Principles of Biomedical Ultrasound and Photoacoustics
hw01: Displacement and Strain

Due on Thursday, Nov 2, 2017

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1 Introduction

For **Focused Ultrasound Thermal Therapy**, an important technique is to estimate the temperature change before and after applying it. The estimation can be derived by the echo-time shift before and after heating. Moreover, the temperature change can be formula as:

$$\Delta T(z) = \frac{C_0}{2} \cdot K \cdot \frac{\partial \Delta t(z)}{\partial z} \tag{1}$$

where $\Delta T(z)$ is the temperature change, C_0 is the speed of sound, K is a constant, $\frac{\partial \Delta t(z)}{\partial z}$ is the **thermal strain**.
In this homework, we need to finish the following requirements:

1. Estimate echo time shift in μs as a function of depth
2. Estimate thermal strain in % as a function of depth

2 Source Code

In this zip archive, there are two matlab source code files:

1. **EE6265_HW1_106061531.m**
2. **Windows.m**

"EE6265_HW1_106061531.m" is the main flow of this homework. It will use the class **Windows** in "Windows.m" to create an object, which can manage each window and makes our code more elegant, and plot figures with our given parameters.

3 Problems

In Equation 1, the term $\Delta t(z)$ is the echo-time shift before and after ultrasound heating. Because $\Delta t(z)$ is a function of z, which means $\Delta t(z)$ will vary at different depth. As a result, we can divide the pre-signal and post-signal into several frames with certain window size and apply cross-correlation for each pre/post window pair. By this way, we can estimate the time shift for each window. For more accurate result, we can upsample the origin signal to get more sample points and higher sample rate. In this homework, I upsample the signal to 10x origin sample rate ($10 \times f_s$) and use a moving average filter to denoise the echo-time shift. Because sliding-window is somehow like sampling, so we can treat it as down-sample process, which will reduce sampling rate. From this assumption, we can find a new sampling rate by

$$f_s' = \frac{\text{number of windows}}{length(signal)} \cdot f_s$$

After we have new f_s, we can treat echo-time shift as a signal and use FFT to analyze it. Figure 1 shows the FFT result. In this figure, the highest peak is located at $\pm 4.398 \cdot 10^4$ Hz, which is the part to tell us where is the focused point of ultrasound. As a result, now our task is to design a moving average filter which cut-off frequency is $\pm 4.398 \cdot 10^4$ Hz. By the cut-off frequency formula

$$\frac{f_{cut}}{f_s} = \frac{0.442947}{\sqrt{N^2 - 1}}$$

we can find the filter size (N) to design appropriate moving average filter. Figure 2 shows the result FFT result after applying moving average filter.

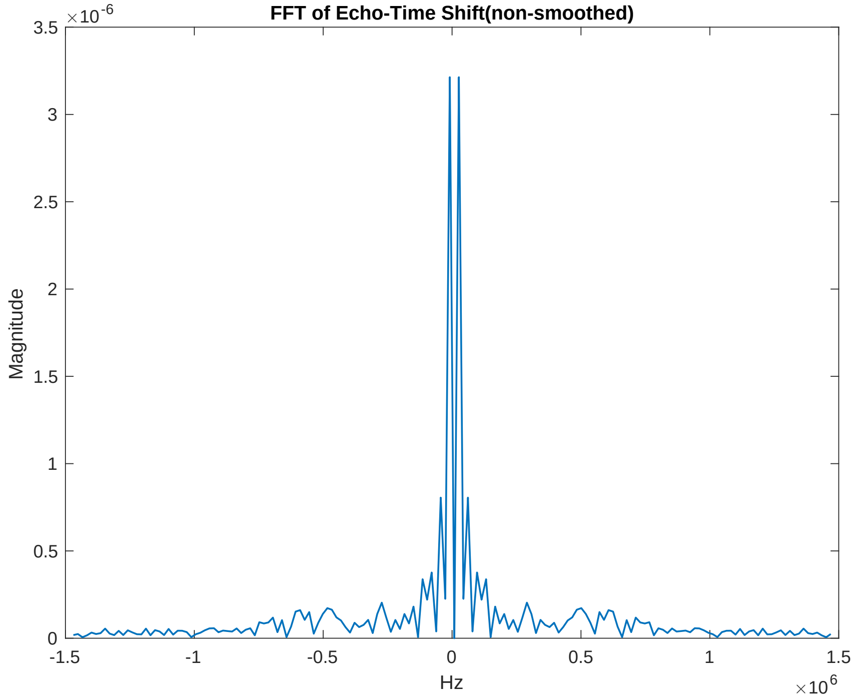

Figure 1: FFT of Echo-Time Shift (non-smoothed)

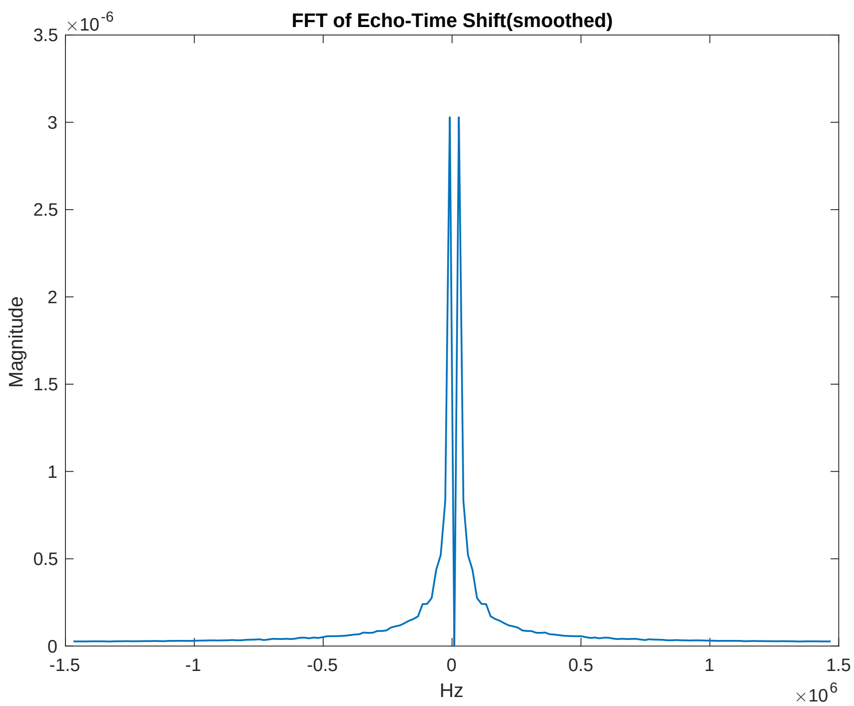

Figure 2: FFT of Echo-Time Shift (smoothed)

From Figure [1, 2], we can find that the highest peak is preserved and the others are well suppressed. Now we can move on to the next tasks, echo-time shift and thermal strain; in the following section, I will show and explain several experiments.

3.1 Echo-Time Shift

In this part, I run experiments with different paramters. Windows size is 2, 6, 10 wavelength, and each combines with different overlap ratio (0, 0.5, 0.75). Figure [3, 4, 5] show the Echo-Time Shift as a function of z for M = 2, 6 and 10, respectively.
When M=2 (Figure 3), echo-time shift is a little bit unstatble, which means the noise is obvious. This is because small window size will focus on **local information**, which make it sensitive to noise. So even if we have smoothen it, we still see noise on the wave. As for the overlap ratio, theoretically small ratio will drop out the variation of signal. So when N=0.75, we can see a more detailed change of slope.
In Figure 4 and 5, the wave become quite stable because window size is getting larger. And when N=0.75, the result looks more realistic and its slope change more continueous.

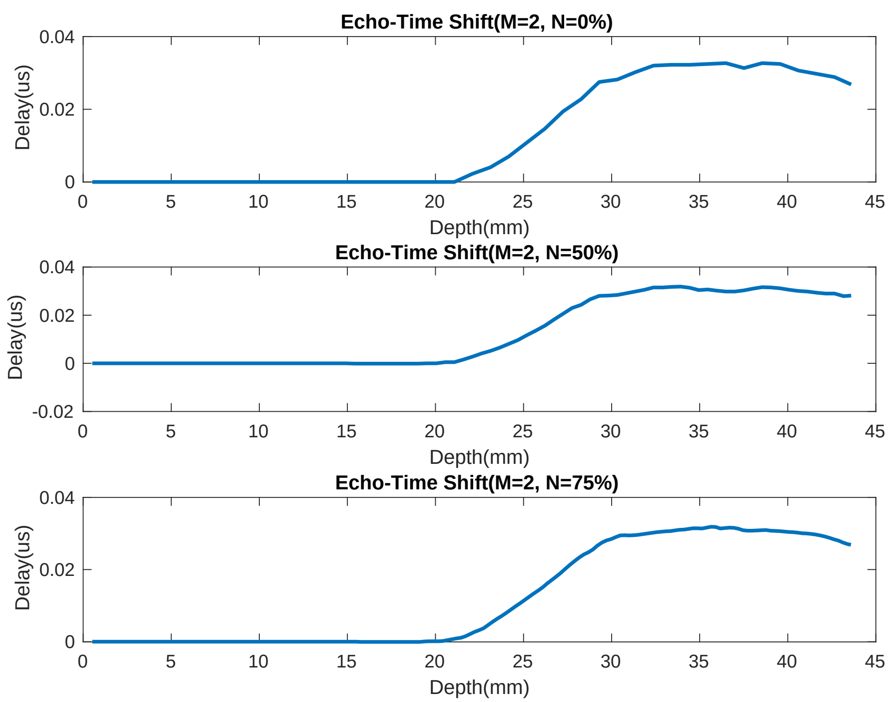

Figure 3: Echo-Time Shift (M=2)

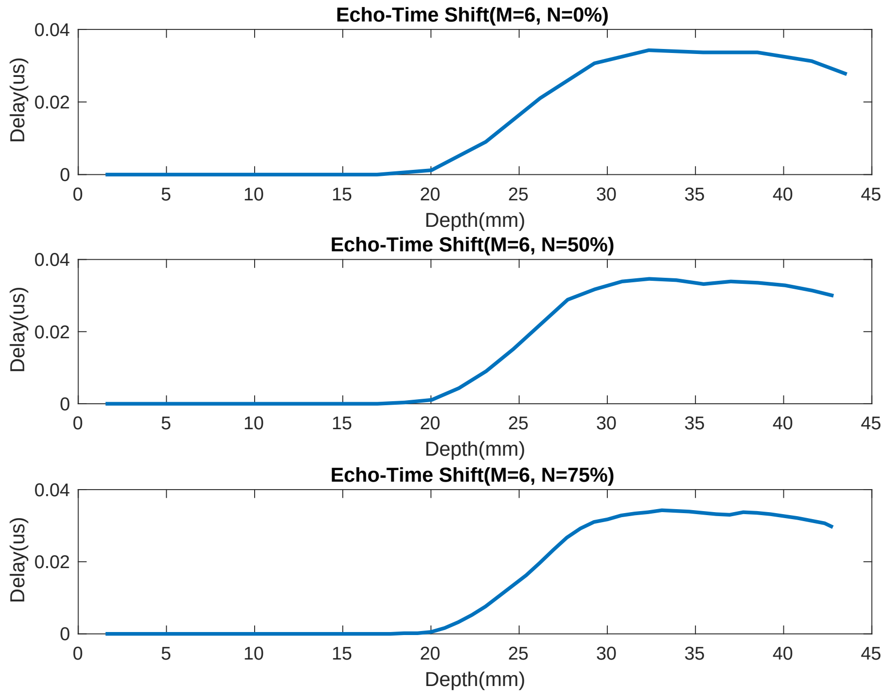

Figure 4: Echo-Time Shift (M=6)

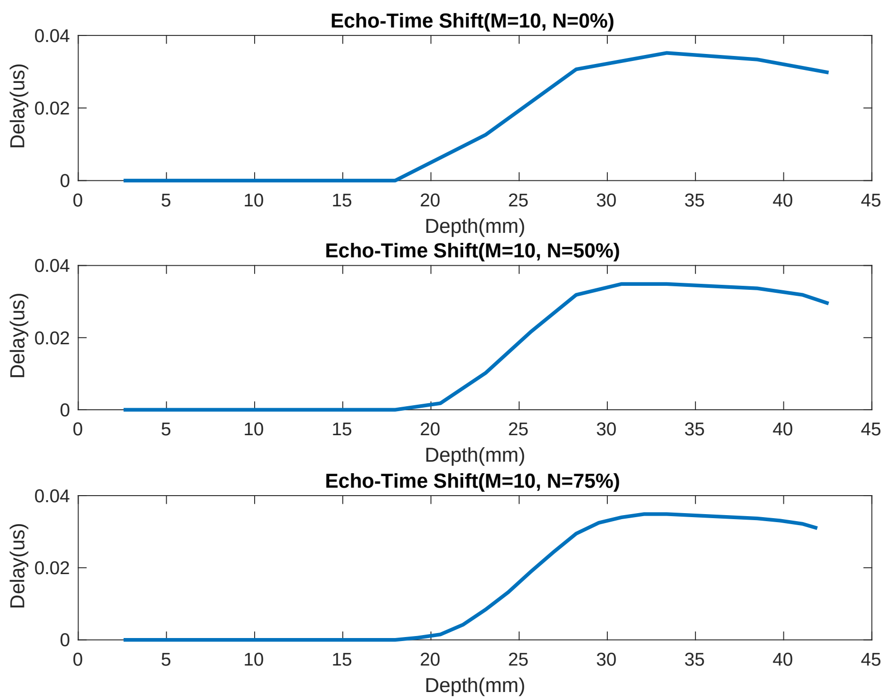

Figure 5: Echo-Time Shift (M=10)

3.2 Thermal Strain

In this section, we need to estimate thermal strain $\frac{\partial \Delta t(z)}{\partial z}$. First, we need to convert the echo-time shift into distance and then use difference approximation to calculate partial differential term as shown in Equation 2:

$$\frac{\partial \Delta t(z)}{\partial z} \approx \frac{diff(\Delta t(z) \cdot \frac{C_0}{2})}{diff(Depth)} \tag{2}$$

Figure [6, 7, 8] show the Thermal Strain (%) as a function of z for M = 2, 6 and 10, respectively.
Because $\frac{\partial \Delta t(z)}{\partial z}$ can be treated as the slope of siganl, if the slope is unstable we will get a bad result; in another words, derivative is very sensitive to noise. When M=2, the waveform (Figure 3) is unstable, so the thermal strain in Figure 6 looks terrible. However, we can stiil find that, the maximun is located at about 25 mm, which should be the focus point of ultrasound.
In Figure 7, the result looks much better because echo-time shift when M=6 (Figure 4) is stabler than M=2. When N is larger, the width of lobe which is location at about 25 mm position become more narrow, which means the quality is better. However, we still see obvious noise at 36 mm.
In Figure 8, the result is better than that in Figure [6, 7]. Because the window size is large enough, it is robust to noise. When N=0.75, we see a clear main lobe centered at about 25 mm. As a result, I think (M=10, N=0.75) is the best parameters in this homework.

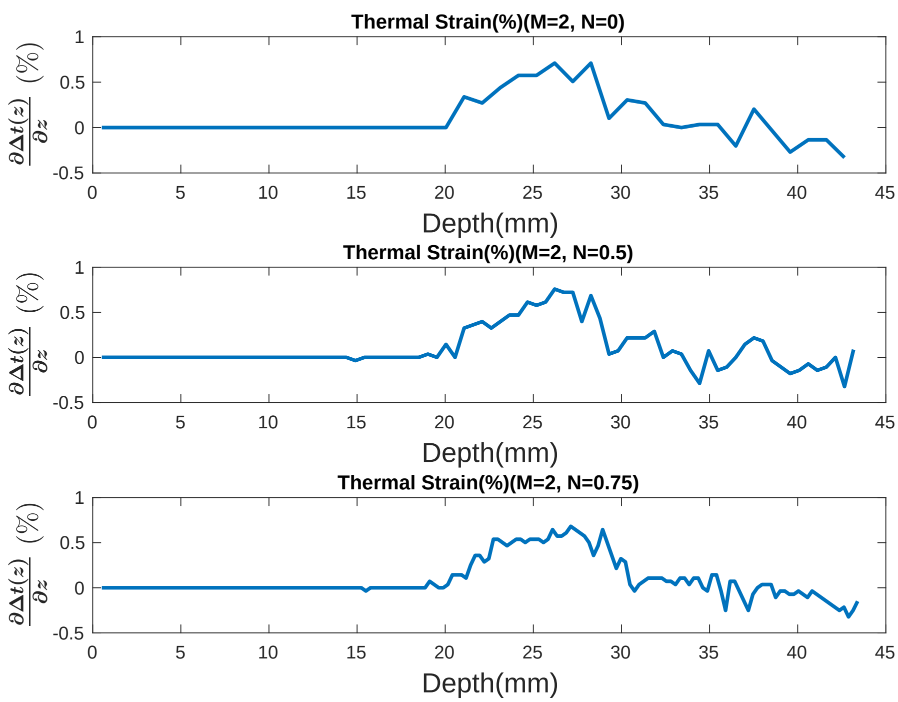

Figure 6: Thermal Strain (M=2)

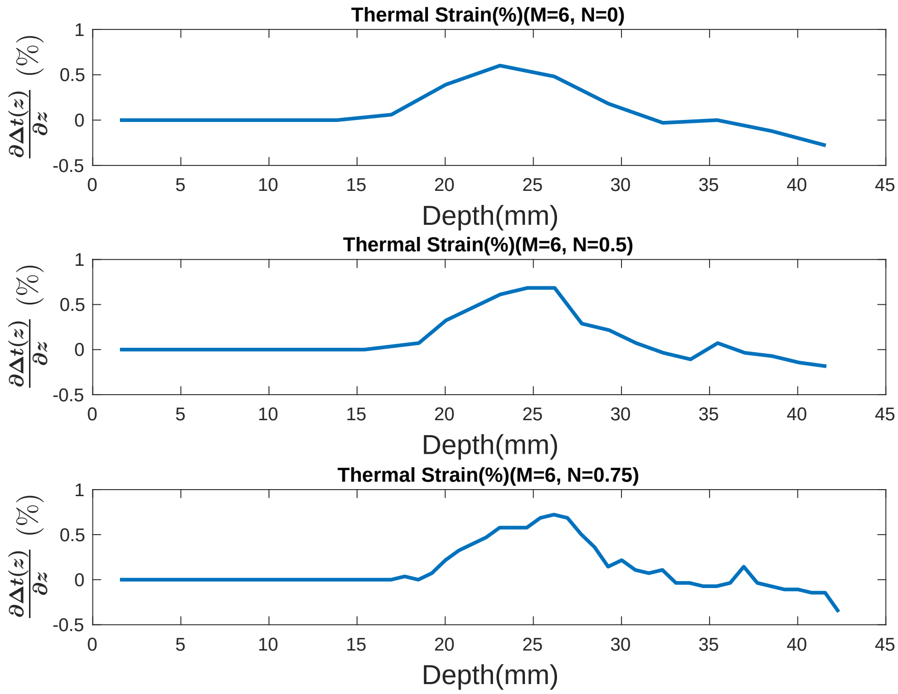

Figure 7: Thermal Strain (M=6)

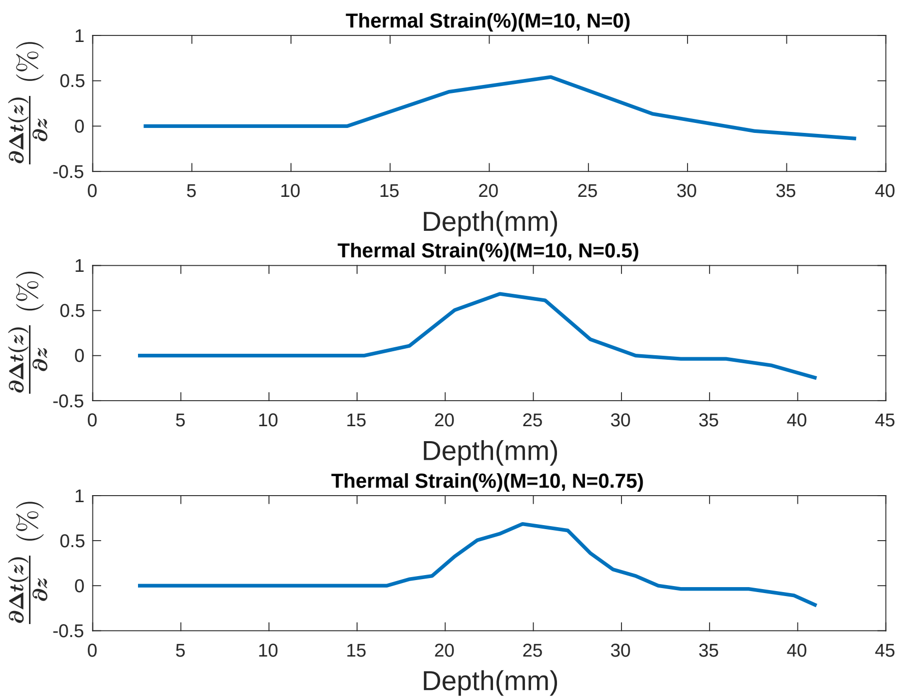

Figure 8: Thermal Strain (M=10)