Physics 4A

# Lab 12: Simple Harmonic Oscillator

**Objective:** To measure the spring constants of an individual spring, springs in parallel and springs in series.

## Theory:

For a simple "spring & mass" oscillating system, the angular frequency of the oscillations is determined by the spring constant, *k*, and the mass, *m*:

$$\omega^2 = \frac{k}{m}$$

We also, by definition, can relate the angular frequency to the period, *T*, of the oscillations:

$$\omega T = 2\pi$$

If we combine these two expressions, eliminating the angular frequency by substitution, we get:

$$\left(\frac{2\pi}{T}\right)^2 = \frac{k}{m}$$

Or:

$$T^2 = \frac{4\pi^2}{k}\, m$$

Note that this expression suggests that for a given spring, i.e. if we hold *k* constant, the period squared should be proportional to the mass of the oscillator. This expression assumes that the spring behaves as an *ideal spring*, so for this experiment we will consider:

- Does our spring behave as an ideal spring, i.e. is $T^2$ proportional to *m* for our system?
- Can we use the results of our data to calculate *k*, the spring constant?

To answer these questions, we will choose several values of *m* and measure the corresponding values of the period. We will then graph $T^2$ vs *m*, and find the best-fit straight line. We can then use the slope of this line to calculate *k*, the spring constant.

## Procedure:

## Part 1: Individual Springs

### 1.1. Introduction

Watch the following video to learn how to collect the data from the videos:
https://youtu.be/kbWoB3wXRMw

### 1.2. Data

To get the mass and times for Spring #1, watch the following video:
https://youtu.be/VB8vIL2751A

Record the mass as well as the start time and end time for twenty oscillations for each of the six trials. For each of the 6 trials, the mass at the end of the spring is different.

Calculate the period, $T$, for each trial, as well as $T^2$. Note $T^2$ is just the square of the period.

### 1.3. Graphs

Create a graph of $T^2$ vs $m$ and find the best-fit line. Your data should be perfectly linear; if it is not, re-measure the errant data.

### 1.4. Calculations

Use the value of your slope to calculate the spring constant of Spring #1.

Note that our equation shows that if $T^2$ is on the y-axis and $m$ is on the x-axis, then the slope of the graph should be:

$$slope = \frac{4\pi^2}{k}$$

And so:

$$k = \frac{4\pi^2}{slope}$$

You should see, from your graph, that the unit of the slope is **$s^2$/kg**, so the unit of $k$ will be **kg/$s^2$**.

### 1.5. Spring #2

Repeat sections 1.2 – 1.4 for Spring #2. To get the mass and times for Spring #2, watch the following video: [https://youtu.be/SUnAqj8b_CY](https://youtu.be/SUnAqj8b_CY)

## Part 2: Springs in Parallel & Series

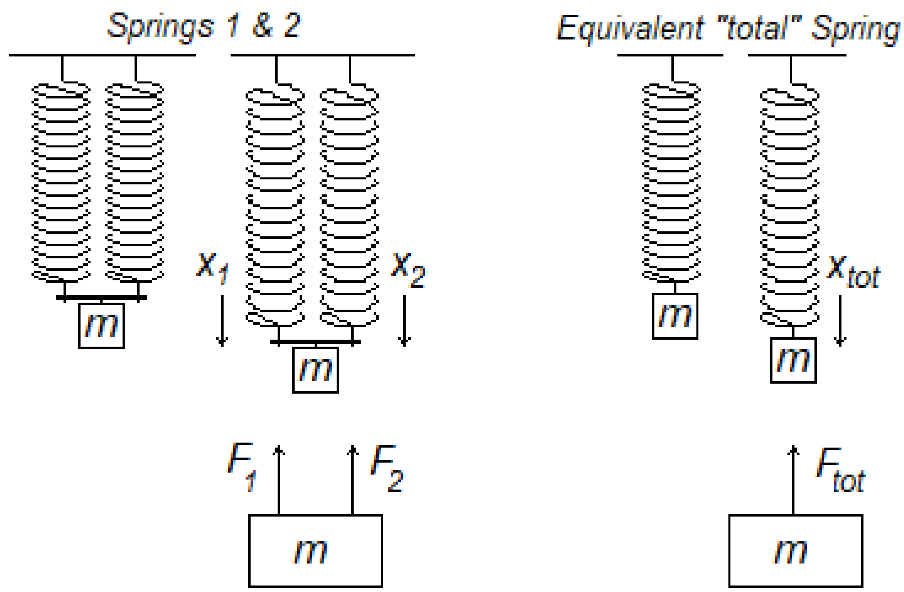
### 2.1. Introduction

We can now take the experiment one step further and consider how the springs behave when they are working together. There are two ways to arrange the springs:

> ***In "parallel"***: *the springs are "side by side"*
> ***In "series"***: *the springs are connected "end to end"*

We first need to create a theory to define what we should *expect*… and this will allow us to compare our measured results to the expectation. To accomplish this, we consider that the springs working together behave as a theoretical "total" spring, i.e. a single spring which would exhibit the same behavior as that of the two springs working together.

In this diagram below, the two springs connected in *parallel* are on the left, and the equivalent "total" spring is on the right:




If we compare the left and right pictures in the diagram, and consider that they are "equivalent" situations, we can see that:

- $F_1$ and $F_2$ combined must be the same as $F_{tot}$
- $x_1$ $x_2$ and $x_{tot}$ must all be equal

We can express these observations as equations, and we can also write equations for the relationship between the force each spring applies, the spring constant of the spring, and the "stretch" of the spring:

$$F_1 = k_1 x_1 \qquad F_2 = k_2 x_2 \qquad F_{tot} = k_{tot} x_{tot}$$

$$F_1 + F_2 = F_{tot} \qquad x_1 = x_2 = x_{tot}$$

We can substitute the expressions for $F_1$ $F_2$ and $F_{tot}$ :

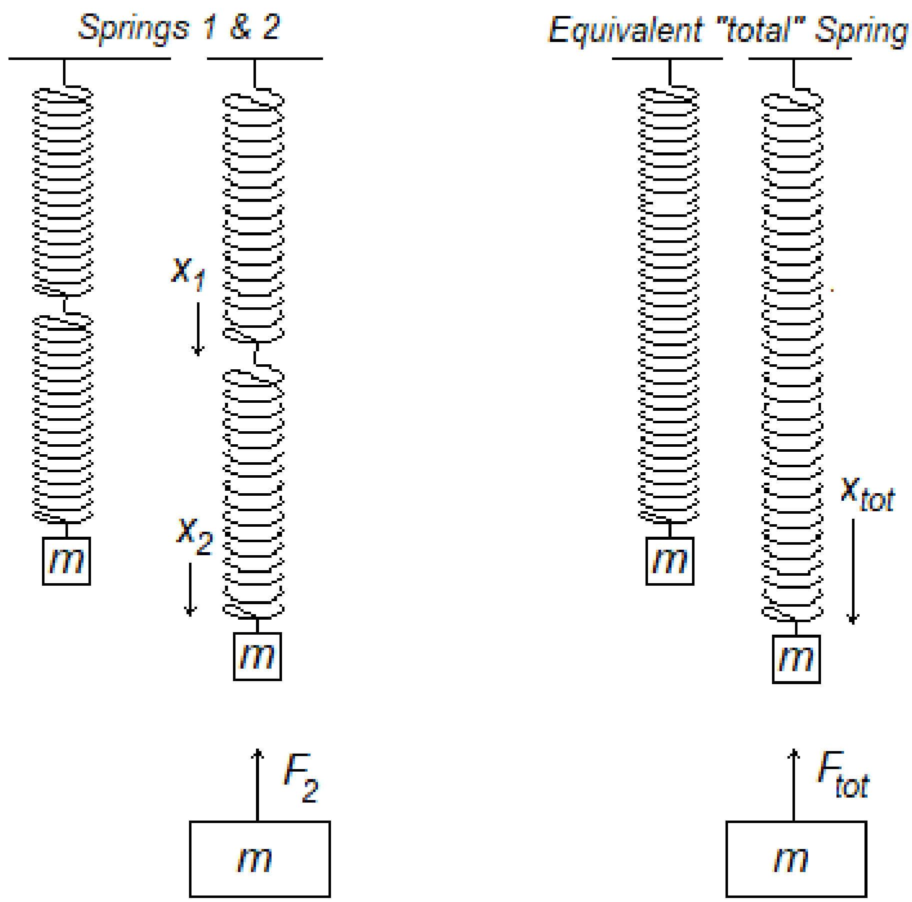
$$k_1 x_1 + k_2 x_2 = k_{tot} x_{tot}$$

And since $x_1$ $x_2$ and $x_{tot}$ are all equal, we can divide both sides by $x_1$ to arrive at the final result:

$$k_{tot} = k_1 + k_2$$ ***springs in parallel***

This result is pretty intuitive: the springs "work together", so the effect of their spring constants add together. In essence this means the two springs act as one "stronger" spring which has the "strength" of the two springs combined. We will use this expression to calculate our *expected* value for the spring constant of our two springs in *parallel*.

We can also consider two springs in *series*. The diagram below shows the two springs together, and the equivalent "total" spring




Notice in this arrangement that Spring #1 is not in direct contact with the block. So the only force acting on the block in the left picture is that of Spring #2. In the right picture, the force acting on the block is the "total" force. *This means that the total force must be equal to the force applied by Spring #2.*

What about Spring #1? Note that Spring #1 pulls on Spring #2, which means Spring #2 also pulls on Spring #1… according to Newton's 3rd Law. This law also tells us that the *force Spring #1 exerts must be equal to the force that Spring #2 exerts*. From these observations, we can write:

$$F_1 = k_1 x_1 \qquad F_2 = k_2 x_2 \qquad F_{tot} = k_{tot} x_{tot}$$

$$x_1 + x_2 = x_{tot} \qquad F_1 = F_2 = F_{tot}$$

We can substitute for $x_1$ $x_2$ and $x_{tot}$ :

$$\frac{F_1}{k_1} + \frac{F_2}{k_2} = \frac{F_{tot}}{k_{tot}}$$

And since $F_1$ $F_2$ and $F_{tot}$ are all equal, we can divide both sides by $F_1$ to arrive at the result:

$$\frac{1}{k_{tot}} = \frac{1}{k_1} + \frac{1}{k_2}$$

It is useful to rearrange this equation to solve for $k_{tot}$ directly by combining the terms on the right side with a common denominator, which would be $k_1$ $k_2$, and then inverting both sides of the equation. The result is:

$$k_{tot} = \frac{k_1\ k_2}{k_1 + k_2} \qquad \textbf{\textit{springs in series}}$$

### 2.2. Data

To get the mass and times for Springs in Parallel, watch the following video:
https://youtu.be/5cuLm8wiQn4

To get the mass and times for Springs in Series, watch the following video:
https://youtu.be/uSWoV-vnDMQ

Calculate the period, $T$, for each trial, as well as $T^2$.

### 2.3. Graphs

Create a graph of $T^2$ vs $m$ and find the best-fit line for Springs in Parallel and Springs in Series. Make an individual graph for each.

Find the *measured* value of the "total" spring constant (i.e. using the slope of the best-fit line from the graph).

### 2.4. Calculations

Calculate the expected value of the "total" spring constant for Springs in Parallel and Springs in Series, then calculate the percent error.

### 2.5. Questions

Answer the questions provided in the Excel spreadsheet, under the tab labelled "Questions", and submit to Canvas.