Physics 4A

Lab 6: Circular Motion

Objective: To verify that the force vector required to keep a body of mass m moving in a circle of radius r with a constant tangential velocity v has a magnitude of $m\frac{v^2}{r}$.

Theory:

An object moving in a circular path is accelerating because its direction is changing, and therefore its velocity is changing. According to theory, the acceleration associated with the circular motion is

$$\boldsymbol{a} = \frac{\boldsymbol{v}^2}{\boldsymbol{r}}$$

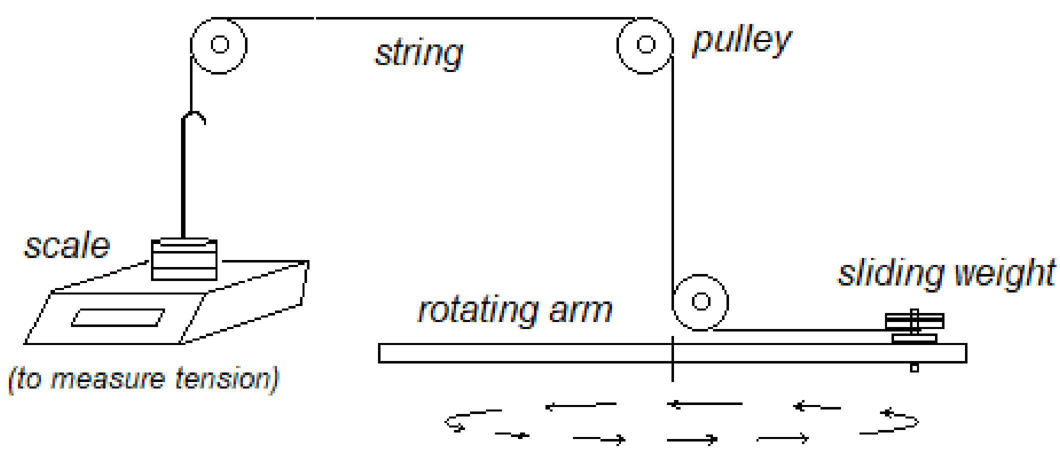
If we combine this expression with Newton's Second Law,

$$\boldsymbol{F} = \boldsymbol{m}\,\boldsymbol{a}$$

We create an expression for the *centripetal force*, i.e. the force acting toward the center of the circle that is responsible for causing the object's acceleration:

$$F_c = \boldsymbol{m}\frac{\boldsymbol{v}^2}{\boldsymbol{r}}$$

We would like to put this theory to the test, by creating an experiment to measure the quantities represented in this expression and verify that the system behaves as we expect. A diagram of the apparatus we will use is given below.



A small weight is free to slide radially on a rotating arm, which is driven by a small electric motor. The sliding weight, which is our "object" that we will consider in circular motion, is attached to a wire, where the **tension of the wire provides the centripetal force**. The wire passes under a small pulley at the center of rotation, and then over two more pulleys. The other end of the wire is attached to a heavy weight that sits on a digital scale. **The tension in the wire**

can be measured by the reading on the scale, as the tension pulls upward on the heavy weight that rests on the scale.

A few definitions for this experiment:

- *m*: mass of the sliding weight, our object in circular motion
- m_{scale}: reading on the scale, which we will use to determine the tension in the wire.
- *T*: tension in the wire.
- *r*: radius of the circle defined by the motion of the sliding weight
- *t*: time for the sliding weight to complete one revolution

We can now use these definitions to modify the equation provided by our theory. We can replace "F" with the more specific "T", the tension we will measure. And we can replace *v*, the speed of our object, with an expression that includes the distance and time for one complete circle of motion:

$$\boldsymbol{T = m\frac{v^2}{r}} \qquad\qquad \boldsymbol{v = \frac{2\pi\, r}{t}}$$

We will be able to measure *t* directly, using a photogate that is mounted underneath the rotating arm. But we cannot measure *v* directly, so we will eliminate *v* from our equation by substitution:

$$\boldsymbol{T = m\frac{\left(\frac{2\pi\, r}{t}\right)^2}{r}}$$

Simplify this expression:

$$\boldsymbol{T = \frac{4\pi^2 mr}{t^2}}$$

Our goal is to create an expression that (a) represents the theory of circular motion and (b) allows us to measure the quantities in the expression. This expression has four quantities: tension in the wire, mass of the object, time for one revolution, and radius of the circular path. We can easily measure the first three of these.

But accurately measuring the radius presents a challenge. This is because as the speed of the apparatus increases, the tension in the wire increases, and the wire stretches slightly. This stretching of the wire allows the radius of the circular path to increase. Which means that the radius cannot be directly determined while the apparatus is in motion.

However, we can measure several values of the radius while the apparatus is motionless and record the corresponding value of tension associated with that radius. This will allow us to indirectly determine the radius of the object's circular path by relating it to the tension we measure while the object is in motion.

We will assume (and confirm from our measured data) that the radius and tension have a linear relationship, i.e. that we can write:

$$\boldsymbol{r = \alpha T + \beta} \qquad \text{where } a \text{ and } b \text{ are constants.}$$

We expect that a graph of our measured data of *r* vs *T* will result in a straight line, with slope and y-intercept of *a* and *b* , respectively. We can combine this equation with our previous equation to eliminate *r* algebraically:

$$\boldsymbol{T = \frac{4\pi^2 m\,(\alpha T + \beta)}{t^2}}$$

This equation looks a little complicated, but we can invoke the power of algebra to tidy it up just a little. By multiplying both sides by t^2, dividing both sides by $4p^2m$, dividing both sides by T, the equation becomes:

$$\boldsymbol{\frac{t^2}{4\pi^2 m} = \frac{\alpha T + \beta}{T}}$$

We can then separate the right side of the equation into two terms:

$$\boldsymbol{\frac{t^2}{4\pi^2 m} = \alpha + \beta T^{-1}}$$

And then we can solve for T^{-1} by subtracting *a* from both sides and dividing everything by *b*:

$$\boldsymbol{T^{-1} = \left(\frac{1}{4\pi^2 \beta m}\right) t^2 - \left(\frac{\alpha}{\beta}\right)}$$

Note that *a* *b* and *m* are all constants for a given trial. Which means that only T and t will change during a trial, i.e. as the speed of the rotation is increased, the tension should increase and the time should decrease (because shorter time, faster speed.) This equation, which is just an algebraic modification of our original expression for the centripetal force, predicts that:

If we measure corresponding values of tension and time and we create a graph of T^{-1} vs t^2 we expect the graph to form a straight line with *slope* = $\boldsymbol{\frac{1}{4\pi^2\beta m}}$ and *y-intercept* = - $\boldsymbol{\frac{\alpha}{\beta}}$.

Procedure:

Part 1: Radius as a Function of Tension

1.1. Data

Watch the following video that gives the calibration for radius as a function of tension:
https://youtu.be/5uXatpwsN3M

In the video, the sliding weight is held at specific points, and the value of r is called out. The camera points at the reading on the scale. As instructed in the video, record the value of r called out, and then pause the video to record the corresponding value of m_{scale}, the reading on the scale. Note that the reading on the scale will be negative; ignore then negative sign.

Insert the data into the table under 'Radius of Circular Motion,' under the appropriate columns.

1.2. Tension Calculation

Calculate the middle column by using the data from the first column. The tension is equal to the reading on the scale times "g." To get the tension in Newtons, convert grams to kilograms. An example of what might be typed into Excel would be "=A4*9.81/1000". Use Excel's fill down feature so you only type the equation in once.

1.3. Graph r v. T

Create a graph of r v. T and find the best-fit line. Display the corresponding equation on the graph. The slope and y-intercept of this equation are the values of α and β. Record these values in the appropriate place on the Excel sheet, and calculate the value of $-\alpha / \beta$ also.

Part 2: Time and Tension

2.1. Measure dimensions of a block of wood

Watch the following videos, giving time and corresponding tension.

Trial 1 https://youtu.be/yRxqWNXPcKU
Trial 2 https://youtu.be/PiLFVn_Jdjo
Trial 3 https://youtu.be/zCgzhRYVXzA
Trial 4 https://youtu.be/PgaB3nixIPw

Within each video, the mass of the object is given for that trial. We then start the apparatus turning at a constant speed. When prompted in the video, pause the video and record m_{scale} and t from the scale and computer screen in the video. Resume playing the video as we increase the speed and then prompt you to pause and take another measurement.

Repeat the above process to collect seven values of m_{scale} and t for each of the four trials, and enter these values into the excel spreadsheet.

Note: All calculations should be completed in Excel!

2.2. Tension Calculation

Calculate the middle column by using the data from the second column. The tension is equal to the reading on the scale times "g." Remember to convert grams to kilograms to get the tension in Newtons. See Section 1.2 for a sample of what would be entered in Excel.

Calculate the fifth column by using the tensions just calculated in the middle column. T^{-1} is a small value, so it will be written as a value $\times 10^{-3}$. To convert, $T^{-1} \times 10^{-3}$ = 1000/T. An example of what would be typed in Excel would be "=1000/C22". Use the fill down function for the remainder of the values for a single trial, and use copy/paste to calculate values for the other trials.

2.3. Time Calculation

Calculate the fourth column by using data from the first column. Because t^2 is a small value, it will be written as a value $\times 10^{-3}$. To convert, $t^2 \times 10^{-3}$ = 1000*t^2. An example of what would be typed in Excel would be "=1000*A22^2". Use the fill down function for the remainder of the values, and use copy/paste to calculate values for the other trials.

2.4. Plot Tension vs Time

Highlight the last two columns in the first table (Trial 1), t^2 $(\times 10^{-3}\ s^2)$ and T^{-1} $(\times 10^{-3}\ N^{-1})$, and create a scatter plot. Make sure the scatter plot has x and y axis, and that they are properly labelled. Keep the plot legend. Include a descriptive title to the plot. Find the best fit line to the data, and make sure the equation is included on the graph.

The data for the other trials will be included on the same graph as Trial 1. Right click on the graph, and choose 'Select Data'. Type in a name for Series 1 as Trial 1. Click on the little '+' icon below the legend entries box, type in the name Trial 2 for the second set of data. Click on the icon to the right of 'X values' (the icon has a small red arrow), the box will minimize and allow you to see the entire spreadsheet. Use your cursor to highlight the t^2 data from Trial 2. Click on the little icon on the right of the collapsed Select Data Source, this will take you back to the full box menu, then click on the icon to the right of 'Y values'. Highlight the T^{-1} data from Trial 2, and click on the little icon on the right of the collapsed Select Data Source. Click 'ok' to return to the graph, where data from the second Trial will appear.

Repeat the above steps to include all 4 trials on the same data sheet. Click on each data set to include a best fit line and equation. Move the equations around so it is clear which data set the equation belongs to. Change the x and y axis on the graph so that the data fills up a majority of the graph.

2.5. Expected Slope

Using the expression for the expected value of the slope, i.e. $slope = \frac{1}{4\pi^2 \beta m}$, calculate the expected value of the slope for each trial. Keep in mind that β is in millimeters and needs to be converted to meters. Also keep in mind that mass is measured in grams and needs to be converted to kilograms. A sample of what this may look like when entered into Excel is "=1000000/(4*PI()^2*F4*14.5)". In order for Excel to use the value of π, parenthesis must follow. If you want to copy/paste this equation and use it for the other trials, it is a good idea to place a $ before F and 4 (looks like F4) so the value of beta does not change.

2.6. Percent Error

Compute the percent error between the measured value of the slope and the expected value of the slope for all four trials. If the percent error is greater than 20%, then there is a problem with your calculations. Take time to track down the error. If you are having trouble finding the error, send me (your instructor) a copy of your Excel spreadsheet.

2.7. Results

Answer the questions provided in the Excel spreadsheet. The spreadsheet will be the only item submitted through Canvas.