Lecture #2: Functions, Expressions, Environments

Administrative

- Reader with discussion and other materials available at Vick Copy (Euclid and Hearst).
- Sign yourself up on Piazza. See course web page:

  `http://inst.cs.berkeley.edu/~cs61a`

- Be sure to get an account form next week in lab, and provide registration data.

Announcement: We're trying to hire a new lecturer. There will be two candidates coming Jan. 27–28 (Josh Hug) and Feb. 3–4 (John DeNero), and you can help evaluate them! For both days:

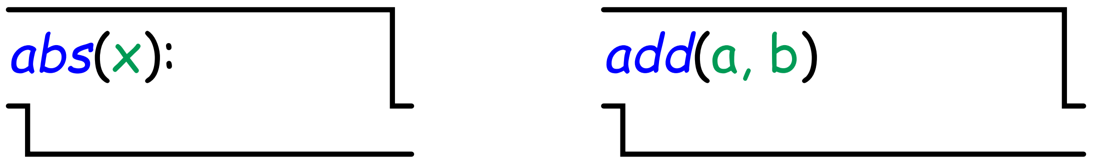
- Mon 01:00pm-02:00pm "Big ideas" talk (in Woz)
- Tue 11:45am-12:45pm Undergrad student lunch on northside (meet in 777 Soda)
- Tue 01:00pm-02:00pm Demo Class talk (in 380 Soda for Josh, Woz for John)
- UG Tue 02:00pm-02:45pm Open Session after demo class (same rooms)

Recap

- From last lecture: *Values* are data we want to manipulate and in particular,
- *Functions* are values that perform computations on values.
- *Expressions* denote computations that produce values.
- Today, we'll look at them in some detail at how functions operate on data values and how expressions denote these operations.
- As usual, although our concrete examples all involve Python, the actual concepts apply almost universally to programming languages.

Functions

- Something like abs *denotes* or *evaluates to* a function.
- To depict the denoted function *values*, we sometimes use this notation:



- Idea: The opening on the left takes in values and one on the right to delivers results.
- The (green) *formal parameter names*—such as x, a, b—show the number of parameters (inputs) to the function.
- The list of formal parameter names gives us the function's *signature*—in Python, this is the number of arguments.
- For our purposes, the blue name is simply a helpful comment to suggest what the function does.
- (Python actually maintains this *intrinsic name* and the parameter names internally, but this is not a universal feature of programming languages, and, as you'll see, can be confusing.)

Functions: Lambda

- I'm often going to use a more venerable notation for function values:

  λ x: $\ll$ | x | $\gg$ $\qquad$ λ a, b: $\ll$ the sum of a and b $\gg$

- Formal parameters go to the left of the colon.
- The part to the right of the colon is an expression that indicates what value is produced.
- I'll use $\ll \cdots \gg$ expressions to indicate non-Python descriptions of values or computations.
- In Python, you can *denote* simple function values like this:

  ```
  lambda a, b :  ≪ the sum of a and b ≫
  ```

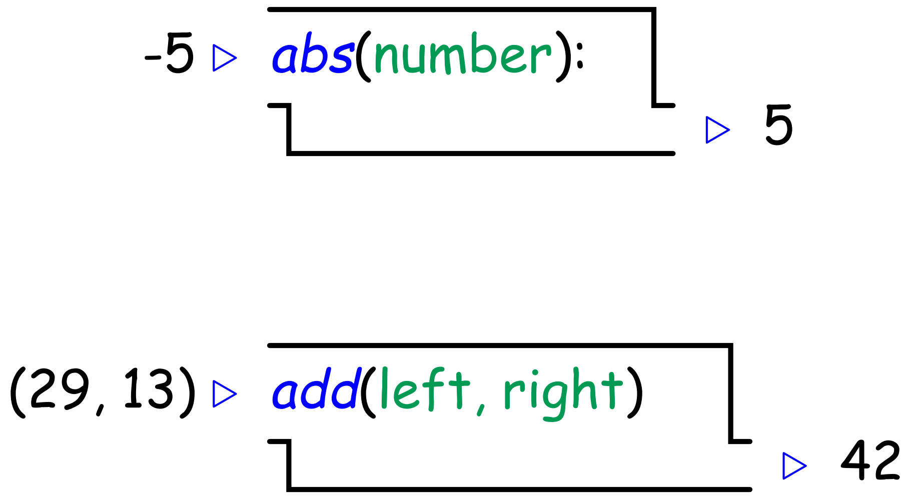
  which evaluates to

  λ a, b: $\ll$ the sum of a and b $\gg$

- (Well, OK: the $\ll \cdots \gg$ isn't really Python, but I'll use it as a place-holder for some computation I'm not prepared to write.)

Calling Functions (I)

- The fundamental operation on function values is to *call* or *invoke* them, which means giving them one value for each formal parameter and having them produce the result of their computation on these values:



Call Expressions

- A call expression denotes the operation of calling a function.
- Consider add(2, 3):

$$\underbrace{\text{add}}_{\textit{Operator}}(\underbrace{2}_{\textit{Operand 0}}, \underbrace{3}_{\textit{Operand 1}})$$

- The operator and the operands are all themselves expressions (recursion again).
- To evaluate this call expression:
  - Evaluate the operator (let's call the value C). It must evaluate to a function.
  - Evaluate the operands (or *actual parameters* in the order they appear (let's call these values P_0 and P_1)
  - Call C with parameters P_0 and P_1.

Calling a Function (I): Substitution

- Once we have the values for the operator and operands, we must still actually evaluate the call.
- A simple way to understand this (which will work for simple expressions) is to think of the process as *substitution*.
- Once you have a value:

  λ a, b: $\ll$ sum of a and b $\gg$

- and values for the operands (let's say 2 and 3),
- *substitute* the operand values for the formal parameters, replacing the whole call with

  $\ll$ sum of 2 and 3 $\gg$

- which in turn evaluates to 5.

Side Trip: Values versus Denotations

- Expressions such as 2 in a programming language are called *literals*.
- To evaluate them, we replace them with whatever values they are supposed to stand for.
- This is confusing:
  - **Q:** What is the value of the literal 2?
  - **A:** 2.
- ...and then you get into long, technical explanations about how the second "2" is really in a different language than the first, and actually is just another notation for some mystical Platonic "2" that is floating off somewhere.
- I'll just try to be practical and distinguish values from literals by surrounding values in a boxes: the value of 2 is $\boxed{2}$.
- One way to see the distinction between literals and values: the literals 0x10 and 16 are obviously different, but both denote the same value: $\boxed{16}$.

Example: From Expression to Value

Let's evaluate the expression mul(add(2, mul(0x4, 0x6)), add(0x3, 005)).
In the following sequence, values are shown in $\boxed{\text{boxes}}$.
Everything outside a box is an expression.

- $\underbrace{\text{mul}}(\underbrace{\underbrace{\text{add}}(\underbrace{2}, \underbrace{\text{mul}(\underbrace{0x4}, \underbrace{0x6})})}, \underbrace{\text{add}(\underbrace{0x3}, \underbrace{005})})$
- $\boxed{\lambda\ \text{a, b:} \ll \text{a} \times \text{b} \gg}$ (add(2, mul(0x4, 0x6)), add(0x3, 005))
- $\boxed{\lambda\ \text{a, b:} \ll \text{a} \times \text{b} \gg}$ ($\boxed{\lambda\ \text{a, b:} \ll \text{a} + \text{b} \gg}$ ($\boxed{2}$, $\boxed{\lambda\ \text{a, b:} \ll \text{a} \times \text{b} \gg}$ ($\boxed{4}$, $\boxed{6}$)), add(0x3, 005))
- $\boxed{\lambda\ \text{a, b:} \ll \text{a} \times \text{b} \gg}$ ($\boxed{\lambda\ \text{a, b:} \ll \text{a} + \text{b} \gg}$ ($\boxed{2}$, $\ll 4 \times 6 \gg$,add(0x3, 005))
- $\boxed{\lambda\ \text{a, b:} \ll \text{a} \times \text{b} \gg}$ ($\boxed{\lambda\ \text{a, b:} \ll \text{a} + \text{b} \gg}$ ($\boxed{2}$, $\boxed{24}$), add(0x3, 005))
- $\boxed{\lambda\ \text{a, b:} \ll \text{a} \times \text{b} \gg}$ ($\ll 2 + 24 \gg$, add(0x3, 005))
- $\boxed{\lambda\ \text{a, b:} \ll \text{a} \times \text{b} \gg}$ ($\boxed{26}$, add(0x3, 005))
- $\boxed{\lambda\ \text{a, b:} \ll \text{a} \times \text{b} \gg}$ ($\boxed{26}$, $\boxed{\lambda\ \text{a, b:} \ll \text{a} + \text{b} \gg}$ ($\boxed{3}$, $\boxed{5}$))
- ... $\boxed{\lambda\ \text{a, b:} \ll \text{a} \times \text{b} \gg}$ ($\boxed{26}$, $\boxed{8}$)
- ... $\boxed{208}$.

Puzzle I

Evaluate

(lambda a: lambda b: a + b)(1)(3)

- First, must understand how it's grouped:

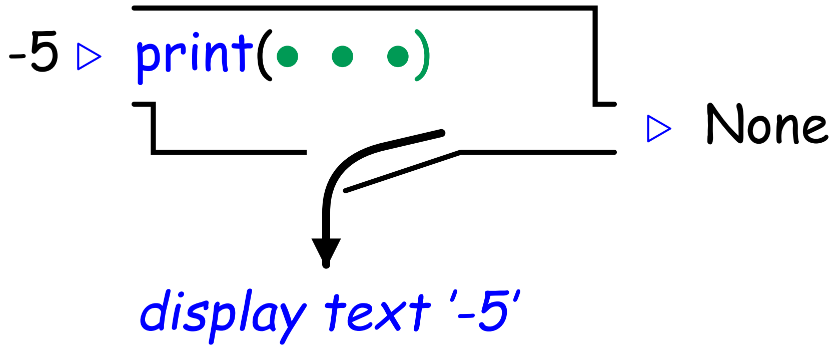
$$\underbrace{(\ \underbrace{(\text{lambda a: lambda b: a + b})}(1)\ )}(3)$$

Puzzle I (contd.)

- (lambda a: lambda b: a + b)(1)(3)
- $\boxed{\lambda\text{ a: lambda b: a + b}}(\boxed{1})(3)$
- (lambda b: $\boxed{1}$ + b)(3)
- $\boxed{\lambda\text{ b: }\boxed{1}\text{ + b}}\ (\boxed{3})$
- $\boxed{1}$ + $\boxed{3}$
- 4

Impure Functions

- The functions so far have been *pure:* their output depends only on their input parameters' values, and they do nothing in response to a call but compute a value.
- Functions may do additional things when called besides returning a value.
- We call such things *side effects.*
- Example: the built-in print function:



- Displaying text is print's side effect. It's value, in fact, is generally useless (always the null value).
- For this lecture (at least), I'll use $\lambda!$ ("lambda bang") to denote function values with side effects.

Example: Print

What about an expression with side effects?

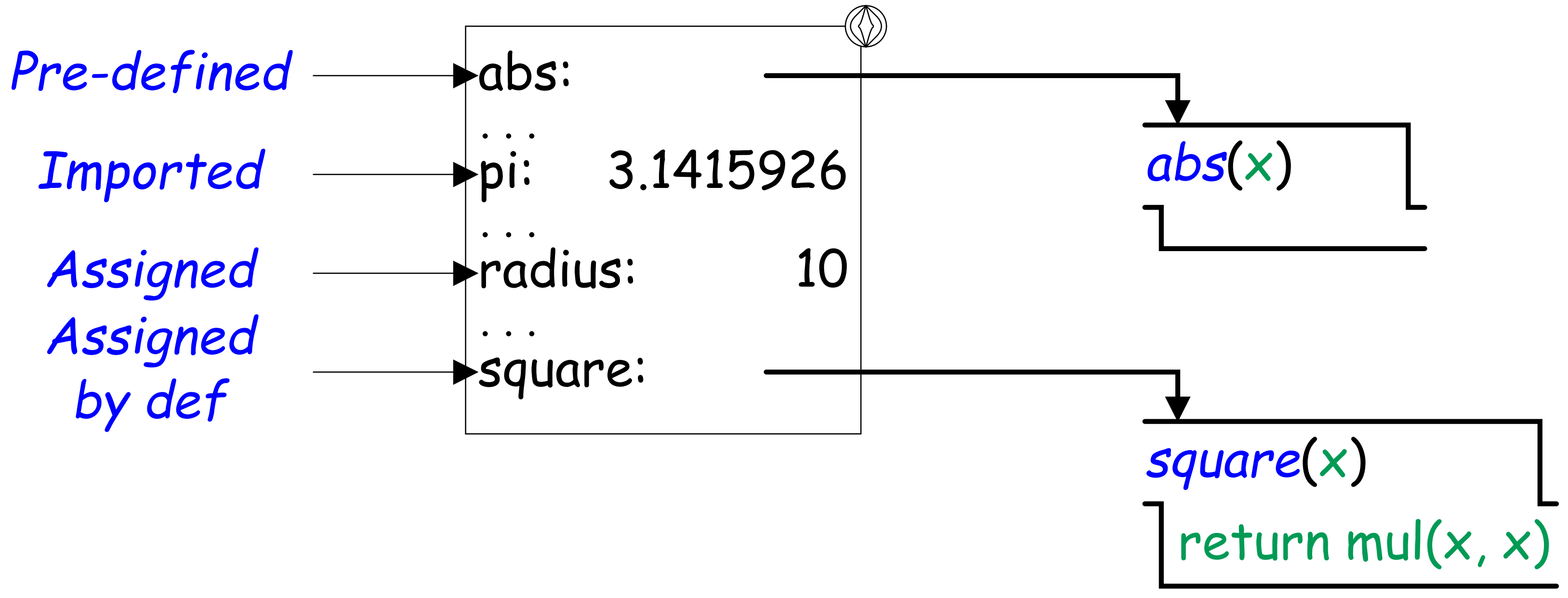
1. print(print(1), print(2))
2. $\boxed{\lambda!\ \ x: \ll \text{print } x \gg}$ ($\boxed{\lambda!\ \ x: \ll \text{print } x \gg}$ ($\boxed{1}$), print(2))
3. $\boxed{\lambda!\ \ x: \ll \text{print } x \gg}$ ($\boxed{\text{None}}$, print(2))
   and print '1'.
4. $\boxed{\lambda!\ \ x: \ll \text{print } x \gg}$ ($\boxed{\text{None}}$, $\boxed{\lambda!\ \ x: \ll \text{print } x \gg}$ ($\boxed{2}$))
5. $\boxed{\lambda!\ \ x: \ll \text{print } x \gg}$ ($\boxed{\text{None}}$, $\boxed{\text{None}}$))
   and print '2'.
6. $\boxed{\text{None}}$
   and print 'None None'.

Names

- Evaluating expressions that are literals is easy: the literal's text gives all the information needed.
- But how did I evaluate names like add, mul, or print?
- Deduction: there must be another source of information.
- We'll use the concept of an *environment* to explain this.

Environments

- An environment is a mapping from names to values.
- We say that a name is *bound to* a value in this environment.
- In its simplest form, it consists of a single *global environment frame*:



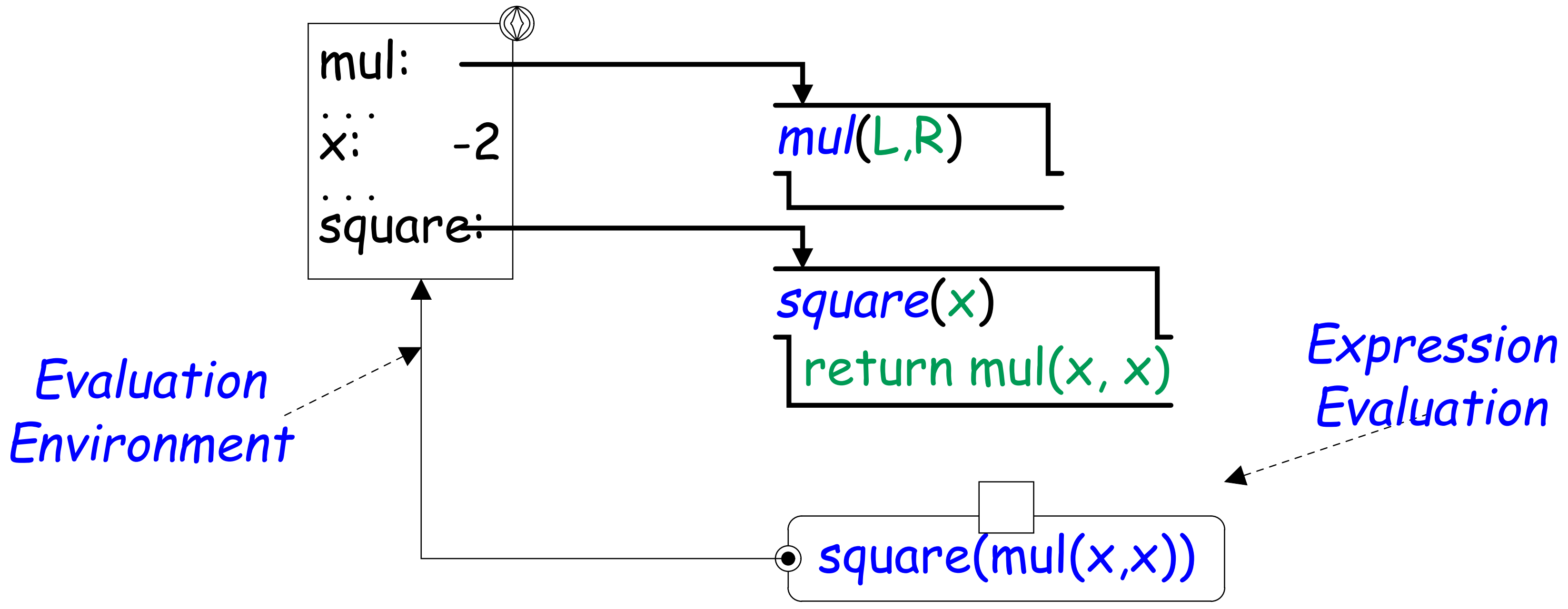
Environments and Evaluation

- Every expression is evaluated in an environment, which supplies the meanings of any names in it.
- Evaluating an expression typically involves first evaluating its subexpressions (the operators and operands of calls, the operands of conventional expressions such as `x*(y+z)`, . . .).
- These subexpressions are evaluated in the same environment as the expression that contains them.
- Once their subexpressions (operator + operands) are evaluated, calls to user-defined functions must evaluate the expressions and statements from the definition of those functions.

Evaluating User-Defined Function Calls

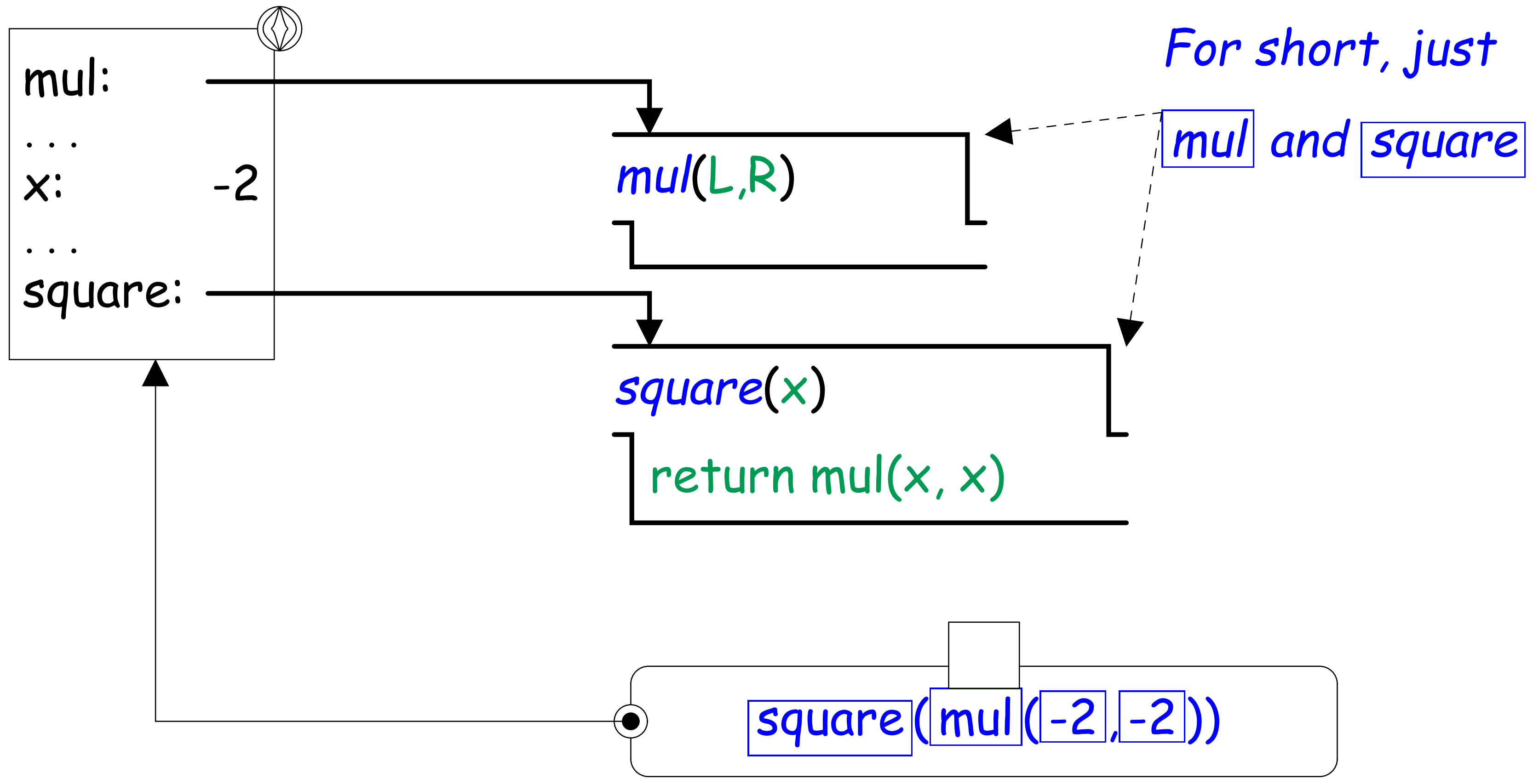
- Consider the expression square(mul(x, x)) after executing

```
from operator import mul
def square(x):
   return mul(x,x)
x = -2
```



Evaluating User-Defined Function Calls (II)

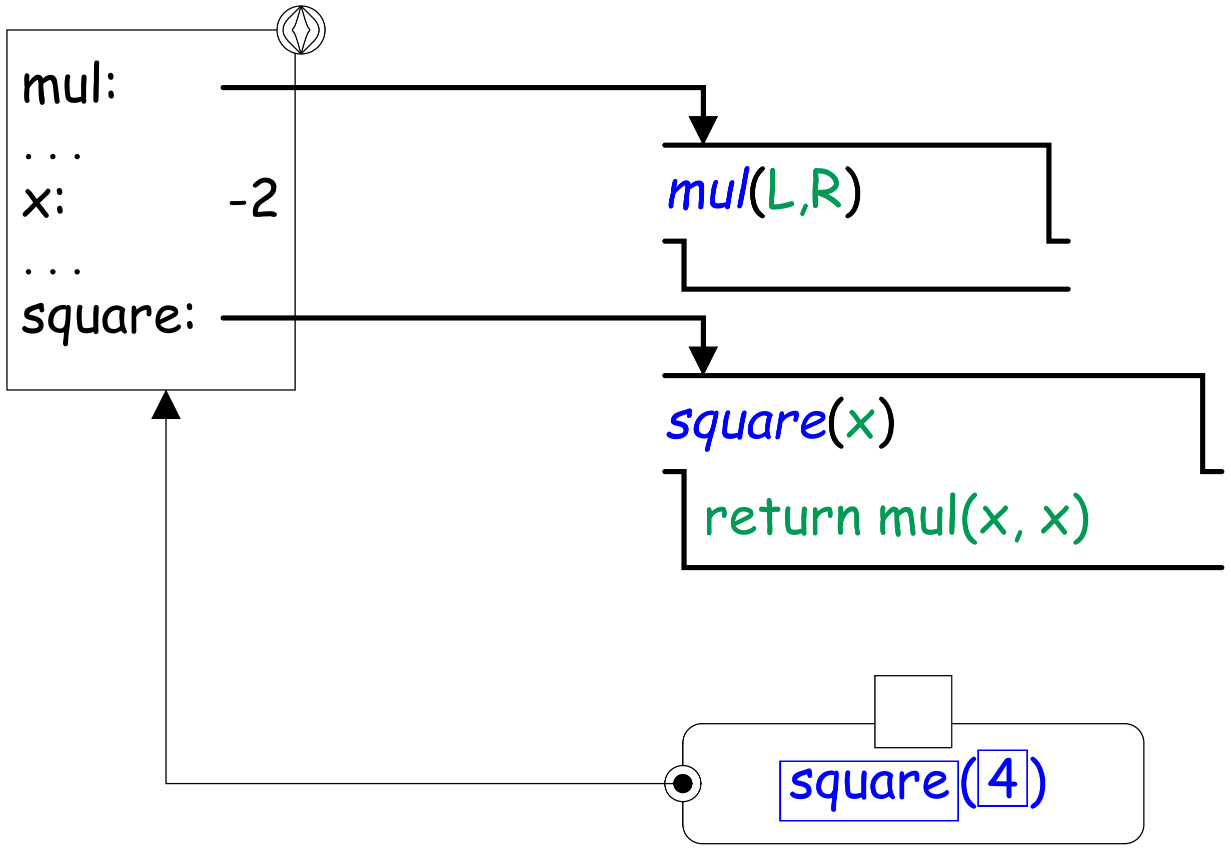
- First evaluate the subexpressions of square(mul(x, x)) in the global environment:



- Evaluating subexpressions x, mul, and square takes values from the expression's environment.

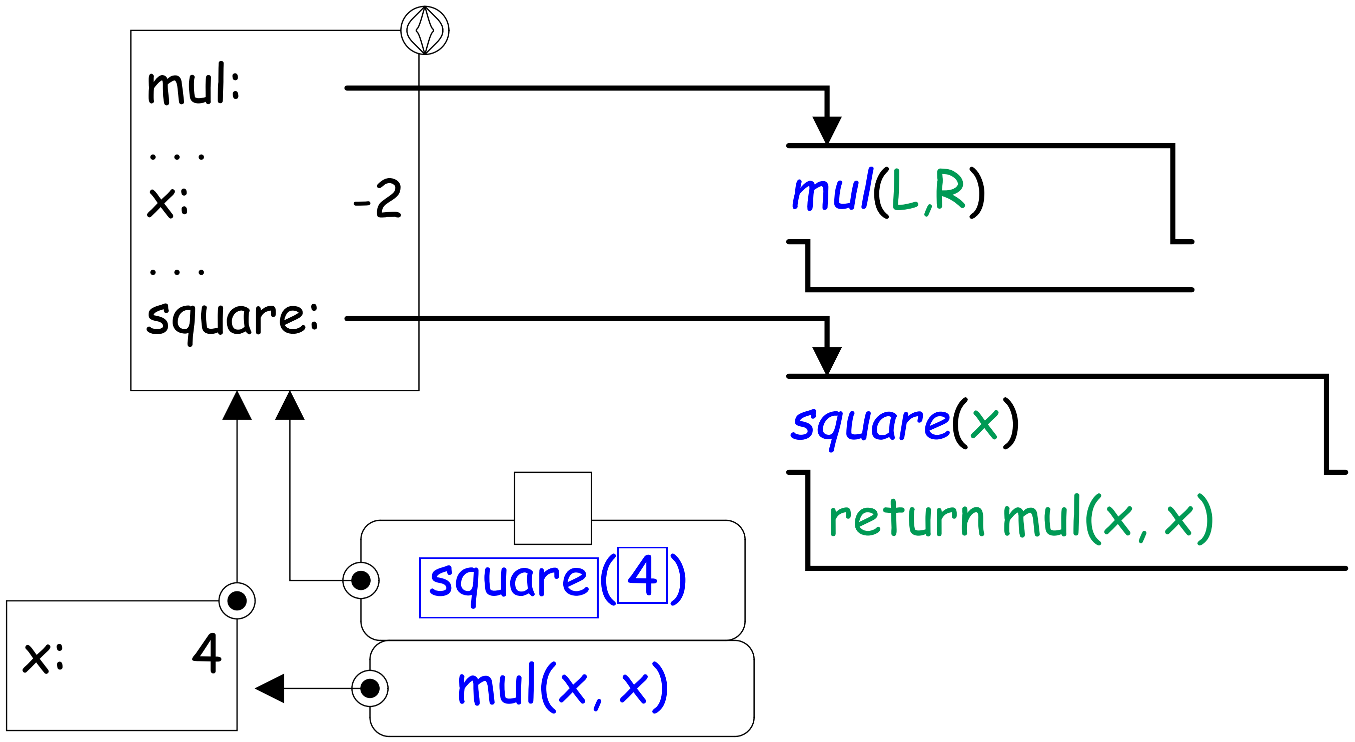
Evaluating User-Defined Functions Calls (III)

- Then perform the primitive multiply function:



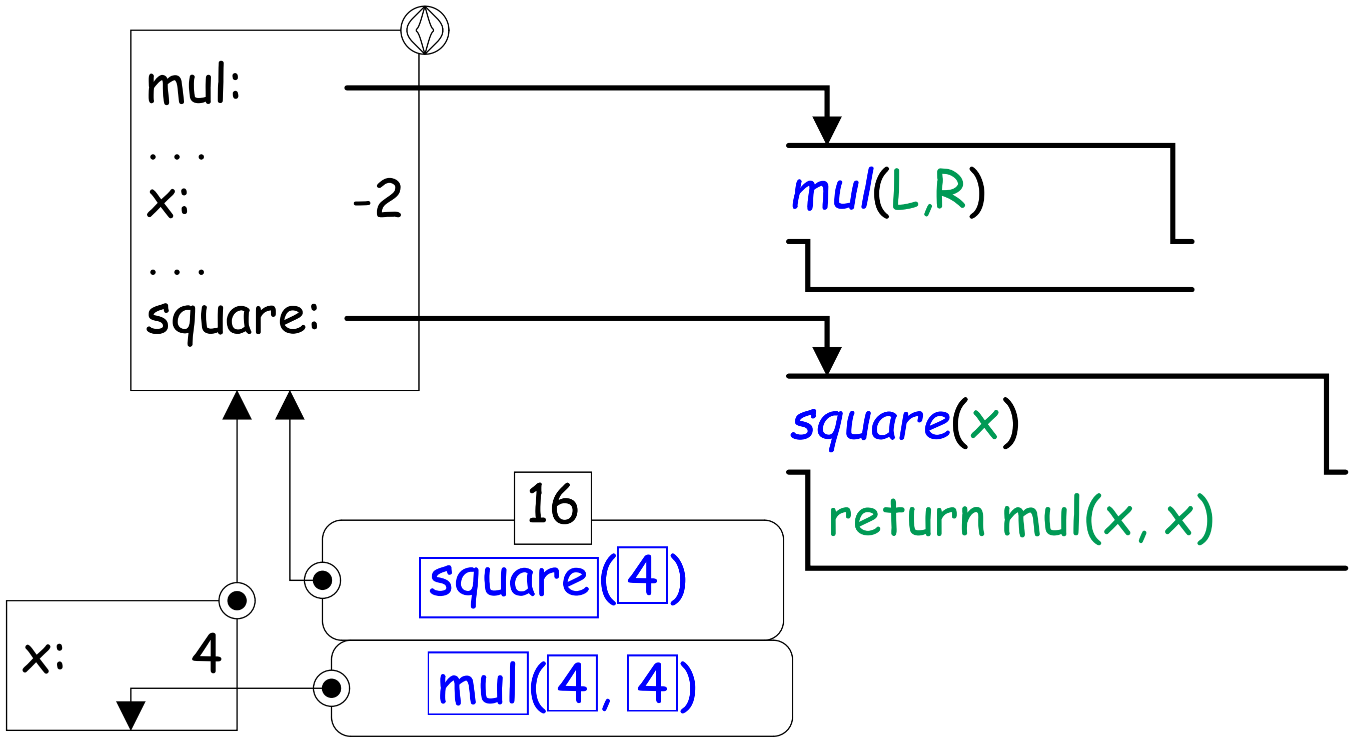
Evaluating User-Defined Functions Calls (IV)

- To explain parameter to user-defined square function, extend environment with a *local environment frame*, attached to the frame in which square was defined (the global one in this case), and giving x the operand value.
- Now replace original call with evaluating body of square in the new local environment.



Evaluating User-Defined Functions Calls (V)

- When we evaluate mul(x, x) in this new environment, we get the same value as before for mul, but the local value for x.


mul:
. . .
x: -2
. . .
square:
mul(L,R)
square(x)
return mul(x, x)
16
square(4)
x: 4
mul(4, 4)