Principles of Biomedical Ultrasound and Photoacoustics
hw03: Photoacoustic Depth Profiling and SO2 Measurement

Due on Tuesday, Dec 12, 2017

106061531 Fu-En Wang

1 Part I

1.1 Repeat Fig.1 in reference paper

In the reference paper, they had simulated an acoustic wave of forward and backward wave. In this problem, we need to reproduce this fig. Figure 1 shows the result.

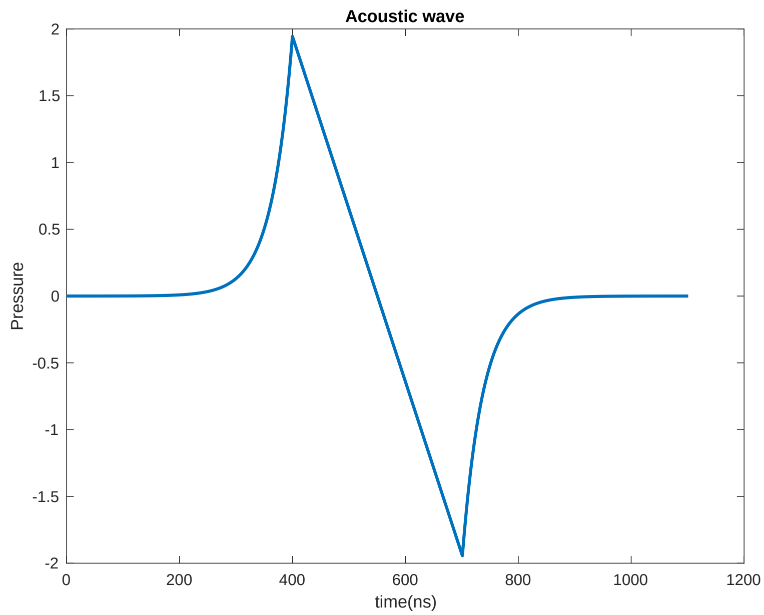

Figure 1: Acoustic Wave

Because the part between positive and negative wave is implicit, I only use a straight line connecting the two peak points by dynamically solving a linear equation and make the space between them 300 ns.

1.2 Repeat Fig.2 in reference paper

Now add a gaussian noise to our signal which the ratio of standard deviation of the noise to the peak of the simulated photoacoustic signal is 5%. Figure 2 shows the noisy signal and Figure 3 shows the exponential decay of siganl.

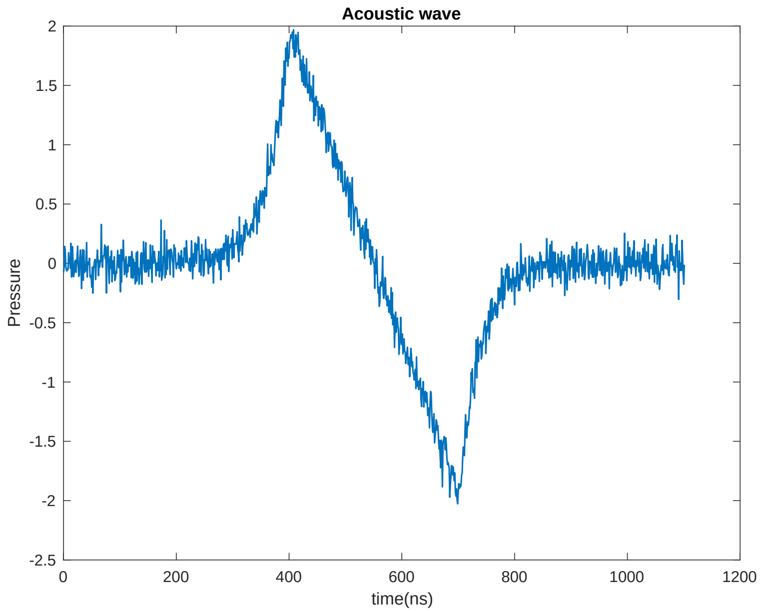

Figure 2: Acoustic Wave with noise

In the reference paper, there is a equation for curve fitting this decay from author's experimental result.

$$p(z) = 9.5 \exp^{-185z}$$

Because the parameters I used is different, I adjust the amplitude from 9.5 to 1.94 as shown in Equation 1.

$$p(z) = 1.94 \exp^{-185z} \tag{1}$$

Now Figure 3 show the noisy decay and curve of Equation 1.

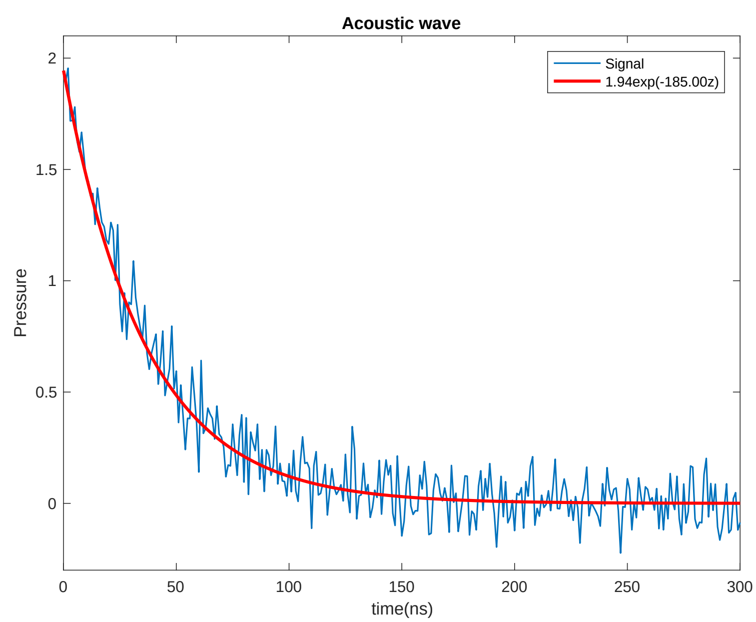

Figure 3: Acoustic Wave with noise

In this figure, we can find that Equation 1 is fitting well.

1.3 Curve fitting for absorbtion coefficient

Now from Figure 3, we apply curve fitting and get our estimated μ_a of noisy signal. Figure 4 shows the estimated curve and noisy signal.

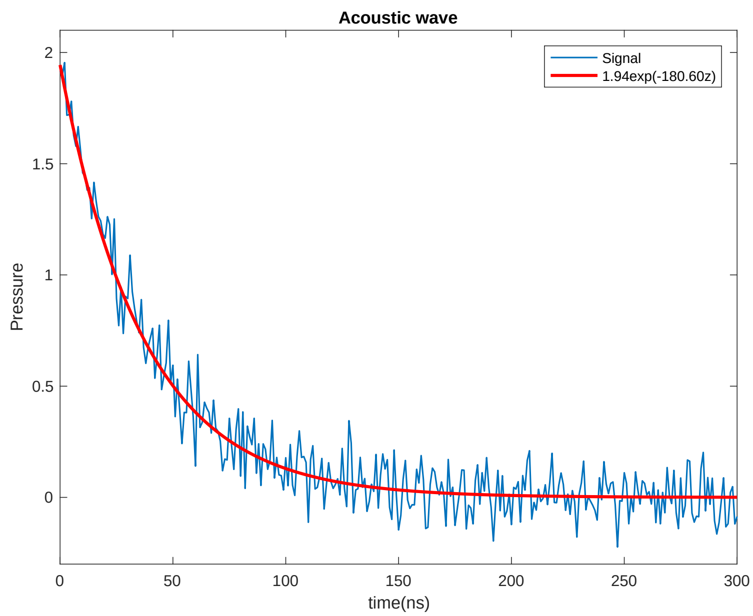

Figure 4: Curve fitting for absorbtion coefficient

From my experimental result, the range of estimated μ_a is from 178 to 182 and the real μ_a I used is 180. As a result, the curve fits the signal pretty well I think.

1.4 Peak value vs absorbtion coefficient

Theoretically, the peak of signal will proportional to μ_a which is $\mu_a \times \Gamma \times H_0 = 0.0108\mu_a$ in my case. So in this part, we need to plot the peak value of different absorbtion coefficient from 10 to 180. Figure 5 shows the result.

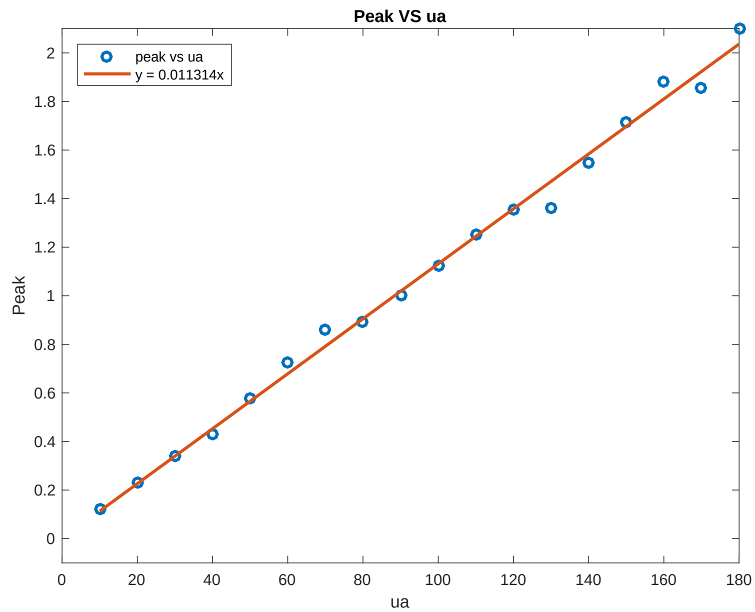

Figure 5: peak vs absorbtion coefficient

For better visualization, I also plot the curve fitting result for peak and μ_a and the slope is 0.011 which is very close to theoretical value 0.0108. So the peak value of noisy signal is still propotional to μ_a. And the reason I think is that theoretically the mean of gaussian noise is zero, so when we add such noise to signal the change of peak value will be bounded in a certain range and the peak will be still proportional with small error.

1.5 μ_a (estimated) vs μ_a (real)

Similar to Section 1.4, now for each μ_a we need to use curve fitting to estimate absorbtion coefficient for them. Figure 6 shows the result.

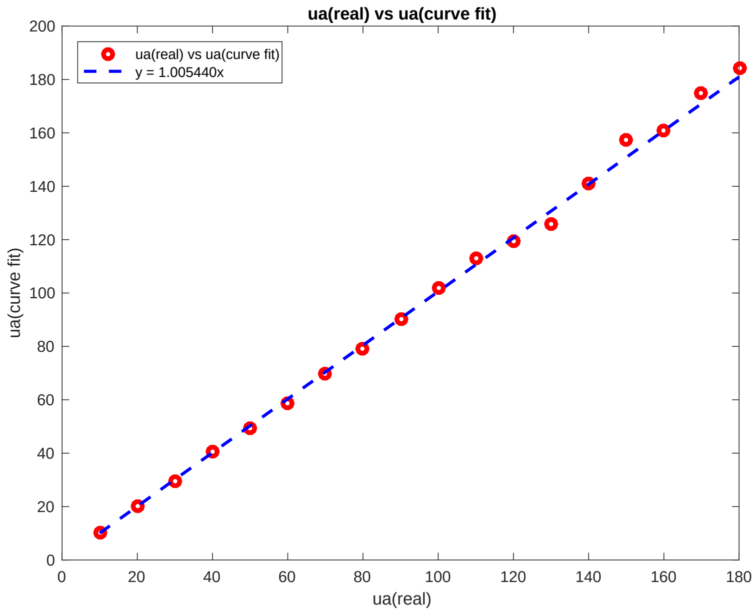

Figure 6: μ_a (estimated) vs μ_a (real)

For better visualization, I use curve fitting for μ_a (estimated) vs μ_a (real) in Figure 6 and the slope is about 1. As a result, our estimated μ_a is really close to real one and surely proportional to it. Because our signal is $\mu_a * H_0 * \Gamma * \exp^{-\mu_a z} + noise$ where mean of noise is zero, to get an accurate estimated μ_a I use an optimization function:

$$f(x) = (x * H_0 * \Gamma * \exp^{-xz}) - (\mu_a * H_0 * \Gamma * \exp^{-\mu_a z} + noise) \quad (2)$$

$$\mu_a' = \min_x |f(x)|^2 \quad (3)$$

And μ_a' is the result I estimate.

1.6 Repeat 4 and 5 with transducer impulse response

Now we repeat 4 and 5 but considering transducer impulse response. Assume the impulse responses of the transducer used are Gaussian pulse centered at 5 MHz, 10 MHz, 25 MHz, and 50 MHz, respectively, with -6 dB fractional bandwidth of 60%, the impulse reponse of transducer is shown in Figure 7

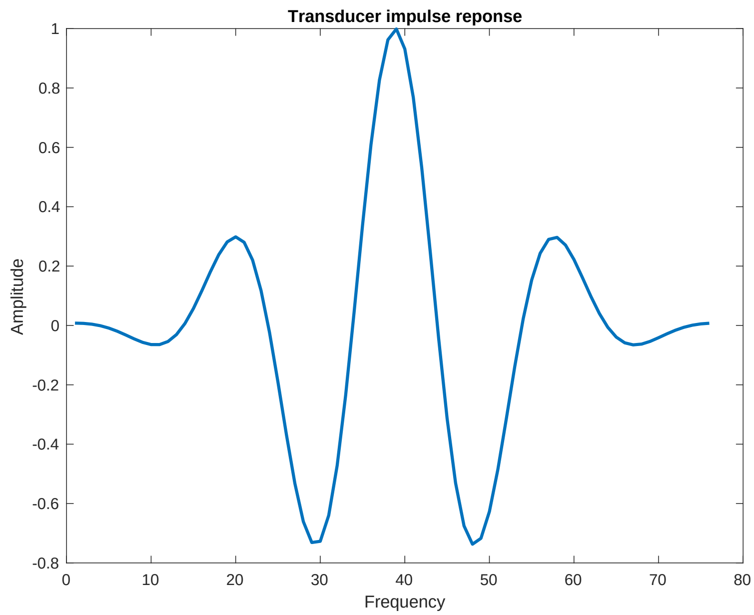

Figure 7: Transducer impulse response

1.6.1 Peak value vs absorbtion coefficient

Figure 8 show the result of Peak value vs absorbtion coefficient.

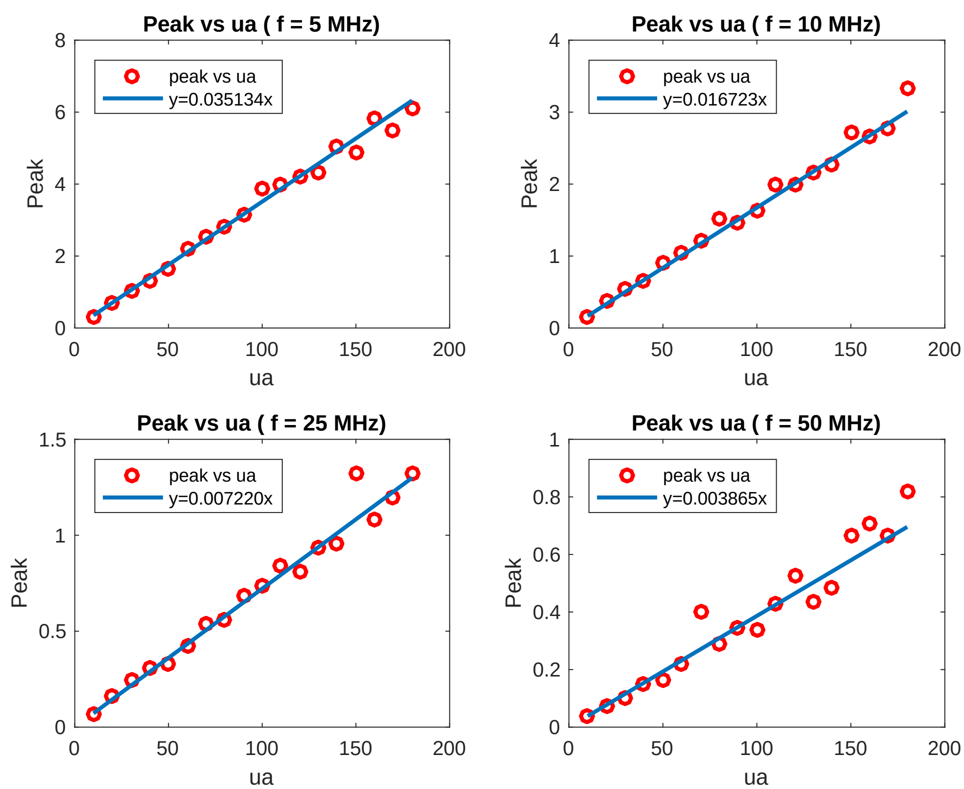

Figure 8: Peak value vs absorbtion coefficient

In Figure 8 we can find that their relation is still proportional from the four sub figures. However, the slope is not 0.011 as Figure 5 anymore because we apply convolution to signal and transducer impulse response which will change our signal magnitude but won't change the relation of them.

1.6.2 μ_a (estimated) vs μ_a (real)

Figure 9 show the result.

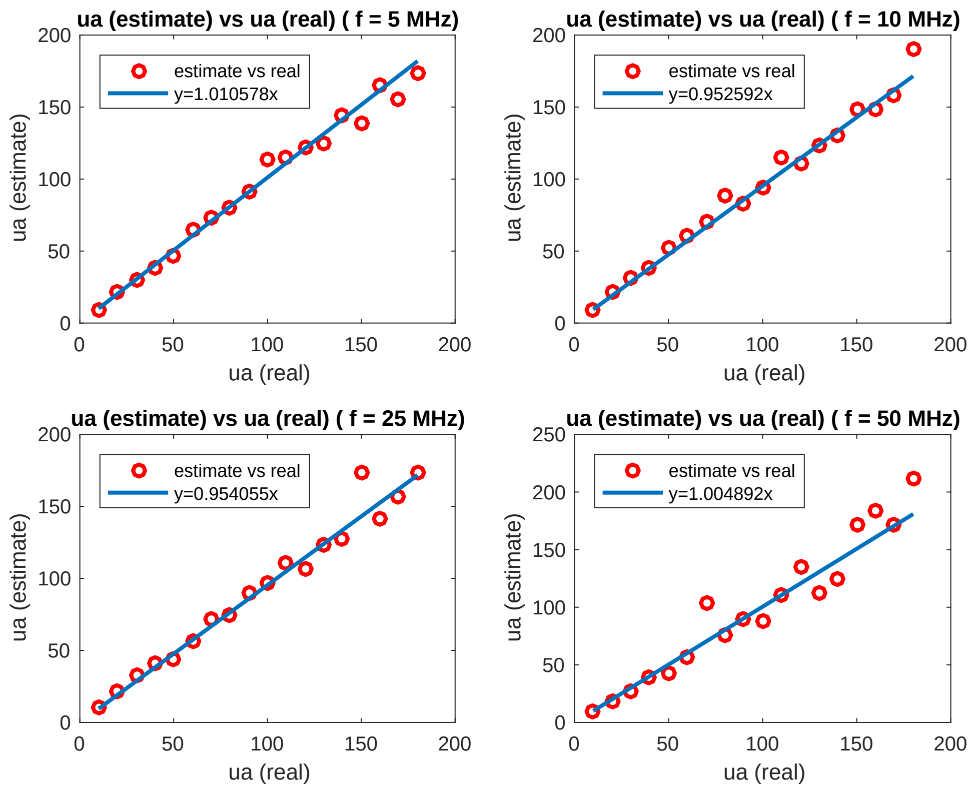

Figure 9: μ_a (estimated) vs μ_a (real)

In Figure 9, we can find that the estimated μ_a is still close to real μ_a and the slope is about 1, but when transducer center frequency gets larger, the points become much noisier because of lower PSR. To get an accurate μ_a, I use an optimization function:

$$f(x) = conv(x * H_0 * \Gamma * \exp^{-xz}, response) - conv(\mu_a * H_0 * \Gamma * \exp^{-\mu_a z} + noise, response) \tag{4}$$

$$\mu_a' = \min_x |f(x)|^2 \tag{5}$$

And μ_a' is the result I get.

2 Part II

Now in the section, we will use the same methodology in Part I for simulating blood sample.

2.1 Estimate SO2 level with peaks of different λ

In this problem, we need to estimate SO2 level with simulated peaks of several λ (= 578, 584, 590, and 596 nm). In class materials, SO2 level can be derived by solving a linear system:

$$k \times \begin{bmatrix} \varepsilon_{HbO2}(\lambda_1) & \varepsilon_{Hb}(\lambda_1) \\ \varepsilon_{HbO2}(\lambda_2) & \varepsilon_{Hb}(\lambda_2) \\ \varepsilon_{HbO2}(\lambda_3) & \varepsilon_{Hb}(\lambda_3) \\ \varepsilon_{HbO2}(\lambda_4) & \varepsilon_{Hb}(\lambda_4) \end{bmatrix} \begin{bmatrix} [HbO2] \\ [Hb] \end{bmatrix} = \begin{bmatrix} P(\lambda_1) \\ P(\lambda_2) \\ P(\lambda_3) \\ P(\lambda_4) \end{bmatrix} \tag{6}$$

Although we don't know what exactly is, but when looking at Equation 6, we can find it is a linear system. So we can move k into linear solution as the following formula:

$$\begin{bmatrix} \varepsilon_{HbO2}(\lambda_1) & \varepsilon_{Hb}(\lambda_1) \\ \varepsilon_{HbO2}(\lambda_2) & \varepsilon_{Hb}(\lambda_2) \\ \varepsilon_{HbO2}(\lambda_3) & \varepsilon_{Hb}(\lambda_3) \\ \varepsilon_{HbO2}(\lambda_4) & \varepsilon_{Hb}(\lambda_4) \end{bmatrix} \begin{bmatrix} k \times [HbO2] \\ k \times [Hb] \end{bmatrix} = \begin{bmatrix} P(\lambda_1) \\ P(\lambda_2) \\ P(\lambda_3) \\ P(\lambda_4) \end{bmatrix} \tag{7}$$

Now we can easily use psuedo inverse to get the solution $\begin{bmatrix} k \times [HbO2] \\ k \times [Hb] \end{bmatrix}$ which k is still unknown. However, the calculation of SO2 level is:

$$\begin{aligned} SO2 &= \frac{[HbO2]}{[HbO2] + [Hb]} \\ &= \frac{k \times [HbO2]}{k \times [HbO2] + k \times [Hb]} \\ &= \frac{[HbO2]}{[HbO2] + [Hb]} \end{aligned}$$

As a result, k wont't affect the calculation of SO2 at all and thus we can just set k to any number.
Now we can estimate SO2 with different transducer impulse and plot it as function of real SO2 as shown in Figure 10.

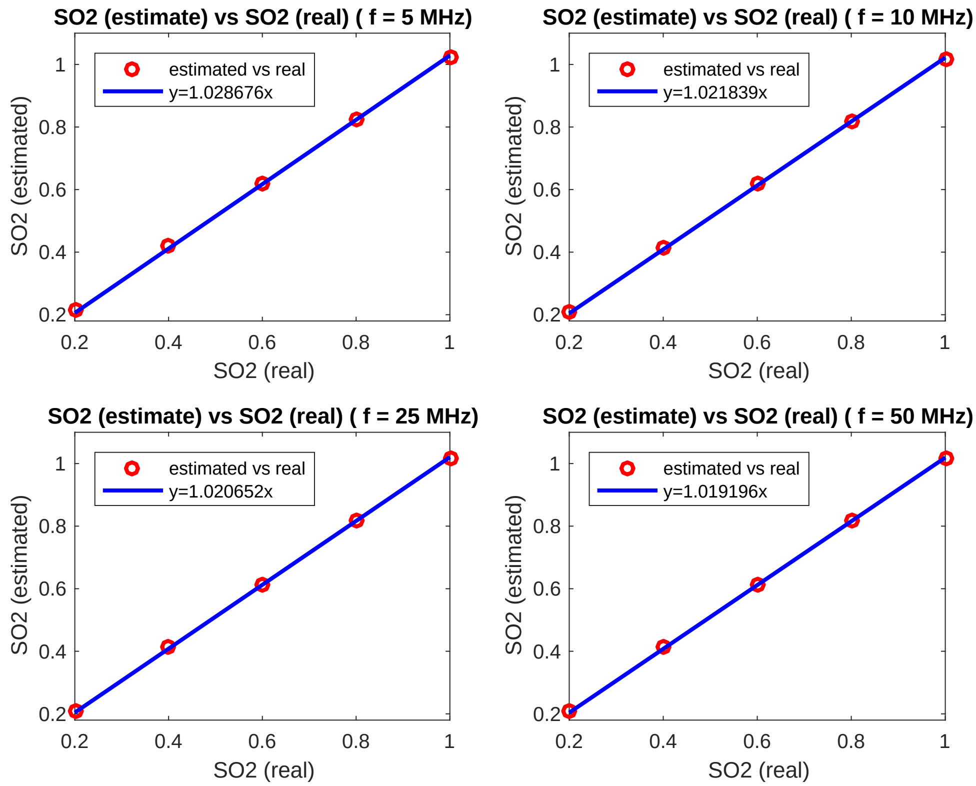

Figure 10: SO2 (estimated) vs SO2 (real)

In Figure 10, we can find that our estimated SO2 is really close to grount truth and slope is almost 1. As a result, I think peak value of four different λ is good enough to get SO2 level.

2.2 Estimate SO2 for different λ

Same as Section 2.1, but now our λ becomes [760, 780, 800, 820] nm. The result is shown in Figure 11.

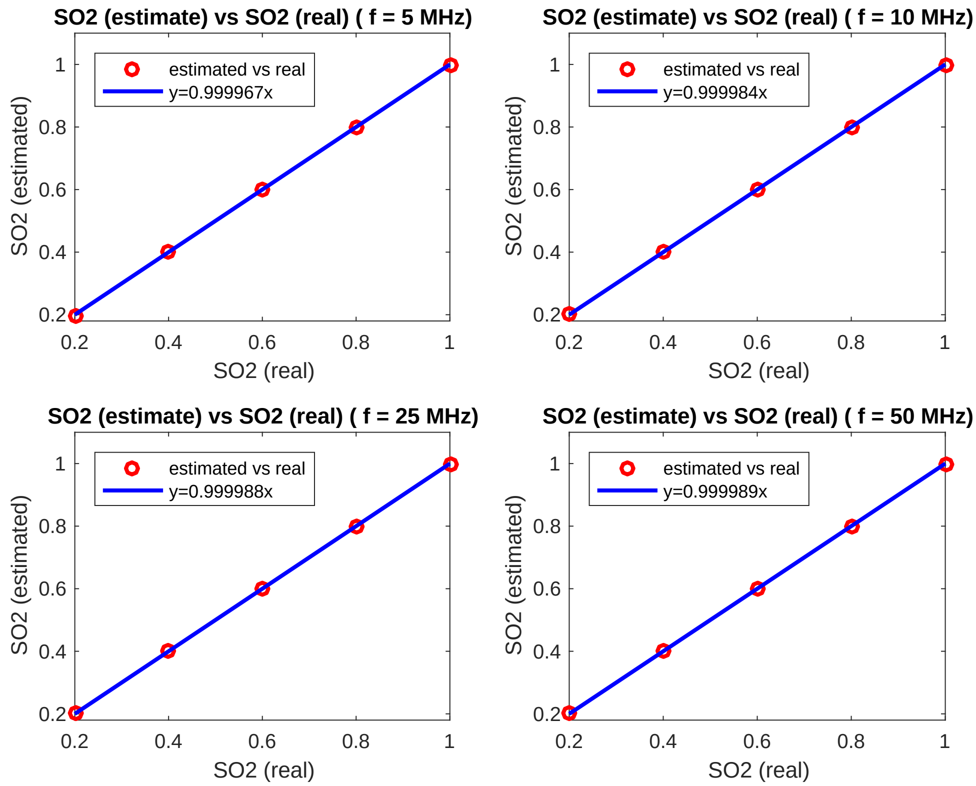

Figure 11: SO2 (estimated) vs SO2 (real)

In Figure 11, we can find that our estimated SO2 is really close to grount truth and slope is almost 1. As a result, I think peak value of four different λ is good enough to get SO2 level.

3 Discussion

After I finish this homework, I find the result in Section 2 is quite different compared with result of other classmates. It seem that my result is **too accurate**. After checking with them, I found the reason is the convolution part. They convolute the result in Figure 1 with transducer impulse reponse, while I use result in Figure 3 and thus get such accurate result. However, I don't think this is a mistake because μ_a will directly affect the exponential decay and the result makes sense when I only use this part to estimate μ_a and SO2 level. But the signal transducer recieves is actually more like Figure 1. As a result, I think the methods used by us are all correct and thus I keep my answer here.

4 Code Usage

To reproduce my result for all problems, just modify the following parameters in the beginning of my code. Just set corresponding config to true or false (only one of p2a and p2b can be true).

```
p1a = true;
p1b = true;
p1d = true;
p1e = true;
p1f = true;

p2a = false; % Only one of p2a and p2b can set to true.
p2b = true;  % Only one of p2a and p2b can set to true.
```

And then just run it.