724 Macroeconomic Theory: Investment Adjustment Costs

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Introduction

- Outline models being compared (no equations)
- Overview of analysis and results

Description

- Description of standard RBC model
- Description of investment adjustment costs
- Methods used to solve RBC model
- Methods used to solve investment adjustment costs
- How was RBC model simulated
- How was investmnet adjustment costs model simulated

Analysis

- Compare the two models
- table of moments reporting sd, sd relative to Y, and autocorrelations, correlation with output for all variables of interest.
- Highlight difference in the moments of the two models
- What is causing these differences?
- Key economic forces driving these results
- Use impulse response functions to clarify explanations and illustrate economics of the system.
- Check sensitivity of results to key parameter choices
- Compare your results to one published article using related models.

Conclusion

- A brief conclusion

Matlab Code

- Provide matlab code

Appendix

- Derivations of first order conditions
- Derivation of steady state.

Rough Work

RBC model

$$\begin{aligned}
\max E_0 \sum_{t=0}^{\infty} \beta^t[\ln(c_t) + \psi \ln(1-n_t)] \text{ s.t.} \\
y_t &= c_t + i_t \\
y_t &= A_t k_t^{\alpha} n_t^{1-\alpha} \\
k_{t+1} &= i_t + (1-\delta)k_t \\
\ln(A_{t+1}) &= \rho \ln(A_t) + \epsilon_{t+1} \text{ where } \epsilon_{t+1} \sim iid(0, \sigma_{\epsilon}^2) \\
k_0 \text{ and } A_0 &\text{ are given and } > 0
\end{aligned}$$

with $\psi > 0, \beta, \delta, \rho \in (0,1)$

Choice variables being consumption, captial in the next period, investment, labour supply and leisure. Have subtituted out leisure for $(1-n_t)$.

Solve through optimal control. Using the constraints for y_t and k_t

$$\begin{aligned}
A_t k_t^{\alpha} n_t^{1-\alpha} &= c_t + i_t \\
A_t k_t^{\alpha} n_t^{1-\alpha} &= c_t + k_{t+1} - (1-\delta)k_t \\
0 &= A_t k_t^{\alpha} n_t^{1-\alpha} - c_t - k_{t+1} + (1-\delta)k_t
\end{aligned}$$

Setting up Lagrange

$$\begin{aligned}
L = E_0 \sum_{t=0}^{\infty} \beta^t[\ln(c_t) + \psi \ln(1-n_t)] + \beta^t \lambda_t [A_t k_t^{\alpha} n_t^{1-\alpha} - c_t - k_{t+1} + (1-\delta)k_t] \\
\frac{\partial L}{\partial c_t} &: \frac{1}{c_t} = \lambda_t \\
\frac{\partial L}{\partial n_t} &: \frac{\psi}{(1-n_t)} = \lambda_t \\
&: \frac{\psi}{(1-n_t)} = \frac{1}{c_t}[(1-\alpha)\frac{y_t}{n_t}] \\
\frac{\partial L}{\partial k_{t+1}} &: \lambda_t = \lambda_{t+1}\beta[\alpha\frac{y_{t+1}}{k_{t+1}} + (1-\delta)] \\
&: \frac{1}{c_t} = \frac{1}{c_{t+1}}\beta[\alpha\frac{y_{t+1}}{k_{t+1}} + (1-\delta)]
\end{aligned}$$

** Need to solve for steady state**

Investment adjustment cost model

This model introduces adjustment cost to changing the level of investment between periods. **Rational for introducing investment adjustment costs**?.

Critical addition is in the capital accumulation equation.

$k_{t+1} = i_t - \theta(\frac{i_t}{i_{t-1}})i_t + (1-\delta)k_t$

Where θ represents the cost of adjusting capital and is a function of both current period investment and previous periods investmnet.

Using the same preferences as RBC **Not so sure this is the model we want to use**

$$
\begin{aligned}
\max E_0 \sum_{t=0}^{\infty} \beta^t [\ln(c_t) + \psi \ln(1-n_t)] \text{ s.t.}& \\
y_t = c_t + i_t& \\
y_t = A_t k_t^{\alpha} n_t^{1-\alpha}& \\
k_{t+1} = [1 - \theta(\frac{i_t}{i_{t-1}})]i_t + (1-\delta)k_t& \\
\ln(A_{t+1}) = \rho \ln(A_t) + \epsilon_{t+1} \text{ where } \epsilon_{t+1} \sim iid(0, \sigma_{\epsilon}^2)& \\
k_0 \text{ and } A_0 \text{ are given and } > 0&
\end{aligned}
$$

$$
\begin{aligned}
L = E_0 \sum_{t=0}^{\infty} & \beta^t [\ln(c_t) + \psi \ln(1-n_t)] \\
& + \beta^t \lambda_t [A_t k_t^{\alpha} n_t^{1-\alpha} - c - i] \\
& + \beta^t \Lambda_t [i_t - \theta_t(i_t, i_{t-1}) + (1-\delta)k_t - k_{t+1}]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial c_t} &: \frac{1}{c_t} = \lambda_t \\
\frac{\partial L}{\partial n_t} &: \frac{\psi}{1-n_t} = \lambda_t [(1-\alpha)\frac{y_t}{n_t}] \\
\frac{\partial L}{\partial i_t} &: \lambda_t = \Lambda_t [1 - \frac{\partial \theta_t(i_t, i_{t-1})}{\partial i_t}] + \Lambda_{t+1} \beta [\frac{\partial \theta_{t+1}(i_{t+1}, i_t)}{\partial i_t}] \\
\frac{\partial L}{\partial k_{t+1}} &: \Lambda_t = \lambda_{t+1} \beta [\alpha \frac{y_{t+1}}{k_{t+1}} + (1-\delta)]
\end{aligned}
$$