① Summary - Decays

parent radio nucleid → spontaneous decay → daughter nucleid

Types: α decay → α particle gets ejected or cluster decay (heavier particle like ^{12}C) or fission

β decay → β^+, β^-, electron capture } p → n or n → p

γ decay: nuclear energy transition → γ or e⁻ gets ejected
usually paired with α or β decay

decays follow an exponential law:

$$\lambda = -\frac{dN/dt}{N}$$

N nuclei at t time
dN decay in dt time
λ desintegration or decay constant

$$N(t) = N_0 e^{-\lambda t}$$

N_0 number of nuclei at $t=0$

half life $t_{1/2} = \frac{0.693}{\lambda}$ ⇒ the time it takes for a quantity of material to reduce to half

α decay: → Coulomb repulsion effect, becomes important at heavy nuclei
↳ Z^2 strong force ~ A

→ energetically the α particle is the „easiest" to eject, largest energy release for light particles

$\alpha = {}^4He$

$${}^{A}_{Z}X_N \to {}^{A-4}_{Z-2}X'_{N-2} + \alpha$$

parent daughter

→ the energy ~~de~~ release can be calculated from the mass difference

$$Q = (m_X - m_{X'} - m_\alpha)c^2 = \underbrace{T_{X'} + T_\alpha}_{E\text{ gets carried away as kinetic }E}$$

T_α → ~ 98% of Q

Q > 0 for decays
λ must be relatively large

$T_{X'}$ ~ 2% of energy = recoile (few cm)

2) $\Rightarrow$ from Q we can measure atomic masses for X'

large $Q \rightarrow$ short half life

- even Z and even $N \rightarrow$ shorter half life
- ~~evens~~ odd-even and odd-odd pairs have ~~have~~ relatively longer $t_{1/2}$
- adding $n \rightarrow$ increases the $t_{1/2}$

Theory: α-particle inside the parent nucleus $\rightarrow$ tunneling $\rightarrow$ decay
- potential well + Coulomb potential

desintegration probability: $\lambda = f P$ f - frequency of presentation at the boundary

P - probability of transit

simplified model is a good representation however some discrepancies with observations

reasons for the discrepancy:
- wave function
- momentum of α
- shape of the nucleus is not symmetric
  - $\rightarrow$ radius is changing

Calculating half lifes $\rightarrow$ shows which decays are less likely to happen e.g. α emission more likely than ^{12}C emission

(proton decay: energetically forbidden, but can happen very rarely)

Angular mom. and parity

^4He spin = 0 $\rightarrow$ only ℓ parity change: $(-1)^{\ell_\alpha}$
- parity selection rule $\rightarrow$ which transitions can occure

if initial & final the same: ℓ_α even

—11— different: ℓ_α odd

α decay: a given initial state can decay into a variety of states in the daughter $\rightarrow$ „fine structure" of α decay

decays to different E levels: (1) centrifugal force (from rot) raises the barrier

(2) excitation E lowers the barrier

relative decay probability: 0^+ is the highest $2^+, 4^+, 6^+, 8^+$ less likely

3 → due to the wave func. being more different
↳ the particle must go from 0^+ to rot or vibrational state
parity: $0 \to 3$ ℓ_α odd $0^+ \to 3^-$ possible
$0^+ \to 3^+$ not possible

if the initial and final spin are not 0 more combination
e.g. $2^+ \to 2^-$ if $\ell_\alpha = 1$ or $\ell_\alpha = 3$
↳ measure from angular destribution of decay
↓
also indicate the shape of the nucleus
3-4 times more emission from the elongated side

→ Applications: - power source
- cancer treatment
- smoke detectors

α-decay spectroscopy → map excited states

β-decay:
β^-: $n^0 \to p^+ + e^- + \bar{\nu}_e$
β^+: $p^+ \to n^0 + e^+ + \nu_e$
e^- capture: $p^+ + e^- \to n + \nu_e$
} only bound p^+

→ A stays the same, Z and N change
↳ unstable nuclei decay untill they reach a stable state

→ β-decay needs to create new particles

Energy release: - energy spectrum of e^- is continous! → not a 2 body decay
↓
neutral ← ν
spin = $\frac{1}{2}$
- α spectrum is discrete

β-decay of free n (half life of ~10 min)

$$n \to p^+ + e^- + \bar{\nu}$$

$$Q = (m_n - m_p - m_e - m_\nu) c^2$$

(4) $Q = \underset{\text{small}}{T_p} + \underbrace{T_e + T_\nu}$

small $\quad$ ↳ share the energy → continous spectrum

from Q and the measured masses we know that ν is very light

small mass of e^- and ν → relativistic motion

<u>β^- decay in nucleus:</u> $\quad {}^A_Z X_N \rightarrow {}^A_{Z+1} X'_{N-1} + e^- + \bar{\nu}$

$$\left(Q_{\beta^-} = \left[m_N\left({}^A_Z X\right) - m_N\left({}^A_{Z+1} X'\right) - m_e \right] c^2 \right)$$

↳ nuclear mass (no e^-)

$$Q_{\beta^-} = \left[m\left({}^A X\right) - m\left({}^A X'\right) \right] c^2$$ → mass measurements

$$Q_\beta = T_e + E_{\bar{\nu}}$$

<u>β^+ decay:</u> $\quad {}^A_Z X_N \rightarrow {}^A_{Z-1} X'_{N+1} + \beta^+ + \nu$

$$Q_{\beta^+} = \left[m\left({}^A X\right) - m\left({}^A X'\right) - 2m_e \right] c^2$$

<u>e^- capture:</u> $\quad {}^A_Z X_N + e^- \rightarrow \underbrace{{}^A_{Z-1} X'_{N+1}}_{\text{excited state}} + \nu$

→ if an inner e- gets captured → γ emission from another e- filling the hole → energy of γ → binding E of e^-

β^+ and e^- capture lead to the same final nucleus but not always are both energetically possible

if β^+ possible → e^- capture possible, but not the other way

<u>Fermi theory of β decay</u> - must create the new particles

- e^- and ν are relativistic
- must explain continous e^- spectrum

→ interpretation as a weak interaction

→ the probability of transitions depends on the density of final states

(5) complete β-decay depends on 3 factors

(1) $p^2(Q-T_e)^2 \to$ availability of final states

(2) Fermi function accounting for the nuclear Coulomb field $F(Z',p)$

(2.) The nuclear matrix element $|M_{fi}|^2 \to$ account for particular initial and final states and the additional momentum from forbidden terms $S(p,q)$ → allowed decay

→ forbidden decay

$$N(p) \propto p^2(Q-T_e)^2 F(Z',p)|M_{fi}|^2 S(p,q)$$

Experimental test: Kurie plot

allowed decay $(Q-T_e) \propto \sqrt{\dfrac{N(p)}{p^2 F(Z',p)}}$ → line againt T_e

forbidden $(Q-T_e) \propto \sqrt{\dfrac{N(p)}{p^2 F(Z',p) S(p,q)}}$

→ large range of β-decay half-life:

- very short half-life → superallowed decay

β decay interaction → weak interaction

Full theory: Fermi model + particle exchange (W, Z bosons)

<u>Neutrino mass</u>: from the shape of the e^- energy spectrum → must be non 0

<u>Angular mom and parity rules</u>:

(1) no orbital ang mom $l=0$

$s_{e^-} = \frac{1}{2}$ $s_{\bar{\nu}} = \frac{1}{2}$ → parallel or antiparallel spin

parallel → $S=1$ Gamow-Teller decay

antiparallel → $S=0$ Fermi-decays

Allowed decay: $\Delta I = 0, 1$ $\Delta\pi = $ no

$0^+ \to 0^+$ Fermi

$0^+ \to 1^+$ G-T

6) first forbidden decay: l is not 0

$l=1$ first forbidden decay $\Delta I = 0,1,2$ $\Delta\pi$ = yes
$l=2$ second forbidden decay $\Delta I = 2,3,$ $\Delta\pi$ = no
$l=3$ third – " – $\Delta I = 3,4$ $\Delta\pi$ = yes
$l=4$ forth – " – $\Delta I = 4,5$ $\Delta\pi$ = no

Inverse β decay: capture of ν or $\bar{\nu}$

$\bar{\nu} + p \rightarrow n + e^+$
$\nu + n \rightarrow p + e^-$

double β decay: $^{48}Ca \rightarrow {}^{48}Ti + 2e^- + 2\bar{\nu}$

simultaneously 2 β-decays

if $\nu = \bar{\nu}$ ⇒ neutrinoless ββ decay

→ very difficult to detect → experiments in mines
→ very long half lifes

β-delayed nucleon emission

precursor → β decay into excited state → nucleon emission (p^+ or n)
↳ needs to be a high excitation state

E of the excited state needs to cover the = separation E + E of emitted particle + small recoil E

→ map excited states: Energy + population

Applications:
- β light
- monitoring thickness
- cancer treatment
- medical imaging (PET)
- radio carbon dating

7) Gamma decay

- most α and β decays are followed by γ decay
-> energy transition of the nucleus trough γ emission
-> very high E 0.1 - 10 MeV -> γ wavelength

Energetics : $E_i = E_f + E_\gamma + T_R$ recoil
initial final

$$\Delta E = E_i - E_f$$

$$\left(\Delta E = E_\gamma + \frac{E_\gamma^2}{2Mc^2}\right) \quad \Delta E \sim E_\gamma$$

actually a bit smaller than ΔE

low E γ rays -> small recoil
high E γ rays -> large recoil -> radiation damage

Angular mom. & parity

orbital ang mom -> L determines the moment : dipole, quadrupole
parity determins electric or magnetic : ΔΠ = no -> even -> electric
ΔΠ = yes -> odd -> magnetic

if L = 0 -> monopole -> no γ emission -> internal conversion
L = 1 -> dipole
L = 2 -> quadrupole

Internal conversion

instead of a γ ray an e^- gets the ΔE and gets ejected
↳ followed by γ emission from e^- filling the hole

$$T_e = \Delta E - B$$

kinetic E, binding E
↳ discrete energies depending on the orbit of the e^- (K, L, M, N)

-> generally both γ emission and internal conversion can happen
Total decay probability : $\lambda_t = \lambda_\gamma + \lambda_e$

(8) internal conversion coefficient $\alpha = \frac{\lambda_e}{\lambda_\gamma}$ γ emission is generally more likely

<u>γ-ray spectroscopy</u>: learn about the excited states

<u>nuclear resonance</u>: the nucleus can also absorb radiation

<u>Applications</u>: - cancer treatment
- scanning containers
- monitoring production in industry