Lecture #2: Functions, Expressions

Administrative

- Reader with discussion and other materials available at Vick Copy (Euclid and Hearst).

- Sign yourself up on Piazza. See course web page:

  `http://inst.cs.berkeley.edu/~cs61a`

- Be sure to get an account form next week in lab, and provide registration data.

Announcement: We're trying to hire a new lecturer. There will be two candidates coming Jan. 27–28 (Josh Hug) and Feb. 3–4 (John DeNero), and you can help evaluate them! For both days:

- Mon 01:00pm-02:00pm "Big ideas" talk (in Woz)

- Tue 11:45am-12:45pm Undergrad student lunch on northside (meet in 777 Soda)

- Tue 01:00pm-02:00pm Demo Class talk (in 380 Soda for Josh, Woz for John)

- UG Tue 02:00pm-02:45pm Open Session after demo class (same rooms)

Recap

- From last lecture: *Values* are data we want to manipulate and in particular,

- *Functions* are values that perform computations on values.

- *Expressions* denote computations that produce values.

- Today, we'll look at them in some detail at how functions operate on data values and how expressions denote these operations.

- As usual, although our concrete examples all involve Python, the actual concepts apply almost universally to programming languages.

Functions

- Something like abs *denotes* or *evaluates to* a function.

- To depict the denoted function *values*, we sometimes use this notation:

$$abs(x): \qquad\qquad add(a, b)$$
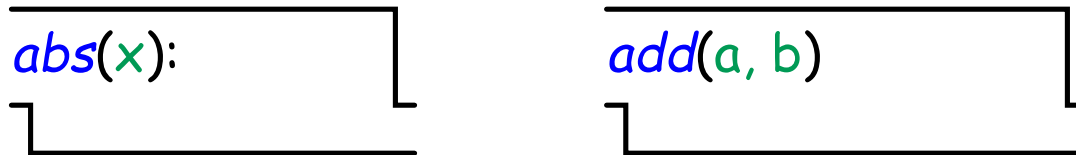
- Idea: The opening on the left takes in values and one on the right to delivers results.

- The (green) *formal parameter names*—such as x, a, b—show the number of parameters (inputs) to the function.

- The list of formal parameter names gives us the function's *signature*—in Python, this is the number of arguments.

- For our purposes, the blue name is simply a helpful comment to suggest what the function does.

- (Python actually maintains this *intrinsic name* and the parameter names internally, but this is not a universal feature of programming languages, and, as you'll see, can be confusing.)

Functions: Lambda

- I'm often going to use a more venerable notation for function values:

  λ x: $\ll$ | x | $\gg$ λ a, b: $\ll$ the sum of a and b $\gg$

- Formal parameters go to the left of the colon.

- The part to the right of the colon is an expression that indicates what value is produced.

- I'll use $\ll \cdots \gg$ expressions to indicate non-Python descriptions of values or computations.

- In Python, you can *denote* simple function values like this:

  ```
  lambda a, b :  ≪ the sum of a and b ≫
  ```

  which evaluates to

  λ a, b: $\ll$ the sum of a and b $\gg$

- (Well, OK: the $\ll \cdots \gg$ isn't really Python, but I'll use it as a place-holder for some computation I'm not prepared to write.)
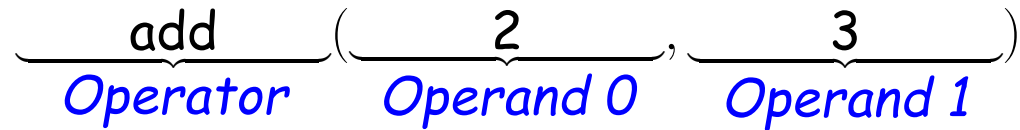
Calling Functions (I)

- The fundamental operation on function values is to *call* or *invoke* them, which means giving them one value for each formal parameter and having them produce the result of their computation on these values:

$$-5 \,\triangleright\, abs(\text{number}): \qquad \triangleright\, 5$$

$$(29, 13) \,\triangleright\, add(\text{left}, \text{right}) \qquad \triangleright\, 42$$

Call Expressions

- A call expression denotes the operation of calling a function.

- Consider add(2, 3):

$$\underbrace{\text{add}}_{\textit{Operator}} (\underbrace{\qquad 2 \qquad}_{\textit{Operand 0}}, \underbrace{\qquad 3 \qquad}_{\textit{Operand 1}})$$

- The operator and the operands are all themselves expressions (recursion again).

- To evaluate this call expression:

  - Evaluate the operator (let's call the value C). It must evaluate to a function.

  - Evaluate the operands (or *actual parameters* in the order they appear (let's call these values P_0 and P_1)

  - Call C with parameters P_0 and P_1.

Calling a Function (I): Substitution

- Once we have the values for the operator and operands, we must still actually evaluate the call.

- A simple way to understand this (which will work for simple expressions) is to think of the process as *substitution*.

- Once you have a value:

  λ a, b: $\ll$ sum of a and b $\gg$

- and values for the operands (let's say 2 and 3),

- *substitute* the operand values for the formal parameters, replacing the whole call with

  $\ll$ sum of 2 and 3 $\gg$

- which in turn evaluates to 5.

Side Trip: Values versus Denotations

- Expressions such as 2 in a programming language are called *literals*.

- To evaluate them, we replace them with whatever values they are supposed to stand for.

- This is confusing:
  - **Q:** What is the value of the literal 2?
  - **A:** 2.

- ...and then you get into long, technical explanations about how the second "2" is really in a different language than the first, and actually is just another notation for some mystical Platonic "2" that is floating off somewhere.

- I'll just try to be practical and distinguish values from literals by surrounding values in a boxes: the value of 2 is $\boxed{2}$.

- One way to see the distinction between literals and values: the literals 0x10 and 16 are obviously different, but both denote the same value: $\boxed{16}$.

Example: From Expression to Value

Let's evaluate the expression mul(add(2, mul(0x4, 0x6)), add(0x3, 005)).
In the following sequence, values are shown in boxes.
Everything outside a box is an expression.

- $\underbrace{mul}(\underbrace{add}(2, \underbrace{mul(0x4, 0x6)})}, \underbrace{add(0x3, 005)})$

- $\boxed{\lambda\, a, b: \ll a \times b \gg}$ (add(2, mul(0x4, 0x6)), add(0x3, 005))

- $\boxed{\lambda\, a, b: \ll a \times b \gg}$ ($\boxed{\lambda\, a, b: \ll a + b \gg}$ ($\boxed{2}$, $\boxed{\lambda\, a, b: \ll a \times b \gg}$ ($\boxed{4}$, $\boxed{6}$)),
add(0x3, 005))

- $\boxed{\lambda\, a, b: \ll a \times b \gg}$ ($\boxed{\lambda\, a, b: \ll a + b \gg}$ ($\boxed{2}$, $\ll 4 \times 6 \gg$, add(0x3, 005))

- $\boxed{\lambda\, a, b: \ll a \times b \gg}$ ($\boxed{\lambda\, a, b: \ll a + b \gg}$ ($\boxed{2}$, $\boxed{24}$), add(0x3, 005))

- $\boxed{\lambda\, a, b: \ll a \times b \gg}$ ($\ll 2 + 24 \gg$, add(0x3, 005))

- $\boxed{\lambda\, a, b: \ll a \times b \gg}$ ($\boxed{26}$, add(0x3, 005))

- $\boxed{\lambda\, a, b: \ll a \times b \gg}$ ($\boxed{26}$, $\boxed{\lambda\, a, b: \ll a + b \gg}$ ($\boxed{3}$, $\boxed{5}$))

- ... $\boxed{\lambda\, a, b: \ll a \times b \gg}$ ($\boxed{26}$, $\boxed{8}$)

- ... $\boxed{208}$.

Puzzle I

Evaluate

    (lambda a: lambda b: a + b)(1)(3)

- First, must understand how it's grouped:

   ((lambda a: lambda b: a + b)(1))(3)

Puzzle I (contd.)

- (lambda a: lambda b: a + b)(1)(3)

- $\boxed{\lambda \text{ a: lambda b: a + b}}(\boxed{1})(3)$

- (lambda b: $\boxed{1}$ + b)(3)

- $\boxed{\lambda \text{ b: } \boxed{1} \text{ + b}}$ ($\boxed{3}$)

- $\boxed{1}$ + $\boxed{3}$

- 4

Impure Functions

- The functions so far have been *pure:* their output depends only on their input parameters' values, and they do nothing in response to a call but compute a value.

- Functions may do additional things when called besides returning a value.

- We call such things *side effects.*

- Example: the built-in print function:

$$-5 \,\triangleright\, \text{print}(\bullet \ \bullet \ \bullet) \qquad \triangleright\ \text{None}$$

display text '-5'
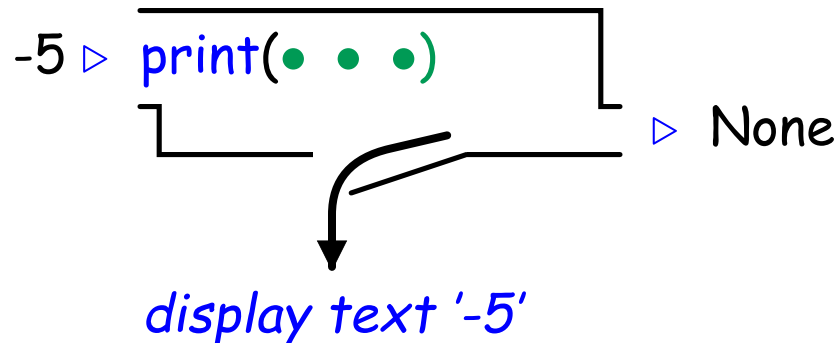
- Displaying text is print's side effect. It's value, in fact, is generally useless (always the null value).

- For this lecture (at least), I'll use λ! ("lambda bang") to denote function values with side effects.

Example: Print

What about an expression with side effects?

1. print(print(1), print(2))

2. $\boxed{\lambda!\ \ x: \ll \text{print}\ x \gg}$ ($\boxed{\lambda!\ \ x: \ll \text{print}\ x \gg}$ ($\boxed{1}$), print(2))

3. $\boxed{\lambda!\ \ x: \ll \text{print}\ x \gg}$ ($\boxed{\text{None}}$, print(2))
      and print '1'.

4. $\boxed{\lambda!\ \ x: \ll \text{print}\ x \gg}$ ($\boxed{\text{None}}$, $\boxed{\lambda!\ \ x: \ll \text{print}\ x \gg}$ ($\boxed{2}$))

5. $\boxed{\lambda!\ \ x: \ll \text{print}\ x \gg}$ ($\boxed{\text{None}}$, $\boxed{\text{None}}$))
      and print '2'.

6. $\boxed{\text{None}}$
      and print 'None None'.