The `smplxn` module *

Alberto Paoluzzi

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Abstract

This module defines a minimal set of functions to generate a dimension-independent grid of simplices. The name of the library was firstly used by our CAD Lab at University of Rome "La Sapienza" in years 1987/88 when we started working with dimension-independent simplicial complexes [PBCF93]. This one in turn imports some functions from the `scipy` package and the geometric library `pyplasm` [].

Contents

*This document is part of the framework [CL13]. November 13, 2013

1 Introduction

2 Signed (co)boundary matrices of a simplicial complex

Importing a library First of all, a modeling application having to deal with simplicial complexes must import the $Simple_X^n$ library, denoted `smplxn` in `python`.

⟨ Inport the $Simple_X^n$ library 2a ⟩ ≡

```
import sys
sys.path.insert(0, 'lib/py/')
from smplxn import *
◇
```

Macro referenced in 3c, 4b.

⟨ Inport a generic 2b ⟩ ≡
module◇

Macro never referenced.

mod
import sys sys.path.insert(0, 'lib/py/') import module as mod

3 Test examples

3.1 Structured grid

3.1.1 2D example

Generate a simplicial decomposition Then we generate and show a 2D decomposition of the unit square $[0,1]^2 \subset \mathbb{E}^2$ into a 3×3 grid of simplices (triangles, in this case), using the `simplexGrid` function, that returns a pair `(V,FV)`, made by the array `V` of vertices, and by the array `FV` of "faces by vertex" indices, that constitute a *reduced* simplicial LAR of the $[0,1]^2$ domain. The computed `FV` array is then dispayed "exploded", being ex, ey, ez the explosion parameters in the x, y, z coordinate directions, respectively. Notice that the `MKPOLS` pyplasm primitive requires a pair `(V,FV)`, that we call a "model", as input — i.e. a pair made by the array `V` of vertices, and by a zero-based array of array of indices of vertices. Elsewhere in this document we identified such a data structure as CSR(M_d), for some dimension d. Suc notation stands for the Compressed Sparse Row representation of a binary characteristic matrix.

⟨ Generate a simplicial decomposition ot the $[0,1]^2$ domain 3a ⟩ ≡

```
V,FV = simplexGrid([3,3])
VIEW(EXPLODE(1.5,1.5,1.5)(MKPOLS((V,FV))))
◇
```

Macro referenced in 3c.

Extract the $(d-1)$-faces Since the complex is simplicial, we can directly extract its facets (in this case the 1-faces, i.e. its edges) by invoking the `simplexFacets` function on the argument `FV`, so returning the array `EV` of "edges by vertex" indices.

⟨ Extract the edges of the 2D decomposition 3b ⟩ ≡

```
EV = simplexFacets(FV)
ex,ey,ez = 1.5,1.5,1.5
VIEW(EXPLODE(ex,ey,ez)(MKPOLS((V,EV))))
◇
```

Macro referenced in 3c.

Export the executable file We are finally able to generate and output a complete test file, including the visualization expressions. This file can be executed by the `test` target of the `make` command.

`"test/py/test01.py"` 3c ≡

⟨ Inport the $Simple_X^n$ library 2a ⟩
⟨ Generate a simplicial decomposition ot the $[0,1]^2$ domain 3a ⟩
⟨ Extract the edges of the 2D decomposition 3b ⟩
◇

3.1.2 3D example

In this case we produce a $2 \times 2 \times 2$ grid of tetrahedra. The dimension (3D) of the model to be generated is inferred by the presence of 3 parameters in the parameter list of the `simplexGrid` function.

⟨ Generate a simplicial decomposition ot the $[0,1]^3$ domain 3d ⟩ ≡

```
V,CV = simplexGrid([2,2,2])
VIEW(EXPLODE(1.5,1.5,1.5)(MKPOLS((V,CV))))
◇
```

Macro referenced in 4b.

and repeat two times the facet extraction:

⟨ Extract the faces and edges of the 3D decomposition 4a ⟩ ≡

```
FV = simplexFacets(CV)
VIEW(EXPLODE(1.5,1.5,1.5)(MKPOLS((V,FV))))
EV = simplexFacets(FV)
VIEW(EXPLODE(1.5,1.5,1.5)(MKPOLS((V,EV))))
◇
```

Macro referenced in 4b.

and finally export a new test file:

"test/py/test02.py" 4b ≡

⟨ Inport the $Simple_X^n$ library 2a ⟩
⟨ Generate a simplicial decomposition ot the $[0,1]^3$ domain 3d ⟩
⟨ Extract the faces and edges of the 3D decomposition 4a ⟩
◇

3.2 Unstructured grid

3.2.1 2D example

3.2.2 3D example

References

[CL13] CVD-Lab, *Linear algebraic representation*, Tech. Report 13-00, Roma Tre University, October 2013.

[PBCF93] A. Paoluzzi, F. Bernardini, C. Cattani, and V. Ferrucci, *Dimension-independent modeling with simplicial complexes*, ACM Trans. Graph. **12** (1993), no. 1, 56–102.