

# Nuclear Physics - Summary - Nuclear Decays

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This summary is based on the book Chapters 6.1, 8-10 from Kenneth Krane: Introductory Nuclear Physics.

## 1 Nuclear Decays

parent radio nucleid  $\rightarrow$  spontaneous decay  $\rightarrow$  daughter nucleid

**Types of decays:**

- $\alpha$  decay  $\rightarrow \alpha$  particle ( ${}^4\text{He}$ ) gets ejected or cluster decay (heavier particle gets ejected)
- $\beta$  decay  $\rightarrow \beta$  particle ( $e^-$ ,  $e^+$ ) +  $\nu$  gets ejected
- $\gamma$  decay  $\rightarrow$  nuclear energy transition  $\rightarrow \gamma$  gets ejected, this is usually following an  $\alpha$  or  $\beta$  decay

Decays follow an **exponential decay law**:

$$\lambda = \frac{dN/dt}{N}$$

where  $\lambda$  is the **decay or disintegration constant**,  $N$  is the number of nuclei at  $t$  time, and  $dN$  nuclei decay in  $dt$  time

$$N(t) = N_0 e^{-\lambda t}$$

where  $N_0$  is the number of nuclei at  $t = 0$ .

The **half life** ( $t_{1/2}$ ) is the time it takes for half of the initial nuclei to decay.

$$t_{1/2} = \frac{0.693}{\lambda}$$

The **average lifetime** ( $\tau$ ) is the average time a nucleus is likely to survive before it decays.

$$\tau = \frac{1}{\lambda}$$

The **activity (A)** is the rate at which decays occur in a sample:

$$A(t) = \lambda N(t) = A_0 e^{-\lambda t}$$

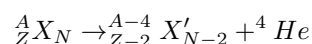
where  $A_0$  is the initial activity in a sample at  $t = 0$ ,  $A_0 = \lambda N_0$ . The units of activity are:

- Curie Ci (1Ci =  $3.7^{10}$  decays/second)
- Becquerel Bq (1Bq = 1 decay/second)

## 2 $\alpha$ decay

- The Coulomb repulsion ( $\sim Z^2$ ) effect becomes important for heavy nuclei compared to the strong force ( $\sim A$ ).
- energetically the  $\alpha$  particle is the best to eject, since it results in the largest energy release for a light particle.
- other nuclei that can be ejected:  ${}^{12}\text{C}$  but these are very rare, they have a much longer half-life compared to the decay through  ${}^4\text{He}$

The basic reaction:



The energy release can be calculated from the mass difference:

$$Q = (m_X - m_{X'} - m_\alpha)c^2 = T_{X'} + T_\alpha$$

- The released energy gets distributed as kinetic energy between the daughter and the  $\alpha$  particle. The  $\alpha$  particle gets most of the energy (98%).
- The small energy for the daughter is effectively a small recoil.
- $Q$  must be  $> 0$  for decays and  $\lambda$  must be not too small.
- from  $Q$  we can measure atomic mass for  $X'$

- **large  $Q \rightarrow$  short half life**
- even  $Z$  and  $N \rightarrow$  short half life
- odd-even and odd-odd nuclei have relatively longer half lives
- **adding  $n^0$  to a nucleus increases the half life**

## 2.1 Theory of $\alpha$ decay

### The one body model:

- The  $\alpha$  particle is already formed inside the nucleus  $\rightarrow$  tunneling  $\rightarrow$  decay
- potential well + Coulomb potential barrier

The desintegration probability:

$$\lambda = fP$$

where  $f$  is the frequency at which the  $\alpha$  particle approaches the edge of the potential and  $P$  is the tunnelling probability.

This is a simplified model, which is a relatively good approach. However, there are some discrepancies due between calculations and measurements. These are due to:

- the exact wave function
- momentum of the  $\alpha$
- the shape of the nucleus is not symmetrical (heavy nuclei are deformed)  $\rightarrow$  the radius is not the same over the nucleus  $\rightarrow$  the tunnelling probability depends strongly on the radius (Coulomb barrier) of the nucleus.

Calculating half lives shows which decays are more likely or less likely to happen.

**Nucleon emission - Proton decay** (decay trough ejecting a  $p^+$ ) for most nuclei energetically forbidden, but can happen for some nuclei with negative proton separation energies ( $S_p$ ).

## 2.2 Angular momentum and parity

- ${}^4He$  spin = 0 (both n and p are paired), only 1 component.
- parity change:  $(-1)^{l_\alpha}$
- parity selection rule  $\rightarrow$  which transition can occur:
  - if initial and final are the same:  $l_\alpha$  even
  - if initial and final are not the same:  $l_\alpha$  odd
- a given initial state can decay into a number of final states (excited states - rotation and vibration) in the daughter  $\rightarrow$  "fine structure" of  $\alpha$  decay
- decays to different E levels:
  - centrifugal force raises the barrier (rotational excitation)
  - excitation energy lowers the barrier
- relative decay probability: decaying into the  $0^+$  state is the most likely compared to the  $2^+, 4^+ \text{ or } 6^+$  states  $\rightarrow$  the wave function is more different for the later states, it is easier to go from  $0^+$  to  $0^+$  because the wave function is very similar.
- parity:  $0 \rightarrow 3$   $l_\alpha = 3$  odd  $\rightarrow 0^+ \rightarrow 3^-$  possible,  $0^+ \rightarrow 3^+$  not possible
- if the initial and final spin are not 0 more combinations are possible
- the angular distribution of the ejected  $\alpha$  particles indicates the shape of the nucleus and the angular momentum of the nucleus.

## 2.3 Applications:

- power source (space crafts, pacemaker)
- cancer treatment
- smoke detectors

## 3 $\beta$ decay

Types:

- $\beta^- : n^0 \rightarrow p^+ + e^- + \bar{\nu}_e$
- $\beta^+ : p^+ \rightarrow n^0 + e^+ + \nu_e$
- $e^-$  capture:  $e^- + p^+ \rightarrow n^0 + \nu_e$
- $\beta^+$  and  $e^-$  capture only occur for bound  $p^+$  in nuclei.
- A stays the same, Z and N can change
- unstable nuclei with too many protons or neutrons decay until they reach a stable state
- $\beta$  decay needs to create new particles ( $e^-, e^+, \nu, \bar{\nu}$ )

### 3.1 Energy release:

The energy spectrum of the  $e^-$  is continuous  $\rightarrow$  not a 2 body interaction  $\rightarrow$  new particle: the  $\nu$  (neutral, spin = 1/2)

#### 3.1.1 $\beta$ decay of free $n$ ( $t_{1/2} \sim 10$ minutes):

$$n \rightarrow p^+ + e^- + \bar{\nu}$$

$$Q = (m_n - m_p - m_e - m_\nu)c^2$$

$$Q = T_p + T_e + T_\nu$$

- $T_p$  is small
- the  $e^-$  and the  $\bar{\nu}$  share most of the energy  $\rightarrow$  continuous spectrum
- from  $Q$  and the measured masses we know that  $\nu$  has a very small mass
- because of the small mass of the  $e^-$  and the  $\nu \rightarrow$  relativistic motion

#### 3.1.2 $\beta^-$ decay in nuclei:

$${}^A_Z X_N \rightarrow {}^A_{Z+1} X_{N-1} + e^- + \bar{\nu}$$

$$Q_{\beta^-} = [m({}^A X) - m({}^A X')]c^2$$

This can be used for mass measurements.

$$Q_{\beta^-} = T_e + E_{\bar{\nu}}$$

#### 3.1.3 $\beta^+$ decay in nuclei:

$${}^A_Z X_N \rightarrow {}^A_{Z-1} X_{N+1} + e^+ + \nu$$

$$Q_{\beta^+} = [m({}^A X) - m({}^A X') - 2m_e]c^2$$

#### 3.1.4 $e^-$ capture in nuclei:

$${}^A_Z X_N + e^- \rightarrow {}^A_{Z-1} X_{N+1} + \nu$$

- if an  $e^-$  gets captured from an  $e^-$  orbit  $\rightarrow \gamma$  emission from other  $e^-$  filling the hole of the captured  $e^-$
- $\beta^+$  and  $e^-$  capture lead to the same final nucleus, but both are not always energetically possible.  $\beta^+$  needs a larger energy difference between the initial and final states.  $\rightarrow$  If  $\beta^+$  is possible  $e^-$  capture is possible too, but not the other way around

### 3.2 Fermi theory of $\beta$ decay:

- $\beta$  decay is interpreted as a weak interaction
- The probability of the transition depends on 3 factors:
  1. The availability of the final states  $p^2(Q - T_e)^2$
  2. The Fermi function, which accounts for the nuclear Coulomb field  $F(Z', p)$
  3. The nuclear matrix element  $|M_{fi}|^2$  which accounts for the exact wave function of the initial and final states, and the additional momentum from the forbidden terms  $S(p, q)$
- there are allowed decays and forbidden decays

The momentum spectrum:

$$N(p) \propto p^2 (Q - T_e)^2 F(Z', p) |M_{fi}|^2 S(p, q)$$

- Experimental test: Kurie plot
- large range of  $\beta$  decay half lives
- the shortest  $t_{1/2}$  decays are called superallowed decays
- The full theory of  $\beta$  decay: Fermi model + particle exchange (W, Z bosons)  $\rightarrow$  interaction through the **weak force**

### 3.3 Angular momentum and parity rules

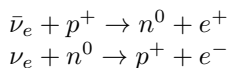
#### 3.3.1 No orbital angular momentum

- $s_e = 1/2, s_\nu = 1/2 \rightarrow$  parallel or antiparallel spin  $\rightarrow$  two types of decays:
  - parallel spins: **Gamow-Teller decay**  $S=1$
  - anti-parallel spins: **Fermi decay**  $S=0$
- allowed decay selection rules:  $l = 0, \Delta I = 0, 1; \Delta\pi = \text{no}$
- first forbidden decay:  $l = 1, \Delta I = 0, 1, 2; \Delta\pi = \text{yes}$
- second forbidden decay:  $l = 2, \Delta I = 2, 3; \Delta\pi = \text{no}$
- third forbidden decay:  $l = 3, \Delta I = 3, 4; \Delta\pi = \text{yes}$

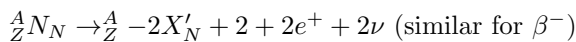
### 3.4 Additional types of $\beta$ decay

#### 3.4.1 Inverse $\beta$ decay

- Capture of  $\nu$  or  $\bar{\nu}$
- Important for detecting neutrinos.



#### 3.4.2 Double $\beta$ decay ( $\beta\beta$ decay)



- two simultaneous  $\beta$  decays
- Interesting because detecting a neutrino less double  $\beta$  decay would prove that the  $\nu$  is its own anti particle.
- very difficult to detect because  $\beta\beta$  decay has very long half lives

#### 3.4.3 $\beta$ delayed nucleon emission

- Precursor:  $\beta$  decay into an excited state  $\rightarrow$  nucleon emission ( $n^0, p^+$ )
- The energy of the excited states must be high.  $\rightarrow$  It needs to cover the separation energy of the nucleon + the kinetic energy of the emitted particle + a small recoil energy
- Can be used to map excited states

### 3.5 Applications:

- $\beta$  light
- monitoring thickness of materials
- cancer treatment
- medical imagery (PET)
- radio carbon dating

## 4 $\gamma$ decay

- $\gamma$  decay is an energy transition of the nucleus through a photon emission
- Typically very high energy: 0.1 - 10 MeV  $\rightarrow$  in the  $\gamma$  wavelength regime
- Most  $\alpha$  and  $\beta$  decays are followed by a  $\gamma$  decay

**Energetics:**

$$E_i = E_f + E_\gamma + T_R$$

where  $E_i$  is the initial state,  $E_f$  is the final state,  $T_R$  is the recoil energy and  $E_\gamma$  is the energy of the emitted  $\gamma$ .

$$\Delta E = E_i - E_f$$

$$\Delta E = E_\gamma + \frac{E_\gamma^2}{2Mc^2}$$

$$\Delta E \sim E_\gamma$$

- low energy  $\gamma$  rays  $\rightarrow$  small recoil
- high energy  $\gamma$  rays  $\rightarrow$  large recoil  $\rightarrow$  radiation damage

**4.1 Angular momentum and parity**

- orbital angular momentum  $\rightarrow$  L determines the moment: dipole, quadrupole, etc.
  - L=0  $\rightarrow$  monopole  $\rightarrow$  no  $\gamma$  emission
  - L=1  $\rightarrow$  dipole
  - L=2  $\rightarrow$  quadrupole
- parity determines if the radiation is electric or magnetic in nature:
  - $\Delta\pi = \text{no}$   $\rightarrow$  even parity  $\rightarrow$  electric
  - $\Delta\pi = \text{yes}$   $\rightarrow$  odd parity  $\rightarrow$  magnetic

**4.2 Internal conversion**

- instead of a  $\gamma$  an  $e^-$  from the atom gets the  $\Delta E$  and gets ejected
- this is then followed by a  $\gamma$  emission from other  $e^-$  filling the hole (much lower energy compared to the  $\gamma$  decay photon)

**Energies:**

$$T_e = \Delta E - B$$

where  $T_e$  is the kinetic energy of the  $e^-$  and  $B$  is the binding energy of the electron.

- $T_e$  has discrete energies, which depend on the orbital shell of the  $e^-$  (e.g. K, L, M)
- generally both  $\gamma$  emission and internal conversion can happen, they both have a decay probability ( $\lambda = \lambda_\gamma + \lambda_e$ )

**4.3 Applications**

- $\gamma$  ray spectroscopy  $\rightarrow$  map excited states
- cancer treatment
- scanning containers
- sterilisation of medical equipment

**Health effects:**  $\gamma$  rays are the worst for our health.  $\gamma$  radiation can only be shielded with thick layers of lead and concrete. Small amount of  $\gamma$  ray radiation can lead to cancer, large amount of  $\gamma$  radiation can lead to radiation sickness and death.