Segun Simpson $3/8$ Para un intervalo $[X_j, X_{j+1}, X_{j+2}, X_{j+3}]$

$$\int_{X_j}^{X_{j+3}} f(x)\,dx \simeq \int_{X_j}^{X_{j+3}} P_3(X)\,dx$$

• donde $P_3(X)$ es un polinomio interpolado por Lagrange

$$P_3(x) = \sum_{j=0}^{3} f(X_j)\,L(X_j) \rightarrow L(X_j) = \prod_{J=0, J\neq j}^{3} \left(\frac{X - X_J}{X_j - X_J}\right)$$

$$P_3(x) = f(X_j)L(X_j) + f(X_{j+1})L(X_{j+1}) + f(X_{j+2})L(X_{j+2}) + f(X_{j+3})L(X_{j+3})$$

$$\Rightarrow \int_{X_j}^{X_{j+3}} P_3(x)\,dx = f(X_j)\int_{X_j}^{X_{j+3}} L(X_j)\,dx + f(X_{j+1})\int_{X_j}^{X_{j+3}} L(X_{j+1})\,dx \pm f(X_{j+2})\int_{X_j}^{X_{j+3}} L(X_{j+2})\,dX$$

$$+ f(X_{j+3})\int_{X_j}^{X_{j+3}} L(X_{j+3})\,dX$$

• Si h es la distancia entre cada punto:

• y expresamos cada punto X en el intervalo de la siguiente forma

$$X = X_j + \lambda h$$

$\Rightarrow$ Para X_j

$$L(X_j) = \frac{(X - X_{j+1})(X - X_{j+2})(X - X_{j+3})}{(X_j - X_{j+1})(X_j - X_{j+2})(X_j - X_{j+3})}$$

$$\Rightarrow X - X_{j+1} = X_j + \lambda h - X_{j+1}$$
$$= X_j + \lambda h - (X_j + h)$$
$$= h(\lambda - 1)$$

• de igual forma para $(X - X_{j+2})$ y $(X - X_{j+3})$, obtenemos $h(\lambda-2)$ y $h(\lambda-3)$ respectivamente.

$$\Rightarrow L(X_j) = \frac{h^3(\lambda-1)(\lambda-2)(\lambda-3)}{-6h^3} = \frac{(\lambda-1)(\lambda-2)(\lambda-3)}{-6}$$

Para x_{j+1}

$$L(x_{j+1}) = \frac{(x-x_j)(x-x_{j+2})(x-x_{j+3})}{(x_{j+1}-x_j)(x_{j+1}-x_{j+2})(x_{j+1}-x_{j+3})} = \frac{\lambda h^3(\lambda-2)(\lambda-3)}{2h^3} = \frac{1}{2}\lambda(\lambda-2)(\lambda-3)$$

Para x_{j+2}

$$L(x_{j+2}) = \frac{(x-x_j)(x-x_{j+1})(x-x_{j+3})}{(x_{j+2}-x_j)(x_{j+2}-x_{j+1})(x_{j+2}-x_{j+3})} = \frac{\lambda h^3(\lambda-1)(\lambda-3)}{-2h^3} = -\frac{1}{2}\lambda(\lambda-1)(\lambda-3)$$

Para x_{j+3}

$$L(x_{j+3}) = \frac{(x-x_j)(x-x_{j+1})(x-x_{j+2})}{(x_{j+3}-x_j)(x_{j+3}-x_{j+1})(x_{j+3}-x_{j+2})} = \frac{\lambda h^3(\lambda-1)(\lambda-2)}{6h^3} = \frac{1}{6}\lambda(\lambda-1)(\lambda-2)$$

* Ahora remplanzando en $\int_{x_j}^{x_{j+3}} P_3(x)\,dx$ – sustituyendo dx por $h\,d\lambda$ y los limites de integracion

$$\int_{x_j}^{x_{j+3}} P_3(x) = f(x_j)\int_0^3 -\frac{h}{6}(\lambda-1)(\lambda-2)(\lambda-3)\,d\lambda + f(x_{j+1})\int_0^3 \frac{h}{2}\lambda(\lambda-2)(\lambda-3)\,d\lambda$$
$$+ f(x_{j+2})\int_0^3 -\frac{h}{2}\lambda(\lambda-1)(\lambda-3)\,d\lambda + f(x_{j+3})\int_0^3 \frac{h}{6}\lambda(\lambda-1)(\lambda-2)\,d\lambda$$

• Expandiendo los polinomios, y resolviendo las integrales, obtenemos.

$$\int_{x_j}^{x_{j+3}} P_3(x) = f(x_j)\frac{\overset{3}{\cancel{9}}h}{4\cdot\cancel{6}^{2}} + f(x_{j+1})\frac{9h}{4\cdot 2} + f(x_{j+2})\frac{9h}{4\cdot 2} + f(x_{j+3})\frac{\overset{3}{\cancel{9}}h}{4\cdot\cancel{6}^{2}}$$

$$= \frac{3h}{8}\left(f(x_j)+3f(x_{j+1})+3f(x_{j+2})+f(x_{j+3})\right)$$

$$\Rightarrow \int_{x_j}^{x_{j+3}} f(x)\,dx \cong \int_{x_j}^{x_{j+3}} P_3(x)\,dx = \sum_{J=0}^{3} f(x_{j+J}) \cdot \int_{x_j}^{x_{j+3}} L(x_{j+J})\,dx$$

Cumpliendo así la formula de Newton-Cotes

Siendo Las integrales de $L(x)$ los pesos