

#### 3.2. Condensation of Vapor Inside High-Finned Trufin Tubes

#### 3.2.1. Vapor-Liquid Two-Phase Flow

1. Phase Relationships of Two-Phase Condensing Flows. Condensation of a vapor is frequently carried out inside the tubes of an air-cooled heat exchanger employing high-finned Trufin tubes. Usually the tubes are horizontal or nearly so, but occasionally inclined or vertical orientations are used. The flow of a vapor-liquid two phase mixture is a good deal more complicated than single phase flow, and the correlations for heat transfer and pressure drop correspondingly less accurate. Since in general the two-phase heat transfer coefficients are quite high, this inaccuracy is not a serious matter for heat transfer in air-cooled condensers. However, the pressure drops are also quite high and care is required in interpreting the predicted values into design considerations.

The first way to characterize a condensing two-phase flow is by its composition. Five cases are discernible:

- a. The liquid and the gas are different pure components. The usual example is an air-water mixture, which is not a common industrial problem, but is very important because a great deal of what is known about two-phase flow has been determined on this system. While in general this information can be carried over to condensation and some boiling work, there are important differences that must be recognized and allowed for. Thermodynamically, the pressure and temperature can be independently varied over wide ranges in this system.
- b. The liquid and gas (vapor) are the same pure component. This is a common case in condenser design, occurring for example in condensers on columns separating and/or purifying a product. The pressure-temperature relationship in this case is the vapor pressure curve for the component.
- c. The liquid and gas (vapor) are multi-component mixtures, each containing some of each of the components. The thermodynamic relationships are more complex, the temperature, for example, being variable over a range of values at a given pressure, but with a changing ratio of total liquid to total vapor and with changing composition of each phase. Prediction of the amount and composition of each phase is relatively well understood and easily done in a few cases, such as mixtures of light hydrocarbons; other cases require laboratory thermodynamic data.
- d. Like case (b) or (c), but with a non-condensable gas present. The most common example cited here is air in steam, but there are also applications as condensation of a solvent from an inert stripping gas.
- e. Intermediate between (c) and (d), typified by condensation of light gasoline fractions from a wet natural gas. Depending on the composition of the gas and the pressure and temperature of the condensation, some of the methane and ethane present will dissolve in the liquid, and a small amount of the heavier components will be present in the vapor.
- 2. Characterization of Two-Phase Vapor-Liquid Flows. This section will concentrate on case (b), condensation of a pure component, but many of the equations and procedures developed will be applicable to the other situations, as discussed later. There are a number of quantitative parameters of two-phase flows that characterize the flow and must be defined:



a. Weight flow rates: The weight flow rates of each phase (in typical units of lb/hr),  $w_{\ell}$  and  $w_{\nu}$ , are generally either known or computable from process specifications. In a condensing flow, the total weight flow rate remains constant, the liquid rate increasing as the vapor rate decreases. The flow mechanisms of the condensation process may change as the ratio changes from entrance to exit. It is convenient to non-dimensionalize the ratio of the two phases by defining the quality x as:

$$x = \frac{W_{\nu}}{W_{\nu} + W_{\ell}} \tag{3.1}$$

b. Volume flow rates: The volume flow rates are occasionally useful quantities and are defined as:

$$\vartheta_{\vartheta} = \frac{w_{\ell}}{\rho_{\ell}}; \ \vartheta_{\vartheta} = \frac{w_{\nu}}{\rho_{\nu}} \tag{3.2}$$

c. Mass velocities: The mass velocity of the total two phase stream, G (lb/hr ft²) is constant in a constant cross-section conduit. It is defined as:

$$G = \frac{w_{\ell} + w_{\nu}}{S} \tag{3.3}$$

where S is the cross-sectional area of the conduit.

d. Superficial mass velocities: The superficial mss velocity of a given phase is defined as:

$$G_{v} = \frac{w_{v}}{S} = G_{x} \tag{3.4}$$

$$G_{\ell} = \frac{w_{\ell}}{S} = G(1 - x) \tag{3.5}$$

The entire cross-sectional area of the conduit is used in calculating  $G_v$  and  $G_\ell$  even though each phase alone occupies only a portion of the cross section. These quantities have no fundamental significance, but they are convenient and useful parameters for correlating two-phase flow data.

e. Velocities: The true mean velocities of each phase are not simply or fundamentally related to the other characterizing parameters of the system. The liquid, being generally preferentially located near the walls of the conduit and being more viscous and dense than the vapor, in general flows more slowly, giving rise to the phenomenon of "slip." if  $V_v$  is the actual mean velocity of the vapor in (ft/hr) at a given point in the conduit and  $V_\ell$ , is the corresponding value for the liquid,  $V_v$  is usually greater than  $V_\ell$ . It is customary to define a quantity, the slip ratio, as  $V_v/V_\ell$ ; as noted above, the slip ratio is almost always greater than 1 and can reach 10 or 20 or even more. If  $V_v = V_\ell$ , the flow is said to be *homogeneous*; this is not a very realistic case, but it is simple to use in that it permits densities, velocities, and weight and volume fractions to be directly related to each



- other. In the lack of better information, homogeneous flow may be assumed taking precaution against non-conservative consequences of this assumption.
- f. Phase volume fractions: The actual volume occupied by the vapor phase divided by the total conduit volume is called the vapor phase volume fraction,  $R_v$ , or perhaps more commonly, the void fraction. The liquid volume fraction,  $R_\ell$ , is equal to  $(1-R_v)$ . For a steady flow, or one averaged over a sufficient period of time, these are also the fractions of the cross-sectional area that are occupied by the respective phases. By use of continuity, we may write

$$R_{v} = \frac{S_{v}}{S} = \frac{\frac{w_{v}}{\rho_{v}V_{v}}}{\frac{w_{\ell}}{\rho_{\ell}V_{\ell}} + \frac{w_{v}}{\rho_{v}V_{v}}} = \frac{x}{x + \frac{\rho_{v}}{\rho_{\ell}}\frac{V_{v}}{V_{\ell}}(1 - x)}$$
(3.6)

In these terms the slip ratio is given by

$$s = \frac{V_{\nu}}{V_{\ell}} = \frac{\rho_{\ell}}{\rho_{\nu}} \left( \frac{1 - R_{\nu}}{R_{\nu}} \right) \left( \frac{x}{1 - x} \right) \tag{3.7}$$

g. Effective density: The effective density for a two-phase mixture,  $\rho_{\it eff}$ , is defined as the mass of a unit Volume of the flowing mixture,

$$\rho_{eff} = R_{\nu} \rho_{\nu} + (1 - R_{\nu}) \rho_{\ell} \tag{3.8}$$

- 3. Correlations of the Vapor Volume Fraction. It is evident that data on the vapor volume fractions are needed in order to calculate some of the characteristic quantities of a two-phase flow Many experimental studies have been made to determine values of R, and a number of correlations have been proposed (Ref. (1) surveys the work done up to about 1965). A generally usable correlation which seems to give reasonable results over the range of process applications was proposed by Martinelli and coworkers, mostly based on air-water systems at about atmospheric pressure. The correlation for volume fractions proposed by Martinelli and Nelson (2) seems to work as well as any other correlation proposed over the entire range of data available. The original correlation was intended for steam-water systems, but is easily generalized in terms of the reduced pressure. The modified correlation is given here as Fig.
- 3. 1. The abscissa of Fig. 3. 1 is  $\sqrt{x_{tt}}$  where

$$x_{tt} = \left(\frac{1-x}{x}\right) \left(\frac{\rho_{\nu}}{\rho_{\ell}}\right)^{0.57} \left(\frac{\mu_{\ell}}{\mu_{\nu}}\right)^{0.11} \tag{3.9}$$

The ordinates are R, and Re; the parameter is the reduced pressure of the vapor

$$P_R = \frac{p}{p_{crit}} \tag{3.10}$$

The form of the parameter  $x_{tt}$  is deduced by Martinelli from a particular model of two phase flow but it seems to be remarkably generally applicable. The limiting curve for  $P_R = 0.005$  is taken from the



experimental results at atmospheric pressure for air-water and the curve for  $P_R = I$  is obtained from the requirement that the slip ratio go to unity as the two phases become more similar. The curves for other reduced pressures are interpolated by eye.

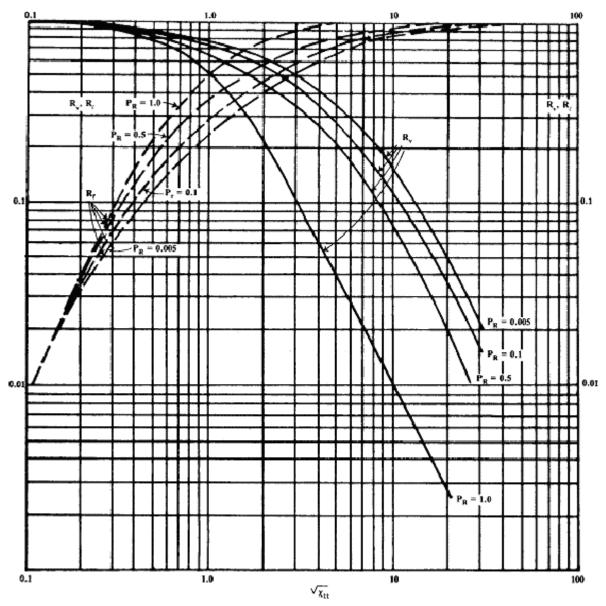


Fig. 3.1  $R_v$  and  $R_f$  as Functions of  $\sqrt{\chi_{tt}}$  (Modified from Martinelli and Nelson (11).)  $R_f \circ \cdots$ ;  $R_v \circ \cdots$ .

4. Regimes of Two-Phase Flow. To this point, we have said nothing of the actual physical appearance of a two phase flow. In some respects, such as void fraction calculations for process condensers, the physical description of the flow is relatively unimportant. In other regards, it is of some importance. The types and the detailed description of two-phase flow configurations (or regimes) depends upon the relative and absolute quantities and the physical properties of the fluids flowing, the geometric configuration of the conduit, and the kind of heat transfer process involved. The flow regimes observed by



Alves (3) in his study of air-water flows in horizontal tubes are diagrammed in rather idealized form in Fig. 3.2. The chief difference between flow regimes studied in a non-heat transfer situation and those existing during condensation is that a liquid film exists on the entire surface of the conduit during condensation; however, we may presume that this film of draining condensate does not cause any vital difference in the interaction between the vapor and the main inventory of liquid.

We may view the flow regime as a consequence of the interaction of two forces-gravity and vapor shear acting in different directions. At low vapor flow rates, gravity dominates and one obtains stratified, slug-plug, or bubble flow depending upon the relative amount of liquid present. At high vapor velocities, vapor shear dominates, giving rise to wavy, annular, or annular-mist flows. It would be desirable to have some

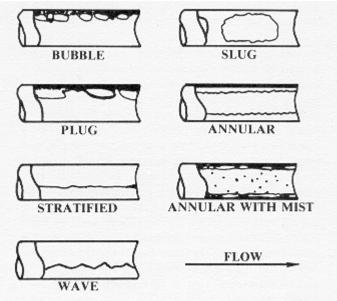


Fig. 3.2 Flow Pattern Sketches for Two-Phase Adiabatic Flow (Slightly modified from Ref.

way to predict the flow regime a priori, and many attempts have been made to do this in a general and consistent way. No attempt has succeeded, but the work of Baker (4) is considered to be generally the best available even though it is a dimensional representation and defies explanation in fundamental terms. The Baker map is shown in modified form as Fig. 3.3 and is useful in giving a general appreciation of the general kind of flow regime existing under given conditions.

During the course of condensation, the relative ratios of vapor and liquid change; these changes may be followed conveniently on the Baker map with the aid of Fig. 3.4, which is used in the following way: Calculate the values of the abscissa and ordinate on the Baker map corresponding to the half-condensed point ( $G_v = G_\ell$ ). Plot this point on Fig. 3.3. Place the 0.5 point (circled) of the curve of Fig. 3.4 on top of the point plotted on Fig. 3.3 (it is helpful if a transparency of Fig. 3.4 is used) so that the coordinates are parallel on- the two figures. Then the curve on Fig. 3.4 traces the sequence of flow regimes as a function of vapor quality from pure vapor at the left infinity point to pure liquid at the bottom in finity point.

It may be assumed that condensing flow patterns in tubes inclined slightly downwards (in the direction of flow) are similar to those in horizontal tubes. Condensation is seriously reduced in tubes inclined slightly upwards (5) and care must be taken to insure that this does not occur, often by deliberately designing the condenser to a slight (2-3°) downward inclination.

For two-phase flow inside vertical tubes, the stratified and wavy flow regimes cannot exist, and the flow regimes generally recognized in this case are bubble (low vapor rate), slug, annular, and annular with mist. There is no generally recognized flow regime map for condensing in vertical tubes at the low heat fluxes generally characteristic of air-cooled condensers. This is not a serious deficiency in predicting heat transfer coefficients and the same is also true of condensing in steeply inclined tubes.



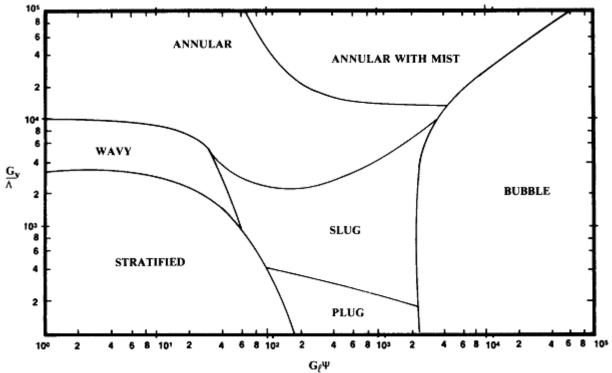


Fig. 3.3 Transformed Baker Flow Regime Map.

- 5. Pressure Drop in Two-Phase In- Tube Flow. Pressure changes in two-phase flow arise from three sources.
  - a. Friction loss, which always causes a pressure decrease in the direction of flow. The pressure loss due to friction in a two-phase flow is generally much higher than in a comparable single phase flow because of the roughness of the vapor-liquid interface. The pressure gradient due to friction depends upon local conditions, which change in a condensing flow. Therefore, the total pressure effect from friction depends upon the path of condensation.
  - b. Momentum effects, resulting from a change in the velocity, and hence kinetic energy, of the stream. The pressure change from this cause is negative (pressure loss) if the flow accelerates as in boiling and positive (pressure loss) if the flow dealers to be accelerated.

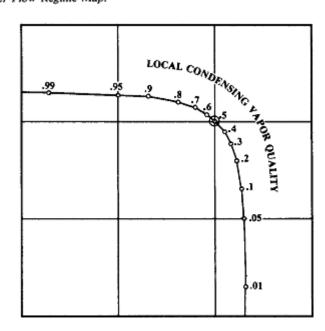


Fig. 3.4 Overlay for Transformed Baker Map.

(pressure gain) if the flow decelerates, as is the usual case in condensation. The total effect on pressure from this cause depends only upon terminal conditions, though in some cases it may be necessary to calculate the local pressure gradient contributed by velocity changes.



c. Hydrostatic effects, resulting from changes in elevation of the fluid. The pressure from this source alone always decreases in an upward direction and is zero in a horizontal tube. The local gradient depends upon the local density; therefore, it is generally necessary to calculate this contribution along the path of condensation.

The algebraic sum of the three contributions is equal to the net total pressure effect. In process applications, the friction loss usually predominates in horizontal configurations and is usually comparable to the hydrostatic effect in vertical designs. However, in very high velocity condensing flows, the momentum effect may be greater than the other contributions, resulting in a pressure *rise* from inlet to outlet (e.g., Ref. (6)).

Calculation of the pressure loss due to friction may be made using the Martinelli-Nelson correlation (2). First, we calculate the pressure gradient as if only the liquid were flowing in the conduit.

$$\left(\frac{dp}{d\ell}\right)_{f,\ell} = -\frac{2f_{\ell}G_{\ell}^{2}}{g_{c}\rho_{\ell}d_{i}} \tag{3.11}$$

where  $f_{\ell}$  is read from the Fanning friction factor chart or Eq.(3.13) at a Reynolds number calculated from:

$$\operatorname{Re}_{\ell} = \frac{d_i G_{\ell}}{\mu_{\ell}} \tag{3.12}$$

if  ${\rm Re}_\ell$  > 2100. If  ${\rm Re}_\ell$  < 2100, use the vapor-phase based correlation given further on in this section. The smooth tube correlation may be used though it is more conservative to use the correlation for the relative roughness of the actual surface at high Reynolds numbers. Over the usual range of process applications,  $f_\ell$  may be computed from

$$f_{\ell} = \frac{0.078}{\left(\frac{d_i G_{\ell}}{\mu_{\ell}}\right)^{1/4}} \tag{3.13}$$

The pressure gradient due to friction for the two-phase flow is then calculated from

$$\left(\frac{dp}{d\ell}\right)_{f,TPF} = \Phi_{\ell tt}^{2} \left(\frac{dp}{d\ell}\right)_{f,\ell} \tag{3.14}$$

where  $\Phi_{\ell yv}^{\phantom{\ell yv}2}$  is read from Fig. 3.5 at the appropriate value of

$$x_{tt} = \left(\frac{1-x}{x}\right) \left(\frac{\rho_{\nu}}{\rho_{\ell}}\right)^{0.57} \left(\frac{\mu_{\ell}}{\mu_{\nu}}\right)^{0.11} \tag{3.15}$$

If  $Re_{\ell}$  < 2100, and  $Re_{\nu}$  > 2100, the calculation proceeds similarly, but based on the vapor phase



$$\left(\frac{dp}{d\ell}\right)_{f,v} = -\frac{2f_v G_v^2}{g_c \mathcal{G}_v d_i} \tag{3.16}$$

where f<sub>v</sub> us taken from the Fanning chart at

$$Re_{v} = \frac{d_i G_{v}}{\mu_{v}} \tag{3.17}$$

or from

$$f_{v} = \frac{0.078}{\left(\frac{d_{i}G_{v}}{\mu_{v}}\right)^{1/4}} \tag{3.18}$$

 $\Phi_{\ell tt}^{2}$  is again read from Fig. 3.5 at the appropriate value of  $x_{tt}$  and the two-phase friction pressure drop is given by

$$\left(\frac{dp}{d\ell}\right)_{f,TPF} = \Phi_{\ell tt}^2 x_{tt}^{1.75} \left(\frac{dp}{d\ell}\right)_{f,v} \tag{3.19}$$

If both  $Re_{\ell}$  and  $Re_{\nu}$  are below 2100, an approximate value may be obtained by using the laminar flow friction factor equation for either phase:

$$f_{\ell} \text{ or } v = \frac{16}{\text{Re}_{\ell} \text{ or } v}$$
 (3.20)

However, in this case, the pressure drop will probably be so low as to be unimportant.

The pressure effect due to momentum changes is given by

$$\left(\frac{dp}{d\ell}\right)_{mTPF} = -\frac{G}{g_c} \frac{d\left[v_\ell(1-x) + v_\nu x\right]}{d\ell} \tag{3.21}$$

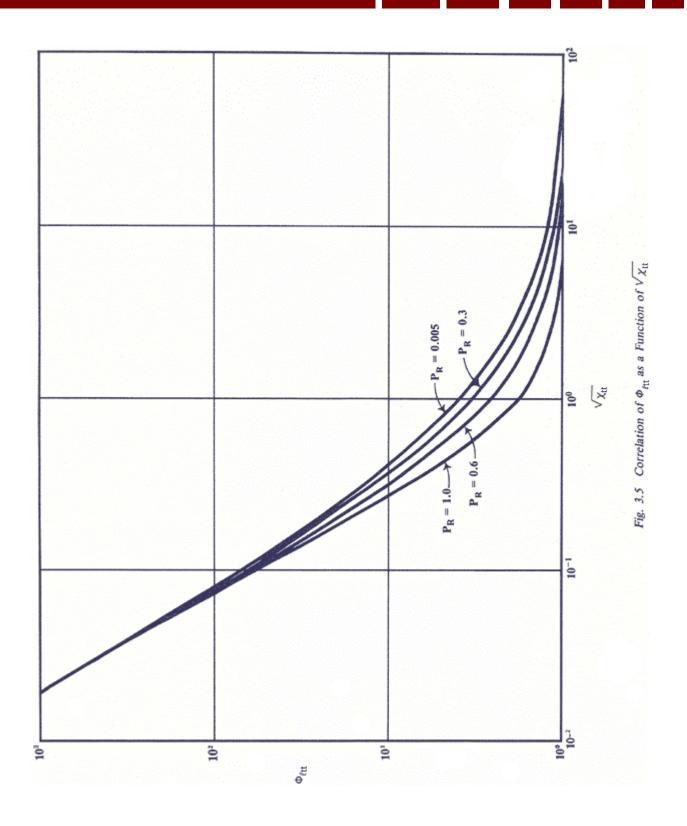
However, the total effect can be calculated from inlet and outlet conditions only:

$$\Delta p_{m,TPF} = \frac{G}{g_c} \left\{ \left[ v_\ell (1 - x) + v_\nu x \right]_{inlet} - \left[ v_\ell (1 - x) + v_\nu x \right]_{outlet} \right\}$$
(3.22)

If a dry saturated vapor enters and is completely condensed, Eq. (3.22) reduces to

$$\Delta p_{m,TPF} = \frac{G}{g_c} \left[ v_{v,inlet} - v_{\ell,outlet} \right] = \frac{G^2}{g_c} \left[ \frac{1}{\rho_v} - \frac{1}{\rho_\ell} \right]$$
(3.23)







Recall that for condensation this is a pressure increase.

The hydrostatic pressure effect is calculated from

$$\left(\frac{dp}{d\ell}\right)_{g,TPF} = \rho_{eff} \, \frac{g \, \cos\Theta}{g_c} \tag{3.24}$$

where  $\mathcal{G}_{e\!f\!f}$  is given by Eq. (3.8) and  $\theta$  is the angle between the vertical and the axis of the tube ( $\theta=0$  for a vertical tube;  $\theta=\pi/2$  for a horizontal tube, see Fig. 3.6). The hydrostatic contribution to the pressure is always negative in the upward direction.

The total local pressure gradient is the algebraic sum of the three effects:

$$\left(\frac{dp}{d\ell}\right)_{T,TPF} = \left(\frac{dp}{d\ell}\right)_{f,TPF} + \left(\frac{dp}{d\ell}\right)_{m,TPF} + \left(\frac{dp}{d\ell}\right)_{g,TPF} \tag{3.25}$$

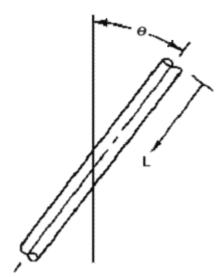


Fig. 3.6 Definition of θ

The total pressure difference from one end of the condenser to the other must be found by integration over the quality range

$$\Delta p_{TPF} = \int_{x_i}^{x_o} \left(\frac{dp}{d\ell}\right)_{T,TPF} \left[1/\left(\frac{dx}{d\ell}\right)\right] dx \tag{3.26}$$

Exact numerical evaluation of this integral is a rather tedious trial-and-error process, even on a computer, because the properties and the local quality depend upon the local pressure implicitly. For a condenser, however, one is usually concerned with complete condensation ( $x_1 = 1$ ;  $x_0 = 0$ ) and with a very nearly linear condensation rate,  $(dx/d\ell) = 1/L$ . Then, a good approximation to  $\Delta p_{TPF}$  is obtained by a "pseudo-Simpson" rule:

$$\Delta p_{TPF} = \left\{ \frac{25}{96} \left[ \left( \frac{dp}{d\ell} \right)_{TPF} + \left( \frac{dp}{d\ell} \right)_{TPF} + \frac{23}{48} \left( \frac{dp}{d\ell} \right)_{TPF} \right] + \frac{23}{48} \left( \frac{dp}{d\ell} \right)_{TPF} \right\} \Delta L$$
(3.27)

6. Flooding Effects in Upward Flow. In some applications, variously termed reflux or knockback condensers, the vapor flows upward in the tube while the liquid flows downward on the walls. In this case, the vapor shear subtracts from the gravitational force causing the liquid to flow downwards, leading to a thickening of the film; if the film is in laminar flow, the heat transfer coefficient is also reduced, an effect analyzed by Nusselt (7). In general, the effect is rather to cause early turbulence, which does thicken the film, but also increases the coefficient. If the film would be turbulent even in the absence of vapor shear, the effect of vapor shear on heat transfer is not clear and probably not of critical importance.

The main concern in knockback condensers is that flooding be avoided, i.e., that at no point in the condenser should the vapor shear be equal to the gravitational force of the liquid. The critical point is at the entrance to the lower end of the tube when both the liquid and vapor flow rates are maximum. Soliman, Schuster, and Berenson (8) have analyzed the problem in very fundamental terms, but the best design correlation for flooding is by Diehl and Koppany (9). In summary, their correlation for incipient flooding is



$$V_{v}^{'} = F_{1}F_{2} \left(\frac{\sigma}{\rho_{v}}\right)^{0.5}$$
 if  $F_{1}F_{2} \left(\frac{\sigma}{\rho_{v}}\right)^{0.5} > 10$  (3.28)

$$V_{v}' = 0.71 \left[ F_{1} F_{2} \left( \frac{\sigma}{\rho_{v}} \right)^{0.5} \right]^{1.15}$$
 if  $F_{1} F_{2} \left( \frac{\sigma}{\rho_{v}} \right)^{0.5} < 10$  (3.29)

where

V'<sub>v</sub> = superficial flooding velocity of vapor, ft/sec

$$F_1 = [(12d_i)/(\sigma/80)]^{0.4} \qquad if [(12d_i)/(\sigma/80)] < 1.0$$
  
= 1.0 \qquad if [(12d\_i)/(\sigma/80)] \leq 1.0 \qquad (3.30)

$$F_2 = (G_v / G_\ell)^{0.25}$$
 (3.31)

The equation is dimensional, so that it is essential to use the units specified ( $\rho_v$  in lb/ft³, d<sub>i</sub> in ft,  $\sigma$  in dyne/cm). Since this is for incipient flooding, conservative design requires that the maximum design vapor velocity be somewhat below this; if the conditions are within the fairly wide range represented by the data in the Diehl-Koppany correlation, design to 70 percent of the predicted flooding velocity appears safe. Otherwise, the velocity should be limited to 50 percent of that calculated.

In a few cases, condensers have been designed so that the vapor shear is great enough to carry the condensate up and out of the condenser. Total condensation is impossible in this case since there must be sufficient vapor flow at the top of the condenser to carry the liquid out. It has been observed that, as one increases the vapor flow above the Diehl-Koppany limit for flooding, the amount of liquid carryover increases slowly until virtually complete carryover occurs at about three times the flooding velocity. This value then becomes the design case at the top of the condenser for relatively foolproof operation. For intermediate values, the liquid that drains back must be carried up into the condenser again and blown through, a situation that offers opportunity for slugging, maldistribution, and generally unstable operation.

#### 3.2.2. Condensation Heat Transfer

Laminar Film Condensation of a Pure Component in a Vertical Tube. Laminar film condensation of a pure component from a saturated vapor was among the first heat transfer problems to be successfully analyzed from a fundamental point of view. The definitive work is by Nusselt (7) in two papers published two weeks apart in 1916. The analysis is readily available in a number of books, of which Jakob (10) and Kern (11) may be especially recommended. The original Nusselt analysis applies specifically to laminar flow of a condensing film on a vertical surface. However, it is possible to generalize the approach to apply to a number of other cases, including in-tube condensation in horizontal tubes. For this reason, it is worthwhile to examine the analysis in some detail here. There are a number of assumptions implicit in the basic Nusselt model. The key assumption is that the liquid film (Fig. 3.7) is in laminar flow and its hydrodynamics are controlled by the viscous terms in the Navier-Stokes equations. This allows the neglect of the inertial or kinetic energy terms and yields a simple equation relating film thickness and velocity profile to gravity and liquid viscosity. Relaxing this assumption yields complex equations that must be solved numerically; this has been done by several authors and the results indicate that the more



complex solution is negligibly different from the simple one for most process applications. Other assumptions of the Nusselt model include:

- 1. Saturated vapor.
- The liquid and the vapor have the same temperature (T<sub>sat</sub>) at the interface. (No interfacial resistance.)
- 3. Heat is transferred by conduction only through the liquid film.
- 4. The temperature profile is linear through the liquid film.
- 5. The liquid and the solid surface are at the same temperature at their interface.
- 6. The solid surface is isothermal.
- 7. The liquid properties are not a function of temperature.
- 8. The vapor exerts neither shear nor normal stresses on the liquid surface.
- 9. The liquid has zero velocity at the liquid-solid interface (no-slip condition).
- 10. The sensible heat of subcooling the liquid is negligible compared to the latent heat load.

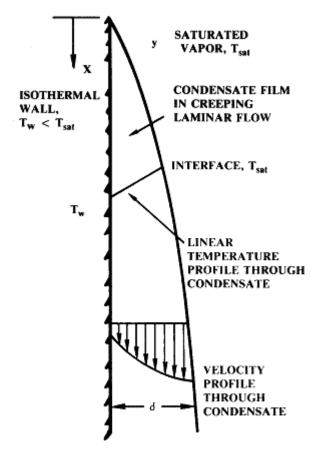


Fig. 3.7 Nusselt Condensation on a Plane Vertical Surface.

Before discussing the validity of these assumptions, let us look at the resulting equations. The local value of the film heat transfer coefficient at a distance x from the start of condensation is

$$h_{x} = \left[ \frac{k_{\ell}^{3} \rho_{\ell} (\rho_{\ell} - \rho_{v}) \lambda g}{4 \mu_{\ell} (T_{sat} - T_{w}) x} \right]^{1/4}$$
(3.32)

A far more useful quantity is the average coefficient for surface of length L, which we identify for convenience as the condensing coefficient h<sub>c</sub>:

$$h_c = \int_0^L h_x dx = 0.943 \left[ \frac{k_\ell^3 \rho_\ell (\rho_\ell - \rho_\nu) \lambda g}{\mu_\ell L (T_{sat} - T_w)} \right]^{1/4}$$
(3.33)



Note that the heat transfer coefficient predicted by (3.33) decreases as L and  $(T_{sat} - T_w)$  increase. This is due to the increased resistance to conduction offered by a thickened film.

The derivation of Eq. (3.33) was carried out in terms of a vertical plane surface. Since the condensate film is so thin compared to typical tube diameters, the result is applicable to condensation on the inside or outside of vertical tubes if the other assumptions of the derivation are satisfied. Strictly speaking, the dependence of  $h_c$  on L and  $(T_{sat} - T_w)$  violates two of the assumptions underlying the validity of the logarithmic mean temperature difference usually employed in heat exchanger design. The effect can be shown to be not serious, and it is actually very slightly conservative to use the conventional F-LMTD formulation.

The coefficient as given by Eq. (3.33) is useful if one knows the condenser tube length, but is awkward if one is trying to design a condenser for a given duty. The equation can be reworked to a more convenient form if one first defines a tube loading per linear foot of tube drainage perimeter  $\Gamma$ . That is, if w lb/hr are to be condensed on each tube,

$$\Gamma = \frac{w}{p_t} \tag{3.34}$$

where  $P_t = \pi d$  for a vertical tube. The total heat load per tube is

$$Q = \lambda w = h_c \pi dL (T_{sat} - T_w)$$
(3.35)

With some rearrangement, we obtain the desired equation:

$$h_c = 0.924 \left[ \frac{k_\ell^3 \rho_\ell (\rho_\ell - \rho_\nu) g}{\mu_\ell \Gamma} \right]^{1/3}$$
 (3.36)

For later purposes, it is also desirable to define a condensate Reynolds number

$$Re_c = \frac{4\Gamma}{\mu_\ell} \tag{3.37}$$

Substituting this into Eq. (3.36) gives

$$h_c = 1.47 \left[ \frac{k_\ell^3 \rho_\ell (\rho_\ell - \rho_\nu) g}{\mu_\ell^2} \right]^{1/3} \text{Re}_c^{-1/3}$$
(3.38)

Before developing the treatment further, it is useful to re-examine the several assumptions of the Nusselt derivation. In general, the validity of the Nusselt equation has been established in experiments in which care has been taken to satisfy the assumptions. But what if those assumptions are not satisfied in an actual application? Does a departure from ideality completely invalidate the equation, or can the equation or its application be modified to still give useful design results? Only by considering each of the assumptions can we answer that.



The consequences of the violation of Nusselt's basic assumption, i.e., larninar flow of the condensate film, are very significant and will be examined at greater length later. Within this assumption, however, many workers have analytically and experimentally tested the effect of violation of the other assumptions listed above. We will now consider these briefly, with attention focused upon the design consequences of the result.

The first assumption, that of saturated vapor, has been studied experimentally and theoretically; the weight of the evidence is that superheating effects in pure vapors are small and Nusselt's equation can be safely applied, if the sensible heat load required to desuperheat the vapor to saturation is added to the latent heat load in calculating condenser duty and if the mean temperature difference for the condenser is calculated using the vapor saturation temperature. This, of course, assumes that the con denser surface temperature is below saturation, so that condensation does occur. The method of calculating surface temperatures will be considered when desuperheating in condensers is discussed.

The second assumption, that of no interfacial resistance, has been a subject of investigation in many areas of transport processes. The effect of an interfacial resistance becomes significant only at extremely high transfer rates for condensation processes; for example, interfacial resistance has been suspected as the phenomenon responsible for liquid metal condensation coefficients being "only" about 10,000 BTU/hr ft²oF, instead of the 100,000 or so predicted by Nusselt's equation. For air cooled condensers, it is completely immaterial whether the assumption is true or not.

The third assumption, concerning the mechanism of heat transfer in the liquid film is exactly as valid as the assumption of laminar flow and breaks down only when the flow is no longer laminar.

The fourth assumption, that of a linear temperature pro file in the film, can be relaxed by replacing  $\lambda$  by

$$\lambda \left[ 1 + \frac{0.68C_{p,\ell}(T_{sat} - T_w)}{\lambda} \right] \tag{3.39}$$

This is a significant correction only when  $(T_{sat} - T_w)$  is relatively quite large, usually out of the range of process practice. Since the effect is to increase the calculated value of  $h_c$  it is generally conservative to neglect this correction.

The fifth assumption, equilibrium at the liquid-solid interface, has not been questioned on theoretical grounds, but in practical cases, allowance must be made for a dirt film resistance to heat transfer. This is handled in the conventional way and does not affect the Nusselt equations.

The sixth assumption, an isothermal condensing surface, is generally not realized in practice. While the remaining features of the Nusselt analysis can be rigorously applied to a non-isothermal surface, the calculation requires a computer, and standard practice is to simply use the average computed surface temperature in a condenser. Even this in principle requires a reiterative calculation; assuming a surface temperature, calculating the coefficient, checking the surface temperature, etc. For the usual process case, the effect of an assumption of a mean wall temperature can be shown to be small and probably conservative.

The seventh assumption, constant liquid properties, is never strictly valid, but is accepted because the alternatives are so formidable. The physical properties are generally taken at the arithmetic mean film temperature. Significant errors in the final result are unlikely to arise unless the temperature difference is



very great or the condensate has a very large temperature coefficient of viscosity. In case of doubt, use the viscosity at the surface temperature in the Nusselt equation.

The assumption concerning vapor shear on the condensate on a vertical surface was examined by Nusselt himself (7) and is described in detail in Jakob (10). If the vapor and the condensate flow together vertically downwards, vapor shear somewhat enhances the condensing coefficient in laminar flow (the role of vapor shear in causing laminar flow to be destroyed is discussed later.) If the vapor and condensate flow in opposite directions, the condensate film is thickened and resistance increases; however, in this case a probable and significant consequence is that the film becomes rippled and/or turbulent, and entrainment, slugging, flooding, etc., occur.

Assumption 9, no slip at the liquid-solid interface, is very strongly supported by many studies on laminar flow.

The last assumption, that the heat load from subcooling the liquid is negligible compared to the latent heat, while generally true, can be easily and conservatively relaxed by adding this amount to the total heat to be transferred. The condensate film is subcooled (on the average) to  $(T_{sc} = 3/8 (T_{sat} - T_w))$ , so the added heat load is

$$Q_{sc} = \frac{3}{8} w c_{p,\ell} (T_{sat} - T_w)$$
 (3.40)

Possibly a more important and interesting point is that this degree of subcooling may be sufficient (if reheating is avoided) to satisfy the NPSH requirements of the condensate pump and eliminate the need for a condensate subcooler.

Turbulent Film Condensation in Vertical Tubes. Since Eqs. (3.33) and (3.38) are valid only for laminar flow of the condensate film in the vertical tube, we now consider the following questions:

- 1. When does the film cease to be laminar?
- 2. What correlation is valid when the film becomes turbulent?
- 3. What is the effect of vapor shear on the condensation mechanisms and correlations?

With reference to the first question, there is no hard and fast answer to the criterion for transition from laminar to turbulent flow. Ripples can appear on the surface at quite low values of  $Re_c$  (as diagrammed in Fig. 3.8) but these seem to have very little effect on the condensing coefficient. There is a definite break in the heat transfer behavior of films at  $Re_c$  of 1600 to 2000 in the absence of vapor shear, and this number can be used here as the critical Reynolds number for the falling film.

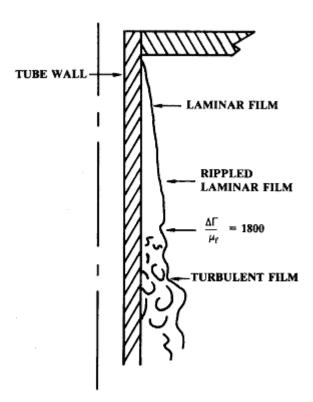


Fig. 3.8 Idealized Vertical Film Condensation.



The presence of vapor shear causes an early appearance of turbulence,  $Re_c$  values as low as 250 to 300 being reported by Carpenter and Colburn (12). As it turns out, from a practical point of view, the question is not a serious one since the magnitude of the calculated coefficient itself indicates what flow regime exists.

Condensation heat transfer coefficients under turbulent flow but low vapor shear conditions are correlated by a fundamental analysis due to Colburn (13) based on analogies between heat and momentum transfer in single phase turbulent flows. The Colburn approach required a numerical integration to get the mean coefficient so that a graphical representation of the final result is usually presented. For this purpose, it is convenient to note that the Nusselt solution can be plotted as

$$h_c \left[ \frac{\mu_\ell^2}{k_\ell^3 \rho_\ell (\rho_\ell - \rho_\nu) g} \right]^{1/3} vs \operatorname{Re}_c$$

with a slope of -1/3 on log-log coordinates and an intercept of 1.47 at  $Re_c = 1$ . The Colburn solution may also be plotted on these coordinates with  $Pr_{\ell}$  as a parameter, giving the graph shown here as Fig. 3.9. The break at  $Re_c = 2100$  shows the point at which the film is presumed to become turbulent; Colburn curves used this value in his computation, and it is somewhat conservative. Notice also that the Colburn curves

used this value in his computation, and it is somewhat conservative. Notice also that the Colburn curves have a Prandtl number dependence. The correlation shown in Fig. 3.9 has been reasonably well verified experimentally, but it seems unwise to extrapolate the Prandtl number dependence to values higher than 5; available data do not go very much higher. (High Prandtl numbers usually arise from higher viscosity, which also causes lower Reynolds numbers, so the problem is in some sense self-limiting. However, the physical structure of a high-flow-rate, high viscosity condensate film may be a thick laminar layer close to the cold wall with a turbulent liquid film cascading down the outside. This could result in a higher heat transfer resistance than expected from a condensate layer whose properties are calculated at a mean temperature.)

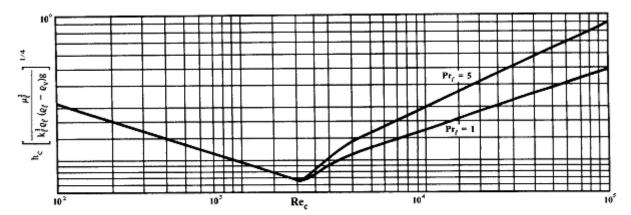


Fig. 3.9 Correlation for Condensation on a Vertical Surface-No Vapor Shear.

Turning now to the case of a vapor-shear controlled condensing situation inside a vertical tube, there are several procedures to choose from in the literature. Carpenter and Colburn (12) did the first really comprehensive experimental study of this problem, correlating their average coefficients by



$$\frac{h_c \mu_\ell}{k_\ell \rho_\ell^{1/2}} = 0.065 \quad \Pr_\ell^{1/2} F_{vc}^{1/2} \tag{3.41}$$

where

$$F_{vc} = \frac{fG_{v,m}^{2}}{2\rho_{v}} \tag{3.42}$$

$$G_{v,m} = \left(\frac{G_{v,i}^2 + G_{v,i}G_{v,o} + G_{v,o}^2}{3}\right)^{1/2}$$
(3.43)

and f is the Fanning friction factor, given by

$$f = \frac{0.078}{\left(\frac{d_i G_{v,m}}{\mu_v}\right)^{1/4}} \tag{3.44}$$

 $G_{v,i}$  and  $G_{v,o}$  are the vapor phase mass velocities at the inlet and outlet, respectively. If the vapor comes in dry and saturated and is completely condensed,  $G_{v,m} = 0.58 G_{v,i}$ .

There are numerous other correlations for condensation in the presence of high vapor shear, and some of them are more accurate. However, the more accurate ones are also harder to use, and usually the additional accuracy is not required in air-cooled condensers, where the condensation is not the controlling resistance.

The Carpenter-Colburn correlation is valid only under the condition that vapor shear controls the liquid film hydrodynamics and hence heat transfer. If vapor shear does not control, this correlation will give an unrealistically low coefficient. Therefore, in deciding which correlation to use, the following rule applies: Calculate condensing coefficients by the gravity-controlled correlation (Eq. 3.36) and by the vapor shear-controlled correlation (Eqs. 3.41 to 3.44) and take the *higher* value.

Larninar Film Condensation Inside Horizontal Tubes. Essentially the same set of assumptions as previously used for vertical tubes may be applied to condensation in a horizontal tube. The only essential difference between the analyses for the two cases is that:

- a. for the horizontal tube, the effective component of the gravitational acceleration, g  $\sin \theta$ , changes with position around the tube, and
- b. the condensate will form a pool in the bottom of the tube (Fig. 3. 10) and render that part of the tube surface ineffective.

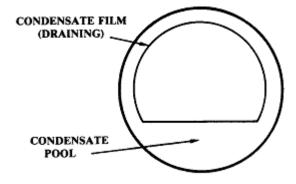


Fig. 3.10 Nusselt Condensation Inside a Horizontal Tube



The fraction of surface thus affected depends upon the properties of the condensate, the rate of condensation, the geometry of the tube, and the provision made for removing the condensate. If there were no condensate pool, Nusselt's analysis gives the heat transfer coefficient as Eq. (3.45)

$$h_c = 0.725 \left[ \frac{k_\ell^3 \rho_\ell (\rho_\ell - \rho_\nu) \lambda g}{\mu_\ell d_i (T_{sat} - T_w)} \right]^{1/4}$$
(3.45)

Kern (11) recast this equation in a form that introduces the condensate weight flow rate per tube  $W_t$ , eliminates the temperature difference, and introduces a penalty for the presence of the pool. His equation is:

$$h_c = 0.761 \left[ \frac{k_\ell^3 \rho_\ell (\rho_\ell - \rho_\nu) gL}{W_t \mu_\ell} \right]^{1/3}$$
 (3.46)

Again, more elegant equations are available, but the presumed additional precision is not essential. Eq. (3.46) is valid only at low vapor shear rates.

At high condensing loads, with vapor shear dominating, the correlations should be independent of tube orientation, and this is in fact found to be the case. Therefore, the high-vapor-shear condensing coefficient may be calculated from the Carpenter-Colburn equations (Eqs. 3.41 to 3.44) previously given for this case.

The selection of which correlation to apply in a given case is based on the following argument: The flow patterns, the hydrodynamics and consequently, the heat transfer processes are dominated by gravity at low tube loadings and by vapor shear at high loadings. Correlations are available for both limiting cases; each of these correlations predicts low coefficients (relative to the appropriate correlation) when it is applied to situations to which it is in fact not applicable. Therefore, in any case in which there is doubt as to the correct correlation, calculate the condensing coefficient by each correlation (Eq. (3.41 to 3.44) and Eq. (3.46)) and choose the *higher* value of  $h_c$ .

Filmwise Condensation Inside Inclined Tubes. In the foregoing discussion of condensation inside vertical and horizontal tubes, the argument was made that tube orientation was unimportant at high velocities where vapor shear controlled, and we would expect the same to be true of condensation in inclined tubes with vapor flow towards the lower end of the tube. In the gravity-controlled regime, however, inclination should make a difference. Very little information is available in the open literature on this problem and the best we can do here is suggest a procedure that will be responsive to the major effects and reduce to the correct limiting cases.

If we start with a vertical tube  $\theta = 0$ , as defined in Fig. 3.6), we would expect an increase in  $\theta$  to reduce the effective component of the gravitational force acting on the draining film by the function g cos  $\theta$ . Therefore, it is suggested that the condensing heat transfer coefficient for an inclined tube in the gravity-controlled regime be calculated from Fig. 3.9, using as the ordinate



$$h_c \left[ \frac{\mu_\ell^2}{k_\ell^3 \rho_\ell (\rho_\ell - \rho_\nu) g \cos \theta} \right]^{1/3}$$

instead of that shown.

But if that procedure is carried to nearly horizontal tubes, where  $\theta \to 90^\circ$ ,  $\cos \theta \to 0$  and  $h_c \to 0$ , which is physically unrealistic. It hardly seems likely that  $h_c$  would fall below the value for a horizontal tube; indeed, even a few degrees of downward slope should aid greatly in the drainage and improve the coefficient above the horizontal case. Therefore, it seems conservative to place the lower bound on  $h_c$  (of an inclined tube) as the value given by Eg. (3.46) for a horizontal tube at the same loading.

Additionally, in each case, the value of  $h_c$  predicted by Eqs. (3.41 to 3.44) should be calculated. If the value so calculated is greater than that obtained in the previous paragraph, the flow may be presumed to be predominantly in the vapor shear-controlled regime and the higher value of  $h_c$  used for further calculations.

#### 3.2.3. Mean Temperature Difference for In-Tube Condensation

MTD for a Pure Saturated Vapor. For the usual conditions of condensing a pure saturated vapor in an air cooled exchanger, the correct value of the Mean Temperature Difference (MTD) is the logarithmic mean temperature difference, using the saturation temperature, T<sub>sat</sub>, of the vapor at the nominal condensing pressure as the constant hot side temperature:

$$MTD = LMTD = \frac{t_o - t_i}{\ell n \left(\frac{T_{sat} - t_i}{T_{sat} - t_o}\right)}$$
(3.47)

Strictly speaking, this equation is only valid if the overall coefficient is constant and if the condensing fluid temperature is isothermal, i.e., if there is no desuperheating or subcooling and the vapor pressure drop is very small compared to the absolute pressure. In fact, these are usually not serious limitations. Since the condensation process is commonly only a relatively small part of the total resistance to heat transfer, the overall coefficient varies little about a mean value calculated on the basis of an average condensing coefficient, or for some correlations, a condensing coefficient calculated at the average quality of the condensing stream. The treatment of desuperheating and subcooling is given later. The other condition negligible pressure drop in the condensing process may be relaxed approximately but satisfactorily by finding the saturation temperatures at the inlet and outlet pressures, T<sub>sat,i</sub> and T<sub>sat,o</sub>, calculating the LMTD as

$$LMTD = \frac{(T_{sat,o} - t_i) - (T_{sat,i} - t_o)}{\ell n \left(\frac{T_{sat} - t_i}{T_{sat} - t_o}\right)}$$
(3.48)

and finally calculating the mean temperature difference as

$$MTD = F(LMTD) (3.49)$$



where F is found from the curves in Chapter 4. This is a reasonably valid procedure only if the temperature approach is not too close and if the pressure drop in the condensing vapor is relatively small. Both of these conditions are usually met in air-cooled condensers.

Most air-cooled condensers are designed so that the condensation takes place in a single pass from one header to the other. If this is not done, the liquid and vapor phases will tend to separate and mal-distribute in the turn around header, resulting in some tubes having a surplus of vapor (and, therefore, possibly not giving complete condensation) and other tubes having a surplus of liquid (and giving excessive subcooling.)

However, it is usually necessary for the single condensing pass to include several vertical tube rows. Since the air becomes progressively hotter from row to row, the local temperature difference and the condensation rate correspondingly decrease. This means that if the vapor in the bottom row of tubes is just completely condensed (no subcooling), the vapor in the upper rows is in completely condensed, leading to a loss of vapor.

In order to avoid this, Mueller (14) has derived a safety factor to be applied to assure that the vapor entering the top rows is completely condensed by the end of the tube (resulting in some subcooling of the condensate in the lower rows). The factor is

$$E = \frac{Total \ tube \ length \ in \ exchanger}{Total \ tube \ length \ required \ for \ condesation} = \frac{n + n(n-1)K}{n + \frac{n(n-1)K}{2}}$$
 (3.50)

where

$$K = 1 - \exp\left[-\frac{UA}{W_{air}C_{p_{air}}}\right]$$
(3.51)

and where U is the overall heat transfer coefficient, based on the total outside heat transfer area A (actually, any consistent combination of U and A may be used) and N is the number of rows of tubes in the exchanger. The heat transfer area (and therefore the tube length) required for complete condensation is calculated assuming a uniform condensing loading for each row of tubes, and is then multiplied by E from Eq. (3.50) to find the total area (and the additional tube length) required to assure complete condensation in the top row.

Desuperheating of vapor. If the vapor entering a condenser is superheated, the sensible heat content of the vapor must be removed and transferred through the cooling surface before that vapor can be condensed. If the cold surface is above the saturation temperature of the vapor, the heat is removed by a convective sensible heat transfer mechanism, the coefficient for which can be calculated from correlations given in Chapter 2 employing vapor physical properties. However, if the cold surface is below the saturation temperature of the vapor at the existing pressure, vapor will condense directly upon the surface with essential thermodynamic equilibrium existing at the condensate-vapor interface and with the temperature gradient from the superheated state to saturation occurring in the vapor immediately adjacent to the interface.

Available information indicates that the heat transfer coefficient for condensation directly from the superheated vapor is within a few percent of that for condensation from the saturated vapor, using in each case the saturation temperature of the vapor as the temperature driving force for heat transfer. Such



differences as have been observed are well within the present ability to predict condensing coefficients for a given situation. This fortuitous agreement is useful in the design of desuperheating condensers.

The first matter is to establish a test to determine whether or not condensation will occur on a cold surface exposed to superheated vapor. If we assume that heat is transferred from the superheated vapor by sensible heat transfer, and if  $T_v$  is the local vapor temperature,  $T_{sat}$  the condensing or saturation temperature, and t the local coolant temperature, the wall temperature on the vapor side,  $T'_w$ , is given by

$$T'_{w} = T_{v} - \frac{U_{s}(T_{v} - t)}{h_{s}}$$
 (3.52)

where  $h_s$  is the sensible heat transfer coefficient for the vapor stream and  $U_s$  is the overall coefficient computed using hs, both referenced to the same surface. If  $T'_w > T_{sat}$ , no condensation will occur,  $T'_w$  is equal to the true wall surface temperature  $T_w$  and the heat transfer rate is given by  $(Q/A) = U_s (T_v - t)$ . If  $T'_w < T_{sat}$ , condensation will occur and the heat transfer rate is given by  $(Q/A) = U_c (T_{sat} - t)$ , where  $U_c$  is the overall coefficient computed assuming condensation does occur, and the true inside tube surface temperature is given by;

$$T_{w} = T_{sat} - \frac{U_{c}(T_{sat} - t)}{h_{c}}$$
 (3.53)

There is a further interesting consequence of the above discussion. Let  $U^+$  be the combined heat transfer coefficient for the wall and fin resistance, the coolant and any dirt films.  $U^+$  is essentially independent of the heat flux and whether or not condensation is occurring inside the tube. Then, the heat flux for the sensible heat transfer desuperheating case is  $(Q/A) = U^+(T_w^- - t)$  and that for the condensing case is  $(Q/A) = U^+(T_w^- - t)$ . Since  $T_w^- \ge T_w$ , we conclude that condensation will occur directly from the superheated vapor, unless a higher heat flux is obtained by the sensible heat transfer mode. A corollary to this is that it is both simpler and more conservative (in the sense of calculating a larger condenser area) to assume that condensation will occur directly from the superheated vapor, using the saturation temperature and a condensing heat transfer coefficient in the rate equation, and of course, including the sensible heat in the heat load.

However, in designing a desuperheating condenser where the desuperheating heat load is an appreciable fraction of the total heat load, the designer may wish to avail himself of the possible surface savings afforded by the higher heat flux of sensible desuperheating. This is a fairly complicated matter, since the heat balance and rate equations must be simultaneously balanced by increments in two directions - vertically through the tube bundle and longitudinally along the length of the tubes. This is strictly a computer solution; proprietary programs are available and they run quite rapidly in both the rating and design modes. It must be remembered that the computer solution is only as good as the data base and the skill of the engineer and systems analyst who put the program together, and even the best of the methods have a substantial, range of uncertainty in the final answer.

Subcooling the Condensate. It is often desirable to further subcool the condensate beyond the few degrees achieved by the condensing process itself. This can be done by passing the condensate through one or more rows of tubes in the bottom of the air cooler.

Again, the exact thermal analysis of this problem is quite complicated and can only be carried out by a computer program. An *approximate* calculation can be carried out by considering the condensing and subcooling heat transfer processes to be carried out in two separate heat exchangers in series. Thus one can calculate the mixed mean air temperature just after passing over the subcooling tubes, using the



methods outlined in Chapter 4 on air cooling. This mixed mean air temperature can then be used as the inlet air temperature to the condensing section, which is designed according to the methods in this chapter.

There are several important cautions to be observed. First, the air off of the subcooling sections varies along the length of the tube, and this variation alters the condensing pattern, increasing the local temperature difference (compared t ' o the mixed mean assumption) and local condensation rate at one end and decreasing it at the other. The effect tends to cancel out, though whether the net effect is conservative or non-conservative depends upon the specifics of each problem. Certainly the absolute error of the method increases as the process stream temperatures approach more closely the air temperatures.

Another consideration is this: Almost certainly the design and the mechanical layout of the air-cooled exchanger will be dominated by the condensing process. Therefore, the subcoofing achieved will be at the mercy of the conditions set for the condensing process and not in dependently controllable. If close condensate subcooling control is required, it is much better to provide a separate heat exchanger, the operating conditions for which can be adjusted as required.