Learning Objective: Small Signal Parameters

Problem 1. Consider the following I_D equation

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2$$

Show the following equality.

(a) $g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH}) = 2K_n(V_{GS} - V_{TH})$

$$\begin{aligned}
g_m &= \frac{\partial I_D}{\partial V_{GS}} \\
&= \frac{\partial}{\partial V_{GS}}\left[\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2\right] \\
&= (2)\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})(1) \\
&= \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH}) \\
&= 2K_n(V_{GS} - V_{TH})
\end{aligned}$$

(b) $g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = 2\sqrt{K_n I_D}$

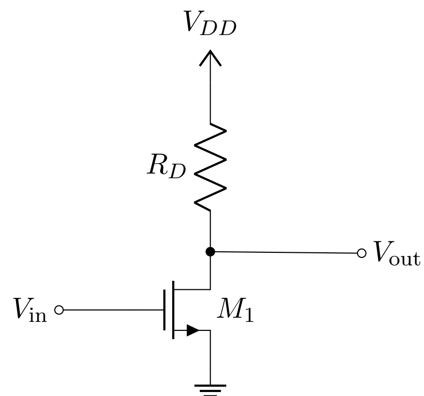
$$\begin{aligned}
g_m^2 &= \left[\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})\right]^2 \\
&= \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2\left[\mu_n C_{ox}\frac{W}{L}\right] \\
&= (2)\left[\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2\right]\left[\mu_n C_{ox}\frac{W}{L}\right] \\
&= 2I_D\left[\mu_n C_{ox}\frac{W}{L}\right] \\
&= 2\mu_n C_{ox}\frac{W}{L}I_D \\
g_m &= \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = 2\sqrt{K_n I_D}
\end{aligned}$$

(c) $g_m = \frac{2I_D}{V_{GS} - V_{TH}}$

$$\begin{aligned}
g_m &= \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH}) \\
&= \frac{1}{2}(2)\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH}) \times \frac{V_{GS} - V_{TH}}{V_{GS} - V_{TH}} \\
&= \frac{(2)\left[\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2\right]}{V_{GS} - V_{TH}} \\
&= \frac{2I_D}{V_{GS} - V_{TH}}
\end{aligned}$$

Learning Objective: Maximum Voltage Swing Configuration

Problem 2. Consider a resistive load common source amplifier below, where $R_D = 1\,\mathrm{k\Omega}$ and $V_{DD} = 5$ V. The n-channel MOSFET has a threshold voltage $V_{\mathrm{TH}} = 0.7$ V, conduction parameter $K_n = 0.5$ mA/V^2. Assume $\lambda = 0$, $\gamma = 0$.



(a) Determine the **range of output voltage V$_{\mathbf{out}}$** to operate the MOSFET M_1 in saturation region.

The maximum V$_{\mathrm{out}}$ occurs when the MOSFET is biased at V$_{\mathrm{in}}$ slightly larger than the threshold voltage, makes a current I$_{\mathrm{DS}}$ close to 0 A, so $\boxed{V_{\mathrm{out}}^{\max} = V_{DD} = 5\ \mathrm{V}}$. The minimum V$_{\mathrm{out}}$ to keep the MOSFET in saturation is when MOSFET is baised at the edge of the saturation region ($V_{\mathrm{DS}} = V_{\mathrm{GS}} - V_{\mathrm{TH}}$), therefore:

$$\begin{aligned}
V_{DD} - I_D R_D &= V_{DS} \\
V_{DD} - K_n (V_{GS} - V_{TH})^2 R_D &= V_{DS} \\
V_{DD} - K_n (V_{DS})^2 R_D &= V_{DS} \\
(5) - (0.5)(V_{DS})^2 (10) &= V_{DS} \\
V_{DS} &= V_{\mathrm{out}}^{\min} = 2.317\ \mathrm{V}
\end{aligned}$$

(b) Determine the **range of input voltage V$_{\mathbf{in}}$** to operate the MOSFET M_1 in saturation region.

The minimum V$_{\mathrm{in}}$ occurs when it's slightly larger than the threshold voltage, so $\boxed{V_{\mathrm{in}}^{\min} = V_{TH} = 0.7\ \mathrm{V}}$. The maximum V$_{\mathrm{in}}$ that keep the MOSFET in saturation occurs when the MOSFET is biased edge of the saturation region ($V_{\mathrm{DS}} = V_{\mathrm{GS}} - V_{\mathrm{TH}}$), with $V_{\mathrm{out}}^{\min}$ calculated above:

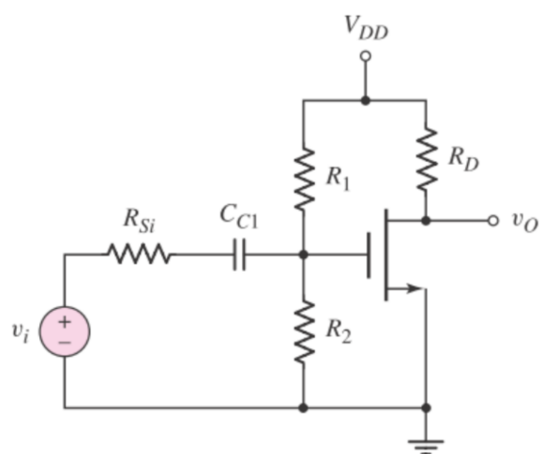
$$\begin{aligned}
V_{DD} - I_D R_D &= V_{\mathrm{out}}^{\min} \\
I_D^{\max} &= \frac{V_{DD} - V_{\mathrm{out}}^{\min}}{R_D} \\
&= \frac{(5) - (2.317)}{1000} = 2.68\ \mathrm{mA} \\
K_n (V_{\mathrm{in}}^{\max} - V_{TH})^2 &= 2.68 \times 10^{-3} \\
(0.5 \times 10^{-3})(V_{\mathrm{in}}^{\max} - 0.7)^2 &= 2.68 \times 10^{-3} \\
V_{\mathrm{in}}^{\max} &= 3.015\ \mathrm{V}
\end{aligned}$$

(c) Assume the MOSFET operates at the center of the saturation region, determine the **Q-point of the MOSFET ($\mathbf{V_{DS}}$, $\mathbf{I_D}$)** and the **input voltage $\mathbf{V_{in}}$**.

$$\begin{aligned}
V_{DS}^Q &= \frac{V_{\text{out}}^{\max} + V_{\text{out}}^{\min}}{2} = 3.659 \text{ V} \\
I_D^Q &= I_D^{\max}/2 = 1.34 \text{ mA} \\
&= K_n(V_{GS}^Q - V_{TH})^2 \\
&= (0.5)(V_{GS}^Q - 0.7)^2 \\
V_{GS}^Q &= 2.337 \text{ V}
\end{aligned}$$

Learning Objective: Common Source Amplifier without R_s

Problem 3. A common-source amplifier circuit has following parameters: $K_n = 0.6$ mA/V^2, $V_{DD} = 3.5$ V, $R_D = 12$ kΩ, $R_1 = 144$ kΩ, $R_2 = 65$ kΩ, $R_{Si} = 12$ kΩ, $V_{TN} = 0.5$ V, and $\lambda = 0.018$ V^{-1}.



(a) Determine I_{DQ}.

$$\begin{aligned}
V_{GSQ} &= V_G - V_S \\
&= (V_{DD})\frac{R_2}{R_1 + R_2} - 0 \\
&= (3.5)\frac{65}{144 + 65} = 1.088 \text{ V} \\
I_{DQ} &= K_n(V_{GSQ} - V_{TH})^2 \\
&= (0.6)(1.088 - 0.5)^2 \\
&= 0.207 \text{ mA}
\end{aligned}$$

(b) Prove that the transistor is biased in the saturation region.

$$\begin{aligned}
V_{DSQ} &= V_{DD} - I_{DQ}R_D \\
&= 3.5 - (0.207)(12) = 1.016 \text{ V} > V_{GSQ} - V_{TH} = 1.088 - 0.5 = 0.588 \text{ V}
\end{aligned}$$

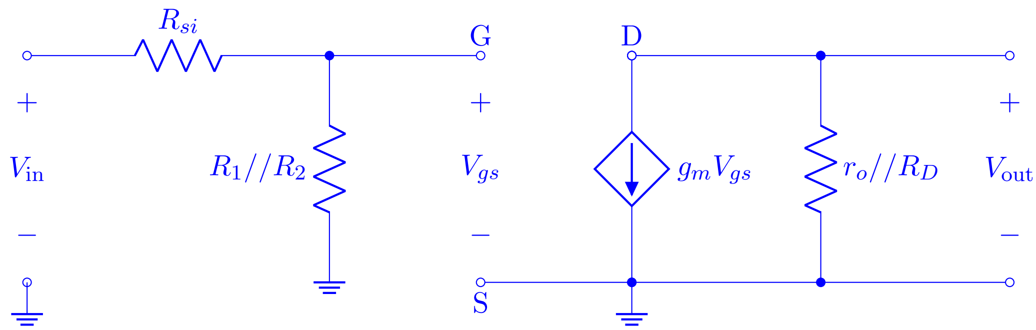
Therefore, the MOSFET is biased in the saturation.

(c) Determine g_m and r_o for the small-signal FET model.

$$g_m = \frac{2I_D}{V_{GS} - V_{TH}} = \frac{2 \times 0.207 \times 10^{-3}}{0.588} = 0.704 \text{ mA/V}$$
$$r_o = [\lambda I_{DQ}]^{-1} = [(0.018)(0.207 \times 10^{-3})]^{-1} = 268.38 \text{ k}\Omega$$

(d) Determine the **input and output resistances** of the common source amplifier.

The given circuit is converted to the following small signal model:



To calculate input resistance R_{in}, make $V_{\text{out}} = 0$ V, then calculate the Thevenin equivalent resistance looking into V_{in} find

$$R_{\text{in}} = R_1//R_2 = 44.784 \text{ k}\Omega$$

To calculate output resistance R_{out}, make $V_{\text{in}} = 0$ V. The dependent current source is open because $V_{\text{in}} = V_{gs} = 0$ V so that $g_m V_{gs} = 0$ A. The Thevenin equivalent resistance looking into V_{out} find

$$R_{\text{out}} = R_D//r_o = 11.486 \text{ k}\Omega$$

(e) Determine the small **signal voltage gain**.

From the small signal model in part (d), do KVL at V_{in} that gives:

$$V_{\text{in}} \times \frac{R_1//R_2}{R_1//R_2 + R_{si}} = V_{gs}$$
$$V_{gs} \left(\frac{R_1//R_2}{R_1//R_2 + R_{si}} \right)^{-1} = V_{\text{in}}$$

Do KCL at V_{out} that gives:

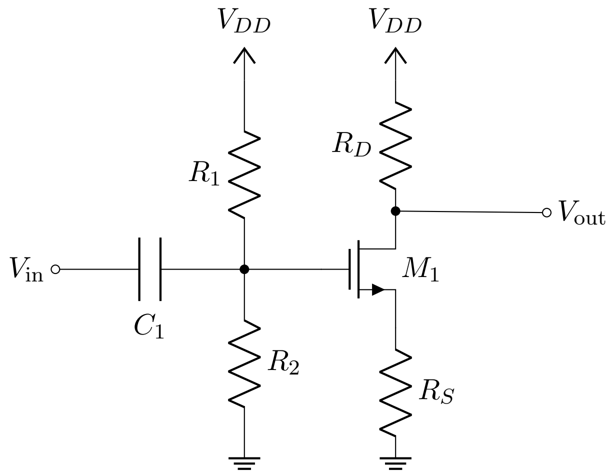
$$V_{\text{out}} = -g_m V_{gs} r_o//R_D$$

Therefore, the small signal gain can be expressed as:

$$\begin{aligned} A_v &= V_{\text{out}}/V_{\text{in}} \\ &= -g_m(r_o//R_D) \left(\frac{R_1//R_2}{R_1//R_2 + R_{si}} \right) \\ &= -6.577 \end{aligned}$$

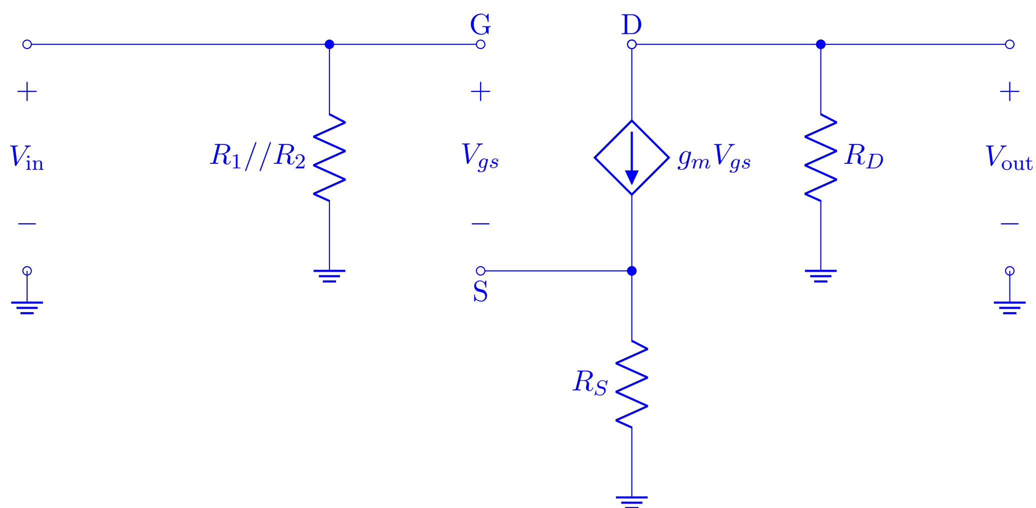
Learning Objective: Common Source Amplifier with R_s

Problem 4. Consider a resistive load common source amplifier below. Assume the MOSFET operates in saturation, $R_S \neq 0$, $\lambda = 0$, $\gamma = 0$.



(a) Plot the **small signal equivalent circuit** of the given amplifier.

The given circuit is converted to the following small signal model:



(b) Determine the **input and output resistances** of the common source amplifier.

To calculate input resistance R_{in}, make $V_{out} = 0$ V, then calculate the Thevenin equivalent resistance looking into V_{in} find

$$R_{in} = R_1 // R_2$$

To calculate output resistance R_{out}, make $V_{in} = 0$ V. Do KVL at V_{in} gives:

$$V_{gs} + g_m V_{gs} R_S = V_{in} = 0$$
$$V_{gs}\left(1 + g_m R_S\right) = 0$$

So that $V_{gs} = 0$ V, due to $(1 + g_m R_S)$ can't be zero. So that the output resistance is:

$$R_{out} = R_D$$

(c) Derive an expression of the **output voltage** and **small-signal voltage gain**.

From the small signal model in part (a), do KCL at V_{out} that gives:

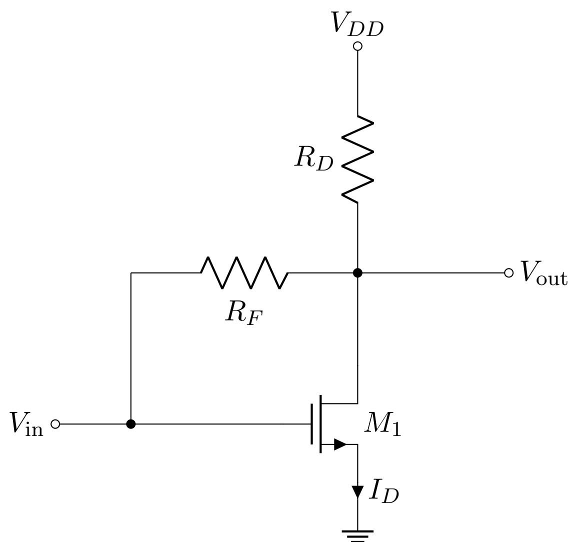
$$V_{\text{out}} = -g_m V_{gs} R_D$$

Therefore, the small signal gain can be expressed as:

$$A_v = V_{\text{out}}/V_{\text{in}} = \frac{-g_m R_D}{1 + g_m R_S}$$

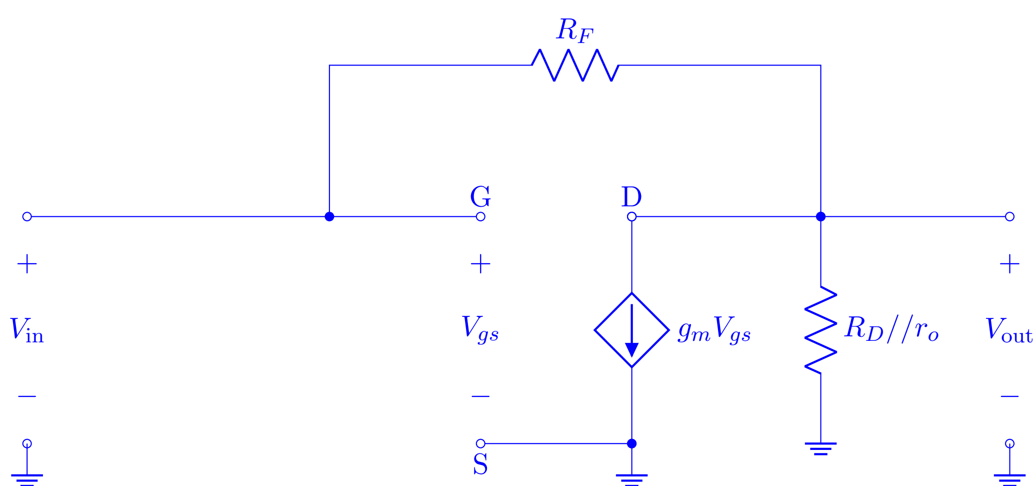
Learning Objective: Common Source Amplifier

Problem 5. Consider a resistive load common source amplifier below. Assume the MOSFET operates in saturation, $\lambda \neq 0$, $\gamma = 0$.



(a) Plot the **small signal equivalent circuit** of the given amplifier.

The given circuit is converted to the following small signal model:



(b) Determine the expression of **output voltage** and **small-signal voltage gain**.

From the small signal model in part (a), do KCL at V_{out} gives:

$$\begin{aligned}
\frac{V_{\text{out}}}{R_D//r_o} + g_m V_{gs} + \frac{V_{\text{out}}}{R_F} - \frac{V_{gs}}{R_F} = 0 \\
V_{\text{out}} \left(\frac{1}{R_D//r_o} + \frac{1}{R_F} \right) = V_{gs} \left(\frac{1}{R_F} - g_m \right) \\
V_{\text{out}} &= V_{gs} \left(\frac{1}{R_F} - g_m \right) \left(\frac{1}{R_D//r_o} + \frac{1}{R_F} \right)^{-1} \\
&= V_{gs} \left(\frac{1}{R_F} - g_m \right) (R_D//r_o//R_F) \\
A_v &= \left(\frac{1}{R_F} - g_m \right) (R_D//r_o//R_F)
\end{aligned}$$