

Textbook notes on Topology

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The following are notes based on the book *Topology* by Munkres.

Chapter 1

Set Theory and Logic

1.1 Fundamental Concepts

We express that an object a belongs to a set A by the notation:

$$a \in A$$

Similarly,

$$a \notin A$$

We denote the inclusion of a set into another set with:

$$A \subseteq B$$

so that $A = B \iff A \subseteq B$ and $B \subseteq A$. If $A \subseteq B$ but A is different from B , we say A is a **proper subset** of B , in notation:

$$A \subsetneq B$$

The relation \subseteq is called **inclusion** and \subsetneq is called proper inclusion.

The Union of Sets and the Meaning of "or"

Given two sets A and B , we can form another set called the union of A and B .

$$A \cup B = \{x | x \in A \text{ or } x \in B\}$$

We will use the concept of exclusive or, if the necessity arises.

The Intersection of Sets, the Empty set, and the Meaning of "If...Then"

Another way to form a set from two existing sets is to take the elements in common, that is:

$$A \cap B = \{x | x \in A \text{ and } x \in B\}$$

The **empty set** is the set with no elements, denoted by \emptyset . We say that two elements are disjoint if:

$$A \cap B = \emptyset$$

Some property of this interesting empty set are:

$$A \cap \emptyset = \emptyset \quad A \cup \emptyset = A$$

Chapter 2

Topological Spaces and Continuous Functions

2.1 Topological Spaces

Definition 1. A **topology** on a set X is a collection τ of subsets of X having the following properties:

- \emptyset and X are in τ
- The union of the elements of any subcollection of τ is in τ
- The intersection of the elements of any finite subcollection of τ is in τ .

A set X for which a topology τ has been specified is called a **topological space**

Properly, a topological space is an ordered pair (X, τ) .

If Z is a topological space with topology τ , we say that a subset U of X is an **open set** of X if U belongs to the collection τ .

Example 1. If X is any set, the collection of all subsets of X is a topology on X ; it is called the **discrete topology**. The collection consisting of X and \emptyset only is also a topology on X ; we shall call it the **indiscrete topology**, or the **trivial topology**

Example 2. Let X be a set; let τ_f be the collection of all subsets of U of X such that $X - U$ is either finite or is all of X . Then τ_f is a topology on X , called the **finite complement topology**. Both X and \emptyset are in τ_f , since $X - X$ is finite and $X - \emptyset$ is all of X . If $\{U_\alpha\}$ is an indexed family of nonempty elements of τ_f , to show that $\cup U_\alpha$ is in τ_f , we compute

$$(\cup U_\alpha)^c = \cap U_\alpha^c$$

and since each U^c is finite, the union of these set is finite. If U_1, \dots, U_n are nonempty elements of τ_f , to show that $\cap U_i$ is in τ_f , er compute:

$$\left(\bigcap_{i=1}^n \right)^c U_i = \bigcup_{i=1}^n U_i^c$$