

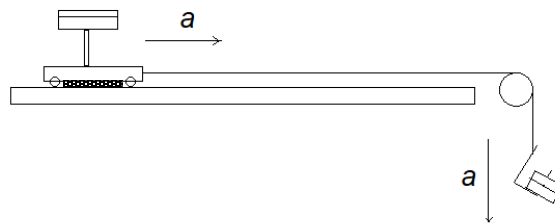
Lab 5: Newton's Second Law with Friction

Objective: To measure the forces and acceleration to validate Newton's Second Law.

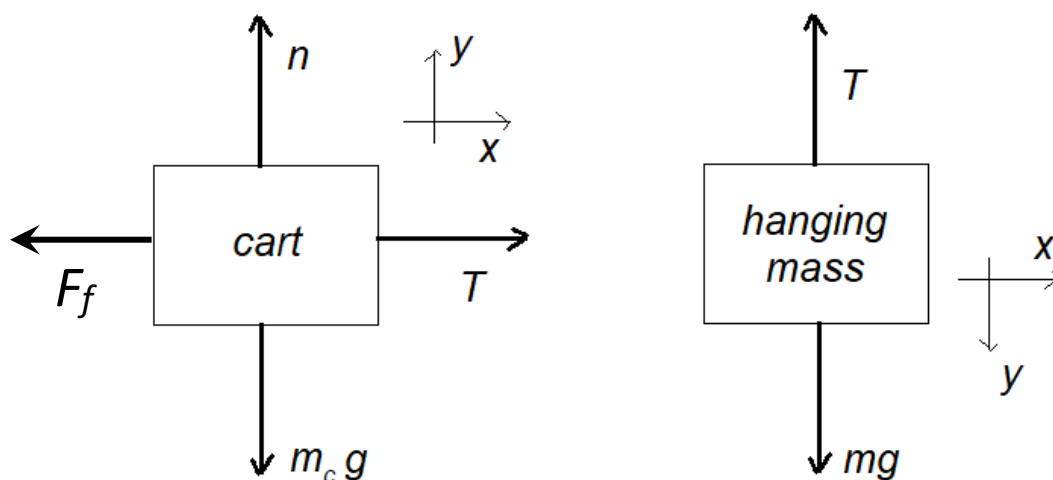
Theory:

In Lab 4, we measured the acceleration of a cart along a horizontal track. The acceleration of the cart was due to the tension from a string that pulls the cart along a horizontal track. Using the length of the tab on the cart, i.e. the tab that passes through the photogate, which is 11.0 cm, we were able to calculate the average speed of the cart passing through each of the two photogates, and then the acceleration of the cart.

For this experiment, we will measure the times and calculate the velocities and acceleration of the cart exactly the same way we did in Lab 4. The experimental setup is nearly identical to Lab 4, where the only difference is that now the cart is sliding along on a friction pad.



We can derive an expression for the acceleration by drawing a *free body diagram* for the cart and another for the hanging mass. The free body diagram of the hanging mass is the same as Lab 4, however now there is friction acting on the cart in the opposite direction of motion.



We can write the equations for each of these *free body diagrams*:

Cart

$$x \text{ equation: } T - F_f = m_c a \qquad y \text{ equation: } N - mg = 0$$

Hanging mass

$$x \text{ equation: } \text{nothing!} \qquad y \text{ equation: } mg - T = ma$$

The force of friction is given by the equation $F_f = \mu_K N$, where the normal force, N , is given by $N = m_c g$. If we rewrite the mass of the cart in terms of the total mass, M , and the hanging mass, m , then the force of friction can be rewritten as $F_f = \mu_K (M - m)g$.

Note that the *x equation* for the cart and the *y equation* for the hanging mass include the acceleration we want to measure, and both equations include the tension in the string. We can combine these equations algebraically, eliminating the tension by substitution:

$$T = m_c a + \mu_K (M - m)g \qquad mg - T = ma \qquad \text{so} \qquad mg - \mu_K (M - m)g = Ma$$

We can rearrange this equation: $g(\mu + 1)m - \mu_K M g = Ma$

Then solve for acceleration: $a = \frac{g(\mu_K + 1)}{M} m - \mu_K g$.

This equation tells us that the acceleration of the system is related to the mass on the hanger and the coefficient of kinetic friction. By plotting the hanging mass v. acceleration the slope of the resulting equation is $g(\mu_K + 1)/M$, and the y-intercept is $-\mu_K g$. If we solve for the coefficient of kinetic friction, then

$$\mu_K = \frac{M(\text{slope})}{g} - 1.$$

We can use this equation to calculate what the coefficient of kinetic friction is for the cart. Note that there is not an expected coefficient of friction for the system. Coefficients of friction depend on the types of the material, the temperature of the system, and the weight of the object that is sliding. This means that coefficients of friction are determined through measurement, not through physical first principles. The coefficients that you will be calculating today are measured values, not theoretical values. Also note that the coefficient of kinetic friction can be calculated if the y-intercept is known.

Procedure:

1. Background

Recall the time t_1 , t_2 and t_3 can be used to calculate:

v_1 : the average speed of the cart as it passes through the first photogate

v_3 : the average speed of the cart as it passes through the second photogate

We can then use v_1 and v_3 to calculate the acceleration of the cart for each run:

$$a = \frac{v_3 - v_1}{t_2 - \frac{1}{2}t_1 + \frac{1}{2}t_3}$$

2. Setup and Equipment

Watch the following video, which introduces the equipment and how data will be collected: <https://youtu.be/OcyA9q84hGA>. Five trials of data will be taken, where each trial increases the mass on the hanging cart, thereby increasing the acceleration of the cart. For each trial, the cart will roll across the track 4 times, for 5 different hanging masses, totaling 20 runs for each cart.

3. Data Collection

Open up the excel spreadsheet from lab 4. Rename and save the spreadsheet for lab 5.

The mass of the cart is obtained in this video: <https://youtu.be/hEy1wRpnW44>. Type the total mass and length of the tab into excel.

Watch the following video and enter the hanging mass, t_1 , t_2 , and t_3 for each of the trials: <https://youtu.be/soAUuSKHtc>.

4. Velocity & Acceleration Calculations

The velocity and acceleration calculations should automatically update when the new times are input into Excel. Double check the equation entered for acceleration is correct, and looks like: “ $=(v_3-v_1)/(t_2 - (t_1/2) + (t_3/2))$ ”. Be sure to include the parenthesis so Excel knows the order of operations. The average and standard deviation of the acceleration should also automatically update.

5. Hanging Mass v. Acceleration Table

Complete the table of hanging mass v. acceleration. You can type the values of hanging mass and acceleration in, or you can type “=” then click on the value you want in that cell. For instance, the hanging mass for trial 1 would just be “=C7”.

6. Hanging Mass v. Acceleration Graph

Based on the new data, the hanging mass v. acceleration graph should automatically update. Set the x and y axis so that the data covers a majority of the graph area.

7. Expected Slope

Within the section labelled 'Calculations', add in a label that states, 'Calculate the coefficient of friction using the slope of the best fit line', then calculate the coefficient of friction using the slope. Keep three significant figures in your answer. Add another label that states, 'Calculate the coefficient of friction using the y-intercept of the best fit line', then calculate the coefficient of friction using the y-intercept. Keep three significant figures in your answer. Hint: coefficients of friction are between 0 and 1. If your coefficient is less than 0, or greater than 1, there is a mathematical error that needs to be fixed.

8. Laboratory Report

The results for this lab will be written in a lab report, in the same style as previous labs. However, for the data section, insert the graph into the word document. Make sure the line equation is included. The best way to ensure the line equation is included is to take a screenshot of the graph, then insert the screenshot into Word. The remainder of the excel spreadsheet will not be submitted.

9. Results

Answer the following questions in the results section of your laboratory report.

- The uncertainty in the coefficient calculated using the slope is ± 0.05 . What was the coefficient of friction using the slope of the line, with uncertainty?
- The uncertainty in the coefficient calculated using the y-intercept is ± 0.02 . What was the coefficient of friction using the y-intercept, with uncertainty?
- Do your two values of the coefficient of friction match within uncertainty?
- There are two main types of error within a laboratory experiment, systematic error and random error. Systematic errors are instrumental or methodological mistakes causing the data to be skewed or lopsided. Random errors are caused by uncontrollable fluctuations of variables that change the experimental results slightly each time the experiment is run. Hypothetically, let us say that the coefficient of friction calculated using the slope of the line did not match the coefficient of friction calculated using the y-intercept within uncertainty. What type of error is the most likely culprit?