Learning Objective: MOSFET Concepts

Problem 1. Consider an p-channel MOSFET with a gate oxide thickness of 10 nm, threshold voltage $V_{\text{TH}} = -0.6$ V, gate width of 25 μm and gate length of 1 μm. Assume the hole channel mobility is $\mu_p = 100$ cm^2/V/s at $T = 300$ K. Use $\epsilon_{ox} = 3.9\epsilon_0$, $n_i = 1.5 \times 10^{10}$ cm^{-3} if needed.

(a) Suppose $V_{\text{GS}} = -0.1$ V, $V_{\text{DS}} = -3$ V, determine the **operating region** and **drain-to-source current** $\mathbf{I_{DS}}$.

$|V_{\text{GS}}| < |V_{\text{TH}}|$, so the MOSFET is in $\boxed{\text{cut-off region}}$, the drain-to-source $\boxed{I_{\text{DS}} = 0 \text{ A}}$.

(b) Suppose $V_{\text{GS}} = -3$ V, $V_{\text{DS}} = -5$ V, determine the **operating region** and **ddrain-to-source current** $\mathbf{I_{DS}}$.

$|V_{\text{GS}}| > |V_{\text{TH}}|$ and $|V_{DS}| > |V_{\text{GS}}| - |V_{\text{TH}}|$, the MOSFET is in $\boxed{\text{saturation region}}$.

$$
\begin{aligned}
I_{\text{SD}}^{\text{sat}} &= \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{\text{SG}} + V_{\text{TH}})^2 \\
&= \frac{1}{2}(100\text{ cm}^2/\text{V/s})\left(\frac{3.9 \times 8.854 \times 10^{-14}\text{ F/cm}}{10 \times 10^{-7}\text{ cm}}\right)\frac{25\ \mu\text{m}}{1\ \mu\text{m}}(3 - 0.6)^2 \\
&= 2.48\text{ mA} \\
I_{\text{DS}}^{\text{sat}} &= \boxed{-2.48\text{ mA}}
\end{aligned}
$$

(c) Suppose $V_{\text{GS}} = -5$ V, $V_{\text{DS}} = -0.1$ V, determine the **operating region** and **drain-to-source current** $\mathbf{I_{DS}}$.

$|V_{\text{GS}}| > |V_{\text{TH}}|$ and $|V_{DS}| < |V_{\text{GS}}| - |V_{\text{TH}}|$, the MOSFET is in $\boxed{\text{linear region}}$.

$$
\begin{aligned}
I_{\text{SD}}^{\text{lin}} &= \mu_p C_{ox}\frac{W}{L}\left[(V_{\text{SG}} + V_{\text{TH}})V_{\text{SD}} - \frac{1}{2}V_{\text{SD}}^2\right] \\
&= (100\text{ cm}^2/\text{V/s})\left(\frac{3.9 \times 8.854 \times 10^{-14}\text{ F/cm}}{10 \times 10^{-7}\text{ cm}}\right)\frac{25\ \mu\text{m}}{1\ \mu\text{m}}\left[(5 - 0.6)(0.1) - \frac{1}{2}(0.1)^2\right] \\
&= 0.375\text{ mA} \\
I_{\text{DS}}^{\text{lin}} &= \boxed{-0.375\text{ mA}}
\end{aligned}
$$

Problem 2. An n-channel MOSFET has parameters: $W = 15\ \mu$m, $L = 2\ \mu$m, $C_{ox} = 6.9 \times 10^{-8}$ F/cm^2. Use $\epsilon_{ox} = 3.9\epsilon_0$, $n_i = 1.5 \times 10^{10}$ cm^{-3} if needed at 300 K.

(a) Calculate the **oxide thickness** of the MOSFET.

$$\begin{aligned} C_{ox} &= \frac{\epsilon_{ox}}{t_{ox}} \\ t_{ox} &= \frac{\epsilon_{ox}}{C_{ox}} = \frac{3.9 \times 8.854 \times 10^{-14}\ \text{F/cm}}{6.9 \times 10^{-8}\ \text{F/cm}^2} \\ &= \boxed{50\ \text{nm}} \end{aligned}$$

(b) Assume the drain current in the saturation region for high V_{DS} is $I_{\text{DS}} = 35\ \mu$A at $V_{\text{GS}} = 2.3$ V and $I_{\text{DS}} = 75\ \mu$A at $V_{\text{GS}} = 2.5$ V. Calculate the **electron channel mobility** and **threshold voltage**.

Plug-in two operating points to the MOSFET $I_{\text{D}}^{\text{sat}}$ equation

$$I_{\text{D}}^{\text{sat}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{\text{GS}} - V_{\text{TH}})^2$$

get:

$$\begin{aligned} 35 \times 10^{-6} &= \frac{1}{2}\mu_n(6.9 \times 10^{-8}\ \text{F/cm}^2)\frac{15\ \mu\text{m}}{2\ \mu\text{m}}(2.3 - V_{\text{TH}})^2 \\ 75 \times 10^{-6} &= \frac{1}{2}\mu_n(6.9 \times 10^{-8}\ \text{F/cm}^2)\frac{15\ \mu\text{m}}{2\ \mu\text{m}}(2.5 - V_{\text{TH}})^2 \end{aligned}$$

Solve the equations above get: $\boxed{\mu_n = 727.58\ \text{cm}^2/\text{V/s}}$ and $\boxed{V_{\text{TH}} = 1.869\ \text{V}}$.

(c) Assume the drain current in the linear region for low $V_{\text{DS}} = 0.10$ V is $I_{\text{DS}} = 35\ \mu$A at $V_{\text{GS}} = 1.5$ V and $I_{\text{DS}} = 75\ \mu$A at $V_{\text{GS}} = 2.5$ V. Calculate the **electron channel mobility** and **threshold voltage**.

Plug-in two operating points to the MOSFET $I_{\text{D}}^{\text{lin}}$ equation

$$I_{\text{D}}^{\text{sat}} = \mu_n C_{ox}\frac{W}{L}\left[(V_{\text{GS}} + V_{\text{TH}})V_{\text{DS}} - \frac{1}{2}V_{\text{DS}}^2\right]$$

get:

$$\begin{aligned} 35 \times 10^{-6} &= \mu_n(6.9 \times 10^{-8}\ \text{F/cm}^2)\frac{15\ \mu\text{m}}{2\ \mu\text{m}}\left[(1.5 - V_{\text{TH}})(0.1) - \frac{1}{2}(0.1)^2\right] \\ 75 \times 10^{-6} &= \mu_n(6.9 \times 10^{-8}\ \text{F/cm}^2)\frac{15\ \mu\text{m}}{2\ \mu\text{m}}\left[(2.5 - V_{\text{TH}})(0.1) - \frac{1}{2}(0.1)^2\right] \end{aligned}$$

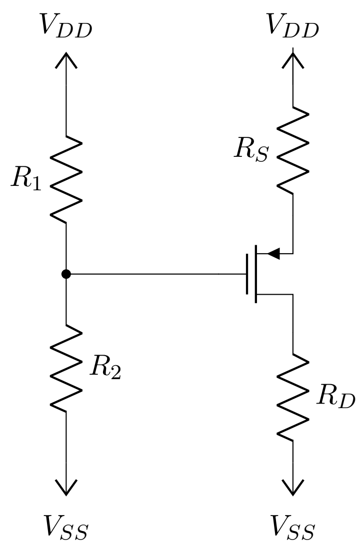
Solve the equations above get: $\boxed{\mu_n = 772.95\ \text{cm}^2/\text{V/s}}$ and $\boxed{V_{\text{TH}} = 0.575\ \text{V}}$.

Learning Objective: MOSFET DC Biasing

Problem 3. Consider the circuit shown below. The transistor parameters are $V_{\mathrm{TH}} = -0.8$ V and $K_p = 0.5\,\mathrm{mA/V^2}$. The component values are $V_{\mathrm{DD}} = 3$ V, $V_{\mathrm{SS}} = -3$ V, $R_1 = 8\,\mathrm{k\Omega}$, $R_2 = 22\,\mathrm{k\Omega}$, and $R_D = 5\,\mathrm{k\Omega}$.

(a) Calculate V_{SG}, I_{SD}, and V_{SD} for $R_S = 0\,\Omega$.

(b) Calculate V_{SG}, I_{SD}, and V_{SD} for $R_S = 0.5\,\mathrm{k\Omega}$.



For $\mathbf{R_S = 0}$ Ω: Assume MOSFET is in saturation:

$$\begin{aligned}
V_{\mathrm{SG}} &= V_{\mathrm{S}} - V_{\mathrm{G}} = V_{DD} - \left[(V_{DD} - V_{SS})\left(\frac{R_2}{R_1 + R_2}\right) + V_{SS}\right] \\
&= 3 - \left[(3+3)\left(\frac{22\times 10^3}{8\times 10^3 + 22\times 10^3}\right) - 3\right] \\
&= \underline{1.6\ \mathrm{V}} \\
I_{\mathrm{SD}} &= K_p(V_{\mathrm{SG}} + V_{\mathrm{TP}})^2 \\
&= (0.5\times 10^{-3}\ \mathrm{A/V^2})(1.6 - 0.8)^2 \\
&= \underline{0.32\ \mathrm{mA}} \\
V_{\mathrm{SD}} &= (V_{DD} - V_{SS}) - I_{\mathrm{SD}}R_D \\
&= 6 - (0.32\ \mathrm{mA})(5\ \mathrm{k\Omega}) \\
&= \underline{4.4\ \mathrm{V}}
\end{aligned}$$

Verify assumption: $|V_{\mathrm{DS}}| = 4$ V $> |V_{\mathrm{GS}}| - |V_{\mathrm{TH}}| = 0.8$ V, the MOSFET is biased in saturation region. Therefore $\boxed{V_{\mathrm{SG}} = 1.6\ \mathrm{V}}$, $\boxed{I_{\mathrm{SD}} = 0.4\ \mathrm{mA}}$, and $\boxed{V_{\mathrm{SD}} = 4\ \mathrm{V}}$.

For $\mathbf{R_S = 0.5}$ kΩ: Assume MOSFET is in saturation:

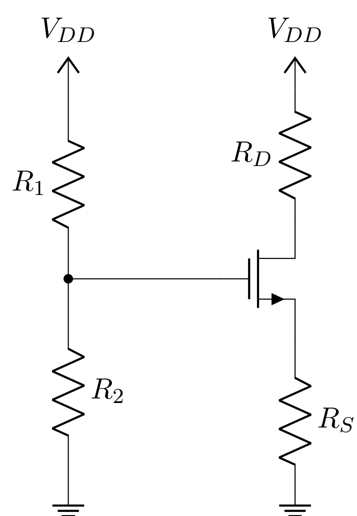
$$
\begin{aligned}
V_{\text{SG}} &= V_{\text{S}} - V_{\text{G}} = (V_{DD} - I_{\text{SD}}R_S) - \left[(V_{DD} - V_{SS})\left(\frac{R_2}{R_1 + R_2}\right) + V_{SS}\right] \\
&= [3 - I_{\text{SD}}(500)] - \left[(3+3)\left(\frac{22\times 10^3}{8\times 10^3 + 22\times 10^3}\right) - 3\right] \\
&= \underline{1.6 - I_{\text{SD}}(500)\ \text{V}} \\
I_{\text{SD}} &= K_p(V_{\text{SG}} + V_{\text{TP}})^2 \\
&= (0.5\times 10^{-3}\ \text{A/V}^2)(1.6 - I_{\text{SD}}(500) - 0.8)^2 \\
&= \underline{\{\ \cancel{10.96},\ 0.23\ \}\ \text{mA}} \\
V_{\text{SD}} &= (V_{DD} - V_{SS}) - I_{\text{SD}}R_D - I_{\text{SD}}R_S \\
&= 6 - (0.23\ \text{mA})(5\ \text{k}\Omega) - (0.23\ \text{mA})(0.5\ \text{k}\Omega) \\
&= \underline{4.735\ \text{V}}
\end{aligned}
$$

Verify assumption: $|V_{\text{DS}}| = 4.735$ V $> |V_{\text{GS}}| - |V_{\text{TH}}| = 0.685$ V, the MOSFET is biased in saturation region. Therefore $\boxed{V_{\text{SG}} = 1.485\ \text{V}}$, $\boxed{I_{\text{SD}} = 0.23\ \text{mA}}$, and $\boxed{V_{\text{SD}} = 4.735\ \text{V}}$.

Problem 4. Consider the circuit shown below. The transistor parameters are $V_{\text{TH}} = 0.4$ V and $K_n = 0.5\,\text{mA/V}^2$. The component values are $V_{\text{DD}} = 5$ V, $R_1 = 20\,\text{k}\Omega$, $R_2 = 5\,\text{k}\Omega$, $R_D = 2\,\text{k}\Omega$.

(a) Calculate V_{GS}, I_{DS}, and V_{DS} for $R_S = 0\,\Omega$.

(b) Calculate V_{GS}, I_{DS}, and V_{DS} for $R_S = 1\,\text{k}\Omega$.



For $\mathbf{R_S = 0}$ Ω: Assume MOSFET is in saturation:

$$\begin{aligned}
V_{\mathrm{GS}} &= V_{\mathrm{G}} - V_{\mathrm{S}} = V_{DD}\left(\frac{R_2}{R_1+R_2}\right) - 0 \\
&= (5)\left(\frac{5\times10^3}{20\times10^3+5\times10^3}\right) - 0 \\
&= \underline{1\ \mathrm{V}} \\
I_{\mathrm{DS}} &= K_n(V_{\mathrm{GS}} - V_{\mathrm{TH}})^2 \\
&= (0.5\times10^{-3}\ \mathrm{A/V^2})(1-0.4)^2 \\
&= \underline{0.18\ \mathrm{mA}} \\
V_{\mathrm{DS}} &= V_{DD} - I_{\mathrm{DS}}R_D \\
&= (5) - (0.18\times10^{-3})(2\times10^3) \\
&= \underline{4.64\ \mathrm{V}}
\end{aligned}$$

Verify assumption: $|V_{\mathrm{DS}}| = 4.64\ \mathrm{V} > |V_{\mathrm{GS}}| - |V_{\mathrm{TH}}| = 0.6\ \mathrm{V}$, the MOSFET is biased in saturation region. Therefore $\boxed{V_{\mathrm{GS}} = 1\ \mathrm{V}}$, $\boxed{I_{\mathrm{DS}} = 0.18\ \mathrm{mA}}$, and $\boxed{V_{\mathrm{DS}} = 4.64\ \mathrm{V}}$.

For $\mathbf{R_S = 1}$ kΩ: Assume MOSFET is in saturation:

$$\begin{aligned}
V_{\mathrm{GS}} &= V_{\mathrm{G}} - V_{\mathrm{S}} = V_{DD}\left(\frac{R_2}{R_1+R_2}\right) - I_{\mathrm{DS}}R_S \\
&= (5)\left(\frac{5\times10^3}{20\times10^3+5\times10^3}\right) - I_{\mathrm{DS}}(1000) \\
&= \underline{1 - I_{\mathrm{DS}}(1000)} \\
I_{\mathrm{DS}} &= K_n(V_{\mathrm{GS}} - V_{\mathrm{TH}})^2 \\
&= (0.5\times10^{-3}\ \mathrm{A/V^2})(1 - I_{\mathrm{DS}}(1000) - 0.4)^2 \\
&= \underline{\{\ \cancel{3.083},\ 0.117\ \}\ \mathrm{mA}} \\
V_{\mathrm{DS}} &= V_{DD} - I_{\mathrm{DS}}R_D - I_{\mathrm{DS}}R_S \\
&= (5) - (0.117\times10^{-3})(2\times10^3) - (0.117\times10^{-3})(1\times10^3) \\
&= \underline{4.649\ \mathrm{V}}
\end{aligned}$$

Verify assumption: $|V_{\mathrm{DS}}| = 4.649\ \mathrm{V} > |V_{\mathrm{GS}}| - |V_{\mathrm{TH}}| = 0.483\ \mathrm{V}$, the MOSFET is biased in saturation region. Therefore $\boxed{V_{\mathrm{GS}} = 0.883\ \mathrm{V}}$, $\boxed{I_{\mathrm{DS}} = 0.117\ \mathrm{mA}}$, and $\boxed{V_{\mathrm{DS}} = 4.649\ \mathrm{V}}$.