The MEA Traversal: A Center-First Cross-Cut Ordering for Finite Index Sets

Abstract

The MEA traversal yields a structurally balanced cross-cut of a finite index set with applications in normalization, sequence comparison, and center-weighted evaluation. Its behavior separates cleanly by parity of n: odd n admits a closed-form permutation with four disjoint arms, while even n admits a distinct closed-form permutation characterized by a strong reflection invariant (every adjacent pair sums to n+1). This paper defines both branches and explains the necessity of the parity split.

1. Introduction

Given an index set {1,…,n}, the MEA traversal emits indices in a center-first, symmetry-respecting order. For odd n it has the informal pattern

c, 1, n, c–1, c+1, 2, n–1, 3, n–2, …

where c = ceil(n/2). For even n the natural behavior instead becomes

m, m+1, 1, n, 2, n–1, 3, n–2, …

where n = 2m. In both cases MEA produces a single permutation of {1,…,n}, but the internal invariant structure differs by parity, forcing two distinct formulas.

2. Preliminaries

Let n ≥ 1 and denote the index set [n] = {1,…,n}. Define

c = ceil(n/2)

When n is odd, c is the unique central index. When n is even, there are two central indices; for convenience we write n = 2m and use

m, m+1

as the lower and upper centers respectively.

3. Odd n Closed Form (n = 2m+1)

When n is odd, the four arms

- Left: 1,2,3,...
- Right: n,n–1,n–2,...
- Below c: c–1,c–2,...
- Above c: c+1,c+2,...

are pairwise disjoint. This allows a clean algebraic definition.

Define MEA(k) for k = 0,…,n–1 by

MEA(0) = c

and for k ≥ 1,

$j = \text{floor}((k-1)/4)$
$r = (k-1) \bmod 4$

then

$\text{MEA}(k) =$
$j + 1$ if $r = 0$
$n - j$ if $r = 1$
$c - (j + 1)$ if $r = 2$
$c + (j + 1)$ if $r = 3$

Examples:

$n = 7$ ($c = 4$) $\rightarrow$ 4, 1, 7, 3, 5, 2, 6
$n = 9$ ($c = 5$) $\rightarrow$ 5, 1, 9, 4, 6, 2, 8, 3, 7
$n = 11$ ($c = 6$) $\rightarrow$ 6, 1, 11, 5, 7, 2, 10, 4, 8, 3, 9

These sequences realize a perfect four-arm expansion from the center with no collisions.

4. Even n Closed Form (n = 2m)

For even n there are two central indices, m and m+1. The natural MEA behavior is to start at the center pair, then walk inwards from both edges, preserving a strict reflection invariant: every adjacent pair sums to n+1.

Formally, define MEA(k) for $k = 0,\ldots,n-1$ as follows.

First, set

$\text{MEA}(0) = m$
$\text{MEA}(1) = m + 1$

Then for $j = 1,\ldots,m-1$, set

$\text{MEA}(2j) = j$
$\text{MEA}(2j+1) = n + 1 - j$

This gives a closed-form permutation of $\{1,\ldots,n\}$. For all j with $0 \leq j \leq m-1$ we have

$\text{MEA}(2j) + \text{MEA}(2j+1) = n + 1$

so every consecutive pair (MEA(2j), MEA(2j+1)) is a reflected pair around the midpoint n+1.

Examples:

$n = 8$ ($m = 4$) $\rightarrow$ 4, 5, 1, 8, 2, 7, 3, 6
$n = 10$ ($m = 5$) $\rightarrow$ 5, 6, 1, 10, 2, 9, 3, 8, 4, 7
$n = 12$ ($m = 6$) $\rightarrow$ 6, 7, 1, 12, 2, 11, 3, 10, 4, 9, 5, 8

In each case MEA is a bijection $[n] \rightarrow [n]$, center-first and pairwise reflecting.

5. Choosing the Correct Branch

The definition of MEA is parity-sensitive:

- If n is odd, use the four-arm closed form of Section 3.
- If n is even, use the reflection-preserving closed form of Section 4.

This split is not an arbitrary design choice but a consequence of how symmetry manifests differently on odd and even discrete intervals.

6. Conclusion

The MEA traversal provides a structurally principled way to order a finite index set from the center outward. Its odd-n form realizes a four-arm expansion with no collisions, while its even-n form realizes a reflection-invariant pairing where every adjacent pair sums to n+1. The parity bifurcation is not a flaw but the mathematically honest reflection of discrete symmetry, and the MEA traversal embraces it fully.