Principles of Biomedical Ultrasound and Photoacoustics
hw02: Speckle Statistics

Due on Thursday, Nov 16, 2017

106061531 Fu-En Wang

1 Introduction

In this homework, we will use Matlab tool **Field2** to simulate speckle scattering.

2 Part I

In this part, we need to create a complex array with 10000 dimension, which magnitude is uniform distribution [0, 1] and phase [0, 2π]. We name this array as **origin array**.

2.a Histogram of the Amplitude and Intensity

Figure 1 shows the historgram of amplitude and intensity of origin array.

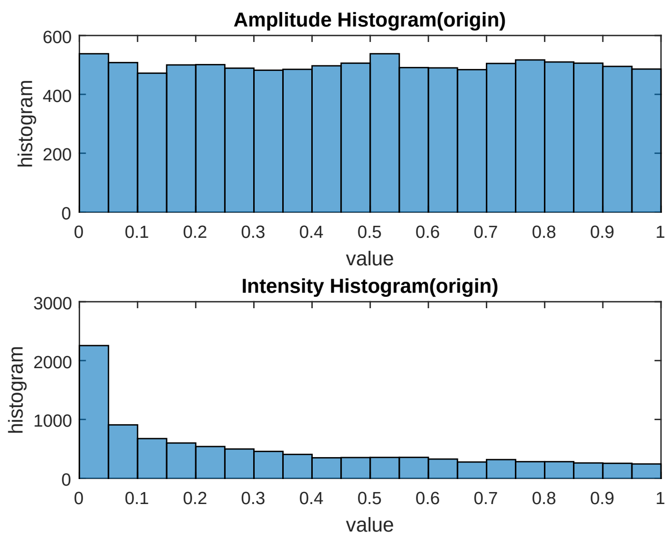

Figure 1: Histogram of amplitude (top) and intensity (bottom) of origin array

In Figure 1, we can see the distribution of amplitude is exactly uniform distribution. Because intensity is $amplitude^2$ and value between [0, 1] will decay exponentially, so the distribution will move left.

2.b Histogram and Ratio of new array

Now we create an new array with size **N (= 10000, 5000, 2000, 1000, 500)**, which value is the sum of **M (= 1, 2, 5, 10, 20)** consecutive data:

$$val(i) = \sum_{k=(i-1)*M}^{i*M} origin(k)$$

And then plot their histogram and calculate ratio of mean and standard deviation as a function of M. Figure [2, 3, 4, 5, 6] show the histogram result and Figure 7 show the ratio as a function of M.

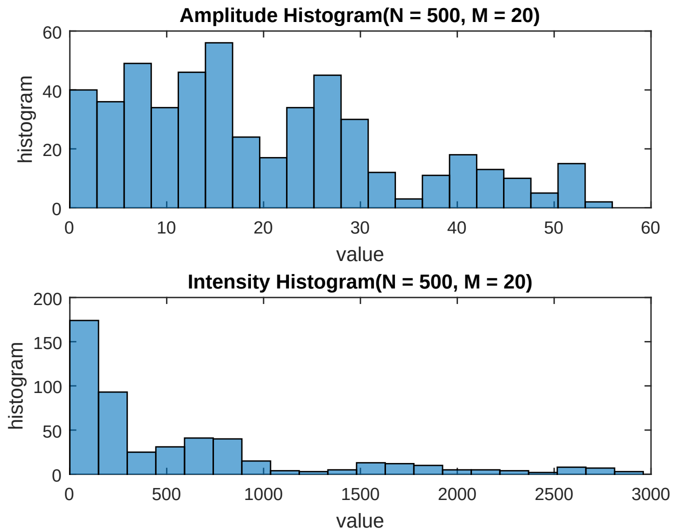

Figure 2: Histogram of amplitude (top) and intensity (bottom) of new array (N = 500, M = 20)

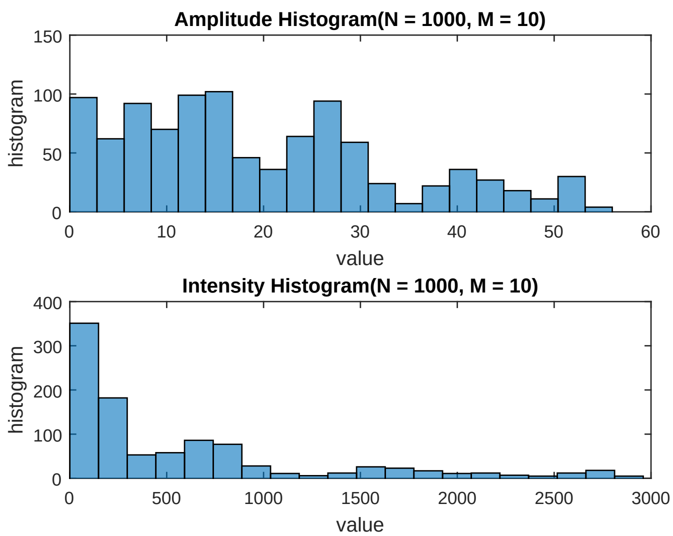

Figure 3: Histogram of amplitude (top) and intensity (bottom) of new array (N = 1000, M = 10)

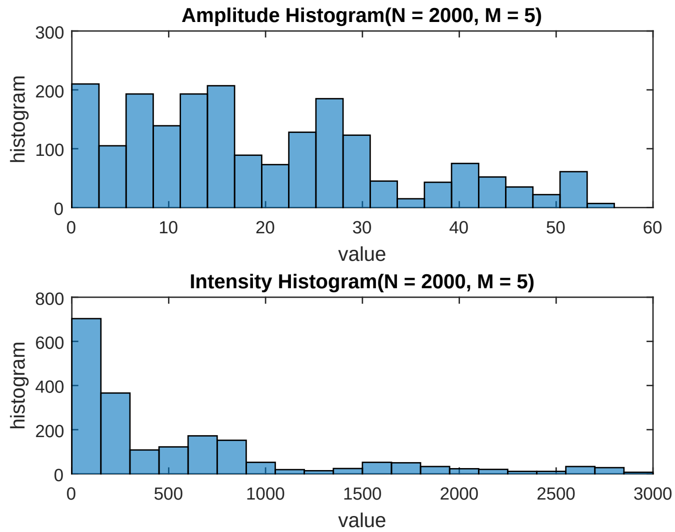

Figure 4: Histogram of amplitude (top) and intensity (bottom) of new array (N = 2000, M = 5)

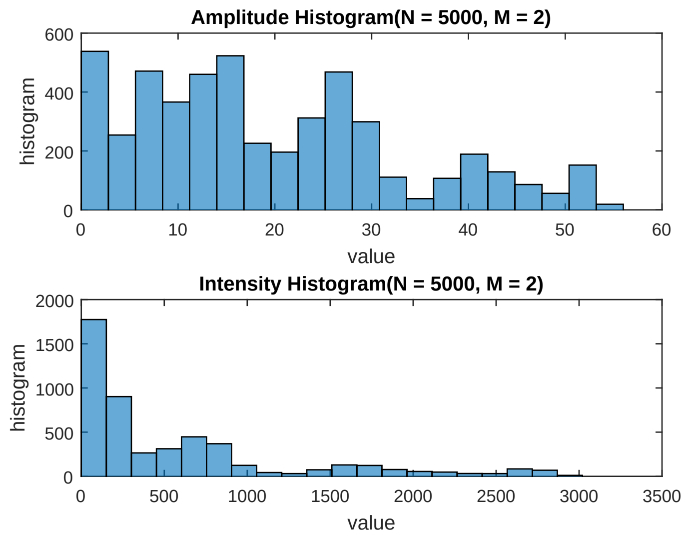

Figure 5: Histogram of amplitude (top) and intensity (bottom) of new array (N = 5000, M = 2)

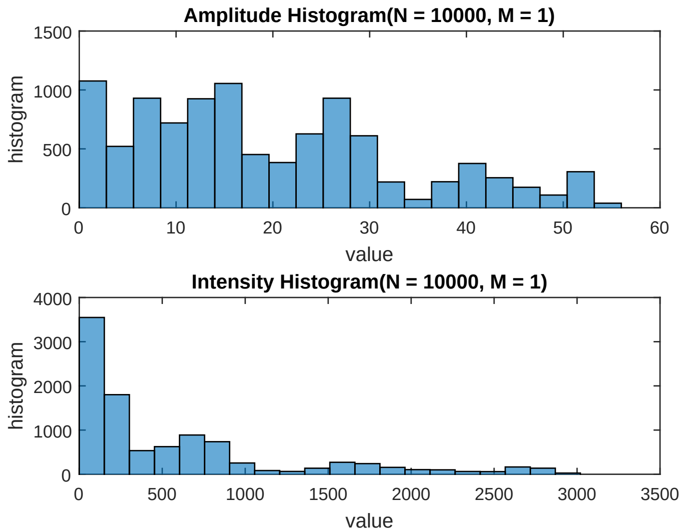

Figure 6: Histogram of amplitude (top) and intensity (bottom) of new array (N = 10000, M = 1)

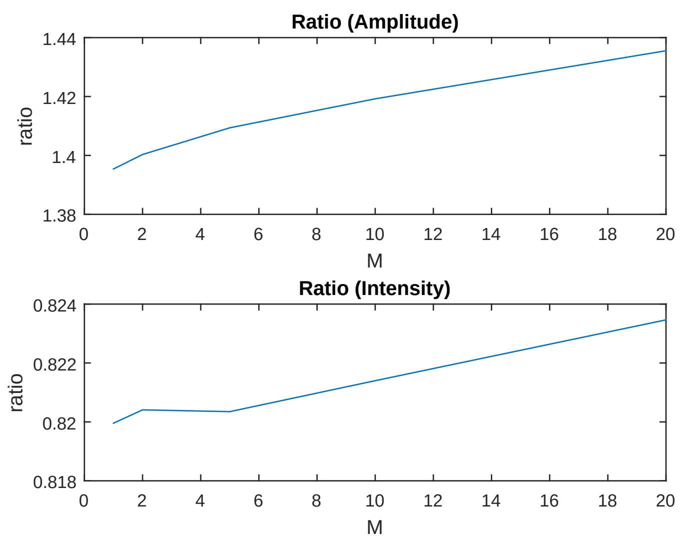

Figure 7: Ratio of mean and standard deviation

2.c Repeat (a) and (b) with phase distribution [0, π]

Now we change the phase distribution of origin array from [0, 2π] to [0, π]. Figure 8 shows the histogram of amplitude and intensity of origin array with phase distribution [0, π]. Figure [9, 10, 11, 12, 13] show histogram for different N and M. Figure 14 show the ratio.

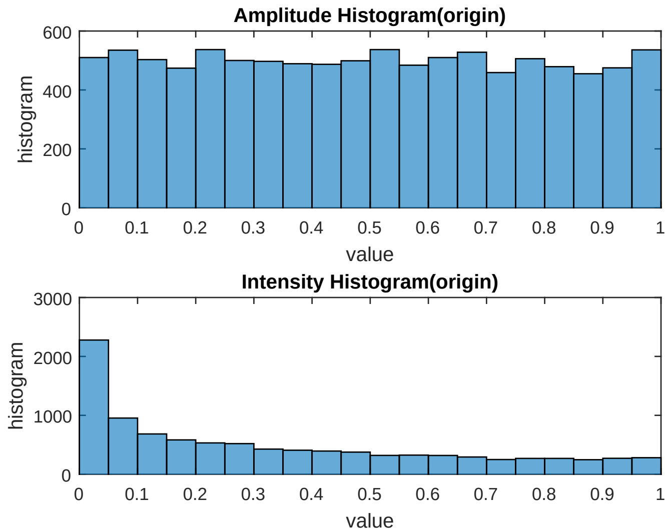

Figure 8: Histogram of amplitude (top) and intensity (bottom) of origin array (phase = $[0, \pi]$)

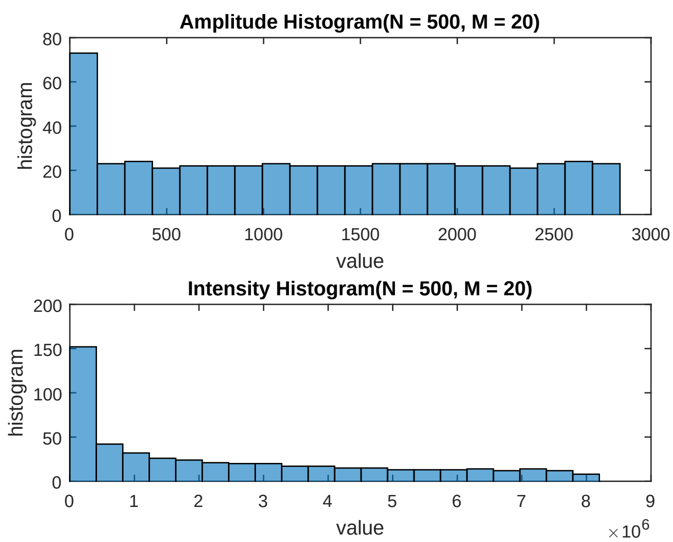

Figure 9: Histogram of amplitude (top) and intensity (bottom) of array (N = 500, M = 20, phase$[0, \pi]$)

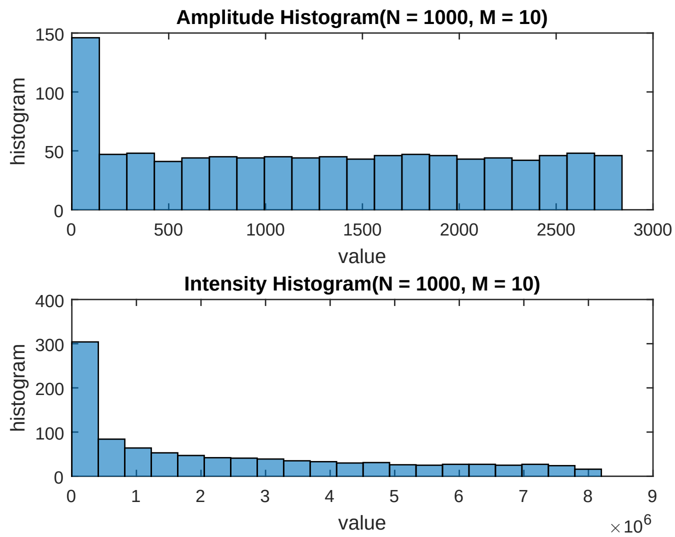

Figure 10: Histogram of amplitude (top) and intensity (bottom) of array (N = 1000, M = 10, phase[0, π])

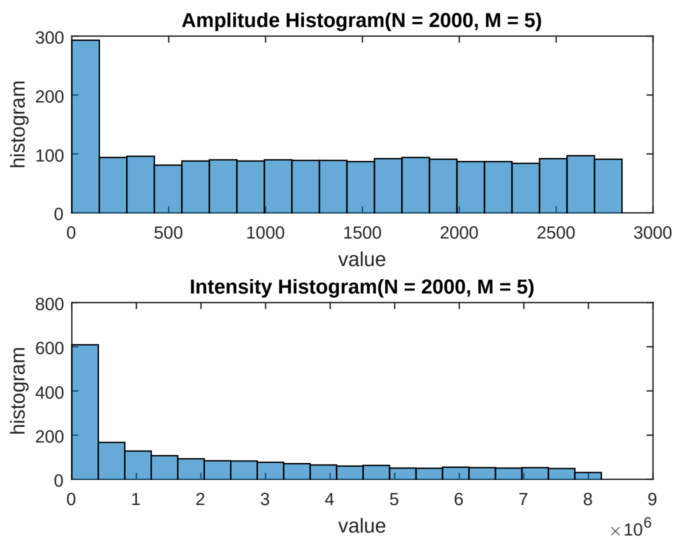

Figure 11: Histogram of amplitude (top) and intensity (bottom) of array (N = 2000, M = 5, phase[0, π])

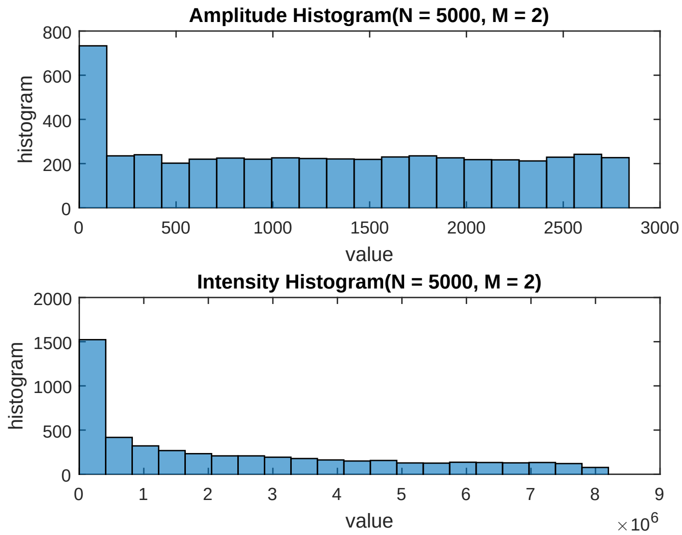

Figure 12: Histogram of amplitude (top) and intensity (bottom) of array (N = 5000, M = 2, phase[0, π])

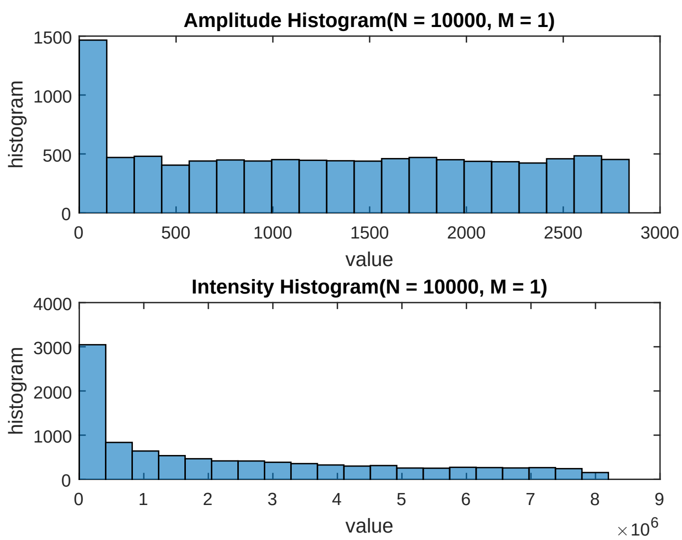

Figure 13: Histogram of amplitude (top) and intensity (bottom) of array (N = 10000, M = 1, phase[0, π])

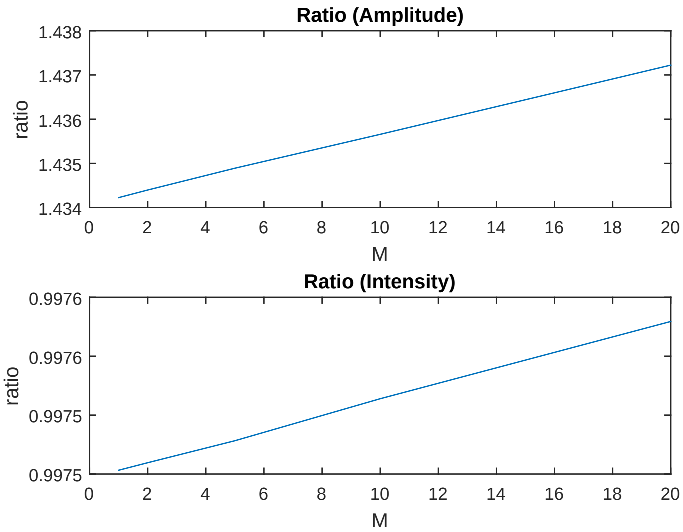

Figure 14: Ratio of mean and standard deviation (phase[0, π])

2.d Smooth amplitude and intensity

Now we will use a moving average filter [0.5, 0.5] to smooth the amplitude and intensity, which is a post-processing. Figure 15 show the ratio as a function of M for different N.

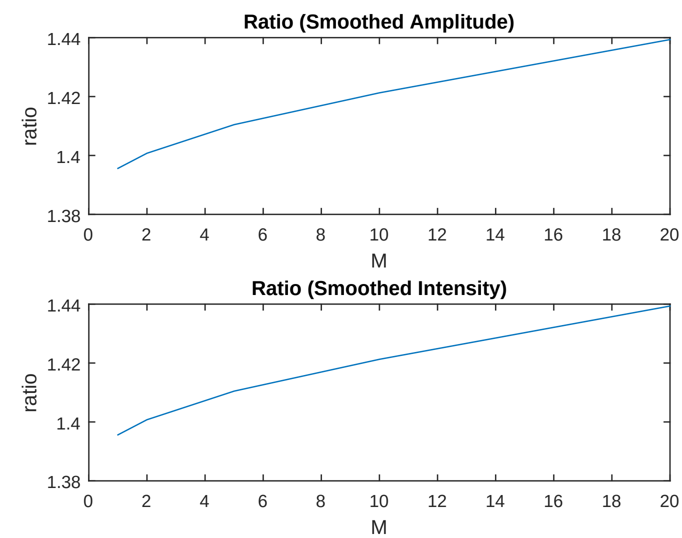

Figure 15: Ratio as function of M with smoothed amplitude and intensity

2.e Smooth new array

Now we will try to use the same filter to smooth new array obtained from (b), which is a pre-processing. Figure 16 show the ratio as a function of M for different N.

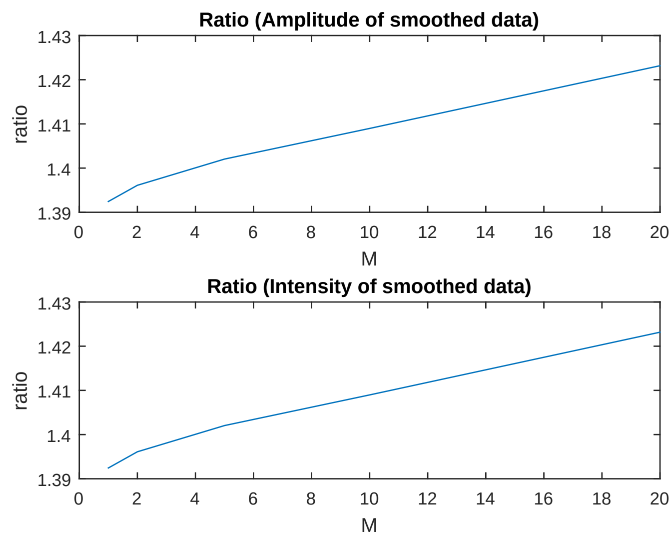

Figure 16: Ratio as function of M with smoothed data array

3 Part II

In this part, we will use Matlab tool **Field2** to simulate phantom scattering.

3.a Ratio of two area

Now we should calculate the ratio of mean to the standard deviation for amplitude and intensity. To divide the image into two part, I manually crop 2 area from the image which one is lighter and the other is darker. The ratio is summarized as Table 1:

	Experimental Ratio	Theoretical Ratio
Amplitude (higher scattering inclusion)	1.6517	1.32
Intensity (higher scattering inclusion)	0.8688	0.71
Amplitude (speckle background)	1.7776	1.32
Intensity (speckle background)	0.8742	0.71

Table 1: Ratio for lighter/darker area

The theoretical value is the ratio of mean and standard deviation of a absolute gaussian distribution. Because higher scattering inclusion is just multiplying a scalar to origin gaussian distribution according to **cyst_phantom.m**, the ratio shouldn't change, which is a property of gaussian distribution. From Table 1, experimental value is not going too far away from theoretical value I think.

3.b Contrast of two area

For the two area, we will calculate their contrast by following formula:

$$contrast = |I_1 - I_2|$$

where I_1 and I_2 are mean value in dB. Table 2 and Table 3 show the result of contrast and standard deviation, respectively.

	Experimental Contrast	Theoretical Contrast
Amplitude	16.9036	10
Intensity	33.8073	20

Table 2: Contrast of amplitude/intensity

	Inclusion	Background
std	6.3	5.6

Table 3: Standard Deviation

Because the contrast of intensity is $20 * \log(amplitude^2) = 2 * 20log(amplitude)$, so the value should be double than amplitude. However, this also means error in amplitude will also be double for intensity. The error in amplitude is about 7 dB which is a little large I think, while error of intensity increase to 14 dB.

3.c Reduce the contrast

Reduce contrast value until we cannot tell which part is lighter or darker in image. Figure [17, 18, 19, 20] show result of contrast [20, 10, 7, 5], respectively.

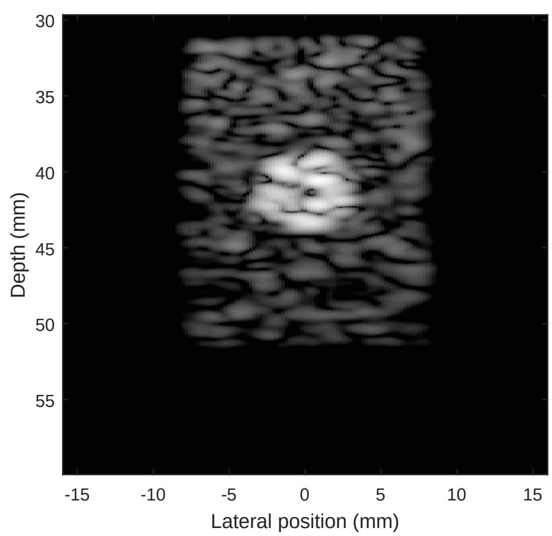

Figure 17: Contrast = 20

When contrast = 20 (Figure 17), the border of lighter and darker area is clear to tell.

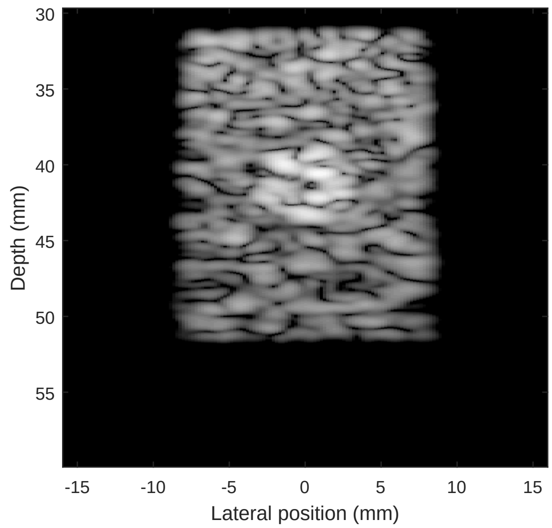

Figure 18: Contrast = 10

When contrast = 10 (Figure 18), the intensity of two area is close but we can see the border clearly.

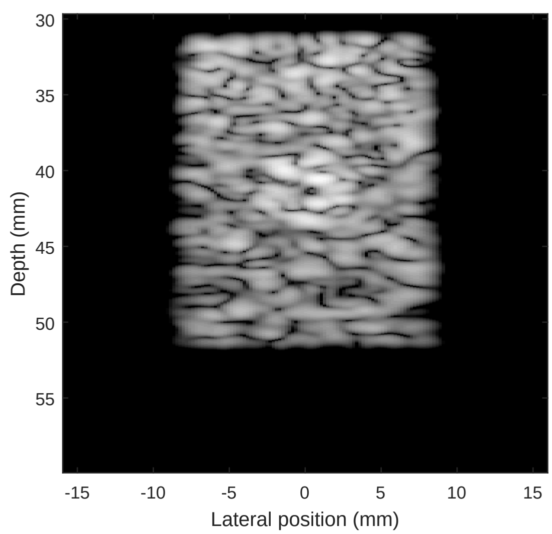

Figure 19: Contrast = 7

When contrast = 7 (Figure 19), the intensity of two area is very close and border is very unclear.

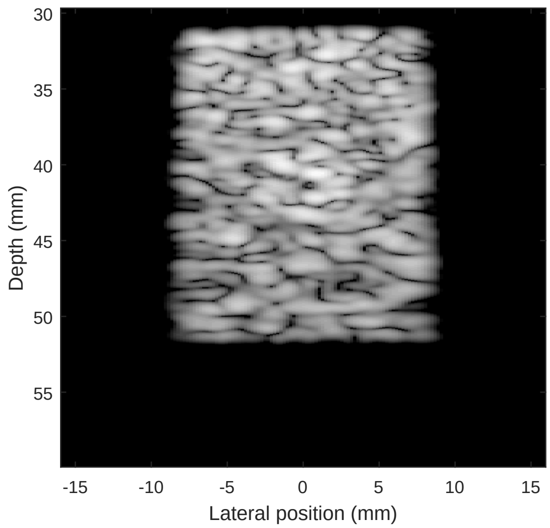

Figure 20: Contrast = 5

When contrast = 5 (Figure 20), the border totally disappear.
As a result, I think the **minimum detectable contrast is 7 dB**.

3.d Increase Diameters

With the minimum contrast in (c), now we try to increase the diameters of higher scattering inclusion and see quality change of image. Figure [21, 22, 23] show the result of diameters [5.0, 5.4, 6.0]mm, respectively.

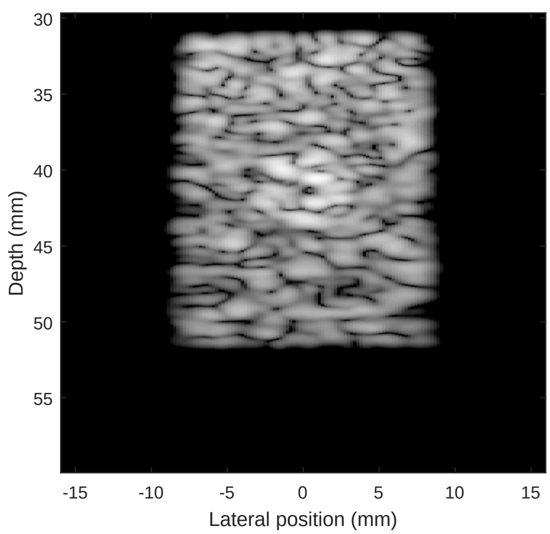

Figure 21: Diameter = 5.0mm

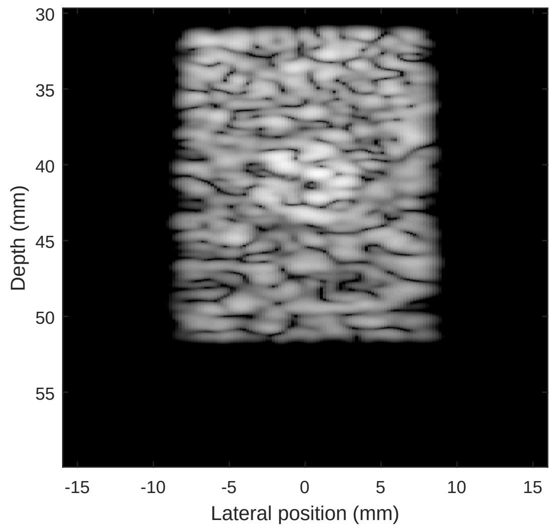

Figure 22: Diameter = 5.4mm

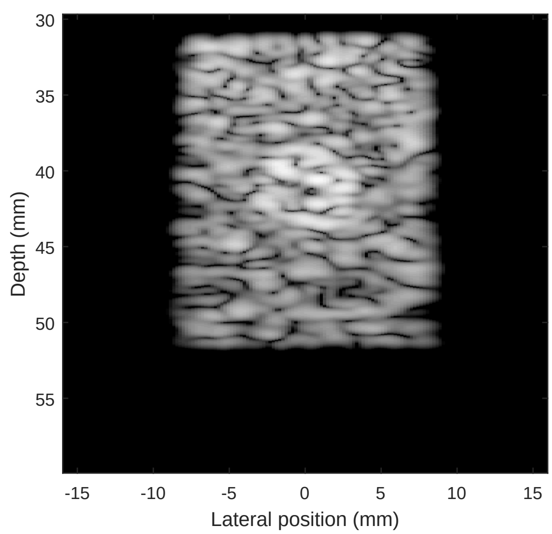

Figure 23: Diameter = 6.0mm

For a larger as in Figure 23, the border is a little more clear than the origin one in Figure 21. I think the reason is that when cysts' diameter is larger, the cross-sectional area of cyst will get larger and this will increase the probability of reflecting ultrasound back to sensor. So the border will be more clear.

3.e Periodically Located Scatterers

The origin cysts is distributed uniformly. Now we make it uniformly located at a grid map and grid size is [1, 2, 10] wavelength. Figure [24, 25, 26] show the result of wavelength [1, 2, 10], respectively.

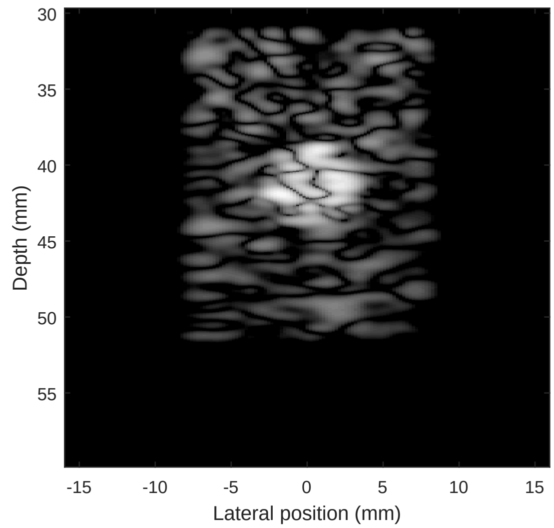

Figure 24: 1 Wavelength

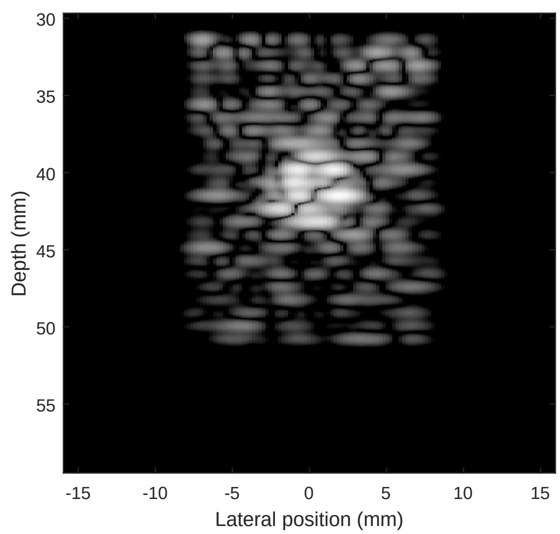

Figure 25: 2 Wavelength

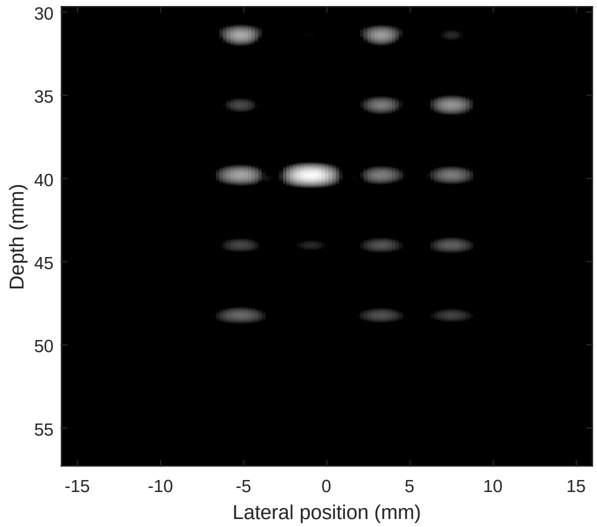

Figure 26: 10 Wavelength

From Figure 24 to 26, the quality become poorer and poorer. The reason is that there are less numbers of cysts in phantom, so the total energy sensor get from reflection is smaller than original one. When distance is large enough as in Figure 26, we can even see the grid distribution of cysts.

3.f RF A-line spectrum comparison

Now let's see the spectrum change from original tissue-mimicking phantom to phantom in (e). Figure 27 and 28 show spectrum of tissue-mimicking phantom and phantom of 2 wavelength in (e) for one sampling line, respectively.

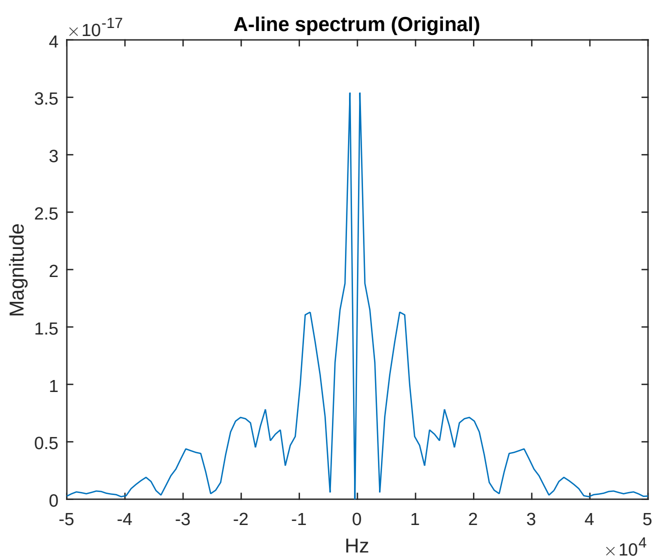

Figure 27: Tissue-mimicking phantom

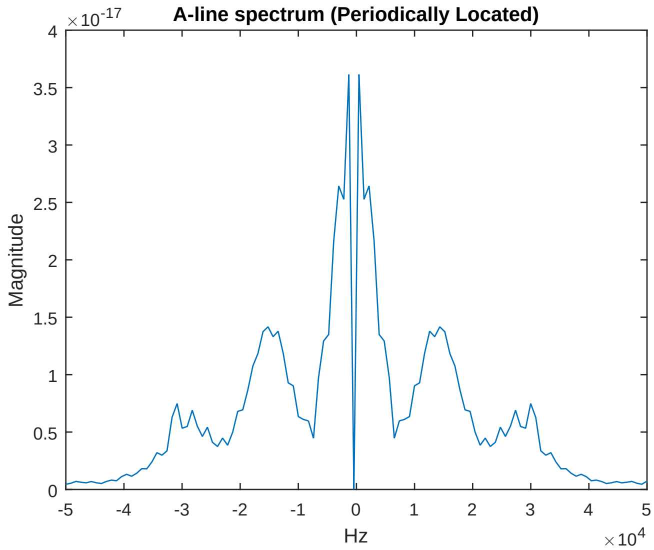

Figure 28: Phantom with 2 wavelength in (e)

The most obvious difference between Figure 27 and 28 is that in Figure 28 there is a small peak attached at the side of highest peak, which is located at about 2608 Hz. Because the grid distance is 2 wavelength which is about $8.46 * 10^{-4}$ m, in another words, 1182 Hz. As a result, the ratio of frequency corresponding to the small peak and 1182 Hz should be an integer. The experimental ratio is $\frac{2608}{1182} \approx 2.2$, which is close to 2.