Nuclear Physics - Summary - Nuclear Decays

by Dr. Helga Dénes (hdenes@yachaytech.edu.ec)

This summary is based on the book Chapters 6.1, 8-10 from Kenneth Krane: Introductory Nuclear Physics.

1 Nuclear Decays

parent radio nucleid $\rightarrow$ spontaneous decay $\rightarrow$ daughter nucleid

Types of decays:

- α decay $\rightarrow$ α particle (^4He) gets ejected or cluster decay (heavier particle gets ejected)
- β decay $\rightarrow$ β particle (e^-, e^+) + ν gets ejected
- γ decay $\rightarrow$ nuclear energy transition $\rightarrow$ γ gets ejected, this is usually following and α or β decay

Decays follow an **exponential decay law**:

$$\lambda = \frac{dN/dt}{N}$$

where λ is the **decay or disintegration constant**, N is the number of nuclei at t time, and dN nuclei decay in dt time

$$N(t) = N_0 e^{\lambda t}$$

where N_0 is the number of nuclei at $t = 0$.
The **half life ($t_{1/2}$)** is the time it takes for half of the initial nuclei to decay.

$$t_{1/2} = \frac{0.693}{\lambda}$$

The **average lifetime (τ)** is the average time a nucleus is likely tu survive befor it decays.

$$\tau = \frac{1}{\lambda}$$

The **activity (A)** is the rate at which decays occur in a sample:

$$A(t) = \lambda N(t) = A_0 e^{-\lambda t}$$

where A_0 is the initial activity in a sample at $t = 0$, $A_0 = \lambda N_0$. The units of activity are:

- Curie Ci (1Ci = 3.7^{10} decays/second)
- Bequerel Bq (1Bq = 1 decay/second)

2 α decay

- The Coulomb repulsion ($\sim Z^2$) effect becomes important for heavy nuclei compared to the strong force ($\sim A$).
- energetically the α particle is the best to eject, since it results in the largest energy release for a light particle.
- other nuclei that can be ejected: ^{12}C but these are very rare, they have a much longer half-life compared to the decay trough ^4He

The basic reaction:

$$^A_Z X_N \rightarrow^{A-4}_{Z-2} X'_{N-2} +^4 He$$

The energy release can be calculated from the mass difference:

$$Q = (m_X - m_{X'} - m_\alpha)c^2 = T_{X'} + T_\alpha$$

- The released energy gets distributed as kinetic energy between the daughter and the α particle. The α particle gets most of the energy (98%).
- The small energy for the daughter is effectively a small recoil.
- Q must be > 0 for decays and λ must be not too small.
- from Q we can measure atomic mass for X'

- **large Q $\rightarrow$ short half life**
- even Z and N $\rightarrow$ short half life
- odd-even and odd-odd nuclei have relatively longer half lives
- **adding n^0 to a nucleus increases the half life** $\rightarrow$ The strong nuclear force from the extra n^0 helps to balance the effects of the Coulomb force.

2.1 Theory of α decay

The one body model:
- The α particle is already formed inside the nucleus $\rightarrow$ tunneling $\rightarrow$ decay
- potential well + Coulomb potential barrier

The desintegration probability:

$$\lambda = fP$$

where f is the frequency at which the α particle approaches the edge of the potential and P is the tunnelling probability.

This is a simplified model, which is a relatively good approach. However, there are some discrepancies due between calculations and measurements. These are due to:
- the exact wave function
- momentum of the α
- the shape of the nucleus is not symmetrical (heavy nuclei are deformed) $\rightarrow$ the radius is not the same over the nucleus $\rightarrow$ the tunnelling probability depends strongly on the radius (Coulomb barrier) of the nucleus.

Calculating half lives shows which decays are more likely or less likely to happen.

Nucleon emission - Proton decay (decay trough ejecting a p^+) for most nuclei energetically forbidden, but can happen for some nuclei with negative proton separation energies (S_p).

2.2 Angular momentum and parity

- ^4He spin = 0 (both n and p are paired), only l component.
- parity change: $(-1)^{l_\alpha}$
- parity selection rule $\rightarrow$ which transition can occur:
  - if initial and final are the same: l_α even
  - if initial and final are not the same: l_α odd
- a given initial state can decay into a number of final states (excited states - rotation and vibration) in the daughter $\rightarrow$ "fine structure" of α decay
- decays to different E levels:
  - centrifugal force raises the barrier (rotational excitation)
  - excitation energy lowers the barrier
- relative decay probability: decaying into the 0^+ state is the most likely compared to the $2^+, 4^+$ or 6^+ states $\rightarrow$ the wave function is more different for the later states, it is easier to go from 0^+ to 0^+ because the wave function is very similar.
- parity: $0 \rightarrow 3$ $l_\alpha = 3$ odd $\rightarrow 0^+ \rightarrow 3^-$ possible, $0^+ \rightarrow 3^+$ not possible
- if the initial and final spin are not 0 more combinations are possible
- the angular distribution of the ejected α particles indicates the shape of the nucleus and the angular momentum of the nucleus.

2.3 Applications:

- power source (space crafts, pacemaker)
- cancer treatment
- smoke detectors

3 β decay

Types:

- $\beta^- : n^0 \to p^+ + e^- + \bar{\nu}_e$
- $\beta^+ : p^+ \to n^0 + e^+ + \nu_e$
- e^- capture: $e^- + p^+ \to n^0 + \nu_e$

- β^+ and e^- capture only occur for bound p^+ in nuclei.
- A stays the same, Z and N can change
- unstable nuclei with too many protons or neutrons decay untill they reach a stable state
- β decay needs to create new particles $(e^-, e^+, \nu, \bar{\nu})$

3.1 Energy release:

The energy spectrum of the e^- is continuous $\to$ not a 2 body interaction $\to$ new particle: the ν (neutral, spin = 1/2)

3.1.1 β decay of free n ($t_{1/2} \sim$ 10 minutes):

$$n \to P^+ + e^- + \bar{\nu}$$
$$Q = (m_n - m_p - m_e - m_\nu)c^2$$
$$Q = T_p + T_e + T_\nu$$

- T_p is a small recoil
- the e^- and the $\bar{\nu}$ share most of the energy $\to$ continuous spectrum
- from Q and the measured masses we know that ν has a very small mass
- because of the small mass of the e^- and the $\nu \to$ relativistic motion

3.1.2 β^- decay in nuclei:

$$^A_ZX_N \to ^A_{Z+1} X_{N-1} + e^- + \bar{\nu}$$
$$Q_{\beta^-} = [m(^AX) - m(^AX')]c^2$$

This can be used for mass measurements.

$$Q_{\beta^-} = T_e + E_{\bar{\nu}}$$

3.1.3 β^+ decay in nuclei:

$$^A_ZX_N \to ^A_{Z-1} X_{N+1} + e^+ + \nu$$
$$Q_{\beta^+} = [m(^AX) - m(^AX') - 2m_e]c^2$$

3.1.4 e^- capture in nuclei:

$$^A_ZX_N + e^- \to ^A_{Z-1} X_{N+1} + \nu$$

- if an e^- gets captured from an e^- orbit $\to \gamma$ emission from other e^- filling the hole of the captured e^-
- β^+ and e^- capture lead to the same final nucleus, but both are not always energetically possible. β^+ needs a larger energy difference between the initial and final states. $\to$ If β^+ is possible e^- capture is possible too, but not the other way around

3.2 Fermi theory of β decay:

- β decay is interpreted as a weak interaction
- The probability of the transition depends on 3 factors:
  1. The availability of the final states $p^2(Q - T_e)^2$
  2. The Fermi function, which accounts for the nuclear Coulomb field $F(Z', p)$
  3. The nuclear matrix element $|M_{fi}|^2$ which accounts for the exact wave function of the initial and final states, and the additional momentum from the forbidden terms $S(p, q)$
- there are allowed decays and forbidden decays depending on the nuclear matrix element. Forbidden decays are not actually forbidden, they are just less likely to happen compared to the allowed decays.

The momentum spectrum:

$$N(p) \propto p^2(Q - T_e)^2 F(Z', p)|M_{fi}|^2 S(p, q)$$

- Experimental test: Kurie plot
- large range of β decay half lives
- the shortest $t_{1/2}$ decays are called superallowed decays
- The full theory of β decay: Fermi model + particle exchange (W, Z bosons) $\rightarrow$ interaction trough the **weak force**

3.3 Angular momentum and parity rules

3.3.1 No orbital angular momentum

- $s_e = 1/2, s_\nu = 1/2 \rightarrow$ parallel or antiparallel spin $\rightarrow$ two types of decays:
  - parallel spins: **Gamow-Teller decay** S=1
  - anti-parallel spins: **Fermi decay** S=0
- allowed decay selection rules: $l = 0, \Delta I = 0, 1; \Delta\pi =$ no
- first forbidden decay: $l = 1, \Delta I = 0, 1, 2; \Delta\pi =$ yes
- second forbidden decay: $l = 2, \Delta I = 2, 3; \Delta\pi =$ no
- third forbidden decay: $l = 3, \Delta I = 3, 4; \Delta\pi =$ yes

3.4 Additional types of β decay

3.4.1 Inverse β decay

- Capture of ν or $\bar{\nu}$
- Important for detecting neutrinos.

$\bar{\nu}_e + p^+ \rightarrow n^0 + e^+$
$\nu_e + n^0 \rightarrow p^+ + e^-$

3.4.2 Double β decay ($\beta\beta$ decay)

${}^A_Z N_N \rightarrow^A_Z -2X'_N + 2 + 2e^+ + 2\nu$ (similar for β^-)

- two simultaneous β decays
- Interesting because detecting a neutrino less double β decay would prove that the ν is its own anti particle.
- very difficult to detect because $\beta\beta$ decay has very long half lives

3.4.3 β delayed nucleon emission

- Precursor: β decay into an excited state $\rightarrow$ nucleon emission (n^0, p^+)
- The energy of the excited states must be high. $\rightarrow$ It needs to cover the separation energy of the nucleon + the kinetic energy of the emitted particle + a small recoil energy
- Can be used to map exited states

3.5 Applications:

- β light
- monitoring thickness of materials
- cancer treatment
- medical imaging, e.g. PET scans (Positron emission tomography)
- radio carbon dating

4 γ decay

- γ decay is an **energy transition** of the nucleus trough a photon emission
- Typically very high energy: 0.1 - 10 MeV $\rightarrow$ in the γ wavelength regime
- Most α and β decays are followed by a γ decay

Energetics:

$$E_i = E_f + E_\gamma + T_R$$

where E_i is the initial state, E_f is the final state, T_R is the recoil energy and E_γ is the energy of the emitted γ.

$$\Delta E = E_i + E_f$$

$$\Delta E = E_\gamma + \frac{E_\gamma^2}{2Mc^2}$$

$$\Delta E \sim E_\gamma$$

- low energy γ rays $\rightarrow$ small recoil
- high energy γ rays $\rightarrow$ large recoil $\rightarrow$ radiation damage

4.1 Angular momentum and parity

- orbital angular momentum $\rightarrow$ L determines the moment: dipole, quadrupole, etc.
  - L=0 $\rightarrow$ monopole $\rightarrow$ no γ emission
  - L=1 $\rightarrow$ dipole
  - L=2 $\rightarrow$ quadrupole
- parity determines if the radiation is electric or magnetic in nature:
  - $\Delta\pi$ = no $\rightarrow$ even parity $\rightarrow$ electric
  - $\Delta\pi$ = yes $\rightarrow$ odd parity $\rightarrow$ magnetic

4.2 Internal conversion

- instead of a γ an e^- from the atom gets the ΔE and gets ejected
- this is then followed by a cascade of γ emission from other e^--s filling the hole left by the decay (much lower energy compared to the γ decay photon)

Energies:

$$T_e = \Delta E - B$$

where T_e is the kinetic energy of the e^- and B is the binding energy of the electron.

- T_e has discrete energies, which depend on the orbital shell of the e^- (e.g. K, L, M)
- generally both γ emission and internal conversion can happen, they both have a decay probability ($\lambda = \lambda_\gamma + \lambda_e$)

4.3 Applications

- γ ray spectroscopy $\rightarrow$ map excited states
- cancer treatment
- scanning containers
- sterilisation of medical equipment
- treating art objects (e.g. killing insects and bacteria)

Health effects: γ rays are the worst for our health. γ radiation can only be shielded with thick layers of lead and concrete. Small amount of γ ray radiation can lead to cancer, large amount of γ radiation can lead to radiation sickness and death.