61A Lecture 7

Monday, September 16

Announcements

Announcements

- Homework 2 due Tuesday at 11:59pm

Announcements

- Homework 2 due Tuesday at 11:59pm
- Project 1 due Thursday at 11:59pm

Announcements

- Homework 2 due Tuesday at 11:59pm
- Project 1 due Thursday at 11:59pm
  - Extra debugging office hours in Soda 405: Tuesday 6-8, Wednesday 6-7, Thursday 5-7

Announcements

- Homework 2 due Tuesday at 11:59pm
- Project 1 due Thursday at 11:59pm
  - Extra debugging office hours in Soda 405: Tuesday 6–8, Wednesday 6–7, Thursday 5–7
  - Readers hold these office hours; they are the ones who give you composition scores!

Announcements

- Homework 2 due Tuesday at 11:59pm
- Project 1 due Thursday at 11:59pm
  - Extra debugging office hours in Soda 405: Tuesday 6-8, Wednesday 6-7, Thursday 5-7
  - Readers hold these office hours; they are the ones who give you composition scores!
- Optional guerrilla section Monday 6pm-8pm, meeting outside of Soda 310

Announcements

- Homework 2 due Tuesday at 11:59pm
- Project 1 due Thursday at 11:59pm
  - Extra debugging office hours in Soda 405: Tuesday 6-8, Wednesday 6-7, Thursday 5-7
  - Readers hold these office hours; they are the ones who give you composition scores!
- Optional guerrilla section Monday 6pm-8pm, meeting outside of Soda 310
- Midterm 1 is next Monday 9/23 from 7pm to 9pm in various locations across campus

Announcements

- Homework 2 due Tuesday at 11:59pm
- Project 1 due Thursday at 11:59pm
  - Extra debugging office hours in Soda 405: Tuesday 6-8, Wednesday 6-7, Thursday 5-7
  - Readers hold these office hours; they are the ones who give you composition scores!
- Optional guerrilla section Monday 6pm-8pm, meeting outside of Soda 310
- Midterm 1 is next Monday 9/23 from 7pm to 9pm in various locations across campus
  - Closed book, paper-based exam.

Announcements

- Homework 2 due Tuesday at 11:59pm
- Project 1 due Thursday at 11:59pm
  - Extra debugging office hours in Soda 405: Tuesday 6-8, Wednesday 6-7, Thursday 5-7
  - Readers hold these office hours; they are the ones who give you composition scores!
- Optional guerrilla section Monday 6pm-8pm, meeting outside of Soda 310
- Midterm 1 is next Monday 9/23 from 7pm to 9pm in various locations across campus
  - Closed book, paper-based exam.
  - You may bring one hand-written page of notes that you created (front & back).

Announcements

- Homework 2 due Tuesday at 11:59pm
- Project 1 due Thursday at 11:59pm
  - Extra debugging office hours in Soda 405: Tuesday 6-8, Wednesday 6-7, Thursday 5-7
  - Readers hold these office hours; they are the ones who give you composition scores!
- Optional guerrilla section Monday 6pm-8pm, meeting outside of Soda 310
- Midterm 1 is next Monday 9/23 from 7pm to 9pm in various locations across campus
  - Closed book, paper-based exam.
  - You may bring one hand-written page of notes that you created (front & back).
  - You will have a study guide attached to your exam.

Announcements

- Homework 2 due Tuesday at 11:59pm
- Project 1 due Thursday at 11:59pm
  - Extra debugging office hours in Soda 405: Tuesday 6-8, Wednesday 6-7, Thursday 5-7
  - Readers hold these office hours; they are the ones who give you composition scores!
- Optional guerrilla section Monday 6pm-8pm, meeting outside of Soda 310
- Midterm 1 is next Monday 9/23 from 7pm to 9pm in various locations across campus
  - Closed book, paper-based exam.
  - You may bring one hand-written page of notes that you created (front & back).
  - You will have a study guide attached to your exam.
  - Midterm information: http://inst.eecs.berkeley.edu/~cs61a/fa13/exams/midterm1.html

Announcements

- Homework 2 due Tuesday at 11:59pm
- Project 1 due Thursday at 11:59pm
  - Extra debugging office hours in Soda 405: Tuesday 6-8, Wednesday 6-7, Thursday 5-7
  - Readers hold these office hours; they are the ones who give you composition scores!
- Optional guerrilla section Monday 6pm-8pm, meeting outside of Soda 310
- Midterm 1 is next Monday 9/23 from 7pm to 9pm in various locations across campus
  - Closed book, paper-based exam.
  - You may bring one hand-written page of notes that you created (front & back).
  - You will have a study guide attached to your exam.
  - Midterm information: http://inst.eecs.berkeley.edu/~cs61a/fa13/exams/midterm1.html
  - Review session: Saturday 9/21 (details TBD)

Announcements

- Homework 2 due Tuesday at 11:59pm
- Project 1 due Thursday at 11:59pm
  - Extra debugging office hours in Soda 405: Tuesday 6-8, Wednesday 6-7, Thursday 5-7
  - Readers hold these office hours; they are the ones who give you composition scores!
- Optional guerrilla section Monday 6pm-8pm, meeting outside of Soda 310
- Midterm 1 is next Monday 9/23 from 7pm to 9pm in various locations across campus
  - Closed book, paper-based exam.
  - You may bring one hand-written page of notes that you created (front & back).
  - You will have a study guide attached to your exam.
  - Midterm information: http://inst.eecs.berkeley.edu/~cs61a/fa13/exams/midterm1.html
  - Review session: Saturday 9/21 (details TBD)
  - HKN Review session: Sunday 9/22 (details TBD)

Announcements

- Homework 2 due Tuesday at 11:59pm
- Project 1 due Thursday at 11:59pm
  - Extra debugging office hours in Soda 405: Tuesday 6-8, Wednesday 6-7, Thursday 5-7
  - Readers hold these office hours; they are the ones who give you composition scores!
- Optional guerrilla section Monday 6pm-8pm, meeting outside of Soda 310
- Midterm 1 is next Monday 9/23 from 7pm to 9pm in various locations across campus
  - Closed book, paper-based exam.
  - You may bring one hand-written page of notes that you created (front & back).
  - You will have a study guide attached to your exam.
  - Midterm information: http://inst.eecs.berkeley.edu/~cs61a/fa13/exams/midterm1.html
  - Review session: Saturday 9/21 (details TBD)
  - HKN Review session: Sunday 9/22 (details TBD)
  - Review office hours on Monday 9/23 (details TBD)

Recursive Functions

Recursive Functions

Recursive Functions

Definition: A function is called *recursive* if the body of that function calls itself, either directly or indirectly.

Recursive Functions

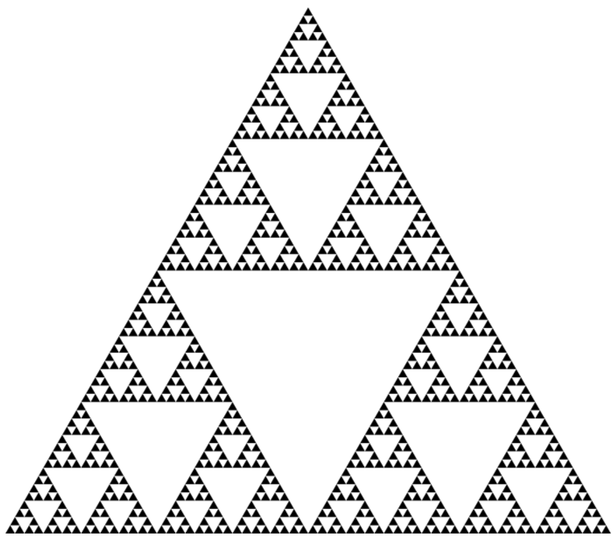
Definition: A function is called *recursive* if the body of that function calls itself, either directly or indirectly.

Implication: Executing the body of a recursive function may require applying that function again.

Recursive Functions

Definition: A function is called *recursive* if the body of that function calls itself, either directly or indirectly.

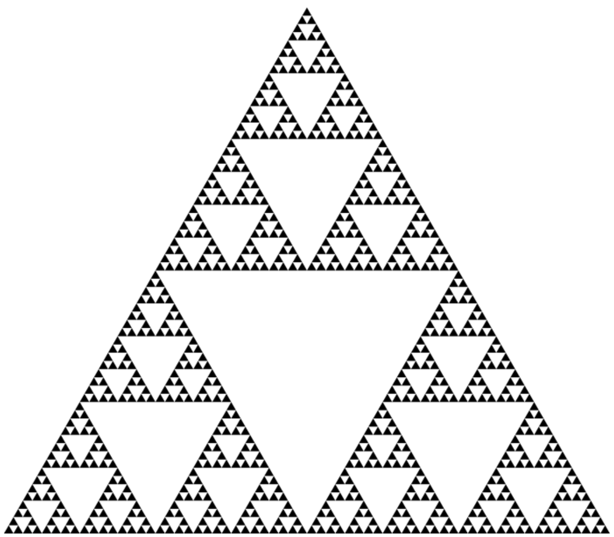
Implication: Executing the body of a recursive function may require applying that function again.

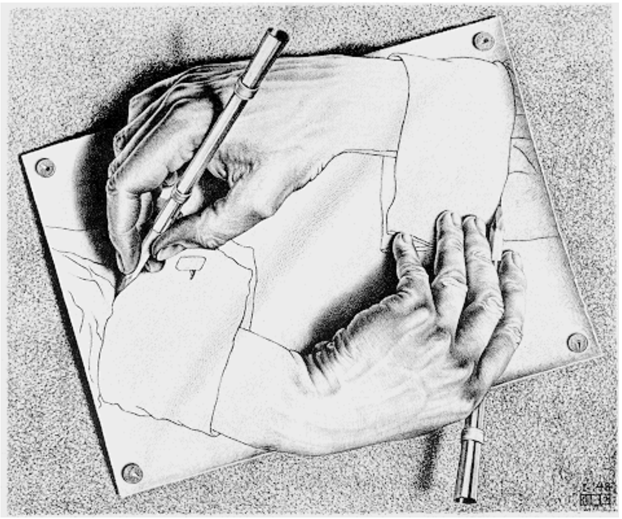


Recursive Functions

Definition: A function is called *recursive* if the body of that function calls itself, either directly or indirectly.

Implication: Executing the body of a recursive function may require applying that function again.





Drawing Hands, by M. C. Escher (lithograph, 1948)

Digit Sums

`2+0+1+3 = 6`

Digit Sums

2+0+1+3 = 6

- If a number `a` is divisible by 9, then `sum_digits(a)` is also divisible by 9.

Digit Sums

2+0+1+3 = 6

- If a number `a` is divisible by 9, then `sum_digits(a)` is also divisible by 9.
- Useful for typo detection!

Digit Sums

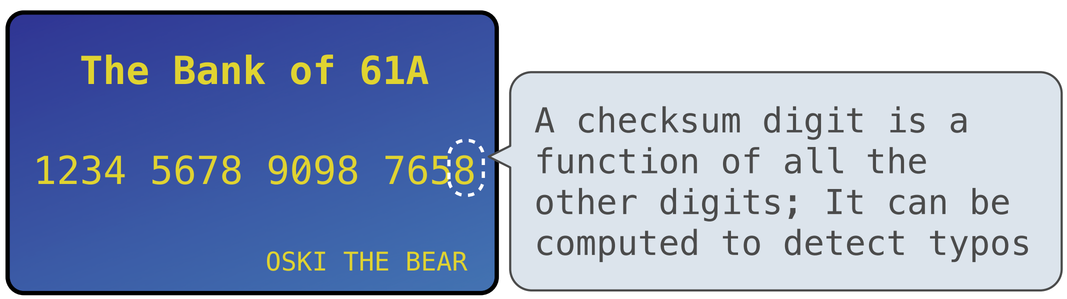
2+0+1+3 = 6

- If a number `a` is divisible by 9, then `sum_digits(a)` is also divisible by 9.
- Useful for typo detection!

Digit Sums

2+0+1+3 = 6

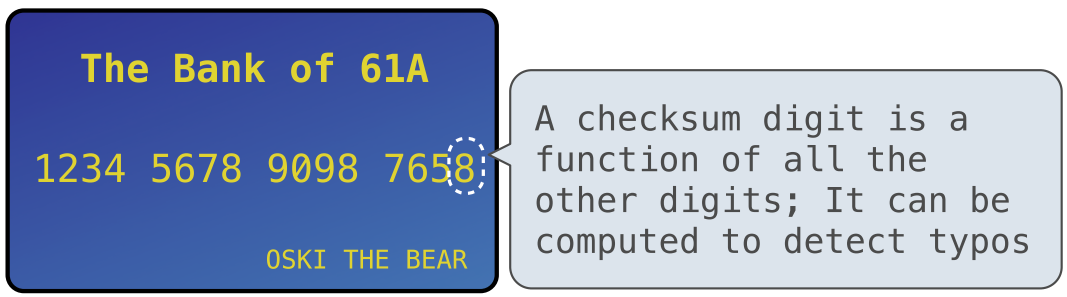
- If a number a is divisible by 9, then sum_digits(a) is also divisible by 9.
- Useful for typo detection!



Digit Sums

2+0+1+3 = 6

- If a number `a` is divisible by 9, then `sum_digits(a)` is also divisible by 9.
- Useful for typo detection!



- Credit cards actually use the Luhn algorithm, which we'll implement after digit_sum.

Sum Digits Without a While Statement

Sum Digits Without a While Statement

```python
def split(n):
    """Split positive n into all but its last digit and its last digit."""
    return n // 10, n % 10
```

Sum Digits Without a While Statement

```
  def split(n):
      """Split positive n into all but its last digit and its last digit."""
      return n // 10, n % 10

def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
```

Sum Digits Without a While Statement

```
 def split(n):
    """Split positive n into all but its last digit and its last digit."""
    return n // 10, n % 10

def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
```

Sum Digits Without a While Statement

```
  def split(n):
      """Split positive n into all but its last digit and its last digit."""
      return n // 10, n % 10

def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
```

Sum Digits Without a While Statement

```python
  def split(n):
     """Split positive n into all but its last digit and its last digit."""
     return n // 10, n % 10

def sum_digits(n):
     """Return the sum of the digits of positive integer n."""
     if n < 10:
         return n
     else:
         all_but_last, last = split(n)
         return sum_digits(all_but_last) + last
```

The Anatomy of a Recursive Function

```
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
```

The Anatomy of a Recursive Function

- The **def statement header** is similar to other functions

```
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
```

The Anatomy of a Recursive Function

- The **def statement header** is similar to other functions

```
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
```

The Anatomy of a Recursive Function

- The **def statement header** is similar to other functions
- Conditional statements check for **base cases**

```
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
```

The Anatomy of a Recursive Function

- The **def statement header** is similar to other functions
- Conditional statements check for **base cases**

```
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
```

The Anatomy of a Recursive Function

- The **def statement header** is similar to other functions
- Conditional statements check for **base cases**
- Base cases are evaluated **without recursive calls**

```
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
```

The Anatomy of a Recursive Function

- The **def statement header** is similar to other functions
- Conditional statements check for **base cases**
- Base cases are evaluated **without recursive calls**

```
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
```

The Anatomy of a Recursive Function

- The **def statement header** is similar to other functions
- Conditional statements check for **base cases**
- Base cases are evaluated **without recursive calls**
- Recursive cases are evaluated **with recursive calls**

```
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
```

The Anatomy of a Recursive Function

- The **def statement header** is similar to other functions
- Conditional statements check for **base cases**
- Base cases are evaluated **without recursive calls**
- Recursive cases are evaluated **with recursive calls**

```
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
```

The Anatomy of a Recursive Function

- The **def statement header** is similar to other functions
- Conditional statements check for **base cases**
- Base cases are evaluated **without recursive calls**
- Recursive cases are evaluated **with recursive calls**

```python
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
```

(Demo)

Recursion in Environment Diagrams

Recursion in Environment Diagrams

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)

fact(3)
```

Example: http://goo.gl/XOP9ps

Recursion in Environment Diagrams

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)

fact(3)
```

(Demo)

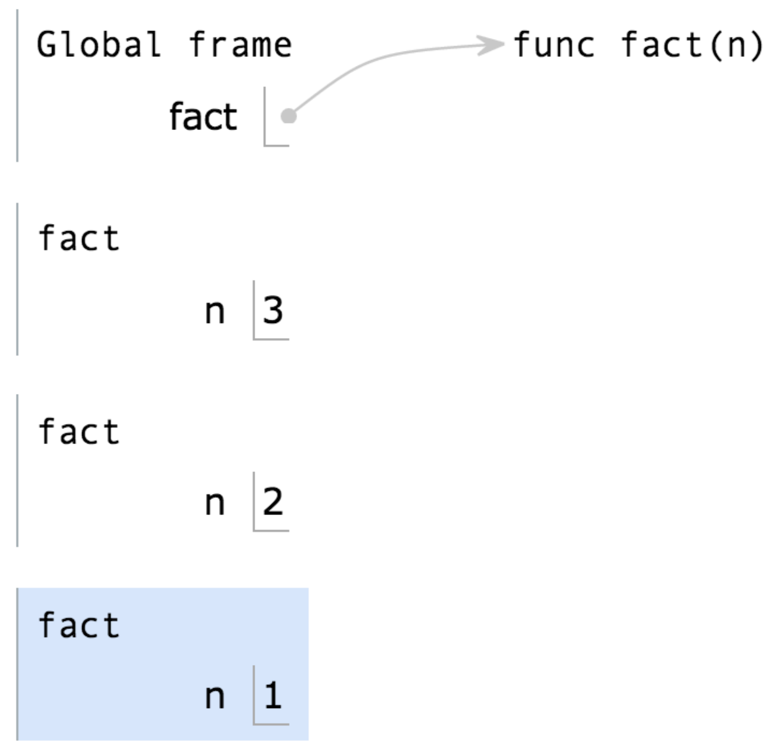
Example: http://goo.gl/XOP9ps

Recursion in Environment Diagrams

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)

fact(3)
```

(Demo)



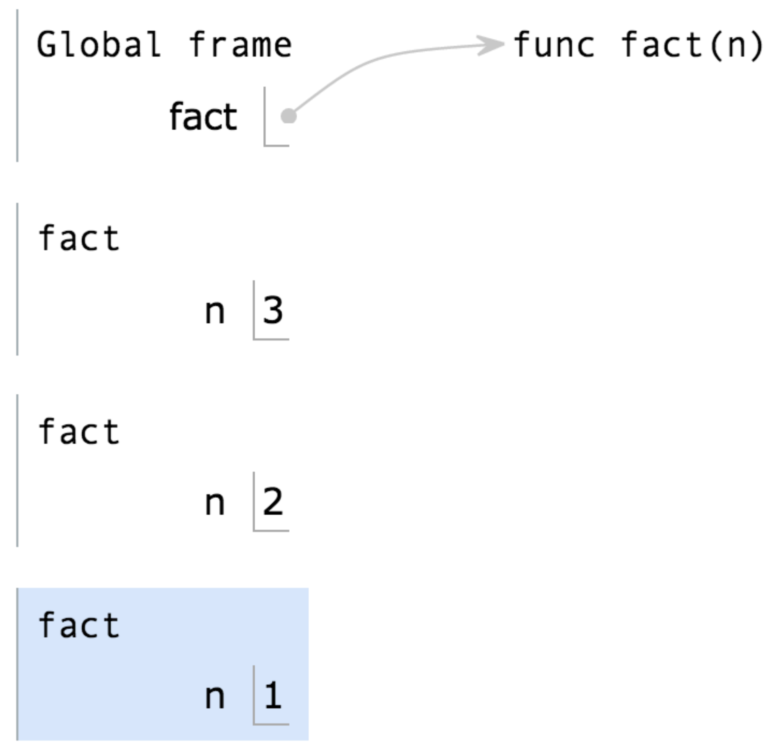
Example: http://goo.gl/XOP9ps

Recursion in Environment Diagrams

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)

fact(3)
```

(Demo)



- The same function **fact** is called multiple times.

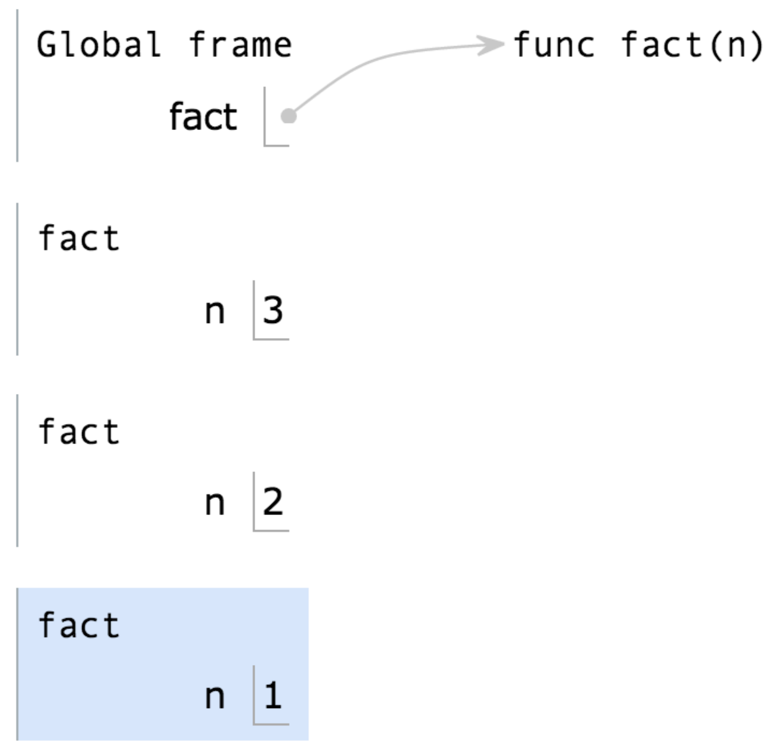
Example: http://goo.gl/XOP9ps

Recursion in Environment Diagrams

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)

fact(3)
```

(Demo)



- The same function **fact** is called multiple times.

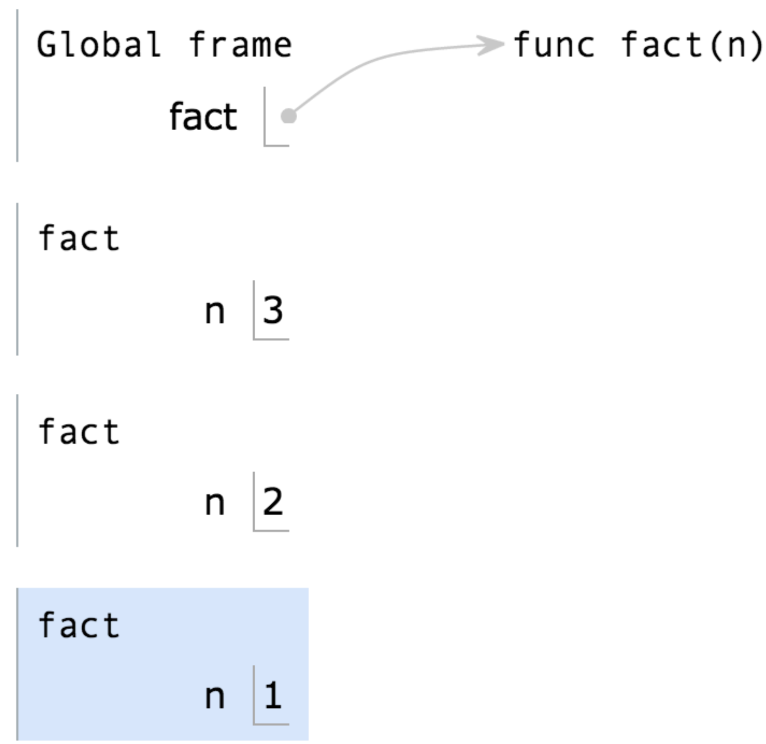
Example: http://goo.gl/XOP9ps

Recursion in Environment Diagrams

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)

fact(3)
```

(Demo)



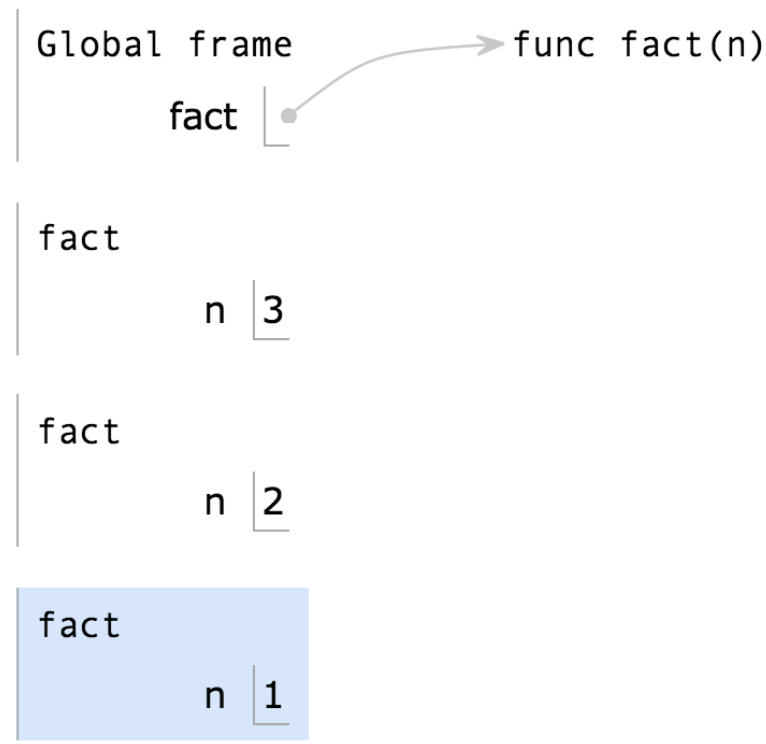
- The same function **fact** is called multiple times.
- Different frames keep track of the different arguments in each call.

Recursion in Environment Diagrams

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)

fact(3)
```

(Demo)



- The same function **fact** is called multiple times.
- Different frames keep track of the different arguments in each call.
- What **n** evaluates to depends upon which is the current environment.

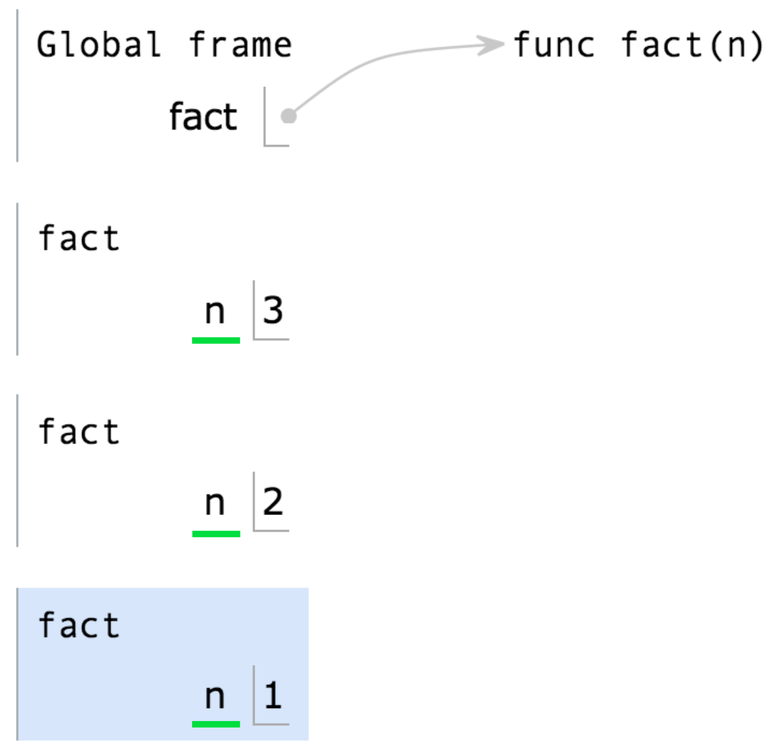
Example: http://goo.gl/XOP9ps

Recursion in Environment Diagrams

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)

fact(3)
```

(Demo)



- The same function **fact** is called multiple times.
- Different frames keep track of the different arguments in each call.
- What **n** evaluates to depends upon which is the current environment.

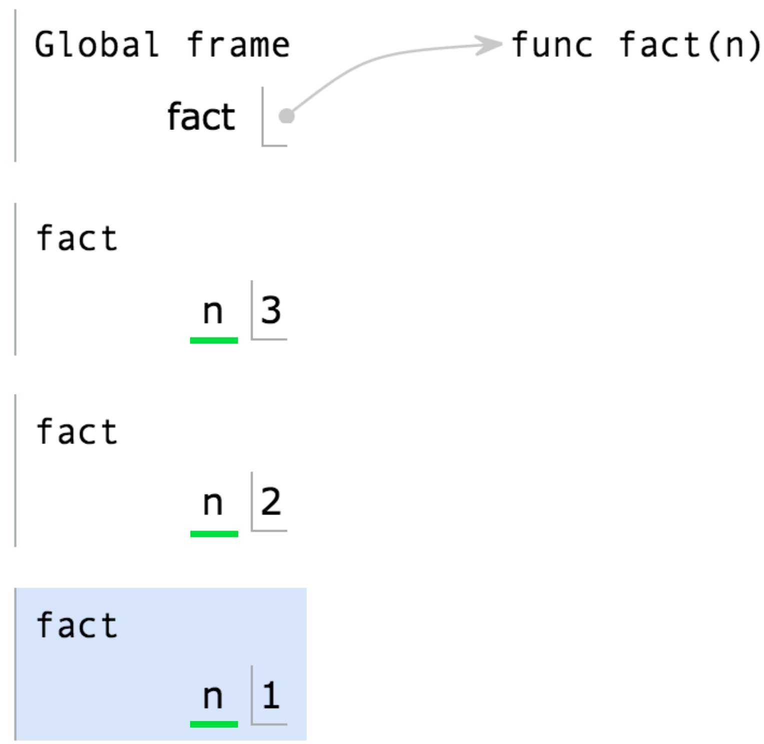
Example: http://goo.gl/XOP9ps

Recursion in Environment Diagrams

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)

fact(3)
```

(Demo)



- The same function **fact** is called multiple times.
- Different frames keep track of the different arguments in each call.
- What **n** evaluates to depends upon which is the current environment.
- Each call to **fact** solves a simpler problem than the last: smaller **n**.

Example: http://goo.gl/XOP9ps

Iteration vs Recursion

Example: Example: http://goo.gl/XOP9ps

Iteration vs Recursion

Iteration is a special case of recursion

Example: Example: http://goo.gl/XOP9ps

Iteration vs Recursion

Iteration is a special case of recursion

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

Example: Example: http://goo.gl/XOP9ps

Iteration vs Recursion

Iteration is a special case of recursion

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

Using iterative control:

Iteration vs Recursion

Iteration is a special case of recursion

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

Using iterative control:

```
def fact_iter(n):
    total, k = 1, 1
    while k <= n:
        total, k = total*k, k+1
    return total
```

Example: Example: http://goo.gl/XOP9ps

Iteration vs Recursion

Iteration is a special case of recursion

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

Using iterative control:

```
def fact_iter(n):
    total, k = 1, 1
    while k <= n:
        total, k = total*k, k+1
    return total
```

Using recursion:

Example: Example: http://goo.gl/XOP9ps

Iteration vs Recursion

Iteration is a special case of recursion

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

Using iterative control:

```
def fact_iter(n):
    total, k = 1, 1
    while k <= n:
        total, k = total*k, k+1
    return total
```

Using recursion:

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```

Example: Example: http://goo.gl/XOP9ps

Iteration vs Recursion

Iteration is a special case of recursion

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

Using iterative control:

```
def fact_iter(n):
    total, k = 1, 1
    while k <= n:
        total, k = total*k, k+1
    return total
```

Using recursion:

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```

Math:

Example: Example: http://goo.gl/XOP9ps

Iteration vs Recursion

Iteration is a special case of recursion

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

Using iterative control:

```
def fact_iter(n):
    total, k = 1, 1
    while k <= n:
        total, k = total*k, k+1
    return total
```

Using recursion:

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```

Math:

$$n! = \prod_{k=1}^{n} k$$

Example: Example: http://goo.gl/XOP9ps

Iteration vs Recursion

Iteration is a special case of recursion

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

Using iterative control:

```
def fact_iter(n):
    total, k = 1, 1
    while k <= n:
        total, k = total*k, k+1
    return total
```

Using recursion:

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```

Math:

$$n! = \prod_{k=1}^{n} k$$

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1)! & \text{otherwise} \end{cases}$$

Example: Example: http://goo.gl/XOP9ps

Iteration vs Recursion

Iteration is a special case of recursion

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

Using iterative control:

```
def fact_iter(n):
    total, k = 1, 1
    while k <= n:
        total, k = total*k, k+1
    return total
```

Using recursion:

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```

Math:

$$n! = \prod_{k=1}^{n} k$$

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1)! & \text{otherwise} \end{cases}$$

Names:

Example: Example: http://goo.gl/XOP9ps

Iteration vs Recursion

Iteration is a special case of recursion

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

Using iterative control:

```
def fact_iter(n):
    total, k = 1, 1
    while k <= n:
        total, k = total*k, k+1
    return total
```

Using recursion:

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```

Math:

$$n! = \prod_{k=1}^{n} k$$

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1)! & \text{otherwise} \end{cases}$$

Names: n, total, k, fact_iter

Example: Example: http://goo.gl/XOP9ps

Iteration vs Recursion

Iteration is a special case of recursion

$$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

Using iterative control:

```
def fact_iter(n):
    total, k = 1, 1
    while k <= n:
        total, k = total*k, k+1
    return total
```

Using recursion:

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```

Math:

$$n! = \prod_{k=1}^{n} k$$

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1)! & \text{otherwise} \end{cases}$$

Names:

n, total, k, fact_iter

n, fact

Example: Example: http://goo.gl/XOP9ps

Verifying Recursive Functions

The Recursive Leap of Faith

The Recursive Leap of Faith

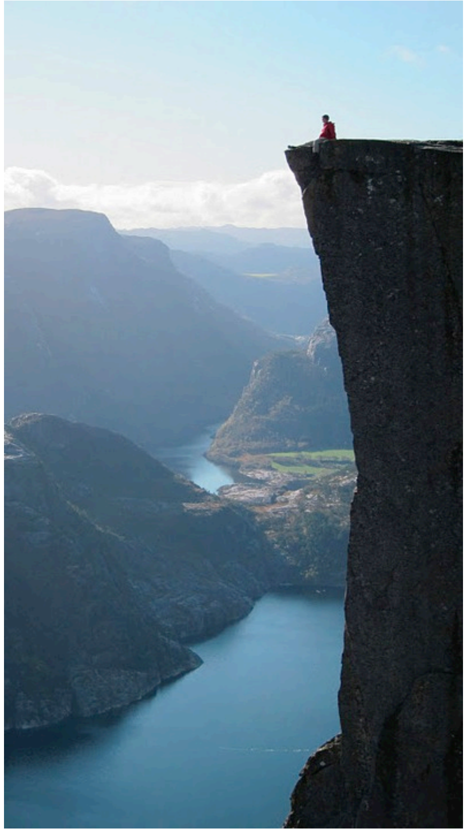


Photo by Kevin Lee, Preikestolen, Norway

The Recursive Leap of Faith

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```

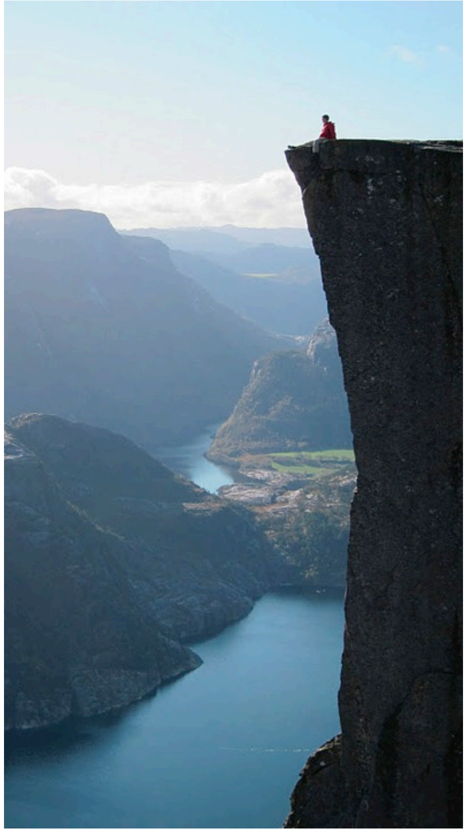


Photo by Kevin Lee, Preikestolen, Norway

The Recursive Leap of Faith

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```

Is fact implemented correctly?

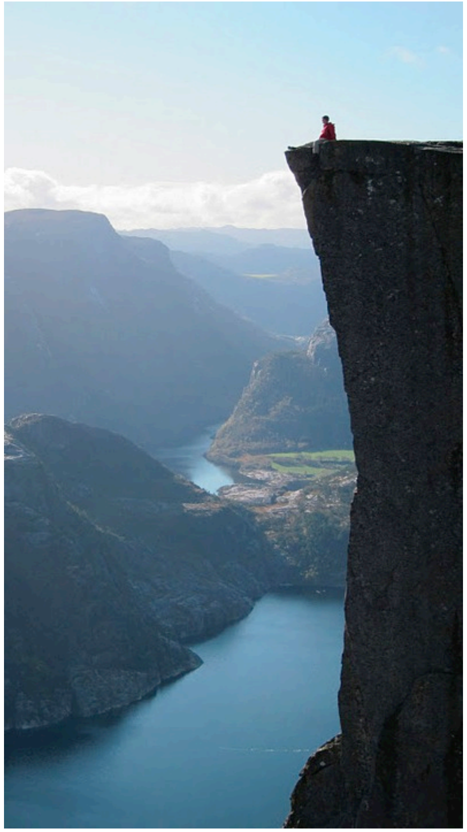
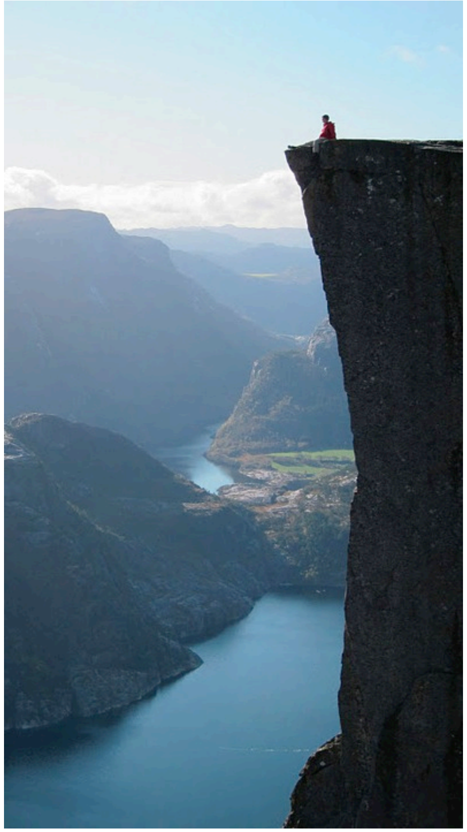


Photo by Kevin Lee, Preikestolen, Norway

The Recursive Leap of Faith

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```



Is fact implemented correctly?

1. Verify the base case.

Photo by Kevin Lee, Preikestolen, Norway

The Recursive Leap of Faith

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```

Is `fact` implemented correctly?

1. Verify the base case.
2. Treat `fact` as a functional abstraction!

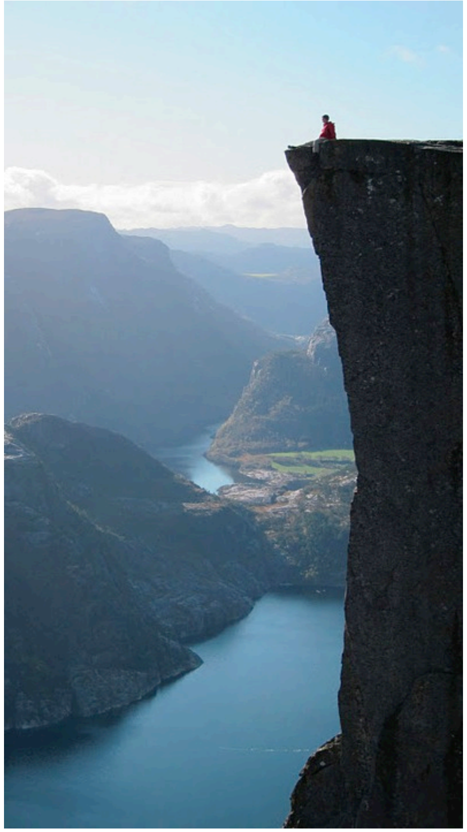
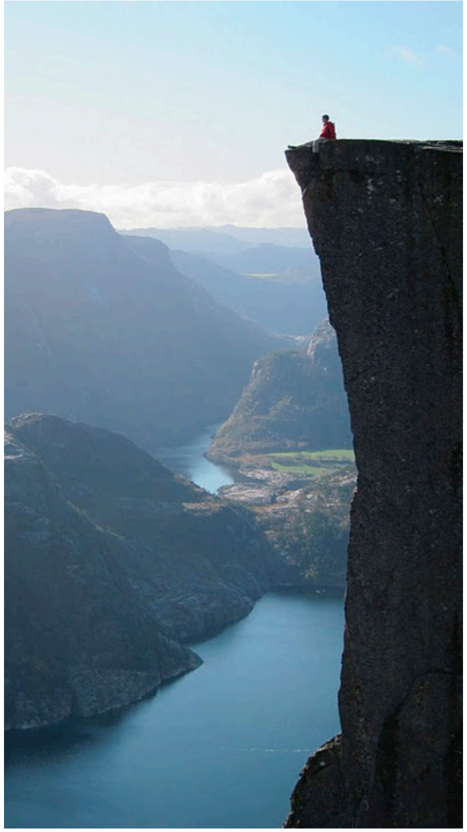


Photo by Kevin Lee, Preikestolen, Norway

The Recursive Leap of Faith

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```



Is fact implemented correctly?

1. Verify the base case.

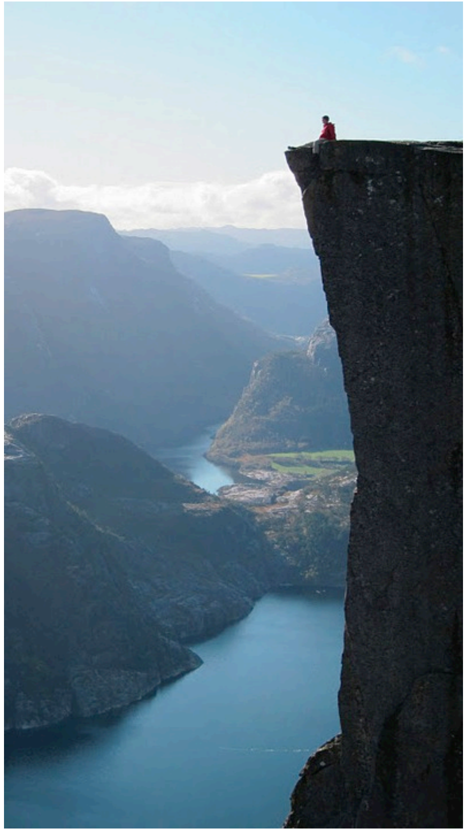
2. Treat fact as a functional abstraction!

3. Assume that fact(n-1) is correct.

Photo by Kevin Lee, Preikestolen, Norway

The Recursive Leap of Faith

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```



Is fact implemented correctly?

1. Verify the base case.

2. Treat fact as a functional abstraction!

3. Assume that fact(n-1) is correct.

4. Verify that fact(n) is correct, assuming that fact(n-1) correct.

Photo by Kevin Lee, Preikestolen, Norway

Mutual Recursion

The Luhn Algorithm

The Luhn Algorithm

Used to verify credit card numbers

The Luhn Algorithm

Used to verify credit card numbers

From Wikipedia: http://en.wikipedia.org/wiki/Luhn_algorithm

The Luhn Algorithm

Used to verify credit card numbers

From Wikipedia: http://en.wikipedia.org/wiki/Luhn_algorithm

1. From the rightmost digit, which is the check digit, moving left, double the value of every second digit; if product of this doubling operation is greater than 9 (e.g., 7 * 2 = 14), then sum the digits of the products (e.g., 10: 1 + 0 = 1, 14: 1 + 4 = 5).

The Luhn Algorithm

Used to verify credit card numbers

From Wikipedia: http://en.wikipedia.org/wiki/Luhn_algorithm

1. From the rightmost digit, which is the check digit, moving left, double the value of every second digit; if product of this doubling operation is greater than 9 (e.g., 7 * 2 = 14), then sum the digits of the products (e.g., 10: 1 + 0 = 1, 14: 1 + 4 = 5).

2. Take the sum of all the digits.

The Luhn Algorithm

Used to verify credit card numbers

From Wikipedia: http://en.wikipedia.org/wiki/Luhn_algorithm

1. From the rightmost digit, which is the check digit, moving left, double the value of every second digit; if product of this doubling operation is greater than 9 (e.g., 7 * 2 = 14), then sum the digits of the products (e.g., 10: 1 + 0 = 1, 14: 1 + 4 = 5).

2. Take the sum of all the digits.

1	3	8	7	4	3

The Luhn Algorithm

Used to verify credit card numbers

From Wikipedia: http://en.wikipedia.org/wiki/Luhn_algorithm

1. From the rightmost digit, which is the check digit, moving left, double the value of every second digit; if product of this doubling operation is greater than 9 (e.g., 7 * 2 = 14), then sum the digits of the products (e.g., 10: 1 + 0 = 1, 14: 1 + 4 = 5).

2. Take the sum of all the digits.

1	3	8	7	4	3
2	3	1+6=7	7	8	3

The Luhn Algorithm

Used to verify credit card numbers

From Wikipedia: http://en.wikipedia.org/wiki/Luhn_algorithm

1. From the rightmost digit, which is the check digit, moving left, double the value of every second digit; if product of this doubling operation is greater than 9 (e.g., 7 * 2 = 14), then sum the digits of the products (e.g., 10: 1 + 0 = 1, 14: 1 + 4 = 5).

2. Take the sum of all the digits.

1	3	8	7	4	3	
2	3	1+6=7	7	8	3	= 30

The Luhn Algorithm

Used to verify credit card numbers

From Wikipedia: http://en.wikipedia.org/wiki/Luhn_algorithm

1. From the rightmost digit, which is the check digit, moving left, double the value of every second digit; if product of this doubling operation is greater than 9 (e.g., 7 * 2 = 14), then sum the digits of the products (e.g., 10: 1 + 0 = 1, 14: 1 + 4 = 5).

2. Take the sum of all the digits.

1	3	8	7	4	3	
2	3	1+6=7	7	8	3	= 30

The Luhn sum of a valid credit card number is a multiple of 10.

The Luhn Algorithm

Used to verify credit card numbers

From Wikipedia: http://en.wikipedia.org/wiki/Luhn_algorithm

1. From the rightmost digit, which is the check digit, moving left, double the value of every second digit; if product of this doubling operation is greater than 9 (e.g., 7 * 2 = 14), then sum the digits of the products (e.g., 10: 1 + 0 = 1, 14: 1 + 4 = 5).

2. Take the sum of all the digits.

1	3	8	7	4	3	
2	3	1+6=7	7	8	3	= 30

The Luhn sum of a valid credit card number is a multiple of 10.

(Demo)

Recursion and Iteration

Converting Recursion to Iteration

Converting Recursion to Iteration

Can be tricky: Iteration is a special case of recursion.

Converting Recursion to Iteration

Can be tricky: Iteration is a special case of recursion.

Idea: Figure out what state must be maintained by the iterative function.

Converting Recursion to Iteration

Can be tricky: Iteration is a special case of recursion.

Idea: Figure out what state must be maintained by the iterative function.

```
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
```

Converting Recursion to Iteration

Can be tricky: Iteration is a special case of recursion.

Idea: Figure out what state must be maintained by the iterative function.

```
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
```

What's left to sum

Converting Recursion to Iteration

Can be tricky: Iteration is a special case of recursion.

Idea: Figure out what state must be maintained by the iterative function.

```
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
```

A partial sum

What's left to sum

Converting Recursion to Iteration

Can be tricky: Iteration is a special case of recursion.

Idea: Figure out what state must be maintained by the iterative function.

```
def sum_digits(n):
    """Return the sum of the digits of positive integer n."""
    if n < 10:
        return n
    else:
        all_but_last, last = split(n)
        return sum_digits(all_but_last) + last
```

A partial sum

What's left to sum

(Demo)

Converting Iteration to Recursion

Converting Iteration to Recursion

More formulaic: Iteration is a special case of recursion.

Converting Iteration to Recursion

More formulaic: Iteration is a special case of recursion.

Idea: The *state* of an iteration can be passed as arguments.

Converting Iteration to Recursion

More formulaic: Iteration is a special case of recursion.

Idea: The *state* of an iteration can be passed as arguments.

```
def sum_digits_iter(n):
    digit_sum = 0
    while n > 0:
        n, last = split(n)
        digit_sum = digit_sum + last
    return digit_sum
```

Converting Iteration to Recursion

More formulaic: Iteration is a special case of recursion.

Idea: The *state* of an iteration can be passed as arguments.

```
def sum_digits_iter(n):
    digit_sum = 0
    while n > 0:
        n, last = split(n)
        digit_sum = digit_sum + last
    return digit_sum


def sum_digits_rec(n, digit_sum):
    if n == 0:
        return digit_sum
    else:
        n, last = split(n)
        return sum_digits_rec(n, digit_sum + last)
```

Converting Iteration to Recursion

More formulaic: Iteration is a special case of recursion.

Idea: The *state* of an iteration can be passed as arguments.

```
def sum_digits_iter(n):
    digit_sum = 0
    while n > 0:
        n, last = split(n)
        digit_sum = digit_sum + last
    return digit_sum
```

Updates via assignment become...

```
def sum_digits_rec(n, digit_sum):
    if n == 0:
        return digit_sum
    else:
        n, last = split(n)
        return sum_digits_rec(n, digit_sum + last)
```

Converting Iteration to Recursion

More formulaic: Iteration is a special case of recursion.

Idea: The *state* of an iteration can be passed as arguments.

```
def sum_digits_iter(n):
    digit_sum = 0
    while n > 0:
        n, last = split(n)
        digit_sum = digit_sum + last
    return digit_sum
```

Updates via assignment become...

```
def sum_digits_rec(n, digit_sum):
    if n == 0:
        return digit_sum
    else:
        n, last = split(n)
        return sum_digits_rec(n, digit_sum + last)
```

...arguments to a recursive call