Particle Physics - Summary - Symmetries

by Dr. Helga Dénes (hdenes@yachaytech.edu.ec)

This summary is based on the book chapters 4.1 - 4.3 of Griffiths: Introduction to Elementary Particles

1 Symmetries

Symmetries are important because they relate to new conservation laws and help discover new physics.

- rotational symmetry - angular momentum, spin
- 'internal' symmetries - isospin SU(3), and flavour SU(6)
- 'discrete' symmetries - parity, charge conjugation, time reversal

Noether theorem: every symmetry in nature yields a conservation law $\leftrightarrow$ every conservation law reflects a symmetry.

Symmetries:

- translation in time $\leftrightarrow$ energy
- translation in space $\leftrightarrow$ momentum
- rotation $\leftrightarrow$ angular momentum
- gauge transformation $\leftrightarrow$ charge

1.1 What is a symmetry?

A symmetry is an operation that can be performed on a system and leaves it invariant. (example: equilateral triangle)

Symmetries can be described systematically trough group theory.

- if all elements commute in a group ($R_i R_j = R_j R_i$) the group is called Abelian. (e.g. transformations in space and time form Abelian groups, rotations on 3D do not form Abelian groups)
- groups can be finite or infinite (depends on the number of elements)
- groups can be continuos (e.g. group of all rotations in a plane, elements depend on one or more continuos parameter) or discrete (all finite groups are discrete)
- most groups in physics can be formulated as groups of matrices
- in elementary particle physics the most common groups are of the U(n) type: a collection of n x n unitary matrices

Summary of common group types in particle physics:

group	matrices in group
U(n)	unitary ($\tilde{U}^* U = 1$)
SU (n)	unitary, determinant =1
O(n)	orthogonal ($\tilde{O} O = 1$)
SO(n)	orthogonal, determinant = 1

- every group G can be represented by a group of matrices: for every group element (a) there is a corresponding matrix (M_a)
- there may be many group elements represented by the same matrix
- trivial case: every group element is represented by a 1 x 1 unit matrix
- if G is a group of matrices, then G is a representation of itself - this is called the fundamental representation
- in general every group has many representations

1.2 Angular momentum

angular momentum = orbital angular momentum (l) + spin angular momentum (s)

- In quantum mechanics we can only measure the magnitude of the angular momentum and one of the components (usually z component).
- there are only certain allowed values: for a measurement of L^2: l(l+1)$\hbar^2$, l = 0, 1, 2, 3 ...

- for a given l a measurement of L_z always gives: $m_l\hbar$, where $m_l = -l, ..0...l \rightarrow$ 2l+1 possibilities
- same for the spin: a measurement of S^2 can only return: $s(s+1)\hbar^2$, however the quantum number can be half integer as well
- for a given value of s, S_z must be: $m_s\hbar$ where $m_s = -s...0...s \rightarrow$ 2s+1 possibilities
- **a given particle can have any orbital angular momentum l, but for each type of particle the values of s is fixed.**
  - e.g. every π or K has s=0,
  - every e^-, p^+, n^0 has s = 1/2,
  - γ, gluon has s=1,
  - Δ, Ω has s=3/2
- particles with half integer spin are **fermions (leptons, baryons, quarks)**
- particles with integer spin are **bosons (mesons and all mediator particles)**

1.2.1 Addition of angular momentum

How do we add $|j_1 m_1\rangle$ and $|j_2 m_2\rangle$?
- the z components adds $m = m_1 + m_2$
- the magnitudes depend on the orientation of the vectors: we get all possible values between $j_1 + j_2$ to $|j_1 - j_2|$

1.2.2 Spin 1/2 particles

Spin 1/2 particles are e^-, p^+, n^0, quarks, leptons. The s=1/2 is called 'spin up', the s=-1/2 is called 'spin down'.

We can represent particles with spinors:

$$|\frac{1}{2}\frac{1}{2}\rangle = \begin{pmatrix}1\\0\end{pmatrix} |\frac{1}{2} - \frac{1}{2}\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

The most general state of a spin 1/2 particle is the linear combination:

$$\begin{pmatrix}\alpha\\\beta\end{pmatrix} = \alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix}$$

In general $|\alpha|^2$ and $|\beta|^2$ are the probability of measuring $1/2\hbar$ or $-1/2\hbar$. And $|\alpha|^2 + |\beta|^2 = 1$.

To each component of $\bar{S}$ we can associate a 2 x 2 matrix:

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}, \hat{S}_y = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\i & 0\end{pmatrix}, \hat{S}_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

A simplified version of these are the Pauli matrices:

$$\sigma_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}, \sigma_y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix}, \sigma_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

So that: $\hat{\bar{S}} = \frac{\hbar}{2}\sigma$

- Spin 1/2 particles transform under rotation according to the 2 dimensional representation of the SU(2) group.
- Spin 1 particles transform under rotation according to the 3 dimensional representation of the SU(2) group.
- Spin 3/2 particles transform under rotation according to the 4 dimensional representation of the SU(2) group.
- SU(2) behaves very similar to the SO(3) group (rotation in 3D)

1.3 Flavour symmetries

The n^0 and the p^+ are very similar particles, and the strong force works the same way for them, which lead Heisenberg to develop the **isospin** quantum number, which is analogous in behaviour to the spin, but it is unrelated to the spin.

We can represent particles the following way:

$$N = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Analogous to $\bar{S}$ we can introduce isospin $\bar{I}$
- $\bar{I}$ is a vector in abstract isospin space and not on regular 3D space
- $\bar{I}$ has 3 components: I_1, I_2, I_3
- **The strong interactions are invariant under rotation in isospin space** $\rightarrow$ this is an **internal symmetry,** because it is unrelated to space and time
- A rotation in isospin space about axis 1 converts a proton into a neutron and vice versa.
- According to the Noether theorem **isospin is conserved in all strong interactions**
- The strong interactions are invariant under an internal symmetry group SU(2) and the nucleons belong to the 2D representation of the group
- The isospin can explain why we can **only have** $p + n$ **as a stable bound state in the deuteron** and that no $n + n$ or $p + p$ bound states exits
- It also explains the multiplet structure of hadrons (book chapter 1.7)