Imaging Morphology with LAR *

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Abstract

In this module we aim to implement the four operators of mathematical morphology, i.e. the *dilation*, *erosion*, *opening* and *closing* operators, by the way of matrix operations representing the linear operators—*boundary* and *coboundary*—over LAR. According to the multidimensional character of LAR, our implementation is dimension-independent. In few words, it works as follows: (a) the input is (the coordinate representation of) a d-chain γ; (b) compute its boundary $\partial_d(\gamma)$; (c) extract the maximal $(d-2)$-chain $\epsilon \subset \partial_d(\gamma)$; (d) consider the $(d-1)$-chain returned from its coboundary $\delta_{d-2}(\epsilon)$; (e) compute the d-chain $\eta := \delta_{d-1}(\delta_{d-2}(\epsilon)) \subset C_d$ *without* performing the mod 2 final transformation on the resulting coordinate vector, that would provide a zero result, according to the standard algebraic constraint $\delta \circ \delta = 0$. It is easy to show that $\eta \equiv (\oplus\gamma) - (\ominus\gamma)$ provides the *morphological gradient* operator. The four standard morphological operators are therefore consequently computable.

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1 Test image generation

Various methods for the input or the generation of a test image are developed in the subsections of this section. The aim is to prepare a set of controlled test beds, used to check both the implementation and the working properties of our topological implementation of morphological operators.

1.1 Small 2D random binary image

A small binary test image is generated here by using a random approach, both for the bulk structure and the small artefacts of the image.

Generation of the gross image First we generate a 2D grid of squares by Cartesian product, and produce the bulk of the random image then used to test our approach to morphological operators via topological ones.

⟨ Generation of random image 2a ⟩ ≡

```
import scipy.misc, numpy
from numpy.random import randint
rows, columns = 100,100
rowSize, columnSize = 10,10

random_array = randint(0, 255, size=(rowSize, columnSize))
image_array = numpy.zeros((rows, columns))
for i in range(rowSize):
   for j in range(columnSize):
      for h in range(i*rowSize,i*rowSize+rowSize):
         for k in range(j*columnSize,j*columnSize+columnSize):
            if random_array[i,j] < 127:
               image_array[h,k] = 0
            else:
               image_array[h,k] = 255
scipy.misc.imsave('./outfile.png', image_array)
◇
```

Macro referenced in 4b.

Generation of random artefacts upon the image Then random noise is added to the previously generated image, in order to produce artifacts at the pixel scale.

⟨ Generation of random artifacts 2b ⟩ ≡

```
noiseFraction = 0.1
noiseQuantity = rows*columns*noiseFraction
k = 0
while k < noiseQuantity:
   i,j = randint(rows),randint(columns)
   if image_array[i,j] == 0: image_array[i,j] = 255
   else: image_array[i,j] = 0
   k += 1
scipy.misc.imsave('./outfile.png', image_array)
◇
```

Macro referenced in 4b.

2 Selection of an image segment

In this section we implement several methods for image segmentation and segment selection. The first and simplest method is the selection of the portion of a binary image contained within a (mobile) image window.

2.1 Selection of a test chain

Here we select the (white) sub-image contained in a given image window, and compute the coordinate representation of the test sub-image.

Image window A window within a d-image is defined by $2 \times d$ integer numbers (2 multi-indices), corresponding to the window `minPoint` (minimum indices) and to the window `maxPoint` (maximum indices). A list of multi-index tuples, contained in the `window` variable, is generated by the macro *Generation of multi-index window* below.

⟨ Generation of multi-index window 3a ⟩ ≡

```
from pyplasm import *
minPoint, maxPoint = (20,20), (40,30)
indexRanges = zip(minPoint,maxPoint)
window = CART([range(min,max) for min,max in indexRanges])
◇
```

Macro referenced in 4b.

From window multi-indices to chain coordinates The set of tuples within the `window` is here mapped to the corresponding set of (single) integers associated to the low-level image elements (pixels or voxels, depending on the image dimension and shape), denoted `windowChain`. Such total chain of the image `window` is then filtered to contain the only coordinates of *white* image elements within the window, and returned as the set of integer cell indices `segmentChain`.

⟨ Window-to-chain mapping 3b ⟩ ≡

```
imageShape = [rows,columns]
d = len(imageShape)
weights = [PROD(imageShape[(k+1):]) for k in range(d-1)]+[1]
imageCochain = image_array.reshape(PROD(imageShape))
windowChain = [INNERPROD([index,weights]) for index in window]
segmentChain = [cell for cell in windowChain if imageCochain[cell]==255]
◇
```

Macro referenced in 4b.

2.2 Show segment chain from binary image

Now we need to show visually the selected `segmentChain`, by change the color of its cells from white (255) to middle grey (127). Just remember that `imageCochain` is the linear representation of the image, with number of cells equal to `PROD(imageShape)`. Then the modified image is restored within `image_array`, and is finally exported to a `.png` image file.

⟨ Change chain color to grey 4a ⟩ ≡

```
for cell in segmentChain: imageCochain[cell] = 127
image_array = imageCochain.reshape(imageShape)
scipy.misc.imsave('./outfile.png', image_array)
◇
```

Macro referenced in 4b.

Test example The macros previously defined are here composed to generate a random black and white image, with a *image segment* (in a fixed position window within the image) extracted, colored in middle grey, and exported to an image file.

`"test/py/morph/test01.py"` 4b ≡

```
⟨ Import the module (4c largrid ) 5a ⟩
⟨ Generation of random image 2a ⟩
⟨ Generation of random artifacts 2b ⟩
⟨ Generation of multi-index window 3a ⟩
⟨ Window-to-chain mapping 3b ⟩
⟨ Change chain color to grey 4a ⟩
⟨ Pyplasm visualisation of an image chain 4d ⟩
◇
```

3 Construction of (co)boundary operators

A d-image is a *cellular d-complex* where cells are k-cuboids ($0 \leq k \leq d$), i.e. Cartesian products of a number k of 1D intervals, embedded in d-dimensional Euclidean space.

A direct construction of cuboidal complexes is offered in `larcc` by the `largrid` module. The `visImageChain` function given by the macro *Visualisation of an image chain* below.

3.1 Visualisation of an image chain

⟨ Pyplasm visualisation of an image chain 4d ⟩ ≡

```
def visImageChain (imageShape,chain):
   model = larCuboids(imageShape)
   imageVerts = model[0]
   imageLAR = model[1]
   chainLAR = [cell for k,cell in enumerate(imageLAR) if k in chain]
   return imageVerts,chainLAR

if __name__== "__main__":
   model = visImageChain (imageShape,segmentChain)
   VIEW(EXPLODE(1.2,1.2,1.2)(MKPOLS(model)))
◇
```

Macro referenced in 4b.

A Utilities

A.1 Importing a generic module

First we define a parametric macro to allow the importing of `larcc` modules from the project repository `lib/py/`. When the user needs to import some project's module, she may call this macro as done in Section **??**.

⟨ Import the module 5a ⟩ ≡

```
import sys
sys.path.insert(0, 'lib/py/')
import @1
from @1 import *
◇
```

Macro referenced in 4b, 5b.

Importing a module A function used to import a generic `lacccc` module within the current environment is also useful.

⟨ Function to import a generic module 5b ⟩ ≡

```
def importModule(moduleName):
   ⟨ Import the module (5c moduleName ) 5a ⟩
◇
```

Macro never referenced.

References

[CL13] CVD-Lab, *Linear algebraic representation*, Tech. Report 13-00, Roma Tre University, October 2013.