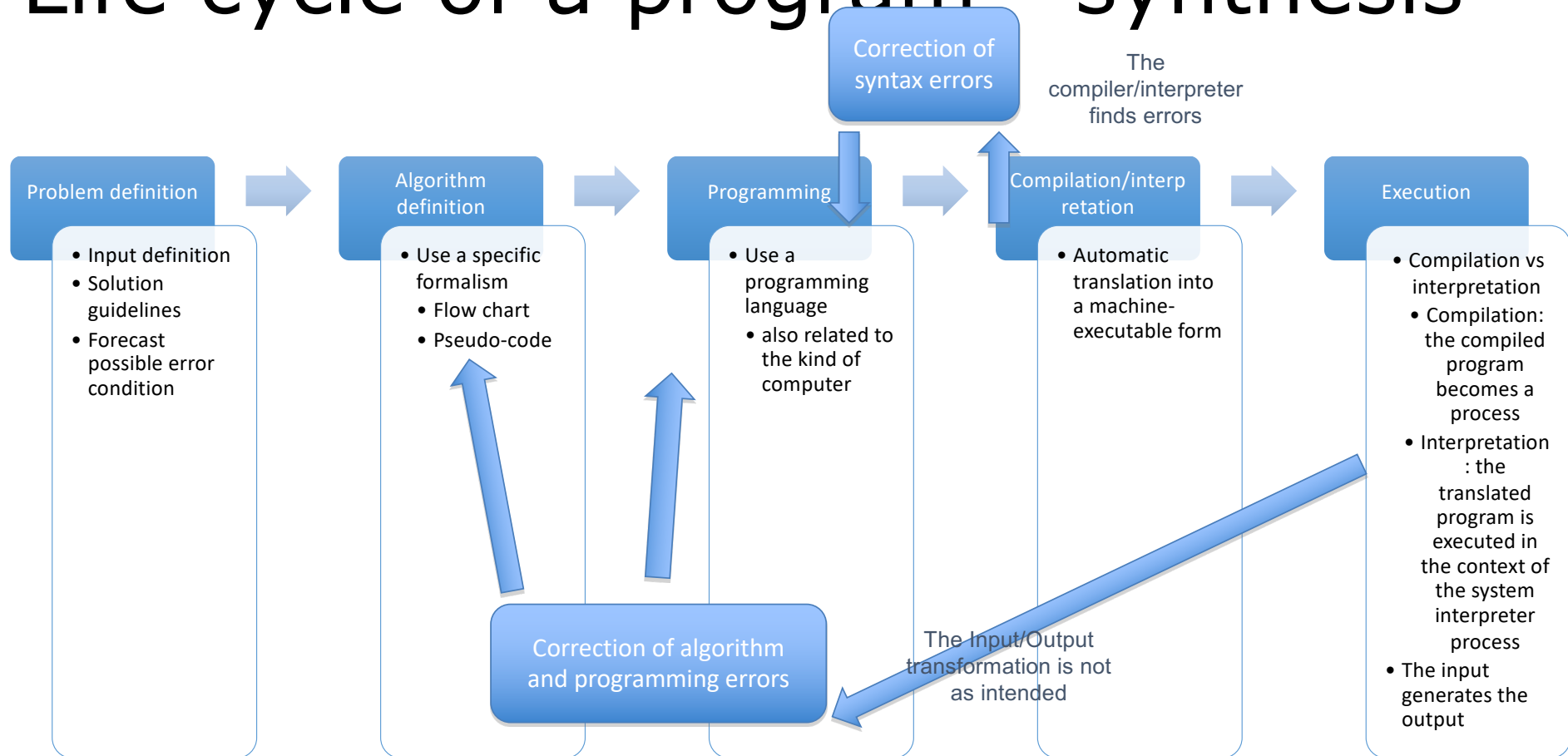
# Life cycle of a program (i)

- Problem definition
  - What are the possible inputs?
  - How can we obtain the solution? Alternatives? Possible errors?
- Algorithm definition
  - Precise definition of the steps necessary to generate the output from the given input
  - Semi-formal description, easy to understand
- Translation of the algorithm into a program
  - High-level programming language
    - It is expressed as a *text*, human-readable, it can be understood by a skilled reader
  - The translation requires a good skill

# Life cycle of a program (ii)

- The designer of the programming language provides also the *translation software*
  - it generates the program expressed in *binary form*, that can be directly executed by the computer
- *Compiler*
  - The binary program can be executed independently, under the control of the operating system
- *Interpreter*
  - The programming language statements are executed in the context of a memory-resident software system

# Life cycle of a program - synthesis



# How can we describe an algorithm?

- What is the building material?
- Where do we store the information?
- How do we specify the operations
- How do we arrange the operations?
- How do we represent sequences
- Is there anything beyond sequences?



# A concrete problem: deposit and interest

- We want to deposit an amount of money, say €100, in a bank
- At the end of each year the bank will pay the interest of 10% of the total amount
- At the end of first year the amount will be  $100 * (1 + 0.1)$
- In the absence of withdrawals, what will be the amount after 5 years?

# Wait and reflect

- Where do we store the values? And the partial results?

## Variable

- ✓ A memory area indicated by a *name*
- ✓ Can be univocally referenced
- ✓ Can contain a *value*
- ✓ The value can be *read/used*
- ✓ The value can be *changed*
- ✓ The value has a *type*

# Compound interest calculation: what variables do we need?

- $A$  = initial amount of money
- $r$  = interest rate
- Do we need to store the new amount at the end of each year?
- $A_1 \leftarrow A * (1+r)$
- $A_2 \leftarrow A_1 * (1+r)$
- $A_3 \leftarrow A_2 * (1+r)$
- $A_4 \leftarrow A_3 * (1+r)$
- $A_5 \leftarrow A_4 * (1+r)$

# Doing better

- Write an algorithm able to work for **any positive amount of money, any positive interest rate, any integer number of years**
  - the *parameters* of the problem
    - $A, r, n$
- The parameters can have different values for every *instance* of the problem
  - i.e. for every execution of the algorithm
- Do we need more variables?

# Operations

- What is the *meaning* of
  - $A_2 \leftarrow A_1 * (1+r)$
- Compute the value of the expression on the right of the arrow, using the current values of variables and the constant values (e.g. the number 1)
- Store the result in the variable on the left of the arrow
- What happens to the variables after the operation?
  - The variables on the right are untouched
  - The variable on the left has a new value
- Can we have on the left something different from a single variable?

**NO!**

# How many variables?

- Do we need to store all the amounts at the end of each year?
  - We are only interested in the final result, say  $A_f$
- At each step the operation could be
  - $A_f \leftarrow A_f * (1 + r)$
- The old value of  $A_f$  is lost after each computation

# Let's try

- $A_f \leftarrow A_f^*(1+r)$
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**What's missing?**  
**The starting point!**

# Try again

- $A_f \leftarrow A$
- $A_f \leftarrow A_f * (1+r)$
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**It works!**



# Sequence

- The sequence is essential
- $A_f \leftarrow A$  must precede all the other operations
  - Otherwise it would not be possible to compute  $A_f \leftarrow A_f * (1+r)$ 
    - The  $A_f$  value is still unknown
- Our computers are *sequential machines*
  - The statements are executed *one after the other*

# Repetition

- In the initial problem definition an operation is repeated five times
- In general the repetition can happen any number  $n$  of times
- We should represent shortly the repetition

Repeat  $n$  times

# Compound interest for n years

- $A_f \leftarrow A$
- Repeat n times
  - $A_f \leftarrow A_f * (1 + r)$

The indentation  
*delimits the scope*  
of the repetition

Add input and  
output

# Compound interest for n years

- Use variables  $A$ ,  $n$ ,  $r$ ,  $A_f$
- Read  $A$ ,  $n$ ,  $r$
- $A_f \leftarrow A$
- Repeat  $n$  times
  - $A_f \leftarrow A_f * (1 + r)$
- Write  $A_f$

# Intersecting $n$ Lists (IL)

Write the friends of Mary, John and Luke in 3 pieces of paper and copy in a new piece of paper the *intersection*

Friends of Mary	Friends of John	Friends of Luke	Intersection
Julie	Susie	Anne	
Anne	Charlie	Denny	
Frank	Bob	Susie	
Susie	Anne	Frank	

# IL - Algorithm

1. Put a marker at the start of each list
2. If all markers show same name, save it
3. Starting from the right, find each list whose marker is at the end, and move the marker to the front
4. When you first find a marker not at the end, advance it and go to step 2.
5. Stop when all markers return to start

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after several steps...

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Anne *	Charlie	Denny	
Frank	Bob	Susie	
Susie	Anne *	Frank	

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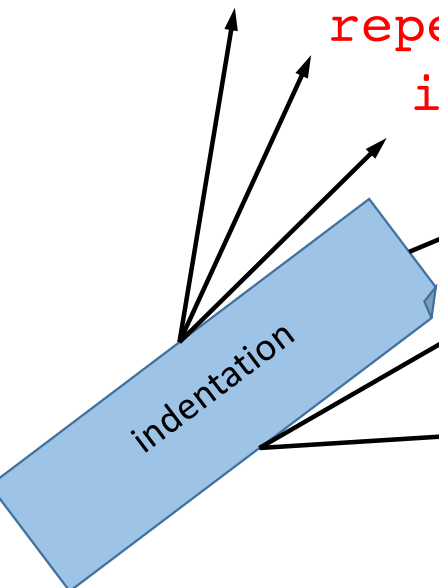
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# Intersecting three lists – algorithm in *pseudo-code*

```
prepare an empty variable intersection, set s to 0
repeat varying i from 1 to the length of list1
  repeat varying j from 1 to the length of list2
    repeat varying k from 1 to the length of list3
      if position i of list1 is equal to position j of
        list2 and position j of list2 is equal
          to position k of list3
        increment s by 1
      store the content of position i of list1 into
        position s of intersection
```



# Intersecting *n* Alphabetized Lists (IAL)

Write the friends of Mary, John and Luke in 3 pieces of paper *in alphabetical order* and copy in a new piece of paper the *intersection*

Friends of Mary	Friends of John	Friends of Luke	Intersection
Anne *	Anne *	Anne *	<b>Anne</b>
Frank	Bob	Denny	<b>Susie</b>
Julie	Charlie	Frank	
Susie	Susie	Susie	

# IAL - Algorithm

Write the friends of Mary, John, ... in  $n$  pieces of paper, *in alphabetical order*

1. Put a marker at start of each list
2. If all markers show the same name, save it
3. Advance the marker(s) for the alphabetically-earliest name
4. Repeat steps 2-3 until some marker reaches the end

# Comparison

- IAL and IL are different algorithms
- IAL and IL accomplish the same thing, starting from the same data
- IL is much slower
- For 5 lists of 10 elements:
  - IAL:  $10+10+10+10+10 = 50$  steps
  - IL:  $10 \times 10 \times 10 \times 10 \times 10 = 100,000$  steps
  - *how many steps do we need to put lists in alphabetical order?*
    - *you will learn it by the end of the course*

# How Do We Know it Works?

- Algorithm solution is clear and simple and efficient
- Then, how do we know it works?
- If there is no loop, the program runs, gets to an end, and we can check the result
- What if there is a loop?
  - Programs with loops cannot be absolutely verified...there are too many possible cases

# Then, what?

- *The way to know that an algorithm works is to know why it works...*
- Strategy for knowing why it works:
  - Find one or more properties that ensure the algorithm works
  - Explain, using the program, why they make it work.

# Why IAL Works

- If a name appears in all lists, it forms a barrier
- Pointers that haven't reached the barrier move sooner than ones that have
- When they all reach the barrier, the name is recorded
- None can pass the next barrier until they all reach it
- Etc.

# Summary

- We use algorithms daily, and we continually create them as we instruct other people in how to do something
- Everyday algorithms can be sometimes be unclear because natural language is imprecise
- Algorithms have five fundamental properties



# Summary

- Algorithms can be given at different levels of detail depending on the abilities of the agent
- Problems can be solved by different algorithms in different ways
- Algorithms always finish—either they give the answer, or say no answer is possible—and they are evaluated on their use of resources such as space and time

try to write the IAL algorithm in pseudo-code