LAB ASSIGNMENT 1: MODELING LINEAR PROGRAMMING TASKS

Heuristics and Optimization

UC3M

Roberto Vaquerizo Icierra

David Rico Menéndez

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1. Description of the models, discussing the decisions carried out.

Model 1: Optimal seat distribution

We are given the following data:

Airplane	Seats	Capacity
AV1	90	1,700 kg.
AV2	120	2,700 kg.
AV3	200	1,300 kg.
AV4	150	1,700 kg.
AV5	190	2,000 kg.

Fair	Allowed baggage	Price
Standard	1 kg.	19€
Leisure plus	20 kg.	49€
Business plus	40 kg.	69€

The following constraints are given

1. It is not allowed to sell more tickets for an airline than its number of available seats.
2. It is strictly forbidden to exceed the maximum capacity of each airplane.
3. At least, 20 leisure plus airplane tickets should be offered for each airplane, and at least 10 business plus airline tickets for each airplane as well.
4. Because this is a low-cost company, the number of standard air tickets should be at least 60 % of the overall number of airline tickets offered.

First, we define out variables (X, Y, Z) where:

X [x1, x2, x3, x4, x5] -> number of Standard seats (x1 corresponding to standard seats on plane 1, while X is the addition of all standard seats of all planes x1 + x2 + x3 + x4 + x5)

Y [y1, y2, y3, y4, y5] -> number of Leisure plus seats(y1 corresponding to standard seats on plane 1, while Y is the addition of all standard seats of all planes y1 + y2 + y3 + y4 + y5)

Z [z1, z2, z3, z4, z5] -> number of Business plus seats (z1 corresponding to standard seats on plane 1, while Z is the addition of all standard seats of all planes z1 + z2 + z3 + z4 + z5)

Now we need to state the objective function:

$$max\ N = 19X + 49Y + 69Z$$

Here we try to maximize the profit overall.

Thirdly, we need to define the constraints. Each of the sentences previously described are reflected into mathematical constraints as it is shown below:

1. $X + Y + Z <= 90 + 120 + 200 + 150 + 190 \ -> \ X + Y + Z <= 750$
2. $x_i + y_i + z_i <= AV_i$ $where\ i\ is\ the\ number\ of\ planes\ (1, 2, 3, 4, 5).\ Example:\ x1 + y1 + z1 = AV1$
3. 20 leisure, 10 business
   a. $y_i >= 20$
   b. $z_i >= 10$
4. $X >= 0.6 \times (X + Y + Z)$

Also, since there can't be any negative seats, we state that $x, y, z >= 0$

Model 2: Optimal plane arrival

In part 2, we are given the following data:

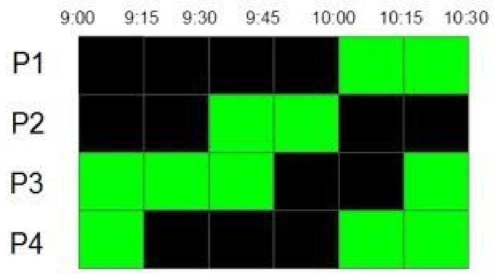

Figure 1

Airplane	Scheduled landing time	Maximum allotted landing time	Additional cost
AV1	9:10	10:15	100€
AV2	8:55	9:30	200€
AV3	9:40	10:00	150€
AV4	9:55	10:15	250€
AV5	10:10	10:30	200€

Figure 2

The following constraints are given:

1. Every airplane should be assigned one slot for landing.
2. No more than one slot has to be assigned to each airplane.
3. Assigned slots should be available, i. e., shown in green in Figure 1.
4. The starting time of an assigned slot shall be either equal or greater than the scheduled landing time.
5. The starting time of an assigned slot shall be either less or equal than the maximum allotted landing time.
6. For safety reasons it is not allowed to assign to consecutive slots in the same track.

The objective function of both problems is the following. Notice that the second part is of the type “max z = -part2”, meaning that is a minimization of the cost of landing the planes subtracted from the incomes of the tickets maximized.

$$max\ profit\ = \sum_{i\ in\ PLANES} (19 * standard_i\ +\ 49 * leisure_i\ +\ 69 * business_i)$$

$$- \sum_{i\ in\ TIMESLOTS} \sum_{j\ in\ PLANES} (unit\_cost_{ij}\ *\ plane\_cost_{ij}\ *\ assign_{ij})$$

Variables

$standard_i$, $leisure_i$ and $business_i$ define the number of tickets of each class for every plane. $assign_{ij}$ is a boolean matrix that represents in which time slot a plane lands.

Parameters

max_seats indicates the maximum seats available for each plane

max_weight indicates the maximum weight of each plane

plane_cost indicates the cost in euros for each minute the plane landing is delayed

The TIMESLOTS will represent each slot of time for each runway, being p01t15 the runway number 1 at the time 9:15, and p04t75 the runway number 4 at the time slot 10:15

p01t00	p01t15	p01t30	p01t45	p01t60	p01t75
p02t00	p02t15	p02t30	p02t45	p02t60	p02t75
p03t00	p03t15	p03t30	p03t45	p03t60	p03t75
p04t00	p04t15	p04t30	p04t45	p04t60	p04t75

The set of PLANES will simply represent all the planes waiting to land

AV1	AV2	AV3	AV4	AV5

The $unit\_cost_{ij}$ matrix represents the cost in minutes of each plane and timeslot. Any infinite value (represented by 99999 in the .dat file) indicates that the plane can not land in that time slot, either because the runway is unavailable at that time, or because the plane is not scheduled to arrive at the airport at that moment in time.

		AV1	AV2	AV3	AV4	AV5
RUNAWAY 1	p01t00	∞	∞	∞	∞	∞
	p01t15	∞	∞	∞	∞	∞
	p01t30	∞	∞	∞	∞	∞
	p01t45	∞	∞	∞	∞	∞
	p01t60	50	∞	∞	5	∞
	p01t75	∞	∞	∞	∞	5

RUNAWAY 2	p02t00	∞	∞	∞	∞	∞
	p02t15	∞	∞	∞	∞	∞
	p02t30	20	∞	∞	∞	∞
	p02t45	35	∞	5	∞	∞
	p02t60	∞	∞	∞	∞	∞
	p02t75	∞	∞	∞	∞	∞
RUNAWAY 3	p03t00	∞	5	∞	∞	∞
	p03t15	5	20	∞	∞	∞
	p03t30	20	∞	∞	∞	∞
	p03t45	∞	∞	∞	∞	∞
	p03t60	∞	∞	∞	∞	∞
	p03t75	65	∞	∞	∞	5
RUNAWAY 4	p04t00	∞	5	∞	∞	∞
	p04t15	∞	∞	∞	∞	∞
	p04t30	∞	∞	∞	∞	∞
	p04t45	∞	∞	∞	∞	∞
	p04t60	50	∞	∞	5	∞
	p04t75	65	∞	∞	∞	5

Constraints

For every plane their class tickets must not exceed maximum seat capacity

$$For\ each\ i\ in\ PLANES:\ \ standard_i\ +\ leisure_i\ +\ business_i\ <=\ max\_seats_i$$

For every plane their combined class ticket luggage weight must not exceed plane max weight

$$For\ each\ i\ in\ PLANES\ :\ \ standard_i\ + 20 *\ leisure_i\ + 40 *\ business_i\ <=\ max\_weight_i$$

For every plane there must be at least 20 leisure tickets

$For\ each\ i\ in\ PLANES:\ \ leisure_i$ >= 20

For every plane there must be at least 10 business tickets

$$For\ each\ i\ in\ PLANES:\ business_i >=\ 10$$

For every plane there must be at least 60% of standard tickets

$$For\ each\ i\ in\ PLANES:\ \sum_{i\ in\ PLANES} standard_i\ >=\ (\sum_{i\ in\ PLANES} max\_seats_i) * 0.6$$

For every plane there must be a time slot assigned

$$For\ each\ j\ in\ PLANES:\ \sum_{i\ in\ TIMESLOTS} assign_{i,j}\ =\ 1$$

For every time slot, there **could** be a plane (but only one at max) assigned to it. That time slot could be empty for that plane, represented by a zero.

$$For\ each\ j\ in\ PLANES:\ \sum_{i\ in\ TIMESLOTS} assign_{i,j}\ <=\ 1$$

These last two constraints restricts the following, respectively:

- There is only one plane landing at a given time slot
- A plane can not be assigned to multiple time slots

2. Analysis of results

Seat distribution problem

Regarding the seats distribution the results obtained are the following (please, note that the results are separated into several parts for the sake of clarity and simplicity when analyzing them)

	Standart	Leisure	Business	All seats	
Decision variables	x	y	z		
AV1	37	23	30	90	Nº Seats
AV2	20	66	34	120	Nº Seats
AV3	160	23	17	200	Nº Seats
AV4	100	20	30	150	Nº Seats
AV5	133	21	36	190	Nº Seats
SUM(ALL PLANES) per seat type	450	153	147	VERDADERO	

First, we will look into the number of seats of each type of each plane in order to check constraints 2, 3, 4 (refer to model 1 description)

We can realize at plain sight that Leisure or Business columns contain values smaller than 20 and 10 respectively. This verifies constraints 3.

Also we can check that the total number of Standard seats (450) is bigger or equal than the 60% of the total number of seats sold by the airline ($0.6 \times (450 + 153 + 147) = 450$). In this case, this looks optimal because the cheaper seats (standard ones) are constrained to the minimum required in order to maximize the earnings. This accomplishes constraint 4.

Finally we can check that none of the values in the column "All Seats", which describes the total amounts of seats assigned in each plane overcomes the maximum capacity of the plane (refer to model 1 Figure 1 to check the maximum capacities). With this, we accomplished constraint 2 successfully.

To prove constraint 1 we can refer to constrain 2 to prove it. If none of the planes overcome the maximum capacity assigned, the sum of these capacities will NOT overcome the sum of all the capacities of the planes by any circumstance. This is proven mathematically in the cell found on the bottom right corner (colored in green , with the logic operation (450+153+147)=(90+120+200+150+190) and equals 750)

Finally, all the constraints are verified.

Let's now analyze the results of the seats maximization:

Coefficients	19	49	69	€
Value (AV1)		3900		€
Value (AV2)		5960		€
Value(AV3)		5340		€
Value (AV4)		4950		€
Value (AV5)		6040		€
Total		26190		

The first results are looking good. At this point, the airline has made a profit of 26190€.

At the beginning of the modeling period, we encountered some problems, since mathprog results were different than the ones achieved with LibreOffice. We added some restrictions that were missing, like optimize using only natural numbers (there aren't "half seats" to be sold), and we got the same results in both, so we can guess that our optimization is - forgive the repetition - optimal. Also the values obtained in each plane profit are similar to the ones obtained when solved by hand.

Runway problem

Now, we must check the runway assignment.

In order to MAXIMIZE the overall profit (that we just computed), we want now to MINIMIZE the costs in this part. In order to do this, we want to schedule our planes efficiently, landing first the more expensive ones, and then the cheaper ones keepen the costs to the minimum.

In order to understand easily the results obtained (since the results given are drawn in a binary matrix, and it will be hard to understand), we are going to draw graphically the landings using the same scheme given in the statement:

	9:00-9:15	9:15-9:30	9:30-9:45	9:45-10:00	10:00-10:15	10:15-10:30
P1					AV4	
P2				AV3		
P3		AV1				AV5
P4	AV2					

As in the statement, green zones are available and black ones are not available. We added in red the time slots where there is a plane landing (name of the plane written as a label)

If we check the name of the planes and compare them one by one with the data given, we will quickly realize that all the plane delay is set to the minimum. Also there are not two planes landing in the same time slot and runway. There are also no consecutive planes landing in the same runway. Of course, there are no planes landing twice, or similar.

Constraints are verified once again.

With this, we get a negative profit of **-4500€ due to delay costs.**

Combining both problems

Last step is to check the final profit calculation, and we can see that the result obtained is **21690 € of profit**, which equals the addition of both profits (26190 + (-4500))

After reviewing the results, we would like to point some things out:

The linear programming task is generalized: we have added two jumbo jets (larger costs and more profit) and a fifth runway. It worked perfectly fine, and we think we could've added far more planes and runways but that would have taken too much time to do so, because data in $unit\_cost_{ij}$ must be realistic so that the optimal solution is also consistent.

A simple optimization on every single time slot can be done, so that the linear programming task, in general, is more optimized. If the plane spends zero time on a given time slot, for example, it has to land between 9:30 and 9:45, it will land at 9:30:01 instead of 9:44:59, saving 15 minutes of fuel. This, we believe is an **optimal substructure** (Cormen, 2009)

Delays on planes

About the challenge proposed of taking into consideration possible delays into part 2 of the problems, and how to handle recomputations.

The delay of 20 minutes in AV1 will result in an arrival time of 9:30. This will mean that AV1 cannot arrive on time slot 9:15-9:30. Recomputing the solution with the delay, this is the resulting schedule

	9:00-9:15	9:15-9:30	9:30-9:45	9:45-10:00	10:00-10:15	10:15-10:30
P1					AV4	
P2				AV3		
P3			AV1			AV5
P4	AV2					

As we can see, it’s quite similar to the one computed previously, but now AV1 is landing in a different slot. The major change is observed in the profit, because now (assuming that this 20 minutes of delay are “free” for the company), the cost of landing AV1 has decreased to 0€, so the profit goes from -4500 to -4000€. This, of course, means an increase in the final profit of 500€, so we will obtain **22190€ of final profit.**

3. Final Remarks

We have learned how to solve linear programming tasks in spreadsheets, which is one of the most useful tools in engineering. Also, now we know how to model real world scenarios to solve real world problems, which is always nice.

We have had a few challenges, like at first how to do multiple planes in LibreOffice, or how to iterate through MathProg.