Carp Count Data

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Background

The common carp *Cyprinus carpiro* (henceforth carp) is one of the most trans-located species in the world, established on every continent except Antarctica. Carp have two basic habitat requirements: 1) a shallow marsh environment with abundant vegetation; 2) a deeper area to retreat to during colder months (McCrimmon 1968). Carp spawn in shallow flooded areas with abundant fixed vegetation on which eggs are deposited (Crivelli 1981). Spawning begins when water temperatures are ~15–16°C (Crivelli 1981). Carp generally spawn in the spring (McCrimmon 1968), but can span March–August and even into October (Crivelli 1981). Most carp show high site fidelity, but a small percentage of the population may also exhibit high mobility (Crook 2004; Stuart and Jones 2006).

Objectives

1) Determine what environmental variables (i.e. temperature and discharge) drive carp migration into the Delta Marsh using camera trap count data.
2) Determine the effect of sampling frequency on model results.

Data exploration

I will loosely follow the protocol by Zuur et al. (2010) for exploring the data to avoid common statistical problems including type I (i.e. rejecting the null hypothesis when it is true) or type II errors (i.e. failure to reject the null hypothesis when it is untrue).

1. Data Distribution

A good place to start exploring the data is looking at the distribution of our response variable—carp counts. The distribution will give us a good indication of what kind of analysis we should use to deal with our data and if any problems may need to be addressed (i.e. many zeros). Because we are modelling count data, a generalized linear model (GLM) is an appropriate analysis (Zuur, Ieno, and Elphick 2010). The Poisson or negative binomial distributions are what we would expect. Looking at Figure 1, we quickly realize we are dealing with many zeros! Therefore, a Poisson or negative binomial GLM will likely produce biased parameter estimates and standard errors as well as over-dispersion. A zero-inflated or zero-altered GLM will likely be an appropriate analyses here.

2. Dealing with the zeros

Lets make sure we are dealing with true zero inflation. I have run the following models to see which ones produce similar number of zeros as our data:

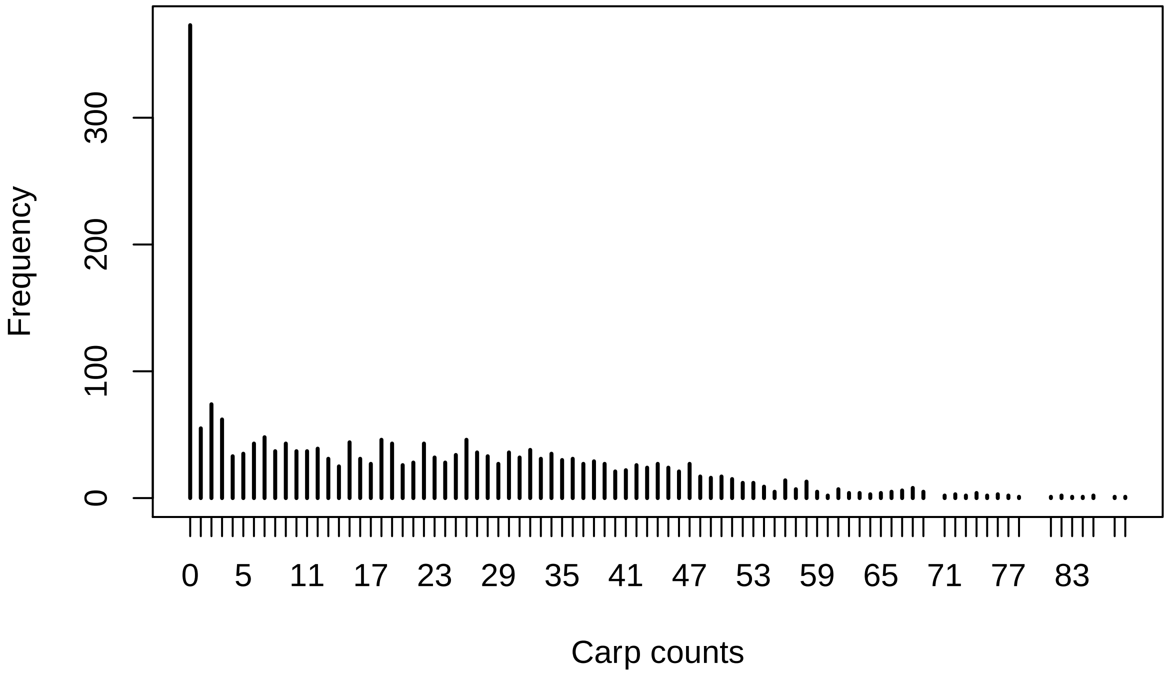


Figure 1: Frequency distribution of raw (unadjusted) carp count data. Notice the large number of zeros—a good indication of zero inflation

```
#Poisson glm
pois<-glm(Count~Discharge+Temp+Pelicans+Time,offset=log(adjArea), family="poisson", data=Z)


#Negative binomial glm model
negb<-glm.nb(Count~Discharge+Temp+Pelicans+Time+offset(log(adjArea)),data=Z)

#zero-inflated formula
fm1<-formula(Count~Discharge+Temp+Pelicans+Time+offset(log(adjArea))
             |Discharge+Temp+Pelicans+Time+fExposure+fWaterClarity+fImageClarity)
#zero inflated poisson
zip<-zeroinfl(fm1,data=Z)

#zero inflated negative binomial
zinb<-zeroinfl(fm1, dist="negbin",data=Z)

#zero altered poisson
zap<-hurdle(fm1,data=Z)

#zero altered negative binomial
zanb<-hurdle(fm1,dist="negbin",data=Z)
```

```
##  Obs Pois    NB  ZIP ZINB  ZAP ZANB
##  373    4  178  373  371  373  373
```

It is quite apparent that only the zero-inflated models (ZIP, ZAP, ZINB and ZANB) have similar zero counts to our observations.

How we deal with the zeros is related to what kind of zeros we have. In general, techniques for dealing with zero inflation have two parts: a binomial part that deals with the zeros and a Poisson (or negative binomial) part that deals with the count data. zero-inflated models, or mixture-models, split the zeros into true zeros (fish truly absent) and false zeros (fish present but not seen). The true zeros are modelled in the Poisson GLM, while the process generating the false zeros is modelled in the binomial GLM. Two-part or hurdle models do not discriminate between true and false zeros, the presence of an animal is the result of some covariate mechanism crossing a hurdle. The hurdle model is most appropriate when there is little chance of missing any items in the counts (Ver Hoef and Jansen 2007). In our case, some zeros may be false zeros (missed counts)—as a result of image obstruction, exposure, water clarity or image clarity—or true zeros (i.e. fish are not present because conditions are not appropriate). Therefore, I feel a zero-inflated mixture-model would best deal with the zero inflation.

3. Outliers

Now that we have an idea of what analysis we can perform lets look at the data in more detail. Outliers can be a problem for Poisson GLM (similarly for zero-inflated models) analyses and may cause over-dispersion. I will define outliers here as observations which values are relatively larger or smaller to the majority of of observations. Here I show a boxplot (Figure 2a) and a Cleveland dotplot (Figure 2b) of 2222 carp count observations. The boxplot visualizes the median and spread of the data. Observations outside of the whiskers are labelled as outliers. Figure 1a shows that there are potentially 9 outliers. A good way to check if these are in fact outliers is a Cleveland dot plot (Figure 2b), in this graph the row number of observations is plotted vs. the frequency of carp, providing more detailed information than a boxplot. Figure 2b reveals that the possible outliers are not really outliers at all because they follow the pattern of peaks displayed by the observations.

Figure 2 displays a multi-panel Cleveland dot plot for our carp count data and all of our potential covariates that could influence the counts, including water discharge, water temperature, and time; as well as covariates that may influence the zeros (or false zeros) including dead fish, pelican counts, exposure factor, image clarity factor, water clarity factor and percent obstructed. For the most part the covariates look fine except for the large values (points far to the right) in the dead fish and percent obstructed panels. It appears the large number of dead fish obstructed 83% of the image. The large number of dead fish is a rare observation and may be influential on our parameter estimates. We will have to investigate how sensitive the model is to these large values and decide if this observation is a candidate for removal.

Over the study duration, the viewable area of the picture frame ranges from 3.6328499–4.3961477 m^2. Further reduction in visible area as a result of obstructions, such as pelicans and dead fish (See Figure 3), required an offset for adjusted area (picture frame area X percent visible area). The results of our model will therefore be a density Carp per m^2.

4. Collinearity among covariates

Ignoring collinearity among covariates may lead to a confusing statistical output with nothing significant. This is because collinearity results in inflated standard errors of parameters which in turn increase *P*-values, making it difficult to detect an effect. We can easily test for collinearity by looking at the variance inflation factors (VIF), covariates with VIF greater than 3 will be sequentially removed. Unfortunately the *vif* function from the *car* package does not allow zero-inflated models—thus, I have split up our covariates into two groups: Poisson covariates including water discharge, water temperature, and time; as well as binomial covariates including water discharge, water temperature, time, presence of pelicans, exposure factor, image clarity factor and water clarity factor. This way I can examine the Poisson covariate VIF in a Poisson GLM and the binomial covariate VIF in a binomial GLM using the *vif* function.

```
##                VIF
## Discharge 1.230574
```

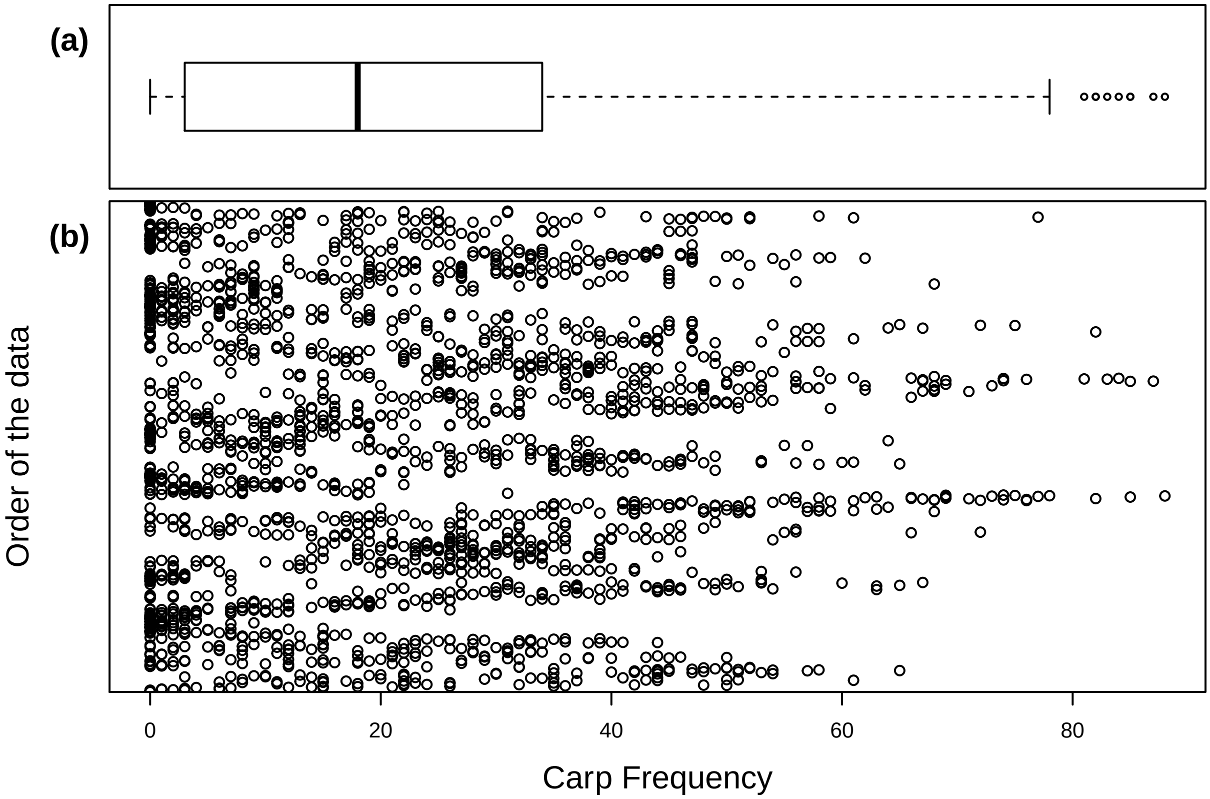


Figure 2: (a) Boxplot of carp frequency counts from 2222 observations taken at the same location. The line in the middle of the box represents the median, and the lower and upper ends of the box are the 25% and 75% quartiles respectively. The lines indicate 1.5 times the size of the hinge, which is the 75% minus 25% quartiles. Points beyond these lines are considered to be outliers. **(b)** Cleveland dot plot of the same data. The horizontal axis represents the carp counts, and the vertical axis corresponds to the order of the data.

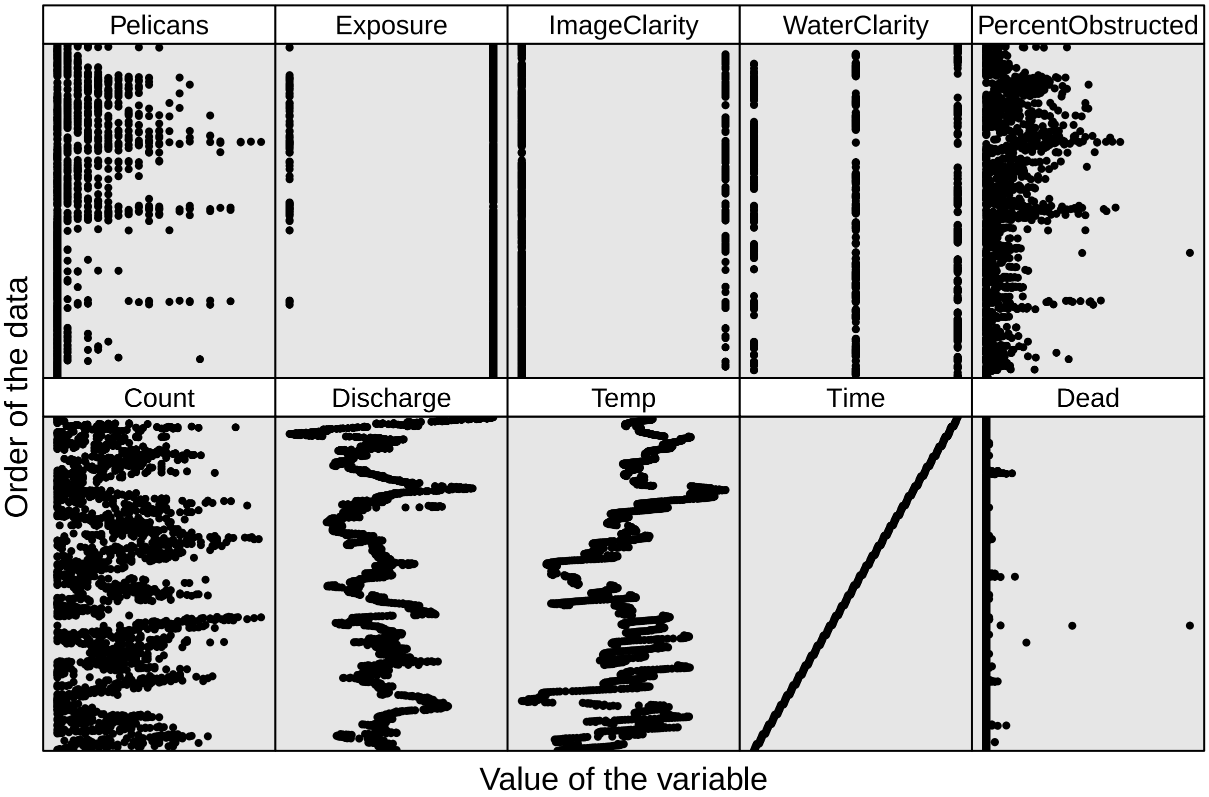


Figure 3: Multi-panel Cleveland dotplot for carp counts and ten covariates that may influence the counts. The plots are ordered by date along the y-axis. Notice the possible influential values in Obstructed and Dead—this observation may be a candidate for removal if model parameters are found to be sensitive to these inputs.

```
## Temp      1.095940
## Pelicans  1.125772
## Time      1.410620
```

Notice that all of our Poisson covariates VIF are < 3. No collinearity! Now let's look at the binomial side.

```
##                   VIF
## Discharge     1.176526
## Temp          1.212129
## Time          1.138461
## Pelicans      1.532177
## fExposure     1.476387
## fWaterClarity 1.137934
## fImageClarity 1.036255
```

Our four covariates do not reveal any collinearity. Next we will examine the relationships between our covariates and the response variable.

5. Relationships Y & X

Looking at the relationship between Y (carp counts) and X covariates (discharge, temperature, pelicans and time) we begin to see strong nonlinear effects of discharge and temperature (Figure 4). Abundance appears to increase at negative discharges close to 0 and temperatures between 16–20°C. These nonlinear trends should be modelled with an additive model (i.e. GAM) or linear model with qudratic relationships.

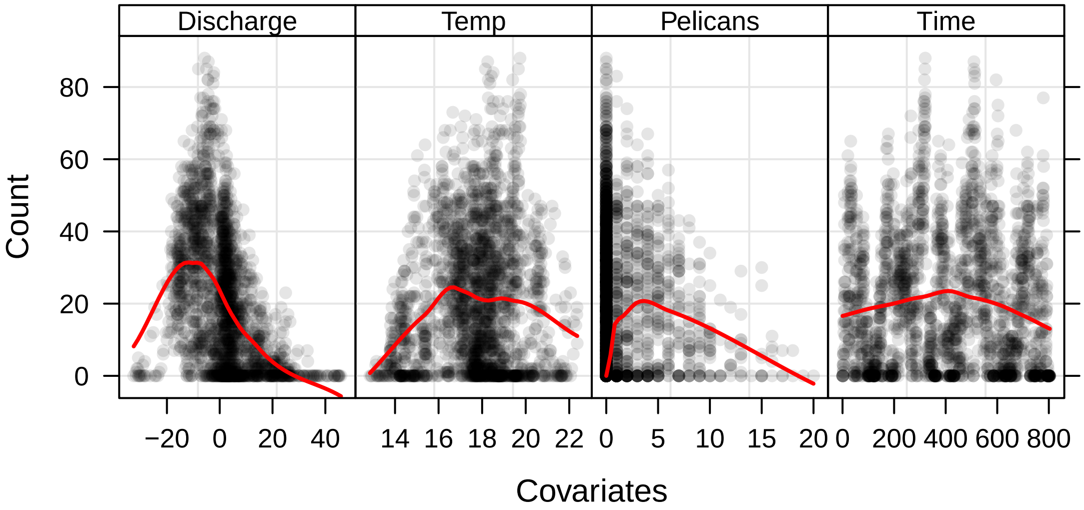


Figure 4: Relationship between carp counts and four potential covariates. The red line represents a LOESS smoother (span=0.67,degree=1) to visualize relationships between carp counts and covariates.

6. Response variable independence

A very important assumption of most statistical techniques is independence among observations. Ignoring dependence among observations (autocorrelation) can lead to underestimates of standard errors and increased false positives. The carp count data is part of a time-series analysis (Figure 5), thus dependence among observations is a potential problem.

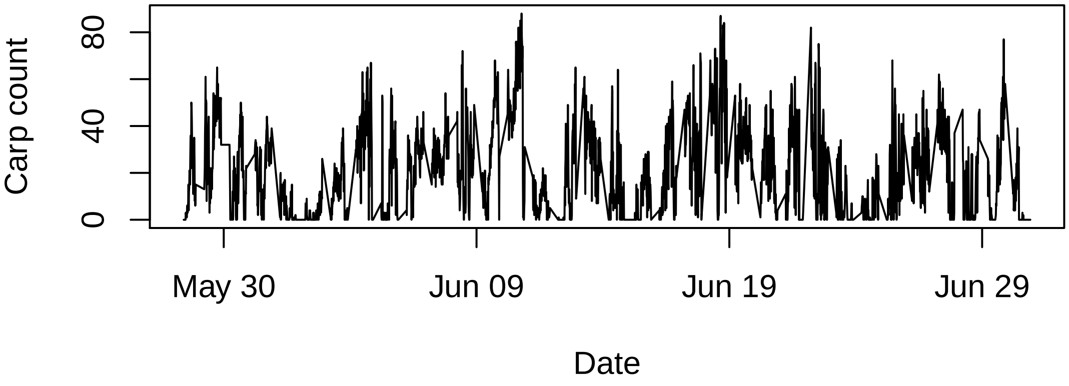


Figure 5: Carp count time-series.

Figure 6 displays the autocorelation between lags for the carp count data and confirms that we have dependence among observations—all lags shown are significantly correlated. At lag 1, the correlation of 0.78 exponentially diminishes at larger lags is suggesting of a stationary autoregressive model.

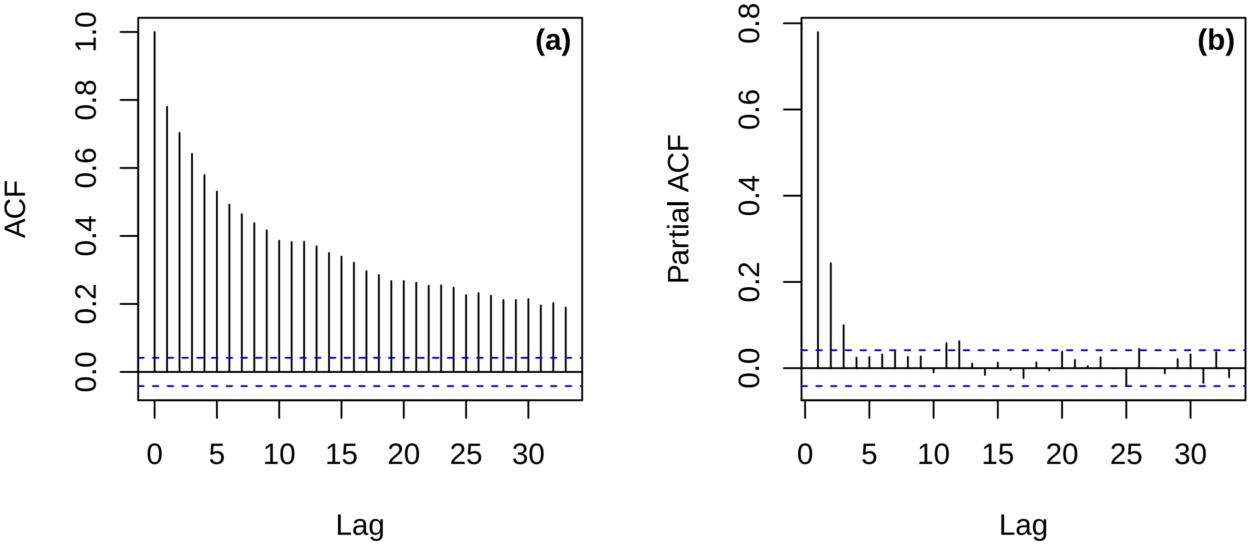


Figure 6: (a) Autocorrelation function plot and (b) Partial autocorrelation function plot for carp count data. The blue dashed lines represents significant correlations such that any lags greater than than the positive blue line or less than the negative blue line are significantly correlated

To determine the order of the autoregressive process we can examine the partial autocorrelation function plot (Figure 6b). We see that significant partial autocorrelation up to lag 3 which suggests that a third order autoregressive process (AR3) could be used to model the data.

7. Model fitting

a) Zero-inflated models

This data set has a few problems to overcome. first, we are dealing zero inflated counts so we need a model that can handle the overdispersion caused by the zeros—I will start with a zero-inflated poisson model. Second, we have dependence among observations which will need to be addressed in order to avoid unrealistic standard errors and false positives—I will start with adding a smoothed trend term for time. Lastly, we have non-linear relationships between our count variable and explanotory vairables discharge and temperature—I have added quadratic terms to account for these relationships. Lets take a look at our intial model:

```
##
## Call:
## zeroinfl(formula = fm1, data = Z)
##
## Pearson residuals:
##     Min      1Q  Median      3Q     Max
## -4.9015 -1.1814 -0.2838  1.1573  9.3164
##
## Count model coefficients (poisson with log link):
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  7.702e-01  5.391e-02   14.29   <2e-16 ***
## Discharge   -3.835e-02  5.376e-04  -71.33   <2e-16 ***
## Temp         6.871e-02  3.102e-03   22.15   <2e-16 ***
## Pelicans    -5.181e-02  2.208e-03  -23.46   <2e-16 ***
## Time        -2.666e-04  2.262e-05  -11.79   <2e-16 ***
##
## Zero-inflation model coefficients (binomial with logit link):
##                 Estimate Std. Error z value Pr(>|z|)
## (Intercept)    -5.529644   0.755741  -7.317 2.54e-13 ***
## Discharge       0.044103   0.006613   6.669 2.58e-11 ***
## Temp            0.062820   0.038923   1.614  0.10653
## Pelicans        0.149875   0.031257   4.795 1.63e-06 ***
## Time            0.001808   0.000321   5.634 1.77e-08 ***
## fExposure2     -0.854030   0.318553  -2.681  0.00734 **
## fWaterClarity2  2.025488   0.313126   6.469 9.89e-11 ***
## fWaterClarity3  3.529759   0.330890  10.667  < 2e-16 ***
## fImageClarity2 -0.349320   0.215413  -1.622  0.10488
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Number of iterations in BFGS optimization: 22
## Log-likelihood: -1.424e+04 on 14 Df
```

We added quadratic terms for discharge and temperature so I will double check the variance inflation factors to ensure we do not have collinearity.

```
##                        VIF
## Discharge         1.329149
## I(Discharge^2)    1.227982
## Temp             17.232253
## I(Temp^2)        17.251432
## Pelicans          1.085463
## ns(Time, df = 3)  1.085332
```

The temperature and temperature^2 covariates are correlated, let's centre these variables and re-test for collinearity.

```
##                       VIF
## Discharge        1.329149
## I(Discharge^2)   1.227982
## Temp.c           1.105827
## I(Temp.c^2)      1.064911
## Pelicans         1.085463
## ns(Time, df = 3) 1.085332
```

Now let's re-run the zero inflated model to see if our results changed.

```
## 
## Call:
## zeroinfl(formula = Count ~ offset(log(adjArea)) + Discharge + I(Discharge^2) + 
##     Temp.c + I(Temp.c^2) + Pelicans + ns(Time, df = 3) | Discharge + 
##     I(Discharge^2) + Temp.c + I(Temp.c^2) + ns(Time, df = 3) + Pelicans + 
##     fExposure + fWaterClarity + fImageClarity, data = Z, dist = "poisson")
## 
## Pearson residuals:
##      Min       1Q   Median       3Q      Max 
## -5.16742 -1.09920 -0.09712  1.19986 20.72422 
## 
## Count model coefficients (poisson with log link):
##                     Estimate Std. Error z value Pr(>|z|)    
## (Intercept)        1.889e+00  1.756e-02 107.600   <2e-16 ***
## Discharge         -5.182e-02  7.819e-04 -66.276   <2e-16 ***
## I(Discharge^2)    -2.625e-03  5.551e-05 -47.288   <2e-16 ***
## Temp.c             7.896e-02  3.386e-03  23.318   <2e-16 ***
## I(Temp.c^2)       -1.697e-02  1.488e-03 -11.409   <2e-16 ***
## Pelicans          -6.677e-02  2.256e-03 -29.592   <2e-16 ***
## ns(Time, df = 3)1  3.094e-01  2.121e-02  14.588   <2e-16 ***
## ns(Time, df = 3)2  3.593e-01  4.287e-02   8.382   <2e-16 ***
## ns(Time, df = 3)3 -2.876e-01  2.123e-02 -13.547   <2e-16 ***
## 
## Zero-inflation model coefficients (binomial with logit link):
##                     Estimate Std. Error z value Pr(>|z|)    
## (Intercept)       -3.8496381  0.5106330  -7.539 4.74e-14 ***
## Discharge          0.0126888  0.0084199   1.507 0.131811    
## I(Discharge^2)     0.0002554  0.0004190   0.610 0.542143    
## Temp.c             0.1192868  0.0451877   2.640 0.008295 ** 
## I(Temp.c^2)        0.0663794  0.0150018   4.425 9.65e-06 ***
## ns(Time, df = 3)1  0.0752994  0.3465954   0.217 0.828010    
## ns(Time, df = 3)2  0.6166158  0.6379996   0.966 0.333803    
## ns(Time, df = 3)3  1.3321822  0.2676777   4.977 6.46e-07 ***
## Pelicans           0.1315330  0.0326904   4.024 5.73e-05 ***
## fExposure2        -1.0982792  0.3296562  -3.332 0.000864 ***
## fWaterClarity2     1.9405022  0.3204797   6.055 1.40e-09 ***
## fWaterClarity3     3.6085895  0.3378961  10.680  < 2e-16 ***
## fImageClarity2    -0.6600696  0.2633723  -2.506 0.012203 *  
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 
## 
## Number of iterations in BFGS optimization: 29 
## Log-likelihood: -1.212e+04 on 22 Df
```

The zero inflation poisson model uses an offset of *adjArea* which is the percent of unobstructed frame multipled by the maximum area of the frame to change our raw count into a density (fish m^{-2}). We can see that our model has several non-significant predictors in the zero-inflation model including discharge, discharge^2 and 2 of the time variables natural splines. Before we start model selection we should check if the zero-inflated poisson model still has significant overdispersion. This is done by comparing the same model formula in a zero-inflated negative binomial model using the likelihood ratio test.

```
zinb<-zeroinfl(Count~offset(log(adjArea))+Discharge+I(Discharge^2)+Temp.c+I(Temp.c^2)
               +Pelicans+ns(Time,df=3)|Discharge+I(Discharge^2)+Temp.c+I(Temp.c^2)
               +ns(Time,df=3)+Pelicans+fExposure+fWaterClarity+fImageClarity,
               dist="negbin", data=Z)
library(lmtest)
lrtest(zip,zinb)
```

```
## Likelihood ratio test
##
## Model 1: Count ~ offset(log(adjArea)) + Discharge + I(Discharge^2) + Temp.c +
##     I(Temp.c^2) + Pelicans + ns(Time, df = 3) | Discharge + I(Discharge^2) +
##     Temp.c + I(Temp.c^2) + ns(Time, df = 3) + Pelicans + fExposure +
##     fWaterClarity + fImageClarity
## Model 2: Count ~ offset(log(adjArea)) + Discharge + I(Discharge^2) + Temp.c +
##     I(Temp.c^2) + Pelicans + ns(Time, df = 3) | Discharge + I(Discharge^2) +
##     Temp.c + I(Temp.c^2) + ns(Time, df = 3) + Pelicans + fExposure +
##     fWaterClarity + fImageClarity
##   #Df   LogLik Df  Chisq Pr(>Chisq)
## 1  22 -12118.0
## 2  23  -8002.5  1 8230.8  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
AIC(zip,zinb)
```

```
##      df      AIC
## zip  22 24279.92
## zinb 23 16051.09
```

The likelihood ratio test and AIC comparison reveal the zero-inflated negative binomial model performs significantly better than the zero-inflated poisson model. Now lets examine the output of our zero-inflated negative binomial model:

```
##
## Call:
## zeroinfl(formula = Count ~ offset(log(adjArea)) + Discharge + I(Discharge^2) +
##     Temp.c + I(Temp.c^2) + Pelicans + ns(Time, df = 3) | Discharge +
##     I(Discharge^2) + Temp.c + I(Temp.c^2) + ns(Time, df = 3) + Pelicans +
##     fExposure + fWaterClarity + fImageClarity, data = Z, dist = "negbin")
##
## Pearson residuals:
##     Min      1Q  Median      3Q     Max
## -1.5831 -0.6786 -0.0962  0.5158 10.2340
##
## Count model coefficients (negbin with log link):
##                      Estimate Std. Error z value Pr(>|z|)
## (Intercept)        1.8238906  0.0549784  33.175  < 2e-16 ***
## Discharge         -0.0465465  0.0020347 -22.876  < 2e-16 ***
## I(Discharge^2)    -0.0019242  0.0001266 -15.196  < 2e-16 ***
## Temp.c             0.0987924  0.0101002   9.781  < 2e-16 ***
## I(Temp.c^2)       -0.0091895  0.0042112  -2.182  0.02910 *
## Pelicans          -0.0717301  0.0065416 -10.965  < 2e-16 ***
## ns(Time, df = 3)1  0.3738051  0.0692895   5.395 6.86e-08 ***
## ns(Time, df = 3)2  0.3488192  0.1332084   2.619  0.00883 **
## ns(Time, df = 3)3 -0.2904917  0.0654607  -4.438 9.09e-06 ***
## Log(theta)         1.0230179  0.0401778  25.462  < 2e-16 ***
##
## Zero-inflation model coefficients (binomial with logit link):
##                     Estimate Std. Error z value Pr(>|z|)
## (Intercept)       -3.9480884  0.5412319  -7.295 2.99e-13 ***
## Discharge          0.0129021  0.0085138   1.515 0.129661
## I(Discharge^2)     0.0005230  0.0004114   1.271 0.203643
## Temp.c             0.1219892  0.0458177   2.662 0.007756 **
## I(Temp.c^2)        0.0663073  0.0151494   4.377 1.20e-05 ***
## ns(Time, df = 3)1  0.1267602  0.3556590   0.356 0.721534
## ns(Time, df = 3)2  0.5440740  0.6514553   0.835 0.403624
## ns(Time, df = 3)3  1.3398428  0.2713473   4.938 7.90e-07 ***
## Pelicans           0.1247192  0.0345787   3.607 0.000310 ***
## fExposure2        -1.1465047  0.3394628  -3.377 0.000732 ***
## fWaterClarity2     2.0799008  0.3633605   5.724 1.04e-08 ***
## fWaterClarity3     3.7312532  0.3797125   9.827  < 2e-16 ***
## fImageClarity2    -0.7313836  0.2748815  -2.661 0.007797 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Theta = 2.7816
## Number of iterations in BFGS optimization: 31
## Log-likelihood: -8003 on 23 Df
```

Looks like we still have some non-significant terms. I will sequentially remove predictors until they are all significant predictors in the model. First, I will remove Discharge^2 from the zero model because it has the highest *P-values* in the zero part of the model. I will continue to sequentially remove insignificant parameters (time and Discharge) until all parameters are significant. Here is the resulting model:

```
##
## Call:
## zeroinfl(formula = Count ~ offset(log(adjArea)) + Discharge + I(Discharge^2) +
##     Temp.c + I(Temp.c^2) + Pelicans + ns(Time, df = 3) | Temp.c +
##     I(Temp.c^2) + Pelicans + fExposure + fWaterClarity + fImageClarity,
##     data = Z, dist = "negbin")
##
## Pearson residuals:
##      Min       1Q   Median       3Q      Max
## -1.57077 -0.68924 -0.09733  0.51812  8.67596
##
## Count model coefficients (negbin with log link):
##                     Estimate Std. Error z value Pr(>|z|)
## (Intercept)        1.8275225  0.0550115  33.221  < 2e-16 ***
## Discharge         -0.0470943  0.0020299 -23.201  < 2e-16 ***
## I(Discharge^2)    -0.0019839  0.0001228 -16.156  < 2e-16 ***
## Temp.c             0.1001531  0.0101141   9.902  < 2e-16 ***
## I(Temp.c^2)       -0.0089944  0.0042157  -2.134   0.0329 *  
## Pelicans          -0.0718087  0.0065290 -10.999  < 2e-16 ***
## ns(Time, df = 3)1  0.3761480  0.0693060   5.427 5.72e-08 ***
## ns(Time, df = 3)2  0.3418135  0.1332763   2.565   0.0103 *  
## ns(Time, df = 3)3 -0.3050934  0.0655296  -4.656 3.23e-06 ***
## Log(theta)         1.0208428  0.0401875  25.402  < 2e-16 ***
## 
## Zero-inflation model coefficients (binomial with logit link):
##                Estimate Std. Error z value Pr(>|z|)    
## (Intercept)    -3.87293    0.46340  -8.358  < 2e-16 ***
## Temp.c          0.23740    0.03970   5.979 2.24e-09 ***
## I(Temp.c^2)     0.07558    0.01401   5.395 6.86e-08 ***
## Pelicans        0.13424    0.03356   4.000 6.34e-05 ***
## fExposure2     -1.05138    0.33766  -3.114  0.00185 ** 
## fWaterClarity2  2.03260    0.34676   5.862 4.58e-09 ***
## fWaterClarity3  3.89854    0.35191  11.078  < 2e-16 ***
## fImageClarity2 -0.54567    0.22902  -2.383  0.01719 *  
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 
## 
## Theta = 2.7755 
## Number of iterations in BFGS optimization: 26 
## Log-likelihood: -8019 on 18 Df
```

Now that all out our predictors are significant let's compare our new model with the full model.

```r
lrtest(zinb,zinb2)
```

```
## Likelihood ratio test
## 
## Model 1: Count ~ offset(log(adjArea)) + Discharge + I(Discharge^2) + Temp.c + 
##     I(Temp.c^2) + Pelicans + ns(Time, df = 3) | Discharge + I(Discharge^2) + 
##     Temp.c + I(Temp.c^2) + ns(Time, df = 3) + Pelicans + fExposure + 
##     fWaterClarity + fImageClarity
## Model 2: Count ~ offset(log(adjArea)) + Discharge + I(Discharge^2) + Temp.c + 
##     I(Temp.c^2) + Pelicans + ns(Time, df = 3) | Temp.c + I(Temp.c^2) + 
##     Pelicans + fExposure + fWaterClarity + fImageClarity
##   #Df  LogLik Df  Chisq Pr(>Chisq)    
## 1  23 -8002.5                         
## 2  18 -8018.7 -5 32.293  5.199e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
AIC(zinb,zinb2)
```

```
##       df      AIC
## zinb  23 16051.09
## zinb2 18 16073.39
```

Both the likelihood ratio test and AIC comparison reveal the full model is a better fit than the updated model, even with non significant parameters. Lets examine the diagnostics plot for the full model to see if the model is valid (Figure 7).

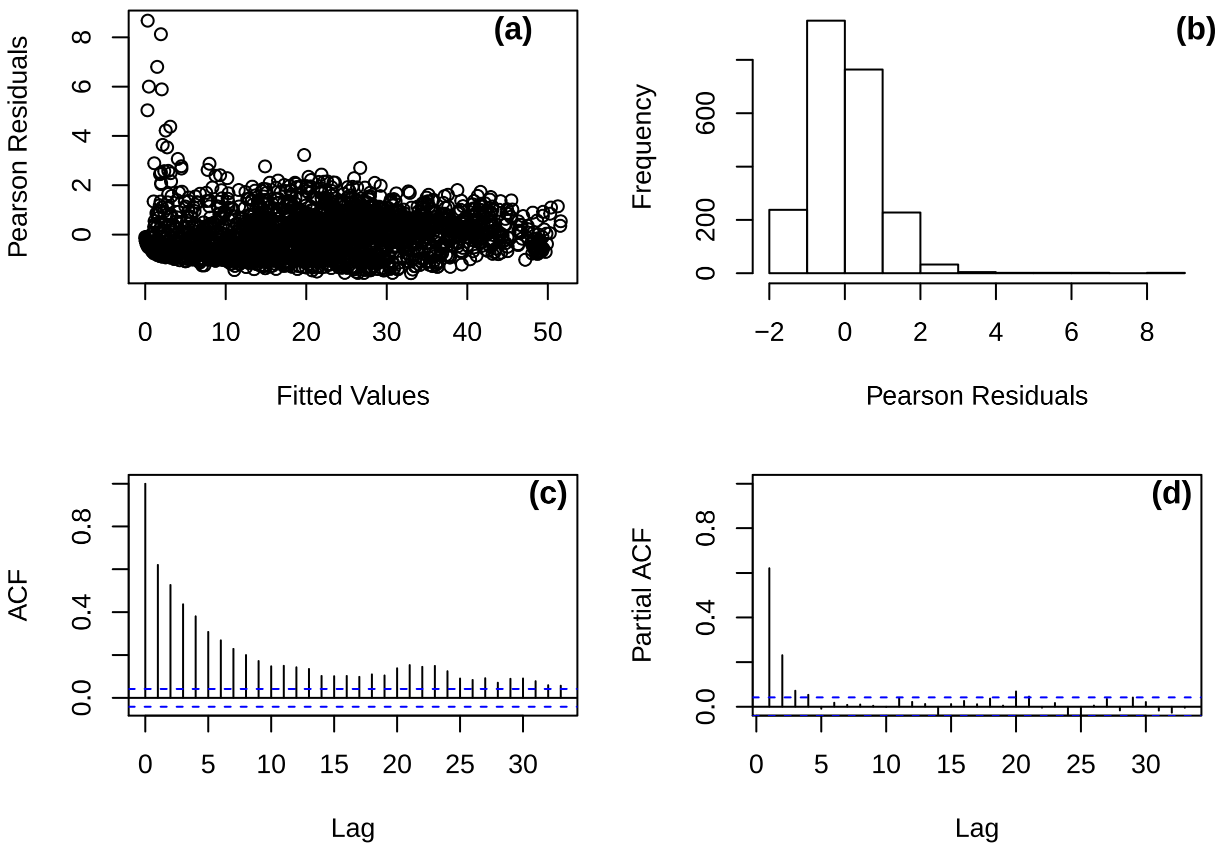


Figure 7: Model diagnostic plots. **a)** Pearson residuals versus fitted values, we should not see any clear pattern. **b)** histogram of pearson residuals, we should see residuals normally distributed around 0. **c)** Auto-correlation function of residuals and **d)** Partial autocorrelation function, vertical bars should be between the two horizontal blue lines if resdiuals are indpendent.

The diagnostic plots in figure 7 reveal that our model has residual problems (figure 7 a and b) and does not meet assumptions of independence (Figure 7 c and d). The time spline does not seem to reduce the autocorrelation in the model likely due to the strong serial correlation. The residual plots reveal that our model is unable to capture some of the variability in the data. We can examine what may be causing this through plotting our response versus our predictors and highllighting the points with large residual errors (Figure 8).

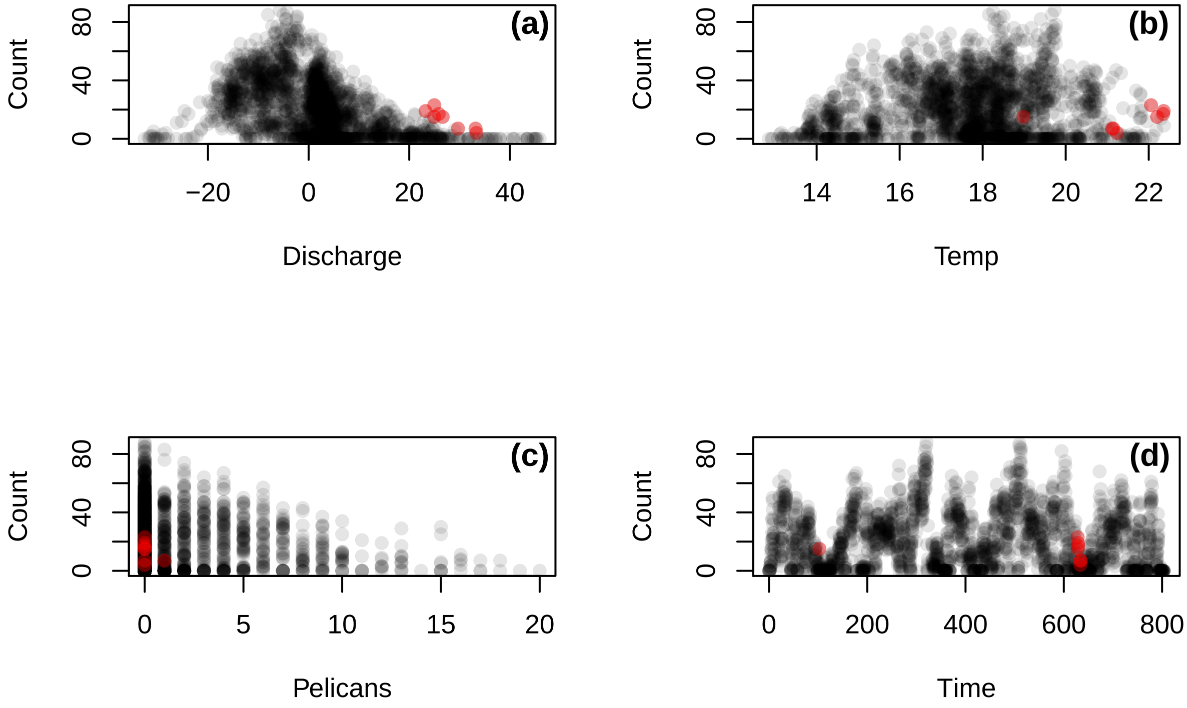


Figure 8: Count vs predictor plots. Black dots are raw data, red dots are observations with residuals > 4 (i.e. poor fitting observsations in the model)

It appears from Figure 8a that all of the residual error is comming from an "interesting anomaly" of higher than expected fish counts at positive discharges >20 cm s^{-1}. Unfortunately we do not have a variable to account for this occurence, therefore high residual error occurs at these points. If we take a closer look at the raw data you will notice that these points for the most part are clustered together in time (ie. lines 1730–1734 and lines 1752–1754), thus accounting for autocorrelation may rectify this problem.

```
##      Count Discharge   Temp DOY Hour Hour.M   Time Dead Pelicans Exposure
## 288     15    26.643 18.983 152   16  16.75 102.75    0        0        2
## 1730    23    24.980 22.056 174   14  14.25 628.25    0        0        2
## 1731    15    24.943 22.203 174   14  14.50 628.50    0        0        2
## 1733    19    23.242 22.359 174   15  15.00 629.00    0        0        2
## 1734    17    25.858 22.348 174   15  15.25 629.25    0        0        2
## 1752     4    33.355 21.237 174   19  19.75 633.75    0        0        2
## 1753     7    29.689 21.145 174   20  20.00 634.00    0        1        2
## 1754     7    33.180 21.119 174   20  20.25 634.25    0        0        2
##      ImageClarity WaterClarity  adjArea PercentObstructed  Obs  Density
## 288             1            3 4.211716               0.0  288 3.561494
## 1730            1            2 3.542491              11.1 1730 6.492606
## 1731            2            2 3.977092               0.0 1731 3.771600
## 1733            2            2 3.979662               0.0 1733 4.774275
## 1734            2            2 3.830908               3.8 1734 4.437590
## 1752           1            3 3.930974            0.0 1752 1.017559
## 1753           1            3 3.827324            2.7 1753 1.828954
## 1754           1            3 3.930974            0.0 1754 1.780729
##      fExposure fImageClarity fWaterClarity bCount fPelicans   Temp.c
## 288          2             1             3      1         0 1.279231
## 1730         2             1             2      1         0 4.352231
## 1731         2             2             2      1         0 4.499231
## 1733         2             2             2      1         0 4.655231
## 1734         2             2             2      1         0 4.644231
## 1752         2             1             3      1         0 3.533231
## 1753         2             1             3      1         1 3.441231
## 1754         2             1             3      1         0 3.415231
```

Next let's add lagged response variables as predicters to the full model which will hopefully account for the autocorrelation among observations. I sequentially removed insignifcant predictors until the resulting model:

```
##
## Call:
## zeroinfl(formula = Count ~ offset(log(adjArea)) + AR1 + AR2 + AR3 +
##     Discharge + I(Discharge^2) + Temp.c + Pelicans + Time | AR1 +
##     Temp.c + I(Temp.c^2) + Pelicans + fWaterClarity + fImageClarity +
##     Time, data = Z, dist = "negbin", x = TRUE)
##
## Pearson residuals:
##     Min      1Q  Median      3Q     Max
## -1.9091 -0.6405 -0.1444  0.4821  6.4606
##
## Count model coefficients (negbin with log link):
##                  Estimate Std. Error z value Pr(>|z|)
## (Intercept)     1.179e+00  4.153e-02  28.397  < 2e-16 ***
## AR1             1.510e-02  1.075e-03  14.049  < 2e-16 ***
## AR2             5.814e-03  1.154e-03   5.036 4.74e-07 ***
## AR3             3.871e-03  1.066e-03   3.630 0.000283 ***
## Discharge      -2.518e-02  1.772e-03 -14.207  < 2e-16 ***
## I(Discharge^2) -1.370e-03  1.082e-04 -12.665  < 2e-16 ***
## Temp.c          3.079e-02  8.389e-03   3.670 0.000242 ***
## Pelicans       -4.067e-02  5.644e-03  -7.205 5.81e-13 ***
## Time            2.367e-04  6.912e-05   3.425 0.000616 ***
## Log(theta)      1.402e+00  4.422e-02  31.698  < 2e-16 ***
##
## Zero-inflation model coefficients (binomial with logit link):
##                  Estimate Std. Error z value Pr(>|z|)
## (Intercept)    -2.3898025  0.4352232  -5.491 4.00e-08 ***
## AR1            -0.1675209  0.0163693 -10.234  < 2e-16 ***
## Temp.c          0.1250615  0.0463652   2.697 0.006990 **
## I(Temp.c^2)     0.0522976  0.0150401   3.477 0.000507 ***
## Pelicans        0.1342999  0.0284863   4.715 2.42e-06 ***
## fWaterClarity2  0.9585414  0.3861823   2.482 0.013061 *
## fWaterClarity3  1.9596312  0.3942031   4.971 6.66e-07 ***
## fImageClarity2 -0.5508982  0.2446019  -2.252 0.024308 *
## Time            0.0010673  0.0003742   2.852 0.004339 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Theta = 4.0619
## Number of iterations in BFGS optimization: 35
## Log-likelihood: -7607 on 19 Df
```

If we look at the diagnostic plots we see that we have much more heteroskedacity in the residuals (figure 9a), but the addition of lagged response predictors has reduced the autocorrelation (Figure 9c and d), although significant autocorrelation still occurs.

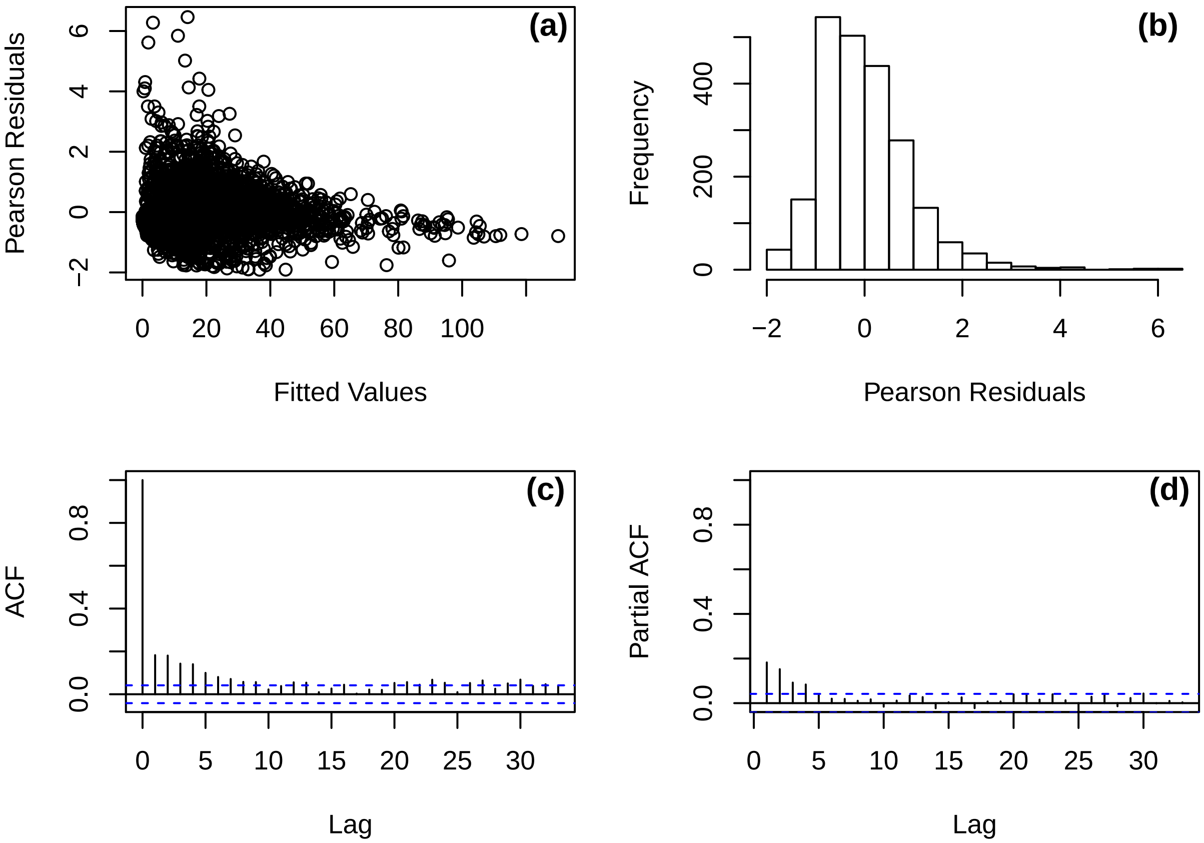


Figure 9: Model diagnostic plots. **a)** Pearson residuals versus fitted values, we should not see any clear pattern. **b)** histogram of pearson residuals, we should see residuals normally distributed around 0. **c)** Auto-correlation function of residuals and **d)** Partial autocorrelation function, vertical bars should be between the two horizontal blue lines if resdiuals are indpendent.

Lets see if our new model (zinb3) is any better than our original full model by compairing AIC.

```
##       df      AIC
## zinb  23 16051.09
## zinb3 19 15252.83
```

Looks like our new model has a better fit. We can use a sandwich estimator to generate more robust standard errors to account for both the heteroskedacity and autocorrelation in the residuals. We can then check that our predictors are still significant with better error estimates.

```
## Loading required package: sandwich
```

```
##
```

```
## t test of coefficients:
##
##                          Estimate  Std. Error  t value  Pr(>|t|)
## count_(Intercept)      1.1792e+00  4.5785e-02  25.7558 < 2.2e-16 ***
## count_AR1              1.5101e-02  1.1295e-03  13.3703 < 2.2e-16 ***
## count_AR2              5.8143e-03  1.1068e-03   5.2535 1.636e-07 ***
## count_AR3              3.8706e-03  1.0592e-03   3.6541 0.0002641 ***
## count_Discharge       -2.5177e-02  1.9862e-03 -12.6764 < 2.2e-16 ***
## count_I(Discharge^2) -1.3701e-03  1.4797e-04  -9.2591 < 2.2e-16 ***
## count_Temp.c           3.0790e-02  8.3930e-03   3.6686 0.0002497 ***
## count_Pelicans        -4.0667e-02  6.3976e-03  -6.3566 2.500e-10 ***
## count_Time             2.3673e-04  6.4682e-05   3.6599 0.0002583 ***
## zero_(Intercept)      -2.3898e+00  5.0493e-01  -4.7329 2.354e-06 ***
## zero_AR1              -1.6752e-01  2.7377e-02  -6.1191 1.111e-09 ***
## zero_Temp.c            1.2506e-01  4.7166e-02   2.6515 0.0080711 **
## zero_I(Temp.c^2)       5.2298e-02  1.4849e-02   3.5219 0.0004372 ***
## zero_Pelicans          1.3430e-01  3.0722e-02   4.3715 1.292e-05 ***
## zero_fWaterClarity2    9.5854e-01  4.1708e-01   2.2982 0.0216441 *
## zero_fWaterClarity3    1.9596e+00  4.3244e-01   4.5315 6.169e-06 ***
## zero_fImageClarity2   -5.5090e-01  2.3870e-01  -2.3079 0.0210961 *
## zero_Time              1.0673e-03  3.8793e-04   2.7512 0.0059867 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Looks like all of the predictors remain significant. Let's compare the fitted model with the real data.

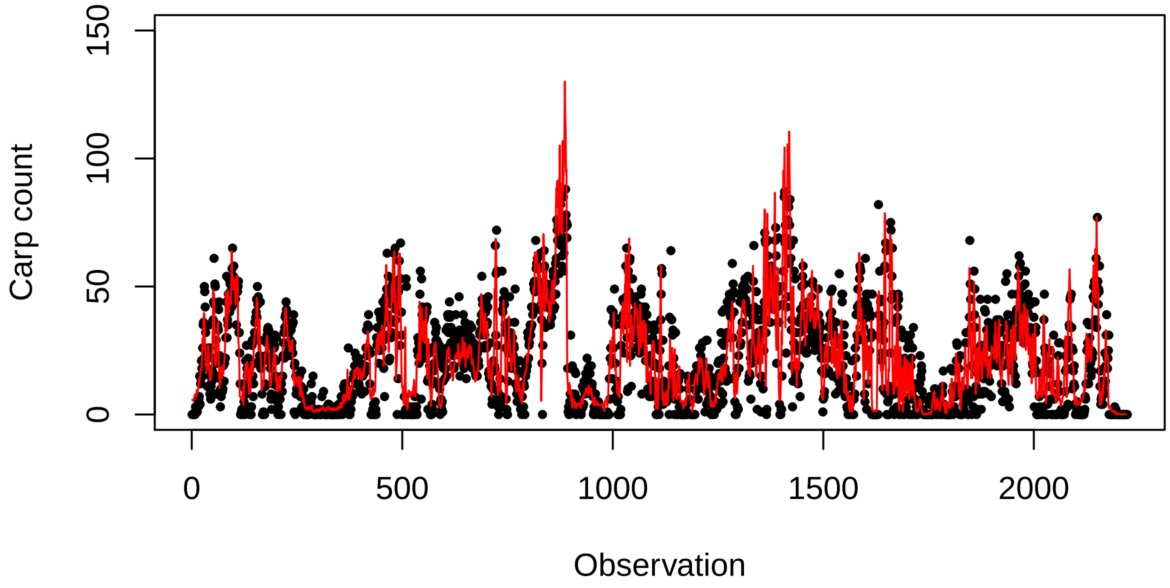


Figure 10: Carp counts (black circles) at each observation index overlayed by the fitted model zinb3 model (red line).

It appears the model does a decent job tracking the raw data (Figure 10 and 11) but over estimates counts when raw carp counts are high (Figure 10). A more appropriate model for the full dataset may be a zero-inflated generalized linear mixed model (ZIGLMM) or zero-inflated generalized additive mixed model (ZIGAMM) with an appropriate correlation error structure. These are complex models and the frontier of current statistical research, which unfortunately means these models are not readily available in R yet. Another approach to dealing with autocorrelated data is thinning the observations until they are independent,

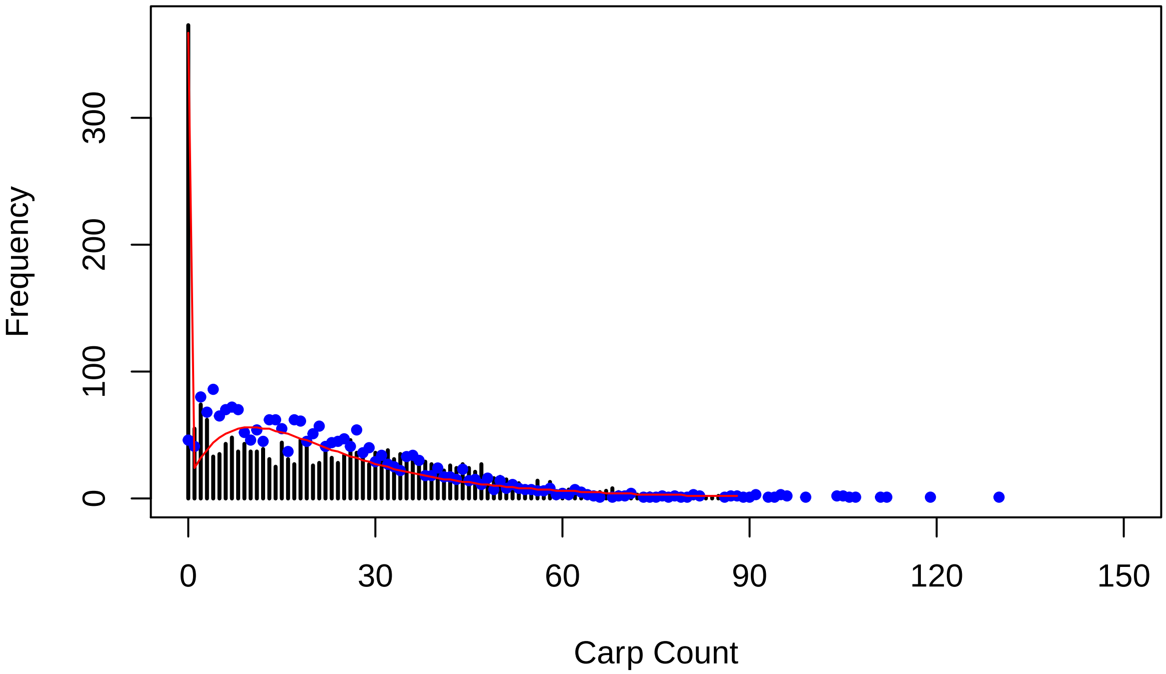


Figure 11: Original carp count histogram (black lines) with zinb3 modelled histogram (blue circles) and zero-inflated negative binomial probability curve overlayed (red line).

I will do this in the next section and compare results to our zero-inflated model with a sandwich estimator for autocorrelated data.

b) Data thinning

I will attempt to deal with the auto-correlation by thinning the data—reducing the observations until they are not auto-correlated any more. Due to the high auto-correlation, the data is not found to be independent until thinned to every 25th observation (summarized in fig 12). This means that observations are indpendent of one another every 6.25 hours and reduces the total number of observations from 2222 to 89.

Now lets look at the thinned data. Figure 13 shows we are still potentally dealing with zero inflated and the relationships between the count data and covariates show similar relationships to the full dataset.

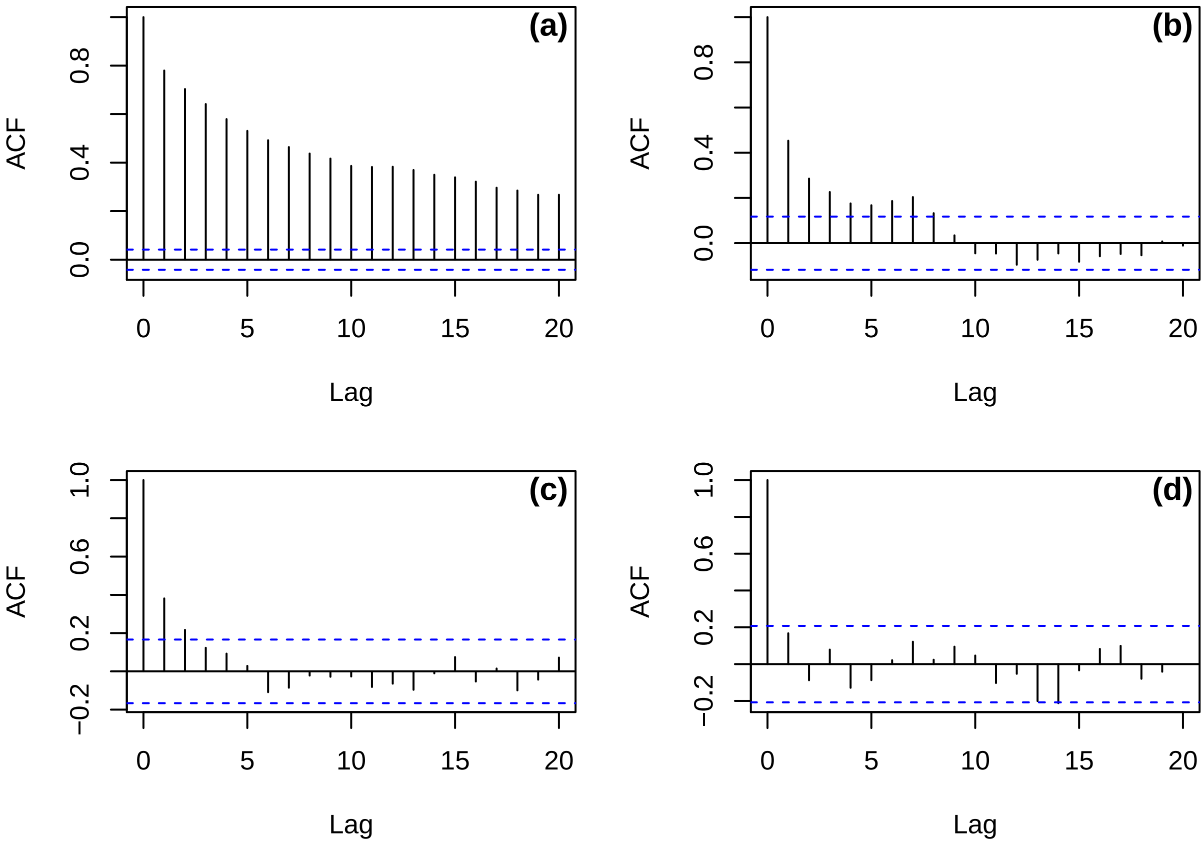
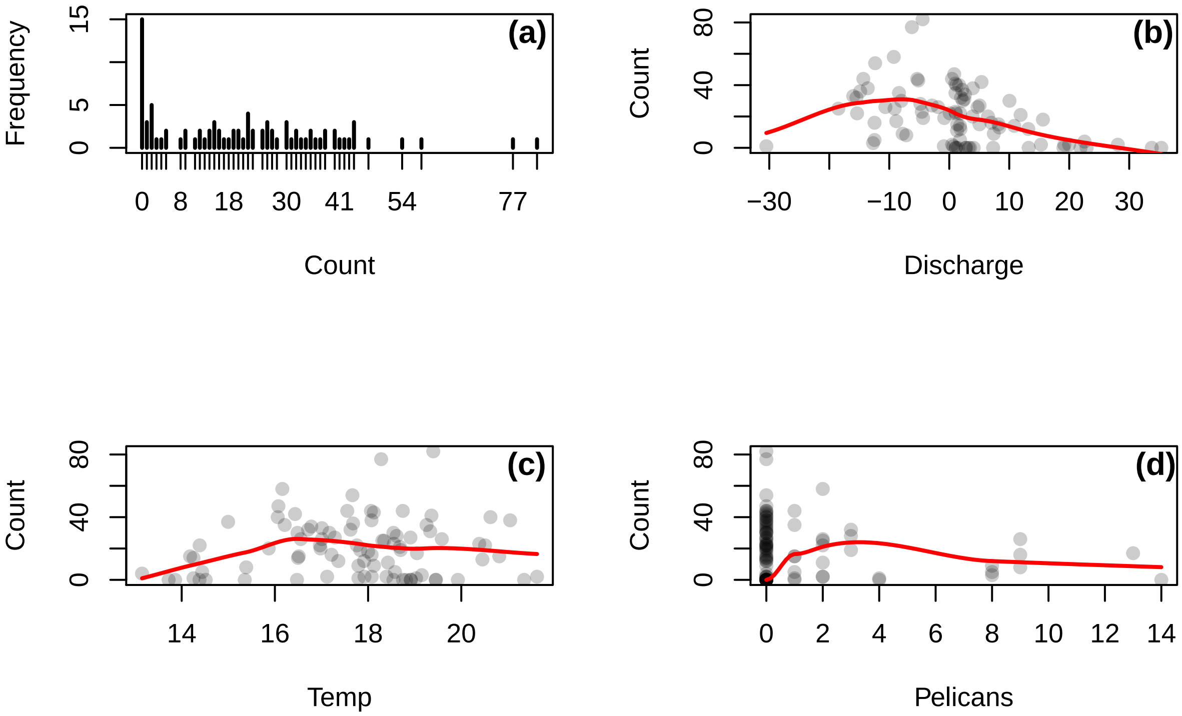


Figure 12: Auto-correlation function for (a) raw data, (b) thined by 8 observations, (c) thinned by 16 observations, and (d) thinned by 25 observations. Vertical bars should be between the two horizontal blue lines if resdiuals are indpendent.



Lets see if we need a zero-inflated model to predict the number of zeros in our dataset

```
#Poisson glm
pois_thin<-glm(Count~Discharge+I(Discharge^2)+Temp.c+I(Temp.c^2)+Pelicans+offset(log(adjArea)), family=

#Negative binomial glm model
negb_thin<-glm.nb(Count~Discharge+I(Discharge^2)+Temp.c+I(Temp.c^2)+Pelicans+offset(log(adjArea)), data=

#zero inflated poisson
zip_thin<-zeroinfl(Count~Discharge+I(Discharge^2)+Temp.c+I(Temp.c^2)+Pelicans+offset(log(adjArea))|Discl
#zero inflated negative binomial
zinb_thin<-zeroinfl(Count~Discharge+I(Discharge^2)+Temp.c+I(Temp.c^2)+Pelicans+offset(log(adjArea))|Disc
```

```
##  Obs Pois   NB  ZIP ZINB
##   15    4    7   14   14
```

Looks like the zero inflated models are more accurate in determining the zeros, let's see if whether the zero-infalted poisson accounts for the overdispersion or if we need a zero-inflated negative binomial by conducting a liklihood ratio test and comparing AIC values.

```
lrtest(zip_thin,zinb_thin)
```

```
## Likelihood ratio test
##
## Model 1: Count ~ Discharge + I(Discharge^2) + Temp.c + I(Temp.c^2) + Pelicans +
##     offset(log(adjArea)) | Discharge + I(Discharge^2) + Temp.c +
##     I(Temp.c^2) + Pelicans + fExposure + fWaterClarity + fImageClarity
## Model 2: Count ~ Discharge + I(Discharge^2) + Temp.c + I(Temp.c^2) + Pelicans +
##     offset(log(adjArea)) | Discharge + I(Discharge^2) + Temp.c +
##     I(Temp.c^2) + Pelicans + fExposure + fWaterClarity + fImageClarity
##   #Df  LogLik Df  Chisq Pr(>Chisq)
## 1  16 -432.20
## 2  17 -309.19  1 246.02  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
AIC(zip_thin,zinb_thin)
```

```
##           df      AIC
## zip_thin  16 896.3922
## zinb_thin 17 652.3758
```

Looks like the ZINB is the better model. Model selection time. count remove temp.c + temp.cˆ2| zero remove image clarity + temp.c +temp.cˆ2+Pelicans + pelicansˆ2+ water clarity

```
#zero inflated negative binomial
zinb_thin2<-zeroinfl(Count~Discharge+I(Discharge^2)+Temp.c+I(Temp.c^2)+Pelicans+offset(log(adjArea))|Di
summary(zinb_thin2)
```

```
##
## Call:
## zeroinfl(formula = Count ~ Discharge + I(Discharge^2) + Temp.c +
##     I(Temp.c^2) + Pelicans + offset(log(adjArea)) | Discharge +
##     I(Discharge^2) + Temp.c + I(Temp.c^2) + Pelicans + fExposure +
##     fWaterClarity + fImageClarity, data = Z[seq(1, dim(Z)[1], by = 25),
##     ], dist = "negbin")
##
## Pearson residuals:
##     Min      1Q  Median      3Q     Max
## -1.6342 -0.6256 -0.1053  0.5243  2.1519
##
## Count model coefficients (negbin with log link):
##                  Estimate Std. Error z value Pr(>|z|)
## (Intercept)     2.1808592  0.1048269  20.804  < 2e-16 ***
## Discharge      -0.0462162  0.0083565  -5.531 3.19e-08 ***
## I(Discharge^2) -0.0033614  0.0005378  -6.250 4.10e-10 ***
## Temp.c          0.0223601  0.0430925   0.519   0.6038
## I(Temp.c^2)    -0.0255916  0.0199961  -1.280   0.2006
## Pelicans       -0.0912952  0.0262302  -3.481   0.0005 ***
## Log(theta)      1.2253776  0.2114862   5.794 6.87e-09 ***
##
## Zero-inflation model coefficients (binomial with logit link):
##                  Estimate Std. Error z value Pr(>|z|)
## (Intercept)      -0.21150    2.59331  -0.082   0.9350
## Discharge         0.47441    0.25842   1.836   0.0664 .
## I(Discharge^2)   -0.02760    0.01517  -1.820   0.0688 .
## Temp.c           -0.05287    0.21177  -0.250   0.8028
## I(Temp.c^2)       0.16025    0.09588   1.671   0.0946 .
## Pelicans          0.22879    0.19158   1.194   0.2324
## fExposure2       -3.30444    2.71637  -1.216   0.2238
## fWaterClarity2   -0.50408    1.36843  -0.368   0.7126
## fWaterClarity3    0.57501    1.39171   0.413   0.6795
## fImageClarity2  -15.82044 1888.65893  -0.008   0.9933
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Theta = 3.4055
## Number of iterations in BFGS optimization: 34
## Log-likelihood: -309.2 on 17 Df
```

```
zinb_thin2<-zeroinfl(Count~Discharge+I(Discharge^2)+Temp.c+I(Temp.c^2)+Pelicans+offset(log(adjArea))|Di
summary(zinb_thin2)
```

```
##
## Call:
## zeroinfl(formula = Count ~ Discharge + I(Discharge^2) + Temp.c +
##     I(Temp.c^2) + Pelicans + offset(log(adjArea)) | Discharge +
##     I(Discharge^2) + Temp.c + I(Temp.c^2) + Pelicans + fExposure +
##     fWaterClarity + fImageClarity, data = Z[seq(1, dim(Z)[1], by = 25),
##     ], dist = "negbin")
##
## Pearson residuals:
##     Min      1Q  Median      3Q     Max
## -1.6342 -0.6256 -0.1053  0.5243  2.1519
##
## Count model coefficients (negbin with log link):
##                 Estimate Std. Error z value Pr(>|z|)
## (Intercept)     2.1808592  0.1048269  20.804  < 2e-16 ***
## Discharge      -0.0462162  0.0083565  -5.531 3.19e-08 ***
## I(Discharge^2) -0.0033614  0.0005378  -6.250 4.10e-10 ***
## Temp.c          0.0223601  0.0430925   0.519   0.6038
## I(Temp.c^2)    -0.0255916  0.0199961  -1.280   0.2006
## Pelicans       -0.0912952  0.0262302  -3.481   0.0005 ***
## Log(theta)      1.2253776  0.2114862   5.794 6.87e-09 ***
##
## Zero-inflation model coefficients (binomial with logit link):
##                 Estimate Std. Error z value Pr(>|z|)
## (Intercept)      -0.21150    2.59331  -0.082   0.9350
## Discharge         0.47441    0.25842   1.836   0.0664 .
## I(Discharge^2)   -0.02760    0.01517  -1.820   0.0688 .
## Temp.c           -0.05287    0.21177  -0.250   0.8028
## I(Temp.c^2)       0.16025    0.09588   1.671   0.0946 .
## Pelicans          0.22879    0.19158   1.194   0.2324
## fExposure2       -3.30444    2.71637  -1.216   0.2238
## fWaterClarity2   -0.50408    1.36843  -0.368   0.7126
## fWaterClarity3    0.57501    1.39171   0.413   0.6795
## fImageClarity2  -15.82044 1888.65893  -0.008   0.9933
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Theta = 3.4055
## Number of iterations in BFGS optimization: 34
## Log-likelihood: -309.2 on 17 Df
```

```
zinb_thin2<-zeroinfl(Count~Discharge+I(Discharge^2)+Temp.c+I(Temp.c^2)+Pelicans+offset(log(adjArea))|Di
summary(zinb_thin2)
```

```
##
## Call:
## zeroinfl(formula = Count ~ Discharge + I(Discharge^2) + Temp.c +
##     I(Temp.c^2) + Pelicans + offset(log(adjArea)) | Discharge +
##     I(Discharge^2) + Temp.c + I(Temp.c^2) + Pelicans + fExposure +
##     fWaterClarity + fImageClarity, data = Z[seq(1, dim(Z)[1], by = 25),
##     ], dist = "negbin")
##
## Pearson residuals:
##     Min      1Q  Median      3Q     Max
## -1.6342 -0.6256 -0.1053  0.5243  2.1519
##
## Count model coefficients (negbin with log link):
##                  Estimate Std. Error z value Pr(>|z|)
## (Intercept)     2.1808592  0.1048269  20.804  < 2e-16 ***
## Discharge      -0.0462162  0.0083565  -5.531 3.19e-08 ***
## I(Discharge^2) -0.0033614  0.0005378  -6.250 4.10e-10 ***
## Temp.c          0.0223601  0.0430925   0.519   0.6038
## I(Temp.c^2)    -0.0255916  0.0199961  -1.280   0.2006
## Pelicans       -0.0912952  0.0262302  -3.481   0.0005 ***
## Log(theta)      1.2253776  0.2114862   5.794 6.87e-09 ***
##
## Zero-inflation model coefficients (binomial with logit link):
##                  Estimate Std. Error z value Pr(>|z|)
## (Intercept)      -0.21150    2.59331  -0.082   0.9350
## Discharge         0.47441    0.25842   1.836   0.0664 .
## I(Discharge^2)   -0.02760    0.01517  -1.820   0.0688 .
## Temp.c           -0.05287    0.21177  -0.250   0.8028
## I(Temp.c^2)       0.16025    0.09588   1.671   0.0946 .
## Pelicans          0.22879    0.19158   1.194   0.2324
## fExposure2       -3.30444    2.71637  -1.216   0.2238
## fWaterClarity2   -0.50408    1.36843  -0.368   0.7126
## fWaterClarity3    0.57501    1.39171   0.413   0.6795
## fImageClarity2  -15.82044 1888.65893  -0.008   0.9933
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Theta = 3.4055
## Number of iterations in BFGS optimization: 34
## Log-likelihood: -309.2 on 17 Df
```

c) Biologically relavent thinning

```
library(mgcv)
```

```
## Loading required package: nlme
```

```
## This is mgcv 1.8-11. For overview type 'help("mgcv-package")'.
```

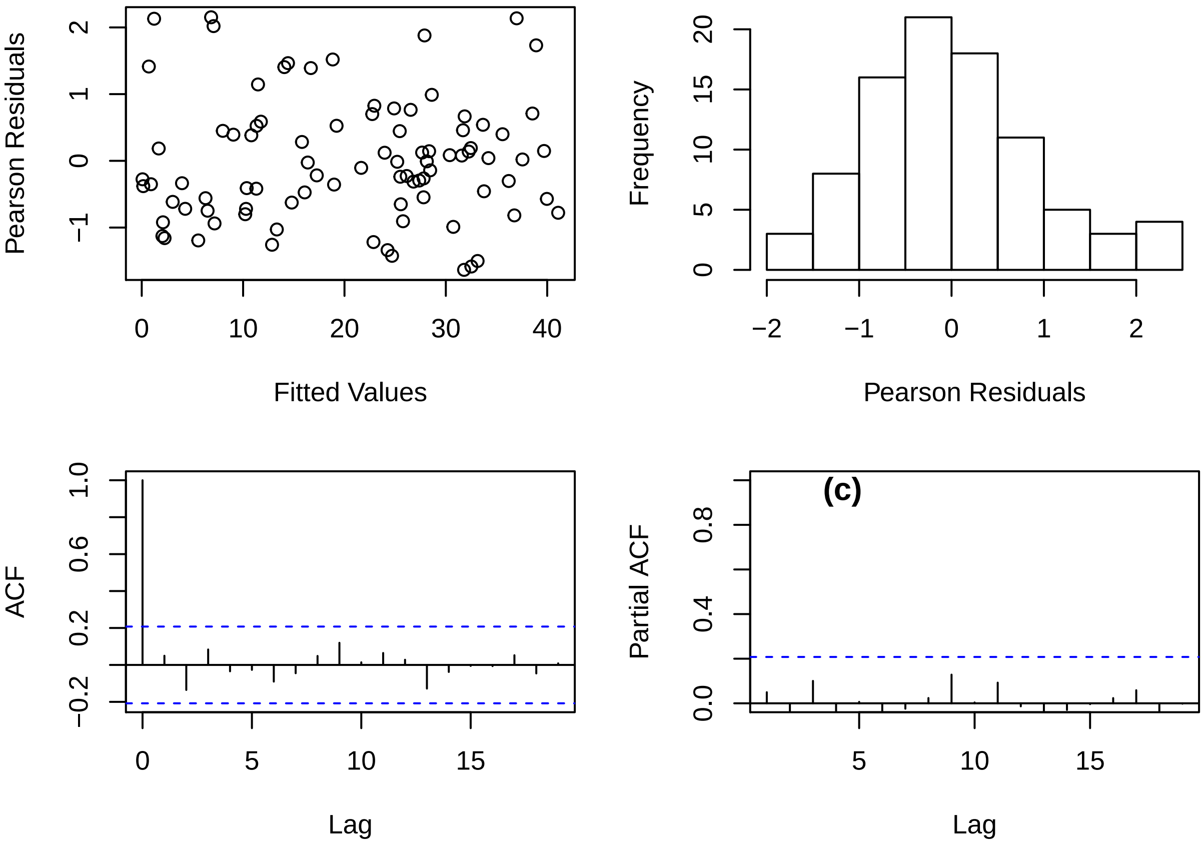


Figure 13: Model diagnostic plots. **a)** Pearson residuals versus fitted values, we should not see any clear pattern. **b)** histogram of pearson residuals, we should see residuals normally distributed around 0. **c)** Auto-correlation function of residuals and **d)** Partial autocorrelation function, vertical bars should be between the two horizontal blue lines if resdiuals are indpendent.

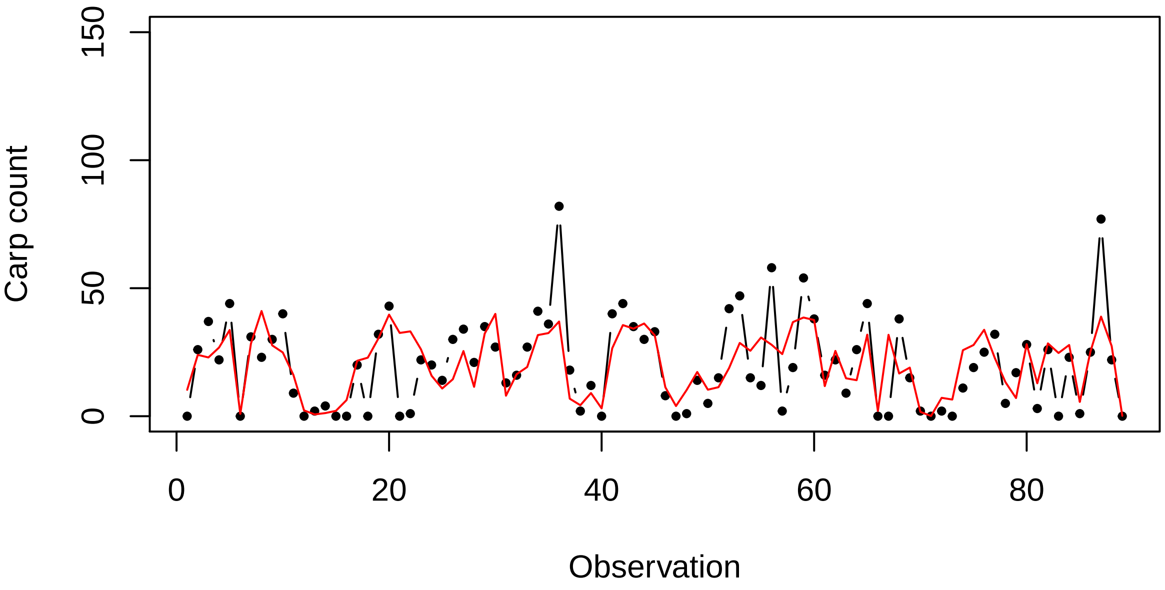


Figure 14: Carp counts (black circles) at each observation index overlayed by the fitted model zinb3 model (red line).

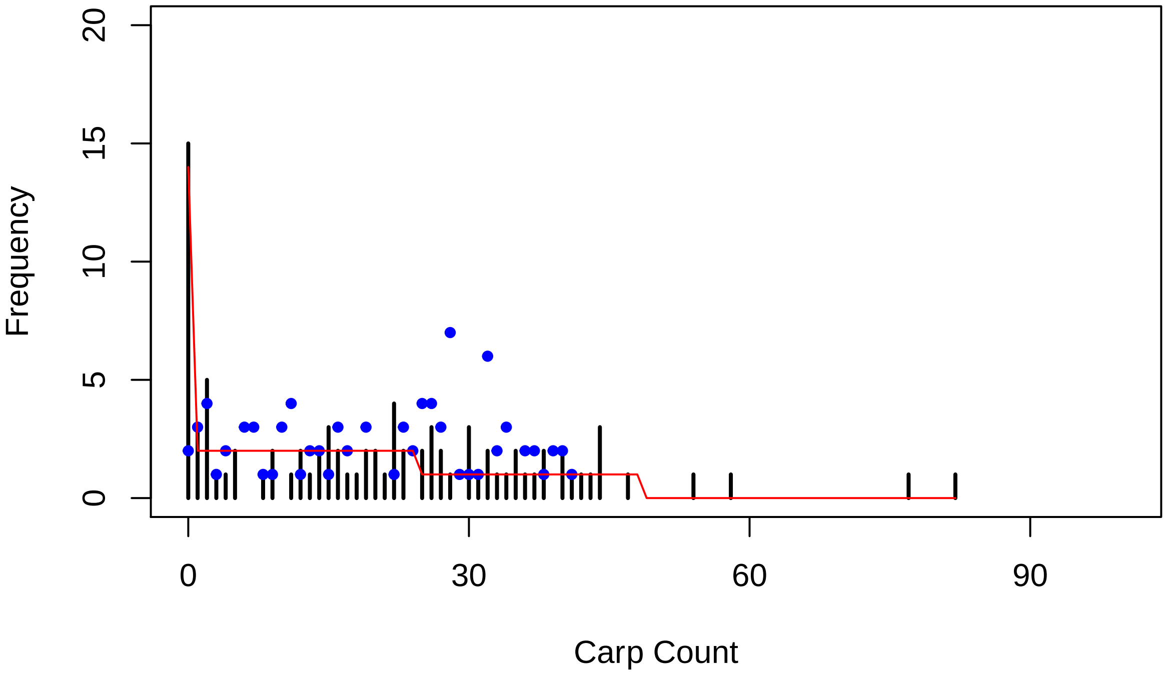
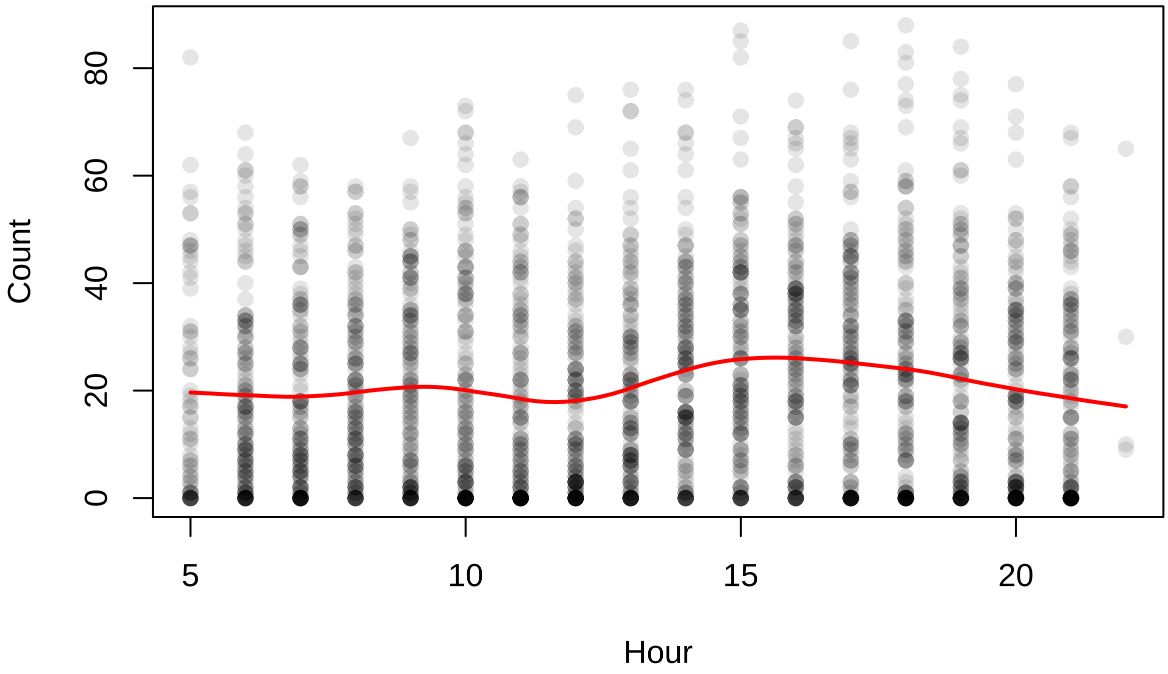


Figure 15: Original carp count histogram (black lines) with zinb3 modelled histogram (blue circles) and zero-inflated negative binomial probability curve overlayed (red line).

```
plot(Count~Hour,data=Z,pch=19,col=rgb(0,0,0,0.1))
fit<-loess(Count~Hour,data=Z,family="gaussian",span=.25, degree=1)
fit2<-gam(Count~s(Hour),data=Z,offset=log(adjArea),family="nb")

fit2<-gam(Count~s(Discharge)+s(Temp)+Pelicans+s(DOY)+s(Hour),data=Z,offset=log(adjArea),family="nb")
fit3<-gam(Count~s(Discharge)+Pelicans+s(DOY)+s(Hour),data=Z,offset=log(adjArea),family="nb")


curve(predict(fit,data.frame(Hour=x),type="response"),add=TRUE,lwd=2,col="red")
```



d) Gamm AR3 model

Summary of results

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