11.4) The Comparison Tests:

Exercises: Determine whether the following series converge or diverge:

a) $\sum_{n=1}^{\infty} \frac{sec^2(n)}{n}$ b) $\sum_{n=1}^{\infty} \frac{n}{n^3+n^2+2n+1}$

a) $\frac{sec^2(n)}{n} > \frac{1}{n}$, so $\sum_{n=1}^{\infty} \frac{sec^2(n)}{n} > \sum_{n=1}^{\infty} \frac{1}{n}$. Divergent.

b) $\frac{n}{n^3+n^2+2n+1} < \frac{n}{n^3} = \frac{1}{n^2}$, so $\sum_{n=1}^{\infty} \frac{n}{n^3+n^2+2n+1} < \sum_{n=1}^{\infty} \frac{1}{n^2}$. Convergent.

Exercises: Determine whether the following series converge or diverge:

a) $\sum_{n=1}^{\infty} \frac{3n^2+n}{n^3+2n^2-4n+8}$ b) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+1}}{n3^n}$

a) $\sum_{n=1}^{\infty} \frac{3n^2+n}{n^3+2n^2-4n+8}$ compare with $\sum_{n=1}^{\infty} \frac{1}{n}$. $\lim_{n\to\infty} \frac{\frac{3n^2+n}{n^3+2n^2-4n+8}}{1/n} = \lim_{n\to\infty} \frac{3n^3+n^2}{n^3+2n^2-4n+8} = \frac{3}{1} = 3$

Both diverge.

b) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+1}}{n3^n}$ compare with $\sum_{n=1}^{\infty} \frac{1}{3^n}$.

$$\lim_{n\to\infty} \frac{\frac{\sqrt{x^2+1}}{n*3^n}}{3^{-n}} = \lim_{n\to\infty} \frac{\sqrt{n^2+1}}{n3^n} * 3^n = \lim_{n\to\infty} \frac{\sqrt{n^2+1}}{n} = 1$$

Both converge.

11.5) Alternating Series and Absolute Convergence:

Exercise: Determine whether the following series converges or diverges:

a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ b) $\sum_{n=1}^{\infty} \frac{cos(\pi n)}{n^2}$ c) $\sum_{n=1}^{\infty} \frac{n^3}{n^4-2}$

a) $\frac{1}{\sqrt{n}} > \frac{1}{\sqrt{n+1}}$ for all n, and $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$. So $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is convergent.

b) $\frac{1}{n^2} > \frac{1}{(n+1)^2}$ for all n, and $\lim_{n\to\infty} \frac{1}{n^2} = 0$. So $\sum_{n=1}^{\infty} \frac{cos(\pi n)}{n^2}$ is convergent.

c) $\sum_{n=1}^{\infty} \frac{n^3}{n^4-2}$ compare with $\sum_{n=1}^{\infty} \frac{1}{n}$. $\lim_{n\to\infty} \frac{\frac{n^3}{n^4-2}}{1/n} = \lim_{n\to\infty} \frac{n^4}{n^4-2} = \frac{1}{1} = 1$

Both Diverge

Exercise: Find the sum of the series $s = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ correct to three decimal places.

First term where $\frac{1}{n^2} < 0.0005$ is n = 45, so $s \approx s_{44} = \sum_{n=1}^{44} \frac{(-1)^n}{n^2} = -0.822$

Exercises: Determine if the following series are absolutely convergent, conditionally convergent, or divergent.

a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ b) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{2n+1}$ c) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$

a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges, so $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is conditionally convergent.

b) $\lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2}$ so the series diverges.

c) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ compare with $\sum_{n=1}^{\infty} \frac{1}{n}$. $\lim_{n\to\infty} \frac{\frac{n}{n^2+1}}{1/n} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = \frac{1}{1} = 1$

Both diverge.

$\lim_{n \to \infty} \frac{n}{n^2+1} = 0$, and $\frac{d}{dn}\left[\frac{n}{n^2+1}\right] = \frac{n^2+1-2n^2}{\left(n^2+1\right)^2} = \frac{1-n^2}{\left(n^2+1\right)^2}$ decreasing for all n > 1.

Conditionally convergent.