Understanding RNNs

Prof. Forrest Davis {fdavis@colgate.edu}

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Contents

1 RNN by Hand

1.1 Central Equation

$$\text{input} = \mathbf{E}(\text{tokenID})$$

$$h_0^t = \text{ReLU}(h_0^{t-1} \cdot \mathbf{W} + \text{input} \cdot \mathbf{U})$$

$$\text{logits} = h_0^t \cdot \mathbf{V}$$

1.2 Central Assumptions

The vocab has 6 words with the following token ids:

word	id
BOS	0
the	1
cat	2
is	3
outside	4
EOS	5

Our $\mathbf{E}$ (embedding matrix) is as follows:

$$\mathbf{E} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 2 & -1 & 3 & 5 \\ -1 & 2 & 1 & 4 \\ 5 & 2 & -1 & 3 \\ 4 & 0 & 1 & 1 \\ 1 & 3 & 4 & -1 \end{pmatrix}$$

Our **U** (input transformation) is as follows:

$$\mathbf{U} = \begin{pmatrix} 3 & -1 & 0 \\ 1 & 4 & -3 \\ 2 & 5 & -1 \\ 4 & 2 & 3 \end{pmatrix}$$

Our **W** (hidden transformation) is as follows:

$$\mathbf{W} = \begin{pmatrix} 2 & -3 & 1 \\ 4 & 1 & 4 \\ -5 & 3 & 2 \end{pmatrix}$$

Our **V** (output transformation) is as follows:

$$\mathbf{V} = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 4 & 2 & 1 & 5 & 2 & 3 \\ 0 & -3 & -1 & 4 & 3 & -1 \end{pmatrix}$$

Finally, our h_0^0 (initial hidden representation) is as follows:

$$h_0^0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

The entire model (ignoring the weight values) can be depicted as in Figure .

1.3 Questions

0. Give a diagram for the model (ignoring the weight values) above. In other words, plot the nodes with their connections, and label the figure with the weight matrix names. Note: Recurrence can be tricky to add, begin first pretending there is no recurrence.

Assume for the following questions, we are only concerned with the sentence "BOS the cat is outside EOS".

1. What is the new hidden state and output after reading in BOS?

2. What is the model predicting as the next token?

3. What is the new hidden state and output after reading in the? And what is the model's favored next token?