724 Macroeconomic Theory: Investment Adjustment Costs

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Introduction

- Outline models being compared (no equations)
- Overview of analysis and results

Description

- Description of standard RBC model
- Description of investment adjustment costs
- Methods used to solve RBC model
- Methods used to solve investment adjustment costs
- How was RBC model simulated
- How was investmnet adjustment costs model simulated

Analysis

- Compare the two models
- table of moments reporting sd, sd relative to Y, and autocorrelations, correlation with output for all variables of interest.
- Highlight difference in the moments of the two models
- What is causing these differences?
- Key economic forces driving these results
- Use impulse response functions to clarify explanations and illustrate economics of the system.
- Check sensitivity of results to key parameter choices
- Compare your results to one published article using related models.

Conclusion

- A brief conclusion

Matlab Code

- Provide matlab code

Appendix

- Derivations of first order conditions
- Derivation of steady state.

Rough Work

RBC model

$$
\begin{aligned}
\max E_0 \sum_{t=0}^{\infty} \beta^t [\ln(c_t) + \psi \ln(1-n_t)] \text{ s.t.}\\
y_t &= c_t + i_t \\
y_t &= A_t k_t^{\alpha} n_t^{1-\alpha} \\
k_{t+1} &= i_t + (1-\delta) k_t \\
\ln(A_{t+1}) &= \rho \ln(A_t) + \epsilon_{t+1} \text{ where } \epsilon_{t+1} \sim iid(0, \sigma_{\epsilon}^2) \\
k_0 \text{ and } A_0 &\text{ are given and } > 0
\end{aligned}
$$

with $\psi > 0, \beta, \delta, \rho \in (0,1)$

Choice variables being consumption, captial in the next period, investment, labour supply and leisure. Have subtituted out leisure for $(1-n_t)$.

Solve through optimal control. Using the constraints for y_t and k_t

$$
\begin{aligned}
A_t k_t^{\alpha} n_t^{1-\alpha} &= c_t + i_t \\
A_t k_t^{\alpha} n_t^{1-\alpha} &= c_t + k_{t+1} - (1-\delta) k_t \\
0 &= A_t k_t^{\alpha} n_t^{1-\alpha} - c_t - k_{t+1} + (1-\delta) k_t
\end{aligned}
$$

Setting up Lagrange

$$
\begin{aligned}
L &= E_0 \sum_{t=0}^{\infty} \beta^t [\ln(c_t) + \psi \ln(1-n_t)] + \beta^t \lambda_t [A_t k_t^{\alpha} n_t^{1-\alpha} - c_t - k_{t+1} + (1-\delta) k_t] \\
\frac{\partial L}{\partial c_t} &: \frac{1}{c_t} = \lambda_t \\
\frac{\partial L}{\partial n_t} &: \frac{\psi}{(1-n_t)} = \lambda_t \\
&: \frac{\psi}{(1-n_t)} = \frac{1}{c_t}[(1-\alpha)\frac{y_t}{n_t}] \\
\frac{\partial L}{\partial k_{t+1}} &: \lambda_t = \lambda_{t+1} \beta [\alpha \frac{y_{t+1}}{k_{t+1}} + (1-\delta)] \\
&: \frac{1}{c_t} = \frac{1}{c_{t+1}} \beta [\alpha \frac{y_{t+1}}{k_{t+1}} + (1-\delta)]
\end{aligned}
$$

-Need to solve for steady state

Investment adjustment cost model

The investment adjustment cost RBC model introduces frictions to changing the level of investment between periods. We will be using the specification for the capital accumulation equation as proposed by Christiano, Eichenbaum and Evans (2001). The capital accumulation equation changes such that next periods capital is equal to the capital from the previous period that did not depreciate and an investment function that translates current investment as well previous investment levels into next periods capital stock.

- **Rationale for introducing investment adjustment costs**

The capital accumulation equation becomes: $k_{t+1} = k_t(1+\delta) + f(i_t, i_{t-1})$

Christiano, Eichenbaum and Evans (2005) prescribe this functional form where $f(i_t, i_{t-1}) = \left[1 - S(\frac{i_t}{i_{t-1}})\right] i_t$ and $S(1) = 0$. Thus ensuring that when $i_t = i_{t-1}$ the capital accumulaton equation reverts back to the one in our basic RBC model.

Using the same preferences as standard RBC model to ensure the changes in paramter valeus and impulse response functions is only driven by the changes in the capital accumulation equation.

We will now use the quadratic investment adjustment term proposed by Schmitt-Groche & Uribe (2005) where

$S(\frac{i_t}{i_{t-1}}) = \frac{\kappa}{2}\left[\frac{i_t}{i_{t-1}} - 1\right]^2$ Where κ is non-negative and represents the cost of adjusting captial.

$$
\begin{aligned}
\max E_0 \sum_{t=0}^{\infty} \beta^t [\ln(c_t) + \psi \ln(1-n_t)] \text{ s.t.} \\
y_t = c_t + i_t \\
y_t = A_t k_t^{\alpha} n_t^{1-\alpha} \\
k_{t+1} = [1 - \frac{\kappa}{2}\left[\frac{i_t}{i_{t-1}} - 1\right]^2] i_t + (1-\delta)k_t \\
\ln(A_{t+1}) = \rho \ln(A_t) + \epsilon_{t+1} \text{ where } \epsilon_{t+1} \sim iid(0, \sigma_\epsilon^2) \\
k_0 \text{ and } A_0 \text{ are given and } > 0 \\
\kappa \geq 0
\end{aligned}
$$

Solving through optimal control

$$
\begin{aligned}
L = E_0 \sum_{t=0}^{\infty} & \beta^t [\ln(c_t) + \psi \ln(1 - n_t)] \\
& + \beta^t \lambda_t [A_t k_t^{\alpha} n_t^{1-\alpha} - c - i] \\
& + \beta^t \Lambda_t \left[[1 - \frac{\kappa}{2} [\frac{i_t}{i_{t-1}} - 1]^2] i_t + (1 - \delta) k_t - k_{t+1} \right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial c_t} &: \frac{1}{c_t} = \lambda_t \\
\frac{\partial L}{\partial n_t} &: \frac{\psi}{1 - n_t} = \lambda_t [(1 - \alpha) \frac{y_t}{n_t}] \\
\frac{\partial L}{\partial k_{t+1}} &: \Lambda_t = \beta [\lambda_{t+1} (\alpha \frac{y_{t+1}}{k_{t+1}}) + \Lambda_{t+1} (1 - \delta)] \\
\frac{\partial L}{\partial i_t} &: \lambda_t = \Lambda_t [1 - \frac{\kappa}{2} (\frac{i_t}{i_{t-1}} - 1)^2 - \kappa [\frac{i_t}{i_{t-1}} - 1] \frac{i_t}{i_{t-1}}] \\
& + \beta \Lambda_{t+1} [\kappa [\frac{i_{t+1}}{i_t} - 1] (\frac{i_{t+1}}{i_t})^2]
\end{aligned}
$$

Schmitt-Grohé, Stephanie, and Martin Uribe. “Optimal fiscal and monetary policy in a medium-scale macroeconomic model.” NBER Macroeconomics Annual 20 (2005): 383-425.

Del Negro and Schorfheide (2008), Fern´andez-Villaverde (2010), and Gerali et al. (2010) all USE SAME CAPITAL adjdustment model as ours might want to check them out.