Physics 4A

Lab 9: Newton's Second Law in Rotation

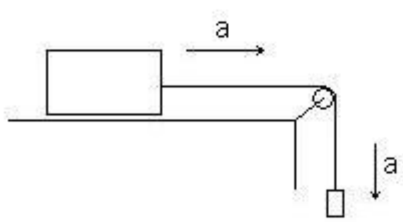
Objective: To measure the forces and accelerations to validate Newton's Second Law in rotation, and to validate the sum of individual moment of inertia gives the total moment of inertia.

Theory:

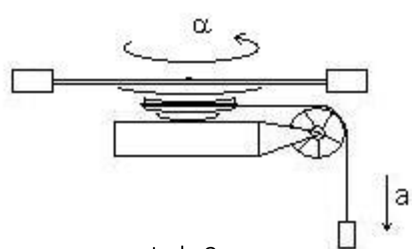
We can describe the motion of objects that rotate (i.e. "spin" on an axis) using the same definitions, adapted for rotational motion, that we have used for objects that move in a straight line. That is, for "linear" motion we defined position, velocity, and acceleration; for rotational motion we define position, angular velocity, and angular acceleration.

For objects in linear motion, Newton's Second Law tells us that forces acting on the object cause its acceleration. For an object in rotational motion, Newton's Second Law tells us that *torque* acting on the object causes its *angular* acceleration.

In Lab 4 we demonstrated Newton's Second Law for an object in linear motion: we attached a string to a cart on a track and a hanging mass and allowed the system to accelerate. In Lab 9, we will attach a string to a rotating apparatus and a hanging mass; as the hanging mass falls, the tension of the string will create a torque on the rotating apparatus that will cause it to rotate.



Lab 4



Lab 9

Newton's Second Law in rotation has exactly the same form as for linear motion. That is:

Linear: *Force* = *mass • acceleration*

Rotation: *Torque* = *moment of inertia • angular acceleration*

In terms of variables, the latter equation is $\tau = I\alpha$ where τ and α are the Greek letters "tau" and "alpha". We use I to represent the moment of inertia, which plays the same role in rotational motion that mass does for linear motion.

The torque applied to a rotating object is defined as: $\tau = rF\sin\phi$

where F is the force applied to the object, r is the distance from the point of rotation to the point where the force is applied, and ϕ is the angle between r and F. Note that if a force is applied at the point (or axis) of rotation, the torque will be zero (because r is zero).

For Lab 9, we can consider three forces acting on the rotating apparatus: gravity, normal force and the tension of the string. Gravity and normal force each act at the center of the apparatus, so r is zero for each of these two forces and they exert no torque (i.e. gravity and normal force do not cause the apparatus to rotate). The tension acts at the edge of the clear plastic spool at a distance equal to the radius of the spool. The tension pulls along the edge of the spool (i.e. in a "tangential" direction), so the angle between r and F is 90 degrees. From these observations, we can claim that the torque acting on the apparatus is:

$\tau = rT$ where r is the radius of the clear spool and T is the tension of the string.

Just as the torque acting on the object depends on the forces and where those forces are applied, **the *moment of inertia* of a rotating object is a measure of the mass of the object and how it is distributed**. Mass that is further from the center of rotation (i.e. the axis of rotation) is more difficult to rotate, because it moves in a larger circle, than mass that is closer to the axis. For this reason, the moment of inertia of an object is defined by its mass and its linear dimensions. The greater the mass and the greater the size of the object, the greater its moment of inertia.

This experiment has two parts:

Part 1: Record initial measurements of the mass and linear dimensions of the rotating apparatus, and calculate the expected value of the moment of inertia for each of three trials.

Part 2: For each of three trials, measure the moment of inertia of the rotating apparatus by applying a torque to the system (by the tension in the string) and measuring the corresponding angular acceleration.

For the rotating apparatus: $rT = I\alpha$ (Obtained by equating previous definitions of torque.)

For the hanging mass: $mg - T = ma$ (Obtained by the summation of forces.)

The string and hanging mass have the same linear acceleration, and the string has the same linear acceleration as a point on the edge of the clear plastic spool. The spool also has the same angular acceleration as the rotating apparatus. We can connect the linear acceleration and angular acceleration of a point on the edge of the spool:

$a = \alpha r$ where r is the radius of the clear spool.

We can eliminate a from the expression for the hanging mass:

$$mg - T = m\alpha r$$

Solve for T for the rotating apparatus:

$$T = \frac{I\alpha}{r}$$

And then eliminate T:

$$mg - \frac{I\alpha}{r} = m\alpha r$$

We will measure several values of m and the corresponding value of α for each. We would like to be able to graph these two variables as a straight line, but the equation above is too complicated. Fortunately, a little bit of algebra can help. First, we'll rearrange the equation to solve for α:

$$mg = m\alpha r + \frac{I\alpha}{r} \qquad\qquad \alpha = \frac{mg}{mr + I/r}$$

This will not work for a straight line graph, as "m" appears in the numerator and denominator on the right side. So, we use a clever algebraic trick: *invert both sides of the equation!*

$$\alpha^{-1} = \frac{mr + I/r}{mg} = \frac{r}{g} + \frac{I}{mgr}$$

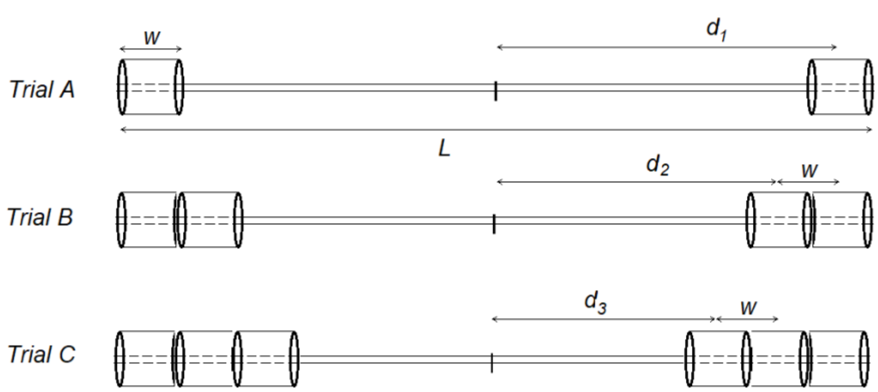
Or:

$$\alpha^{-1} = \frac{I}{gr} m^{-1} + \frac{r}{g}$$

This equation has the form $y = (slope)x + (y - int)$ where $\boldsymbol{\alpha^{-1}}$ is the "y" and $\boldsymbol{m^{-1}}$ is the "x". So, if we graph $\boldsymbol{\alpha^{-1}}$ vs $\boldsymbol{m^{-1}}$, we expect to get a straight line with a slope of $I\,/\,gr$.

We will conduct three "trials" for the experiment. For each trial, the rotating "apparatus" will be a long thin rod with brass weights at each end. The difference between the trials will be the number of brass weights:

Trial A: long thin rod with ***one*** brass weight on each end
Trial B: long thin rod with ***two*** brass weights on each end
Trial C: long thin rod with ***three*** brass weights on each end



Note that if we measure L, the length of the rod, and w, the width of one piece of brass, we can calculate:

$$d_1 = \frac{1}{2}L - \frac{1}{2}w = \frac{1}{2}(L - w)$$

$$d_2 = d_1 - w$$

$$d_3 = d_2 - w$$

The moment of inertia for each trial will be the sum of the moment of inertia of the rod and the moment of inertia of the masses, as moments of inertia can be added to obtain the total moment of inertia of the system.

If we also measure m_{rod}, the mass of the rod, and m_{brass}, the mass of a brass weight, we can calculate the moment of inertia for each trial:

For Trail A: $I_A = I_{rod} + 2I_1$ where $I_{rod} = \frac{1}{12}m_{rod}L^2$ and $I_1 = m_{brass}d_1^2$
Note that I_{rod} is the same for all three trials.

For Trail B: $I_B = I_A + 2I_2$ where $I_2 = m_{brass}d_2^2$.

For Trail C: $I_C = I_B + 2I_3$ where $I_3 = m_{brass}d_3^2$.

Procedure:

Part 1: Expected Moment of Inertia

1.1. Data

To learn what a caliper is, and how to read it, watch this video: https://youtu.be/mY4Jq9eg5r0

Watch the following video to obtain the width of the brass weight, and the diameter of the pulley: https://youtu.be/nBj-OCV670s

Record the width of the brass weight, w, and both diameters of the pulley. Calculate the average diameter of the pulley, divide by 2 to obtain the radius, and record the pulley's radius in the Excel datasheet.

Watch the following video to obtain the length of the rod, and record the length in the Excel datasheet: https://youtu.be/U5GlDxiXUic

Watch the following video to obtain the masses of the brass weight and rod and record them in the Excel datasheet: https://youtu.be/crIWZ7XorMo

Note: All calculations should be completed in Excel!

1.2. Calculations

Calculate the moment of inertia for the rod using the measured mass and length. An example of what might be typed in Excel is "=(1/12)*m*L^2".

Calculate the distance of the brass for Trials A, B, and C. These distances were labelled d_1, d_2, and d_3 in the Theory section.

Calculate the moment of inertia of the brass weights for each of the trials (these were labelled I_1, I_2, and I_3 in the Theory section).

Calculate the expected moment of inertia for each of the trials (these were labelled I_A, I_B, and I_C in the Theory section).

Part 2: Measured Moment of Inertia

2.1. Introduction

Watch the introduction to the equipment and procedure video: https://youtu.be/muIg0eFhaYM

2.2. Data

For each run, additional mass is added to the hanging mass, resulting in an increase in angular acceleration. Watch the following videos, where each video runs through 6 different hanging masses:

Trial A, one brass weight: https://youtu.be/W4qUjDcexxE
Trial B, two brass weights: https://youtu.be/a92WEWjU28A
Trial C, three brass weights: https://youtu.be/kbhUAuzbXwo

For each of the trials:

- Record the 6 hanging masses.
- Record the associated 6 angular accelerations.

2.3. Calculations

For each of the three runs, calculate α^{-1} and m^{-1}. Remember, once the values of a trial have been calculated, all you need to do is copy/paste the cells into the next trial, and the values will update.

2.4. Graphs

Highlight α^{-1} and m^{-1} in Trial A, and create a scatter plot. Include axis labels, a legend, and a linear fit with an equation on the plot. Add data from Trail B and Trial C onto the same plot, each with their own linear fits and equations. Move the equations around so it is clear which equation goes with which linear fit. Change the axis so that the data covers most of the plot area.

2.5. Comparison

Enter the slope of each graph into the Excel sheet, then calculate the expected moment of inertia for each trial. The moment of inertia is equal to gravity multiplied by the radius of the pulley multiplied by the slope. An example of what you might type in Excel is "=980*r*slope" (the acceleration due to gravity must be in cm/s^2 to match the other units).

Calculate the percent error between the measured value of the moment of inertia and the expected value of the moment of inertia for each trial. If the percent error is greater than 5% there is a calculation error that needs to be fixed. If you are having trouble finding the error, send me (your instructor) a copy of your Excel spreadsheet.

2.6. Questions

Answer the questions provided in the Excel spreadsheet, under the tab labelled "Questions".

Answer the following question on a separate sheet of paper, and submit to Canvas along with the Excel spreadsheet.

So far, we have made the approximation that each brass weight on the end of the aluminum rod is a point mass. This is not, however, entirely true. The brass weight has a width that we are not considering. For a rigid object, with a linear density λ, the moment of inertia is given by

$$I = \int_{x_1}^{x_2} \lambda x^2 \, dx$$

This accounts for the fact that the brass weight is not a point mass, but has a width. For Trial C, there are three weights on each end of the rod, and this deviates the most from three point masses. Because there are weights on either end of the rod, the moment if inertia for the brass masses is:

$$I_{brass} = 2 \int_{x_1}^{x_2} \lambda x^2 \, dx$$

where $\lambda = \frac{m_{brass}}{w}$ (*w* is the width of the brass). Integrate this equation for Trial C where the brass is located ($x_1 = L/2 - 3w$ and $x_2 = L/2$) and numerically solve for I_{brass}. Calculate the percent error between this value and the value of the moment of inertia calculated for Trial C, I_C. Based on the result, can you say that the approximation of point masses is a good or bad approximation?