

Lecture #9: More Functions

Another Tree Recursion: Hog Dice

- What are the odds of rolling at least k in hog with n s -sided dice?
($n > 0$ and for us, $s > 0$ is 4 or 6)

$$\frac{\text{\# rolls of } n \text{ } s\text{-sided dice totaling } \geq k}{s^n}$$

- If $k \leq 1$, then clearly the numerator is just s^n .
- For $k > 1$, we consider only rolls that include dice values 2- s , since any 1-die “pigs out.” Let’s call this quantity `rolls2(k, n, s)`.
- The number of ways to score $\geq k$ is 0 if _____. This is a base case.
- If $n > 0$ then the number of ways to score at least $k \leq 1$ with n dice none of which is 1 is _____. This is also a base case.
- If the first die comes up d ($2 \leq d \leq s$), then there are _____ ways to throw the remaining $n - 1$ dice to get a total of at least k with all n dice.
- This gives us a tree recursion. How would you modify it for the “swine swap” rule?

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- If the first die comes up d ($2 \leq d \leq s$), then there are $\text{rolls2}(k - d, n - 1, s)$ ways to throw the remaining $n - 1$ dice to get a total of at least k with all n dice.
- This gives us a tree recursion. How would you modify it for the “swine swap” rule?

Back to Numeric Pairs: Find the Number

- A *numeric pair* is either an empty tuple, an integer, or a tuple consisting of two numeric pairs (slight revision from last time).
- Problem: does the number x occur in a given numeric pair?

```
def occurs(x, pair):  
    """X occurs at least once in numeric pair PAIR.  
>>> occurs(3, ((2, 1), ((), (3, ())))  
True  
>>> occurs(5, ((2, 1), ((), (3, ())))  
False  
"""  
    if _____:  
        return True  
    elif _____:  
        return False  
    else:  
        return _____
```

- What is the time required by this function proportional to? A:

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    if x == pair:  
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False  
"""  
  
    if x == pair:  
        return True  
    elif pair == () or type(pair) is int:  
        return False  
    else:  
        return _____
```

- What is the time required by this function proportional to? A:

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    """
    if x == pair:
        return True
    elif pair == () or type(pair) is int:
        return False
    else:
        return occurs(x, pair[0]) or occurs(x, pair[1])
```

- What is the time required by this function proportional to? A:

Back to Numeric Pairs: Find the Number

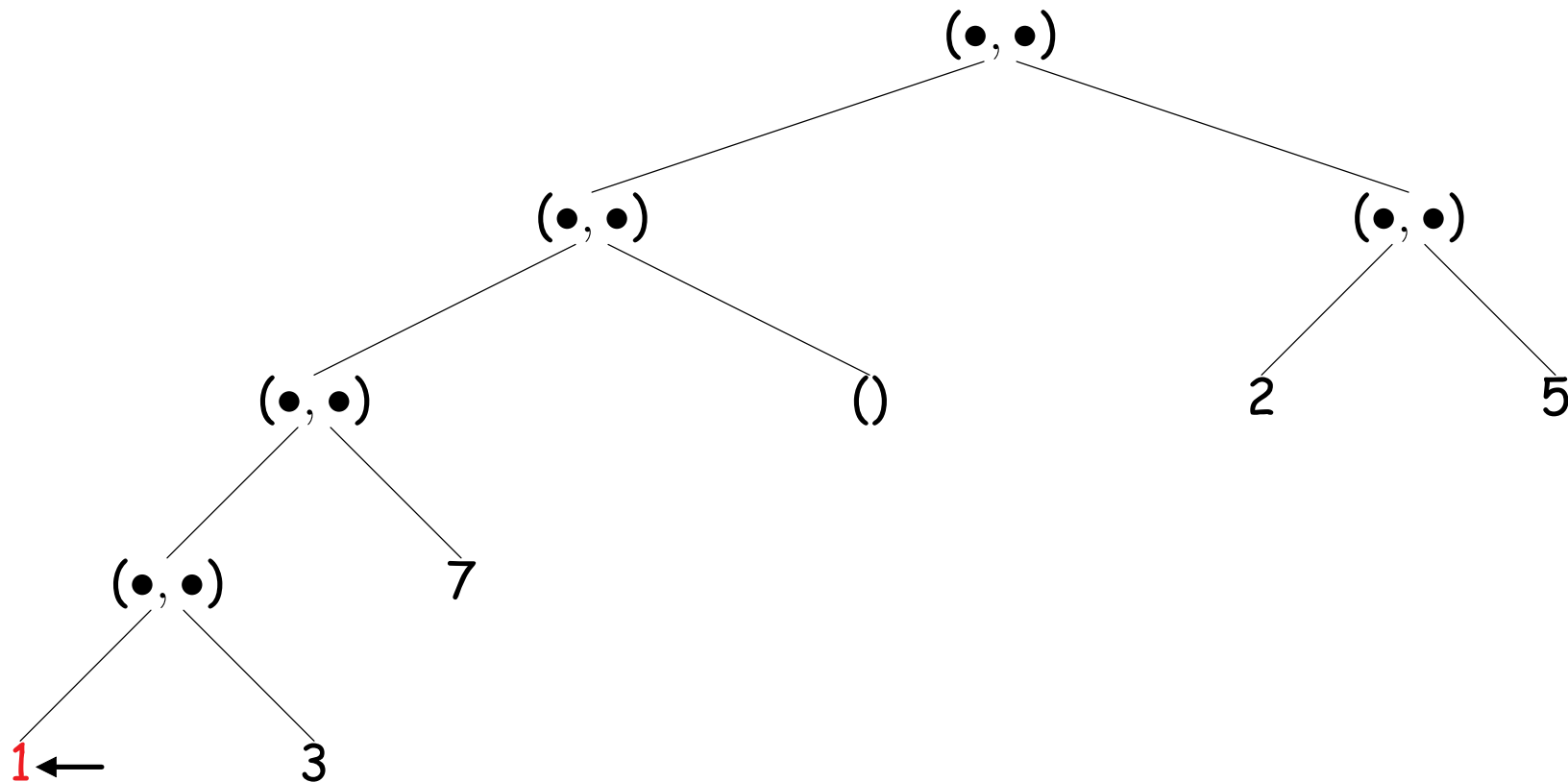
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```

- What is the time required by this function proportional to? A:
The total number of tuples and integers in pair.

Numeric Pairs: First Leaf

- A *leaf* in a numeric pair is the empty tuple or an integer.
- Define the *first leaf* as the leftmost leaf in the Python expression that denotes a tree.
- Example: the first leaf of $((((1, 3), 7), ()), (2, 5))$ is 1:



First Leaf Code

```
def first_leaf(pair):  
    """The first leaf in PAIR, reading left to right.  
>>> first_leaf(())  
()  
>>> first_leaf(5)  
5  
>>> first_leaf(((3, ()), (2, 1)), ()))  
3  
>>> first_leaf(((((), 3), (2, 1)), ()))  
()  
"""  
if _____:  
    return pair  
else:  
    return _____
```

What kind of a recursive process is this? A: _____

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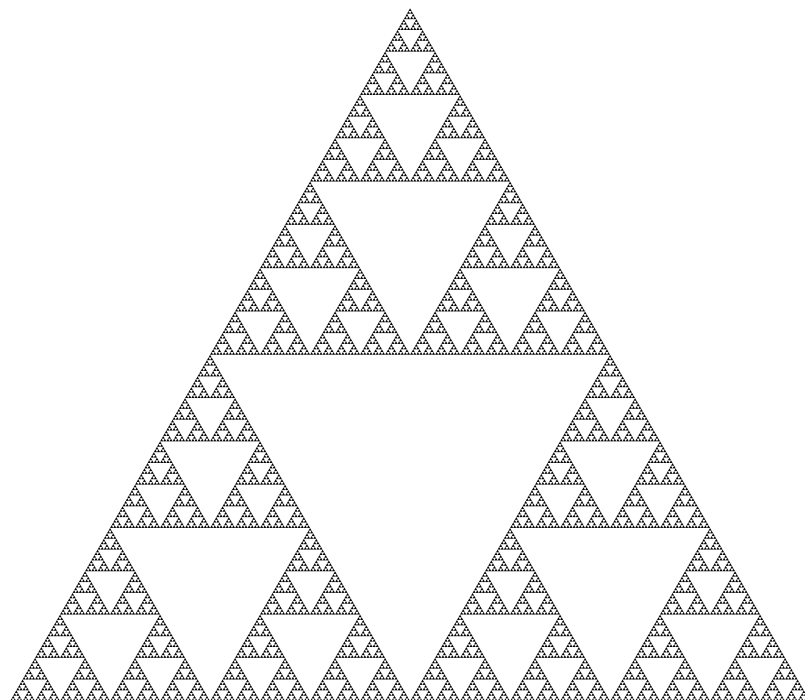
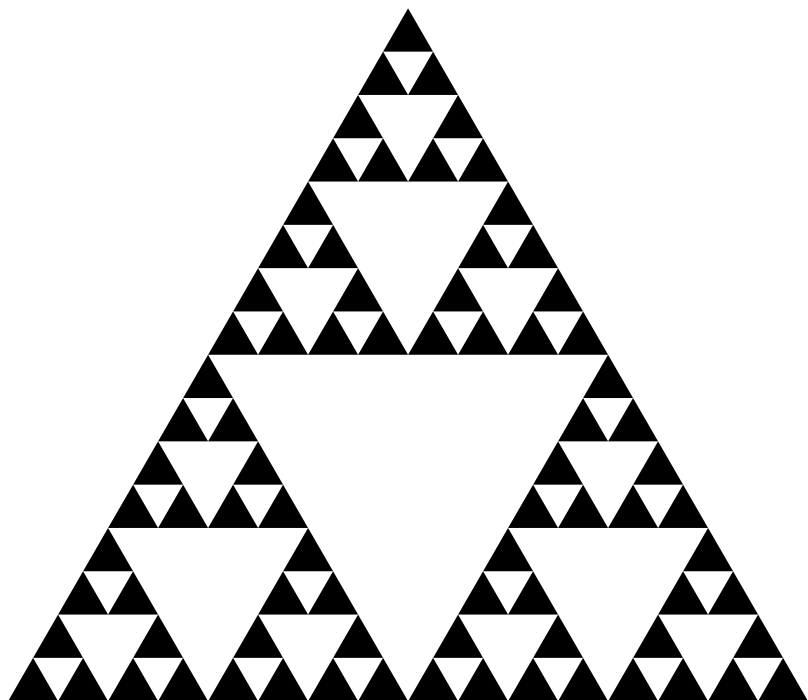
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    >>> first_leaf(((((), 3), (2, 1)), ()))
    ()
    """
    if type(pair) is int or pair == ():
        return pair
    else:
        return first_leaf(pair[0])
```

What kind of a recursive process is this? A: Iterative process (tail recursion)

Sierpinski Triangle

- No discussion of recursion is complete without a mention of *fractal patterns*, which exhibit self-similarity when scaled.
- We'll define a "Sierpinski Triangle of depth k and side s " to be
 - A filled equilateral triangle with sides of length s , if $k = 0$, else
 - Three Sierpinski Triangles of depth $k - 1$ and side $s/2$ arranged in the three corners of an equilateral triangle with side s .
- Here are triangles of degree 4 and 8:



Drawing Sierpinski Triangles

- Assume the existence of the function `triangle`:

```
def triangle(x, y, side):  
    """Draw a filled equilateral triangle with its lower-left corner  
    at (X, Y) and with given SIDE. The base is aligned with the x-axis."""
```

- We can now read off the definition of the triangle:

```
def sierpinski(x, y, side, depth):  
    """Draw a Sierpinski triangle of given DEPTH with given SIDE and  
    lower-left corner at (X, Y)."""
```

```
    if depth == 0:
```

```
        _____  
    else:
```

```
        height = 0.25 * sqrt(3) * side
```

```
        _____  
        _____  
        _____
```

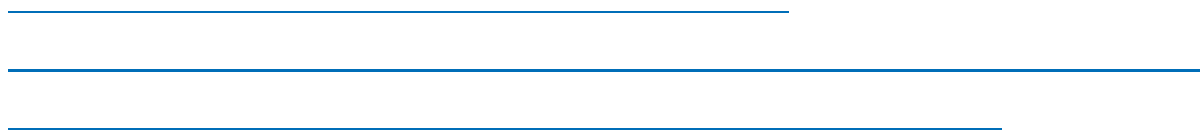
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```

```
    if depth == 0:  
        triangle(x, y, side)  
    else:  
        height = 0.25 * sqrt(3) * side  
  
        sierpinski(x, y, side/2, depth-1)
```



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    if depth == 0:  
        triangle(x, y, side)  
    else:  
        height = 0.25 * sqrt(3) * side  
  
        sierpinski(x, y, side/2, depth-1)  
        sierpinski(x + side/4, y + height, side/2, depth-1)
```

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        sierpinski(x, y, side/2, depth-1)  
        sierpinski(x + side/4, y + height, side/2, depth-1)  
        sierpinski(x + side/2, y, side/2, depth-1)
```