

Numeric types in Python:

```
>>> type(2)
<class 'int'>

>>> type(1.5)
<class 'float'>

>>> type(1+1j)
<class 'complex'>
```

Represents integers exactly

Represents real numbers approximately

User-defined complex type:

```
>>> z = ComplexRI(-1, 0)
>>> z.real
-1
>>> z.magnitude
1
>>> z.angle
3.141592653589793
```

```
class ComplexRI:
    def __init__(self, real, imag):
        self.real = real
        self.imag = imag
    @property
    def magnitude(self):
        return (self.real**2 + self.imag**2)**0.5
    @property
    def angle(self):
        return math.atan2(self.imag, self.real)
    def __repr__(self):
        return 'ComplexRI({0}, {1})'.format(self.real, self.imag)
```

Property decorator: "Call this function on attribute look-up"

math.atan2(y,x): Angle between x-axis and the point (x,y)

**Type dispatching:** Look up a cross-type implementation of an operation based on the types of its arguments

**Data-directed programming:** Look up a cross-type implementation based on both the operator and types of its arguments

**Type coercion:** Look up a function for converting one type to another, then apply a type-specific implementation.

Rational number:  $\frac{\text{numerator}}{\text{denominator}}$

- Exact representation of fractions
- A pair of integers
- As soon as division occurs, the exact representation may be lost!
- Assume we can compose and decompose rational numbers:

Constructor: `rational(n, d)` returns a rational number `x`

Selectors: `num(x)` returns the numerator of `x`  
`denom(x)` returns the denominator of `x`

These functions implement an abstract data type for rational numbers

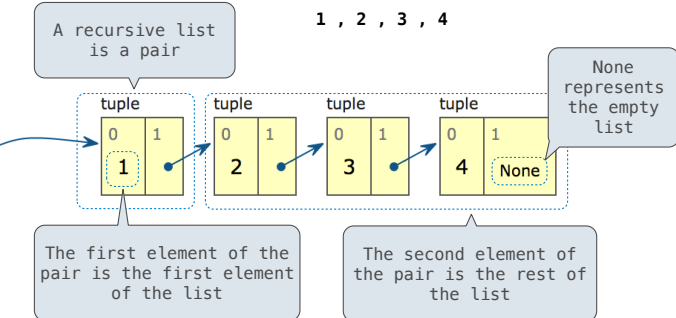
There isn't just one sequence class or abstract data type (in Python or in general).

The sequence abstraction is a collection of behaviors:

**Length.** A sequence has a finite length.

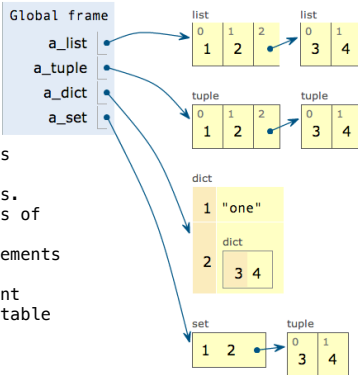
**Element selection.** A sequence has an element corresponding to any non-negative integer index less than its length, starting at 0 for the first element.

We can implement recursive lists as pairs. We'll use two-element tuples to encode pairs.



```
1 a_list = [1, 2, [3, 4]]
2 a_tuple = (1, 2, (3, 4))
3 a_dict = {'one': 1, 'two': 2, 'three': 3}
4 a_set = {1, 2, (3, 4)}
```

- Lists are mutable sequences
- Tuples are immutable sequences
- Dictionaries are unordered collections of key-value pairs.
- Sets are unordered collections of values.
- Two dictionary keys or set elements cannot be equal.
- A dictionary key or set element cannot be an instance of a mutable built-in type.



Executing a for statement: `for <name> in <expression>:`  
`<suite>`

- Evaluate the header `<expression>`, which must yield an iterable value.
- For each element in that sequence, in order:
  - Bind `<name>` to that element in the first frame of the current environment.
  - Execute the `<suite>`.

A range is a sequence of ..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ... consecutive integers.

Length: ending value - starting value  
 range(-2, 2)

Element selection: starting value + index

>>> tuple(range(-2, 2))  
 (-2, -1, 0, 1)

Tuple constructor

>>> tuple(range(4))  
 (0, 1, 2, 3)

With a 0 starting value

List comprehensions: `[<map exp> for <name> in <iter exp> if <filter exp>]`  
 Short version: `[<map exp> for <name> in <iter exp>]`

- A combined expression that evaluates to a list by this procedure:
  - Add a new frame extending the current frame.
  - Create an empty `result` list that is the value of the expression.
  - For each element in the iterable value of `<iter exp>`:
    - Bind `<name>` to that element in the new frame from step 1.
    - If `<filter exp>` evaluates to a true value, then add the value of `<map exp>` to the result list.

Strings are sequences too:

```
>>> city = 'Berkeley'
>>> len(city)
8
>>> city[3]
'k'

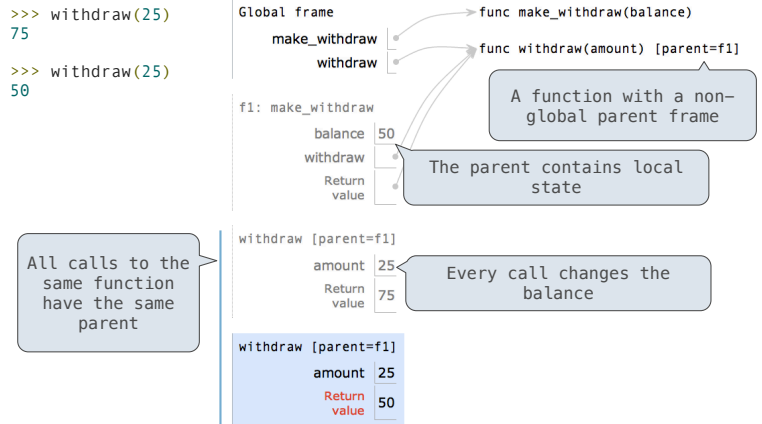
The "in" and "not in" operators match substrings
>>> 'here' in "Where's Waldo?"
True
>>> 234 in (1, 2, 3, 4, 5)
False
```

An element of a string is itself a string, but with only one character!

```
def make_withdraw(balance):
    """Return a withdraw function with a starting balance."""
    def withdraw(amount):
        if amount > balance:
            return 'Insufficient funds'
        balance = balance - amount
        return balance
    return withdraw
```

Declare the name "balance" nonlocal at the top of the body of the function in which it is re-assigned

Re-bind balance in the first non-local frame in which it was bound previously



Status	Effect
<code>x = 2</code>	Create a new binding from name "x" to object 2 in the first frame of the current environment.
No nonlocal statement "x" is not bound locally	Re-bind name "x" to object 2 in the first frame of the current env.
No nonlocal statement "x" is bound locally	Re-bind "x" to 2 in the first non-local frame of the current environment in which it is bound.
nonlocal x "x" is bound in a non-local frame	SyntaxError: no binding for nonlocal 'x' found
nonlocal x "x" is bound in a non-local frame "x" also bound locally	SyntaxError: name 'x' is parameter and nonlocal

```
class <name>:
    <suite>
```

The suite is executed when a class statement is evaluated.

A class statement **creates** a new class and **binds** that class to **<name>** in the first frame of the current environment. Statements in the **<suite>** create attributes of the class. As soon as an instance is created, it is passed to **\_\_init\_\_**, which is a class attribute called the *constructor method*.

Objects receive messages via dot notation. Dot notation accesses attributes of the instance or its class.

**<expression> . <name>**  
The **<expression>** can be any valid Python expression. The **<name>** must be a simple name. Evaluates to the value of the attribute **looked up** by **<name>** in the object that is the value of the **<expression>**.

```
tom_account.deposit(10)
```

Dot expression      Call expression

- To evaluate a dot expression:
1. Evaluate the **<expression>** to the left of the dot, which yields the object of the dot expression.
  2. **<name>** is matched against the instance attributes of that object; **if an attribute with that name exists**, its value is returned.
  3. If not, **<name>** is looked up in the class, which yields a class attribute value.
  4. That value is returned **unless it is a function**, in which case a *bound method* is returned instead.

Assignment statements with a dot expression on their left-hand side affect attributes for the object of that dot expression

- For an instance, then assignment sets an instance attribute
- For a class, then assignment sets a class attribute

Account class attributes	interest: <del>0.02</del> <del>0.04</del> (withdraw, deposit, __init__)	Instance attributes of tom_account
Instance attributes of jim_account	balance: 0 holder: 'Jim' interest: 0.08	balance: 0 holder: 'Tom'

```
>>> jim_account = Account('Jim')
>>> tom_account = Account('Tom')
>>> tom_account.interest
0.02
>>> jim_account.interest
0.02
>>> Account.interest = 0.04
>>> tom_account.interest
0.04
>>> jim_account.interest
0.08
```

```
>>> a = Account('Jim')
>>> a.holder
'Jim'
```

When a class is called:

1. A new instance of that class is created: {balance: 0, holder: 'Jim'}
2. The constructor **\_\_init\_\_** of the class is called with the new object as its first argument (named **self**), along with any additional arguments provided in the call expression.

```
class Account:
    def __init__(self, account_holder):
        self.balance = 0
        self.holder = account_holder
```

Every object that is an instance of a user-defined class has a unique identity:

```
>>> a = Account('Jim')
>>> b = Account('Jack')
```

Every call to Account creates a new Account instance. There is only one Account class.

Identity testing is performed by "is" and "is not" operators:

```
>>> a is a
True
>>> a is not b
True
```

Binding an object to a new name using assignment **does not** create a new object:

```
>>> c = a
>>> c is a
True
```

All invoked methods have access to the object via the **self** parameter, and so they can all access and manipulate the object's state.

```
class Account:
    ...
    def deposit(self, amount):
        self.balance = self.balance + amount
        return self.balance
```

Defined with two arguments

Dot notation automatically supplies the first argument to a method.

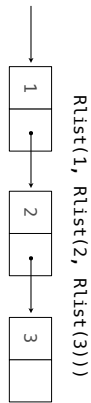
```
>>> tom_account = Account('Tom')
>>> tom_account.deposit(100)
100
```

Invoked with one argument

```
class Rlist:
    class EmptyList:
        def __len__(self):
            return 0
    empty = EmptyList()
    def __init__(self, first, rest=empty):
        assert type(rest) is Rlist or rest is Rlist.empty
        self.first = first
        self.rest = rest
    def __getitem__(self, index):
        if index == 0:
            return self.first
        else:
            return self.rest[index-1]
    def __len__(self):
        return 1 + len(self.rest)
    def extend_rlist(s1, s2):
        if s1 is Rlist.empty:
            return s2
        else:
            return Rlist(s1.first, extend_rlist(s1.rest, s2))
```

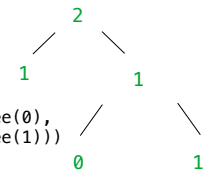
There's the base case!

Yes, this call is recursive



```
class Tree:
    def __init__(self, entry, left=None, right=None):
        self.entry = entry
        self.left = left
        self.right = right
    def sum_entries(t):
        if t is None:
            return 0
        else:
            return t.entry + sum_entries(t.left) + sum_entries(t.right)
```

left and right are Trees or None



```
def memo(f):
    cache = {}
    def memoized(n):
        if n not in cache:
            cache[n] = f(n)
        return cache[n]
    return memoized
```

$\Theta(b^n)$  Exponential growth! Incrementing the problem scales  $R(n)$  by a factor.

$\Theta(n^2)$  Quadratic growth. Incrementing  $n$  increases  $R(n)$  by the problem size  $n$ .

$\Theta(n)$  Linear growth. Resources scale with the problem size.

$\Theta(\log n)$  Logarithmic growth. Doubling the problem only increments  $R(n)$ .

$\Theta(1)$  Constant. Independent of problem size.

$n$ : size of the problem  
 $R(n)$ : Measurement of some resource  
 $R(n) = \Theta(f(n))$   
 means that there are positive constants  $k_1$  and  $k_2$  such that  
 $k_1 \cdot f(n) \leq R(n) \leq k_2 \cdot f(n)$   
 for sufficiently large values of  $n$ .

**Tree set:** A set is a Tree. Each entry is:

- Larger than all entries in its left branch, and
- Smaller than all entries in its right branch

Right! Left! Right! Stop!