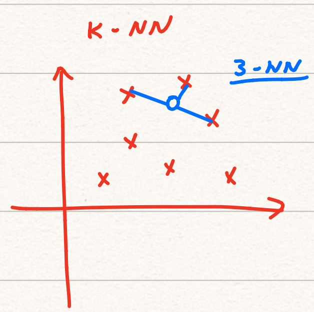
1. For the following dataset:

apple	*ibm*	*lemon*	*sun*	CLASS
TRAINING INSTANCES				
4	0	1	1	FRUIT
5	0	5	2	FRUIT
2	5	0	0	COMPUTER
1	2	1	7	COMPUTER
TEST INSTANCES				
2	0	3	1	?
1	2	1	0	?

(i). Using the **Euclidean distance** measure, classify the test instances using the 1-NN method.

Euclidean (2-norm) : vectors A & B

$$d_E(A,B) = \sqrt{\Sigma_k (a_k - b_k)^2}$$



1-NN for T_1 : $\langle 2, 0, 3, 1 \rangle$

$$d_E(T_1, A) = \sqrt{(4-2)^2 + (0-0)^2 + (1-3)^2 + (1-1)^2}$$

$$= \sqrt{4+0+4+0}$$

$$= \sqrt{8}$$

$$d_E(T_1, B) = \sqrt{(5-2)^2 + (0-0)^2 + (5-3)^2 + (2-1)^2}$$

$$= \sqrt{9+0+4+1}$$

$$= \sqrt{14}$$

$$d_E(T_1, C) = \sqrt{35}$$

$$d_E(T_1, D) = \sqrt{49}$$

$\Rightarrow$ Classify T_1 as Fruit.

For T_2 :

$$d_E(T_2, A) = \sqrt{14}$$

$$d_E(T_2, B) = \sqrt{40}$$

$$d_E(T_2, C) = \sqrt{11}$$

$$d_E(T_2, D) = \sqrt{19}$$

$\Rightarrow$ Classify T_2 as computer (ii). Using the **Manhattan distance** measure, classify the test instances using the 3-NN method, for the three weightings we discussed in the lectures: *majority class*, *inverse distance*, *inverse linear distance*.

$$d_M(A,B) = \Sigma_k |a_k - b_k|$$

$$d_M(T_1, A) = |4-2| + |0-0| + |1-3| + |1-1|$$

$$= 2 + 0 + 2 + 0$$

$$= 4$$

$$d_M(T_1, B) = 6$$

$$d_M(T_1, C) = 9$$

3-NN : A, B, C

$$d_M(T_1, D) = 11$$

Classification : (3 weightings)

① Majority Class (equal weights)

2 Fruit , 1 Computer

⇒ Classify as Fruit

② Inverse distance ($w = \frac{1}{d+\varepsilon}$) ← avoid $\frac{1}{0}$

Let $\varepsilon = 1$

for A (fruit) : $\frac{1}{4+1} = 0.2$

for B (fruit) : $\frac{1}{6+1} = 0.14$

"Score" for fruit = 0.2+0.14 = 0.34

for C (comp) : $\frac{1}{9+1} = 0.1$

0.34 (fruit) > 0.1 (comp) ⇒ Classify as fruit

③ Inverse linear distance ($w_j = \frac{d_3 - d_j}{d_3 - d_1}$) (rescaling dist) ⇒ $w_1 = \frac{d_3 - d_1}{d_3 - d_1} = 1$

↖ furthest ↖ nearest (in NN)

$w_3 = \frac{d_1 - d_1}{d_3 - d_1} = 0$

⇒ $w_j \in [0, 1]$

$$\text{3-NN}\left\{\begin{array}{l} d_M(T_1, A) = 4 \\ d_M(T_1, B) = 6 \\ d_M(T_1, C) = 9 \end{array}\right. \Rightarrow \left.\begin{array}{l} \text{For A (fruit)}: \frac{9-4}{9-4} = 1 \\ \text{For B (fruit)}: \frac{9-6}{9-4} = \frac{3}{5} = 0.6 \\ \text{For C (comp)}: \frac{9-9}{9-4} = 0 \end{array}\right\} 1+0.6 = 1.6$$

$\cancel{d_M(T_1, D) = 11}$

1.6 (fruit) > 0 (comp) ⇒ Classify as fruit.

(iii). Can we do weighted k-NN using **cosine similarity**?

Yes, easier than distance, use cos similarity as weights directly.

(score)
weight : cosine similarities.

All predictions:

Inst	Measure	k	Weight	Prediction
T_1	d_E	1	-	FRUIT
		3	Maj	FRUIT
		3	ID	FRUIT
		3	ILD	FRUIT
	d_M	1	-	FRUIT
		3	Maj	FRUIT
		3	ID	FRUIT
		3	ILD	FRUIT
	cos	1	-	FRUIT
		3	Maj	FRUIT
		3	Sum	FRUIT
T_2	d_E	1	-	COMPUTER
		3	Maj	FRUIT
		3	ID	FRUIT
		3	ILD	COMPUTER
	d_M	1	-	COMPUTER
		3	Maj	COMPUTER
		3	ID	COMPUTER
		3	ILD	COMPUTER
	cos	1	-	COMPUTER
		3	Maj	FRUIT
		3	Sum	FRUIT

2. Approximately 1% of women aged between 40 and 50 have breast cancer. 80% of mammogram screening tests detect breast cancer when it is there. 90% of mammograms DO NOT show breast cancer when it is **NOT** there[1]. Based on this information, complete the following table.

Cancer	Probability
No	99%
Yes	1%

Cancer	Test	Probability
Yes	Positive	80%
Yes	Negative	?
No	Positive	?
No	Negative	90%

$\rightarrow$ $P(P|C) = 0.8$ TP

$P(N|C) = 1 - P(P|C) = 0.2$ FN

$P(P|NC) = 1 - P(N|NC) = 0.1$ FP

$P(N|NC) = 0.9$ TN

3. Based on the results in question 2, calculate the **marginal probability** of 'positive' results in a Mammogram Screening Test.

$P(P) = ?$

$\Rightarrow$ Law of total probability

$$P(p) = \sum_{i \in \{C, NC\}} P(p|i)\,P(i) = \sum_{i \in \{C, NC\}} P(p, i)$$

$$= P(p|C)\,P(C) + P(p|NC)\,P(NC)$$

$$= 0.8 \times 0.01 + 0.1 \times 0.99$$

$$= 0.008 + 0.099$$

$$= 0.107$$

4. Based on the results in question 2, calculate *P(Cancer = 'Yes' | Test = 'Positive')*, using the Bayes Rule.

$$P(c|p) = \frac{P(p|c)\,P(c)}{P(p)}$$

$$= \frac{0.8 \times 0.01}{0.107}$$

$$= 0.075 = 7.5\%$$