7.1) **Exercises**: Find the integrals of the following:

a) $\int x^4 ln(x^3)dx$ b) $\int xe^{4x}dx$

a) $u = ln(x^3)$, $du = \frac{3}{x}dx$, $dv = x^4dx$, $v = \frac{1}{5}x^5$

$$\int x^4 ln(x^3)dx = \frac{ln(x^3)x^5}{5} - \int \frac{3}{x}\frac{1}{5}x^5dx = \frac{ln(x^3)x^5}{5} - \int \frac{3}{5}x^4dx = \frac{ln(x^3)x^5}{5} - \frac{3}{25}x^5 + C$$

b) $u = x$, $du = dx$, $dv = e^{4x}dx$, $v = \frac{1}{4}e^{4x}$

$$\int xe^{4x}dx = \frac{xe^{4x}}{4} - \int \frac{1}{4}e^{4x}dx = \frac{xe^{4x}}{4} - \frac{1}{16}e^{4x} + C$$

Exercise: Integrate the following:

a) $\int xe^{2x}dx$ b) $\int sin(x)e^{2x}dx$

a) $u = x$, $du = dx$, $dv = e^{2x}dx$, $v = \frac{1}{2}e^{2x}$

$$\int xe^{2x}dx = \frac{xe^{2x}}{2} - \int \frac{1}{2}e^{2x}dx = \frac{xe^{2x}}{2} - \frac{1}{4}e^{2x} + C$$

b) $u = e^{2x}$, $du = 2e^{2x}dx$, $dv = sin(x)dx$,
$v = - cos(x)$

$$\int sin(x)e^{2x}dx = - cos(x)e^{2x} + \int 2cos(x)e^{2x}dx$$

$u = e^{2x}$, $du = 2e^{2x}dx$, $dv = cos(x)dx$, $v = sin(x)$

$$\int sin(x)e^{2x}dx = - cos(x)e^{2x} + \int 2cos(x)e^{2x}dx = - cos(x)e^{2x} + 2sinxe^{2x} - 4\int sin(x)e^{2x}dx$$

$$\int sin(x)e^{2x}dx = - cos(x)e^{2x} + 2sin(x)e^{2x} - 4\int sin(x)e^{2x}dx$$

$$5\int sin(x)e^{2x}dx = \; - cos(x)e^{2x} + 2sin(x)e^{2x}$$

$$\int sin(x)e^{2x}dx = \; -\frac{1}{5}cos(x)e^{2x} + \frac{2}{5}sin(x)e^{2x} + C$$

Exercise: Evaluate $\int\limits_0^1 sin^{-1}(x)dx$

$$u = sin^{-1}(x), \qquad du = \frac{dx}{\sqrt{1-x^2}}, \qquad dv = dx \qquad v = x$$

$$\int\limits_0^1 sin^{-1}(x)dx = \; xsin^{-1}(x) - \int\limits_0^1 x\frac{1}{\sqrt{1-x^2}}dx \; = \; xsin^{-1}(x) - \int\limits_0^1 \frac{x}{\sqrt{1-x^2}}dx \; = xsin^{-1}(x) + \frac{1}{2}\int\limits_0^1 \frac{-2x}{\sqrt{1-x^2}}dx$$

$$= \; xsin^{-1}(x) + \frac{1}{2}(2 * \sqrt{1 - x^2})\,\Big|_0^1 \; = \left(xsin^{-1}(x) + \sqrt{1 - x^2}\right)\Big|_0^1 \; = \left(1\left(\frac{\pi}{2}\right) + 0\right) - (0(0) + \sqrt{1}) = \frac{\pi}{2} - 1$$

Exercise: Integrate the following:

$$a) \int sin^3(x)dx \qquad\qquad b) \int sin^4(x)dx$$

a) $\int sin^3(x)dx = \; -\frac{1}{3}cos(x)sin^2(x) + \frac{2}{3}\int sin(x)dx = \; -\frac{1}{3}cos(x)sin^2(x) - \frac{2}{3}cos(x) + C$

$b) \int sin^4(x)dx = \; -\frac{1}{4}cos(x)sin^3(x) + \frac{3}{4}\int sin^2(x)dx =$

$$-\frac{1}{4}cos(x)sin^3(x) + \frac{3}{4}\left(-\frac{1}{2}cos(x)sin(x) + \frac{1}{2}\int sin^0(x)dx\right) = \; -\frac{1}{4}cos(x)sin^3(x) - \frac{3}{8}cos(x)sin(x) + \frac{3}{8}x$$