Learning Objective: Carrier, Dopant, Carrier Concentration, Doping

Problem 1. Silicon (Si) is doped with 2×10^5 atoms/cm^3 of Boron and 2×10^{15} atoms/cm^3 of Phosphorous at $T = 300$ K.

(a) The doped Silicon is p-type or n-type?

The doped Silicon is $\boxed{\text{n-type}}$, because Phosphorous doped with Silicon creates a majority carrier of electrons.

(b) Calculate the **intrinsic carrier concentration** of Silicon, using the constants given in Table 1.

$$
\begin{aligned}
n_i &= BT^{3/2} \exp\left(\frac{-E_g}{2kT}\right) \\
&= (5.32 \times 10^{15}\ \text{cm}^{-3}\text{K}^{-3/2})(300\ \text{K})^{3/2} \exp\left(\frac{-1.1\ \text{eV}}{2(86 \times 10^{-6}\ \text{eV/K})(300\ \text{K})}\right) \\
&= 1.5 \times 10^{10}\ \text{cm}^{-3}
\end{aligned}
$$

(c) Calculate the **electron and hole concentration**.

$$
\begin{aligned}
n &= N_D = d_{\text{Phosphorous}} = 2 \times 10^{15}\ \text{cm}^{-3} && \text{(n-type semiconductor)} \\
p &= \frac{n_i^2}{n} = \frac{(1.5 \times 10^{10}\ \text{cm}^{-3})^2}{2 \times 10^{15}\ \text{cm}^{-3}} = 1.125 \times 10^5\ \text{cm}^{-3}
\end{aligned}
$$

(d) Suppose the doped Silicon described above has a length of 2 μm and a cross-sectional area of 1.5 μm^2. Calculate the **current through the Silicon sample** if 2.5 V voltage is applied. (use $\mu_n = 1320$ cm^2/V/s, $\mu_p = 460$ cm^2/V/s)

$$
\begin{aligned}
\sigma &= q(n\mu_n + p\mu_p) \simeq qn\mu_n && (n >> p) \\
&= (1.602 \times 10^{-19}\ \text{A} \cdot \text{s})(2 \times 10^{15}\ \text{cm}^{-3})(1320\ \text{cm}^2/\text{V/s}) \\
&= 0.4229\ \Omega^{-1} \cdot \text{cm}^{-1} \\
\rho &= \frac{1}{\sigma} = \frac{1}{0.4229\ \Omega^{-1} \cdot \text{cm}^{-1}} = 2.3646\ \Omega \cdot \text{cm} \\
R &= \rho \frac{L}{A} = (2.3646\ \Omega \cdot \text{cm}) \frac{(2 \times 10^{-4}\ \text{cm})}{(1.5 \times 10^{-8}\ \text{cm}^2)} \\
&= 31.528\ \text{k}\Omega \\
I &= \frac{V}{R} = \frac{2.5\ \text{V}}{31.528\ \text{k}\Omega} = 79.29\ \mu\text{A}
\end{aligned}
$$

Learning Objective: Resistivity, Conductivity, Drift Current

Problem 2. Suppose a doped Silicon (Si) sample has a resistivity $\rho = 0.67\ \Omega$·cm at $T = 300$ K. (Use $\mu_n = 1000\ \text{cm}^2/\text{V/s}$, $\mu_p = 400\ \text{cm}^2/\text{V/s}$)

(a) Calculate the **conductivity** of the doped Silicon material.

$$\sigma = \frac{1}{\rho} = \frac{1}{(0.67\ \Omega \cdot \text{cm})} = 1.493\ \Omega^{-1} \cdot \text{cm}^{-1}$$

(b) Suppose the doped Silicon sample is *n-type*, calculate the **minimum concentration of Boron and Phosphorous atoms** being added.

For *n-type* Silicon, the minimum concentration of Boron atoms

$$d_{\text{Boron}} = 0\ \text{cm}^{-3}$$

because Boron dope with Silicon creates holes. The minimum concentration of Phosphorous atoms is:

$$\begin{aligned}
\sigma &= qn\mu_n \\
(1.493\ \Omega^{-1} \cdot \text{cm}^{-1}) &= (1.602 \times 10^{-19}\ \text{A} \cdot \text{s})n(1000\ \text{cm}^2/\text{V/s}) \\
n &= \frac{(1.493\ \Omega^{-1} \cdot \text{cm}^{-1})}{(1.602 \times 10^{-19}\ \text{A} \cdot \text{s})(1000\ \text{cm}^2/\text{V/s})} \\
&= 9.3196 \times 10^{15}\ \text{cm}^{-3} = d_{\text{Phosphorous}}
\end{aligned}$$

(c) Suppose the doped Silicon sample is *p-type*, calculate the **minimum concentration of Boron and Phosphorous atoms** being added.

For *p-type* Silicon, the minimum concentration of Phosphorous atoms

$$d_{\text{Phosphorous}} = 0\ \text{cm}^{-3}$$

because Phosphorous dope with Silicon creates electrons. The minimum concentration of Boron atoms is:

$$\begin{aligned}
\sigma &= qp\mu_p \\
(1.493\ \Omega^{-1} \cdot \text{cm}^{-1}) &= (1.602 \times 10^{-19}\ \text{A} \cdot \text{s})p(400\ \text{cm}^2/\text{V/s}) \\
n &= \frac{(1.493\ \Omega^{-1} \cdot \text{cm}^{-1})}{(1.602 \times 10^{-19}\ \text{A} \cdot \text{s})(400\ \text{cm}^2/\text{V/s})} \\
&= 2.330 \times 10^{16}\ \text{cm}^{-3} = d_{\text{Boron}}
\end{aligned}$$

Learning Objective: Diffusion Current

Problem 3. The total current in a semiconductor sample is composed of electron drift current and hole diffusion current at 300 K, where the total current density is 4.8 A/cm^2. The electron concentration is 10^{16} cm^{-3}. The hole concentration is given by

$$p(x) = 10^{15} \exp\left(-\frac{x}{L}\right) \text{ cm}^{-3}$$

where $x \geq 0$ and $L = 12$ μm. (Use $\mu_n = 1000$ cm^2/V/s, $\mu_p = 480$ cm^2/V/s)

(a) Determine the **hole diffusion current density** as a function of x.

$$\begin{aligned}
J_p^{\text{diffusion}} &= -qD_p \frac{\partial p}{\partial x} \\
&= -(1.602 \times 10^{-19} \text{ A} \cdot \text{s})(12.5 \text{ cm}^2/\text{s}) \left[10^{15} \exp\left(-\frac{x}{L}\right)\left(-\frac{1}{L}\right) \text{ cm}^{-4}\right] \\
&= 1.67 \exp\left(-\frac{x}{L}\right) \text{ A/cm}^2
\end{aligned}$$

(b) Determine the **electron drift current density** as a function of x.

$$\begin{aligned}
J^{\text{total}} &= J_n^{\text{drift}} + J_p^{\text{diffusion}} \\
\left(4.8 \text{ A/cm}^2\right) &= J_n^{\text{drift}} + \left[1.67 \exp\left(-\frac{x}{L}\right) \text{ A/cm}^2\right] \\
J_n^{\text{drift}} &= 4.8 - 1.67 \exp\left(-\frac{x}{L}\right) \text{ A/cm}^2
\end{aligned}$$

Table 1: Semiconductor Constants (Textbook Table 1.3)

Material	E_g (eV)	B (cm^{-3} K$^{-3/2}$)
Silicon (Si)	1.1	5.23×10^{15}
Gallium Arsenide (GaAs)	1.4	2.10×10^{14}
Germanium (Ge)	0.66	1.66×10^{15}