Lecture 32: Declarative Programming (Under the Hood)

Review: A "Schemish" Prolog

- Programs in our language define subsets of Scheme expressions that will be considered "true."

  `(fact` *CONCLUSION)* means that *CONCLUSION* is to be taken as true, for any replacement of its logical variables.

  `(fact` *CONCLUSION HYPOTHESIS . . .)* means that *CONCLUSION* is to be taken as true for all replacements of the logical variables that cause each of the the *HYPOTHESES* to be true.

  logical variables, represented as symbols starting with '?', stand for operands that may be replaced by other expressions (including other logical variables).

Another Example: Lists

- In ordinary Scheme, `append` (or `extend` in Python) is a function taking two lists and returning a list.
- In our Scheme Prolog, it is a *relation between three lists*, which we define by writing two facts about it that cover all cases:

```
;;; (append-to-form A B C) means "appending list B to list A produces
;;; list C.

; Fact about the empty list.
(fact (append-to-form () ?x ?x))
; Fact about a general non-empty list
(fact (append-to-form (?a . ?r) ?b (?a . ?s))  ; assuming that
      (append-to-form ?r ?b ?s))
```

Applying append-to-form

```
logic> (fact (append-to-form () ?x ?x))
logic> (fact (append-to-form (?a . ?r) ?b (?a . ?s))
             (append-to-form ?r ?b ?s))
logic> (query (append-to-form (a b c) (d e f) (a b c d e f)))
Success!
logic> (query (append-to-form (a b c) (d e f) ?x))
Success!
x: (a b c d e f)
logic> (query (append-to-form ?x (d e f) (a b c d e f)))
Success!
x: (a b c)
logic> (query (append-to-form (a b c) ?y (a b c d e f)))
Success!
y: (d e f)
logic> (query (append-to-form (a . ?r) ?x (a b c d e f)))
???
```

Permutations (Anagrams)

- When is list B a permutation (reordering) of A?
- An obvious fact:

```
logic> (fact (permutation () ()))
```

- Key fact: every permutation of $(a\ .\ R)$ consists of a permutation of R with a inserted somewhere in that permutation:

```
(0 1 2 3 4)  ===>   (4  3      1  2)
```

$$\Uparrow$$
$$0$$

- Or, in our logic language:

```
logic> (fact (permutation (?a . ?r) ?s)
             (permutation ?r ?t) (insert ?a ?t ?s))
```

where we intend (insert x $L0$ $L1$) to mean that inserting x into $L0$ (at the right place) gives $L1$:

```
logic> (fact (insert ?a ?r (?a . ?r)))
logic> (fact (insert ?a (?b . ?r) (?b . ?s)) (insert ?a ?r ?s))
```

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Operational and Declarative Meanings

- An assertion

  ```
  (fact (eats ?P ?F) (hungry ?P) (has ?P ?F) (likes ?P ?F))
  ```

  means that for any replacement of `?P` (e.g., 'brian') and `?F` (e.g., 'potstickers') throughout the rule:

  Declarative Meaning If brian is hungry and has potstickers and likes potstickers, then brian will eat potstickers.

  Operational Meaning To show that brian will eat potstickers, show that brian is hungry, then that brian has potstickers, and then that brian likes potstickers.

- The *declarative meaning* allows us to look at our Scheme-Prolog program as a logical specification of a problem for which the system is to find a solution.
- The *operational meaning* allows us to look at our Scheme-Prolog specification as an executable program for searching for a solution.
- *Closed Universe Assumption:* We make only positive statements. The closest we come to saying that something is false is to say that we can't prove it.

Unification

- In general, our system, given a target expression involving a predicate to prove, must find a fact that might assert that target, given a suitable replacement of logical variables.
- To do this, we try to pattern-match the conclusions of all our facts against the target expression.
- The pattern matching is called *unification,* [J. A. Robinson].

```
(likes  brian  potstickers)
(likes    ?P       ?F)
```

True: {P: brian, F: potstickers}

- The substitution itself (the dictionary on the right) is called a *unifier*.

Unification (II)

- The substitution has to be uniform:

```
(le    0    1)
(le   ?x   ?x)
```

False

- And logical variables may appear in either expression (unification is *symmetric*).

```
(related (a b c)     ?x     )
(related   ?x     (a .  ?r))
```

True: { x: (a b c), r: (b c) }

- It is possible for logical variables to be unified with each other:

```
(likes ?P yams)
(likes ?Q  ?F )
```

True: { P: ?Q, F: yams }, or { Q: ?P, F: yams }

Implementing Unification

- A plain, unbound logical variable will unify with anything. Must record this unification in the unifier we construct.
- Before unifying other (bound) logical variables, first must replace them with their recorded bindings, in order to make sure we bind consistently.
- To unify two atoms (numbers, booleans, symbols that are not logical variables), just compare them.
- To unify two lists: recursively unify their heads and tails.

Implementing Unification: Code

A simple tree recursion with side-effects:

```
def unify(e, f, env):
    """Destructively extend ENV so as to unify (make equal) E and F, returning
    True if this succeeds and False otherwise.  ENV may be modified in either
    case (its existing bindings are never changed)."""
    e = lookup(e, env)
    f = lookup(f, env)
    if scheme_eqvp(e, f):
        return True
    elif isvar(e):
        env.define(e, f)
        return True
    elif isvar(f):
        env.define(f, e)
        return True
    elif scheme_atomp(e) or scheme_atomp(f):
        return False
    else:
        return unify(e.first, f.first, env) and unify(e.second, f.second, env)
```

Using Unification to Search for Proofs

- The process of attempting to demonstrate an assertion (answer a query) is a systematic *depth-first search* of facts.

```
def search(clauses, env):
    if clauses is nil:
        yield env
    for fact in fact database:
        fact = rename_variables(fact, ...)
        env_head = new environment that extends env
        if unify(conclusion of fact, first clause, env_head):
           for env_rule in search(hypotheses of fact, env_head):
               for result in search(rest of clauses, env_rule):
                   yield result
```

- In the actual program, we put on a *depth limit*: a limit on how deeply the recursive calls on search may go.
- This prevents us from going down infinite paths when there is a finite path that will work.