11.7) Strategy for Testing Series:

-After all the methods for determining when a series is convergent, divergent, absolutely or conditionally convergent, it can be hard to know when to use one test as opposed to another. It's not a good idea to just start with your favorite test and if that one doesn't work try the next one, and the next one, and so on. Instead, consider following a certain hierarchy of tests, some tests are better to attempt first than others, especially if the term in the series is of a certain type:

1) **Test for Divergence**: You can always take the time first to see if $\lim_{n\to\infty} a_n$ goes to zero first. If $\lim_{n\to\infty} a_n$ is not zero, then the series diverges. Otherwise....

2) **p-Series**: See if the series is a p-series of the form $\sum_{n=1}^{\infty} \frac{1}{n^p}$. If p > 1 the series converges, if $p \leq 1$ the series diverges. Otherise....

3) **Geometric Series**: See if the series is a geometric series of the form $\sum_{n=1}^{\infty} ar^{n-1}$. If $r \geq 1$ the series diverges, if r<1 the series converges. You can also find the sum by dividing the first term by 1 minus the common ratio. Otherwise....

4) **Comparison Tests**: If the series has a form similar to a p-series or geometric series, try using a comparison test. If the series $\sum_{n=1}^{\infty} a_n$ is larger than a divergent series, then it is also divergent. If the series $\sum_{n=1}^{\infty} a_n$ is smaller than a convergent series, then it is also convergent. Otherwise....

5) **Alternating Series Test**: If the series has an alternating factor and is of the form $\sum_{n=1}^{\infty} (-1)^{n-1} b_n$ then check for absolute convergence. If $\sum_{n=1}^{\infty} |b_n|$ converges, then the alternating series is absolutely convergent and also convergent in general. Otherwise, if b_n is decreasing and $\lim_{n\to\infty} b_n = 0,$ then the series is conditionally convergent. Otherwise, the series is divergent.

6) **Ratio Test**: If the series contains factorials and products, including constants raised to powers of n, then find $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$. If it's greater than 1 or infinite, the series is divergent. If it's less than 1, it's absolutely convergent. If it's equal to 1, try something else.

7) **Root Test**: If the series involves functions of n raised to the power of n, find $\lim_{n\to\infty} \sqrt[n]{|a_n|}$ as in the Ratio Test.

8) **Integral Test**: If $a_n = f(x)$ and you can easily evaluate $\int_1^{\infty} f(x)dx,$ then if the integral is convergent, so is the series. Make sure the series is also decreasing, positive, and continuous.