Imaging Morphology with LAR *

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February 26, 2014

Abstract

In this module we aim to implement the four operators of mathematical morphology, i.e. the *dilation*, *erosion*, *opening* and *closing* operators, by the way of matrix operations representing the linear operators—*boundary* and *coboundary*—over LAR. According to the multidimensional character of LAR, our implementation is dimension-independent. In few words, it works as follows: (a) the input is (the coordinate representation of) a d-chain γ; (b) compute its boundary $\partial_d(\gamma)$; (c) extract the maximal $(d-2)$-chain $\epsilon \subset \partial_d(\gamma)$; (d) consider the $(d-1)$-chain returned from its coboundary $\delta_{d-2}(\epsilon)$; (e) compute the d-chain $\eta := \delta_{d-1}(\delta_{d-2}(\epsilon)) \subset C_d$ *without* performing the mod 2 final transformation on the resulting coordinate vector, that would provide a zero result, according to the standard algebraic constraint $\delta \circ \delta = 0$. It is easy to show that $\eta \equiv (\oplus\gamma) - (\ominus\gamma)$ provides the *morphological gradient* operator. The four standard morphological operators are therefore consequently computable.

Contents

1 Test image generation

Various methods for the input or the generation of a test image are developed in the subsections of this section. The aim is to prepare a set of controlled test beds, used to check both the implementation and the working properties of our topological implementation of morphological operators.

*This document is part of the *Linear Algebraic Representation with CoChains* (LAR-CC) framework [CL13]. February 26, 2014

1.1 Small 2D random binary image

A small binary test image is generated here by using a random approach, both for the bulk structure and the small artefacts of the image.

Generation of the gross image First we generate a 2D grid of squares by Cartesian product, and produce the bulk of the random image then used to test our approach to morphological operators via topological ones.

⟨ Generation of random image 2a ⟩ ≡

```
import scipy.misc, numpy
from numpy.random import randint
rows, columns = 100,100
rowSize, columnSize = 10,10

random_array = randint(0, 255, size=(rowSize, columnSize))
image_array = numpy.zeros((rows, columns))
for i in range(rowSize):
   for j in range(columnSize):
      for h in range(i*rowSize,i*rowSize+rowSize):
         for k in range(j*columnSize,j*columnSize+columnSize):
            if random_array[i,j] < 127:
               image_array[h,k] = 0
            else:
               image_array[h,k] = 255
scipy.misc.imsave('./outfile.png', image_array)
```

◇

Macro never referenced.

Generation of random artefacts upon the image Then random noise is added to the previously generated image, in order to produce artifacts at the pixel scale.

⟨ Generation of random artifacts 2b ⟩ ≡

```
noiseFraction = 0.1
noiseQuantity = rows*columns*noiseFraction
k = 0
while k < noiseQuantity:
   i,j = randint(rows),randint(columns)
   if image_array[i,j] == 0: image_array[i,j] = 255
   else: image_array[i,j] = 0
   k += 1
scipy.misc.imsave('./outfile.png', image_array)
```

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Macro never referenced.

2 Selection of an image segment

In this section we implement several image segmentation and selection of a segment methods. The first and simplest method is the selection of the portion of a binary image contained within a (mobile) image window.

2.1 Selection of a test chain

Here we select the (white) sub-image contained in a given image window, and compute the coordinate representation of the test sub-image.

References

[CL13] CVD-Lab, *Linear algebraic representation*, Tech. Report 13-00, Roma Tre University, October 2013.