Sylow:

$$|G| = p^{\alpha} m, p \not| m$$

we have $\varnothing \neq \mathrm{Syl}_p(G) \ni P$

$$0 < n_p = \left|\mathrm{Syl}_p(G)\right| |m$$

$$n_p = 1 + kp = [G : N_G(P)]$$

Now consider: $|G| = p^2 q$, p, q are distinct primes. Looking for nomal sylow subgroups. $P \in \mathrm{Syl}_P, Q \in \mathrm{Syl}_q$

- $$p < q \implies \begin{cases} n_q = 1 \implies Q \triangleleft G \\ n_q > 1 \implies n_q = 1 + tq | p^2 \implies 1 < 1 + tq | p^2 \end{cases}$$

From the second

$$\implies \begin{cases} 1 + tq = p \text{ not posible} \\ 1 + tq = p^2 \implies q | p^2 - 1 = (p-1)(p+1) \implies \begin{cases} q | p - 1 \implies \text{ not posible} \\ q | p + 1 \implies q = p + 1 \implies p = 2, q = 3 \to |G| = 12 \end{cases} \end{cases}$$

Now consider $|G| = 12$ Either G has a normal sylow 3-subgroup or $G \cong A_4$.

$$1 < n_3 = 1 + 3k | 4 \implies n_3 = 4$$

so that:

$$\mathrm{Syl}_3(G) = \{P = P_1, P_2, P_3, P_4\}$$

$$P_i \cap P_j = \{1\} \text{ if } i \neq j$$

G has 8 elements of order 3.

$$[G : N_G(P)] = n_3 = 4 \implies N_G(P) = P$$

$$= [G : P]$$

Now act by conjugation:

$$\phi : G \xrightarrow{Conjugation} S_4$$

we can show that the above map is an injection. (Prove it)

$$K = \mathrm{Ker}\, \phi \leq N_G(P) = P$$

P is not normal Conjugation on

$$\mathrm{Syl}_3(G) \implies K = 1$$

Now by 1st isomorphism theorem:

$$G \cong \phi(G) \leq S_4 \implies \phi(G) = A_4$$

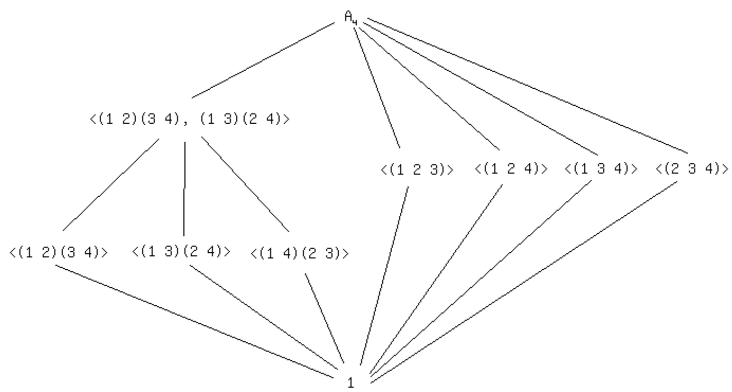

Figure 1:

We identify: Sylow 2-subgroup $< (12)(34), (13)(24) >$, and $< (123) >< (124) >< (134) >$

We observer that normal Sylow 2-subgroup A_4 has 8 elements of order 3-Complement

About the exam:

- Class equation: very likely
- Semi-direct Product (Favorite)
- Automorphisms of D_8
- Inductive argument ascending and descending chain (pag. 195)
- Lot of Sylow Stuff
- Iso for Rings
- Read the examples for Ring sections.
- One very difficult question