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Fuzzy Control of a Real Time Inverted Pendulum System

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Abstract. In this study, a real-time control of the cart-pole inverted pendulum system was developed using fuzzy logic controller. Swing-up and stabilization of the inverted pendulum were implemented directly in fuzzy logic controller. The fuzzy logic controller designed in the Matlab-Simulink environment was embedded in a dSPACE DS1103 DSP controller board. Swing-up algorithm brings the pendulum near to its inverted position in 10 seconds from downward position. In order to test the robustness of the fuzzy logic controller internal (changing model parameters) and external disturbances (applying external forces) were applied on the inverted pendulum. The inverted pendulum system was shown to be robust to the external and internal disturbances. The maximum errors of the pendulum angle to the impulse input were between 1.89° and 4.6449° in the robustness tests.

Keywords: Inverted pendulum, swing up, stabilization, fuzzy logic.

1 Introduction

Inverted pendulum is very common and interesting nonlinear system in the control applications. Nonlinear, unstable inverted pendulum system is used to test performance of the different control algorithms. Inverted pendulum problem is a very useful system to utilize in the control education due to simplicity of establishing the system. It is therefore very widely used laboratory tool in the control laboratories.

Fuzzy set theory is employed to model the amount of ambiguity or uncertainty subject to parameter inaccuracy and unmodeled dynamics. It offers a very attractive way to extract information from data for designing a controller. Comparison of classical set theory with fuzzy set theory helps one to understand the basic differences between them. The main difference is that with fuzzy set theory a set membership is represented as a possibility distribution, while the classical set theory defines sets with a clear-cut decision that a feature either belongs to a set or not [1].

In the literature, different inverted pendulum systems and their control methods have been studied. The inverted pendulum systems can be grouped structurally as cart-pole single pendulum systems [2-4], cart-pole double pendulum systems [5], rotary single and double pendulum systems [6-7]. The control methods applied to inverted pendulum systems can be summarized to linear methods such as PID, state

feedback [2,4,6,8,9] and nonlinear methods such as energy based, fuzzy logic, feedback linearization and etc. [2,3,5,6,10-14]. Muskinja and Tovornik [3] proposed energy based and fuzzy logic controllers for swing-up of inverted pendulum. Adaptive state controller is also suggested for the stabilization of inverted pendulum. Ji, Lei and Kin [12] presented simulation results of inverted pendulum using fuzzy logic controller for swing-up and stabilization routines.

2 The Structure of the Inverted Pendulum

In the cart-pole single inverted pendulum system, pendulum is attached to the cart which can move in the limited horizontal track. The rod is attached to an optical encoder, so it can rotate very small friction. Inverted pendulum system has two equilibrium points: downward and upright positions. Stable downward equilibrium point corresponds to $\theta = 0$, $\dot{\theta} = 0$. Unstable upright equilibrium point corresponds to $\theta = \pi$, $\dot{\theta} = 0$.

The inverted pendulum developed here consists of several parts such as servo motor providing movement of the cart and applying the desired force to the pendulum, the sensors measuring the state variables (cart position x and pendulum angle θ) and the controller supplying control signals. These parts are shown in Fig. 1. The control algorithm is embedded in dSPACE DS1103 DSP controller board. This board takes the sensor outputs and supply control signal. Photography of the inverted pendulum system developed here is shown in Fig. 1.

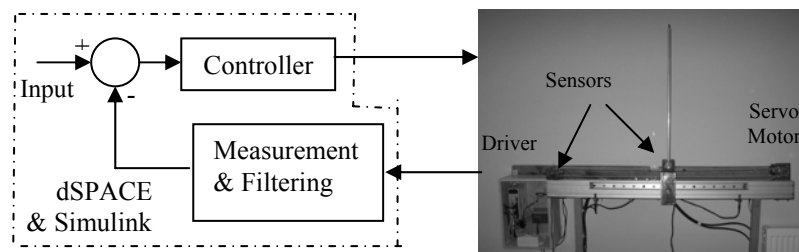


Fig. 1. Block diagram and setup of the inverted pendulum system

3 Controller Design

Generally, hybrid control approach is used in the control of inverted pendulum systems. The hybrid approach is implemented in two steps: swing-up and stabilization routines. The intention of swing-up routine is to bring up the pendulum from downward position to upright position. In stabilization routine, the pendulum is balanced in a limited track. In hybrid approach, the controller decides which routines switch on based on pendulum angle. However, the fuzzy logic control can handle both routines in rules base. Swing-up and stabilization routines are explained below.

3.1 Swing-Up Routine

In this study, a cart motion strategy was determined to swing up the pendulum based on work effect of a given acceleration [3]. The basic strategy is to move the cart in such a motion that energy is gradually pumped to the pendulum. How to add the best energy to the pendulum should be explored and the strategy should be determined based on the analysis. The pendulum model based on Newtonian approach is shown in Fig. 2-a. Throughout this analysis, bold characters are used to denote vector quantities, while non-bold characters represent scalar quantities.

The forces act on the pendulum is,

$$\begin{aligned}\mathbf{F}_r &= F_r (\sin \theta \hat{\mathbf{i}} + \cos \theta \hat{\mathbf{j}}) \\ \mathbf{F}_g &= -mg \hat{\mathbf{j}}\end{aligned}\quad (1)$$

Using Newton's law F_r is found,

$$F_r = m(\sin \theta \ddot{x} + l\dot{\theta}^2 + g \cos \theta). \quad (2)$$

Using F_r the work is found,

$$\delta W = \mathbf{F}_r \cdot \partial x = F_r \sin \theta \partial x = m(\sin^2 \theta \ddot{x} + \sin \theta l\dot{\theta}^2 + g \sin \theta \cos \theta) \partial x. \quad (3)$$

The term $\sin^2 \theta \ddot{x}$ in Equation 3 is important, because it describes the work effect of a given acceleration at any angle. This term is shown in Fig. 2-b. For θ close to $\pm 90^\circ$, a given acceleration and displacement does maximum work. For θ close to 0° , hardly any work is done. The work is positive when the acceleration and displacement are in the same direction. In this case, the energy is added to the system.

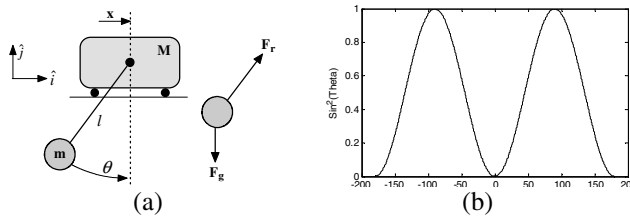


Fig. 2. (a) Coordinate system and free body diagram of the system. (b) Work effect of the system.

Considering the work effect of the system, the strategy for a cart trajectory that is driven by pendulum angle is developed. To maximize the positive work done on the pendulum, the cart must be accelerated when θ near $\pm 90^\circ$ to achieve high work transfer and decelerated when θ near 0° since there is low work transfer. Acceleration and deceleration regions are shown in Fig. 3. The cart must be accelerated when the pendulum is in region I, cart must be decelerated in region II and it must be waited while pendulum reaches its maximum point in region III. This process is repeated until pendulum reaches its inverted position. Because the pendulum begins at rest, hanging

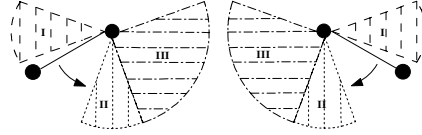


Fig. 3. Swing-up strategy

downward position in region II, this strategy does not produce an output, so swing-up controller should include a series of quick cart movements that begin the pendulum swinging. Finally, to bring the pendulum close to its inverted position and in low angular velocity, cart movement amplitude should be gradually reduced when the pendulum swings higher.

The rules mentioned above are very suitable to design a fuzzy controller. In the proposed fuzzy swing-up controller, two input variables are used: pendulum angle θ and pendulum angular velocity $\dot{\theta}$. The system has a limited track length but it is enough to apply this control strategy. Seven fuzzy subsets [NLS (Negative large), NBS (Negative big), SALN (Start swing negative), Z (Zero), SALP (Start swing positive), PBS (Positive big) and PLS (Positive large)] are used for the error of pendulum angle. Three fuzzy subsets are chosen for the error of pendulum angular velocity. The input membership functions are shown in Fig. 4, the rule base is given in Table 1 and the output membership functions are shown in Fig. 6. The output membership functions has nine fuzzy sets [NVVB (Negative very very big), NVB (Negative very big), N (Negative), NB (Negative big), Z (Zero), P (Positive), PB (Positive big), PVB (Positive very big), PVVB (Positive very very big)].

Response of the swing-up controller used here is shown in Fig. 5. Pendulum is swung-up from downward position to inverted position in about 10 seconds. The number of fluctuations and swing-up time can be reduced or increased by changing amplitude of the output membership functions.

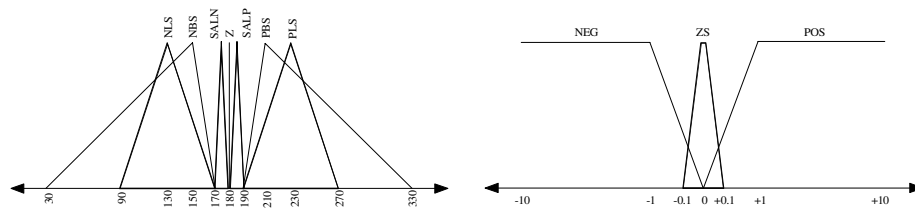

 Fig. 4. Membership functions for θ and $\dot{\theta}$

Table 1. Fuzzy rule base for swing-up

$e\dot{\theta}$	$e\theta$						
	NLS	NBS	SALN	Z	SALP	PBS	PLS
NEG					P	Z	PB
ZS				P			
POS	NB	Z	N				

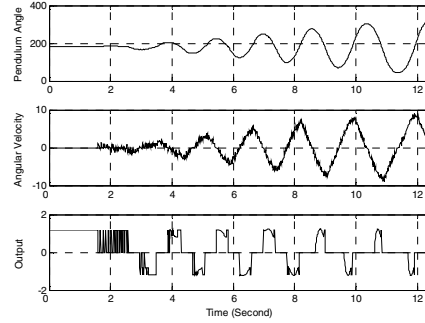


Fig. 5. Response of the system during swing-up

3.2 Stabilization

In the proposed fuzzy stabilization controller, four input variables are used: pendulum angle θ , pendulum angular velocity $\dot{\theta}$, cart position x and cart velocity \dot{x} . In order to balance the pendulum in the range of $\pm 30^\circ$, seven fuzzy subsets [NVB, NB, N, ZO, P, PB, PVB] are chosen for the error of pendulum angle and five fuzzy subsets [NB, N, ZO, P, PB] are chosen for the error of angular velocity. Also, five fuzzy subsets [NBIG, NEG, Z, POS, PBIG] are chosen for the error of cart position and three fuzzy subsets [NEG, Z, POS] are chosen for the error of cart velocity to return the cart to its home position. Swing-up and stabilization rules use the same output fuzzy subsets. These membership functions are shown in Fig. 6. The rule bases for cart position control and pendulum angle control are given in Table 2 and Table 3, respectively. Totally, 49 rules are used to control the whole system.

Table 2. Fuzzy rule base for cart position

	ex				
$e\dot{x}$	NBIG	NEG	Z	POS	PBIG
NEG	PVVB	PVB	PB		
ZERO			Z		
POS			NB	NVB	NVVB

Table 3. Fuzzy rule base for stabilization

	$e\theta$						
$e\dot{\theta}$	NVB	NB	N	ZO	P	PB	PVB
NB	NVVB	NVVB	NVB	NB	N	Z	P
N	NVVB	NVB	NB	N	Z	P	PB
ZO	NVB	NB	N	Z	P	PB	PVB
P	NB	N	Z	P	PB	PVB	PVVB
PB	N	Z	P	PB	PVB	PVVB	PVVB

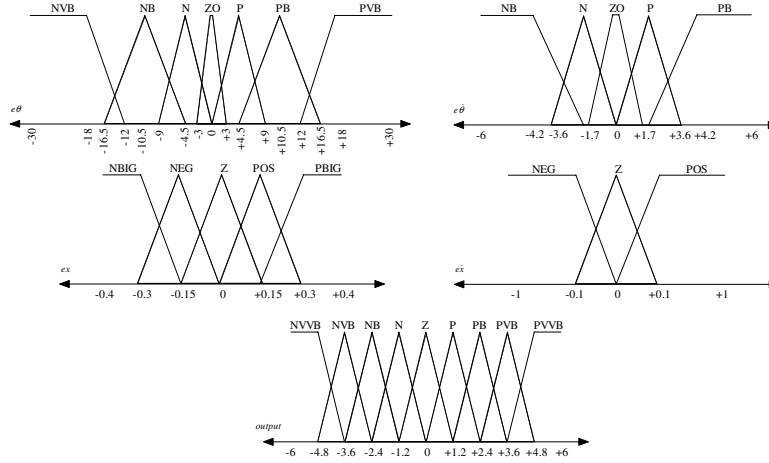


Fig. 6. Membership functions for $\theta, \dot{\theta}, x, \dot{x}$ and output

Response of the controller developed here is shown in Fig. 7. In the figure, the fuzzy logic controller based on the rules switches from swing-up to stabilization. After the switching stabilization routine, pendulum angle and cart position are brought to their desired values.

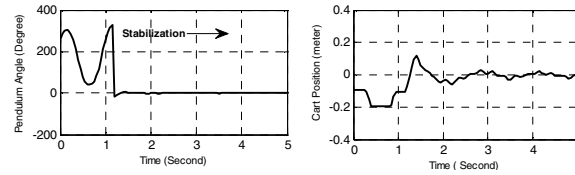


Fig. 7. Controller response during switching from swing-up to stabilization

4 Experimental Results

In order to test the robustness of the designed fuzzy controller, several experiments were implemented using the rods with different lengths and masses. In these experiments, two types of disturbances were applied to the system: external and internal disturbances. Internal disturbances were implemented by changing the system parameters: rod mass and rod length. Minimum and maximum values of the state variables are given in Table 4 as the system parameters change. The fuzzy controller was designed for the inverted pendulum with the rod shown in Fig. 1 (0.65m and 0.2 kg). This rod is called as a normal. Its physical properties were changed by:

- choosing heavier mass of the rod
- same mass and different length of the rod
- increasing length of the normal rod
- decreasing length of the normal rod
- attaching a moving mass to the end of the normal rod.

According to Table 4, longer and lighter pendulum has a minimum error, and shorter and heavier pendulum has a maximum error in state variables of the system. Natural frequency of inverted pendulum is based on length of the rod about equilibrium point so longer pendulum has a lower natural frequency. Longer pendulum is controlled more easily because of its low natural frequency. It causes a high rotational inertia giving a high resistance to a rotational change.

Table 4. Errors in the state variables

Rod spec		θ peak	x peak	$\int \theta $	$\int x $	$\int u $
Pendulum length	Pendulum mass					
0.65m	0.2 kg	-2.6952	0.0655	0.0540	0.0444	0.0492
0.97 m	0.3 kg	-1.89	0.0384	0.0426	0.0065	0.0410
0.325 m	0.1 kg	-4.6449	0.1482	0.1568	0.0816	0.1388
0.42 m	0.2 kg	-3.8655	0.1881	0.0988	0.1131	0.0731
0.65 m	0.3 kg	-2.79	0.1027	0.0782	0.0680	0.0643
0.65 m	0.4 kg (moving mass)	-2.3400	0.0440	0.0575	0.0121	0.0435

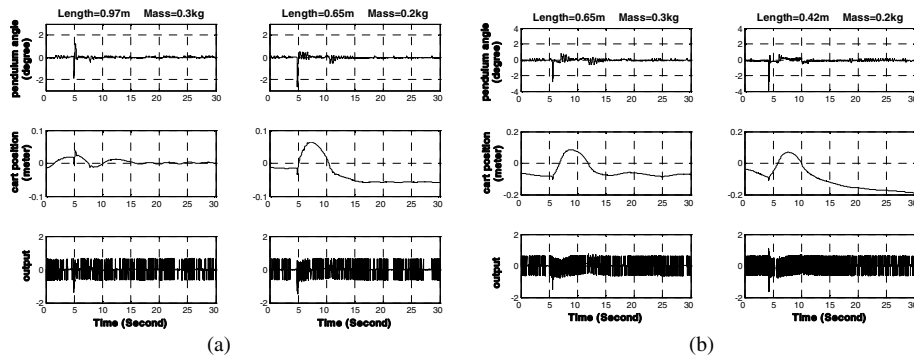


Fig. 8. (a) System responses for longest and normal rod experiments. (b) System responses for heavier and shorter rod experiments.

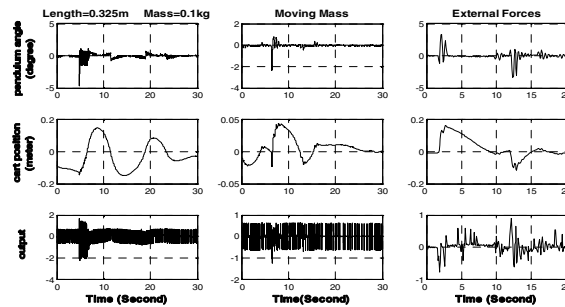


Fig. 9. System responses for shortest rod, moving mass and external disturbances experiments

5 Conclusion

The real-time fuzzy logic control of the cart-pole inverted pendulum system was developed for swing-up and stabilization. The controller was implemented in dSPACE-1103 development board. Several experiments were conducted to verify the robustness of the fuzzy controller under external and internal disturbances. Based on these experiments, the inverted pendulum system was seen to be robust to the disturbances. During disturbance experiments, the peak errors of the pendulum angle to the impulse input were obtained between 1.89° and 4.6449° .

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