

Lecture #4: Higher-Order Functions

Control

- The expressions we've dealt with recently evaluate *all* of their operands *in order*.
- While there are very clever ways to do everything with just this [challenge!], it's generally clearer to introduce constructs that *control* the order in which their components execute.
- A *control expression* evaluates some or all of its operands in an order depending on the kind of expression, and typically on the values of those operands.
- A *statement* is a construct that produces no value (not even *None*, but is used solely for its side effects.
- A *control statement* is a statement that, like a control expression, evaluates some or all of its components, in an order that may depend on the these components.
- We typically speak of statements being *executed* rather than evaluated, but the two concepts are essentially the same, apart from the question of a value.

Conditional Expressions (I)

- The most common kind of control is *conditional evaluation (execution)*.

- In Python, to evaluate

TruePart if *Condition* else *FalsePart*

- First evaluate *Condition*.
- If the result is a "*true value*," evaluate *TruePart*; its value is then the value of the whole expression.
- Otherwise, evaluate *FalsePart*; its value is then the value of the whole expression.

- **Example:**

If x is 2:

1 / x if x != 0 else 1
1 / x if 2 != 0 else 1
 \Rightarrow 1 / x if True else 1
 \Rightarrow 1 / x
 \Rightarrow 1 / 2
 \Rightarrow 0.5

If x is 0:

1 / x if x != 0 else 1
1 / x if 0 != 0 else 1
 \Rightarrow 1 / x if False else 1
 \Rightarrow 1
 \Rightarrow 1

“True Values”

- Conditions in conditional constructs can have any value, not just True or False.
- For convenience, Python treats a number of values as indicating “false”:
 - False
 - None
 - 0
 - Empty strings, sets, lists, tuples, and dictionaries.
- All else is a “true value” by default.
- So, for example: `13 if 0 else 5` and `13 if [] else 5` both evaluate to 5.

Conditional Expressions (II)

- To evaluate

Left and *Right*

- Evaluate *Left*.
 - If it is a false value, that becomes the value of the whole expression.
 - Otherwise the value of the expression is that of *Right*.
- This is an example of something called "*short-circuit evaluation*."
 - For example,

5 and "Hello" \Rightarrow .

0 and print(6) \Rightarrow + side-effects: .

[] and 1 / 0 \Rightarrow .

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[] and 1 / 0 \Rightarrow *[]*.

Conditional Expressions (III)

- To evaluate

Left or *Right*

- Evaluate *Left*.
 - If it is a true value, that becomes the value of the whole expression.
 - Otherwise the value of the expression is that of *Right*.
- Another example of "*short-circuit evaluation*."
 - For example,

5 or "Hello" \Rightarrow .

2 or print(6) \Rightarrow + side-effects: .

[] or 1 / 0 \Rightarrow .

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 or *1 / 0* \Rightarrow .

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 - For example,

5 or "Hello" \Rightarrow 5.

2 or print(6) \Rightarrow 2 + side-effects: *None*.

[] or 1 / 0 \Rightarrow error.

Chained Comparisons

- An interesting feature of Python (quite rare; Cobol has something like it) involves the relational operators:

`== != < > <= >= is is not in not in`

- Ordinarily, `3<4` yields `True` and `4<3` yields `False`.
- But what does `4 >= 3 > 1` produce? In Java, it's an error, and in C, it doesn't do what you probably want.
- In Python, it's a special control expression and works as expected.
- To evaluate `First > Second >= Third`, for example,
 - Evaluate `First` and `Second`.
 - If the first value is not larger than the second, stop and yield `False` for the entire expression.
 - Otherwise, compute the value of `Third` and compare against the value previously computed for `Second`, and yield `True` or `False` as appropriate.
 - In any case, no expression is evaluated more than once.

Chained Comparisons (II)

- So what is

`(print("A") or 3) < (print("B") or 2) < (print("C") or 4)`

and what does it print?

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`(print("A") or 3) < (print("B") or 2) < (print("C") or 4)`

and what does it print?

- Prints A and B, evaluates to False.

Conditional Statement

- Finally, this all comes in statement form:

```
if Condition1:  
    Statements1  
    ...  
elif Condition2:  
    Statements2  
    ...  
...  
else:  
    Statementsn  
    ...
```

- Execute (only) *Statements1* if *Condition1* evaluates to a true value.
- Otherwise execute *Statements2* if *Condition2* evaluates to a true value (optional part).
- ...
- Otherwise execute *Statementsn* (optional part).

Example

```
def signum(x):  
    if x > 0:  
        return 1  
    elif x == 0:  
        return 0  
    else:  
        return -1
```

Alternative Definition

```
def signum(x):  
    return 1 if x > 0 else 0 if x == 0 else -1
```

A Puzzle: Define compare3

What goes here?

```
from operator import lt, gt # Comparison functions
```

```
gt(gt(3,2), 1)           # Yields False, not like 3>2>1 (why?)
```

```
compare3(gt)(3)(2)(1)    # This should yield True
```

```
compare3(gt)(3)(2)(4)    # This should yield False
```

```
compare3(lt)(1)(2)(3)    # This should yield True
```

```
# etc.
```

Some Solutions

```
def compare3(op):  
    def f(a):  
        def g(b):  
            return lambda c: op(a,b) and op(b, c)  
        return g  
    return f
```

```
def compare3(op):  
    def f(a):  
        def g(b):  
            if op(a,b):  
                return lambda c: op(b, c)  
            else:  
                return lambda c: False  
        return g  
    return f
```

Indefinite Repetition

- With conditionals and function calls, we can conduct computations of any length.
- For example, to sum the squares of all numbers from 1 to N (a parameter):

```
def sum_squares(N):  
    """The sum of K**2 for K from 1 to N (inclusive)."""  
    if N < 1:  
        return 0  
    else:  
        return N**2 + sum_squares(N - 1)
```

- This will repeatedly call `sum_squares` with decreasing values (down to 1), adding in squares:

```
sum_squares(3) => 3**2 + sum_squares(2)  
                => 3**2 + (2**2 + sum_squares(1))  
                => 3**2 + (2**2 + (1**2 + sum_squares(0)))  
                => 3**2 + (2**2 + (1**2 + 0)) => 14
```

Explicit Repetition

- But in the Python, C, Java, and Fortran communities, it is more usual to be explicit about the repetition.
- The simplest form is **while**

```
while Condition:  
    Statements
```

means "If condition evaluates to a true value, execute statements and repeat the entire process. Otherwise, do nothing."

- So our sum-of-squares becomes:

```
def sum_squares(N):  
    """The sum of K**2 for K from 1 to N (inclusive)."""  
    result = 0  
    while N >= 1:  
        result += N**2    # Or result = result + N**2  
        N -= 1           # Or N = N-1  
    return result
```

- (Actually, this isn't quite right. What's different from the first version?)

Going Backwards

- OK: I cheated. In the recursive version, you actually add up the squares starting from the small end.
- So to be true to the original, I would write:

```
def sum_squares(N):  
    """The sum of K**2 for K from 1 to N (inclusive)."""  
    result = 0  
    k = 1  
    while k <= N:  
        result += k**2  
        k += 1  
    return result
```

Definite Repetition

- In most programming languages, we write “counting loops” like the preceding with a specialized kind of loop. In Python:

```
def sum_squares(N):  
    """The sum of K**2 for K from 1 to N (inclusive)."""  
    result = 0  
    # Original:  
    # k = 1  
    # while k <= N:  
    #     result += k**2  
    #     k += 1  
    for k in range(1, N+1):  
        result += k**2  
    return result
```

- This actually means “execute `result += k**2` for every value of `k` in the range 1 (inclusive) to `N+1` (exclusive).”
- Special case of a more general version that we’ll see later.