



HOMEWORK I: MODELLING OF A DYNAMIC SYSTEM

The objective of this homework is to model and simulate a dynamical system that will be used as virtual plant for the controllers to be developed during the course. For this exercise set, choose ONE OF THE FOLLOWING: System 1 (S1), System 2 (S2), System 3 (S3), System 4 (S4)¹ or System 5 (S5).

S1: Three-tank system.

S2: Inverted pendulum.

S3: Hot and cold water mixing tank.

S4: Continuous bioreactor.

S5: A system of your choice².

Regardless of your choice, you must:

1. Describe the system of your choice and obtain a mathematical model for it. Verify / derive the differential equations describing the system dynamics. Discuss and justify the assumptions/simplifications you adopt. Build the model in a simulation environment of your choice using the numerical values provided in the class material. Define the states, the inputs and the outputs of the system, along with the objectives of the control system to be developed in the next HWs.
2. Linearise the mathematical model obtained in the first task around a convenient steady state point. Use Taylor series for the approximation. Represent it as input-output and state-space. Simulate the linearised model and compare it with the one you obtained in the first task (using some input signal, e.g., series of steps and/or sin wave). Plot the comparison results.
3. Transform the state space model into the transfer function that describes the input-output system. Plot the response of the system of a step input of amplitude A , defined based on your system. Use the figure and identify, if possible, (i) the steady state value; (ii) the overshoot in % of the final value; (iii) the rise time; (iv) the settling time.
4. Analyse the stability of the system by studying the poles position of the linearised model (in open loop configuration). Compute and plot the pole-zero map. Identify the open-loop system poles and zeros. Discuss their effect on the process response and the important information you get from the poles-zeros map. Define the Lyapunov's

¹Details and assumptions on each system have been given and discussed in class. They can be retrieved from the lecture slides.

²This system should have the similar characteristics in terms of number of inputs/output, nonlinearities and difficulty of the ones proposed above.

stability of the given system. Briefly explain the concept of stability according to Lyapunov. Define the input and output variables and the corresponding controllability and observability of the system.

GUIDELINES

Regardless of your choice of dynamical system, you must generate the following:

- REPORT/PRESENTATION: You must provide a clear and exhaustive description of the models and their derivation, the associated plot/tables of the simulation results and your comments. In particular, you should include at least the following sections:
 - Introduction: Here, you provide some context and background. Briefly, explore the literature in order to understand how your chosen system has been modelled and analysed. Discuss some application examples (if any) and provide the references.
 - System description: Here, you describe the system and the associated model. In particular, you must provide the theoretical background that brought to the definition of the nonlinear model of the system and its the state-space representation. You simulate the nonlinear model and comment on the results.
 - System linearisation: Here, you motivate the need of a linear model, describe the theoretical background of the linearisation and represent the nonlinear model as input-output and in the state-space. Also, you simulate the linearised model and compare it with the nonlinear one graphically and numerically in terms of RMSE.
 - System analysis: Here, you provide the theoretical background for the main properties of the dynamical models. In particular, you must define and compute the controllability/observability of the system. You evaluate the its BIBO and Lyapunov's stability, briefly explaining the concepts and highlighting the differences existing within the different definitions of stability.
 - References: Here, you provide bibliographic references. Report the books and/or articles that you used for defining and analysing the system. Each reference reported in this section must be cited in the main text.
- CODE LISTING: You must provide the executable/functioning code produced to model, simulate and perform the analysis on the system. The code can be packaged together with the report and sent trough SIGAA as a zip file.

The work will be evaluated based on: adherence to the instructions, clear and critical argumentation, formatting and orthography.

The work can be done individually or in pair. You can write your report/present either in English or Portuguese. You can base your work on the resources you might find on the web but you must adequately reference to them. The presentation of the first homework and deadline for submission of the report is set on APRIL 9, 2024.