Lecture #4: Higher-Order Functions

Control

- The expressions we've dealt with recently evaluate *all* of their operands *in order*.
- While there are very clever ways to do everything with just this [challenge!], it's generally clearer to introduce constructs that *control* the order in which their components execute.
- A *control expression* evaluates some or all of its operands in an order depending on the kind of expression, and typically on the values of those operands.
- A *statement* is a construct that produces no value (not even None, but is used solely for its side effects.
- A *control statement* is a statement that, like a control expression, evaluates some or all of its components, in an order that may depend on the these components.
- We typically speak of statements being *executed* rather than evaluated, but the two concepts are essentially the same, apart from the question of a value.

Conditional Expressions (I)

- The most common kind of control is *conditional evalutation (execution)*.
- In Python, to evaluate

  TruePart `if` *Condition* `else` *FalsePart*

  - First evaluate *Condition*.
  - If the result is a "*true value,*" evaluate *TruePart*; its value is then the value of the whole expression.
  - Otherwise, evaluate *FalsePart*; its value is then the value of the whole expression.
- Example:

If x is 2:	If x is 0:
1 / x if x != 0 else 1	1 / x if x != 0 else 1
1 / x if 2 != 0 else 1	1 / x if 0 != 0 else 1
⟹ 1 / x if True else 1	⟹ 1 / x if False else 1
⟹ 1 / x	⟹ 1
⟹ 1 / 2	⟹ 1
⟹ 0.5	

"True Values"

- Conditions in conditional constructs can have any value, not just True or False.
- For convenience, Python treats a number of values as indicating "false":
  - False
  - None
  - 0
  - Empty strings, sets, lists, tuples, and dictionaries.
- All else is a "true value" by default.
- So, for example: 13 if 0 else 5 and 13 if [] else 5 both evaluate to 5.

Conditional Expressions (II)

- To evaluate

  Left `and` *Right*

  - Evaluate *Left*.
  - If it is a false value, that becomes the value of the whole expression.
  - Otherwise the value of the expression is that of *Right*.
- This is an example of something called "*short-circuit evaluation*."
- For example,

  5 and "Hello" ⟹ "Hello".

  0 and print(6) ⟹ 0 + side-effects: None.

  [] and 1 / 0 ⟹ [].

Conditional Expressions (III)

- To evaluate

  Left `or` *Right*

  - Evaluate *Left*.
  - If it is a true value, that becomes the value of the whole expression.
  - Otherwise the value of the expression is that of *Right*.
- Another example of "*short-circuit evaluation*."
- For example,

  5 or "Hello" ⟹ 5.

  2 or print(6) ⟹ 2 + side-effects: None.

  [] and 1 / 0 ⟹ error.

Chained Comparisons

- An interesting feature of Python (quite rare; Cobol has something like it) involves the relational operators:

  `== != < > <= >= is is not in not in`

- Ordinarily, 3<4 yields True and 4<3 yields False.
- But what does 4 >= 3 > 1 produce? In Java, it's an error, and in C, it doesn't do what you probably want.
- In Python, it's a special control expression and works as expected.
- To evaluate `First > Second >= Third`, for example,
  - Evaluate First and Second.
  - If the first value is not larger than the second, stop and yield False for the entire expression.
  - Otherwise, compute the value of Third and compare against the value previously computed for Second, and yield True or False as appropriate.
  - In any case, no expression is evaluated more than once.

Chained Comparisons (II)

- So what is

  ```
  (print("A") and 3) < (print("B") and 2) < (print("C") and
  4)
  ```

  and what does it print?
- Prints A and B, evaluates to False.

Conditional Statement

- Finally, this all comes in statement form:

```
if Condition1:
    Statements1
    ...
elif Condition2:
    Statements2
    ...
...
else:
    Statementsn
    ...
```

- Execute (only) *Statements1* if *Condition1* evaluates to a true value.
- Otherwise execute *Statements2* if *Condition2* evaluates to a true value (optional part).
- ...
- Otherwise execute *Statementsn* (optional part).

Example

```
def signum(x):
    if x > 0:
        return 1
    elif x == 0:
        return 0
    else:
        return -1
```

```
# Alternative Definition
def signum(x):
    return 1 if x > 0 else 0 if x == 0 else -1
```


A Puzzle: Define compare3

```
# What goes here?

from operator import lt, gt  # Comparison functions

gt(gt(3,2), 1)          # Yields False, not like 3>2>1 (why?)

compare3(gt)(3)(2)(1)   # This should yield True
compare3(gt)(3)(2)(4)   # This should yield False
compare3(lt)(1)(2)(3)   # This should yield True
# etc.
```


One solution

```
def compare3(op):
    def normal(a):
        def comp1(b):
            if op(a,b):
                return lambda c: op(b, c)
            else:
                return lambda c: False
        return comp1
    return normal
```

Indefinite Repetition

- With conditionals and function calls, we can conduct computations of any length.
- For example, to sum the squares of all numbers from 1 to N (a parameter):

```
def sum_squares(N):
    """The sum of K**2 for K from 1 to N (inclusive)."""
    if N < 1:
        return 0
    else:
        return N**2 + sum_squares(N - 1)
```

- This will repeatedly call `sum_squares` with decreasing values (down to 1), adding in squares:

```
sum_squares(3) => 3**2 + sum_squares(2)
               => 3**2 + (2**2 + sum_squares(1))
               => 3**2 + (2**2 + (1**2 + sum_squares(0)))
               => 3**2 + (2**2 + (1**2 + 0)) => 14
```


Explicit Repetition

- But in the Python, C, Java, and Fortran communities, it is more usual to be explicit about the repetition.
- The simplest form is **while**

```
while Condition:
    Statements
```

means "If condition evaluates to a true value, execute statements and repeat the entire process. Otherwise, do nothing."

- So our sum-of-squares becomes:

```
def sum_squares(N):
    """The sum of K**2 for K from 1 to N (inclusive)."""
    result = 0
    while N >= 1:
        result += N**2    # Or result = result + N**2
        N -= 1            # Or N = N-1
    return result
```

- (Actually, this isn't quite right. What's different from the first version?)

Going Backwards

- OK: I cheated. In the recursive version, you actually add up the squares starting from the small end.
- So to be true to the original, I would write:

```
def sum_squares(N):
    """The sum of K**2 for K from 1 to N (inclusive)."""
    result = 0
    k = 1
    while k <= N:
        result += k**2
        k += 1
    return result
```


Definite Repetition

- In most programming languages, we write "counting loops" like the preceding with a specialized kind of loop. In Python:

```
def sum_squares(N):
    """The sum of K**2 for K from 1 to N (inclusive)."""
    result = 0
    # Original:
    # k = 1
    # while k <= N:
    #     result += k**2
    #     k += 1
    for k in range(1, N+1):
        result += k**2
    return result
```

- This actually means "execute `result += k**2` for every value of `k` in the range 1 (inclusive) to `N+1` (exclusive)."
- Special case of a more general version that we'll see later.