

① Summary - Decays

parent radio nuclide \rightarrow spontaneous decay \rightarrow daughter nuclide

Types: α decay \rightarrow α particle gets ejected or cluster decay (heavier particle hole ^{12}C)
 β decay \rightarrow β^+ } $p \rightarrow n$
 β^- } or $n \rightarrow p$
 electron capture

γ decay: nuclear energy transition \rightarrow γ or e^- gets ejected
 usually paired with α or β decay

decays follow an exponential law:

$$\lambda = - \frac{dN/dt}{N}$$

N nuclei at t time
 dN decay in dt time

λ disintegration or decay constant

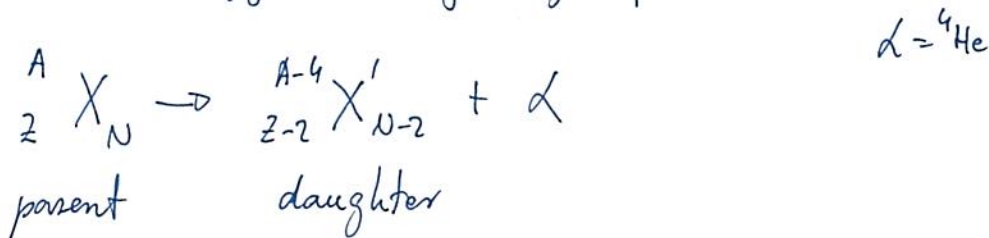
$$N(t) = N_0 e^{-\lambda t}$$

N_0 number of nuclei at $t=0$

half life $t_{1/2} = \frac{0.693}{\lambda} \Rightarrow$ the time it takes for a quantity of material to reduce to half

α decay: \rightarrow Coulomb repulsion effect, becomes important at heavy nuclei $\rightarrow Z^2$ strong force $\sim A$

\rightarrow energetically the α particle is the "easiest" to eject, largest energy release for light particles



\rightarrow the energy release can be calculated from the mass difference $\sim 98\%$ of Q

$$Q = (m_X - m_{X'} - m_\alpha) c^2 = \underbrace{T_{X'} + T_\alpha}$$

$Q > 0$ for decays

λ must be relatively large

E gets carried away as kinetic E

$T_{X'} \sim 2\%$ of energy = recoil (few cm)

2) \Rightarrow from Q we can measure atomic masses for X'

large $Q \rightarrow$ short half life

- even Z and even $N \rightarrow$ shorter half life
- ~~even~~ odd-even and odd-odd pairs have ~~less~~ relatively longer $t_{1/2}$
- adding $n \rightarrow$ increases the $t_{1/2}$

Theory: α -particle inside the parent nucleus \rightarrow tunneling \rightarrow decay

- potential well + Coulomb potential

disintegration probability: $\lambda = f P$ f - frequency of presentation at the boundary

P - probability of transit

simplified model is a good representation however some discrepancies with observations

reasons for the discrepancy {

- wave function
- momentum of α
- shape of the nucleus is not symmetric
 \hookrightarrow radius is changing

Calculating half lives \rightarrow shows which decays are less likely to happen
e.g. α emission more likely than ^{12}C emission

(proton decay: energetically forbidden, but can happen very rarely)

Angular mom. and parity

^4He spin = 0 \rightarrow only l

parity change: $(-1)^{l_{\alpha}}$

- parity selection rule \rightarrow which transitions can occur

if initial & final the same: l_{α} even

- " - different: l_{α} odd

α decay: a given initial state can decay into a variety of states in the daughter \rightarrow "fine structure" of α decay

decays to different E levels: ① centrifugal force (from rot) raises the barrier

② excitation E lowers the barrier

relative decay probability: 0^+ is the highest $2^+, 4^+, 6^+, 8^+$ less likely

③ \rightarrow due to the wave func. being more different
 \rightarrow the particle must go from 0^+ to rot or vibrational state
 parity: $0 \rightarrow 3$ l_x odd $0^+ \rightarrow 3^-$ possible
 $0^+ \rightarrow 3^+$ not possible

if the initial and final spin are not 0 more combination

e.g. $2^+ \rightarrow 2^-$ if $l_x = 1$ or $l_x = 3$

\rightarrow measure from angular distribution of decay

\downarrow
 also indicate the shape of the nucleus
 3-4 times more emission from the elongated side

\rightarrow Applications : - power source
 - cancer treatment
 - smoke detectors

α -decay spectroscopy \rightarrow map excited states

β -decay : β^- : $n^0 \rightarrow p^+ + e^- + \bar{\nu}_e$
 β^+ : $p^+ \rightarrow n^0 + e^+ + \nu_e$ } only bound p^+
 e^- capture : $p^+ + e^- \rightarrow n + \nu_e$

\rightarrow A stays the same, Z and N change

\rightarrow unstable nuclei decay until they reach a stable state

\rightarrow β -decay needs to create new particles

Energy release : - energy spectrum of e^- is continuous \rightarrow not a 2 body decay
 - α spectrum is discrete

neutral \downarrow
 \uparrow
 spin = $\frac{1}{2}$

β -decay of free n (half life of ~ 10 min)

$$n \rightarrow p^+ + e^- + \bar{\nu}$$

$$Q = (m_n - m_p - m_e - m_{\bar{\nu}}) c^2$$

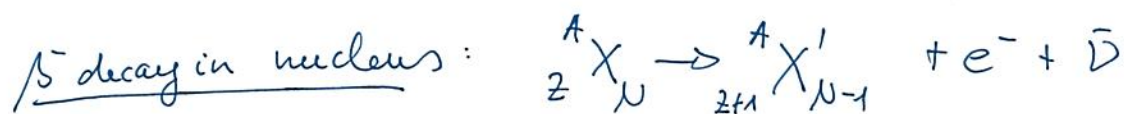
④

$$Q = T_p + T_e + T_\nu$$

↑
small

↳ share the energy → continuous spectrum

from Q and the measured masses we know that ν is very light
 small mass of e^- and ν → relativistic motion

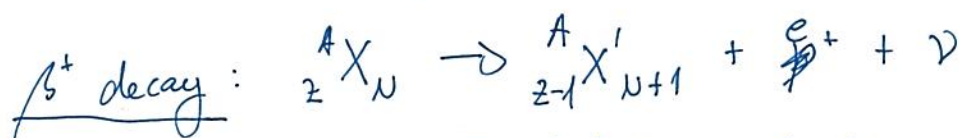


$$\left(Q_{\beta^-} = [m_N({}_Z^AX) - m_N({}_{Z+1}^AX') - m_e] c^2 \right)$$

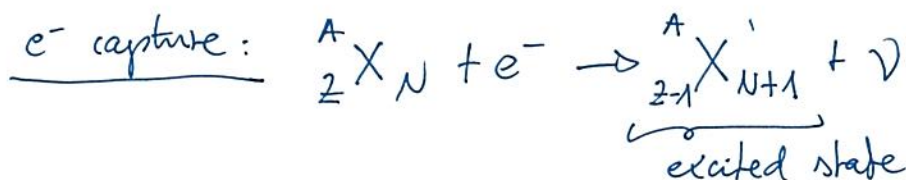
↳ nuclear mass (no e^-)

$$Q_{\beta^-} = [m({}_Z^AX) - m({}_{Z+1}^AX')] c^2 \rightarrow \text{mass measurements}$$

$$Q_{\beta^-} = T_e + E_{\bar{\nu}}$$



$$Q_{\beta^+} = [m({}_Z^AX) - m({}_{Z-1}^AX') - 2m_e] c^2$$



→ if an inner e^- gets captured → γ emission from another e^- filling the hole → energy of γ → binding E of e^-

β^+ and e^- capture lead to the same final nucleus but not always
 are both energetically possible

if β^+ possible → e^- capture possible, but not the other way

Fermi theory of β decay - must create the new particles
 - e^- and ν are relativistic
 - must explain continuous e^- spectrum

→ interpretation as a weak interaction

→ the probability of transitions depends on the density of final states

⑤ complete β -decay depends on 3 factors

- ① $p^2(Q - T_e)^2 \rightarrow$ availability of final states
- ② Fermi function accounting for the nuclear Coulomb field $F(Z', p)$
- ② The nuclear matrix element $|M_{fi}|^2 \rightarrow$ account for particular initial and final states and the additional momentum from forbidden terms $S(p, q) \rightarrow$ allowed decay
 \rightarrow forbidden decay

$$N(p) \propto p^2(Q - T_e)^2 F(Z', p) |M_{fi}|^2 S(p, q)$$

Experimental test: Kurie plot

allowed decay $(Q - T_e) \propto \sqrt{\frac{N(p)}{p^2 F(Z', p)}} \rightarrow$ line against T_e

forbidden $(Q - T_e) \propto \sqrt{\frac{N(p)}{p^2 F(Z', p) S(p, q)}}$

\rightarrow large range of β -decay half-life:

- very short half-life \rightarrow superallowed decay

β decay interaction \rightarrow weak interaction

Full theory: Fermi model + particle exchange (W, Z bosons)

Neutrino mass: from the shape of the e^- energy spectrum \rightarrow must be non 0

Angular mom and parity rules:

① no orbital ang mom $l = 0$

$$s_{e^-} = \frac{1}{2} \quad s_{\bar{\nu}} = \frac{1}{2}$$

\Rightarrow parallel or antiparallel spin

$$\downarrow$$

$$S = 1$$

Gamow-Teller decay

$$\downarrow$$

$$S = 0$$

Fermi-decay

Allowed decay: $\Delta l = 0, 1 \quad \Delta \pi = \text{no}$

$0^+ \rightarrow 0^+$ Fermi

$0^+ \rightarrow 1^+$ GT

⑥ first forbidden decay: l is not 0

$l=1$	first forbidden decay	$\Delta l = 0, 1, 2$	$\Delta \pi = \text{yes}$
$l=2$	second forbidden decay	$\Delta l = 2, 3,$	$\Delta \pi = \text{no}$
$l=3$	third - " -	$\Delta l = 3, 4$	$\Delta \pi = \text{yes}$
$l=4$	forth - " -	$\Delta l = 4, 5$	$\Delta \pi = \text{no}$

Inverse β decay: capture of γ or $\bar{\nu}$



double β decay: $^{48}\text{Ca} \rightarrow ^{48}\text{Ti} + 2e^- + 2\bar{\nu}$

simultaneously 2 β -decays \rightarrow

if $\nu = \bar{\nu} \Rightarrow$ neutrinoless $\beta\beta$ decay

\rightarrow very difficult to detect \rightarrow experiments in mines

\rightarrow very long half lives

β -delayed nucleon emission

precursor $\rightarrow \beta$ decay into excited state \rightarrow nucleon emission (p or n)
 \hookrightarrow needs to be a high excitation state

E of the excited state needs to cover the separation $E + E$ of emitted particle
+ small recoil E

\rightarrow map excited states: Energy + population

Applications:

- β light
- monitoring thickness
- cancer treatment
- medical imaging (PET)
- radio carbon dating

7) Gamma decay

- most α and β decays are followed by γ decay
- energy transition of the nucleus through γ emission
- very high E 0.1 - 10 MeV \rightarrow γ wavelength

Energetics: $E_i = E_f + E_\gamma + T_{\text{recoil}}$
initial final

$$\Delta E = E_i - E_f$$

$$\left(\Delta E = E_\gamma + \frac{E_\gamma^2}{2Mc^2} \right) \quad \Delta E \sim E_\gamma$$

\nearrow actually a bit smaller than ΔE

low E γ rays \rightarrow small recoil
high E γ rays \rightarrow large recoil \rightarrow radiation damage

Angular mom. & parity

orbital ang mom $\rightarrow L$ determines the moment: dipole, quadrupole
parity determines electric or magnetic: $\Delta\pi = \text{no} \rightarrow \text{even} \rightarrow \text{electric}$
 $\Delta\pi = \text{yes} \rightarrow \text{odd} \rightarrow \text{magnetic}$

if $L=0 \rightarrow$ monopole \rightarrow no γ emission \rightarrow internal conversion
 $L=1 \rightarrow$ dipole
 $L=2 \rightarrow$ quadrupole

Internal conversion

instead of a γ ray an e^- gets the ΔE and gets ejected
 \rightarrow followed by γ emission from e^- filling the hole

kinetic $E \rightarrow T_e = \Delta E - B_{\text{binding } E}$

\rightarrow discrete energies depending on the orbit of the e^- (K, L, M, N)

\rightarrow generally both γ emission and internal conversion can happen

Total decay probability: $\lambda_t = \lambda_\gamma + \lambda_e$

⑧ internal conversion coefficient $\alpha = \frac{A_e}{A_\gamma}$ γ emission is generally more likely

γ -ray spectroscopy: learn about the excited states

nuclear resonance: the nucleus can also absorb radiation

Applications:
- cancer treatment
- scanning containers
- monitoring production in industry