Informatics

The R language and system
with programming examples
Part 3

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Determinant of a square matrix

- Definition well known from algebra
  - e.g. Wikipedia https://en.wikipedia.org/wiki/Determinant
- Laplace expansion theorem: given matrix A
  - cofactor
    - $A_{i,j} = (-1)^{(i+j)} * M_{ij}$
  - minor
    - submatrix obtained by elimination of row i and column j from A
  - for n=1 det(a) = a
  - for n>1, for any k between 1 and n the expansion can be done along column or row k

$$det(A) = \sum_{i=1}^{n} a_{ik}(-1)^{(i+k)} det(M_{ik}) = \sum_{j=1}^{n} a_{kj}(-1)^{(k+j)} det(M_{kj})$$

Determinant with Laplace formula: recursive solution for n>=2

- if n = 2 return (a11 * a22) – (a21 * a12)
- choose expansion line, e.g. column 1
- initialize sign to 1^(expansion row/column +1)
- initialize d to 0
- repeat varying i from 1 to n
  - if ai1 different from 0
    - d <- d + sign * ai1 * determinant(Mi1)
  - sign <- -sign
- return d

RREF and the determinant (n×n only)

- the determinant of the matrix in reduced form is either 1, if the rank is n, or 0
- the determinant of the matrix in reduced form is equal to the determinant of the original matrix multiplied by a scale factor
  - each row swap generates a change of sign in the scale
  - each scalar multiplication causes a multiplication of the scale with the inverse of the scalar

det(m) = det(Rref(m)) * scale(Rref(m))

Intersecting *n* Alphabetized Lists (ii)

algorithms

- Working hypothesis
  - the Lists to be intersected are stored as text files in a directory one word per row, not in alphabetical order
- Caveat
  - we will use the general word "List", to indicate the sequences of words stored in the text files, with the data type "list", used for the variable l below;
  - we use a list of vectors, instead of a bi-dimensional array since each vector can have a different length

From Module 07

Intersecting *n* Alphabetized Lists

Write the friends of Mary, John, ... in *n* pieces of paper, *in alphabetical order*

1. Put a marker at start of each list
2. If all markers show the same name, save it
3. Advance the marker(s) for the alphabetically-earliest name
4. Repeat steps 2-3 until some marker reaches the end

Intersecting *n* Alphabetized Lists (iii)

algorithms

Uses:

l:	list of vectors, each vector contains the words read from the text file
fileNames:	vector of the filenames of the files found in the directory
listsLength:	vector of the dimension of each word List
marker:	vector of the current marker for each word List initialized to all 1, since we start examining each List from the beginning
intersection:	vector of the resulting List
same:	logical variable used to test if all the words currently pointed by each List marker are equal

Intersecting *n* Alphabetized Lists (iv)

algorithms

Algo:

- read in fileNames the names of the files containing the Lists
- initialize an empty list l
- initialize the result intersection as an empty vector
- initialize the marker vector to all 1
- repeat varying i from 1 to the number of names in fileNames
  - read the contents of the i-th file in fileNames into the i-th component of l
  - set the length of the i-th List in listsLength
  - sort the i-th component of l
- repeat if none of the markers has reached the end, as stored in listsLength
  - set same to true if all the elements pointed by the markers are equal, otherwise set to false
  - if same is true
    - add the element of first List pointed by first marker to the result
    - increment the first marker
  - else
    - (look for the element of minimum value among those pointed by the markers)
    - set m to 1, as temporary List with the minimum pointed by its marker
    - repeat varying i from 2 to the number of Lists
      - if element pointed by i is less than the element pointed by m
        - set m to i
    - increment marker of List m
- output intersection

algorithms

Running the algorithm - 1

	l[[1]]	marker[1]	l[[2]]	marker[2]	l[[3]]	marker[3]
1	betty	1	betty	1	alan	1
2	james		michael		betty	
3	richard		richard		linda	
4			thomas		richard	

intersection

elements pointed by the markers are not equal: move the first marker corresponding to the minimum value

Running the algorithm - 2

algorithms

	l[[1]]	marker[1]	l[[2]]	marker[2]	l[[3]]	marker[3]
1	betty	1	betty	1	alan	
2	james		michael		betty	2
3	richard		richard		linda	
4			thomas		richard	

intersection

elements pointed by the markers are equal
copy the pointed element to intersection and advance first marker

Running the algorithm - 3

algorithms

	l[[1]]	marker[1]	l[[2]]	marker[2]	l[[3]]	marker[3]
1	betty		betty	1	alan	
2	james	2	michael		betty	2
3	richard		richard		linda	
4			thomas		richard	

intersection
betty

elements pointed by the markers are not equal: move the first marker corresponding to the minimum value

Running the algorithm - 4

algorithms

	l[[1]]	marker[1]	l[[2]]	marker[2]	l[[3]]	marker[3]
1	betty		betty		alan	
2	james	2	michael	2	betty	2
3	richard		richard		linda	
4			thomas		richard	

intersection
betty

elements pointed by the markers are not equal: move the first marker corresponding to the minimum value

algorithms

Running the algorithm - 5

	l[[1]]	marker[1]	l[[2]]	marker[2]	l[[3]]	marker[3]
1	betty		betty		alan	
2	james	2	michael	2	betty	
3	richard		richard		linda	3
4			thomas		richard	

intersection
betty

elements pointed by the markers are not equal: move the first marker corresponding to the minimum value

algorithms

Running the algorithm - 6

	l[[1]]	marker[1]	l[[2]]	marker[2]	l[[3]]	marker[3]
1	betty		betty		alan	
2	james		michael	2	betty	
3	richard	3	richard		linda	3
4			thomas		richard	

intersection
betty

elements pointed by the markers are not equal: move the first marker corresponding to the minimum value

Running the algorithm - 7

algorithms

	l[[1]]	marker[1]	l[[2]]	marker[2]	l[[3]]	marker[3]
1	betty		betty		alan	
2	james		michael	2	betty	
3	richard	3	richard		linda	
4			thomas		richard	4

intersection
betty

elements pointed by the markers are not equal: move the first marker corresponding to the minimum value

algorithms

Running the algorithm - 8

	l[[1]]	marker[1]	l[[2]]	marker[2]	l[[3]]	marker[3]
1	betty		betty		alan	
2	james		michael		betty	
3	richard	3	richard	3	linda	
4			thomas		richard	4

intersection
betty

elements pointed by the markers are equal
copy the pointed element to intersection and advance first marker

algorithms

Running the algorithm - 9

	l[[1]]	marker[1]	l[[2]]	marker[2]	l[[3]]	marker[3]
1	betty		betty		alan	
2	james		michael		betty	
3	richard		richard	3	linda	
4		4	thomas		richard	4

intersection
betty
richard

one of the markers is greater than the corresponding length:
algorithm ends

algorithms

Useful R functions

```
list.files(path = "<path>")
           pattern = "*.txt")
```

- produces a vector with the names of the files in the *path* directory that satisfy the *pattern*

```
rep.int(<integer number>,<number of repetitions>)
```

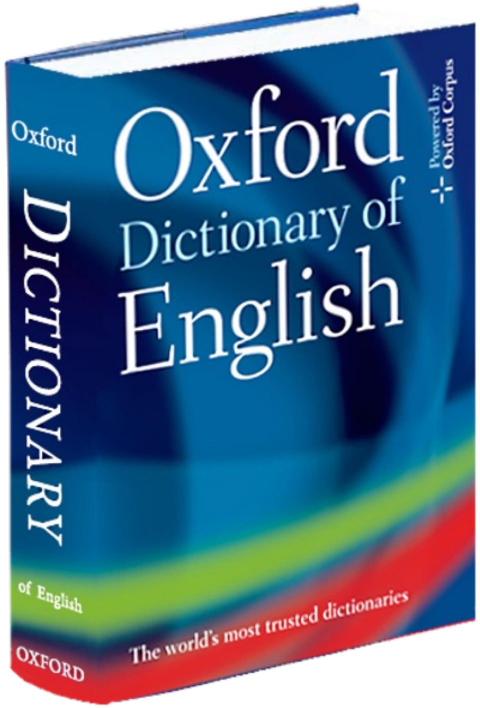
- produces a vector with a repeated integer, first argument is the integer, second is the repetition
- suitable for vector initialization

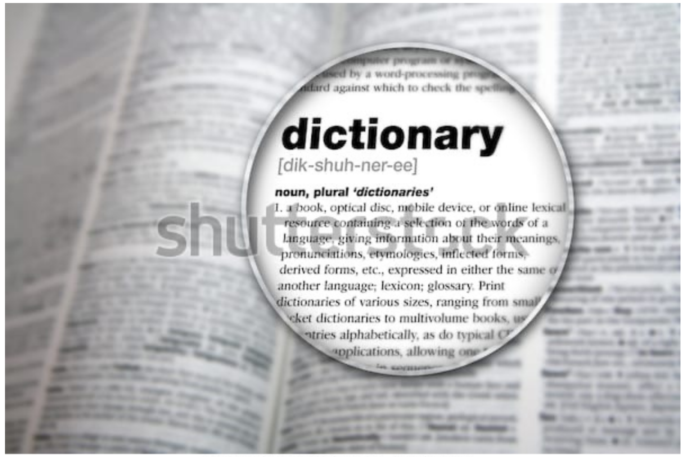
```
all(<logical vector>)
```

- produces TRUE if and only if **all** the elements of the logical vector are TRUE
- suitable for comparing two vectors

See the R algorithm and implementation in the R examples

Looking for a word in a dictionary





www.shutterstock.com · 392723320

Looking for a word in a dictionary

Algorithm – first attempt

1. You are looking for the word *thing*
2. Open the dictionary at some position
3. If the word is in the open pages stop and *return the position*
4. If the word is *smaller* than the smaller word in the open pages
5. then open the dictionary at some position preceding the current one
6. else open the dictionary at some position following the current one
7. go back to step 3

Looking for a word in a dictionary
Algorithm – first attempt

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5. then open the dictionary at some position preceding the current one
6. else open the dictionary at some position following the current one
7. go back to step 3

What's wrong with this algorithm?

Looking for a word in a dictionary
Algorithm – second attempt

- You are looking for the word *thing*, *the pages to search are from the start to the end of the dictionary*
- Repeat while there are pages to search
  - Open the dictionary at some position inside the pages to search
  - If *thing* is in the open pages
    - stop and *return the position of the open pages*
  - If the word is *smaller* than the smaller word in the open pages
    - open the dictionary at some position preceding the current one: the pages to search are from the current start to the preceding position
  - else
    - open the dictionary at some position following the current one: the pages to search are from the following position to the current end
- stop and return not found

Looking for a word in a dictionary
Algorithm – second attempt

- You are looking for the word *thing, the pages to search are from the start to the end of the dictionary*
- Repeat while there are pages to search
  - Open the dictionary at some position inside the pages to search
  - If *thing* is in the open pages
    - stop and *return the position of the open pages*
  - If the word is *smaller* than the smaller word in the open pages
    - open the dictionary at some position preceding the current one
  - else
    - open the dictionary at some position following the current one
- stop and return not found

exit point: found

exit point: not found

How to test if *"there are pages to search"*

- Define the search area with two variables
  - start of the search area
  - end of the search area
  - after each failed search inside the open pages update either the start or the end of the search area

Looking for a word in a dictionary

Algorithm – third attempt

- You are looking for the word *thing, the pages to search are from the start to the end of the dictionary*
- Repeat while *the set of pages to search is not empty*
  - Open the dictionary at some position inside the pages to search
  - If the word is in the open pages
    - stop and *return the position of the open pages*
  - If *thing* is *smaller* than the smaller word in the open pages
    - update the end of the search area to the pages immediately before the open pages
  - else
    - update the start of the search area to the pages immediately after the open pages
- stop and return not found

Looking for a word in a dictionary
Algorithm – third attempt

- You are looking for the word *thing*, *the pages to search are from the start to the end of the dictionary*
- Repeat while *the set of pages to search is not empty*
  - Open the dictionary at some position inside the pages to search
  - If the word is in the open pages
    - stop and *return the position of the open pages*
  - If *thing* is *smaller* than the smaller word in the open pages
    - update the end of the search area to the pages immediately before the open pages
  - else
    - update the start of the search area to the pages immediately after the open pages
- stop and return not found

now we need only this "open"

left part	center	right part
start	*(start+end)/2*	*end*

how can we specify now the test

the set of pages to search is not empty?

Looking for a word in a dictionary
Algorithm – fourth attempt

- You are looking for the word *thing*,
- *set start to the first page of the dictionary*
- *set end to the last page of the dictionary*
- Repeat while *start <= end*
  - Open the dictionary at some position inside the pages to search
  - If the word is in the open pages
    - stop and *return the position of the open pages*
  - If *thing* is *smaller* than the smaller word in the open pages
    - update the end of the search area to the pages immediately before the open pages
  - else
    - update the beginning of the search area to the pages immediately after the open pages
- stop and return not found

Looking for a word in a dictionary
Algorithm – fourth attempt

- You are looking for the word *thing,*
- *set start to the first page of the dictionary*
- *set end to the last page of the dictionary*
- Repeat while *start* <= *end*
  - Open the dictionary at some position between start and end
  - If the word is in the open pages
    - stop and *return the position of the open pages*
  - If *thing* is *smaller* than the smaller word in the open pages
    - update the end of the search area to the pages immediately before the open pages
  - else
    - update the beginning of the search area to the pages immediately after the open pages
- stop and return not found

how can we specify now

at some position between start and end?

We could use any point between start and end, but the best strategy is using the central point!
In other words, the algorithm works with any choice, but using the center it *works better*

Challenge: can you prove it?

Looking for a word in a dictionary

Algorithm – final

- You are looking for the word *thing,*
- *set start to the first page of the dictionary*
- *set end to the last page of the dictionary*
- Repeat while *start* <= *end*
  - Open the dictionary in the position center = (start+end)/2
  - If the word is in the open pages
    - stop and *return the position of the open pages*
  - If *thing* is *smaller* than the smaller word in the open pages
    - update the end of the search area to the pages immediately before the open pages
  - else
    - update the beginning of the search area to the pages immediately after the open pages
- stop and return not found

Searching a target in a sorted (e.g. alphabetized) list: binary search

- Sequential search
  - it can be done on a list, no requirement on sorting
- If the list is sorted the search can be done in a more efficient way

```
parameters:
searchList: vector containing the
   sorted values to be searched
target: value to search
use:
start, end: integers indicating the
    start and the ending indexes of
    the scope
center: index of the center of the
    current scope
result:
found: boolean
position: index where value is equal to
    target, if found, or where the
    target could be inserted keeping the
    list sorted, if not found
```

```
- set start, end to the first and last element of searchList
- repeat if start<=end  (i.e. the scope is not empty)
  - compute center as (start+end)/2 using integer arithmetic
  - if target==searchList[center]
    - found: exit
    else
      if target < searchList[center]
        - continue search with scope start:center-1
      else
        - continue search with scope center+1:end
- if searchList[center] < target
  - the output position is center+1
  else the output position is center
- output: not found
```

initialization

```
- set start, end to the first and last element of searchList
- repeat if start<=end  (i.e. the scope is not empty)
  - compute center as (start+end)/2 using integer arithmetic
  - if target==searchList[center]
    - found: exit
    else
      if target < searchList[center]
        - continue search with scope start:center-1
      else
        - continue search with scope center+1:end
- if searchList[center] < target
  - the output position is center+1
  else the output position is center
- output: not found
```

main loop

finalization

Why should we use *integer arithmetic?*
What happens if start+end is odd?

left part	center	right part
start	*(start+end)/2*	*end*

Binary search (1)

Target: Eddy

	searchList	
1	Ann	start
2	Eddy	
3	Fred	
4	Mary	center
5	Oliver	
6	Peter	
7	Terry	
8	Victoria	end

```
algo:
- set start, end to the first and
  last element of searchList
- repeat if start<=end
  i.e. the scope is not empty
  - compute center as (start+end)/2
    using integer arithmetic
  - if target==searchList[center]
    - found: exit
    else
      if target < searchList[center]
        - continue search with scope
          start:center-1
      else
        - continue search with scope
          center+1:end
- if searchList[center] < target
  - the output position is center+1
  else the output position is center
- output: not found
```

Binary search (2)

Target: Eddy

	searchList	
1	Ann	start
2	Eddy	center
3	Fred	End
4	Mary	
5	Oliver	
6	Peter	
7	Terry	
8	Victoria	

```
algo:
- set start, end to the first and
  last element of searchList
- repeat if start<=end
  i.e. the scope is not empty
  - compute center as (start+end)/2
    using integer arithmetics
  - if target==searchList[center]
    - found: exit
    else
      if target < searchList[center]
        - continue search with scope
          start:center-1
      else
        - continue search with scope
          center+1:end
- if searchList[center] < target
  - the output position is center+1
  else the output position is center
- output: not found
```

Binary search (3)

Target: **Ada**

	searchList	
1	Ann	Start
2	Eddy	
3	Fred	
4	Mary	Center
5	Oliver	
6	Peter	
7	Terry	
8	Victoria	End

```
algo:
- set start, end to the first and
  last element of searchList
- repeat if start<=end
  i.e. the scope is not empty
  - compute center as (start+end)/2
    using integer arithmetics
  - if target==searchList[center]
    - found: exit
    else
      if target < searchList[center]
        - continue search with scope
          start:center-1
      else
        - continue search with scope
          center+1:end
- if searchList[center] < target
  - the output position is center+1
  else the output position is center
- output: not found
```

Binary search (4)

Target: Ada

	searchList	end
1	Ann	Start
2	Eddy	Center
3	Fred	End
4	Mary	
5	Oliver	
6	Peter	
7	Terry	
8	Victoria	

```
algo:
- set start, end to the first and
  last element of searchList
- repeat if start<=end
  i.e. the scope is not empty
  - compute center as (start+end)/2
    using integer arithmetics
  - if target==searchList[center]
    - found: exit
    else
      if target < searchList[center]
        - continue search with scope
          start:center-1
      else
        - continue search with scope
          center+1:end
- if searchList[center] < target
  - output position is center+1: exit
  else output position is center: exit
- target not found: exit
```

Binary search

Target: Ada

	searchList	
1	Ann	Start End Center
2	Eddy	
3	Fred	
4	Mary	
5	Oliver	
6	Peter	
7	Terry	
8	Victoria	

```
algo:
- set start, end to the first and
  last element of searchList
- repeat if start<=end
  i.e. the scope is not empty
  - compute center as (start+end)/2
    using integer arithmetics
  - if target==searchList[center]
    - found: exit
    else
      if target < searchList[center]
        - continue search with scope
          start:center-1
      else
        - continue search with scope
          center+1:end
- if searchList[center] < target
  - the output position is center+1
  else the output position is center
- output: not found
```

Binary search

Target: Ada

	searchList	End
1	Ann	Start
2	Eddy	
3	Fred	
4	Mary	
5	Oliver	
6	Peter	
7	Terry	
8	Victoria	

```
algo:
- set start, end to the first and
  last element of searchList
- repeat if start<=end
  i.e. the scope is not empty
  - compute center as (start+end)/2
    using integer arithmetics
  - if target==searchList[center]
    - found: exit
    else
      if target < searchList[center]
        - continue search with scope
          start:center-1
      else
        - continue search with scope
          center+1:end
- if searchList[center] < target
  - the output position is center+1
  else the output position is center
- output: not found
```

If the list has n elements and

- we repeat the execution several times with different targets,
- we keep track of the number of loop iterations in each experiment

<u>what is the maximum number of loop iterations we should expect?</u>

Search area size along the iterations

Search area size – n=1023	Iteration	$\log_2(n)$-iteration
1023	1	9
511	2	8
255	3	7
127	4	6
63	5	5
31	6	4
15	7	3
7	8	2
3	9	1
1	10	0

In general, the number of iteration is

`ceiling(log2(n+1))`

Binary search is a

divide-and-conquer

algorithm

Recap:

What is the purpose of this algorithm?

- input t, v
- set f to 0
- repeat varying i from 1 to length of v
  - if the position i of v is equal to t
    - set f to i and stop repeating
- if f is zero
  - display "no""
- else
  - display the value of f

Recap:

If the length of v is **n**, what is the **maximum** number of repetitions?

- input t, v
- set f to 0
- repeat varying i from 1 to length of v
  - if the position i of v is equal to t
    - set f to i and stop repeating
- if f is zero
  - display "no""
- else
  - display the value of f

Recap:

If the length of v is **n**, what is the **minimum** number of repetitions?

```
- input t, v
- set f to 0
- repeat varying i from 1 to length of v
  - if the position i of v is equal to t
    - set f to i and stop repeating
- if f is zero
  - display "no""
- else
  - display the value of f
```

Recap:

If the length of v is **n**, what is the **average** number of repetitions?

- input t, v
- set f to 0
- repeat varying i from 1 to length of v
  - if the position i of v is equal to t
    - set f to i and stop repeating
- if f is zero
  - display "no""
- else
  - display the value of f

Continue to module 8:
Computational complexity