

Lab 4: Newton's Second Law

Objective: To measure the forces and acceleration to validate Newton's Second Law.

Theory:

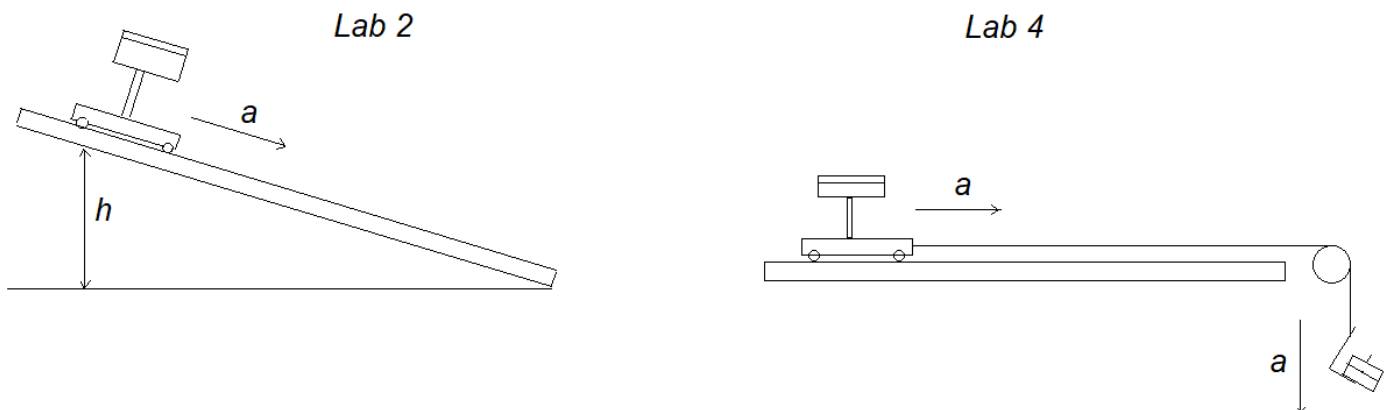
In Lab 2, we measured the acceleration of a cart on an inclined track. The acceleration was due to the component of gravity that was parallel to the track; we used two photogates to measure three times, which we labeled t_1 t_2 t_3 .

Recall that these three times were:

- t_1 time for the cart to pass through the first photogate
- t_2 time the cart enters the first photogate to the time cart enters the second photogate
- t_3 time for the cart to pass through the second photogate

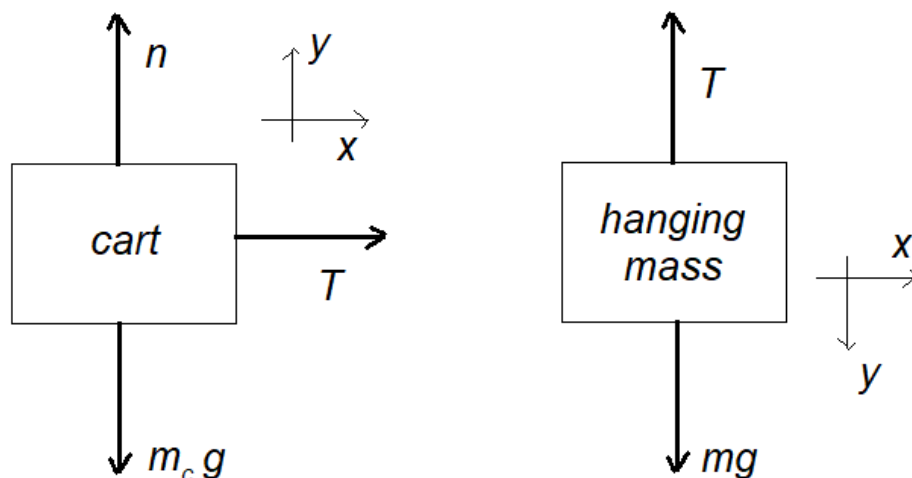
Using the length of the tab on the cart, i.e. the tab that passes through the photogate, which is 11.0 cm, we were able to calculate the average speed of the cart passing through each of the two photogates, and then the acceleration of the cart.

For today's experiment, we will measure the times and calculate the velocities and acceleration of the cart exactly the same way we did in Lab 2. The only difference is that now the acceleration of the cart will be due to the tension from a string that pulls the cart along the horizontal track:



The other end of the string will be attached to a hanging mass which will accelerate downward at the same rate the cart accelerates along the track.

We can derive an expression for the acceleration by drawing a *free body diagram* for the cart and another for the hanging mass.



We can write the equations for each of these *free body diagrams*:

Cart

$$x \text{ equation: } T = m_c a$$

$$y \text{ equation: } n - m_c g = 0$$

Hanging mass

$$x \text{ equation: } \text{nothing!}$$

$$y \text{ equation: } mg - T = ma$$

Note that the x equation for the cart and the y equation for the hanging mass include the acceleration we want to measure, and both equations include the tension in the string. We can combine these equations algebraically, eliminating the tension by substitution:

$$T = m_c a \quad mg - T = ma \quad \text{so} \quad mg - (m_c a) = ma$$

We can rearrange this equation and: $mg = ma + m_c a$

$$\text{Or: } mg = (m + m_c) a$$

We can now define M as the **total mass**, i.e. that of the cart and the hanging mass combined, and simplify the equation to read:

$$mg = Ma \quad \text{or} \quad a = \frac{g}{M} m$$

where m is the hanging mass.

This equation tells us that the acceleration of the system should be proportional to the mass on the hanger. We can use this to direct how we collect our data, and the process will be nearly identical to that of Lab 2:

Lab 2: $a = \frac{g}{L} h$ We varied h for five trials and measured a for each trial.

Lab 4: $a = \frac{g}{M} m$ We will vary m for five trials and measure a for each trial.

This equation tells us that the acceleration of the system is equal to (g/M) times the mass of the hanging cart. Therefore, if the mass of the hanging cart is changed, the acceleration of the system will change as well. By plotting the hanging mass v. acceleration, the slope is given by g/M . To ensure the total mass of the system remains the same, masses on the cart will be transferred to the hanging mass.

Procedure:

1. Background

Recall the time t_1 t_2 and t_3 can be used to calculate:

v_1 : the average speed of the cart as it passes through the first photogate

v_3 : the average speed of the cart as it passes through the second photogate

We can then use v_1 and v_3 to calculate the acceleration of the cart for each run:

$$a = \frac{v_3 - v_1}{t_2 - \frac{1}{2}t_1 + \frac{1}{2}t_3}$$

2. Setup and Equipment

Watch the following video, which introduces the equipment and how data will be collected: <https://youtu.be/k6oAhNDs5Rs>. Five trials of data will be taken, where each trial increases the mass on the hanging cart, thereby increasing the acceleration of the cart. For each trial, the cart will roll across the track 4 times, for 5 different hanging masses, totaling 20 runs for each cart.

3. Data Collection

Open up the excel spreadsheet from lab 2. Rename and save the spreadsheet for lab 4. Delete the 'heavy cart on an incline' data and graph.

The mass of the cart and hanging mass system is obtained in this video: <https://youtu.be/Qc6Y7vun2CM>. Type the total mass and length of the tab into excel.

For each run, the hanging mass value changes (instead of the height). Relabel the 'height (cm)' as 'hanging mass (g)'.

Watch the following video and enter the hanging mass, t_1 , t_2 , and t_3 for each of the trials: <https://youtu.be/DiJ34FHoFao>.

4. Velocity & Acceleration Calculations

The velocity and acceleration calculations should automatically update when the new times are input into Excel. Double check the equation entered for acceleration is correct, and looks like: “ $= (v_3 - v_1) / (t_2 - (t_1/2) + (t_3/2))$ ”. Be sure to include the parenthesis so Excel knows the order of operations. The average and standard deviation of the acceleration should also automatically update.

5. Hanging Mass v. Acceleration Table

Complete the table of hanging mass v. acceleration. You can type the values of hanging mass and acceleration in, or you can type “=” then click on the value you want in that cell. For instance, the hanging mass for trial 1 would just be “=C7”.

6. Hanging Mass v. Acceleration Graph

Based on the new data, the hanging mass v. acceleration graph should automatically update. Set the x and y axis so that the data covers a majority of the graph area.

7. Expected Slope

Add in a section to the spreadsheet that is labelled ‘Calculations’. Within this section of the spreadsheet, add in a label for expected slope, then calculate the expected slope of the line. Include units as well.

8. Percent Error

In the section under calculations, add in a label for percent error. The percent error calculates the difference between a measured and accepted value, and is given by the equation

$$\% \text{ error} = \frac{\text{measured} - \text{expected}}{\text{expected}} * 100\%.$$

Calculate the percent error between the measured and accepted value of the slope.

9. Laboratory Report

The results for this lab will be written in a lab report, in the same style as previous labs. However, for the data section, insert the graph into the word document. Make sure the line equation is included. The best way to ensure the line equation is included is to take a screenshot of the graph, then insert the screenshot into Word. The remainder of the excel spreadsheet will not be submitted.

10. Results

Answer the following questions in the results section of your laboratory report.

- What was the expected value of the slope, with units?
- What was the measured value of the slope, with units?
- What was the percent error between the expected and measured slope?
- Does the data show that increasing the hanging mass increased the overall acceleration?
- The uncertainty in the slope works out to be about $\pm 0.15 \text{ cm/g}\cdot\text{s}^2$. What does this uncertainty tell us about the data and the procedure used to collect it?
- Within uncertainty, do the measured and expected slope agree? Should this be the case?