

Algebra Cheat Sheet

Arithmetic Operations

$$ab+ac=a(b+c)$$

$$\frac{(a)}{c}=\frac{a}{bc}$$

$$\frac{a+c}{b+d}=\frac{ad+bc}{bd}$$

$$\frac{a-b}{c-d}=\frac{b-a}{d-c}$$

$$\frac{ab+ac}{a}=b+c, \quad a \neq 0$$

Exponent Properties

$$a^n a^m = a^{n+m}$$

$$(a^n)^m = a^{nm}$$

$$(ab)^n = a^n b^n$$

$$a^{-n} = \frac{1}{a^n}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad a^{\pm} = \left(a^{\frac{1}{n}}\right)^{\pm} = \left(a^n\right)^{\pm}$$

Properties of Radicals

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

$$\sqrt[n]{\sqrt[n]{a}} = \sqrt[n]{a}$$

$$\sqrt[n]{a^n} = a, \quad \text{if } n \text{ is odd}$$

$$\sqrt[n]{a^n} = |a|, \quad \text{if } n \text{ is even}$$

Basic Properties & Facts

Properties of Inequalities

If $a < b$ then $a+c < b+c$ and $a-c < b-c$

If $a < b$ and $c > 0$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

If $a < b$ and $c < 0$ then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

Properties of Absolute Value

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

$$|a| \geq 0 \quad | -a | = |a|$$

$$|ab| = |a||b| \quad \left| \frac{a}{b} \right| = \frac{|a|}{|b|}$$

$$|a+b| \leq |a| + |b| \quad \text{Triangle Inequality}$$

Distance Formula

If $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ are two points the distance between them is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Complex Numbers

$$i = \sqrt{-1} \quad i^2 = -1 \quad \sqrt{-a} = i\sqrt{a}, \quad a \geq 0$$

$$(a+bi) + (c+di) = a+c+(b+d)i$$

$$(a+bi) - (c+di) = a-c+(b-d)i$$

$$(a+bi)(c+di) = ac - bd + (ad+bc)i$$

$$(a+bi)(a-bi) = a^2 + b^2$$

$$|a+bi| = \sqrt{a^2 + b^2} \quad \text{Complex Modulus}$$

$$\overline{(a+bi)} = a - bi \quad \text{Complex Conjugate}$$

$$\overline{(a+bi)(a+bi)} = |a+bi|^2$$

Logarithms and Log Properties

Definition

$y = \log_b x$ is equivalent to $x = b^y$

Example

$\log_5 125 = 3$ because $5^3 = 125$

Special Logarithms

$\ln x = \log_e x$ natural log

$\log x = \log_{10} x$ common log

where $e = 2.718281828\dots$

Logarithm Properties

$$\log_b b = 1 \quad \log_b 1 = 0$$

$$\log_b b^x = x \quad b^{\log_b x} = x$$

$$\log_b(x^r) = r \log_b x$$

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

The domain of $\log_b x$ is $x > 0$

Factoring and Solving

Quadratic Formula

$$\text{Solve } ax^2 + bx + c = 0, \quad a \neq 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac > 0$ - Two real unequal solns.

If $b^2 - 4ac = 0$ - Repeated real solution.

If $b^2 - 4ac < 0$ - Two complex solutions.

Square Root Property

If $x^2 = p$ then $x = \pm\sqrt{p}$

Absolute Value Equations/Inequalities

If b is a positive number

$$|p| = b \Rightarrow p = -b \quad \text{or} \quad p = b$$

$$|p| < b \Rightarrow -b < p < b$$

$$|p| > b \Rightarrow p < -b \quad \text{or} \quad p > b$$

Completing the Square

$$\text{Solve } 2x^2 - 6x - 10 = 0$$

(4) Factor the left side

$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

(5) Use Square Root Property

$$x - \frac{3}{2} = \pm \sqrt{\frac{29}{4}} = \pm \frac{\sqrt{29}}{2}$$

(6) Solve for x

$$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$

Functions and Graphs		Common Algebraic Errors	
		Error	Reason/Correct/Justification/Example
Constant Function $y = a$ or $f(x) = a$	Parabola/Quadratic Function $x = ay^2 + by + c$ $g(y) = ay^2 + by + c$	$\frac{2}{0} \neq 0$ and $\frac{2}{0} \neq 2$	Division by zero is undefined!
Graph is a horizontal line passing through the point $(0, a)$.	The graph is a parabola that opens right if $a > 0$ or left if $a < 0$ and has a vertex at $\left(g\left(-\frac{b}{2a}\right), -\frac{b}{2a}\right)$.	$-3^2 \neq 9$	$-3^2 = -9$, $(-3)^2 = 9$ Watch parenthesis!
Line/Linear Function $y = mx + b$ or $f(x) = mx + b$	Circle $(x - h)^2 + (y - k)^2 = r^2$	$(x^2)^3 \neq x^5$	$(x^2)^3 = x^2 x^2 x^2 = x^6$
Graph is a line with point $(0, b)$ and slope m .	Graph is a circle with radius r and center (h, k) .	$\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$	$\frac{1}{2+1} = \frac{1}{1+1} \neq \frac{1}{1} + \frac{1}{1} = 2$
Slope Slope of the line containing the two points (x_1, y_1) and (x_2, y_2) is	Ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{1}{x^2+x^3} \neq x^{-2} + x^{-3}$	A more complex version of the previous error.
Slope – intercept form The equation of the line with slope m and y -intercept $(0, b)$ is	Graph is an ellipse with center (h, k) with vertices a units right/left from the center and vertices b units up/down from the center.	$\frac{a+bx}{a} \neq 1+bx$	$\frac{a+bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$ Beware of incorrect canceling!
$y = mx + b$	Hyperbola $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$-a(x-1) \neq -ax-a$	$-a(x-1) = -ax+a$
Point – Slope form The equation of the line with slope m and passing through the point (x_1, y_1) is	Graph is a hyperbola that opens left and right, has a center at (h, k) , vertices a units left/right of center and asymptotes that pass through center with slope $\pm \frac{b}{a}$.	$(x+a)^2 \neq x^2 + a^2$	Make sure you distribute the “-“!
$y = y_1 + m(x - x_1)$	Hyperbola $\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$	$\sqrt{x^2 + a^2} \neq x+a$	$(x+a)^2 = (x+a)(x+a) = x^2 + 2ax + a^2$
Parabola/Quadratic Function $y = a(x-h)^2 + k$ $f(x) = a(x-h)^2 + k$	Graph is a hyperbola that opens up and down, has a center at (h, k) , vertices b units up/down from the center and asymptotes that pass through center with slope $\pm \frac{b}{a}$.	$\sqrt{x+a} \neq \sqrt{x} + \sqrt{a}$	$5 = \sqrt{25} = \sqrt{3^2 + 4^2} \neq \sqrt{3^2} + \sqrt{4^2} = 3 + 4 = 7$
The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at (h, k) .		$(x+a)^n \neq x^n + a^n$ and $\sqrt[n]{x+a} \neq \sqrt[n]{x} + \sqrt[n]{a}$	See previous error. More general versions of previous three errors.
Parabola/Quadratic Function $y = ax^2 + bx + c$ $f(x) = ax^2 + bx + c$		$2(x+1)^2 \neq (2x+2)^2$	$2(x+1)^2 = 2(x^2 + 2x + 1) = 2x^2 + 4x + 2$
The graph is a parabola that opens up if $a > 0$ or down if $a < 0$ and has a vertex at $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.		$(2x+2)^2 \neq 2(x+1)^2$	$(2x+2)^2 = 4x^2 + 8x + 4$ Square first then distribute!

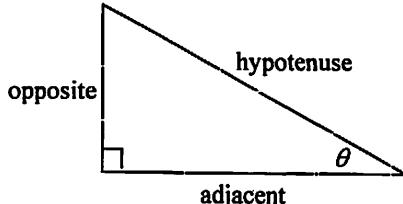
Trig Cheat Sheet

Definition of the Trig Functions

Right triangle definition

For this definition we assume that

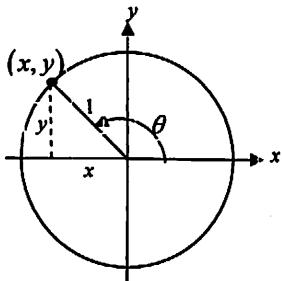
$$0 < \theta < \frac{\pi}{2} \text{ or } 0^\circ < \theta < 90^\circ.$$



$$\begin{aligned}\sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} & \csc \theta &= \frac{\text{hypotenuse}}{\text{opposite}} \\ \cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} & \sec \theta &= \frac{\text{hypotenuse}}{\text{adjacent}} \\ \tan \theta &= \frac{\text{opposite}}{\text{adjacent}} & \cot \theta &= \frac{\text{adjacent}}{\text{opposite}}\end{aligned}$$

Unit circle definition

For this definition θ is any angle.



$$\begin{aligned}\sin \theta &= \frac{y}{1} = y & \csc \theta &= \frac{1}{y} \\ \cos \theta &= \frac{x}{1} = x & \sec \theta &= \frac{1}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y}\end{aligned}$$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

$$\begin{aligned}\sin \theta, \quad \theta &\text{ can be any angle} \\ \cos \theta, \quad \theta &\text{ can be any angle} \\ \tan \theta, \quad \theta &\neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ \csc \theta, \quad \theta &\neq n\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ \sec \theta, \quad \theta &\neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \dots \\ \cot \theta, \quad \theta &\neq n\pi, \quad n = 0, \pm 1, \pm 2, \dots\end{aligned}$$

Range

The range is all possible values to get out of the function.

$$\begin{aligned}-1 \leq \sin \theta \leq 1 & \quad \csc \theta \geq 1 \text{ and } \csc \theta \leq -1 \\ -1 \leq \cos \theta \leq 1 & \quad \sec \theta \geq 1 \text{ and } \sec \theta \leq -1 \\ -\infty \leq \tan \theta \leq \infty & \quad -\infty \leq \cot \theta \leq \infty\end{aligned}$$

Period

The period of a function is the number, T , such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\begin{aligned}\sin(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \cos(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \tan(\omega\theta) &\rightarrow T = \frac{\pi}{\omega} \\ \csc(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \sec(\omega\theta) &\rightarrow T = \frac{2\pi}{\omega} \\ \cot(\omega\theta) &\rightarrow T = \frac{\pi}{\omega}\end{aligned}$$

Formulas and Identities

Tangent and Cotangent Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Reciprocal Identities

$$\begin{aligned}\csc \theta &= \frac{1}{\sin \theta} & \sin \theta &= \frac{1}{\csc \theta} \\ \sec \theta &= \frac{1}{\cos \theta} & \cos \theta &= \frac{1}{\sec \theta} \\ \cot \theta &= \frac{1}{\tan \theta} & \tan \theta &= \frac{1}{\cot \theta}\end{aligned}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Even/Odd Formulas

$$\begin{aligned}\sin(-\theta) &= -\sin \theta & \csc(-\theta) &= -\csc \theta \\ \cos(-\theta) &= \cos \theta & \sec(-\theta) &= \sec \theta \\ \tan(-\theta) &= -\tan \theta & \cot(-\theta) &= -\cot \theta\end{aligned}$$

Periodic Formulas

If n is an integer.

$$\begin{aligned}\sin(\theta + 2\pi n) &= \sin \theta & \csc(\theta + 2\pi n) &= \csc \theta \\ \cos(\theta + 2\pi n) &= \cos \theta & \sec(\theta + 2\pi n) &= \sec \theta \\ \tan(\theta + \pi n) &= \tan \theta & \cot(\theta + \pi n) &= \cot \theta\end{aligned}$$

Double Angle Formulas

$$\begin{aligned}\sin(2\theta) &= 2\sin \theta \cos \theta \\ \cos(2\theta) &= \cos^2 \theta - \sin^2 \theta \\ &= 2\cos^2 \theta - 1 \\ &= 1 - 2\sin^2 \theta \\ \tan(2\theta) &= \frac{2\tan \theta}{1 - \tan^2 \theta}\end{aligned}$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \Rightarrow t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Half Angle Formulas

$$\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$$

$$\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$$

$$\tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Sum and Difference Formulas

$$\begin{aligned}\sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\ \cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta\end{aligned}$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

Product to Sum Formulas

$$\sin \alpha \sin \beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \sin \beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$$

Sum to Product Formulas

$$\sin \alpha + \sin \beta = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin \alpha - \sin \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha + \cos \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha - \cos \beta = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

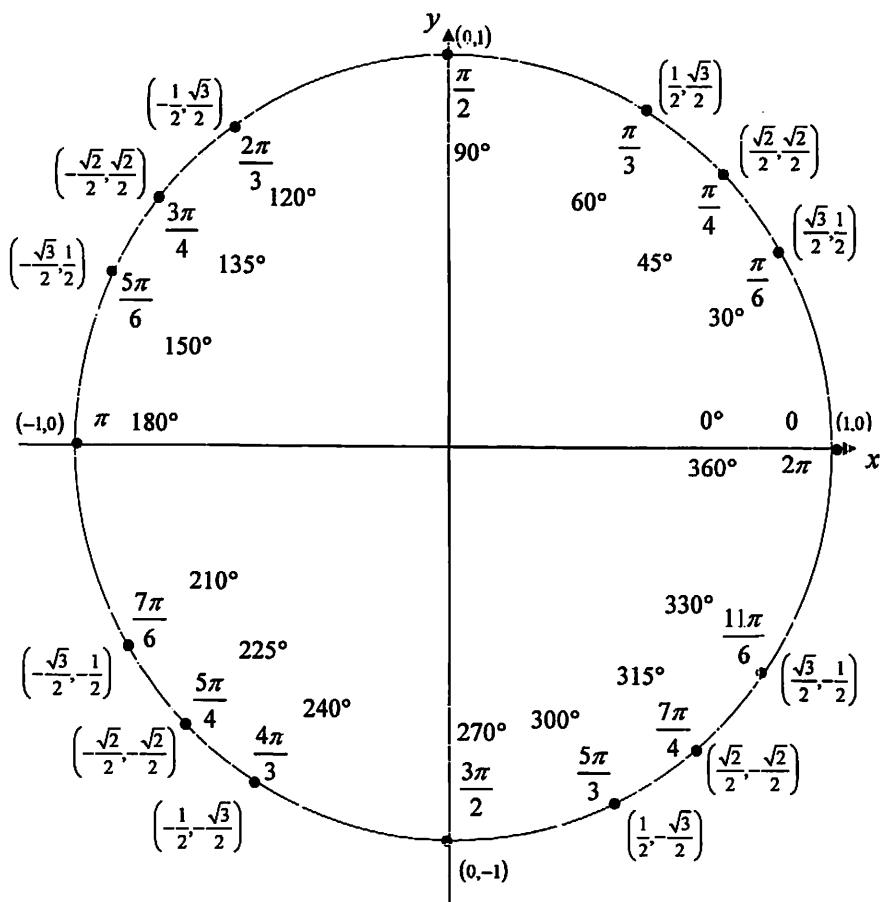
Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta \quad \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta \quad \sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta \quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$$

Unit Circle



For any ordered pair on the unit circle (x, y) : $\cos \theta = x$ and $\sin \theta = y$

Example

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \quad \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Inverse Trig Functions

Definition

$y = \sin^{-1} x$ is equivalent to $x = \sin y$

$y = \cos^{-1} x$ is equivalent to $x = \cos y$

$y = \tan^{-1} x$ is equivalent to $x = \tan y$

Inverse Properties

$\cos(\cos^{-1}(x)) = x$ $\cos^{-1}(\cos(\theta)) = \theta$

$\sin(\sin^{-1}(x)) = x$ $\sin^{-1}(\sin(\theta)) = \theta$

$\tan(\tan^{-1}(x)) = x$ $\tan^{-1}(\tan(\theta)) = \theta$

Domain and Range

Function	Domain	Range
----------	--------	-------

$y = \sin^{-1} x$ $-1 \leq x \leq 1$ $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$y = \cos^{-1} x$ $-1 \leq x \leq 1$ $0 \leq y \leq \pi$

$y = \tan^{-1} x$ $-\infty < x < \infty$ $-\frac{\pi}{2} < y < \frac{\pi}{2}$

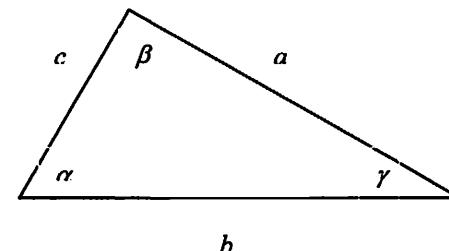
Alternate Notation

$\sin^{-1} x = \arcsin x$

$\cos^{-1} x = \arccos x$

$\tan^{-1} x = \arctan x$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan \frac{1}{2}(\alpha-\beta)}{\tan \frac{1}{2}(\alpha+\beta)}$$

$$\frac{b-c}{b+c} = \frac{\tan \frac{1}{2}(\beta-\gamma)}{\tan \frac{1}{2}(\beta+\gamma)}$$

$$\frac{a-c}{a+c} = \frac{\tan \frac{1}{2}(\alpha-\gamma)}{\tan \frac{1}{2}(\alpha+\gamma)}$$

Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos \frac{1}{2}(\alpha-\beta)}{\sin \frac{1}{2}\gamma}$$

Derivatives

Basic Properties/Formulas/Rules

$$\begin{aligned} \frac{d}{dx}(cf(x)) &= cf'(x), c \text{ is any constant. } (f(x) \pm g(x))' = f'(x) \pm g'(x) \\ \frac{d}{dx}(x^n) &= nx^{n-1}, n \text{ is any number. } \quad \frac{d}{dx}(c) = 0, c \text{ is any constant.} \\ (fg)' &= f'g + fg' \text{ - (Product Rule)} \quad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} \text{ - (Quotient Rule)} \\ \frac{d}{dx}(f(g(x))) &= f'(g(x))g'(x) \text{ (Chain Rule)} \\ \frac{d}{dx}(e^{g(x)}) &= g'(x)e^{g(x)} \quad \frac{d}{dx}(\ln g(x)) = \frac{g'(x)}{g(x)} \end{aligned}$$

Common Derivatives

Polynomials

$$\frac{d}{dx}(c) = 0 \quad \frac{d}{dx}(x) = 1 \quad \frac{d}{dx}(cx) = c \quad \frac{d}{dx}(x^n) = nx^{n-1} \quad \frac{d}{dx}(cx^n) = ncx^{n-1}$$

Trig Functions

$$\begin{aligned} \frac{d}{dx}(\sin x) &= \cos x & \frac{d}{dx}(\cos x) &= -\sin x & \frac{d}{dx}(\tan x) &= \sec^2 x \\ \frac{d}{dx}(\sec x) &= \sec x \tan x & \frac{d}{dx}(\csc x) &= -\csc x \cot x & \frac{d}{dx}(\cot x) &= -\csc^2 x \end{aligned}$$

Inverse Trig Functions

$$\begin{aligned} \frac{d}{dx}(\sin^{-1} x) &= \frac{1}{\sqrt{1-x^2}} & \frac{d}{dx}(\cos^{-1} x) &= -\frac{1}{\sqrt{1-x^2}} & \frac{d}{dx}(\tan^{-1} x) &= \frac{1}{1+x^2} \\ \frac{d}{dx}(\sec^{-1} x) &= \frac{1}{|x|\sqrt{x^2-1}} & \frac{d}{dx}(\csc^{-1} x) &= -\frac{1}{|x|\sqrt{x^2-1}} & \frac{d}{dx}(\cot^{-1} x) &= -\frac{1}{1+x^2} \end{aligned}$$

Exponential/Logarithm Functions

$$\begin{aligned} \frac{d}{dx}(a^x) &= a^x \ln(a) & \frac{d}{dx}(e^x) &= e^x \\ \frac{d}{dx}(\ln(x)) &= \frac{1}{x}, \quad x > 0 & \frac{d}{dx}(\ln|x|) &= \frac{1}{x}, \quad x \neq 0 & \frac{d}{dx}(\log_a(x)) &= \frac{1}{x \ln a}, \quad x > 0 \end{aligned}$$

Hyperbolic Trig Functions

$$\begin{aligned} \frac{d}{dx}(\sinh x) &= \cosh x & \frac{d}{dx}(\cosh x) &= \sinh x & \frac{d}{dx}(\tanh x) &= \operatorname{sech}^2 x \\ \frac{d}{dx}(\operatorname{sech} x) &= -\operatorname{sech} x \tanh x & \frac{d}{dx}(\operatorname{csch} x) &= -\operatorname{csch} x \coth x & \frac{d}{dx}(\coth x) &= -\operatorname{csch}^2 x \end{aligned}$$

Integrals

Basic Properties/Formulas/Rules

$$\begin{aligned} \int cf(x) dx &= c \int f(x) dx, c \text{ is a constant. } \quad \int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx \\ \int_a^b f(x) dx &= F(x)|_a^b = F(b) - F(a) \text{ where } F(x) = \int f(x) dx \\ \int_a^b cf(x) dx &= c \int_a^b f(x) dx, c \text{ is a constant. } \quad \int_a^b f(x) \pm g(x) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx \\ \int_a^a f(x) dx &= 0 & \int_a^b f(x) dx &= - \int_b^a f(x) dx \\ \int_a^b f(x) dx &= \int_a^c f(x) dx + \int_c^b f(x) dx & \int_a^b c dx &= c(b-a) \\ \text{If } f(x) \geq 0 \text{ on } a \leq x \leq b \text{ then } \int_a^b f(x) dx &\geq 0 \\ \text{If } f(x) \geq g(x) \text{ on } a \leq x \leq b \text{ then } \int_a^b f(x) dx &\geq \int_a^b g(x) dx \end{aligned}$$

Common Integrals

Polynomials

$$\begin{aligned} \int dx &= x + c & \int k dx &= kx + c & \int x^n dx &= \frac{1}{n+1}x^{n+1} + c, \quad n \neq -1 \\ \int \frac{1}{x} dx &= \ln|x| + c & \int x^{-1} dx &= \ln|x| + c & \int x^{-n} dx &= \frac{1}{-n+1}x^{-n+1} + c, \quad n \neq 1 \\ \int \frac{1}{ax+b} dx &= \frac{1}{a} \ln|ax+b| + c & \int x^{\frac{p}{q}} dx &= \frac{1}{\frac{p}{q}+1}x^{\frac{p+q}{q}} + c = \frac{q}{p+q}x^{\frac{p+q}{q}} + c \end{aligned}$$

Trig Functions

$$\begin{aligned} \int \cos u du &= \sin u + c & \int \sin u du &= -\cos u + c & \int \sec^2 u du &= \tan u + c \\ \int \sec u \tan u du &= \sec u + c & \int \csc u \cot u du &= -\csc u + c & \int \csc^2 u du &= -\cot u + c \\ \int \tan u du &= \ln|\sec u| + c & \int \cot u du &= \ln|\sin u| + c \\ \int \sec u du &= \ln|\sec u + \tan u| + c & \int \sec^3 u du &= \frac{1}{2}(\sec u \tan u + \ln|\sec u + \tan u|) + c \\ \int \csc u du &= \ln|\csc u - \cot u| + c & \int \csc^3 u du &= \frac{1}{2}(-\csc u \cot u + \ln|\csc u - \cot u|) + c \end{aligned}$$

Exponential/Logarithm Functions

$$\begin{aligned} \int e^u du &= e^u + c & \int a^u du &= \frac{a^u}{\ln a} + c & \int \ln u du &= u \ln(u) - u + c \\ \int e^{au} \sin(bu) du &= \frac{e^{au}}{a^2+b^2}(a \sin(bu) - b \cos(bu)) + c & \int ue^u du &= (u-1)e^u + c \\ \int e^{au} \cos(bu) du &= \frac{e^{au}}{a^2+b^2}(a \cos(bu) + b \sin(bu)) + c & \int \frac{1}{u \ln u} du &= \ln|\ln u| + c \end{aligned}$$

Inverse Trig Functions

$$\int \frac{1}{\sqrt{a^2 - u^2}} du = \sin^{-1} \left(\frac{u}{a} \right) + c \quad \int \sin^{-1} u du = u \sin^{-1} u + \sqrt{1 - u^2} + c$$

$$\int \frac{1}{a^2 + u^2} du = \frac{1}{a} \tan^{-1} \left(\frac{u}{a} \right) + c \quad \int \tan^{-1} u du = u \tan^{-1} u - \frac{1}{2} \ln(1 + u^2) + c$$

$$\int \frac{1}{u\sqrt{u^2 - a^2}} du = \frac{1}{a} \sec^{-1} \left(\frac{u}{a} \right) + c \quad \int \cos^{-1} u du = u \cos^{-1} u - \sqrt{1 - u^2} + c$$

Hyperbolic Trig Functions

$$\begin{aligned} \int \sinh u du &= \cosh u + c & \int \cosh u du &= \sinh u + c & \int \operatorname{sech}^2 u du &= \tanh u + c \\ \int \operatorname{sech} \tanh u du &= -\operatorname{sech} u + c & \int \operatorname{csch} \coth u du &= -\operatorname{csch} u + c & \int \operatorname{csch}^2 u du &= -\operatorname{coth} u + c \\ \int \tanh u du &= \ln(\cosh u) + c & \int \operatorname{sech} u du &= \tan^{-1} |\sinh u| + c \end{aligned}$$

Miscellaneous

$$\int \frac{1}{a^2 - u^2} du = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + c \quad \int \frac{1}{u^2 - a^2} du = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + c$$

$$\int \sqrt{a^2 + u^2} du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln \left| u + \sqrt{a^2 + u^2} \right| + c$$

$$\int \sqrt{u^2 - a^2} du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln \left| u + \sqrt{u^2 - a^2} \right| + c$$

$$\int \sqrt{a^2 - u^2} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{u}{a} \right) + c$$

$$\int \sqrt{2au - u^2} du = \frac{u-a}{2} \sqrt{2au - u^2} + \frac{a^2}{2} \cos^{-1} \left(\frac{a-u}{a} \right) + c$$

Standard Integration Techniques

Note that all but the first one of these tend to be taught in a Calculus II class.

 u Substitution

Given $\int_a^b f(g(x))g'(x)dx$ then the substitution $u = g(x)$ will convert this into the

$$\text{integral, } \int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u) du.$$

Integration by Parts

The standard formulas for integration by parts are,

$$\int u dv = uv - \int v du \quad \int_a^b u dv = uv \Big|_a^b - \int_a^b v du$$

Choose u and dv and then compute du by differentiating u and compute v by using the fact that $v = \int dv$.

Trig Substitutions

If the integral contains the following root use the given substitution and formula.

$$\sqrt{a^2 - b^2 x^2} \Rightarrow x = \frac{a}{b} \sin \theta \quad \text{and} \quad \cos^2 \theta = 1 - \sin^2 \theta$$

$$\sqrt{b^2 x^2 - a^2} \Rightarrow x = \frac{a}{b} \sec \theta \quad \text{and} \quad \tan^2 \theta = \sec^2 \theta - 1$$

$$\sqrt{a^2 + b^2 x^2} \Rightarrow x = \frac{a}{b} \tan \theta \quad \text{and} \quad \sec^2 \theta = 1 + \tan^2 \theta$$

Partial Fractions

If integrating $\int \frac{P(x)}{Q(x)} dx$ where the degree (largest exponent) of $P(x)$ is smaller than the degree of $Q(x)$ then factor the denominator as completely as possible and find the partial fraction decomposition of the rational expression. Integrate the partial fraction decomposition (P.F.D.). For each factor in the denominator we get term(s) in the decomposition according to the following table.

Factor in $Q(x)$	Term in P.F.D	Factor in $Q(x)$	Term in P.F.D
$ax+b$	$\frac{A}{ax+b}$	$(ax+b)^k$	$\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \dots + \frac{A_k}{(ax+b)^k}$
$ax^2 + bx + c$	$\frac{Ax+B}{ax^2 + bx + c}$	$(ax^2 + bx + c)^k$	$\frac{A_1 x + B_1}{ax^2 + bx + c} + \dots + \frac{A_k x + B_k}{(ax^2 + bx + c)^k}$

Products and (some) Quotients of Trig Functions

$$\int \sin^n x \cos^m x dx$$

1. If n is odd. Strip one sine out and convert the remaining sines to cosines using $\sin^2 x = 1 - \cos^2 x$, then use the substitution $u = \cos x$
2. If m is odd. Strip one cosine out and convert the remaining cosines to sines using $\cos^2 x = 1 - \sin^2 x$, then use the substitution $u = \sin x$
3. If n and m are both odd. Use either 1. or 2.
4. If n and m are both even. Use double angle formula for sine and/or half angle formulas to reduce the integral into a form that can be integrated.

$$\int \tan^n x \sec^m x dx$$

1. If n is odd. Strip one tangent and one secant out and convert the remaining tangents to secants using $\tan^2 x = \sec^2 x - 1$, then use the substitution $u = \sec x$
2. If m is even. Strip two secants out and convert the remaining secants to tangents using $\sec^2 x = 1 + \tan^2 x$, then use the substitution $u = \tan x$
3. If n is odd and m is even. Use either 1. or 2.
4. If n is even and m is odd. Each integral will be dealt with differently.

Convert Example : $\cos^6 x = (\cos^2 x)^3 = (1 - \sin^2 x)^3$