

## Lecture #7: Recursion (and a data structure)

### Announcements:

- A message from the AWE:

"The Association of Women in EECS is hosting a 61A party this Sunday (2/9) from 1-3PM in the Woz! Come hang out, befriend other girls in 61A and meet AWE members who have taken it before! There will be lots of food, games, and fun!"

- Guerrilla Sections this weekend. Extra, optional sections to practice HOF and Environment Diagrams this weekend. You'll be expected to work in groups on questions that range from basic to midterm-level. Details will be announced on Piazza.

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## Data Structures

- To date, we've dealt with numbers and functions for the most part.
- Although one can do just about anything with these, it's not exactly convenient.
- Example: encode a *pair of integers* as a single integer:

$$(x, y) \Leftrightarrow 2^x \cdot 3^y$$

- Every  $(x, y)$  pair can be encoded, but extracting  $x$  and  $y$  is a chore.
- So Python (like most languages) provides a set of additional *data structures* for representing *collections of values*.

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## Creating Tuples

- To create (*construct*) a tuple, use a sequence of expressions in parentheses:

```
()          # The tuple with no values
(1, 2)       # A pair: tuple with two items
(1, )       # A singleton tuple: use comma to distinguish from (1)
(1, "Hello", (3, 4)) # Any mix of values possible.
```

- When unambiguous, the parentheses are unnecessary:

```
x = 1, 2, 3      # Same as x = (1,2,3)
return True, 5   # Same as return (True, 5)
for i in 1, 2, 3: # Same as for i in (1,2,3):
```

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## Selecting from Tuples

- Can compare, print, or *select* values from a tuple; little else.
- Selection is by explicit item number or "unpacking":

```
>>> x = (1, 7, 5)
>>> print(x[1], x[2])
7 5
>>> from operator import getitem
>>> print(getitem(x, 1), getitem(x, 2))
7 5
>>> x = (1, (2, 3), 5)
>>> print(len(x))
3
>>> a, b, c = x
>>> print(b, c)
(2, 3) 5
>>> d, (e, f), g = x
>>> print(e, g)
2, 5
>>> x, y = y, x
???
```

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## More Selection

Selecting sub tuples (*slices*) is also possible:

```
>>> x = (1, 7, 5, 6)
>>> print(x[1:3], x[0:2], x[:2], x[1:4], x[1:], x[1:2])
(7, 5) (1, 7) (1, 7) (7, 5, 6) (7, 5, 6) (7,)
>>> from operator import getitem
>>> print(getitem(x, slice(1,3)), getitem(x, slice(0,2)))
(7, 5) (1, 7)
>>> a, *b, c = x
>>> print(a, b, c)
1 (7, 5) 6
>>> a, *b = x
>>> print(a, b)
1 (7, 5, 6)
```

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## Multiple Returns

Tuples provide a useful way to return multiple things from a function:

```
>>> divmod(38, 5) # Returns (38//5, 38%5)
(7, 3)

>>> def sumprod(x, y):
...     return x+y, x*y
>>> sumprod(3, 5)
(8, 15)
```

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## Tuple is a Recursive Type

- Tuple is one type of *value*.
- Values thus include integers, booleans, strings, and tuples (among others).
- Tuples are sequences of 0 or more *values*.
- Therefore, the definitions of "value" and "tuple" are *recursive*: they refer to themselves.
- In this case, we'd say that their definitions are *mutually recursive*, since they each refers to the other.
- Recursive data types and recursive algorithms go together.

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## Example: How Many Numbers?

- Let's consider a restricted tuple (call it a "numeric pair") consisting of:
  - The empty tuple: `()`,
  - Or a tuple containing two values, each of which is an integer or a numeric pair (still more recursion!)
- Given such a numeric pair, how many numbers are in it?

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## Example: Code

```
def count_vals(pair):
    """Assuming PAIR is a numeric pair, the total number of integers
    contained in the pair.
    >>> count_vals(())
    0
    >>> count_vals( (1, ()) )
    1
    >>> count_vals( (1, 2) )
    2
    >>> count_vals( ((1, 2), ((3, 4), ())) )
    4
    """
    if pair == ():
        return 0
    elif type(pair) is int:
        return 1
    else return count_vals(pair[0]) + count_vals(pair[1])
```

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## The Recursive Leap of Faith

- To implement *count\_vals*, we trusted its comment to be correct, even as we implemented it.
- This is the essence of *recursive thinking*.
- If we can show that
  - Our implementation is correct *given* that the comment is correct,
  - And if we can show that the process must terminate,then the comment (the specification of the function) is correct.
- For recursive data structures, showing termination involves using a form of *Noetherian induction*.

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## Noetherian Induction



(Source: [http://en.wikipedia.org/wiki/Emmy\\_Noether](http://en.wikipedia.org/wiki/Emmy_Noether))

- A relation on values is *well-founded* if there are no *infinite descending chains*:
- That is, if you start at some value and keep stepping to smaller values (according to the relation), then you must always get to a minimal value after finite steps.
- E.g., natural or positive numbers under  $<$ .
- Or numeric pairs under "is an element of."
- Principle of Noetherian induction (named after Emmy Noether):
  - If  $P(x)$  is statement about values  $x$  from a well-founded set, and
  - If  $P(x)$  is true whenever  $P(y)$  is true for all  $y < x$ ,
  - Then  $P(x)$  is true for all  $x$ .

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## Induction and Recursion

- Recursive programs are justified (and constructed) by inductive reasoning.
- Basic structure:

```
def f(x):
    if There are no valid values  $< x$ :
        # The 'base case'
        return A value that's correct when x is minimal
    else:
        # Use 'The inductive hypothesis'
        return A solution constructed using f(y) where y < x
```
- The meaning of  $<$  depends on the application.
- In place of "return" might also use side-effect-producing code.

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