Sparse modeling of risk factors in insurance analytics

Sander Devriendt
Joint work with K. Antonio, T. Reynkens, E. Frees, R. Verbelen

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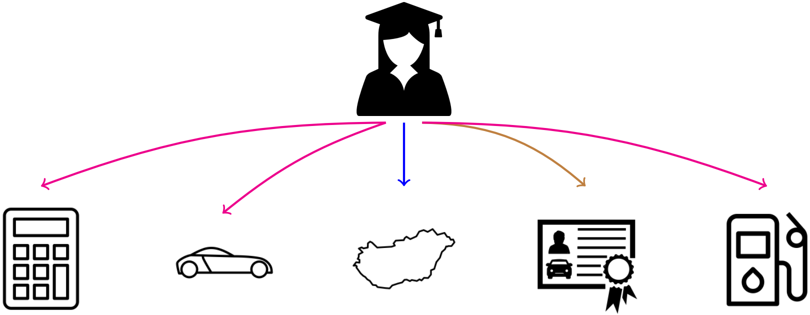
Motivation



Claim frequency and claim severity

as function of

nominal / numeric $\sim$ ordinal / spatial

features

Research questions

- Generalized Linear Models (GLMs) for frequency ($\sim$ Poisson) and severity ($\sim$ Gamma).
- How to:
  - (1) select risk factors or features?
  - (2) cluster (or bin or fuse) levels within a risk factor?

    age groups / postal code clusters / clusters of car models
- Procedure should be data driven, scalable to large (big) data.
- End product is interpretable, within actuarial comfort zone.

Research questions rephrased

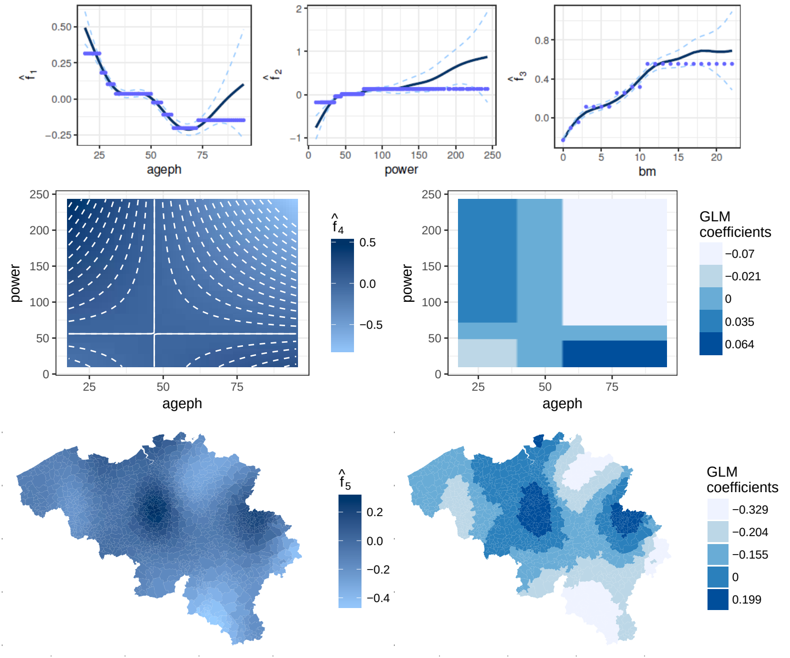
- Generalized Linear Models (GLMs) for frequency ($\sim$ Poisson) and severity ($\sim$ Gamma).
- How to:
  (1) avoid overfitting with too many risk factors or levels?
  (2) avoid underfitting with a priori binning/selection?

Henckaerts, Antonio et al., 2018 (accepted)

Stepwise procedure

1. Do an exhaustive search through variables to find best GAM model.
2. Use well-chosen clustering algorithm to bin 2D spatial effect.
3. Use evolutionary trees to bin 1D continuous effects and interactions.
4. Fit GLM with bins and clusters obtained in previous steps.

`R` packages: `mgcv`, `classInt`, `evtree`, `rpart`



$\hat{f}_1$
0.50
0.25
0.00
−0.25
25
50
75
ageph
$\hat{f}_2$
2
1
0
−1
0
50
100
150
200
250
power
$\hat{f}_3$
0.8
0.4
0.0
0
5
10
15
20
bm
power
250
200
150
100
50
0
25
50
75
ageph
$\hat{f}_4$
0.5
0.0
−0.5
GLM coefficients
−0.07
−0.021
0
0.035
0.064
$\hat{f}_5$
0.2
0.0
−0.2
−0.4
GLM coefficients
−0.329
−0.204
−0.155
0
0.199

Sparse modeling of risk factors in insurance analytics

Devriendt, Antonio, et al., 2018 (in progress)

- Standard GLM:
  - fit data as good as possible,
  - no constraint on parameters.

$$\updownarrow$$

- Regularized GLM:
  - tradeoff between fit and interpretability/sparsity/stability,
  - constraint on parameters.

- Less is more: (Hastie, Tibshirani & Wainwright, 2015)

  a sparse model is easier to estimate and interpret than a dense model.

- Regularize (with budget constraint t, or regularization parameter λ):

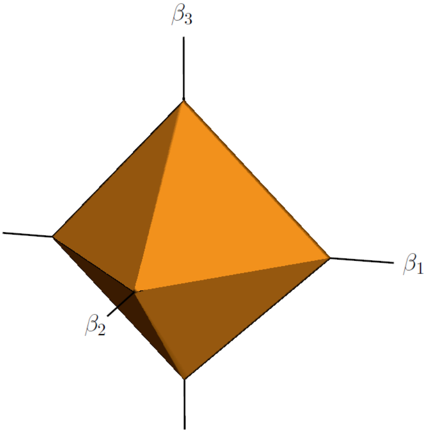
$$\min_{\beta_0,\boldsymbol{\beta}} \{-\mathcal{L}(\beta_0,\boldsymbol{\beta})\} \text{ subject to } \|\boldsymbol{\beta}\|_1 \leq t,$$

  or equivalenty

$$\min_{\beta_0,\boldsymbol{\beta}} \left\{-\mathcal{L}(\beta_0,\boldsymbol{\beta}) + \lambda \cdot \sum_{j=1}^{p} |\beta_j|\right\}.$$

  Shrinks coefficients and even sets some to zero.

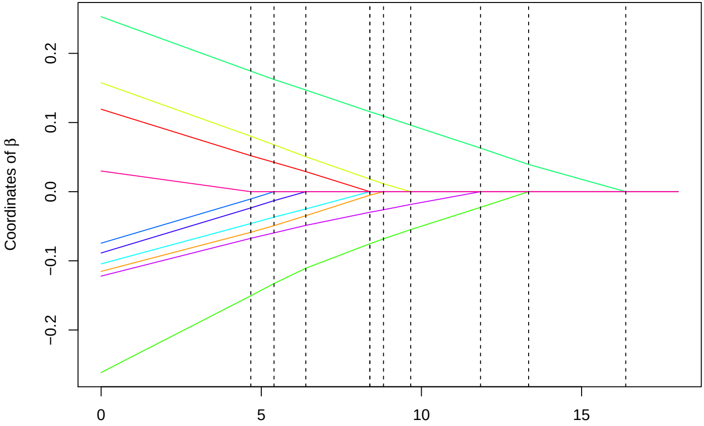
Regularization = limited budget for $\beta_1, \beta_2, \beta_3$.



'Statistical Learning with Sparsity' - Hastie et al. (2015)

Package `glmnet`

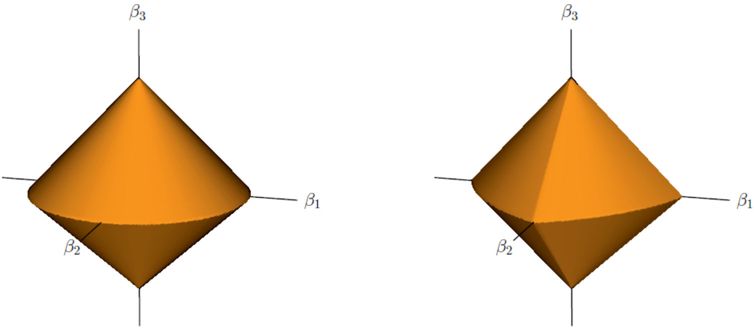
overfitting $\longleftarrow$ λ $\longrightarrow$ underfitting



- Adjust lasso regularization to the type of risk factor:
  - Determine type (nominal / numeric $\sim$ ordinal / spatial);
  - Allocate logical penalty.
- Thus, for J risk factors, each with regularization term $P_j(.)$, we want to optimize:

$$-\mathcal{L}\left(\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_J\right) + \lambda \cdot \sum_{j=1}^{J} P_j\left(\boldsymbol{\beta}_j\right).$$

Different variable type $\rightarrow$ different penalty budget.

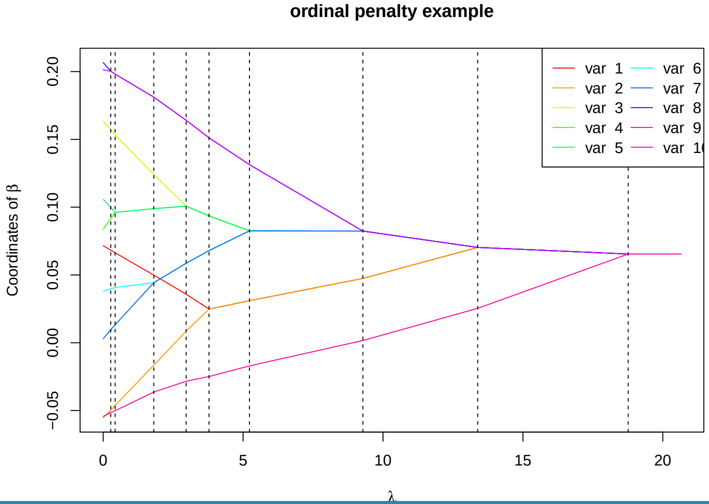


'Statistical Learning with Sparsity' - Hastie et al. (2015)

Fused Lasso

Package `genlasso`

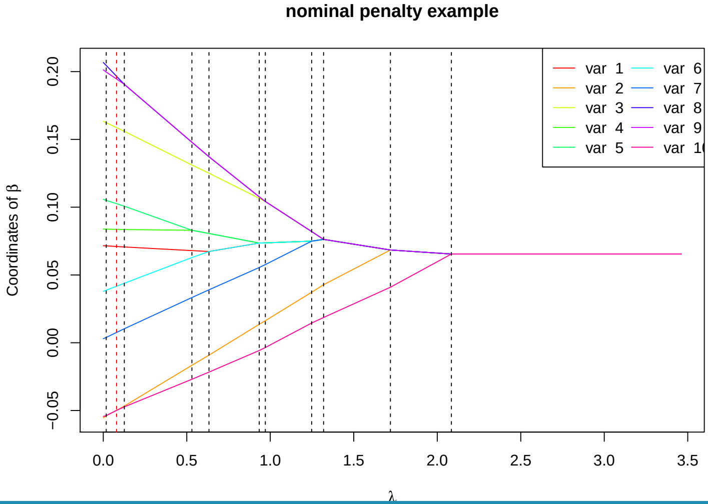
overfitting $\longleftarrow$ λ $\longrightarrow$ underfitting



Generalized Fused Lasso

Package `genlasso`

overfitting $\longleftarrow$ λ $\longrightarrow$ underfitting



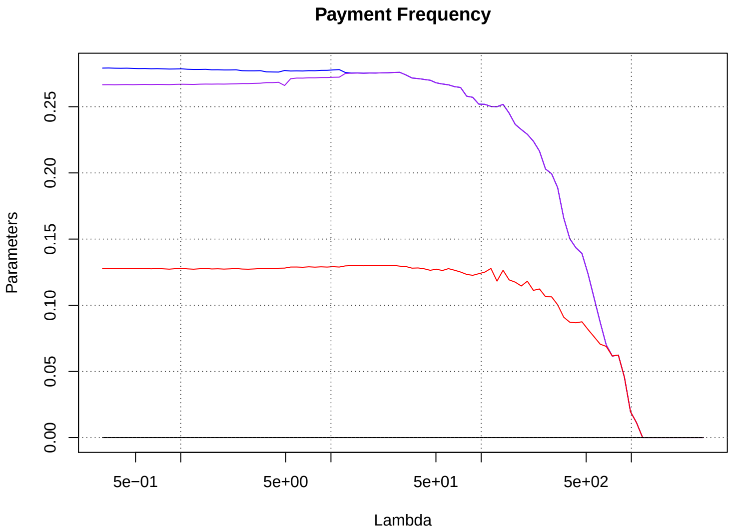
Unified GLM framework with multiple type of penalties

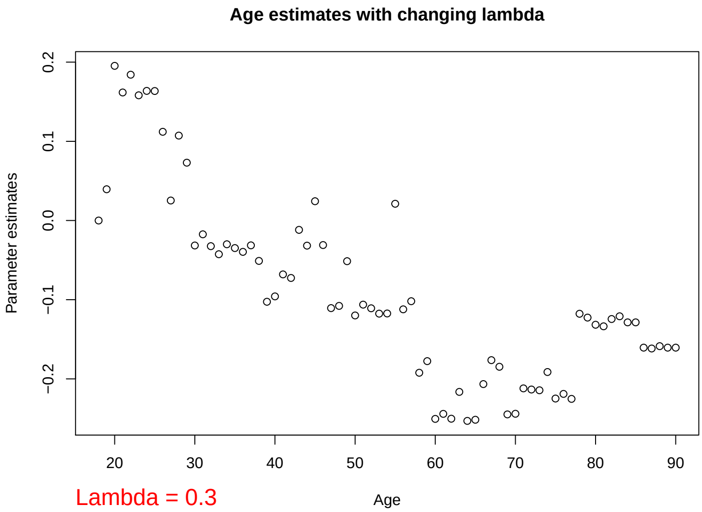
- Gertheiss & Tutz (2010) and Oelker & Gertheiss (2017):
  - GLMs with various penalties.
  - R package available: `gvcm.cat` (not maintained).
- Uses local quadratic approximations of penalties and PIRLS:
  - non-exact selection or fusion;
  - computationally intensive.

- Our contribution:
  - implements an efficient algorithm (with proximal operators);
    - code bottleneck in C++ (`Rcpp`)
    - efficient linear algebra (`RcppArmadillo`)
    - parallel computations (`parallel`)
  - scalable to big data (splits into smaller sub-problems);
  - flexible regularization
    - penalty takes type of risk factor into account;
    - works for all popular penalties;

$\Rightarrow$ Package `mtppga` under construction.

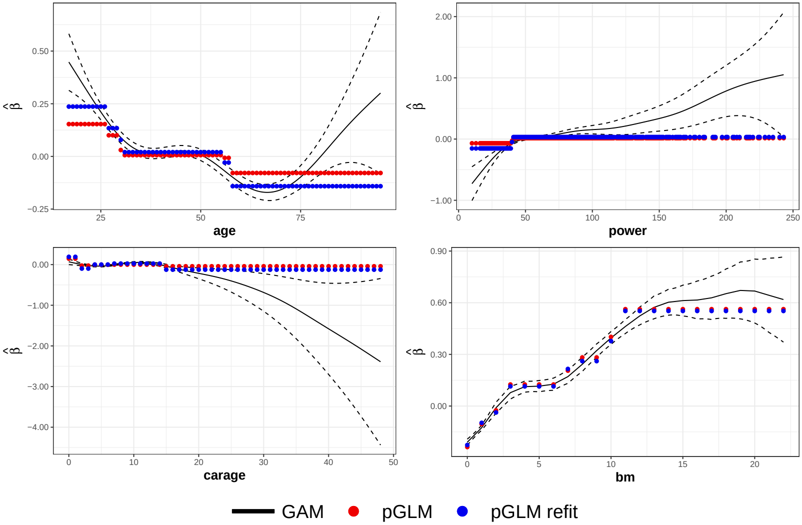
- Frequency (and severity) information for $n = 163,234$ policyholders.
- 15 risk factors: binary, ordinal and nominal.
- Exposure modeled as offset.
- Fit Poisson GLM for frequency data with different penalties.


Payment Frequency
Parameters
0.00
0.05
0.10
0.15
0.20
0.25
5e−01
5e+00
5e+01
5e+02
Lambda

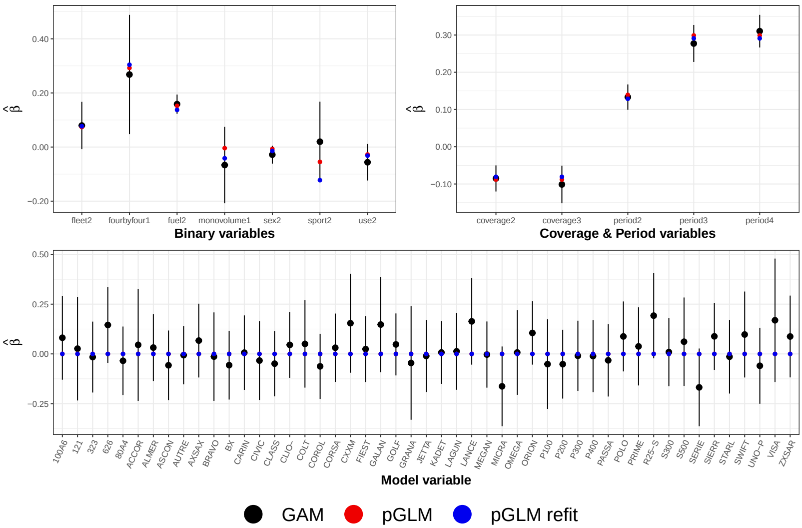

Age estimates with changing lambda
Parameter estimates
0.2
0.1
0.0
−0.1
−0.2
20
30
40
50
60
70
80
90
Lambda = 0.3
Age

- Settings:
  - Incorporate adaptive and standardization weights for better consistency and predictive performance.
  - Tune λ with out-of-sample MSE
- Re-estimate the final sparse GLM with standard GLM routines (from 146 to 30 params.).

MTPL claim frequency with multiple type of penalties



MTPL claim frequency with multiple type of penalties



- less is more.
- Flexible regularization can help predictive modeling
- R package `mtppga` combines general framework with efficient algorithm.
- Package and working paper to be finalized.

Thank you

- Ageas Continental Europe
- Tom Reynkens and colleagues
- You, the public

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