Lecture #9: More Functions

Another Tree Recursion: Hog Dice

- What are the odds of rolling at least k in hog with n s-sided dice? ($n > 0$ and for us, $s > 0$ is 4 or 6)

$$\frac{\text{\# rolls of } n \text{ } s\text{-sided dice totaling} \geq k}{s^n}$$

- If $k \leq 1$, then clearly the numerator is just s^n.
- For $k > 1$, we consider only rolls that include dice values 2–s, since any 1-die "pigs out." Let's call this quantity rolls2(k, n, s).
- The number of ways to score $\geq k$ is 0 if ______. This is a base case.
- If $n > 0$ then the number of ways to score at least $k \leq 1$ with n dice none of which is 1 is ______. This is also a base case.
- If the first die comes up d ($2 \leq d \leq s$), then there are ________________ ways to throw the remaining $n-1$ dice to get a total of at least k with all n dice.
- This gives us a tree recursion. How would you modify it for the "swine swap" rule?

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- If the first die comes up d ($2 \leq d \leq s$), then there are ____________________ ways to throw the remaining $n-1$ dice to get a total of at least k with all n dice.
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- If the first die comes up d ($2 \leq d \leq s$), then there are <u>rolls2(k - d, n - 1, s)</u> ways to throw the remaining $n-1$ dice to get a total of at least k with all n dice.
- This gives us a tree recursion. How would you modify it for the "swine swap" rule?

Back to Numeric Pairs: Find the Number

- A *numeric pair* is either an empty tuple, an integer, or a tuple consisting of two numeric pairs (slight revision from last time).
- Problem: does the number x occur in a given numeric pair?

```
def occurs(x, pair):
    """X occurs at least once in numeric pair PAIR.
    >>> occurs(3, ((2, 1), ((), (3, ()))))
    True
    >>> occurs(5, ((2, 1), ((), (3, ()))))
    False
    """
    if __________:
        return True
    elif __________________________________:
        return False
    else:
        return ____________________________________________
```

- What is the time required by this function proportional to? *A:*

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    if x == pair:
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    False
    """
    if x == pair:
        return True
    elif pair == () or type(pair) is int:
        return False
    else:
        return ________________________________
```

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____

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    """
    if x == pair:
        return True
    elif pair == () or type(pair) is int:
        return False
    else:
        return occurs(x, pair[0]) or occurs(x, pair[1])
```

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    else:
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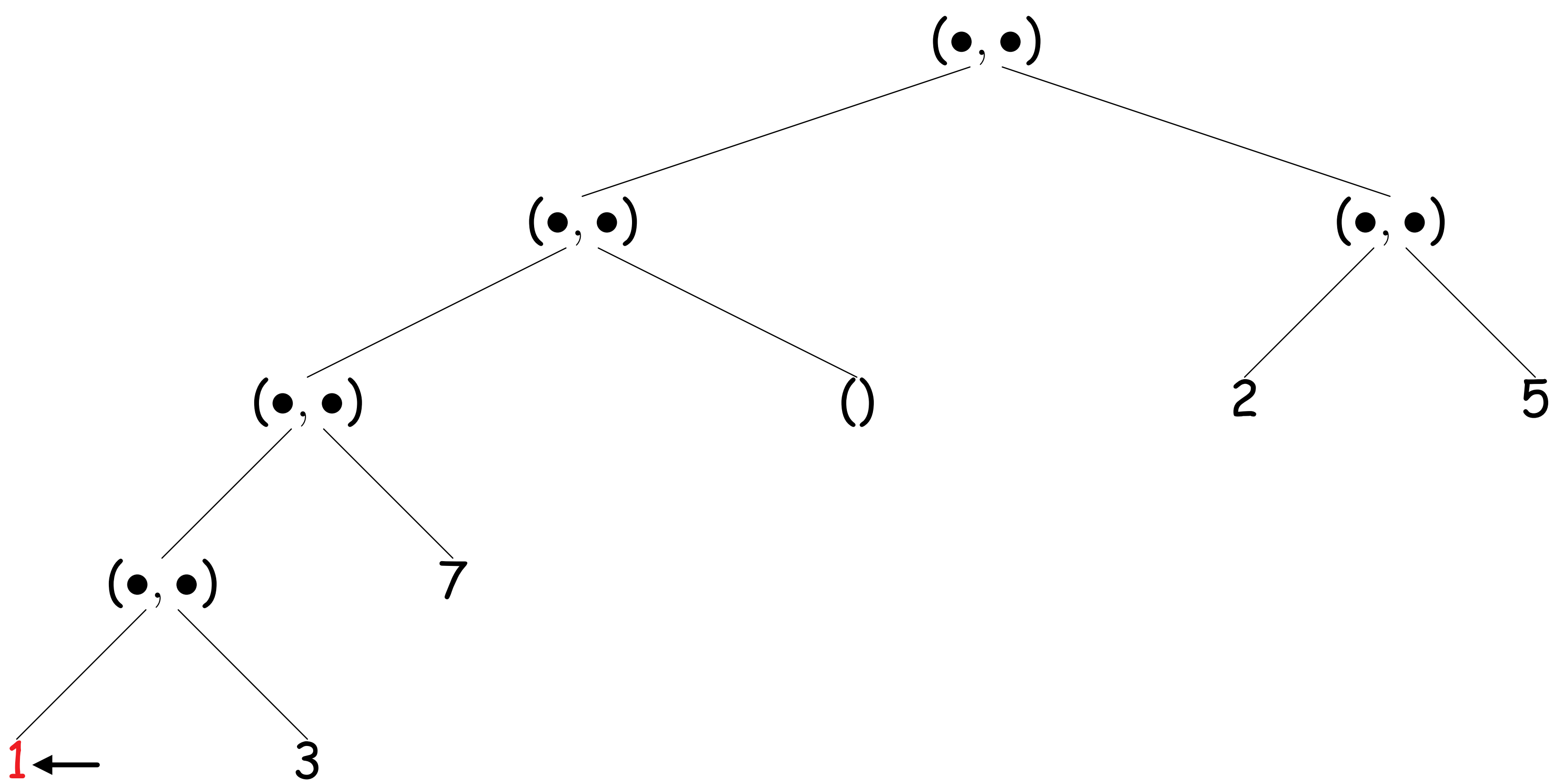
```

- What is the time required by this function proportional to? *A*: The total number of tuples and integers in pair.

Numeric Pairs: First Leaf

- A *leaf* in a numeric pair is the empty tuple or an integer.
- Define the *first leaf* as the leftmost leaf in the Python expression that denotes a tree.
- Example: the first leaf of ((((1, 3), 7), ()), (2, 5)) is 1:



First Leaf Code

```
def first_leaf(pair):
    """The first leaf in PAIR, reading left to right.
    >>> first_leaf(())
    ()
    >>> first_leaf(5)
    5
    >>> first_leaf((((3, ()), (2, 1)), ()))
    3
    >>> first_leaf(((((), 3), (2, 1)), ()))
    ()
    """
    if ______________________________:
        return pair
    else:
        return ___________________
```

What kind of a recursive process is this? *A*: ______________________________

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    >>> first_leaf(((((), 3), (2, 1)), ()))
    ()
    """
    if type(pair) is int or pair == ():
        return pair
    else:
        return ____________________
```

What kind of a recursive process is this? A: ______________________________

First Leaf Code

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    ()
    """
    if type(pair) is int or pair == ():
        return pair
    else:
        return first_leaf(pair[0])
```

What kind of a recursive process is this? *A*: ____________________

First Leaf Code

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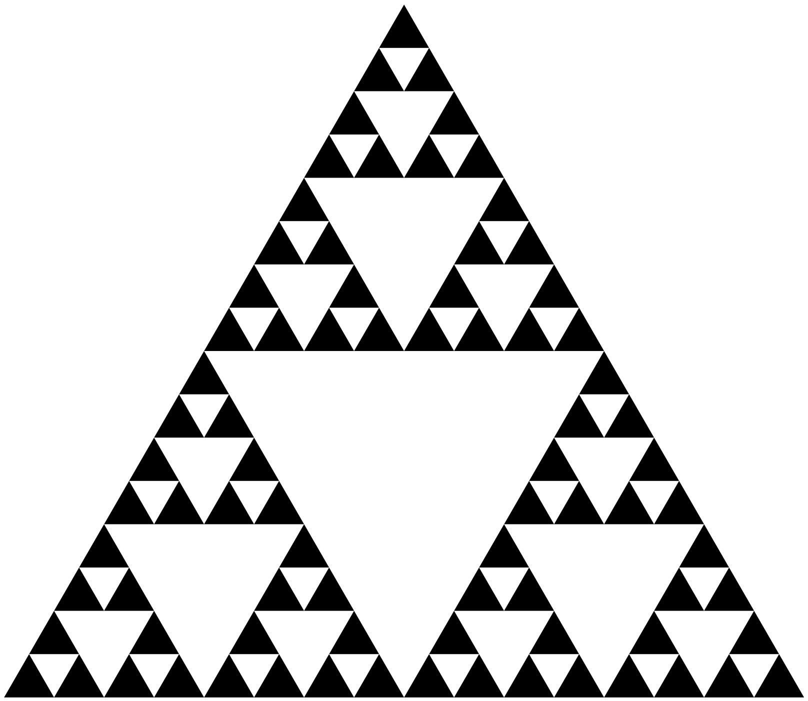
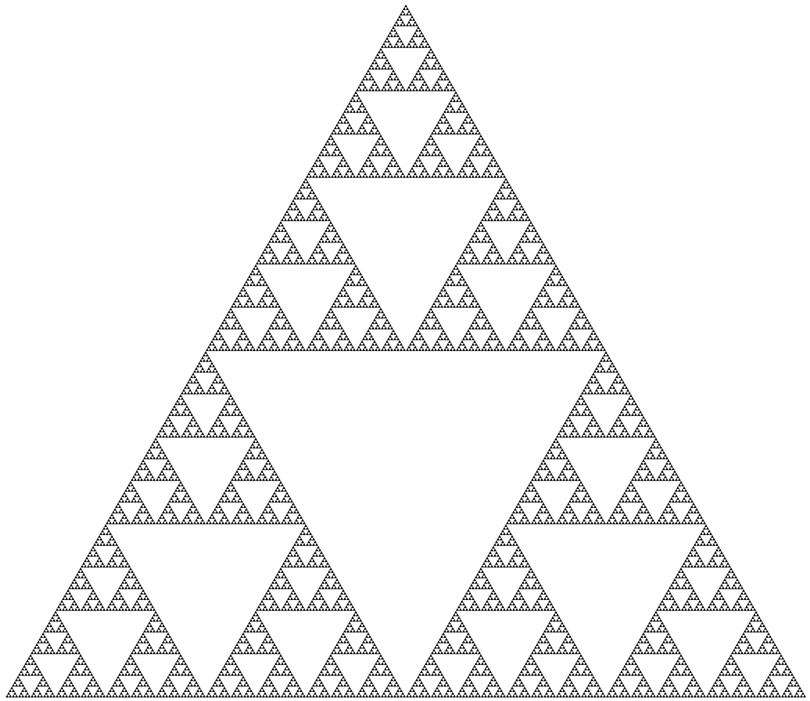
    ()
    """
    if type(pair) is int or pair == ():
        return pair
    else:
        return first_leaf(pair[0])
```

What kind of a recursive process is this? *A*: Iterative process (tail recursion)

Sierpinski Triangle

- No discussion of recursion is complete without a mention of *fractal patterns,* which exhibit self-similarity when scaled.
- We'll define a "Sierpinski Triangle of depth k and side s" to be
  - A filled equilateral triangle with sides of length s, if $k = 0$, else
  - Three Sierpinski Triangles of depth $k - 1$ and side $s/2$ arranged in the three corners of an equilateral triangle with side s.
- Here are triangles of degree 4 and 8:





Drawing Sierpinski Triangles

- Assume the existence of the function triangle:

```
def triangle(x, y, side):
    """Draw a filled equilateral triangle with its lower-left corner
    at (X, Y) and with given SIDE.  The base is aligned with the x-axis."""
```

- We can now read off the definition of the triangle:

```
def sierpinski(x, y, side, depth):
    """Draw a Sierpinski triangle of given DEPTH with given SIDE and
    lower-left corner at (X, Y)."""

    if depth == 0:
        ____________________
    else:
        height = 0.25 * sqrt(3) * side

        _________________________________
        ___________________________________________________
        __________________________________________
```

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    if depth == 0:
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    else:
        height = 0.25 * sqrt(3) * side

        sierpinski(x, y, side/2, depth-1)
        ____________________________________________________
        ___________________________________________
```

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    if depth == 0:
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    else:
        height = 0.25 * sqrt(3) * side

        sierpinski(x, y, side/2, depth-1)
        sierpinski(x + side/4, y + height, side/2, depth-1)
        ________________________________________
```

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    if depth == 0:
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    else:
        height = 0.25 * sqrt(3) * side

        sierpinski(x, y, side/2, depth-1)
        sierpinski(x + side/4, y + height, side/2, depth-1)
        sierpinski(x + side/2, y, side/2, depth-1)
```

Functions: Separation of Concerns

- The sierpinski routine used triangle.
- To write sierpinski, I needed only to know:
  - The *syntactic specification* of triangle: its name and number of arguments (given by its **def** header), and
  - Its *semantic specification*: what a call does or means (given by its documentation comment).
- I did not need to know how triangle works or who else calls it.
- Likewise, triangle does not need to know
  - where its arguments come from,
  - who calls it, or
  - what use is made of its return value or side effects.
- There is a *separation of concerns* between these functions.
- This is a fundamental concept in software engineering: organize programs so that you can work on one thing at a time in isolation.

Names

Semantically, names are arbitrary; to the reader, they are part of the documentation.

Bad:	**Better:**
number true_false	dice_rolls pigged_out
d	dice, die
helper	take_turn, find_repeat
do_stuff	rescale_figure
random obscenity	report_error
l, I, O	k, m, n

Names convey meaning or purpose to the programmer (not to the machine).

Function names should convey their value (abs, sqrt) or effect (print)

Use the documentation comments of functions to elaborate where necessary, to indicate the types of arguments and return values, and to indicate assumptions or limitations on the arguments.

Names and Comments

- I generally limit comments to
  - Docstrings on functions (or later, on classes)
  - Comments and documentation at the beginning of a module describing its purpose, conventions, authorship, copyright permissions, etc.
  - Comment names of significant constants.
- Avoid internal comments: they indicate places where you could make a function shorter or use a better name:

Rather than	*Use*
`# Compute the discriminant` `d = b**2 - 4*a*c`	`discriminant = b**2 - 4*a*c`

Function Comments

Comments on a function should suffice to tell the reader everything needed to use it.

Rather than

```
def largest(L):
    """Find the largest value"""
    k = 0
    for i in range(1, len(L)):
        if L[i] > L[k]:
            k = i
    return k
```

Use

```
def largest(L):
    """Return the index of the largest
    value in L."""
    k = 0
    for i in range(1, len(L)):
        if L[i] > L[k]:
            k = i
    return k
```

Refactoring

- Your comments can suggest to you that things are getting too big, or that a function is doing to much.
- When that happens, it is time to *refactor:* break functions up into more coherent pieces.
- Consider the function:

```
def print_averages(grade_book, out):
    """Compute the average scores for each student in
    GRADE_BOOK and prints on OUT."""
```

- What if we just want to know the averages?
- What if we also want a different format, including other information?
- Makes more sense, e.g., to have a get_averages function, and a more general print routine that will print any information about students.

Unit Testing

- The docstring tests that you execute with python3 -m doctest are examples of *unit tests.*
- That is, tests on the smallest testable units of your program (functions).
- *Test-driven development* refers to the practice of creating tests *ahead of* implementation.
- You don't have to wait for your program to be implemented to test it.
- The doctest Python module makes it possible to run all your tests cumulatively, watching for inadvertant errors and tracking how much still needs to be done.

Decorators

- You've seen functions on functions. They can also be used for testing or debugging:

```
def trace1(fn):
    """Return a function equivalent to FN, a one-argument
    function, that also prints trace output.
    """
    def traced(x):
        print('Calling', fn, 'on argument', x)
        return fn(x)
    return traced
```

- To use this:

```
def triple(x):
    return 3*x
triple = trace1(triple)
```

- Or, more conveniently, use Python's decorators:

```
@trace1
def triple(x):
    return 3*x
```