Nuclear Physics - Summary - Nuclear Decays

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This summary is based on the book Chapters 6.1, 8-10 from Kenneth Krane: Introductory Nuclear Physics.

1 Nuclear Decays

parent radio nucleid $\rightarrow$ spontaneous decay $\rightarrow$ daughter nucleid

Types of decays:
- α decay $\rightarrow$ α particle (^4He) gets ejected or cluster decay (heavier particle gets ejected)
- β decay $\rightarrow$ β particle (e^-, e^+) + ν gets ejected
- γ decay $\rightarrow$ nuclear energy transition $\rightarrow$ γ gets ejected, this is usually following and α or β decay

Decays follow an **exponential decay law**:

$$\lambda = \frac{dN/dt}{N}$$

where λ is the **decay or disintegration constant**, N is the number of nuclei at t time, and dN nuclei decay in dt time

$$N(t) = N_0 e^{\lambda t}$$

where N_0 is the number of nuclei at $t = 0$.
The **half life ($t_{1/2}$)** is the time it takes for half of the initial nuclei to decay.

$$t_{1/2} = \frac{0.693}{\lambda}$$

The **average lifetime (τ)** is the average time a nucleus is likely tu survive befor it decays.

$$\tau = \frac{1}{\lambda}$$

The **activity (A)** is the rate at which decays occur in a sample:

$$A(t) = \lambda N(t) = A_0 e^{-\lambda t}$$

where A_0 is the initial activity in a sample at $t = 0$, $A_0 = \lambda N_0$. The units of activity are:
- Curie Ci (1Ci = 3.7^{10} decays/second)
- Bequerel Bq (1Bq = 1 decay/second)

2 α decay

- The Coulomb repulsion ($\sim Z^2$) effect becomes important for heavy nuclei compared to the strong force ($\sim A$).
- energetically the α particle is the best to eject, since it results in the largest energy release for a light particle.
- other nuclei that can be ejected: ^{12}C but these are very rare, they have a much longer half-life compared to the decay trough ^4He

The basic reaction:

$$^A_Z X_N \rightarrow ^{A-4}_{Z-2} X'_{N-2} +^4 He$$

The energy release can be calculated from the mass difference:

$$Q = (m_X - m_{X'} - m_\alpha)c^2 = T_{X'} + T_\alpha$$

- The released energy gets distributed as kinetic energy between the daughter and the α particle. The α particle gets most of the energy (98%).
- The small energy for the daughter is effectively a small recoil.
- Q must be > 0 for decays and λ must be not too small.
- from Q we can measure atomic mass for X'

- **large Q $\rightarrow$ short half life**
- even Z and N $\rightarrow$ short half life
- odd-even and odd-odd nuclei have relatively longer half lives
- **adding n^0 to a nucleus increases the half life**

2.1 Theory of α decay

The one body model:
- The α particle is already formed inside the nucleus $\rightarrow$ tunneling $\rightarrow$ decay
- potential well + Coulomb potential barrier

The desintegration probability:

$$\lambda = fP$$

where f is the frequency at which the α particle approaches the edge of the potential and P is the tunnelling probability.

This is a simplified model, which is a relatively good approach. However, there are some discrepancies due between calculations and measurements. These are due to:
- the exact wave function
- momentum of the α
- the shape of the nucleus is not symmetrical (heavy nuclei are deformed) $\rightarrow$ the radius is not the same over the nucleus $\rightarrow$ the tunnelling probability depends strongly on the radius (Coulomb barrier) of the nucleus.

Calculating half lives shows which decays are more likely or less likely to happen.

Nucleon emission - Proton decay (decay trough ejecting a p^+) for most nuclei energetically forbidden, but can happen for some nuclei with negative proton separation energies (S_p).

2.2 Angular momentum and parity

- ^4He spin = 0 (both n and p are paired), only l component.
- parity change: $(-1)^{l_\alpha}$
- parity selection rule $\rightarrow$ which transition can occur:
  - if initial and final are the same: l_α even
  - if initial and final are not the same: l_α odd
- a given initial state can decay into a number of final states (excited states - rotation and vibration) in the daughter $\rightarrow$ "fine structure" of α decay
- decays to different E levels:
  - centrifugal force raises the barrier (rotational excitation)
  - excitation energy lowers the barrier
- relative decay probability: decaying into the 0^+ state is the most likely compared to the $2^+, 4^+ or 6^+$ states $\rightarrow$ the wave function is more different for the later states, it is easier to go from 0^+ to 0^+ because the wave function is very similar.
- parity: $0 \rightarrow 3$ $l_\alpha = 3$ odd $\rightarrow 0^+ \rightarrow 3^-$ possible, $0^+ \rightarrow 3^+$ not possible
- if the initial and final spin are not 0 more combinations are possible
- the angular distribution of the ejected α particles indicates the shape of the nucleus and the angular momentum of the nucleus.

2.3 Applications:

- power source (space crafts, pacemaker)
- cancer treatment
- smoke detectors

3 β decay

Types:

- $\beta^- : n^0 \to p^+ + e^- + \bar{\nu}_e$
- $\beta^+ : p^+ \to n^0 + e^+ + \nu_e$
- e^- capture: $e^- + p^+ \to n^0 + \nu_e$

- β^+ and e^- capture only occur for bound p^+ in nuclei.
- A stays the same, Z and N can change
- unstable nuclei with too many protons or neutrons decay untill they reach a stable state
- β decay needs to create new particles $(e^-, e^+, \nu, \bar{\nu})$

3.1 Energy release:

The energy spectrum of the e^- is continuous $\to$ not a 2 body interaction $\to$ new particle: the ν (neutral, spin = 1/2)

3.1.1 β decay of free n ($t_{1/2} \sim$ 10 minutes):

$$n \to P^+ + e^- + \bar{\nu}$$

$$Q = (m_n - m_p - m_e - m_\nu)c^2$$

$$Q = T_p + T_e + T_\nu$$

- T_p is small
- the e^- and the $\bar{\nu}$ share most of the energy $\to$ continuous spectrum
- from Q and the measured masses we know that ν has a very small mass
- because of the small mass of the e^- and the $\nu \to$ relativistic motion

3.1.2 β^- decay in nuclei:

$${}^A_Z X_N \to {}^A_{Z+1} X_{N-1} + e^- + \bar{\nu}$$

$$Q_{\beta^-} = [m({}^A X) - m({}^A X')]c^2$$

This can be used for mass measurements.

$$Q_{\beta^-} = T_e + E_{\bar{\nu}}$$

3.1.3 β^+ decay in nuclei:

$${}^A_Z X_N \to {}^A_{Z-1} X_{N+1} + e^+ + \nu$$

$$Q_{\beta^+} = [m({}^A X) - m({}^A X') - 2m_e]c^2$$

3.1.4 e^- capture in nuclei:

$${}^A_Z X_N + e^- \to {}^A_{Z-1} X_{N+1} + \nu$$

- if an e^- gets captured from an e^- orbit $\to \gamma$ emission from other e^- filling the hole of the captured e^-
- β^+ and e^- capture lead to the same final nucleus, but both are not always energetically possible. β^+ needs a larger energy difference between the initial and final states. $\to$ If β^+ is possible e^- capture is possible too, but not the other way around

3.2 Fermi theory of β decay:

- β decay is interpreted as a weak interaction
- The probability of the transition depends on 3 factors:
  1. The availability of the final states $p^2(Q - T_e)^2$
  2. The Fermi function, which accounts for the nuclear Coulomb field $F(Z', p)$
  3. The nuclear matrix element $|M_{fi}|^2$ which accounts for the exact wave function of the initial and final states, and the additional momentum from the forbidden terms $S(p, q)$
- there are allowed decays and forbidden decays

The momentum spectrum:

$$N(p) \propto p^2(Q-T_e)^2 F(Z',p)|M_{fi}|^2 S(p,q)$$

- Experimental test: Kurie plot
- large range of β decay half lives
- the shortest $t_{1/2}$ decays are called superallowed decays
- The full theory of β decay: Fermi model + particle exchange (W, Z bosons) $\rightarrow$ interaction trough the **weak force**

3.3 Angular momentum and parity rules

3.3.1 No orbital angular momentum

- $s_e = 1/2, s_\nu = 1/2 \rightarrow$ parallel or antiparallel spin $\rightarrow$ two types of decays:
  - parallel spins: **Gamow-Teller decay** S=1
  - anti-parallel spins: **Fermi decay** S=0
- allowed decay selection rules: $l = 0, \Delta I = 0, 1; \Delta\pi =$ no
- first forbidden decay: $l = 1, \Delta I = 0, 1, 2; \Delta\pi =$ yes
- second forbidden decay: $l = 2, \Delta I = 2, 3; \Delta\pi =$ no
- third forbidden decay: $l = 3, \Delta I = 3, 4; \Delta\pi =$ yes

3.4 Additional types of β decay

3.4.1 Inverse β decay

- Capture of ν or $\bar{\nu}$
- Important for detecting neutrinos.

$\bar{\nu}_e + p^+ \rightarrow n^0 + e^+$
$\nu_e + n^0 \rightarrow p^+ + e^-$

3.4.2 Double β decay ($\beta\beta$ decay)

${}^A_Z N_N \rightarrow^A_Z -2X'_N + 2 + 2e^+ + 2\nu$ (similar for β^-)

- two simultaneous β decays
- Interesting because detecting a neutrino less double β decay would prove that the ν is its own anti particle.
- very difficult to detect because $\beta\beta$ decay has very long half lives

3.4.3 β delayed nucleon emission

- Precursor: β decay into and excited state $\rightarrow$ nucleon emission (n^0, p^+)
- The energy of the excited states must be high. $\rightarrow$ It needs to cover the separation energy of the nucleon + the kinetic energy of the emitted particle + a small recoil energy
- Can be used to map exited states

3.5 Applications:

- β light
- monitoring thickness of materials
- cancer treatment
- medical imagery (PET)
- radio carbon dating

4 γ decay

- γ decay is an energy transition of the nucleus trough a photon emission
- Typically very high energy: 0.1 - 10 MeV $\rightarrow$ in the γ wavelength regime
- Most α and β decays are followed by a γ decay

Energetics:

$$E_i = E_f + E_\gamma + T_R$$

where E_i is the initial state, E_f is the final state, T_R is the recoil energy and E_γ is the energy of the emitted γ.

$$\Delta E = E_i + E_f$$

$$\Delta E = E_\gamma + \frac{E_\gamma^2}{2Mc^2}$$

$$\Delta E \sim E_\gamma$$

- low energy γ rays $\rightarrow$ small recoil
- high energy γ rays $\rightarrow$ large recoil $\rightarrow$ radiation damage

4.1 Angular momentum and parity

- orbital angular momentum $\rightarrow$ L determines the moment: dipole, quadrupole, etc.
  - L=0 $\rightarrow$ monopole $\rightarrow$ no γ emission
  - L=1 $\rightarrow$ dipole
  - L=2 $\rightarrow$ quadrupole
- parity determines if the radiation is electric or magnetic in nature:
  - $\Delta\pi$ = no $\rightarrow$ even parity $\rightarrow$ electric
  - $\Delta\pi$ = yes $\rightarrow$ odd parity $\rightarrow$ magnetic

4.2 Internal conversion

- instead of a γ an e^- from the atom gets the ΔE and gets ejected
- this is then followed by a γ emission from other e^- filling the hole (much lower energy compared to the γ decay photon)

Energies:

$$T_e = \Delta E - B$$

where T_e is the kinetic energy of the e^- and B is the binding energy of the electron.
- T_e has discrete energies, which depend on the orbital shell of the e^- (e.g. K, L, M)
- generally both γ emission and internal conversion can happen, they both have a decay probability ($\lambda = \lambda_\gamma + \lambda_e$)

4.3 Applications

- γ ray spectroscopy $\rightarrow$ map excited states
- cancer treatment
- scanning containers
- sterilisation of medical equipment

Health effects: γ rays are the worst for our health. γ radiation can only be shielded with thick layers of lead and concrete. Small amount of γ ray radiation can lead to cancer, large amount of γ radiation can lead to radiation sickness and death.