

# 724 Macroeconomic Theory: Investment Adjustment Costs

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*April 30, 2017*

## Introduction

- Outline models being compared ( no equations)
- Overview of analysis and results

## Description

- Description of standard RBC model
- Description of investment adjustment costs
- Methods used to solve RBC model
- Methods used to solve investment adjustment costs
- How was RBC model simulated
- How was investment adjustment costs model simulated

## Analysis

- Compare the two models
- table of moments reporting sd, sd relative to Y, and autocorrelations, correlation with output for all variables of interest.
- Highlight difference in the moments of the two models
- What is causing these differences?
- Key economic forces driving these results
- Use impulse response functions to clarify explanations and illustrate economics of the system.
- Check sensitivity of results to key parameter choices
- Compare your results to one published article using related models.

## Conclusion

- A brief conclusion

## Matlab Code

- Provide matlab code

## Appendix

- Derivations of first order conditions
- Derivation of steady state.

## Rough Work

### RBC model

$$\begin{aligned} \max E_0 \sum_{t=0}^{\infty} \beta^t [\ln(c_t) + \psi \ln(1 - n_t)] \text{ s.t.} \\ y_t &= c_t + i_t \\ y_t &= A_t k_t^\alpha n_t^{1-\alpha} \\ k_{t+1} &= i_t + (1 - \delta)k_t \\ \ln(A_{t+1}) &= \rho \ln(A_t) + \epsilon_{t+1} \text{ where } \epsilon_{t+1} \sim iid(0, \sigma_\epsilon^2) \\ k_0 \text{ and } A_0 &\text{ are given and } > 0 \end{aligned}$$

with  $\psi > 0, \beta, \delta, \rho \in (0, 1)$

Choice variables being consumption, capital in the next period, investment, labour supply and leisure. Have substituted out leisure for  $(1 - n_t)$ .

Solve through optimal control. Using the constraints for  $y_t$  and  $k_t$

$$\begin{aligned} A_t k_t^\alpha n_t^{1-\alpha} &= c_t + i_t \\ A_t k_t^\alpha n_t^{1-\alpha} &= c_t + k_{t+1} - (1 - \delta)k_t \\ 0 &= A_t k_t^\alpha n_t^{1-\alpha} - c_t - k_{t+1} + (1 - \delta)k_t \end{aligned}$$

Setting up Lagrange

$$\begin{aligned} L &= E_0 \sum_{t=0}^{\infty} \beta^t [\ln(c_t) + \psi \ln(1 - n_t)] + \beta^t \lambda_t [A_t k_t^\alpha n_t^{1-\alpha} - c_t - k_{t+1} + (1 - \delta)k_t] \\ \frac{\partial L}{\partial c_t} &: \frac{1}{c_t} = \lambda_t \\ \frac{\partial L}{\partial n_t} &: \frac{\psi}{(1 - n_t)} = \lambda_t \\ &: \frac{\psi}{(1 - n_t)} = \frac{1}{c_t} [(1 - \alpha) \frac{y_t}{n_t}] \\ \frac{\partial L}{\partial k_{t+1}} &: \lambda_t = \lambda_{t+1} \beta [\alpha \frac{y_{t+1}}{k_{t+1}} + (1 - \delta)] \\ &: \frac{1}{c_t} = \frac{1}{c_{t+1}} \beta [\alpha \frac{y_{t+1}}{k_{t+1}} + (1 - \delta)] \end{aligned}$$

\*\* Need to solve for steady state\*\*

## Investment adjustment cost model

This model introduces adjustment cost to changing the level of investment between periods. **Rational for introducing investment adjustment costs?**

Critical addition is in the capital accumulation equation.

$$k_{t+1} = \theta(i_t, i_{t-1})i_t + (1 - \delta)k_t$$

Solve for the steady state of a quadratic cost model as proposed by *Christiansen 2005*. Where theta reflects the adjustment costs associated with changing the level of investment.

$$\theta = \frac{i_t}{i_{t-1}}$$

Using the same preferences as RBC **Not so sure this is the model we want to use**

$$\begin{aligned} \max E_0 \sum_{t=0}^{\infty} \beta^t [\ln(c_t) + \psi \ln(1 - n_t)] \text{ s.t.} \\ y_t &= c_t + i_t \\ y_t &= A_t k_t^\alpha n_t^{1-\alpha} \\ k_{t+1} &= [1 - \theta(\frac{i_t}{i_{t-1}})]i_t + (1 - \delta)k_t \\ \ln(A_{t+1}) &= \rho \ln(A_t) + \epsilon_{t+1} \text{ where } \epsilon_{t+1} \sim iid(0, \sigma_\epsilon^2) \\ k_0 \text{ and } A_0 &\text{ are given and } > 0 \end{aligned}$$