9.5) Linear Equations:

Exercise: Find the following solutions:

a) $x^2\frac{dy}{dx} + 3xy = 4$ for x>0 and y(2) = 4

b) $\frac{dy}{dx} + 4x^3y = 12x^3$

a)

$$\frac{dy}{dx} + \frac{3}{x}y = \frac{4}{x^2}$$

$$\frac{dy}{dx}e^{\int\frac{3}{x}dx} + \frac{3}{x}e^{\int\frac{3}{x}dx} * y = \frac{4}{x^2} * e^{\int\frac{3}{x}dx}$$

$$\frac{dy}{dx}e^{3ln(x)} + \frac{3}{x}e^{3ln(x)} * y = \frac{4}{x^2} * e^{3ln(x)}$$

$$\frac{dy}{dx}(x^3) + \frac{3}{x}(x^3) * y = \frac{4}{x^2} * (x^3)$$

$$\frac{dy}{dx}(x^3) + 3x^2 * y = 4x$$

$$\int\frac{d}{dx}(x^3y) = \int 4xdx$$

$$x^3y = 2x^2 + C$$

$$y = \frac{2}{x} + \frac{C}{x^3}$$

$$4 = \frac{2}{2} + \frac{C}{2^3}$$

$$3 = \frac{C}{8}$$

$$C = 24$$

$$y = \frac{2}{x} + \frac{24}{x^3}$$

b)

$$\frac{dy}{dx} + 4x^3y = 12x^3$$

$$\frac{dy}{dx}e^{\int 4x^3dx} + 4x^3e^{\int 4x^3dx} * y = 12x^3 * e^{\int 4x^3dx}$$

$$\frac{dy}{dx}e^{x^4} + 4x^3e^{x^4} * y = 12x^3 * e^{x^4}$$

$$\int\frac{d}{dx}(e^{x^4}y) = \int 12x^3e^{x^4}$$

$$e^{x^4}y = 3e^{x^4} + C$$