① Summary - Decays

parent radio nucleid → spontaneous decay → daughter nucleid

Types: α decay → α particle gets ejected or cluster decay (heavier particle here ^{12}C) or fission

β decay → $β^+$
$β^-$
electron capture $\Big\}$ p → n or n → p

γ decay : nuclear energy transition → p or e^- gets ejected usually paired with α or β decay

decays follow an exponential law:

$$\lambda = -\frac{dN/dt}{N}$$

N nuclei at t time
dN decay in dt time
λ desintegration or decay constant

$$N(t) = N_0 e^{-\lambda t}$$

N_0 number of nuclei at $t = 0$

half life $t_{1/2} = \frac{0.693}{\lambda}$

the time it takes for
⇒ a quantity of material to reduce to half

α decay : → Coulomb repulsion effect, becomes important at heavy nuclei $\hookrightarrow Z^2$ strong force ~ A

→ energetically the α particle is the „easiest" to eject, largest energy release for light particles

$α = {}^4He$

$$^A_Z X_N \to {}^{A-4}_{Z-2} X'_{N-2} + α$$

parent   daughter
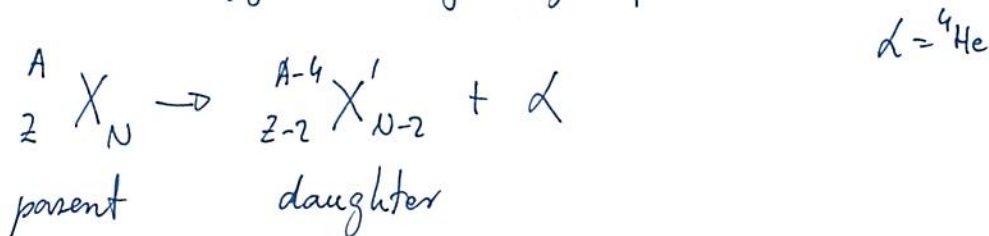
→ the energy release can be calculated from the mass difference

$$Q = (m_X - m_{X'} - m_α)c^2 = \underline{T_{X'} + T_α}$$

→ ~ 98% of Q

Q > 0 for decays
λ must be relatively large

E gets carried away as kinetic E

T_X ~ 2% of energy = recoile (few cm)

2) ⇒ from Q we can measure atomic masses for X'
   large Q → short half life
   - event and even N → shorter half life
   - ~~event~~ odd - even and odd - odd pairs have ~~less~~ relatively longer $t_{1/2}$
   - adding n → increases the $t_{1/2}$

Theory: α -particle inside the parent nucleus → tunneling → decay
         - potential well + Coulomb potential

disintegration probability : $\lambda = f P$ f - frequency of presentation
                                                      at the boundary
                                                  P - probability of transit

simplified model is a good representation however some
discrepancies with observations
reasons for the ⎰ - wave function
discrepancy   ⎱ - momentum of α
                  - shape of the nucleus is not symmetric
                       ↳ radius is changing

Calculating half lifes → shows which decays are less likely to happen
   e.g. α emission more likely than ^{12}C emission

(proton decay : energetically forbidden , but can happen very rarely)

Angular mom. and parity   parity change : $(-1)^{l_\alpha}$
   ^4He spin = 0 → only l
   - parity selection rule → which transitions can occure
              if initial & final the same : l_α even
              - || -   different : l_α odd

α decay: a given initial state can decay into a variety of states in the
         daughter → "fine structure" of α decay
   decays to different E levels : ① centrifugal force (from rot) raises the barrier
                                  ① excitation E lowers the barrier
   relative decay probability : 0^+ is the highest $2^+, 4^+, 6^+, 8^+$ less likely

③ → due to the wave func. being more different
     ↳ the particle must go from 0^+ to rot or vibrational state
     parity: $0 → 3$ ℓ_α odd $0^+ → 3^-$ possible
                                    $0^+ → 3^+$ not possible

if the initial and final spin are not 0 more combination
   e.g. $2^+ → 2^-$ if $\ell_\alpha = 1$ or $\ell_\alpha = 3$
                     ↳ measure from angular distribution of decay
                                          ↓
                           also indicate the shape of the nucleus
                           3-4 times more emission from the
                           elongated side

→ <u>Applications</u> :- power source
                 - cancer treatment
                 - smoke detectors

α-decay spectroscopy → map excited states   ∘ ——

<u>β-decay</u> : β^- : $n^0 → p^+ + e^- + \bar{\nu}_e$
            β^+ : $p^+ → n^0 + e^+ + \nu_e$ } only bound p^+
        e^- capture : $p^+ + e^- → n + \nu_e$

→ A stays the same ,   Z and N change
   ↳ unstable nuclei decay untill they reach a stable state

→ β-decay needs to create new particles

<u>Energy release</u> :- energy spectrum of e^- is continous! → not a 2 body decay
             - α spectrum is discrete
                                        ↓
                                    neutral $\leftarrow \nu$
                                    spin $= \frac{1}{2}$

β-decay of free n (half life of ~10 min)
           $n → p^+ + e^- + \bar{\nu}$
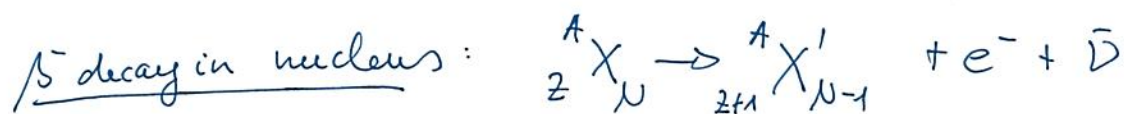 $Q = (m_n - m_p - m_e - m_\nu) c^2$

④ $Q = T_p + T_e + T_\nu$

small $\hookrightarrow$ share the energy $\to$ continous spectrum

from Q and the measured masses we know that ν is very light

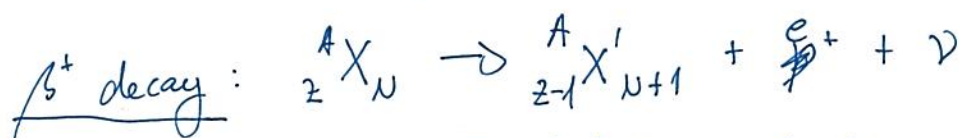small mass of e^- and $\nu \to$ relativistic motion

β^- decay in nucleus: $^A_Z X_N \to\, ^A_{Z+1} X'_{N-1} + e^- + \bar{\nu}$

$$\left(Q_{\beta^-} = \left[m_N \left(^A_Z X \right) - m_N \left(^A_{Z+1} X' \right) + m_e \right] c^2 \right.$$
$$\left. \qquad\qquad\qquad \hookrightarrow \text{nuclear mass (no } e^-\text{)} \right)$$

$$Q_{\beta^-} = \left[m \left(^A X \right) - m \left(^A X' \right) \right] c^2 \quad \to \text{mass measurements}$$

$$Q_\beta = T_e + E_{\bar{\nu}}$$

β^+ decay: $^A_Z X_N \to\, ^A_{Z-1} X'_{N+1} + \beta^+ + \nu$

$$Q_{\beta^+} = \left[m \left(^A X \right) - m \left(^A X' \right) - 2 m_e \right] c^2$$

e^- capture: $^A_Z X_N + e^- \to\, \underbrace{^A_{Z-1} X'_{N+1}}_{\text{excited state}} + \nu$
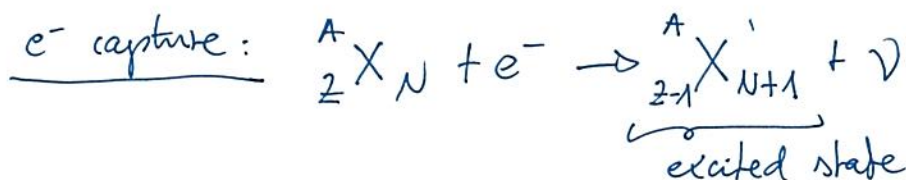
$\to$ if an inner e^- gets captured $\to \gamma$ emission from another
e^- filling the hole $\to$ energy of $\gamma \to$ binding E of e^-

β^+ and e^- capture lead to the same final nucleus but not always
are both energetically possible
    if β^+ possible $\to e^-$ capture possible, but not the other way

Fermi theory of β decay   – must create the new particles
                            – e^- and ν are relativistic
                            – must explain continuous e^- spectrum

$\to$ interpretation as a weak interaction
※ the probability of transitions depends on the density of
                                             final states

⑤ complete β-decay depends on 3 factors

① $p^2(Q-T_e)^2$ → availability of final states

② Fermi function accounting for the nuclear Coulomb field $F(z',p)$

②. The nuclear matrix element $|M_{fi}|^2$ → account for particular initial and final states and the additional momentum from forbidden terms $S(p,q)$
  └→ allowed decay

  → forbidden decay

$$N(p) \propto p^2(Q-T_e)^2 F(z',p)|M_{fi}|^2 S(p,q)$$

Experimental test: Kurie plot

allowed decay $(Q-T_e) \propto \sqrt{\dfrac{N(p)}{p^2 F(z',p)}}$ ⟹ line against T_e

forbidden $(Q-T_e) \propto \sqrt{\dfrac{N(p)}{p^2 F(z',p) S(p,q)}}$

→ large range of β-decay half-life:
  — very short half-life → superallowed decay

β decay interaction → weak interaction
  Full theory: Fermi model + particle exchange $(W, Z$ bosons$)$

Neutrino mass: from the shape of the e^- energy spectrum → must be non 0

Angular mom and parity rules:
  ① no orbital ang mom $l = 0$
    $s_{e^-} = \frac{1}{2}$ $s_{\bar\nu} = \frac{1}{2}$ ⟹ parallel or antiparallel spin

      $S=1$ (↓) $S=0$ (↓) Fermi-decay
      Gamow-Teller decay

Allowed decay: $\Delta l = 0,1$ $\Delta \pi = no$
    $0^+ \to 0^+$ Fermi
    $0^+ \to 1^+$ G-T

6) first forbidden decay: ℓ is not 0

$\ell = 1$ first forbidden decay $\quad \Delta I = 0,1,2 \quad \Delta \pi = $ yes
$\ell = 2$ second forbidden decay $\quad \Delta I = 2,3, \quad \Delta \pi = $ no
$\ell = 3$ third $\quad - \text{ '' } - \quad \Delta I = 3,4 \quad \Delta \pi = $ yes
$\ell = 4$ forth $\quad - \text{ '' } - \quad \Delta I = 4,5 \quad \Delta \pi = $ no

Inverse β decay : capture of ν or $\bar{\nu}$

$$\bar{\nu} + p \rightarrow n + e^+$$
$$\nu + n \rightarrow p + e^-$$

double β decay :

$$^{48}Ca \rightarrow {}^{48}Ti + 2e^- + 2\bar{\nu}$$

simultaneously 2 β-decays ⬆

if $\nu = \bar{\nu} \Rightarrow$ neutrinoles $\beta\beta$ decay

→ very difficult to detect → experiments in mines
→ very long half lifes

β - delayed nucleon emission

precursor $\rightarrow \beta$ decay into excited state → nucleon emission (p^+ or n)
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ ↳ needs to be a high excitation state

E of the excited state needs to cover the = separation E + E of emitted particle
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ + small recoil E

→ map excited states: Energy + population

Applications :

- β light
- monitoring thickness
- cancer treatment
- medical imaging (PET)
- radio carbon dating